BOOK REVIEWS

The fractional quantum Hall effect

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T. Chakraborty and P. Pietiläinen. *The Fractional Quantum Hall Effect: Properties of an Incompressible Quantum Fluid*. Springer-Verlag. Berlin; Heidelberg; New York; London; Paris; Tokyo, 1988. 175 pp. (Springer Series in Solid State Sciences. V. 85)

"The experimental discovery of the fractional quantum Hall effect at the end of 1981 by Tsui, Störmer, and Gossard, was absolutely unexpected and intriguing... A critical analysis of the available experimental data demonstrated that the ideas most appropriate to reality turn out to be those of Laughlin on the transformation of a two-dimensional magnetized system of electrons into an incompressible quantum fluid. The theoretical work collected in this book demonstrates that the Laughlin wave function forms a very good basis for a discussion of the fractional quantum Hall effect.. . This book is the first general review of the work of different groups in this field... I hope that this book will inspire scientists to new ideas."

This brief characteristic of the book under review which belongs to Klaus von Klitzing (the author of the discovery of the integer quantum Hall effect) is in its own way an indication of its high quality.

The book contains a brief review of experimental data on the integer and fractional quantum Hall effects in different two-dimensional charged systems, and also a detailed discussion of the main theoretical aspects of the phenomenon of the fractional quantum Hall effect (FQHE):

1. Introduction
2. Ground State
3. Elementary Excitations
4. Collective Modes (intra-Landau level)
5. Collective Modes (Inter-Landau level)
6. Further Topics
7. Open Problems and New Directions

Appendices

The introduction gives a comparative analysis of experimental data on the integer and fractional quantum Hall effects. It is emphasized that while the integer QHE which is manifested in the appearance of fundamental sharp plateaus $\sigma_{xy} = -le^2/h$ (e is the electron charge, h is the Planck constant, l is an integer) in the dependence of the Hall conductivity σ_{xy} on the filling factor ν for $\nu = l$, and also in the vanishing in the same regions with respect to ν of the diagonal part of the conductivity, can be understood using the concepts of a system of two-dimensional, noninteracting electrons in a strong magnetic field, the situation with the fractional quantum Hall effect is qualitatively different. For the FQHE the numerical factor ν in the definition of σ_{xy} turns out to be fractional, with the denominator of the fraction capable of being both odd and (the latest of the new aspects) even. The presence of these special features and the behavior of the conductivity tensor σ_{ik} is entirely due to the correlation effects in the system of interacting particles with Fermi statistics. Thus, the first step in understanding the FQHE must consist of a thorough study of the properties of 2D-systems of interacting electrons in the presence of a neutralizing homogeneous background and a strong magnetic field H normal to the plane of the electron system corresponding to an incomplete filling of the Landau ground level, i.e., to a filling factor $\nu < 1$. By definition the factor ν is given by

$$\nu = 2\pi l_H^2 n_s, \quad \text{(A)}$$

where l_H is the so-called magnetic length, and $l_H^2 = \hbar c/eH, n_s$ is the average density of the electrons in the system.

A detailed discussion of the properties of the ground state of the system of interacting electrons in the lowest Landau level using the Laughlin variational function is the content of Ch. 2 of this book. An analysis shows that for certain special values of the filling factor $\nu = 1/m$ where m is an odd number, the energy of the system of electrons as a function of its density using the Laughlin approximation shows definite breaks, and the corresponding behavior of the chemical potential at these points turns out to be discontinuous. These facts indicate that at these points the electron system is in a state of incompressible fluid.

Laughlin proposed further that near the distinguished stable states in the system there arise electron and hole excitations with fractional charge. Chapter 3 of this book discusses details of how such excitations are structured and what are the energies of their coming into existence. In addition a brief review is given of the most significant experimental studies on the measurement of the energy gaps based on the data of the activation temperature dependence of the diagonal part of the resistance, and also with the aid of different spectroscopic methods.

The next logical step is a study of the lowest neutral excitations—quasiexcitons arising as a result of pairing of fractional electrons and holes. Chapter 4 contains quite detailed information on the spectrum of collective exciton excitations. In particular it is noted that this problem has much in common with the problem of the roton spectrum in liquid ^{4}He.

Chapter 5 discusses magnetoplasmic oscillations taking into account the effects of electron correlations in the neighborhood of cyclotron resonance. It should be noted that in accordance with Kohn's theorem the electron-electron interaction in systems with translational symmetry should not affect cyclotron resonance. However, in the presence of im-

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purities this assertion becomes invalid, and cyclotron resonance becomes a valuable source of information on the correlation properties of an electron gas under the conditions of FQHE. Here also a discussion is given of spin waves under the conditions of FQHE.

Finally Ch. 6 contains a brief review of the different phenomena which are of considerable interest for a general understanding of the FQHE. The most interesting of these is a discussion of the FQHE for fractional values of ν with even denominators.

The seventh section is very brief. It is simpler to read it in the original, than to comment on it.

In spite of the fact that the authors do not touch upon a number of well-known directions in the theory of the FQHE (for example, the use of the method of the renormalization group), the book is without doubt very useful by the breadth of the collected information, by the variety and persuasiveness of the arguments, by the discussion of alternative possibilities for describing the FQHE, etc. The book is written in clear language, and contains a large number of diagrams.

Without doubt the book under review is very necessary for scientists, and graduate and undergraduate students who are interested in the newest achievements in the field of physics of low-dimensional electron systems.