Hydrodynamic cumulative processes in plasma physics

I. V. Sokolov

Institute of General Physics of the Academy of Sciences of the USSR
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This review is devoted to cumulative hydrodynamic processes in a plasma and to the possibility of using them for controlled thermonuclear fusion. The cumulation of convergent shock waves and the mechanisms of their limitation are discussed in greatest detail. Results are presented of study of non-one-dimensional cumulative shock waves, which had practically not yielded to analysis until recently.

INTRODUCTION

This review is devoted to cumulative hydrodynamic processes in a plasma and the possibility of using them for *purposes of controlled thermonuclear fusion.*

We mean by cumulation here the property characteristic of certain hydrodynamic flows of a sharp increase (by an order of magnitude or more) in the local energy density in a certain zone of the flow. For example, it is known that a high-velocity jet of metal is formed upon fast compression of a metallic conical shell (the cumulative effect[1]). The velocity of the jet being formed can exceed by severalfold the velocity of compression of the shell,[2,3] and hence the cumulation in the given case is manifested in a concentration of kinetic-energy density in the cumulative jet. This effect is discussed sometimes in connection with problems of controlled thermonuclear fusion (CTF)—for creating plasma jets[4] or for accelerating macroparticles for impact initiation of CTF,[5] but here only minimal attention has been paid to it.

Of far greater interest for plasma physics are convergent shock waves,[6] as well as other examples of cumulative flows[7] that lead to a considerable increase in the temperature of the plasma. Some of these effects function in real physical equipment, and others prospectively can be used to attain thermonuclear temperatures. The possibility would be most fascinating of creating a plasma flow with a relatively low initial temperature (e.g., $T \sim 1$ eV) so that the cumulation would increase the temperature to the values necessary for thermonuclear fusion. This approach to the problem of inertial thermonuclear fusion (ITF) has both a certain experimental history and definite prospects.

To discuss the possibilities of using the ideas of cumulation to solve the fundamental problem of heating a thermonuclear plasma with inertial confinement is one of the principal goals of this review.

While concretizing the set of phenomena to be discussed, we stress that the topic only covers the cases in which the cumulation arises from concentration of the intrinsic energy of the flow, rather than from unbounded growth of an external agent. For comparison we present two examples.[7–11] In the isentropic compression of a spherical target under the action of a sharpened laser pulse (see Refs. 7, 10, and 11 and the studies cited there), the energy density is maximal at the boundary of the mass being compressed. Its growth is caused only by the increase in power of the laser pulse, rather than by hydrodynamic cumulation.

We shall be interested in flows of another type. For example, the propagation of a shock wave in a medium with a power-law dependence of the density ρ_0 on the coordinate x ($\rho_0 \sim x^\delta$, $\delta = \text{const}$) is accompanied by a temperature increase following the front as the coordinate of the front approaches zero. The effect for $x_f \to 0$ ceases to depend on the external agent (on the initial and boundary conditions).[8,9] We shall take cumulative hydrodynamic processes to mean precisely those phenomena in which the increase in energy density is of purely hydrodynamic nature, i.e., caused by the redistribution of the energy of the medium as it moves.

Among all the forms of cumulative flows, here we shall discuss in greatest detail convergent shock waves.[12–14] Such waves arise in targets for ITF (spherical convergent shock waves), in pinches,[15] and in a plasma being compressed by an exploding shell[16] (cylindrical convergent waves), and at a plasma focus[17] (axially symmetric noncylindrical waves). Apparently they can be used to obtain thermonuclear temperatures.[18] All of this warrants the interest in this form of cumulative flows.

We add the idea that cumulative shock waves—especially inhomogeneous ones—are one of the most interesting objects of study for the contemporary theory of nonlinear waves.[9]

The structure of this paper is as follows. Section 1 briefly reviews the potentialities of using the known cumulative flows except convergent shock waves in plasma physics. Section 2 treats one-dimensional convergent shock waves (spherically and cylindrically symmetric). Section 3 is devoted to the more complex case of non-one-dimensional cumulative shock waves, which until recently had practically not been studied.

1. EXAMPLES OF CUMULATIVE FLOWS AND THEIR ROLE IN PLASMA PHYSICS

Here we shall discuss the role of various cumulative hydrodynamic processes that can occur in a plasma. We must note that these effects are well known, but the possibility (or more often the impossibility) of realizing them in a plasma is rarely discussed.

Henceforth we shall assume the plasma to be ideal and treat it in the hydrodynamic single-fluid approximation. The latter is valid if the linear dimension L of the system exceeds the dissipative scale l_d associated with nonhydrodynamic energy transport. Depending on the conditions, l_d can arise from electronic heat conduction, deviation of the electronic temperature from the ionic temperature, and also from viscosity, radiative heat conduction, or transport of α-particles. The influence of these processes on cumulative shock waves is discussed in Sec. 2.

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1.1. Cumulative jets in a compressible medium

We shall show that in a plasma the effect mentioned above of formation of cumulative jets is weakly marked.

First we recall the fundamental formula of the theory of cumulative jets in an incompressible liquid.[2,3] Let two plane layers of liquid collide at the velocities V_0 at the angle 2α with respect to one another (Fig. 1a). The process is a steady-state one in a reference system that moves at the velocity $V_0/\sin\alpha$, while the colliding layers have the velocity $V_0\cot\alpha$ (Fig. 1b). Hence the velocity of the cumulative jet emerging forward in the laboratory system of reference equals

$$V_1 = V_0 \operatorname{ctg}\alpha + \frac{V_0}{\sin\alpha} = V_0 \operatorname{ctg}\frac{\alpha}{2}. \qquad (1.1)$$

For small α we have $V_1 \gg V_0$, so that the cumulation is manifested in a strong concentration of kinetic energy in the cumulative jet.

In a plasma the analogous effect is limited by compressibility. For example, let us study the collision in vacuo of two plane layers of plasma moving in the same way as in the previous example (see Fig. 1a). We shall assume that

$$V_0 \gg c_0; \qquad (1.2)$$

where c_0 is the velocity of sound in the plasma.

We note that, if the condition (1.2) is violated, we must take account of the expansion of the plasma. In expansion into a vacuum, the gas at the boundary is cooled, and the velocity of sound in it becomes much smaller than the velocity of the gas.[13] Therefore, even if the inequality (1.2) fails in the bulk of the plasma, nevertheless in the collision of the boundary layers the inequality (1.2) is unavoidably fulfilled.

When (1.2) is fulfilled, the regime of interaction of the colliding layers depends only on the angle α. If $\alpha < \alpha_{cr} = \sin^{-1}(1/\gamma)$ ($\gamma = 5/3$ is the adiabatic index for a plasma), then a cumulative jet is not formed, and the fluxes at the collision point of the layers are diverted into inclined shock waves (Fig. 1c). When

$$\alpha > \alpha_{cr} \qquad (1.3)$$

such a configuration is impossible.[13] Precisely in this case a cumulative jet is formed whose velocity can be estimated by

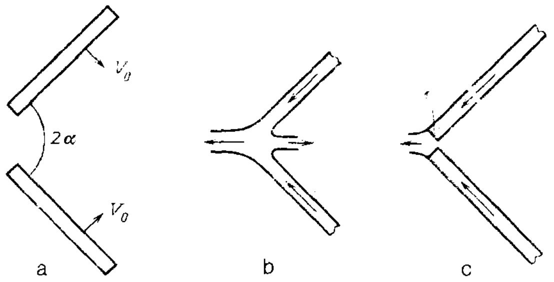

FIG. 1. Formation of cumulative jets in compressible and incompressible media. a—Collision of plane layers. b—Diagram of the formation of jets in a system of reference in which the process is a steady-state one. c—Appearance of inclined shock fronts (*1*) in a compressible medium with $\alpha < \alpha_{cr}$.

Eq. (1.1). The condition (1.3) leads to a limitation of the velocity of the jet:

$$V_1 < V_0(1+\cos\alpha_{cr})(\sin\alpha_{cr})^{-1} = V_0[\gamma + (\gamma^2-1)^{1/2}] = 3V_0, \qquad (1.4)$$

which is a consequence of compressibility. We note that, for media of low compressibility—water, metals—high values of the effective adiabatic index are typical ($\gamma \sim 3$–7), and the condition (1.4) becomes less rigid for them: $V_1 < (10$–$15)\,V_0$.

