Reflection of light from a moving mirror and related problems

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The Maxwell equations, the Minkowski material equations, and the conditions which must be obeyed by fields on moving interfaces are used to obtain expressions for the frequencies and wave vectors of all the waves that appear on reflection and refraction by moving interfaces, by a tangential discontinuity of velocities of two media, and by a moving mirror. The main reported investigations of these topics are discussed. Calculations are made of a large frequency shift, of a considerable increase in the amplitudes, and of a change in the direction of propagation of waves reflected by a rapidly moving mirror or by the front of a parameter traveling in a medium at rest. Allowance is made for the Fresnel transmission of waves across such an interface and for the finite thickness of the transition layer. The change in the parameters of the reflected wave packets is considered together with the problem of the exchange of energy and momentum between an electromagnetic field and a moving mirror. In the case of a tangential discontinuity the rotation of the plane of polarization of reflected and refracted waves is dealt with, as well as the amplification of these waves on reflection from a medium moving faster than light.

INTRODUCTION

We shall consider electromagnetic fields (in the absence of their sources) in moving media containing interfaces. These interfaces are surfaces (in the simplest case, planes) separating media with different electromagnetic properties (i.e., with different values of the permittivity and magnetic permeability). An interface itself can move as well. We shall assume that both media and the interface between them are traveling at constant velocities. As a special case we shall consider an interface between media at rest. The simplest example is a medium with a permittivity ε_1 separated by a plane boundary from a medium with a permittivity ε_2 when the two media are at rest and the interface between them is traveling at a constant velocity. These are known as systems with a traveling parameter or with a wave of a parameter. In the case under discussion there is a traveling discontinuity of a parameter, which is the permittivity. The problem of the interaction of electromagnetic waves with a traveling wave of a parameter has much in common with the boundary-value problems in electrodynamics of moving media and we shall consider problems of both kinds.

The reflection and refraction of waves by moving interfaces has a number of special properties, so that in many respects the situation is different from the interaction of waves with an interface at rest. We can illustrate this by recalling the main features of the thoroughly investigated and well-known effect, which is the reflection and refraction of plane electromagnetic waves by an immobile interface between two media at rest (Fresnel problem). It is well known that in this case we have an incident wave, as well as two other waves: a reflected wave in the same medium as the incident wave and a refracted wave transmitted to the second medium. All three waves have the same frequency. The directions of the wave vectors of the reflected and refracted waves are governed by the Descartes–Snell law (the angle of incidence is equal to the angle of reflection and the sines of the angles of incidence and refraction are inversely proportional to the refractive indices of the two media).[1)] The amplitudes of the reflected and refracted waves are governed by the Fresnel formulas and depend on the polarization, angle of incidence, and optical properties of the media on both sides of the interface.

The motion of an interface gives rise to several important departures from the case just discussed. First of all, the frequencies of the incident, reflected, and refracted waves are all different. The angle of incidence is no longer equal to the angle of reflection and the familiar Descartes–Snell law governing the direction of propagation of the reflected and refracted waves is replaced with a more complex law. The relationship between the amplitudes of the incident, reflected, and refracted waves is also more complex and, in addition to dependences on the angle of incidence, on the polarization, and on the optical properties of the media, we have also a dependence on the velocities of the interface and of both media. In some cases the reflected wave may be completely absent and instead a second refracted wave may appear on the other side of the interface. Allowance for the dispersion of the media in question complicates matters even further (several reflected or several refracted waves may appear). A quantitative analysis of all these phenomena is given below.

It is sometimes said that the mathematical apparatus of electrodynamics of moving media is not necessary in calculations dealing with specific physical effects. According to this point of view any problem in electrodynamics of moving media can be solved in a reference (coordinate) system in which the media are at rest and then the Lorentz transformation can be used to convert the results to media in motion. In the case of unbounded media[27,29] this approach is to some extent justified, but even then all is sacrificed to the conven-

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ience of calculations. It is sometimes simpler to carry out the necessary calculations at the onset for the case of moving media rather than begin with the problem of media at rest, and then make the necessary transformations. If moving media have interfaces, it is often found that the use of the electrodynamics of moving media provides the only way of solution. By way of example, we shall consider two moving media separated by a plane interface and we shall assume that the velocities of the media are different. The simplest example is a tangential discontinuity of velocities, known from hydrodynamics, when the velocities of flow on both sides of the interface lie in the plane of this interface and differ in magnitude or direction. We can consider also a normal discontinuity of the velocity such as that encountered in a shock wave. In such cases there is in general no reference system in which the interface and the media separated by it could be simultaneously at rest. This range of problems will also be discussed below. At this point we shall mention an interesting property of the interaction of electromagnetic waves with a tangential velocity discontinuity. We shall show below that in this case the reflection and refraction of waves are accompanied by a rotation of the plane of polarization and the angle of rotation is proportional to the relative velocity of motion of the two media.

These distinguishing features of the interaction of waves with moving interfaces are of practical importance in many cases. The applications include transformation of the frequency and amplification of electromagnetic waves by reflection and refraction from a moving interface. We shall mention also here the possibility of using transition radiation generated at a moving interface for the purpose of acceleration of charged particles. This is possible because of inversion of the losses due to the emission of transition radiation in the case of a moving interface: if an interface is at rest, the moving charge loses its energy as a result of generation of transition radiation, whereas in the case of a moving interface the energy of a moving charge can sometimes increase.

Another possible application is that the properties of reflected and refracted waves can be used to determine the properties of an interface and also of the media on both sides of it. We can thus determine, for example, the velocity of an interface and the velocities and optical properties of media on both sides of an interface. These possibilities are of specific interest, particularly in the case of diagnostics of plasma streams (both under laboratory conditions as well as in space near the Earth and far from it).

We shall now consider briefly the history of these problems. The first boundary-value problem in electrodynamics of moving media was considered by A. Einstein within the framework of the theory of relativity in the well-known paper of 1905 in which he formulated the fundamentals of the special theory of relativity.[2]

Einstein illustrated his approach by solving the problem of reflection of a plane electromagnetic wave incident obliquely on a moving mirror. He determined the kinematic (phase) relationships between the wave vectors and the frequencies of the reflected and incident waves, as well as the amplitude relationships. Subsequently (1929) Eropkin[3] obtained the phase relationships for a moving boundary of an insulator on the assumption that the velocities of the boundary and the medium are nonrelativistic (obtaining expressions of the first order in respect of the ratio u/c, representing the ratio of the velocity of both media and of the interface to the velocity of light in vacuum). In 1952 Landecker[4] discussed the reflection of electromagnetic waves incident normally on an abrupt leading edge of a beam of relativistic electrons moving opposite to an electromagnetic wave. He demonstrated the possibility of a considerable increase in the frequency and amplitude of the reflected signal at relativistic velocities of the interface. However, the experiment discussed theoretically by Landecker was difficult to perform because of the need to generate relativistic electron beams of sufficiently high density and this was impossible at the time. In 1956 Lampert[5] drew attention to the fact that the relativistic effects of an increase in the frequency and amplitude on reflection of waves may be observed even if the interface does not travel at the velocity close to that of light in vacuum. Lampert pointed out that the relativistic effects can be observed on reflection of waves by a moving interface if use is made of "retarding" media in which the phase velocity of waves was much less than the velocity of light in vacuum. Examples of such retarding systems are waveguides partly filled with an insulator, stopped down and helical waveguides, as well as various comb-like and interdigital structures. In media of this kind the degree of approach to relativistic motion is governed not by the ratio u/c, where u is the velocity of the interface and c is the velocity of light in vacuum, but by u/c', where c' is the phase velocity of waves in a retarding system. The latter ratio can reach unity at much lower velocities of a medium or an interface.

Lampert discussed the problem of reflection of a wave incident normally on a moving interface in a retarding system. He calculated the frequencies and amplitudes of the transmitted and reflected waves. He found that when the velocity of the interface exceeds the phase velocity of waves in a retarding medium, there is no reflected wave and instead an additional refracted wave appears behind the interface.

Very soon after the publication of Lampert's paper, Totaro reported investigations of the reflection and refraction of waves by moving interfaces between two media.[6] One of the media (containing the incident wave) was assumed to be at rest and the other, as well as the interface, were postulated to be moving at the same velocity. Totaro obtained expressions for the amplitudes, frequencies, and wave vectors of the reflected and refracted waves. In the derivation he considered the refracted wave in a reference system in which the interface and one of the media were at rest, and then applied the Lorentz transformation. The expressions obtained by Totaro are cumbersome and they hinder a physical analysis of the results. We shall show below that in this case a simpler and clearer result can be obtained by applying the mathematical apparatus of the electrodynamics of moving media.

Ideas similar to those considered by Lampert were later analyzed in detail by Faĭnberg[7] and his colleagues.[2)] They put foward the idea of an increase in the efficiency of frequency and amplitude transformations by multiple reflection of waves from a moving interface. One variant of this approach shows that a wave reflected by a moving interface and exhibiting an increase in the frequency and amplitude is "turned back" by a mirror at rest and is incident again on the same moving interface. The second reflection from the moving interface increases again the frequency and amplitude. The process can be repeated many times.

