How to measure complexity?

I. M. Sokolov

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Measures of Complexity: Proceedings of the Conference. Rome. September 30–October 2, 1987. Eds. L. Peliti and A. Vulpiani. Springer-Verlag, Berlin; Heidelberg; New York; London; Paris; Tokyo, 1988–150 pp. (Lecture Notes in Physics. V. 314)

It is worthwhile to begin the review of this collection of articles by quoting from its first article: "There have always been three major directions along which the frontiers of physics have advanced: towards the very large, towards the very small and towards the complex." (P. Grassberger). While the criteria developed by physics make it possible to understand what should be regarded as very big or very small it is not clear what should be regarded as very complex or even just simply complex. There is no well defined procedure for measuring complexity and even at the informal and intuitive level it is not well understood what properties must characterize a quantity laying claim to serve as a measure of complexity. The standard definitions of complexity in encyclopedic dictionaries, as is indicated in the preface to the collection of articles, treat three aspects of the word "complex": 1) composed of interconnected parts; compound; composite; 2) characterized by a very complicated or involved arrangement of parts, units, etc.; 3) so complicated or intricate as to be hard to understand or deal with. It is clear that definitions 1 and 2 are not appropriate, for example, to dynamic chaos—complex (in the sense of 3) behavior of simple dynamic systems.

The wish to clarify this problem at least to a small extent led the organizers of the conference "Measures of Complexity" to invite specialists in different fields of knowledge—mathematicians, physicists, biologists—in order to discuss how complexity is qualitatively defined in their discipline and how this concept relates to those that are used in other sciences. Ideally the results of these intellectual endeavors should be the formulation of a procedure for measuring complexity, after which complexity would be utilized as a physical characteristic of an object even if complete understanding has not been attained. One might deal with complexity in the same manner as one dealt with temperature in the course of those two hundred years until its interpretation became clear within the framework of statistical physics.

Naturally this maximal program was not carried out. Some of the authors fulfilled the rules of the game only formally, having used the word "complexity" either in the introductory or the concluding part of their papers, but the majority of those presenting papers took a serious approach to the problem and in attempting to answer the posed questions formulated still more new ones. The main variants of the definitions that are being used are given below.

Complexity is associated with information (or entropy). This definition is unsatisfactory because intuitively a random sequence of zeros and units appears to be just as simple as a sequence of units alone. Attempts to improve the situation force one at once to attempt to define the concepts "significance" or "meaning" which is not much more simple. A more sensible approach seems to be the definition of complexity through the difficulty of carrying out some task associated with the system. The difficulty of the task is measured by the number of resources required for carrying it out, in which case the taking into account of the different types of resources being utilized leads to different definitions of complexity. For example, in application to sequences of numbers the minimal length of the algorithm (the number of symbols in the program) generating the sequence corresponds to the concept of algorithmic complexity according to Kolmogorov–Chaitin, the minimal time (number of operations required to obtain the given portion of the sequence) corresponds to the criterion of complexity involving logical depth according to Bennett. However, both criteria are unsatisfactory because with their aid it is not possible to define complexity in terms of the sequence itself, and not in terms of the algorithm which gives rise to it. Yet another criterion is associated with the difficulty of predicting the subsequent terms in terms of the preceeding ones. The criterion loses its academic nature and becomes vitally important as soon as we leave aside the abstract sequences and go over to a discussion of predictability of behavior of a complex technical system—a uranium enrichment facility or an atomic electrical power station. A definite place has been reserved for the investigation of the concept of complexity in biological systems. Here the problems turn out to be even more complicated than in the case of algorithmically generated sequences, lattice automata or even large technical objects, since in distinction from the case of communication or a technical system it is not clear what is a significant and what is an inessential part, and also what is it specifically that is coded on a given information carrier (say, in DNA)—the program or the initial data for the program.

Having indicated the area of the basic problems examined in the articles of the collection I shall cite the titles of the articles, the set of which reflects those concrete problems which are used as examples to discuss problems associated with complexity. Altogether there are 12: P. Grassberger, Complexity and Forecasting in Dynamical Systems; G. Parisi, On Complexity; S. Patarnello and P. Carnevali, Boolean Networks Which Learn to Compute; U. Krey and P. Pöppel, A Dynamical Learning Process for the Recognition of Correlated Patterns in Symmetric Spin Glass Models; J.-P. Nadal, Neural Networks That Learn Temporal Sequences; C. P. Bachas, Hierarchical Diffusion; Ph. de Forcrand, K. Koukiou and D. Petritis, Random Walks, Random Surfaces and Complexity; G. B. Scuricini, Complexity in Large Technological Systems; D. P. Bovet and P. L. Crescenzi, An Introduction to the Theory of Computational Complexity; H. Atlan, Measures of Biologically Meaningful Complexity; G. Weisbuch, Complex Systems, Organization and Networks of Automata; J.-A. Meyer, Complexity in Ecological Systems.

As can be seen, the area containing these problems is very broad. And although no final answer was given to the question "How to measure complexity?", and also in spite of a certain amount of confusion and chaos characteristic of

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proceedings of interdisciplinary conferences, the majority of the articles of the collection, and the collection as a whole, are, in my opinion, of undoubted interest for specialists in the fields of computational mathematics, physics, biology and many others, who have thought about the question: after all, what *is* complexity?

Translated by G. M. Volkoff