The results of numerical simulation of cumulative jets in compressible media and in gases have been presented in Ref. 19, while a calculation of the critical angle α_{cr} for different equations of state is given in Ref. 20.

It would be incorrect to deny the possible role of formation of cumulative jets (not necessarily plasma jets) in certain experiments on a plasma, or to neglect the interesting physical effects involving high-velocity plasma jets (not necessarily cumulative). For example, it was proposed[21] to use a plasma jet from a coaxial magnetoplasma compressor to accelerate granules of tungsten carbide to velocities $\sim 10^6$ cm/s. The possibility was noted in Ref. 22 of forming a jet of metal particles with a velocity $\sim 10^7$ cm/s in the interaction of a plasma flux with a thin-walled metallic hemisphere. A numerical calculation of the compression of the plasma in a conical cavity with a copper piston accelerated to $\sim 10^6$ cm/s demonstrates the formation of a cumulative jet of metal at the site of contact of the piston with the wall of the cavity and an influence of these jets on the compression of the plasma.[23]

Nevertheless, in a plasma itself, we stress again, the effect of formation of a cumulative jet is restricted by compressibility and can hardly lead to substantially new results.

1.2. Gradient acceleration of a shock wave

In the Introduction we have already mentioned that the propagation of a shock wave in a medium having the power-law density profile $\rho_0 \sim x^\delta$, $\delta = \text{const}$ in the direction $x \to +0$ (against the density gradient) is accompanied by an unbounded growth in the velocity of the wave, and correspondingly, of the temperature behind the front. A self-similar solution with reference to the original literature has been presented in Ref. 8. A number of studies have been devoted to density profiles of another form (e.g., Refs. 24–27).

A simple but crude physical explanation of the effect of gradient acceleration can be obtained if we take into account the fact that the propagation of a shock wave is accompanied by transport of hydrodynamic energy in the direction of propagation. Transport of kinetic energy into a region of declining density leads to increase in the velocity of the plasma, and hence in the velocity of the shock wave.

A generalization of a large amount of original results, and also some nonrigorous analytical calculations, have permitted the conclusion[9,24] that the increase in the velocity D of a shock wave moving against the gradient of the density ρ_0 (gradient acceleration) is a rather universal effect. For a strong wave in many cases it is described well by the simple formula:

$$D \sim \rho_0^{-k}, \quad k = \left[2 + \left(\frac{2\gamma}{\gamma-1}\right)^{1/2}\right]^{-1} \approx 0{,}24. \qquad (1.5)$$

The temperature following the front of the wave also increases:

$$T \sim \rho_0^{-0.48}, \tag{1.6}$$

This can be important in many problems of plasma physics.

For example, in Ref. 28 the role was studied of gradient acceleration of a shock wave in the explosions of supernovas, It was shown that the emergence of the wave from the core of the star into the stellar atmosphere can be accompanied by an increase in the energy of particles following the front up to ultrarelativistic values. The astrophysical consequences of this conclusion have been discussed in Refs. 8, 29, and 30.

References 31–34 have discussed gradient acceleration of various types of magnetohydrodynamic shock waves.

A direct application of the idea of gradient acceleration of a shock wave to obtain a hot plasma has been achieved in Refs. 35–37. In the experiment a laser spark was created at the center of a gas-filled glass sphere. After the laser pulse at the center of the sphere, a spherical volume remained of cooling laser plasma from which a spherical shock wave diverged (Fig. 2). Then the shock wave was reflected from the glass wall of the cavity and returned to the high-temperature region, where it was accelerated and heated the plasma further.

For ITF this and similar schemes seem not very promising, since one must pay for a considerable increase in the temperature of the plasma with a decrease in its density, whereas the realization of ITF requires high values of both the density and the temperature.

1.3. Cumulation in cavity collapse

The historically first hydrodynamic cumulative effect is the unbounded increase in pressure in the collapse of a spherical cavity in an incompressible liquid.[7,13] An analogous phenomenon arises also in a cylindrically symmetric geometry, and is widely applied to generate pulsed magnetic fields.[38] High pressures can be created in the collapse of a spherical cavity in a solid upon heating with a neutron flux.[39]

Among the different factors that limit this effect,[7,40] the most substantial for a plasma is compressibility. Calculations for a medium with $\gamma = 5/3$ show that, in the collapse of a spherical cavity in a plasma, the velocity V of the boundary increases with decreasing radius of the cavity, albeit extremely slowly:[41]

$$V \sim r^{-0.064}. \tag{1.7}$$

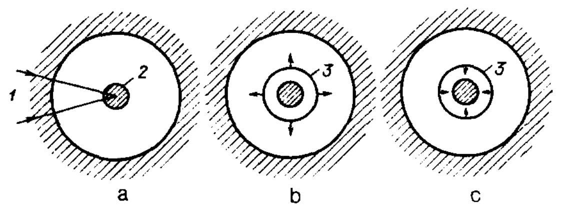

FIG. 2. Laser spark in a gas-filled glass sphere. a—The laser beam (*1*) forms a laser spark (*2*). b—The shock wave (*3*) breaks away from the region of hot plasma. c—The wave is reflected from the walls of the cavity and returns to the central region.

Thus this effect in a plasma is very weak and can be neglected.

1.4. Cumulation in multiliner systems

Let there be a system of plane-parallel alternating layers of light and heavy substances such that the successively numbered thicknesses of heavy layers b_n and the successively numbered thicknesses of the light layers a_n form a geometric progression:

$$a_n \sim q^n, \quad b_n \sim q^n, \quad 0 < q < 1. \tag{1.8}$$

The theoretical study of this system performed by E. I. Zababakhin showed that the energy density increases in a shock wave propagating in the direction from the thicker to the thinner layers.[42] This conclusion was confirmed experimentally.[43] The phenomenon is discussed in detail in Ref. 7.

The effect is especially clear in the limiting case in which the heavy layers (e.g., metallic) can be considered incompressible, the density ρ_l of the light layers is negligibly small in comparison with that of the heavy layers ρ_h, while the velocity D of the shock wave satisfies the condition

$$\left(\frac{P_0 a_n}{\rho_h b_n}\right)^{1/2} \ll D \ll \left(\frac{P_0}{\rho_l}\right)^{1/2}. \tag{1.9}$$

Here P_0 is the unperturbed pressure in the system, and the value of a_n/b_n does not depend on n. One can easily show that in this case the interaction of the heavy layers is equivalent to their successive pairwise elastic collision through an intermediate layer of light gas.

Actually, owing to the condition $D \ll (P_0/\rho_l)^{1/2}$, we can assume that the pressure in the light layers during approach of the heavy layers varies adiabatically: $P = P_0 (S_{n,n-1}/\alpha_n)^{-5/3}$. Here $s_{n,n-1}$ is the time-dependent distance between the nth and the $(n-1)$th heavy layers. Thus the interaction between the heavy layers hinders their approach to an infinitesimally small distance ($P \to \infty$ as $s_{n,n-1} \to 0$). On the other hand, the long-range interaction of the layers is negligibly small: owing to the condition $P_0 \alpha_n / \rho_h b_n \ll D^2$, the pressure P_0 in the characteristic time of compression of a light layer a_n/D cannot substantially alter the momentum of the heavy layer $\rho_h b_n D$. Consequently the heavy layers move freely all the time apart from short periods of strong pairwise interaction as any of the $s_{n,n-1}$ approaches zero.

This remark allows us to express the velocity of the nth heavy layer V_n in terms of V_{n-1} by using the conservation laws $b_n V_n + b_{n-1} V'_{n-1} = b_{n-1} V_{n-1}$ and $b_n V_n^2 + b_{n-1} V'^2_{n-1} = b_{n-1} V^2_{n-1}$, where V'_{n-1} is the velocity of the $(n-1)$th layer after collision with the nth layer (after reflection). Hence we obtain

$$V_n = \frac{2\rho_h b_{n-1}}{\rho_h b_{n-1} + \rho_h b_n} V_{n-1} = V_1 \left(\frac{2}{1+q}\right)^{n-1}. \tag{1.10}$$

V_n increases with increasing n.