The recent years have seen many theoretical papers on the reflection and refraction of light by moving interfaces and those at rest, separating moving media (Bolotovskiĭ and Stolyarov,[8] Mergelyan,[9] and Stolyarov[10,11]). The results obtained by these authors are repeated and partly extended in a number of more recent papers usually without any reference to the preceding work (see Refs. 12–17). These papers give the phase, amplitude, and energy relationships for the interaction of electromagnetic waves with interfaces and moving media of different kinds, or they deal with a traveling wave of a parameter in a medium at rest.[18]

These theoretical relationships were checked in a number of experiments. Hey, Pinson, and Smith[19] observed a shift of the frequency of electromagnetic waves reflected by the front of a shock wave propagating in argon. The low velocity of the shock wave produced a small frequency shift (of the order of $10^{-3}\%$). In spite of the fact that the change in the frequency was very small (and, consequently, the ratio u/c of the velocity of the shock wave front to the velocity of light was also low), the experimental method used made it possible to measure accurately the shock wave velocity. Moreover, it was possible to study changes in the shock wave velocity as it traveled in a shock tube. A change in the frequency as a result of single reflection was also investigated experimentally by Zagorodnov, Faĭnberg, and Egorov.[20] In this case the reflecting surface was the front of a plasma moving at a velocity of the order of 10^7 cm/s. The experiments were carried out in a retarding medium (in the form of a helical waveguide), where the velocity of electromagnetic waves was approximately 1/200 of the velocity of light in vacuum. The relative shift of the frequency was then approximately 20% and this was confirmed experimentally. It should be pointed out that at the plasma densities attainable at the time the transparency of the plasma was high. The reflection coefficient of a moving plasma was low and, therefore, no relativistic increase in the amplitude on reflection was observed.

A change in the frequency as a result of multiple reflection by a moving plasma had been investigated experimentally on several occasions. Linhart and Ornstein[21] measured the increase in the frequency as a result of multiple reflection of a wave from the approaching walls of a vacuum cavity in a plasma. Zagorodnov, Faĭnberg, Egorov, and Bolotin[22] discussed the frequency change resulting from multiple reflection of an electromagnetic wave confined between a mirror wall and an approaching plasma front. The whole system was inside a waveguide and the reflecting wall was a junction between this waveguide and a terminating waveguide with a smaller diameter. The primary wave could not penetrate this terminating waveguide and the appearance of a wave inside it indicated an increase in the frequency. These experiments demonstrated an increase in the frequency by a factor exceeding 2 (precisely 2.3). The velocity of a plasma cylinder was 2×10^7 cm/s. Since a single reflection altered the frequency in these experiments by an amount of the order of $u/c\approx0.7\times10^{-3}$, the electromagnetic wave clearly underwent thousands of reflections.

The circumstances mentioned above (the low density and the consequently high transparency of the plasma) made the experiments involving multiple reflection more difficult to carry out because each reflection from a moving plasma reduced the wave amplitude considerably.

We have not mentioned yet any of the numerous theoretical and experimental investigations of the scattering of electromagnetic waves by single charged particles. In this case the change in the frequency of a wave due to the scattering is described by the familiar expressions from the theory of the Compton effect. If we ignore the recoil of a particle as a result of the scattering of an electromagnetic wave, the treatment becomes purely classical and we find that the frequencies and wave vectors of the incident and scattered waves obey exactly the same relationship as in the case of reflection of a wave by a moving interface. We shall now discuss this question in greater detail. Let us assume that a moving particle with a momentum $\mathbf{p}_1$ and an energy E_1 collides with a photon with a momentum $\hbar\mathbf{k}_1$ and an energy $\hbar\omega_1$ ($\mathbf{k}_1$ is the wave vector and ω_1 is the frequency of the incident photon). The scattering creates a photon with a momentum $\hbar\mathbf{k}_2$ and an energy $\hbar\omega_2$, whereas the momentum and energy of the particle become $\mathbf{p}_2$ and E_2, respectively. Let us write down the laws of conservation of energy and momentum for this process:

$$\mathbf{p}_1-\mathbf{p}_2=\hbar(\mathbf{k}_2-\mathbf{k}_1),\quad E_1-E_2=\hbar(\omega_2-\omega_1).$$

We shall multiply the first of these equations scalarly by the particle velocity $\mathbf{v}$. We then obtain

$$(\mathbf{v},\Delta\mathbf{p})=-\hbar(\Delta\mathbf{k},\mathbf{v}),$$

where $\Delta\mathbf{p}=(\mathbf{p}_2-\mathbf{p}_1)$ is the change in the momentum of a particle due to the scattering of waves and $\hbar\cdot\Delta\mathbf{k}=\hbar(\mathbf{k}_2-\mathbf{k}_1)$ is the change in the photon momentum. If the change in the particle momentum is sufficiently small, so that its motion can be regarded as uniform, we can employ the expression

$$(\mathbf{v},\Delta\mathbf{p})=\Delta E,$$

where $\Delta E=E_2-E_1$ is the change in the energy of the charged particle due to the scattering of a wave. When this expression is used, it follows from the laws of conservation that the frequencies and wave vectors of the incident and scattered waves are related by:

$$(\mathbf{k}_1,\mathbf{v})-\omega_1=(\mathbf{k}_2,\mathbf{v})-\omega_2.$$

We shall show below that an exactly the same relationship is satisfied when an electromagnetic wave is reflected or refracted by a plane interface moving at a constant velocity $\mathbf{v}$. However, there are major differences between the two cases in respect of the amplitude. The difference can be demonstrated as follows. Let us consider a monoenergetic beam of fast electrons on which a plane monochromatic wave is incident. Let the wavelengths of the incident and scattered waves be small (in a reference system in which the beam is at rest) compared with an average distance between the beam particles $N^{-1/3}$ in the same reference system (N is the beam density). Then, the separate particles in the beam scatter the wave independently and the total intensity of the scattered wave is equal to the sum of the intensities of the waves scattered by each individual particle, i.e., the intensity of the scattered wave is proportional to the density in the beam where the particle concentration is N. We are dealing with a set of independently scattering particles. The beam can be regarded as a medium if the average distance between the particles is short compared with the wavelength of the inci-

dent or scattered wave. In this case we can expect coherent scattering, i.e., the particles within a certain volume scatter in phase. Therefore, the amplitude of the scattered wave is proportional to the number of particles N per unit volume and the intensity is proportional to the square of the density N^2. This is precisely the situation when electromagnetic waves are reflected and refracted by a moving interface.[42]

We shall recall also some of the investigations of the transformation of waves in media with a traveling parameter. Theoretical and experimental investigations of these effects were carried out by the Gor'kiĭ school of radiophysicists. An example of a medium with a traveling parameter is a nonlinear medium in which a solitary pulse of a strong field (soliton) is traveling at a constant velocity. Since the permittivity or the magnetic permeability of a nonlinear medium depend on the field, it follows that in the strong field region (i.e., where at a given moment the strong field pulse is located) the values of the permittivity or the magnetic permeability differ from the values in the surrounding space. If such a pulse interacts with an electromagnetic wave, the equations for the field can be linearized in terms of this weak wave.[18,23,24] We thus have the problem of transformation (reflection or refraction) of an electromagnetic wave by a wave of a parameter and the latter wave does not change in the course of the interaction, so that it can be regarded as given. Another example of a medium with a parameter wave is a transmission line representing a section of an RLC circuit (or a line characterized by running values of R, L, and C per unit length). In a transmission line of this kind a parameter wave can be created either externally (by altering R, L, and C in accordance with a given law, in particular, in accordance with a traveling wave law) or because of a nonlinearity of the line resulting in the excitation of a strong wave.[18,24] A moving ionization front in a gas is another example of a parameter wave. On both sides of a moving front the media are at rest and the conductivity of the medium ahead of the front is zero, whereas behind the front we have an ionized gas. Daume and Freĭdman[26] investigated experimentally the reflection of a weak electromagnetic wave by a strong magnetization wave in a ferrite. The velocity of the strong wave was of the order of 10^9 cm/s. When the weak electromagnetic wave propagated opposite to the strong wave, the weak wave was reflected from the moving magnetization discontinuity. The frequency of the reflected wave should then increase by about 5–6%, as confirmed by measurements. The power of the reflected wave was 0.01% of the power of the incident wave and this was due to the strong absorption of the reflected wave in the ferrite.

We shall conclude with the following comments. Although the theoretical ideas underlying the experiments described above are not in any doubt, a large change in the frequency and the wave amplitude as a result of single reflection by a moving interface has not yet been achieved. This has been either because of a low velocity of the interface or a low density (and, consequently, a high transparency) of the reflecting medium, which in the published experiments was a relativistic electron beam or a plasma flux. Clearly, the main hope for the more effective increase in the frequency and amplitude lies in the progress being made in high-current electron and plasma accelerators.

Some idea of the current experimental possibilities is given in Refs. 48 and 49. The experiments reported in Ref. 48 achieved a sixfold increase in the frequency and doubling of the energy of a wave as a result of its reflection from the front of an electron beam with a current of the order of 2 kA and an electron energy of the order of 1 MeV. The frequency of the incident wave was 9.3 GHz (corresponding to the wavelength of $\lambda \sim 3$ mm). Similar results were reported in Ref. 49.

We shall now consider the problems of transformation of waves by moving interfaces quantitatively.

1. MAIN RELATIONSHIPS: INITIAL EQUATIONS AND BOUNDARY CONDITIONS[30]

The Maxwell equations for a homogeneous isotropic medium moving at a constant velocity $\mathbf{u}$ can be written in the form

$$\operatorname{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \operatorname{curl}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{j},$$
$$\operatorname{div}\mathbf{D} = 4\pi\rho, \quad \operatorname{div}\mathbf{B} = 0; \tag{1.1}$$

Here, $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields; $\mathbf{D}$ and $\mathbf{B}$ are the electric and magnetic inductions; ρ and $\mathbf{j}$ is the density of external charges and of the current.

The system of equations (1.1) must be supplemented by the Minkowski material equations which contain explicitly the velocity of the medium $\mathbf{u}$:

$$\mathbf{D} + \frac{1}{c}[\mathbf{u}, \mathbf{H}] = \varepsilon\left\{\mathbf{E} + \frac{1}{c}[\mathbf{u}, \mathbf{B}]\right\},$$
$$\mathbf{B} - \frac{1}{c}[\mathbf{u}, \mathbf{E}] = \mu\left\{\mathbf{H} - \frac{1}{c}[\mathbf{u}, \mathbf{D}]\right\}, \tag{1.2}$$

where ε and μ are the permittivity and magnetic permeability measured in a reference system in which the medium is at rest. Equations (1.1) and (1.2) allow us to determine completely the field in a homogeneous moving medium.

