Introduction to string theory

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Usp. Fiz. Nauk **160**, 197–200 (November 1990)

D. Lüst and S. Theisen. *Lectures on String Theory*; (*Lecture Notes in Physics*, Vol. 346), Springer-Verlag, Berlin; Heidelberg; New York, 1989, 346 pp.

The book being reviewed is a version of lectures presented by the authors in the Federal Republic of Germany in the winter of 1987–1988 and revised in 1989. It is devoted to the field of theoretical and mathematical physics which has developed vigorously in recent years that is usually called "string theory". The attempt to introduce into consideration one-dimensional, extended objects that are called strings serves as the key idea of this theory. Here the excitations of a string, with quantization that is described by a discrete spectrum, correspond to particles that are actually observed. The lowest excitations correspond to massless particles, and then both the mass and the spin of the excitations increase. Thus, a theory with an infinite number of particle spins is obtained. Obviously, a string is described by a single significant dimensional parameter, its tension. If we want to describe all existing interactions, including gravitation, by strings, one must introduce a characteristic scale of tension of the order of the Planck mass $M_P = (hc/G)^{1/2} \approx 10^{19}$ GeV. Thus, one must not expect an experimental detection of the higher string excitations with present-day accelerator energies. Therefore, the development of the theory is determined basically by certain internal criteria. As a consequence, string theory initiated the development of several related fields of theoretical physics, even though at present it has not fulfilled its original task (1984) of unifying all known interactions, including gravitation. Two-dimensional conformal theories, which have many applications also in the field of solid state physics, since they describe the critical points of phase transitions in two-dimensional systems, serve as one of the most important new fields. Due to string theory, interest has arisen towards many new directions. Unfortunately, even a slightly detailed description of them with a simultaneous consideration of all the problems of interest does not appear to be possible. From this point of view, the authors of the book under review managed to maintain the exact middle position while retaining a breadth of coverage of the material with a detailed presentation of it; this enables one to use the book as a good text on string theory. The authors of the book presented their general views on the subject in question in the introductory Chapter 1.

Chapter 2 is devoted to a detailed description of a classical boson string. And specifically, the different possible forms of classical action (Polyakov and Nambu–Goto) and their symmetries are discussed. In order to get a feel for the dynamics of a string, several examples of classical solutions for the string equations of motion are presented.

The following Ch. 3 is devoted to the quantization of strings. This is done both by a covariant method and also by choosing a suitable gauge for a light cone. Two new concepts having very great significance for future development arise in string quantization. These are the Virasoro algebra (a special appendix to Ch. 3 is devoted to a more detailed description of it), and the critical dimensionality of a string which, as is well-known, is determined by the condition of the splitting off of ghosts (or of unitarity in ghostless gauges) along with the requirement of Lorentz invariance. Of course, in the critical dimensionality $D = 26$, where these complications are absent, there nevertheless remains the problem connected with the fact that the ground state of a string corresponds to a particle having a negative square of the mass (a tachyon). In this chapter the spectrum of the states of a string are studied in great detail for the first several levels of excitation; also questions connected with the inclusion of the internal degrees of freedom, which are necessary in order to describe the gauge particles, are examined. Finally, a general introduction to a method for the continual integration of strings, a problem to which the authors later repeatedly return, is also contained here. Ghosts connected with the reparametrization invariance of a string arise in this framework for the first time in the book.

As was already noted, the development of string theory also required further progress in studying two-dimensional conformal field theories. The point is that the fundamental dynamical principle in string theory is its conformal invariance; the ground state of a string vacuum (in the second-quantization sense) must be described by some kind of two-dimensional conformal field theory. Therefore, the task of investigating and classifying such theories acquires prime significance, and the introductory explanation of the methods of conformal field theories in Ch. 4 of the present book seems natural. After starting with a general description in the framework of the bootstrap approach suggested by Belavin, Polyakov, and Zamolodchikov, the authors of the book then apply the method that has been developed directly to a free boson string, which is in itself the simplest example of a conformal theory.

The already introduced reparameterization ghosts are a less trivial example of conformal theory. The next Ch. 5 of the book is devoted to studying them both from the point of view of conformal theory and also from the point of view of its use for constructing a BRST formalism in string theory.

The description of the "classical" results of string theory obtained up to 1984, from which time a new round of interest in the problem started, is completed in Chs. 7 and 8 of the book (Ch. 6 is discussed below). Both the classical and also the quantum closed fermion string are studied in these chapters. Here the course that has already been gone over once in the case of the boson string is repeated with insignificant differences. Thus, the critical dimensionality of a fermion string is $D = 10$, and there exists a sector (called the

Ramon sector) where there are no tachyons. Evidently the observation of (ten-dimensional) supersymmetry for both sectors of a fermion string with a definite cutting of Fock space, which is called the GSO projection, is the only new feature. Such a string with supersymmetry is called a superstring.