Replacement of the plane-parallel layers with concentric spherical shells leads to the well known concept of multishell targets for ITF, which enable one to increase considerably the energy flux density in the inner shell and are widely applied in experiments to obtain a hot plasma. To accelerate the outer shell, besides the traditional beam and laser methods, it has been proposed to use the energy of chemical explosives.[44]

A detailed review of the studies on shell targets lies outside our topic, and we wish only to stress their theoretical connection with Ref. 42.

We note also that that the cumulative process in a multiliner system is highly unstable. The reason is that the propagation of a shock wave in a multilayer system is accompanied by multiple reflections and passage of the shock wave through contact breaks, and this gives rise to and amplifies the Richtmyer-Meshkov instability.[45-47]

In the first passage of the wave through the phase boundary of two media, the boundary undergoes a pulsed acceleration, and small perturbations on it begin to increase. In contrast to the Rayleigh-Taylor instability, the perturbations increase both in the case of passage from the light to the heavy gas, and vice versa. The repeated interaction (after reflection) of the shock wave with the perturbed boundary can considerably distort the form of the front.

Thus we have discussed here some known cumulative effects and noted the difficulties involved in realizing these effects in a plasma.

In closing this section, we wish to stress again that here we have adopted a restricted view of cumulation only as the self-compression of the intrinsic energy of gasdynamic flow. For this reason we have not discussed such interesting phenomena as multiple compression of a plasma by a sequence of shock waves as two plane-parallel walls converge,[48] the analogous effect in the convergence of walls at a small angle,[49,50] which is of practical significance,[51] many phenomena in the compression of a plasma by a shell,[52] etc.

2. ONE-DIMENSIONAL CONVERGENT SHOCK WAVES

Here a cumulative flow will be discussed that is of greatest interest for purposes of plasma physics and CTF—convergent shock waves having spherical or cylindrical symmetry.

2.1. Hydrodynamic theory

As is well known, in the geometric-optics approximation a linear convergent wave with a spherically or cylindrically symmetric front is amplified according to the law

$$A^2 \sim S^{-1}. \tag{2.1}$$

Here A is the amplitude of the wave, and S is the area of the wave front. In particular, the increase in the parameters (e.g., the jump in the temperature T) at the front of a weak spherical shock wave obeys the relationship (2.1):

$$T(R) \sim R^{-1}; \tag{2.2}$$

Correspondingly, for a cylindrical wave we have

$$T(r) \sim r^{-0,5}; \tag{2.3}$$

Hereinafter R is the spherical and r the cylindrical radial coordinate.

Equation (2.1) expresses simply the law of conservation of the energy of the wave. In a linear wave, energy is transported with the constant velocity of propagation of the wave (in acoustics—at the velocity of sound). Since the energy in a volume bounded by two infinitely close wave fronts is proportional for a linear wave to the area of the front and the square of the amplitude, the condition of conservation of energy leads directly to (2.1).

In the opposite limiting case of a very strong shock wave ($T \gg T_0$, where T_0 is the temperature ahead of the front), we should also expect an enhancement of the wave as it converges toward a center (or axis) of symmetry. Actually the transport of part of the hydrodynamic energy in the direction of propagation of the shock wave (toward the center or axis) is accompanied by a redistribution of the energy to an ever diminishing area of the front, and the intensity of the shock wave must increase.

Mathematically this is expressed in the existence of self-similar solutions of the system of gasdynamic equations of spherically or cylindrically symmetric motion of the gas obtained by Guderley[14] and by Landau and Stanyukovich.[12,13] These solutions include a strongly converging shock wave propagating through an unperturbed cold homogeneous gas, and yield power-function laws of growth of the amplitude of the wave ($\gamma = 5/3$):

$$T(R) \sim R^{-0,9} \tag{2.4}$$

for the spherical case, and

$$T(r) \sim r^{-0,45} \tag{2.5}$$

for the cylindrical case.[7,8,12,13] The exponents in (2.4) and (2.5) are expressed in terms of the self-similarity index, which in turn is determined from the condition of physically reasonable solution of the hydrodynamic equations and which depends on γ.

Since the self-similarity index is determined not by considerations of dimensionality and is expressed by an irrational number, the inclusion of new physical effects without loss of self-similar character is possible only in special cases. Thus, in Ref. 53 a cylindrically symmetric solution for a convergent shock wave was found with account taken of the magnetic field, including the action of the concentrated current I at the symmetry axis with a special dependence of I on the time t. The case was also studied in which a convergent wave is reflected from a cylindrical piston that expands according to a power-function law chosen such that self-similarity was conserved. However, even without generalizations, the Guderley-Landau-Stanyukovich solutions are very interesting and have been the object of many studies. For example, in Ref. 41 the problem was discussed of multiple values of the solution for $\gamma > 1.7$. Numerical calculations[54-56] have studied the transition of non-self-similar convergent waves to a self-similar regime.

With all the significance of the self-similar solutions, the existence of simple concepts that treat the increase in the amplitude of a convergent wave as the consequence of redistribution of the energy of the wave over the decreasing area S of the front leads to the idea of the possible construction of a theory that would directly associate the amplitude of the wave with S. An important step in this direction has been taken by Chester,[57] Chisnell,[24] and Whitham.[9] They proposed on the basis of analyzing a large number of special and limiting cases a general and rather precise expression for the amplification of a shock wave owing to decreasing area S of the front upon convergence:

$$T \sim S^{-2/n}, \quad n = 1 + \frac{2}{\gamma} + \left(\frac{2\gamma}{\gamma - 1}\right)^{1/2} \approx 4.5. \tag{2.6}$$

(Here $T \gg T_0$, where T_0 is the temperature ahead of the front). In particular, Eq. (2.6) approximates (2.4) and (2.5) well, and can be applied to calculate the amplification of a shock wave in slowly narrowing channels having solid walls.[57,58]

This approach can be generalized to shock waves converging in an inhomogeneous medium in which the amplification of the wave owing to convergence is intensified by gradient acceleration.[59,60] Upon combining Eqs. (1.6) and (2.6) we obtain

$$T \sim S^{-0.45}\rho_0^{-0.48}. \tag{2.7}$$

A good review devoted to calculations of convergent waves with self-similar approximate numerical calculations is presented in Refs. 61 and 62.

A convergent wave elevates not only the temperature following the front. The density also increases, although not by an unbounded law of the type of (2.4), but by a finite factor. After convergence of a spherical wave to the center it is reflected and the density following the front reaches $\rho_2 \approx 30\rho_0$, where ρ_0 is the initial density of the plasma.

We shall try to estimate what initial radius R_0 a convergent spherical shock wave must have to initiate ignition in a D–T mixture, if the initial temperature behind the front is $T_{in} = T\ (R_0) \sim 1$ eV, and the density is $\rho_0 = 0.2$ g/cm^3 (condensed D–T). To initiate ignition, one must increase the temperature of the plasma to ~ 10 keV in a region of spatial scale R_e. Here, according to various sources, the parameter (ρR_e) must exceed approximately 0.3–0.5 g/cm^2. Upon determining the final radius R_e from the relationship

$$R_e \sim \frac{0.5\ \text{g/cm}^2}{\rho_2} \sim 10^{-1}\ \text{cm}, \tag{2.8}$$

we find

$$R_0 \sim \left(\frac{10\ \text{keV}}{T_{in}}\right)^{1,1} R_e \sim 10^3\ \text{cm}. \tag{2.9}$$

The estimate is impressive, but still too low, since, with the adopted values of R_0 and T_{in}, the law (2.4) loses force before thermonuclear temperatures are reached. The corresponding physical effects are discussed below.

2.2. On dissipative limitations of cumulation of a shock wave in a plasma

The restricted applicability of the cumulative laws of temperature increase (2.4) and (2.5) involve, first, the failure to take account of various physical processes that lie outside the single-fluid approximation—radiation, thermal conduction, and viscosity. Second, the laws were derived from the one-dimensional solutions, whereas real non-one-dimensional flows always differ to some extent from their one-dimensional models.

The estimate of the relative role of these factors has been gradually changing in recent time. Earlier it was emphasized[63,64] that taking account of dissipation in cases in which it was possible to trace it to the end yielded no bound on cumulation. Conversely, the instability of convergent spherical and cylindrical shock waves discussed in Sec. 3 seemed to be a universal mechanism of limitation of cumulation.