We shall now consider two media separated by an interface. We shall assume that one of these media is characterized by the parameters ε_1 and μ_1 and the velocity of its motion is $\mathbf{u}_1$. The fields in this medium will be identified by the index 1: $\mathbf{E}_1$, $\mathbf{H}_1$, $\mathbf{B}_1$, $\mathbf{D}_1$. The second medium is characterized by ε_2 and μ_2 and by the velocity $\mathbf{u}_2$. The fields in this medium will be identified by the index 2: $\mathbf{E}_2$, $\mathbf{H}_2$, $\mathbf{B}_2$, $\mathbf{D}_2$. The velocity of the interface between these two media is $\mathbf{v}$. We shall assume that the velocity $\mathbf{v}$ is generally different from the velocities $\mathbf{u}_1$ and $\mathbf{u}_2$ of the media on both sides of the interface.

In the presence of interfaces the systems of equations (1.1) and (1.2) have to be supplemented by the boundary conditions for the fields and inductions. In the monograph of Landau and Lifshitz[47] these boundary conditions are obtained on the assumption that the interface and media on both sides of it travel at the same velocity. The boundary conditions can then be derived by a simple (Lorentz) transformation of the familiar boundary conditions for a reference system in which the interface and the media are at rest. In general, when the velocities of the interface and the media on both sides of it are different, a detailed derivation of the boundary conditions can be made on the basis of a paper by Costen and Adamson.[41] The boundary conditions are found to contain only the velocity of the interface, but not the velocities of the media. Consequently, both approaches give

the same boundary conditions, which in the absence of surface charges or currents become

$$[\mathbf{n}, \mathbf{E}_2 - \mathbf{E}_1] = \frac{v_n}{c}(\mathbf{B}_2 - \mathbf{B}_1),$$
$$[\mathbf{n}, \mathbf{H}_2 - \mathbf{H}_1] = -\frac{v_n}{c}(\mathbf{D}_2 - \mathbf{D}_1), \tag{1.3}$$

where v_n is the projection of the velocity of the interface along the normal $\mathbf{n}$ to it.

The projections of these vector equalities along the coordinate axes give six boundary conditions. In particular, the conditions of continuity of the normal components of the vectors $\mathbf{B}$ and $\mathbf{D}$ are obtained if we use the projections along the normal to the interface.

We must bear in mind that if the interface is not plane, the boundary conditions of Eq. (1.3) are local since the direction of the normal to the interface $\mathbf{n}$ is different at different points. This makes it difficult to find the general solutions of the boundary-value problems. In the next sections we shall discuss mainly the transformation of waves on plane interfaces.

2. BOUNDARY CONDITIONS FOR PLANE WAVES. PHASE INVARIANTS

We shall discuss two homogeneous moving media, which are isotropic in the system at rest. The velocity of the first medium will be denoted by $\mathbf{u}_1$, and its permittivity and magnetic permeability in the system at rest by ε_1 and μ_1. Similar quantities for the second medium will be denoted by $\mathbf{u}_2$, ε_2, and μ_2, respectively. We shall assume that the two media are separated by a plane interface and that this interface is traveling, parallel to itself, at a constant velocity $\mathbf{v}$ directed at right-angles to the plane of the interface.

We shall assume that a plane electromagnetic wave incident on the interface of a medium described by ε_1, μ_1, and $\mathbf{u}_1$ (we shall call it the first medium) is given by

$$\mathrm{E}_{\mathrm{In}} = \mathrm{E}_0 \exp[i(\mathbf{k}_0\mathbf{r} - \omega_0 t)]. \tag{2.1}$$

Then the field in the first medium is the sum of the incident wave described by Eq. (2.1) and of a reflected wave, which is

$$\mathbf{E}_r = \mathrm{E}_{1,1} \exp[i(\mathbf{k}_{1,1}\mathbf{r} - \omega_{1,1} t)]. \tag{2.2}$$

The total field $\mathbf{E}_1$ in the first medium is therefore

$$\mathrm{E}_1 = \mathbf{E}_0 \cdot \exp[i(\mathbf{k}_0\mathbf{r} - \omega_0 t)] + \mathrm{E}_{1,1} \exp[i(\mathbf{k}_{1,1}\mathbf{r} - \omega_{1,1} t)]. \tag{2.3}$$

The representation of the field in the first medium by just two waves, incident and reflected, is not self-evident. In fact, in the presence of spatial dispersion more than one reflected wave may exist in a medium at rest. However, for the sake of simplicity we shall assume that the media on both sides of the interface do not exhibit spatial dispersion in a reference system in which they are at rest. Then, we shall show that on each side of the interface there are no more than two waves. Therefore, the field $\mathbf{E}_2$ in the second medium can be represented by

$$\mathbf{E}_2 = \mathbf{E}_{2,1} \exp[i(\mathbf{k}_{2,1}\mathbf{r} - \omega_{2,1} t)] + \mathrm{E}_{2,2} \exp[i(\mathbf{k}_{2,2}\mathbf{r} - \omega_{2,2} t)], \tag{2.4}$$

where $\mathbf{E}_{2,i}$ is the amplitude of the ith wave (where $i = 1$ or 2), whereas ω_2 and $\mathbf{k}_{2,i}$ are the frequencies and wave vectors of these waves.

Some of the amplitudes $\mathbf{E}_{1,1}$, $\mathbf{E}_{2,1}$, and $\mathbf{E}_{2,2}$ in general vanish, depending on the physical conditions. In particular, the amplitude $\mathbf{E}_{1,1}$ of the reflected wave may vanish if the amplitudes of the transmitted waves $\mathbf{E}_{2,i}$ are nonzero. Conversely, one of the amplitudes $\mathbf{E}_{2,i}$ of the transmitted waves may vanish for a nonzero amplitude $\mathbf{E}_{1,1}$ of the reflected wave. The conditions under which a particular solution is realized will be considered in detail later. The fields (2.3) and (2.4) should satisfy the boundary conditions of Eq. (1.3). This is achieved if all the "matched" waves at each point on the interface have the same phase.

We shall write down the equations for a moving plane containing an interface:

$$(\mathrm{r}, \mathrm{n}) = vt, \tag{2.5}$$

where $\mathbf{n}$ is the normal to the interface and v is the magnitude of the velocity of the interface in the direction of its normal. It is assumed that the moving plane crosses the origin of the coordinate (reference) system at $t = 0$.

We shall now consider a plane monochromatic wave characterized by

$$\exp[i(\mathbf{k}\mathbf{r} - \omega t)]. \tag{2.6}$$

We shall write down the wave vector $\mathbf{k}$ in the form of a vector sum of two terms in which the tangential component $\mathbf{k}_t$ is parallel to the interface and the normal component $\mathbf{k}_n$ is perpendicular to it. Obviously, we can write down

$$\mathbf{k} = \mathbf{k}_t + \mathbf{k}_n = \mathbf{k}_t + k_n\mathbf{n}. \tag{2.7}$$

Substitution of this expression into Eq. (2.7) subject to allowance for Eq. (2.5) gives the phase of the plane wave on the moving interface

$$\exp[i(\mathbf{k}_t, \mathbf{r}_t)] \exp[i(k_n(\mathbf{n}, \mathbf{v}) - \omega) t]$$
$$= \exp[i(\mathbf{k}_t, \mathbf{r}_t)] \exp[i(\mathbf{k}, \mathrm{v}) - \omega) t], \tag{2.8}$$

where $\mathbf{r}_t$ is the vector in the plane of the interface.

If the phases of all the "matched" waves are represented in this form at the interface and if we assume that the phases are equal there, we obtain

$$\mathbf{k}_{0t} = \mathbf{k}_{1,1t} = \mathbf{k}_{2,1t} = \mathbf{k}_{2,2t} = \mathbf{I}_t. \tag{2.9}$$

This condition represents the equality of the tangential components of the wave vectors of all the waves interacting at the interface and we find that

$$I_1 = (\mathbf{k}_0, \mathrm{v}) - \omega_0 = (\mathbf{k}_{1,1}, \mathrm{v}) - \omega_{1,1}$$
$$= (\mathbf{k}_{2,1}, \mathbf{v}) - \omega_{2,1} = (\mathbf{k}_{2,2}, \mathrm{v}) - \omega_{2,2}. \tag{2.10}$$

This condition represents equality of the frequencies of all the interacting waves in a reference system in which the interface is at rest. This can be easily demonstrated by assuming that the velocity of the interface is zero in Eq. (2.10).

In this reference system both media generally travel at different velocities. Hence, it is clear that the frequency transformation at moving interfaces is entirely due to the motion of the interface itself and not due to the motion of the media on both sides of it. In particular, the frequency transformation occurs when a wave of some parameter of the me-

dium is traveling in a medium at rest (this parameter may be the density, magnetization, etc.[18,24]).