A more modern approach to the fermion string is explained in Ch. 9; the partition function is calculated for a free fermion string. In particular, the introduced spin structure concept, which enables one to generalize the concept of the GSO projection, has key significance.

Chapter 10 of the book is constructed entirely on ideas from about 1984 as to by what methods one may construct new string models. Specifically, one may consider a boson string for which one of its 26 components is compactly fitted onto a circle, i.e., its coordinates take values in a space with a non-trivial topology. The spectrum of such a string varies as a function of the radius of the circle. A somewhat more general case of such a string is described by the compact fitting of several coordinates onto a multidimensional torus. One can imagine the torus as an $\mathbb{R}^n$ space factored by some lattice. For such a lattice we choose one formed by the roots of some kind of Lie algebra. From requirements of consistency (of the modular invariance type), this lattice must be even and self-dual. There are very few algebras with such lattices (with a dimensionality of 16, there are SO(32), $SO(16)\times SO(16)$, and $E_8\times E_8$; see below). Naturally, this leads the authors of the book to the concept of a heterotic string formed from 10 superstring modes of right (of course, the orientation is purely conventional) chirality and from 10 boson modes of left chirality. Here the remaining 16 modes of a boson string are assumed to be compactly fitted onto the lattice of an $E_8\times E_8$ group (a group picked out entirely from phenomenological arguments) or of SO(32).

While the previous Ch. 10 was practically entirely devoted to a discussion of the spectrum of a string, its interaction is studied in Ch. 11. Actually, a description of the interactions of a heterotic string requires studying conformal theories with an additional continuous two-dimensional symmetry that is described by current algebra with spin 1, of the infinite-dimensional Kac–Moody algebras. It turns out that the operators which generate physical particles from strings realize the representations of such algebras. Here a Kac–Moody $E_8\times E_8$ algebra corresponds to a heterotic string that is compactly fitted onto the lattice of the roots of an $E_8\times E_8$-algebra. Thus, the connection of strings, lattices, and conformal theories with the additional Kac–Moody symmetry is studied in Ch. 11.

Superconformal theories, another example of a conformal theory with an additional symmetry, are studied in Ch. 12. Many objects which appeared even earlier in the study of a fermion string now find a natural interpretation in the framework of the conformal approach.

One must notice that calculations of correlators in an arbitrary (non-free) conformal theory (or of amplitudes in string theory, for example, of fermion strings) is a very complicated task. Therefore, the possibility of bosonizing fermion strings, i.e., the possibility of representing correlators as averages of the exponents of free boson fields, that has been discoverd by Knizhnik and Friedan, Martinec, and Schenker is an important technical step. An approach of this kind, which is explained in detail in Ch. 13 of the book, leads the authors towards the significant concept of a covariant lattice.

It should be noted that the entire theory of a ten-dimensional heterotic string that has been presented above can be easily generalized for a four-dimensional string. The large number of allowable lattices and, consequently, also the impossibility of the general (kinematic) requirements for determining the theory more or less unambiguously is the only drawback of the four-dimensional string theory. Nevertheless, the authors present a great deal of "experimental" material about four-dimensional heterotic strings in Ch. 14.

Since the last question is the special field of the authors' scientific interests, it is easy to understand that it has been presented in greater completeness and detail. One may draw the conclusion that the authors considered the introduction of the reader specifically to the last problem, which required the entire method of modern string theory to explain it, as their ultimate objective. Naturally, such a method of approaching the material is not free from certain drawbacks connected with the arrangement of the emphases for modern themes. Thus, since the problem of describing four-dimensional heterotic strings was of current interest two years ago when this book was written, several themes that are now of more current interest were left out of the scope of questions covered by the authors; still others, even though the authors touched on them superficially, were not considered to the proper degree. Multiple loop calculations, to which the authors devoted Ch. 6 of the book, are among the latter. Indeed, unlike field theory, in string theory there are serious expectations of being able to summarize the entire set of perturbation theories associated with an explicit geometric treatment of a g-loop contribution as originating from a two-dimensional Riemann surface of genus g (i.e., with g-handles), furthermore, the measure in a continual integral serves as the density over the space of the moduli of such surfaces.

It should be noted that, in the last Ch. 15, the authors of the book touched on one more important theme, an effective field theory by which massless string excitations are described. Since one can hope to detect only such excitations experimentally, this seems to be very important.

Thus, one may draw the conclusion that the book by Lüst and Theisen is a very useful review of string theory from the point of view of studying a large number of different models. One must also note a number of very useful appendices, which contain summaries of necessary equations, as an undoubted virtue of the book.

Undoubtedly the book will be useful for theoretical physicists, including those not directly studying the problems of string theory, for scientists studying mathematical physics, and also for all those wishing to get an idea of modern string theory.

Translated by Frederick R. West