However, to counterpose these statements, it was finally shown[65] (and earlier in Refs. 15 and 17 and other studies by the same authors) that taking account of plasma dissipative processes limits the growth of the parameters of converging shock waves. Yet the role of non-one-dimensional instability effects now seems more modest, although it is still difficult to agree with the extreme statement that a convergent wave is stable.[37]

The physical limitations introduced by dissipative involve the fact that, when the radius of the front of a convergent shock wave is comparable with the mean free path, nonhydrodynamic energy transport begins to predominate over the hydrodynamic process and suppresses the mechanism of cumulative growth of the wave amplitude.

Let us present estimates characterizing the limitation of cumulation in an equal-component D–T plasma on scales of the order of the mean free path. We recall that the structure of a shock wave includes a jump in the ionic temperature whose width is of the order of the mean free path in ion-ion collisions:[8]

$$l_{ii} \sim \frac{(T/1\ \text{eV})^2}{\Lambda \cdot 4{,}4 \cdot 10^{-14}\,(4N_0)} \sim 10^{-5}\ \text{cm} \cdot \frac{4{,}5 \cdot 10^{22}}{N_0}\left(\frac{T}{1\ \text{keV}}\right)^2. \tag{2.10}$$

Here Λ is the Coulomb logarithm, while the characteristic concentration N_0 is given as its value for a condensed D–T mixture. A wave of electronic heating propagates ahead of the jump in ionic temperature in the front of the shock wave. Its width λ_e exceeds l_{ii} severalfold:

$$\lambda_e \sim 10^{-2} D\tau_{ei} \sim (10^{-4} - 10^{-3})\,\text{cm} \cdot \frac{4{,}5 \cdot 10^{22}}{N_0}\left(\frac{T}{1\ \text{keV}}\right)^2. \tag{2.11}$$

Here τ_{ei} is the time of equalization of the electronic and ionic temperatures (a detailed calculation is given in Refs. 8 and 66).

Owing to the difference between the scales l_{ii} and λ_e, the limitation of the cumulative growth of the electronic and ionic temperatures occurs with different values of the radius x_f of the front. Specifically for the electronic temperature, the relationships (2.4) and (2.5) lose force at

$$x_f \approx \lambda_e, \tag{2.12}$$

when the transport of energy of the electrons owing to electronic thermal conduction predominates over hydrodynamic transport. In the region of (2.12) the cumulative growth of the electronic temperature is replaced by a slower increase. The maximum value of T_e exceeds $T_e(\lambda_e)$ by a factor of ~ 3 in the cylindrical case[15] and by a factor of ~ 5 in the spherical case[18] (the former number is for deuterium, and the latter for D–T).

When $x_f < \lambda_e$ cumulation of the ionic shock wave continues. The law of growth of the jump in the ionic temperature $T_i(x_f)$ must be somewhat slower than (2.4) and (2.5) owing to the cooling of the ionic by the electronic component. Cumulation ceases when

$$x_f \sim l_{ii}. \tag{2.13}$$

Owing to continuation of cumulation in the region between (2.12) and (2.13), the maximum ionic temperature exceeds the electronic temperature by a factor of 2 to 4.[15,18]

The limitation of cumulation of shock waves owing to various dissipative processes has been discussed also in Refs. 56 and 67.

Interestingly, in calculating the possibilities of using cumulative shock waves to ignite a D–T mixture, we can apparently neglect the mechanism discussed here of limitation of cumulation. The reason is that the dimension R_c at which thermonuclear temperatures T must be reached substantially exceeds the dissipative scale.

Actually, according to (2.8) the dimension R_c satisfies the condition: $R_c \gg \lambda_e \gg l_{ii}$ for $T < 5$ keV. Yet, beginning at temperatures $T > 5$ keV, the transport of heat in thermonuclear α-particles begins to predominate over the electronic heat conduction and becomes the dominant mechanism of limitation of cumulation in D–T.

2.3. Radiative losses in convergent shock waves

This effect was analyzed in Ref. 68, and detailed studies using numerical methods have been performed in Refs. 18 and 69–71,

The role of radiation depends strongly on the constant in the amplitude growth laws (2.4) and (2.5). This constant determines the magnitude of the amplitude of the wave for a given value of the radius of the front. The action of radiation on the limitation of cumulation has a qualitatively different character that depends on which of the two conditions is first satisfied in the course of the cumulation process, i.e., with decreasing x_f and increasing $T\ (x_f)$:

$$x_f = l_r(T) \tag{2.14}$$

or

$$T = T_c; \tag{2.15}$$

Here $l_r(T)$ is the mean free path for equilibrium radiation, and T_c is the so-called critical temperature,[8] which is determined by the relationship

$$\sigma T_c^4 = D(T_c)\, e_*(T_c), \tag{2.16}$$

i.e., equality of the hydrodynamic and radiative energy fluxes. In Eq. (2.16) σ is the Stefan-Boltzmann constant, and $D(T)$ is the velocity of the shock wave. In a D–T mixture we have

$$D(T) = 0{,}64 \cdot 10^8 \text{ cm/s.} \left(\frac{T}{1 \text{ keV}}\right)^{1/2}. \tag{2.17}$$

Further, $e_*(T)$ is defined as the energy density in the plasma at a concentration N equal to the concentration N_0 ahead of the front of the shock wave, and at the temperature T. A calculation of T_c in certain media is given in Sec. 2.5; in particular, in condensed D–T we have $T_c \approx 190$ eV.

We shall assume first that the equality (2.14) is attained earlier than (2.15). That is, under the condition (2.14) the relationship $T \ll T_c$ is fulfilled. Hence we find that the power of the bulk losses due to radiation $\sigma T^4/l_r$ multiplied by the characteristic time x_f/D is small in comparison with the energy density:

$$\frac{x_f}{l_r}\frac{\sigma T^4}{De_*(T)} \ll 1, \tag{2.18}$$

Here, as x_f decreases and T increases, the left-hand side of (2.18) continues to decrease: $l_r \sim T^{7/2}$, and moreover, $x_f < l_r$. This means that the radiative losses from the region of cumulation are of bulk type and that these losses are negligibly small and do not affect the cumulation up to the instant when $x_f \to 0$.

Here, indeed, we must note that the left-hand side of (2.18) is proportional to N and can increase as $x_f \to 0$ if the unperturbed concentration N_0 is inhomogeneous and increases strongly near the center (or axis) of symmetry. A calculation of the convergent shock wave for this case is given in Ref. 72.

Now, conversely, let the temperature T behind the front reach the critical value T_c for $x_f > l_r(T_c)$. In this case we can assume that the characteristic spatial scale of the flow considerably exceeds the mean free path of photons, and the radiative-heat-conduction approximation holds. As is known,[8] in supercritical shock waves ($T > T_c$), radiative heat conduction blurs the front of the wave by the width

$$r_* \sim \frac{l_r \sigma T^4}{De_*(T)} \sim l_r(T_c)\left(\frac{T}{T_c}\right)^6. \tag{2.19}$$

This implies that a rise in the temperature above the critical value is accompanied by an extremely rapid ($\sim (T/T_c)^6$) increase in the width of the front r_*. When r_* becomes comparable with x_f, cumulative temperature increase ceases (and is replaced by cumulative density increase). In particular, in the numerical example of Sec. 2.1. the maximum attainable temperature is not 10 keV, but ~ 0.3 keV.

An estimate of the energy, which must be supplied only into the region of limited cumulation, is:

$$\mathscr{E} \sim e_*(T)\, r_*^3 \sim e_*(T_c)\, l_r^3(T_c)\, (TT_c^{-1})^{19}. \tag{2.20}$$

It shows that the attainment of temperatures $T \gg T_c$ in this regime seems to be unrealistic owing to energy considerations.

We can conclude that radiative losses limit the possibility of attaining temperatures $T \gg T_c$. In the latter regime the realization of such temperatures requires unimaginable energy expenditures ($\sim (T/T_c)^{19}$), while in the former regime these temperatures can be attained only in a small region, namely in the spherical case for

$$x_f < l_r(T_c)\left(\frac{T_c}{T}\right)^{1,1}. \tag{2.21}$$

[Otherwise the condition $T < T_c$ cannot be fulfilled when $x_f = l_r(T)$.] For a condensed D–T mixture Eq. (2.21) has the form $x_f < (T/200 эB)^{-1,1}$, and the value of the parameter ρx_f proves to be clearly insufficient to ignite the reaction in D–T.