If the incident wave is given, i. e., if we know the frequency ω_0 and the components of the wave vector $\mathbf{k}_0$, we find that the relationships (2.9) and (2.10) allow us to determine the frequencies and wave vectors of all the other waves. For this purpose we need to use not only the relationships given above, but also the dispersion equations for each of the moving media. We shall consider, for example, the dispersion equations for waves in the first medium[30]:

$$\mathbf{k}^2 - \frac{\omega^2}{c^2} - \frac{\varkappa_1}{c^2}\frac{(\omega - \mathbf{k}\mathbf{u}_1)^2}{1-(u_1^2/c^2)} = 0. \tag{2.11}$$

For a given incident wave, we know the following quantities in the above equation: the tangential component of the wave vector $\mathbf{k}_t = \mathbf{I}_t$ and the combination $I_1 = (k_n v - \omega)$, where $k_n = (\mathbf{k}, \mathbf{v})$ and $\mathbf{n}$ is a unit vector along the normal to the interface, which we shall regard as directed from the first to the second medium. Substituting $\mathbf{k}_t = \mathbf{I}_t$ and $k_n = (\omega + I_1)/v$ into Eq. (2.11), we obtain a quadratic equation for the frequency ω and the solution of this equation is

$$(\omega_1)_{1,2} = (-I_1)\frac{[1+\varkappa_1\gamma_1^2(\beta-\beta_{1n})(\beta_{1n}+\beta(\boldsymbol{\beta}_{1t},\mathbf{d}))] \pm \beta Q_1^{1/2}}{[(1-\beta^2)-\varkappa_1\gamma_1^2(\beta-\beta_{1n})^2]}, \tag{2.12}$$

$$Q_1 = \{[1+\varkappa_1\gamma_1^2(1-\beta_{1n}^2)] - \mathbf{d}^2[(1-\beta^2)-\varkappa_1\gamma_1^2(\beta-\beta_{1n})^2] + \varkappa_1\gamma_1^2(\mathbf{d},\boldsymbol{\beta}_{1t})[2(1-\beta\beta_{1n}) + (1-\beta^2)(\mathbf{d},\boldsymbol{\beta}_{1t})]\},$$

where

$$\mathbf{d} = c\frac{\mathbf{I}_t}{I_1}, \quad \varkappa_1 = (\varepsilon_1\mu_1 - 1), \quad \beta = \frac{v}{c}, \quad \beta_{1n} = \frac{u_{1n}}{c},$$

$$\boldsymbol{\beta}_{1t} = \frac{\mathbf{u}_{1t}}{c}, \quad \gamma_1 = \frac{1}{(1-\beta_1^2)^{1/2}}, \quad \beta_1^2 = \frac{\mathbf{u}_1^2}{c^2} = \beta_{1n}^2 + \beta_{1t}^2,$$

and u_{1n} and $\mathbf{u}_{1t}$ are, respectively, the normal and tangential (to the interface) components of the velocity of the first medium.

Equation (2.12) gives two values of the frequency in the first medium, which are expressed in terms of the invariants I_1 and $\mathbf{I}_t$, and also in terms of the parameter of the medium $(\varepsilon_1, \mu_1, \mathbf{u}_1)$ and the velocity of the interface v. One frequency is ω which governs the frequency of the incident wave, and the other the frequency of the reflected wave. Knowing the frequency $(\omega_1)_{1,2}$ we find from Eq. (2.10) the following values of the normal component of the wave vector

$$(k_{1n})_{1,2} = [(\omega_1)_{1,2} + I_1]v^{-1} = \frac{(-I_1)}{c}\frac{[\beta+\varkappa_1\gamma_1^2(\beta-\beta_{1n})(1+(\boldsymbol{\beta}_{1t},\mathbf{d}))] \pm Q_1^{1/2}}{(1-\beta^2)-\varkappa_1\gamma_1^2(\beta-\beta_{1n})^2}. \tag{2.13}$$

It is therefore clear that the relationships (2.9), (2.10), (2.12), and (2.13) determine completely the values of the frequency and the wave vector for waves in the first medium. The relationships for the waves in the second medium are exactly the same if we replace ε_1 with ε_2, μ_1 with μ_2, and $\mathbf{u}_1$ with $\mathbf{u}_2$.

For given values of the invariants I_1 and $\mathbf{I}_t$ the dispersion equations for each of the media in contact have only two solutions (we recall that this is true if we neglect the dispersion of the moving media). This justifies the representation of the total field on both sides of the interface as a superposition of two waves described by Eqs. (2.3) and (2.4).

In the derivation of the relationships given by Eqs. (2.12) and (2.13) and describing the frequencies and components of the wave vectors we had assumed that the velocities of the media $\mathbf{u}_1$ and $\mathbf{u}_2$ on both sides of the interface are oriented in an arbitrary manner. Such a general formulation of the problem leads to very complex expressions not only for the frequencies and wave vectors, but also for the field amplitudes. The physical features of the reflection and refraction of the waves by an interface between two moving media are best identified by considering the example of two special cases which are in fact those discussed usually in the literature.

In the case of an interface in the form of a normal discontinuity the velocities of the media on both sides are directed along the normal to the interface. We shall call this the case of a normal velocity discontinuity. It is realized, for example, in shock waves. The interface is then the front of a shock wave and the velocity of the interface v is the velocity of the front of this wave. The velocity u_1 can then be assumed to be the velocity of matter in front of the shock wave and the velocity u_2 is the velocity of matter behind the front. Naturally, all these velocities should satisfy the condition of continuity of matter across the interface. Another example of a normal discontinuity is the propagation of a strong field pulse in a nonlinear medium at rest. In this case the role of the moving interface is performed by the front (leading edge) of a pulse. In view of the nonlinearity of the medium, its electromagnetic properties are different on either side of the front.

In the case of a normal discontinuity of the velocity the expressions for the frequencies and wave vectors are simpler. These expressions can be obtained from Eqs. (2.12) and (2.13) by assuming that $\beta_{1t} = 0$. We then obtain the following expressions for the first medium:

$$(\omega_1)_{1,2} = (-I_1)\frac{[1+\varkappa_1\beta_1\gamma_1^2(\beta-\beta_1)] \pm \beta Q_1^{1/2}}{(1-\beta^2)-\varkappa_1\gamma_1^2(\beta-\beta_1)^2},$$

$$Q_1 = \{(1+\varkappa_1) - \mathbf{d}^2[(1-\beta^2)-\varkappa_1\gamma_1^2(\beta-\beta_1)^2]\}, \quad \mathbf{d} = c\frac{\mathbf{I}_t}{I_1}, \tag{2.14}$$

$$(k_{1n})_{1,2} = \frac{(-I_1)}{c}\frac{[\beta+\varkappa_1\gamma_1^2(\beta-\beta_1)] \pm Q_1^{1/2}}{[(1-\beta^2)-\varkappa_1\gamma_1^2(\beta-\beta_1)^2]}.$$

These expressions give the frequency and the component of a wave vector k_{1n} for two possible waves in the first medium. They are expressed in terms of the invariants I_1 and $\mathbf{I}_t$, and in terms of the parameters of the first medium and of the interface. If the plus sign in front of the square root is selected in the system (2.14), the frequency and wave vector of the incident wave are obtained. Selection of the minus sign gives the frequency and the wave vector of the reflected wave (we recall that the tangential component of the wave vector of the reflected wave is identical with the tangential component of the wave vector of the incident wave).

The corresponding expressions describing the frequencies and the wave vectors of two waves in the second medium are obtained from the expressions in Eq. (2.14) by the replacement of $\varkappa_1$, β_1, and γ_1 with $\varkappa_2$, β_2, and γ_2.

Next, we shall consider some special cases and discuss in detail the expressions obtained for the frequency and wave vector of the reflected and refracted waves. We shall begin with the problem of determination of the amplitudes $E_{1,1}$,

$E_{2,1}$, and $E_{2,2}$. These amplitudes are found from the boundary conditions. If, for the sake of simplicity, we assume that no surface charges or currents appear at the interface, the boundary conditions are given by the system (1.3). We shall consider specifically that the interface is plane and the velocity of its motion is along the normal to the interface. We shall also assume that the velocities of the media on both sides of the interface are again directed along the normal to the interface. Finally, for the sake of simplicity, we shall consider the case of normal incidence of the waves, i.e., we shall assume that the vectors $\mathbf{k}_0$, $\mathbf{k}_{1,1}$, $\mathbf{k}_{2,1}$, and $\mathbf{k}_{2,2}$ are all perpendicular to the interface. This relatively simple case is discussed in Refs. 18 and 32. We shall follow the analysis given in these two papers.

Let us assume that a plane wave of amplitude E_0 is incident from a medium 1 opposite to a moving interface. An analysis of the phase relationships shows that a reflected wave may exist in the medium 1 and two refracted waves in a medium 2. We can determine the amplitudes of these waves by using the boundary conditions of Eq. (1.3). In the case under discussion (normal incidence of the plane of the wave) the boundary conditions of (1.3) yield only two independent equations. We can show this by considering the Maxwell equations (1.1) in the absence of charges and currents:

$$\operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \operatorname{div} \mathbf{D} = 0,$$
$$\operatorname{curl} \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \quad \operatorname{div} \mathbf{B} = 0, \tag{2.15}$$

for plane monochromatic waves of the type described by Eq. (2.6). Hence, the vector amplitudes **E**, **B**, **H**, and **D** are described by

$$\mathbf{B} = \left[\frac{c\mathbf{k}}{\omega}, \mathbf{E}\right], \quad (\mathbf{k}, \mathbf{D}) = 0,$$
$$\mathbf{D} = -\left[\frac{c\mathbf{k}}{\omega}, \mathbf{H}\right], \quad (\mathbf{k}, \mathbf{B}) = 0. \tag{2.16}$$

We shall substitute these values of **B** and **D** into the boundary conditions given by Eq. (1.3). We then obtain the conditions of continuity for the following two quantities:

$$(\omega - \mathbf{kv})\frac{E_t}{\omega} \text{ and } (\omega - \mathbf{kv})\frac{H_t}{\omega}. \tag{2.17}$$

We recall that these conditions of continuity are valid in the specific case when the wave vector **k** is perpendicular to the interface.

Since the quantity $(\omega - \mathbf{kv})$ has the same value for all the waves interacting at the interface, we can obtain the conditions of continuity of the quantities E_t/ω and H_t/ω at the interface. Thus, in the case of normal incidence of a plane wave we obtain just two conditions for finding three waves $E_{1,1}$, $E_{2,1}$, and $E_{2,2}$. Therefore, this problem can be solved if we invoke some additional conditions. One of them, which makes it possible to obtain a unique solution of the boundary-value problem in a number of cases, is the radiation principle. According to this principle,[1] we have to select those of the reflected and refracted waves which can carry energy away from a moving interface. If one of the secondary waves (reflected or refracted) does not satisfy this requirement, i.e., if it does not carry energy away from the interface, the boundary-value problem becomes uniquely determinate. This is the situation observed in two extreme cases.

In the first case, which we shall call subluminal, the velocity of the interface is less than the group velocities of the waves in the two media next to the interface. Then, out of the two refracted waves $E_{2,1}$ and $E_{2,2}$ we have to retain that for which the group velocity is directed away from the interface. Consequently, in the subluminal case we have just one reflected wave and one refracted wave.

In the second case (which we shall call superluminal) the velocity of an interface exceeds the group velocities of the waves in both media. We shall then assume that the interface moves opposite to the incident wave. In the superluminal case there is no reflected wave, but there are two refracted waves behind the interface.

The intermediate case when the velocity of an interface lies between the group velocities of the two media is more complex. The boundary-value problems must then be solved subject not only to the radiation principle, but also subject to additional assumptions about the structure of the interface, stability of the interface, role of nonlinearities, etc.[23-25,32,33] Allowance for these circumstances may, in particular, have the effect that on both sides of the interface we have to allow for either of the possible solutions. Therefore, on one side of the interface the total field consists of the incident and reflected waves, whereas on the other side it consists of two refracted waves.

In determination of the frequency and wave vector of the reflected and refracted waves we assumed above that the values of ε and μ are constant, independent of the frequency ω, and of the wave vector **k**. In other words, we ignored the dispersion. If the dispersion is allowed for, the number of solutions of Eq. (2.11) for given invariants $\mathbf{I}_i$ of Eq. (2.9) and I_1 of Eq. (2.10) can generally be more than two. The amplitudes of the new waves can be determined if we have some additional boundary conditions, because the number of the boundary conditions given by Eq. (1.3) is then generally insufficient.

If the wave is incident obliquely on a moving interface, once again all the discussion given above remains valid. However, we must bear in mind that the concepts of the subluminal and superluminal cases, etc., are no longer governed by the absolute values of the group velocities, but by their projections along the normal to the interface.

3. REFLECTION OF WAVES FROM A MOVING MIRROR, FROM AN INSULATOR, AND FROM A TRAVELING PARAMETER

The problem of the reflection of light from a moving mirror was first discussed by the well-known paper of Einstein, published in 1905 (Ref. 2), where he formulated the fundamentals of the special theory of relativity. The solution of the problem in question was given by Einstein in two stages. First, the characteristics of an incident electromagnetic wave (frequency, angle of incidence, and amplitude) were transformed to a reference system in which the mirror is at rest and this system was used to find the quantities describing the reflected wave. Einstein followed this by transformation back to the laboratory coordinate system. The approach presented in the preceding sections can be used readily to solve this problem directly in the laboratory

coordinate system without the need to transform from one inertial reference system to another.

We shall consider a mirror moving in vacuum at a constant velocity v and assume that this velocity is directed at right-angles to the mirror plane. We shall introduce a Cartesian coordinate system (x, y, z) in such a way that the z axis is directed along the normal to the mirror plane. We shall assume that the mirror is moving in the negative direction of the z axis and that its velocity is v. Therefore, the position of the mirror at a moment t is described by the equation $z = -vt$ (Fig. 1). We shall assume that a plane electromagnetic wave of Eq. (2.1) is incident on this mirror:

$$\mathbf{E}_{\text{in}} = \text{Re}\,\{\mathbf{F}_0 \exp[i(\mathbf{k}_0\mathbf{r} - \omega_0 t)]\}. \tag{3.1}$$

Here, the symbol Re denotes the real part of the expression following it. Therefore, the field $\mathbf{E}_{\text{in}}$ can be written in the form

$$\mathbf{E}_{\text{in}} = \frac{1}{2}\{\mathbf{E}_0 \exp[i(\mathbf{k}_0\mathbf{r} - \omega_0 t)] + \mathbf{E}_0^* \exp[-i(\mathbf{k}_0\mathbf{r} - \omega_0 t)]\}.$$

We shall select the x and y axes of the Cartesian system in such a way that the vector $\mathbf{k}_0$ lies in the (x, z) plane which we shall call the plane of incidence (Fig. 1). In the case of a perfectly reflecting mirror there is no transmitted wave. Therefore, in solving the boundary-value problem it is sufficient to allow only for the reflected wave, which can be described by

$$\mathbf{E}_r = \text{Re}\,\{\mathbf{E}_1 \exp[i(\mathbf{k}_1\mathbf{r} - \omega_1 t)]\}. \tag{3.2}$$

It follows from the boundary conditions of Eq. (2.9) that the wave vector $\mathbf{k}_1$ of the reflected wave lies in the plane of incidence (x, z).

We shall first determine the wave vector $\mathbf{k}_1$ and the frequency ω_1 of the reflected wave. This can be done using the following relationships:

$$k_0 = \frac{\omega_0}{c}, \quad k_1 = \frac{\omega_1}{c}, \tag{3.3}$$

$$I_t = k_0 \sin\vartheta_0 = k_1 \sin\vartheta_1, \tag{3.4}$$

$$(-I_1) = (\omega_0 + k_0 v \cos\vartheta_0) = (\omega_1 - k_1 v \cos\vartheta_1); \tag{3.5}$$

here, ϑ_0 is the angle of incidence and ϑ_1 is the angle of reflection (Fig. 1). The relationships given by Eq. (3.3) represent the dispersion equations for the propagation of light in vacuum. They follow from Eq. (2.11) if we assume that $\varepsilon_1 = \mu_1 = 1$ $(\varkappa_1 = 0)$. The relationships given by Eq. (3.4) are the consequences of Eq. (2.9), whereas the relationship (3.5) follows from the expressions in Eq. (2.10).

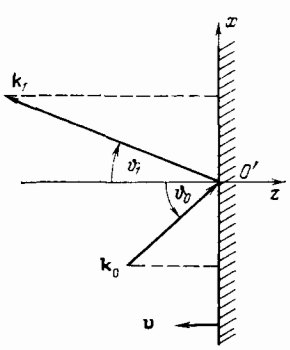

FIG.

Using Eqs. (3.3)–(3.5), we can express the frequency of the reflected wave ω_1 and the angle of reflection ϑ_1 in terms of ω_0 and ϑ_0, respectively:[3)]

$$\omega_1 = \omega_0 \frac{[1 + (v^2/c^2)] + 2(v/c)\cos\vartheta_0}{1 - (v^2/c^2)}, \tag{3.6}$$

$$\sin\vartheta_1 = \frac{\omega_0}{\omega_1}\sin\vartheta_0 = \frac{[1 - (v^2/c^2)]\sin\vartheta_0}{1 + (v^2/c^2) + 2(v/c)\cos\vartheta_0}. \tag{3.7}$$

These expressions are interesting not only in connection with the problem under discussion, which is the reflection of waves from a moving mirror, but also give some idea on the properties of the reflected waves in more complex cases. In particular, we shall show that these expressions are in a sense valid also in those cases when the wave passes partly through a moving interface or a mirror is moving in a refracting medium. In the latter case the quantity c is the velocity of light in the medium under consideration.

We shall now discuss Eqs. (3.6) and (3.7). If the mirror is moving opposite to the incident wave $(v > 0)$, it follows from Eq. (3.6) that the frequency of the reflected wave ω_1 exceeds the frequency ω_0 of the incident wave. If the incident wave catches up with the mirror $(v < 0)$, the frequency of the reflected wave is always less than that of the incident wave. Equation (3.6) becomes particularly simple in the normal incidence case $(\vartheta_0 = 0)$:

$$\omega_1 = \omega_0 \frac{1 + (v/c)}{1 - (v/c)}. \tag{3.8}$$

It is clear from Eqs. (3.6) and (3.8) that at relativistic velocities of a mirror the reflection changes the frequency considerably.

Equation (3.8) can be derived on the basis of simple physics. Let us assume that the wave incident on a moving mirror can be described by

$$\exp[i(k_0 z - \omega_0 t)], \quad k_0 = \frac{\omega_0}{c}.$$

At the point $z = -vt$, where the mirror is located, the phase of this wave is

$$-(k_0 v + \omega_0)t = -\omega_0\left(1 + \frac{v}{c}\right)t.$$

This means that the field on the mirror surface oscillates at a frequency

$$\omega' = \omega_0\left(1 + \frac{v}{c}\right).$$

Therefore, the mirror is a moving oscillator oscillating at the frequency ω'. This oscillator emits in the forward direction a wave whose frequency is shifted relative to ω' in accordance with the Doppler effect[34]

$$\omega_1 = \frac{\omega'}{1 - (v/c)} = \omega_0 \frac{1 + (v/c)}{1 - (v/c)}.$$

This relationship is identical with Eq. (3.8). The same relationships can be derived by the Lorentz transformation using the results of the conventional problem of the reflection of light by a mirror at rest. In fact, if an electromagnetic wave described by

$$\exp[i(kz - \omega t)]$$

is incident along the normal on a mirror at rest, the reflected wave is proportional to

$$\exp[-i(kz+\omega t)];$$

since the frequencies of the incident and reflected waves are in this case identical while the directions of propagation are opposite. We shall consider the same problem in a system of coordinates moving at a velocity v in the positive direction of the z axis. In this system the mirror velocity is $-v$, whereas the frequencies of the incident and reflected waves are, respectively,

$$\omega_1' = \frac{\omega - kv}{[1-(v^2/c^2)]^{1/2}} \quad \text{and} \quad \omega_2' = \frac{\omega + kv}{[1-(v^2/c^2)]^{1/2}}.$$

Therefore, the ratio of the frequencies of the incident ω_1' and reflected ω_2' waves in the system in which the mirror moves toward the incident wave at the velocity v is

$$\frac{\omega_1'}{\omega_2'} = \frac{\omega - kv}{\omega + kv} = \frac{1-(v/c)}{1+(v/c)},$$

since $k = \omega/c$. The above expression is identical with Eq. (3.8).