We recall that the realization of the former or latter regimes is determined by the value of the proportionality constant in (2.4) and (2.5). The growth law in the case delimiting the regimes is

$$T \approx T_c\left(\frac{l_r(T_c)}{R}\right)^{0,9}. \tag{2.22}$$

In line with what is said above, in shock waves weaker than (2.22) the radiative losses are insubstantial, while in stronger waves radiation limits the temperature growth to a value somewhat exceeding T_c.

In completing the discussion of the role of radiative losses, we must make two qualifications. First, we have implicitly assumed here that at the onset of the cumulation process we have $T_{in} < T_c$ and $x_f > l_r(T_{in})$. Both of these assumptions can prove inapplicable to shock waves in targets for CTF. In this case the conclusions drawn above require more precise definition. Thus, if the initial radius of the target is small enough ($R_{in} \ll l_r(T_{in})$), then the condition (2.18) can prove to be satisfied even for shock waves with an intensity exceeding (2.22). Further, if the initial temperature following the front of the shock wave created by a compressed shell exceeds T_c, while the radius R_{in} of the target satisfies the condition $R_{in} \ll r_*$, then the radiative losses strongly affect the shock wave, but the energy estimate (2.20) is incorrect (one must replace r_* with R_{in}).

The second qualification is: even in supercritical shock waves there is a narrow zone of the flow (the temperature peak[8]) in which the radiation is not at equilibrium with the material and does not prevent the cumulative growth of the temperature of the plasma.[18]

However, with all the remarks, the attainment of supercritical temperatures $T \gg T_c$ by using only the effect of cumulation of convergent shock waves by itself seems problematic.

2.4. Experimental possibilities of generating a hot plasma by using cumulative shock waves

A considerable number of studies has been devoted to experiments with cumulative shock waves, while many of them are not associated with plasmas. Therefore we present here only a cursory review of the methods of generating shock waves, and present in barely more detail only the studies that are oriented toward obtaining a hot plasma producing thermonuclear neutrons.

In the first of the published experimental studies,[73] a convergent shock wave was created in the gap between two plane walls by using a complex conversion of a plane shock wave. This method has been used also in later studies.[74] One can also create a cylindrical wave in the gap between two disks by using a peripheral explosion of a detonating material or the electrical explosion of a metal foil.[75–77]

A convergent cylindrical wave of large extent along the axis was created with an induction electrical discharge in a cylindrical gas-filled tube.[78–80] One can generate the same kind of wave in cylindrical charges of explosives with surface initiation. Convergent detonation waves have been created and studied in detonating gas mixtures.[81] In Ref. 82 it was proposed to use a cylindrical wave arising upon breakdown of a gas by an annular laser beam.

The most serious technical applications of convergent shock waves have involved using a spherical charge of solid explosive initiated at the surface.[83] The same or a similar scheme of generation of a spherical wave is sometimes used in physical experimentation.[84,85] Interestingly, spherical shock waves themselves have been hardly studied until the present (apart from Refs. 35–37), since the source of such a wave is not transparent for diagnostics.

If a shock wave is propagating in a wedge-shaped (or conical) cavity in the direction toward the edge (or vertex) (Fig. 3), in a crude approximation we can treat this wave as part of a cylindrical (or spherical) convergent wave. Such an experiment has been performed for a wedge-shaped cavity.[85] Theoretical studies show[86,87] that the mechanism of enhancement of the shock wave in the cavity has a considerable resemblance, but also some differences, from cumulation in the cylindrically and spherically symmetric cases.

Experiments on conical targets are described in Refs. 88–92. A conical cavity with a characteristic dimension of ~ 0.1 cm was created in a heavy material (usually lead) and was filled with deuterium at a pressure of ~ 1 atm. The convergent shock wave arose upon pressing into the cavity a piston accelerated under the action of x-ray[88] or laser[90,91] radiation or with an explosion[89,92] to velocities of the order of several tens of km/s.

The measured yield of thermonuclear neutrons at the level of 10^4–$10^{7.5}$ counts/s indicates that the temperature of the plasma in the target reaches 0.3–1 keV. It is not clear whether one can associate the generation of neutrons only with the cumulation of the first shock wave—for example, the data of numerical calculation in Ref. 91 show that, after the front of the shock wave converges toward the vertex of the cone and is reflected, the piston still continues for some time to compress the plasma and to heat it.

We note that a study[89] performed in Poland (IFPiLM) on compression of a plasma in a conical target with a liner accelerated with an explosive device is of fundamental significance. This is the first publication that has experimentally proved the possibility of using ordinary explosives for ITF. Earlier such a possibility had only been briefly mentioned in 1958 in a report by L. A. Artsimovich.[93]

Only after Ref. 93, a report was published of a Soviet experiment to detect thermonuclear neutrons arising upon cumulation of a convergent spherical shock wave (the information is given in Ref. 94 with a reference to the data of A. S. Kozyrev, B. A. Aleksandrov, and N. A. Popov). The wave was created with a spherical explosive device with an outer radius of 70 cm. In targets filled with gaseous deuterium or uranium deuteride, UD_3, the neutron yield reached 3×10^{11}.

We can mention further that the popular literature contains statements that an analogous experiment was performed in Germany during the Second World War.

Recently a series of proposals has been published on experimental schemes for realizing a convergent spherical shock wave specially for purposes of CTF—e.g., Refs. 119 and 120. They envisage the creation of a convergent spherical wave in a spherical chamber by using energy release at the periphery of a plasma (e.g., by gas discharge)[119] or by using magnetohydrodynamic acceleration of the plasma toward the center. In principle, such an acceleration can be realized by using a system of spurting pinches capable of bouncing the plasma off the wall at high velocity.[120]

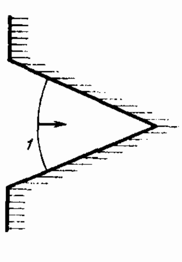

FIG. 3. A shock wave (*1*) entering a wedge-shaped or conical cavity can be treated approximately as a portion of a cylindrical (or spherical) wave.

2.5. On the possibility of using convergent shock waves for initiating thermonuclear fusion

The results presented here seem unpromising from the standpoint of their possible application for ITF. Actually, taking account of the radiative losses points out the difficulty of obtaining supercritical temperatures $T \gg T_c$ via the effect of cumulation of a convergent wave. Here two ways out are envisioned—either to create a wave by using high-velocity shells so that it would have a supercritical temperature even at the initial instant, or to elevate the critical temperature itself to several keV by preliminary compression of a D–T mixture by a factor of 10^3–10^4 [$T_c \sim N_0^{2/5}$, as we see from (2.16)]. However, both of them are difficult to realize. The experimental results existing in the literature, as we have seen, also cannot be considered encouraging.

Apparently extensive possibilities arise from combining the effect of cumulation of shock waves with the most promising modern concept of ITF—conversion of the energy into x-ray radiation.

Usually the conversion of laser radiation[10] or the energy of a relativistic electron beam[88] into soft x rays allows one to increase the pressure on the thermonuclear target. Moreover, the conversion enables a more uniform irradiation and compression of the target.[95]

Let us discuss the question of whether one can use the thermal radiation of a convergent spherical shock wave as a source for irradiating the target. It was noted[70] that the radiation of a convergent shock wave can create high pressure at the surface of a metal sphere. Here it is proposed to study a convergent spherical shock wave in a material (e.g., in the same D–T mixture, which offers extra possibilities) as the generator of x rays for compression of a thermonuclear target.

We recall that the intensity of a cumulative wave is characterized by the proportionality constant in the law (2.4). As the radiation source it seems optimal to use a convergent spherical wave with a value of the constant corresponding to Eq. (2.22).

Actually, in less powerful shock waves the flux of radiation energy is always smaller than the flux of hydrodynamic energy [see (2.18)]. This is identical to the statement that a shock wave has low efficiency as an emitter. In more powerful waves, as we have already stressed, the increase in the temperature of the plasma—and of the radiation—does not compare favorably with the energy expenditures.

Finally, in a wave with the growth law (2.22) the critical temperature $T = T_c$ is reached. That is, the condition (2.16) is fulfilled and the flux of hydrodynamic energy is effectively converted into radiation, with $T \sim T_c$ and the radius of the front being $x_f = l_r(T_c)$.