The angular dependence of the reflected radiation is described by Eq. (3.7). If the mirror moves toward the incident wave ($v>0$) the angle of reflection ϑ_1 is always less than the angle of incidence ϑ_0. Irrespective of the value of ϑ_0, the angle ϑ_1 tends to zero as the mirror velocity approaches the velocity of light. If the mirror moves in the same direction as the incident wave ($v<0$), the angle of reflection ϑ_1 is always greater than the angle of incidence. In this case ($v<0$) there is a range of the angles of incidence ϑ_0 and the velocities of the mirror v in which the angle of reflection ϑ_1 exceeds $\pi/2$. This can be demonstrated by writing down the expression for $\cos\vartheta_1$, which follows from Eq. (3.7) for $\sin\vartheta_1$ when $v<0$:

$$\cos\vartheta_1 = \frac{(1+\beta^2)\cos\vartheta_0 - 2\beta}{(1+\beta^2) - 2\beta\cos\vartheta_0}, \tag{3.9}$$

where $\beta = v/c > 0$. The denominator in the above expression is always positive and the numerator changes its sign when $\cos\vartheta_0 = 2\beta/(1+\beta^2)$. If the angle of incidence ϑ_0 becomes greater than $\cos^{-1}[2\beta/(1+\beta^2)]$, we find from Eq. (3.9) that the angle of reflection ϑ_1 becomes greater than $\pi/2$. It is then found that the wave vector of the reflected wave meets at an acute angle with the direction of the mirror motion. The reflected wave propagates behind the mirror, but does not catch up with it. In fact, the projection of the phase velocity of the reflected wave along the normal to the mirror surface (along the z axis) is in this case ($\cos\vartheta_1<0$) equal to $c|\cos\vartheta_1|$ (Fig. 1). Using Eq. (3.9), we can readily show that if

$$\beta \leqslant \cos\vartheta_0 \leqslant \frac{2\beta}{1+\beta^2} \tag{3.10}$$

we find that

$$v - c|\cos\vartheta_1| \geqslant 0, \tag{3.11}$$

i.e., the projection of the phase and group velocities (which are equal in our case because waves propagate in free space) of the reflected wave along the direction of the mirror motion is positive and its absolute value is less than the mirror velocity. This means that the reflected wave still carries energy away from the moving mirror.

The left-hand side of the inequality (3.10) means that the incident wave delivers energy to the mirror moving away from it ($c\cos\vartheta_0 \geqslant v$), which is exactly as expected.

We shall now calculate the reflected wave amplitude. We shall consider two cases discussed separately below.

a) The electric vector of the incident wave is perpendicular to the plane of incidence, i.e., $\mathbf{E}_0 = E_0\mathbf{e}_y$, where $\mathbf{e}_y$ is a unit vector in the direction of the y axis (Fig. 1). Then, the reflected wave has the same polarization as the incident wave, i.e., $\mathbf{E}_1 = E_1\mathbf{e}_y$. We shall use the first of the boundary conditions in Eq. (1.3) and express the magnetic induction $\mathbf{B}$ in this condition using the field $\mathbf{E}$ taken from the Maxwell system of equations (2.16) for plane monochromatic waves. If the first boundary condition of Eq. (1.3) is projected along the normal to the interface, we obtain

$$\frac{k_{0x}}{\omega_0}E_0 + \frac{k_{1x}}{\omega_1}E_1 = 0. \tag{3.12}$$

If we now consider the projection of the same boundary condition on the mirror plane, we obtain

$$\frac{\omega_0 - \mathbf{k}_0\mathbf{v}}{\omega_0}E_0 + \frac{\omega_1 - \mathbf{k}_1\mathbf{v}}{\omega_1}E_1 = 0. \tag{3.13}$$

These relationships can be simplified using the fact that $(\omega_0 - \mathbf{k}_0\mathbf{v}) = (\omega_1 - \mathbf{k}_1\mathbf{v})$ and $k_{0x} = k_{1x}$ [see Eqs. (2.9) and (2.10) for the invariants I_1 and $\mathbf{I}_t$]. Therefore, both Eqs. (3.12) and (3.13) give the same ratio which can be used to find the amplitude of the reflected wave:

$$\frac{E_1}{\omega_1} = -\frac{E_0}{\omega_0}. \tag{3.14}$$

It follows from the above ratio that the amplitudes of the incident and reflected waves differ by the same factor as the corresponding frequencies. Using Eq. (3.6), we readily obtain

$$E_1 = -\frac{\omega_1}{\omega_0}E_0 = -\frac{(1+\beta^2)+2\beta\cos\vartheta_0}{1-\beta^2}E_0. \tag{3.15}$$

We recall that the mirror velocity $v = c\beta$ is positive in the case when the mirror moves opposite to the wave.

b) The magnetic vector of the incident wave is perpendicular to the plane of incidence, i.e., $\mathbf{H}_0 = H_0\mathbf{e}_y$ (Fig. 1). In this case the second boundary condition of Eq. (1.3) yields two relationships which can be obtained from Eqs. (3.12) and (3.13) if we replace E_0 with H_0 and E_1 with H_1. These relationships yield the amplitude H_1 of the magnetic vector of the reflected wave:

$$H_1 = -\frac{(1+\beta^2)+2\beta\cos\vartheta_0}{1-\beta^2}H_0. \tag{3.16}$$

The relationships given by Eqs. (3.15) and (3.16) are similar although they apply to cases of different polarizations of the incident wave. This is not surprising because in vacuum the amplitudes of the electric and magnetic fields in a plane monochromatic wave are identical.

It is clear from Eqs. (3.6) and (3.14)–(3.16) that reflection of an electromagnetic wave from a mirror moving in vacuum transforms its frequency and amplitude in accordance with the same law.

We considered here the laws of reflection from a mov-

ing ideal mirror. If behind a moving interface we replace the reflecting metal with a transparent insulator, the expressions for the frequencies and directions of propagation of the reflected wave remain unaffected. However, the amplitude of the reflected wave is now different from that given by Eqs. (3.15) and (3.16). In the simplest case when the wave is incident on an interface along the normal and the moving medium behind the interface exhibits no dispersion, the expressions (3.15) and (3.16) for the amplitude of the reflected wave now have an additional factor equal to the Fresnel reflection coefficient of an insulator at rest. In the case of oblique incidence these expressions become more complex.[32]

We mentioned earlier (in the Introduction) a system with a parameter wave traveling in a medium at rest. The velocity of a parameter wave can have any value and, in particular, it can be higher than the velocity of light in the medium or even in vacuum. We shall consider this possibility by discussing an example of a transmission line representing a chain of cells with given values of the parameters, which can be the capacitance, resistance, and inductance. Such a transmission line is characterized by a definite velocity of a signal traveling along it. The parameters of each of the cells can be varied by an external agency. We shall consider the simplest case when a transmission line is composed of cells with the same parameters, for example, when the capacitance of each cell is C_0. We shall assume that at some initial moment the capacitance of the first cell changes from C_0 to C_1, whereas the capacitances of the other cells are unaffected. After a time interval Δt the capacitance of the first cell is restored to its initial value C_0 and at the same moment the capacitance of the second cell is altered from C_0 to C_1. After the same time interval Δt, the capacitance of the second cell recovers its initial value C_0 and the capacitance of the next (third) cell is altered from C_0 to C_1, and so on. Obviously, a parameter wave travels along this transmission line and in this case the capacitance is the parameter. The velocity of this wave can have any value and, in particular, if Δt tends to zero, the parameter wave velocity tends to infinity because in this case the capacitance of all the cells in the line changes simultaneously. Obviously, a parameter wave can be created not only in a transmission line, but also in a continuous medium and this can be done by, for example, altering the refractive index of the medium with the aid of the Kerr effect in a strong external electromagnetic field.

Any plane monochromatic wave propagating in a medium at rest where there is a traveling parameter will be transformed at the traveling discontinuity of the parameter by partly passing across the discontinuity and partly becoming reflected by it. Expressions for the amplitudes of the transformed waves can be found in Ref. 32 for some specific laws governing variation of the traveling parameter. We shall not consider these results in detail, but we shall discuss one important feature of the transformation of waves by a discontinuity of a traveling parameter. If the velocity of the parameter is less than the velocity of waves in a medium at rest, then on one side of the parameter discontinuity there are two waves (incident and reflected), whereas on the other there is only one (transmitted) wave. The frequencies and components of the wave vectors of the reflected and transmitted waves can be found from Eqs. (2.12) and (2.13) for the reflected wave and from the corresponding equations for the transmitted wave [see the discussion after Eq. (2.13)]. In the case of the transmitted wave, expressions of the type given by Eqs. (2.12) and (2.13) yield two possible solutions. We have to select that solution which corresponds to the removal of energy from a moving parameter discontinuity. Usually this solution is in the form of a wave traveling in the same direction as the incident wave. The amplitudes of the reflected and transmitted waves can be found from the condition of continuity of Eq. (2.17) at the moving parameter discontinuity.

When the velocity of a parameter discontinuity exceeds the velocity of waves in both media, the set of solutions described above becomes invalid. We shall assume that the parameter discontinuity travels toward the incident wave and the velocity v of this discontinuity is greater than the velocity of light c/n_1 in the medium at rest in front of the discontinuity and greater than the velocity c/n_2 in the medium at rest behind it. Since the phase velocity c/n_1 is now less than the velocity of the parameter v, the wave cannot catch up with the discontinuity. On the other hand, in the medium at rest behind the moving parameter discontinuity both possible waves, given by solutions of Eqs. (2.12) and (2.13), transfer energy away from the parameter discontinuity because its velocity v is higher than the phase velocity c/n_2 of each of the waves (they cannot catch up with the moving interface). Therefore, the structure of the solution is as follows. On the one side of the parameter discontinuity there is only the incident wave, and on the other there are two transmitted waves. In this case there is no reflected wave.

If the velocity v of the traveling parameter is higher than the phase velocity of light in one medium and lower than the phase velocity of light in the other medium (for example, if $c/n_1 < v < c/n_2$ and $c/n_2 < v < c/n_1$), the selection of the solutions of both sides of the discontinuity is much more difficult.[33] In particular, in the case when $c/n_1 < v < c/n_2$, there is only one solution on either side of the parameter discontinuity: in front of the discontinuity it represents the incident wave, and behind the discontinuity it represents the transmitted wave moving away from it. The reflected wave and the second transmitted wave do not remove energy from the moving parameter discontinuity and, therefore, should be ignored. Consequently, we are faced with an overdetermined problem: we have to determine the amplitude of just one transmitted wave and for this we have two independent equations [the boundary conditions given by Eq. (2.17)]. It was shown by Ostrovskiĭ[33] that in this case the parameter discontinuity becomes unstable.

In the other case when $c/n_1 > v > c/n_2$, the problem becomes underdetermined. In fact, in this case on one side of the parameter discontinuity we have not only the incident but also a reflected wave, because the latter moves energy away from the interface ($c/n_1 > v$). On the other side of the discontinuity both possible waves also remove energy from the moving parameter discontinuity because the velocity of the discontinuity v exceeds the velocity c/n_2 of both waves and, therefore, these waves cannot catch up with the discontinuity. It follows that there are three waves: one reflected and two transmitted. Their amplitudes can be found if we have just two boundary conditions [see Eq. (2.17)]. These conditions are insufficient and they should be supplemented

by a further condition selected on the basis of physical considerations.[33]

The problem of transformation of waves by a discontinuity of a parameter traveling at a superluminal velocity is related to the problem of transformation of waves in a homogeneous medium which is far from equilibrium. In fact, a medium at rest has a refractive index n_1 ahead of the parameter discontinuity and the index behind the discontinuity is n_2. In this case we have $v > c/n_1$ and $v > c/n_2$ and, as pointed out already, in front of the discontinuity there is one incident wave, whereas behind it there are two transmitted waves. If the velocity of this discontinuity now tends to infinity, we in fact obtain an instantaneous change in the refractive index throughout all space from the initial value n_1 to n_2. Hence, in particular, it follows that in the case of such a discontinuity one plane monochromatic wave splits into two plane monochromatic waves (of different frequency) and the latter travel in two opposite directions.[35,36]

We shall now return to the problem of reflection by a moving mirror. If we know the laws of reflection of monochromatic waves, we can consider the reflection of a wave packet. Let us assume that a wave packet incident on a moving mirror is characterized by an electric field

$$\mathbf{E}(z, t) = \mathbf{E}_0 f\left(t - \frac{z}{c}\right). \tag{3.17}$$

For simplicity, we shall assume that the wave packet is traveling along the z axis at a velocity c opposite to the moving mirror; the vector $\mathbf{E}_0$ is perpendicular to the z axis. The function $f(\xi)$ describes the shape of the packet. We shall expand the packet of Eq. (3.17) as a Fourier integral in terms of plane monochromatic waves:

$$\mathbf{E}(z, t) = \mathbf{E}_0 \int_{-\infty}^{+\infty} d\omega f(\omega) \exp\left[-i\omega\left(t - \frac{z}{c}\right)\right]. \tag{3.18}$$

Each of the components of the Fourier expansion is a wave of frequency ω and of amplitude $f(\omega)\mathbf{E}_0$. The above expressions allow us to find, for each of these waves, the corresponding reflected wave

$$\mathbf{E}_1(z, \omega_1) \exp\left[-i\omega_1\left(t + \frac{z}{c}\right)\right] = \frac{1+\beta}{1-\beta} f(\omega) \mathbf{E}_0 \exp\left[-i\omega \frac{1+\beta}{1-\beta}\left(t + \frac{z}{c}\right)\right]. \tag{3.19}$$

We recall that in our case we have $\omega_1(\omega) = \omega(1+\beta)/(1-\beta)$. Summing the reflected waves at all frequencies, we obtain the following expression for the wave packet reflected by the moving mirror:

$$\begin{aligned}\mathbf{E}_1(z, t) &= \int_{-\infty}^{+\infty} d\omega_1 \mathbf{E}_1(z, \omega_1(\omega)) \exp\left[-i\omega_1(\omega)\left(t + \frac{z}{c}\right)\right] \\ &= \frac{1+\beta}{1-\beta} \mathbf{E}_0 \int_{-\infty}^{+\infty} d\omega f(\omega) \exp\left[-i\omega \frac{1+\beta}{1-\beta}\left(t + \frac{z}{c}\right)\right] \\ &= \frac{1+\beta}{1+\beta} \mathbf{E}_0 f\left(\frac{1+\beta}{1-\beta}\left(t + \frac{z}{c}\right)\right).\end{aligned} \tag{3.20}$$

It therefore follows that the reflected wave packet may form from the incident packet in the following way. Let the distribution of the field in the incident packet be described by a function $f(z)$ at some moment in time. Then, the field distribution in the reflected packet is described by $\alpha f(\alpha z)$, where $\alpha = (1+\beta)/(1-\beta)$. Hence, it follows in particular that if the mirror is moving toward the incident packet of spatial dimensions of the order of l, the spatial size of the reflected packet decreases by a factor $\alpha = (1+\beta)/(1-\beta)$ and the field within the packet increases by the same factor. It is clear from Eq. (3.20) that the change in the spatial dimensions of the packet is related to the frequency transformation as a result of reflection. The following conclusion can be deduced from these expressions. If we select some point z and measure the duration of the passage of the wave packets across this point, we find that the time taken by the incident packet to cross this point is τ and the time taken by the reflected packet is $\tau/\alpha = \tau(1-\beta)/(1+\beta)$, i.e., in this case (with the mirror moving toward the incident packet) the duration of the reflected signal is $1/\alpha$ times less than the duration of the incident signal.

For the sake of simplicity we limited ourselves above to the case of the normal incidence of waves (and packets) on a mirror. However, we can equally easily consider also the case of oblique incidence. All the qualitative conclusions drawn above remain valid. As far as the quantitative relationships are concerned, the quantity α for the oblique incidence case does not represent the ratio $(1+\beta)/(1-\beta)$, but $\alpha(\vartheta_0) = [1+\beta^2+2\beta\cos\vartheta_0]/(1-\beta^2)$.

Formally, the same approach can be used to show that an increase in the mirror velocity can be used to create pulses of any duration, which can be as small as we wish. However, we must bear in mind that short duration pulses contain high-frequency components and for these components a correct description requires allowance for the dispersion. For example, it is known that if the mirror is ideal at optical frequencies, then beginning from wavelengths corresponding to soft x rays, the window becomes increasingly transparent.

The problem of generation of short light pulses by reflection from a mirror moving in an accelerated manner was considered by Ostrovskiĭ.[33,37] It should be noted that the limitation due to the dispersion of the mirror at high frequencies remains valid in this case as well.

In our analysis it is assumed that an interface is perfectly abrupt, i.e., that the size of the transition layer at the interface is much less than all the other characteristic lengths of the problem. If the mirror is moving toward the incident wave, in the relativistic limit the wavelength of the reflected wave can become very short, i.e., it may be comparable or even less than the size of the transition layer. In the extreme case when the reflected wavelength is much less than the thickness of the transition layer, the reflection coefficient becomes exponentially small.[38,39]

Since the reflection alters the frequency and amplitude of the incident wave, it follows that the energy flux and density in the reflected wave differ from the corresponding parameters of the incident wave. This in turn means that exchange of energy occurs between the radiation field and the mirror. If reflection increases the wave energy, this means that the mirror does work on the field. In the opposite case the field does work on the mirror. Exchange of energy is, of course, impossible without a corresponding exchange of momentum and, therefore, strictly speaking the mirror should exhibit acceleration. However, we shall assume that the

change in the mirror velocity is negligible either because the mass of the mirror is sufficiently high or because there is some mechanism which compensates for the change in the mirror energy (for example, the mirror may be set in motion by an external force and the work done by this force may maintain a constant mirror velocity).

The energy balance on interaction of a moving mirror with the incident radiation can be made clearer by invoking the law of conservation of energy of an electromagnetic field. It follows from the system of Maxwell equations (1.1) written down for vacuum ($\mathbf{D} = \mathbf{E}$, $\mathbf{B} = \mathbf{H}$) that for an arbitrary volume V bounded by a surface σ, we have

$$\frac{d}{dt}\left[\int_V \frac{E^2+H^2}{8\pi}dV\right] = -\int_V (\mathbf{j}, \mathbf{E})\,dV - \frac{c}{4\pi}\oint_\sigma ([\mathbf{E}, \mathbf{H}], d\sigma). \tag{3.21}$$

We shall assume that the surface σ is in the form of two planes parallel to the reflecting plane of the mirror and located on both sides of this plane.