Let us present the results of calculating the critical temperature T_c taking into account the ionization losses for shock waves in condensed materials—frozen hydrogen, deuterium, D–T mixtures, lithium deuteride, and in diamond. The data for T_c and $J_c = \sigma T_c^4$ are given in Table I.

Ahead of the density jump in a shock wave of critical amplitude (details given in Ref. 8), the plasma is heated to the temperature T_c, but is compressed weakly. This permits us to estimate the mean free path $l_r(T_c)$ by the well-known formula

$$l_r(T_c) \approx 2{,}5\left(\frac{T_c}{100\ \text{eV}}\right)^{7/2}\left(\frac{N_0}{10^{22}}\right)^{-2} z^{-3} \tag{2.23}$$

with the temperature and concentration of T_c and N_0, respectively. These values are also given in the Table I.

For a gas (hydrogen isotopes) whose state ahead of the front of the shock wave corresponds to room temperature and the pressure P_0, the critical temperature, the flux J_c, and the mean free path $l_r(T_c)$ are described by the formulas

$$T_c \approx 100\ \text{eV}\cdot\left(\frac{P_0}{130\ \text{atm}}\right)^{0{,}4} M^{-0{,}2},$$

$$J_c \approx 10^{13}\ \text{W/cm}^2\cdot\left(\frac{P_0}{130\ \text{atm}}\right)^{1{,}6} M^{-0{,}8}, \tag{2.24}$$

$$l_r(T_c) \approx 7\ \text{cm}\left(\frac{P_0}{130\ \text{atm}}\right)^{-0{,}6} M^{-0{,}7},$$

M is the atomic weight.

The results of calculation indicate the possibility of attaining high (10^{13}–10^{14} W/cm^2) values of the flux of radiation onto the target, which lies in the region $R < l_r(T_c)$. In condensed D–T, ~ 4 MJ of radiation is incident on a target of 1 cm radius in 3 ns, while a target of 0.7-cm radius receives more than 20 MJ of radiation, but over a longer time. Apparently these values suffice for a thermonuclear target to give a considerable energy yield.

If a convergent shock wave is created in D–T, then the capability of the D–T mixture for ignition[96] creates a great possibility for multiplying the energy yield of the thermonuclear target. Actually, a cumulative spherical shock wave not only yields radiation to compress the target, but also compresses the D–T mixture by a factor of ~ 30. If now the energy release $\mathscr{E}$ in the target suffices to heat to ~ 10 keV a volume of the D–T mixture of radius (2.8) $\mathscr{E} \gtrsim 30$ MJ, then a burning wave that elevates the energy yield arises in the D–T outside the target.

Perhaps adding Li to the D–T mixture will enable one to decrease somewhat the energetics of the convergent wave owing to lowering of $l_r(T_c)$. However, the addition must be small; otherwise the mixture loses the capability for ignition. Under optimistic assumptions, estimates of the overall energy of the experiment (wave generation—fusion in the target—ignition of the mixture outside the target) yield 10^8–10^9 J.

TABLE I.

	H	D	D–T	Li	DLi	C
T_c, eV	220	190	185	240	310	520
J_c, W/cm^2	$2\cdot10^{14}$	10^{14}	10^{14}	$3\cdot10^{14}$	$9\cdot10^{14}$	$7\cdot10^{15}$
$l_r(T_c)$, cm	2	1	1	0.1	0.1	10^{-2}

3. NON-ONE-DIMENSIONAL CUMULATION OF SHOCK WAVES

The use in theory and in practice of the cumulation laws of growth of the amplitude of convergent shock waves always involves the problem of how applicable these laws, which were derived from one-dimensional (spherically or cylindrically symmetric) solutions, are to actual, non-one-dimensional shock waves. For a plasma and other gases the problem is complicated by the instability of the one-dimensional solutions for convergent waves with respect to small non-one-dimensional perturbations.

Another problem involves the possibility of cumulation of a noncylindrical (and nonspherical) axially symmetric shock wave. For example, the compressed current shell of a Z-pinch can give rise to a convergent shock wave and, since the shell is subject to instability with respect to formation of sausage instabilities, analogous perturbations arise also in the shock wave (Fig. 4). It is convenient to start the discussion of cumulation of non-one-dimensional shock waves specifically with this problem.

3.1. Cumulation of an axially symmetric noncylindrical shock wave

For theoretical description of non-one-dimensional cumulative shock waves, G. B. Whitham[9] proposed a special method of geometric dynamics. If one describes the position of the front at each instant of time with the relationship

$$\Phi(x, y, z) + t = 0 \tag{3.1}$$

(x, y, and z are the coordinates), and applies Eq. (2.6) for each element of area of the front, then one obtains the following nonlinear equation for the unknown function Φ (while taking n from (2.6)):

$$\operatorname{div} \frac{\operatorname{grad} \Phi}{|\operatorname{grad} \Phi|^{n+1}} = 0. \tag{3.2}$$

In particular, a cylindrical convergent shock wave is described by the function[9]

$$\Phi \sim r^{1+(1/n)}. \tag{3.3}$$

The velocity of the front derived from (3.1) and (3.3) is found to agree with Eq. (2.6) if we consider that $T \sim D^2$.

Now let small sausage-type perturbations be applied to the cylindrical convergent wave at the initial instant of time: $r = r_0(1 + \varepsilon \cos qz)$, where r, z, and ϕ are the cylindrical coordinates, and r_0, ε, and q are constants, with $\varepsilon \ll 1$. Linearization of (3.2) with respect to (3.3) allows one to obtain a solution for the convergent perturbed cylindrical wave in the form[97]

$$\Phi = \left[\left(\frac{r}{r_0}\right)^{1+(1/n)} - \frac{n+1}{n} \varepsilon \cos (qz)\right] \cdot \text{const}. \tag{3.4}$$

Equations (3.1) and (3.4) imply that, as the wave converges toward the axis (in the region of small r), the perturbation increases as $r^{-1/n}$, and at the instant of arrival at the axis the amplitude becomes of the order of $r_0 \varepsilon^{n/(n+1)}$. Thus the perturbation increases by an unbounded factor ($\varepsilon^{n/(n+1)} \gg \varepsilon$), although it remains infinitesimally small ($\varepsilon^{n/(n+1)} \ll 1$).

The physical reason for the instability is directly associated with the cumulative, accelerated character of the motion of the front: the regions of the front closest to the axis are accelerated more strongly than those remote, whereby the amplitude of the perturbation of the front increases.

The instability has the result that the front of a perturbed wave, in contrast to an unperturbed one, reaches the axis at separate, isolated points (see Fig. 4a). The same property is inherent in a shock wave at a plasma focus (see Fig. 4b), as well as an annular shock wave in a gas, which is formed upon energy release in a region having the form of a thin circular ring (see Fig. 4c).

The behavior of an axially symmetric convergent wave near an isolated point where it arrives at the axis was first studied in Ref. 98 by numerical methods for the case of a shock wave at a plasma focus. References 99 and 100 experimentally proved the cumulation of a convergent noncylindrical wave with the example of an annular shock wave in a gas (air) (Fig. 5). Theoretical analysis of Eq. (3.2) shows that such a cumulation with dissipation neglected is unbounded in character and the law of growth of the shock wave near the axis asymptotically goes over into (2.5).[101]

Let us present the local expansion found in Ref. 101 of the solution of Eq. (3.2) near an isolated point at which the shock wave reaches the axis:

$$\Phi = \frac{R_0}{V_0}\left[\frac{n}{n+1}\left(\frac{r}{R_0}\right)^{1+1/n} - \frac{1}{2}\left(\frac{z}{R_0}\right)^2 - \frac{1}{2n-2}\left(\frac{r}{R_0}\right)^2\right]. \tag{3.5}$$

Here V_0 and R_0 are arbitrary constants, with V_0 having the dimensions of velocity and R_0 the dimensions of length. In particular, Eq. (3.5) implies that the local equation (valid near the axis) of the surface of the front at the instant of arrival at the axis is

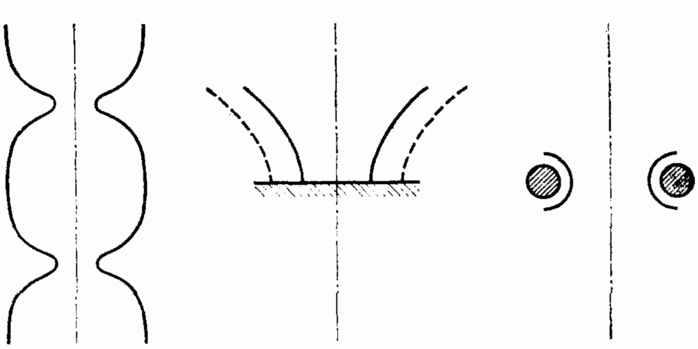

FIG. 4. Axially symmetric, noncylindrical convergent shock waves. a—Cylindrical front with sausage-type perturbations. b—Shock wave in a plasma focus. c—Annular (toroidal) shock wave.

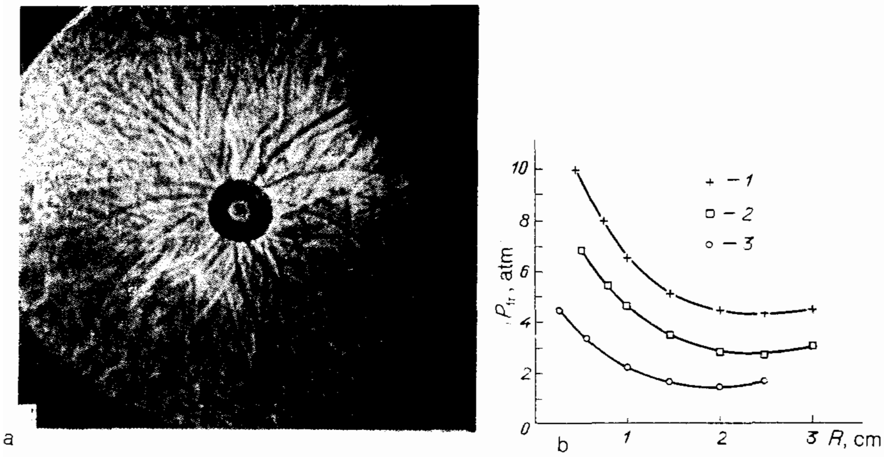

FIG. 5. Results of studying an annular shock wave created in air by using a special annular discharger. The radius of the discharger is 5 cm. a—Form of the wave near the axis (shadow photography; the plane of the diagram coincides with the plane of the ring). b—The pressure P_{fr} behind the front of the wave as a function of the distance r to the axis at different pressures P of the unperturbed gas. The increase in P_{fr} at small r indicates cumulation. P (atm) = 1 (*1*), 0.4 (*2*), and 0.1 (*3*).

$$r \sim z^{2n/(n+1)}. \qquad (3.6)$$

It amounts to a nonquadratic paraboloid of rotation $r \sim z^k$, where $1 < k < 2$. The radius of curvature of the parabola at the vertex ($z = 0$) vanishes. This sharpening is caused by the accelerated character of the motion of the front toward the axis, to which we have called attention above.

The law of approach of the velocity of the shock wave at the vertex to infinity as the wave approaches the axis ($r \to 0$) is implied by (3.1) and (3.5) ($D = -(\mathrm{d}\Phi/\mathrm{d}r)^{-1}$):

$$D = V_0 \left(\frac{r}{R_0}\right)^{-1/n} \left[1 + \frac{1}{n-1}\left(\frac{r}{R_0}\right)^{1-(1/n)}\right]. \qquad (3.7)$$

The velocity increases somewhat more slowly than in the case of a cylindrical shock wave [for which $D \sim (r/R_0)^{-1/n}$, as is clear from (2.5)]. The difference involves the noncylindrical form of the front in (3.6). However, the principal terms remain the same as $r \to 0$.

The fact of cumulation of a noncylindrical wave that reaches the axis at an isolated point (Fig. 4) is not trivial since, in contrast to a cylindrical wave, hydrodynamic energy can escape in the direction along the axis,[99] and since the local form of the front near the axis substantially differs from cylindrical.

A numerical calculation for an annular shock wave is given in Refs. 100, 102, and 103. We note that, in view of the universal character of the behavior of a shock wave near a point of cumulation,[101] the laws of behavior of a perturbed cylindrical wave, a wave at a plasma focus, and an annular wave near the axis are fully analogous. Therefore the experiment with an annular shock wave[99,100] and the numerical calculation can also be considered as a hydrodynamic simulation of a shock wave at a plasma focus.[104]

This approach enables one, for example, to confirm the hypothesis[97] that a convergent shock wave at a plasma focus gives rise to a plasma jet in the axial direction, whose velocity exceeds severalfold the velocity of the convergent wave. The appearance of such a plasma flux might explain both the yield of thermonuclear neutrons and their anisotropy. A detailed experimental study of the reflection of an annular wave from the symmetry axis[104,105] actually enabled detecting such a jet (Fig. 6).

It was shown that the appearance of jets involves the specific ("Mach") character of the reflection of an annular

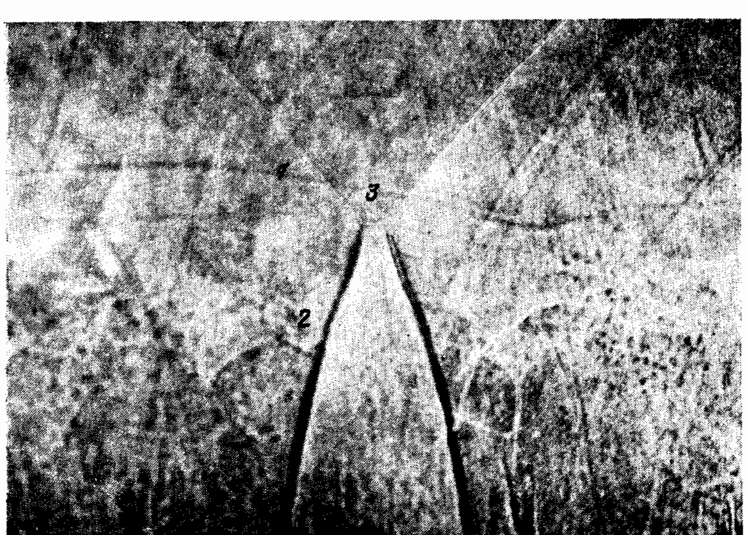

FIG. 6. Appearance of a Mach configuration in the reflection of an annular shock wave from the axis: shadow photography in a direction perpendicular to the axis. *1*—converging front of the shock wave; *2*—reflected wave; *3*—high-velocity jet of gas propagating along the axis.

(or generally noncylindrical) axially symmetric shock wave from the axis.[106] The problem of such a reflection is better known for the case of a conical shock wave.[107-110] Unfortunately, here we cannot present in any detail the rather complicated physics of the Mach interaction of shock waves,[58,107] and all the more so—the features of this interaction with axially symmetric geometry.[106,107,110] We note only that the temperature of the jet behind a Mach shock wave (Fig. 6) exceeds by approximately two- or threefold the temperature behind the reflected shock wave.

The complicated connection between one-dimensional and non-one-dimensional cumulation is manifested in the example of a noncylindrical, axially symmetric shock wave. Actually, the amplitude of axially symmetric perturbations of the sausage type increases, but remains infinitesimally small. While remaining infinitesimally small, the perturbation qualitatively alters the type of cumulation. The altered law of growth of the amplitude proves to be slower than that for a cylindrical wave, but it asymptotically goes over into the latter. Finally, the weaker noncylindrical cumulation is accompanied by formation of jets with higher temperature than in the cylindrical case.

3.2. The influence of azimuthal perturbations on the process of cumulation of a shock wave

As we have already mentioned, strongly differing opinions have been expressed in the literature on the question of the influence of non-one-dimensional perturbations of the front on the cumulation of shock waves. They range from the assertion that convergent waves are stable with respect to perturbations[37] to the conclusion that instability with respect to non-one-dimensional perturbations offers a universal mechanism of limitation of cumulation.[7,63,64]

We can consider it to be proved that convergent cylindrical and spherical shock waves are unstable with respect to infinitesimally small perturbations that depend on the azimuthal angle.[9,59,97,111] For a cylindrical wave, if we linearize Eq. (3.2) against the background of (3.3) and solve it, we can easily find the law of evolution, which depends on the perturbation as the wave converges on the axis. The solution has the form

$$\Phi = \text{const} \cdot \left[1 + \varepsilon \left(\frac{r}{r_0} \right)^{-1-(1/n)} \Psi \right] \left(\frac{r}{r_0} \right)^{1+(1/n)}, \ \varepsilon \ll 1, \tag{3.8}$$

where

$$\Psi \propto \cos(m\varphi) \cdot \exp\left[\left(\ln \frac{r}{r_0} \right) \frac{n+1}{2n} \left\{ 1 - \left[1 - \frac{4m^2 n}{(n+1)^2} \right]^{1/2} \right\} \right]. \tag{3.9}$$

If we introduce the quantity δr, the small deviation of the distorted surface of the front from the cylindrical surface of radius $\langle r \rangle$ (Fig. 7), the distortion of shape characterized by the ratio $\delta r / \langle r \rangle$ increases with decrease of $\langle r \rangle$:

$$\left| \frac{\delta r}{\langle r \rangle} \right| \propto \langle r \rangle^{-0.6} \tag{3.10}$$

($\gamma = 5/3$; $n \approx 4.5$). For a spherical wave we have

$$\left| \frac{\delta r}{\langle r \rangle} \right| \propto \langle r \rangle^{-0.7}. \tag{3.11}$$

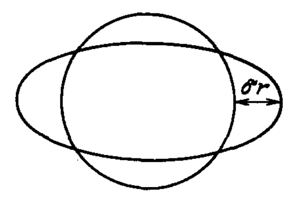

FIG. 7. The magnitude of δr characterizes the deviation of the form of the front from cylindrical symmetry.

Reference 111 is very important, where the result (3.11) was obtained within the framework of a rigorous gasdynamic approach without adducing the approximate equation (3.2). This conclusion confirms the applicability of the theory of Whitham for describing non-one-dimensional cumulative shock waves.

The fact of growth of a linear perturbation leads to the need to analyze the nonlinear effects, the first of which is the formation of shock waves transverse to the main front of the convergent wave.[9,112-114] The reason for the effect is simple: since the velocity of the shock wave with respect to the medium following its front is subsonic, then compression and rarefaction waves can propagate behind the front in a direction transverse to the front. Therefore the growth of the distortions of the front of the cumulative wave leads to propagation of compression and rarefaction waves transverse to the front, and the reversal of the compression waves—an effect well known in hydrodynamics[13]—forms shock waves transverse to the main wave (Fig. 8; taken from Ref. 113).

Reference 112 experimentally demonstrated the formation of transverse fronts at an artificially perturbed convergent cylindrical wave. Numerical calculations of a weakly perturbed convergent wave[113] indicate the possibility of formation of nonlinear structures with the form of a convergent front resembling a polygonal prism. The law of amplification of such a wave on the average coincides with Eq. (2.4) for a cylindrical wave.

In was shown experimentally[114] that a convergent elliptical wave gives rise in the region of the foci of the ellipse to a complicated configuration of fronts recalling in form the geometry of linear waves near a caustic, but modified by nonlinear effects (an excellent study of a nonlinear caustic of a shock front is presented in Ref. 115) (Fig. 9). Here the amplitude of the shock wave remains bounded, instead of the unbounded growth according to Eq. (2.4). Thus, sufficiently strong perturbations with the azimuthal number $m = 2$

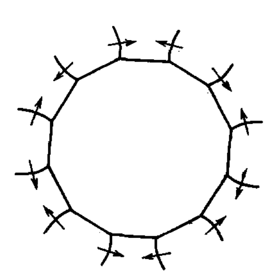

FIG. 8. The appearance of transverse shock waves upon perturbation of an axially symmetric convergent shock wave.

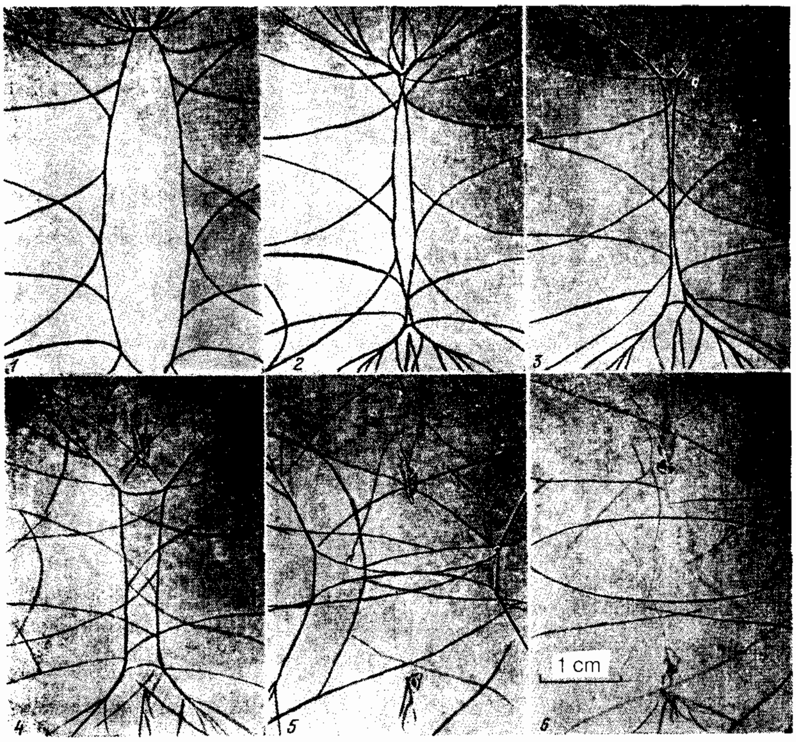

FIG. 9. Shadowgrams of a convergent elliptical shock wave. *1–6*—sequential stages of formation of the convergent (*1*) and divergent (*2*) fronts.

(i.e., depending on φ as $\cos 2\varphi$) can in principle prevent unbounded cumulation.

However, it was shown in the same article that even strong perturbations with large m do not prevent unbounded cumulation. In general there are grounds for assuming that the formation of transverse shock fronts is a factor that strongly stabilizes the instability of a convergent shock wave with respect to azimuthal perturbations.

From the practical standpoint the cumulation of an annular wave has proved to be rather stable. At a radius of the source, and correspondingly, the initial radius of the shock wave of $R_0 = 5$ cm, the cumulation of a wave was observed[114] down to a distance from the axis of $r < 10^{-1}$ cm, while in Ref. 104, according to indirect data, cumulation was established at $r < 10^{-2}$ cm. In other words, 50- and 500-fold degrees of stable convergence of the front were achieved (the degree of convergence is taken to be the ratio R_0/r). In this regard we note that, to create a spherical convergent wave in D–T with critical amplitude, the stability of the cumulation must be conserved at a degree of convergence of $R_{in}/R_e \sim 10^2$.

As regards the general conclusion already mentioned above of the universal role of non-one-dimensional perturbations in limiting cumulation that was made in Refs. 7, 63, and 64, it seems to be not fully proved. First, there are special results that explicitly contradict this general conclusion. For example, models are known if media (a liquid with $\gamma = 7$ (Ref. 117) and an ideally inelastic medium[118]) in which a convergent shock wave is stable even in the linear approximation, and infinitesimally small perturbations are known not to be able to limit cumulation. Second, the possibility seems to be disputable of identifying the mathematical property of cumulative flow proved in Ref. 7 (it was shown that the mapping of the set of initial states of the flow onto the set of states in which unbounded cumulation is attained cannot be in a one-to-one relationship and be mutually continuous) with the property of stability of cumulation.

In any case, as we have seen, the instability has not proved to be catastrophic for experiment.

CONCLUSION

Thus we have attempted here to draw attention to a set of interesting phenomena—cumulative hydrodynamic processes—and to the prospects of using them in plasma physics.

We can hope that the results presented here are of separate scientific interest independent of applications. However, in the review we have repeatedly touched on the question of the possibility of using cumulative shock waves to initiate ITF, which obliges us to formulate here the final conclusion on this problem.

We must acknowledge that a final solution of the problem of igniting a plasma by using convergent shock waves has not yet been presented in the literature. At the same time, possibilities can be seen of overcoming the main difficulties (radiation, instability) that impede the attainment of this result. Apparently, despite repeated attempts to act along this line, the path has not yet been pursued to the end.

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