We shall apply Eq. (3.21) to a part of the volume corresponding to a unit surface of the mirror. We recall that the electrical and magnetic fields vanish behind the mirror. Under these conditions obviously the left-hand side of Eq. (3.21) is equal to $-vW$, where v is the mirror velocity and W is the electromagnetic energy density:

$$W = \frac{1}{4\pi}(E_0^2 + E_1^2) \tag{3.22}$$

(we recall that E_0 and E_1 are the amplitudes of the incident and reflected waves, respectively). The second term on the right-hand side of Eq. (3.21) can be denoted by S_z (we are assuming that the vector element of the surface $d\sigma$ is directed along the outer normal). We then obtain

$$-\int_V (\mathbf{j}, \mathbf{E})\,dV = -vW - S_z. \tag{3.23}$$

The quantity $\mathbf{j}$ on the left-hand side of Eq. (3.23) is the density of the surface currents induced on the mirror by the incident wave. It therefore follows that the volume integral on the left-hand side of Eq. (3.23) is essentially a surface integral. Moreover, the following relationship is obeyed on the mirror surface:

$$\mathbf{E}_t = -\frac{1}{c}[\mathbf{v}, \mathbf{H}]_t, \tag{3.24}$$

which follows from the fact that in a reference system in which the mirror is at rest the tangential component of the electric field vanishes on the mirror surface. Substituting the above relationship on the left-hand side of Eq. (3.23), we obtain

$$-\int_V (\mathbf{j}, \mathbf{E})\,dV = \left(\mathbf{v}, \int_\sigma \frac{1}{c}[\mathbf{j}, \mathbf{H}]\,d\sigma\right) = (\mathbf{v}, \mathbf{F}_L) = pv, \tag{3.25}$$

where the surface of integration σ is identical with the mirror surface and the positive direction of the velocity $\mathbf{v}$ is identified in Fig. 1. Equation (3.25) contains

$$\mathbf{F}_L = \frac{1}{c}\int_\sigma [\mathbf{j}, \mathbf{H}]\,d\sigma. \tag{3.26}$$

The quantity $\mathbf{F}_L$ is the Lorentz force exerted by the magnetic field of the wave on the currents flowing on the mirror surface. It is clear from Eq. (3.25) that the projection of the force $\mathbf{F}_L$ on the mirror velocity $\mathbf{v}$ is equal to the optical pressure on the moving mirror:

$$\mathbf{F}_L = p. \tag{3.27}$$

Using Eqs. (3.25) and (3.27), we find that Eq. (3.23) becomes

$$pv = -S_z - vW. \tag{3.28}$$

Equation (3.28) is an analytic expression of the statement that the work pv performed by a unit surface area of the mirror per unit time against the forces of the field consists of two parts: the work $(-S_z)$ used to alter the energy flux in the wave on reflection and the work $(-vW)$ associated with the change in the volume occupied by the field.

4. REFLECTION AND REFRACTION OF ELECTROMAGNETIC WAVES BY A TANGENTIAL VELOCITY DISCONTINUITY

We shall now consider two moving media separated by a plane interface. If the velocities of the media on both sides of the interface are parallel to it, obviously, a discontinuity of the velocity on transition across the interface has only a tangential component. Examples of tangential discontinuities of the velocity can be found in hydrodynamics and aerodynamics (ocean and air currents) and in plasma physics (ionospheric flows and laboratory plasma jets). In all these examples we shall assume that the interfaces are at rest, i.e., that the normal components of their velocities are zero.

We shall now assume that a plane monochromatic electromagnetic wave is traveling in one of the moving media and this wave is incident on a tangential discontinuity surface. We have to determine the characteristics of the reflected and refracted waves. We shall direct the z axis along the normal to the interface between these media (Fig. 2). We shall assume that the medium in the half-space $z < 0$ has the permittivity ε_1 and the magnetic permeability μ_1 in a system at rest. The corresponding parameters of the medium located at $z > 0$ in the reference system at rest are ε_2 and μ_2. We shall denote the velocities of the media by $\mathbf{u}_1$ and $\mathbf{u}_2$, respectively. Obviously, the vectors representing these velocities lie in the (x, y) plane. We can always select such a reference system in which one of the media (for example, that characterized by ε_1, μ_1, and $\mathbf{u}_1$) is at rest. Then, the other medium (characterized by ε_2, μ_2, and $\mathbf{u}_2$) must be in motion, i.e., it must "glide" against the medium at rest. Therefore, in order

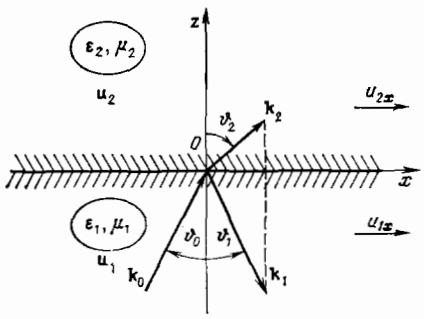

FIG. 2.

to solve the problem of the reflection and refraction of waves by such an interface we must (for fundamental reasons) employ the mathematical apparatus of the electrodynamics of moving media. For the sake of simplicity, we shall assume that the medium with ε_1 and μ_1 containing the incident and reflected waves is at rest, whereas the medium with ε_2 and μ_2 containing the transmitted wave is moving at a velocity $\mathbf{u}$ in the plane of the interface, i.e., we shall postulate that $\mathbf{u} = u_x\mathbf{e}_x + y_y\mathbf{e}_y$, where $\mathbf{e}_{x,y,z}$ are unit vectors. We shall postulate that the wave vector $\mathbf{k}_0$ of the incident wave is located in the (x, z) plane and makes an angle ϑ_0 (angle of incidence) with the z axis. It then follows from the above discussion [see Eq. (2.9)] that the wave vectors of the reflected $\mathbf{k}_1$ and transmitted $\mathbf{k}_2$ waves also lie in the (x, z) plane, i.e., in the plane of incidence. The frequencies of the incident ω_0, reflected ω_1, and transmitted ω_2 waves are also equal. This follows from the relationships given by Eq. (2.10) in which the velocity of the interface is in our case zero. The invariants I_t and I_1 introduced earlier now become

$$I_t = k_{0x} = k_{1x} = k_{2x}, \quad I_1 = -\omega_0 = -\omega_1 = -\omega_2 = -\omega. \quad (4.1)$$

Substituting these values in Eqs. (2.12) and (2.13), we obtain the following expressions for the components k_{1z} and k_{2z} of the wave vectors of the reflected and transmitted waves, respectively:

$$k_{1z} = -\left(\frac{\omega^2}{c^2}\varepsilon_1\mu_1 - k_{0x}^2\right)^{1/2} = -k_{0z}, \quad (4.2)$$

$$(k_{2z})_{1,2} = \pm\frac{\omega}{c}\left[\left(1-\frac{c^2k_{0x}^2}{\omega^2}\right) + \varkappa_2\gamma^2\left(1-\frac{k_{0x}}{\omega}u_x\right)^2\right]^{1/2},$$

where

$$\varkappa_2 = \varepsilon_2\mu_2 - 1, \quad \gamma^{-2} = (1-\beta_t^2), \quad c\beta_t = \mathbf{u}_t = u_x\mathbf{e}_x + u_y\mathbf{e}_y.$$

We obtained the above expressions from Eqs. (2.12) and (2.13) by substituting $\beta = 0$, $\beta_{1n} = 0$, $\beta_{2n} = 0$, $\beta_{1t} = 0$, and $\beta_{2t} = \mathbf{u}/c$. There are two possible signs in front of the square root in the expression for k_{2z} and we have to select that which corresponds to a wave with a group velocity directed away from the interface.[1,40]

When we know the components of the wave vectors of the incident, reflected, and transmitted waves, we can find the angles of reflection and refraction:

$$\operatorname{tg}\vartheta_0 = \frac{k_{0x}}{k_{0z}}, \quad \operatorname{tg}\vartheta_1 = \frac{k_{1x}}{k_{1z}} = -\operatorname{tg}\vartheta_0, \quad \operatorname{tg}\vartheta_2 = \frac{k_{2x}}{k_{2z}}. \quad (4.3)$$

Hence, it is clear (Fig. 2) that the angle of incidence ϑ_0 is equal to the angle of reflection ϑ_1 and the angle of refraction ϑ_2 can be found using Eqs. (4.3) and (4.2). We have thus determined the wave vectors of the reflected and refracted waves.

The vector amplitudes $(\mathbf{E}_1, \mathbf{H}_1)$ of the reflected $(\mathbf{E}_2, \mathbf{H}_2)$ and refracted waves can be expressed in terms of the vector amplitudes $(\mathbf{E}_0, \mathbf{H}_0)$ of the incident wave, subject to the boundary conditions given by Eq. (1.3). The actual calculations and the final expressions[9,11,41,43] are quite cumbersome. Therefore, we shall not give the final results in their complete form, but consider only some simple special cases. We note first of all one important feature which distinguishes the problem of reflection and refraction by a tangential discontinuity from the corresponding problem of media at rest.

In the problem of reflection and refraction of light at an interface between two media at rest (classical Fresnel problem), we shall consider separately two main cases. In one of them the electric vector of the incident wave lies in the plane of incidence (which is the x, z plane in Fig. 2), whereas the magnetic vector is perpendicular to the plane of incidence. It is then found that the electric vectors of the reflected and refracted waves also lie in the plane of incidence, and the magnetic vectors are perpendicular to it. In the other case the electric vectors of all the three waves—incident, reflected, and refracted—are perpendicular to the plane of incidence and the magnetic vectors lie in this plane. Therefore, the distribution of the field vectors relative to the plane of incidence is a property which is invariant for all three waves in the sense defined above. The situation is different in the case of reflection and refraction of waves by a tangential discontinuity of the velocities of two media. For example, if the electric vector of the incident wave is perpendicular to the plane of incidence, the electric vectors of the reflected and refracted waves are no longer perpendicular to the plane of incidence, i.e., they have nonzero (and generally different) projections on the plane of incidence. We may therefore say that the reflection and refraction of waves by a tangential discontinuity rotates the plane of polarization of the reflected and refracted waves. The angle of rotation of the plane of polarization depends on the optical parameters of the two media, on the velocity discontinuity on the tangential discontinuity, and on the angle of incidence. Therefore, measurement on the angle of rotation can provide additional information on the parameters of the media.

We shall now illustrate these results by considering a simple example.[11] Let us assume that a plane monochromatic wave is incident from vacuum on an interface with a moving medium and the velocity of the medium lies in the plane of the interface. The situation is still described by Fig. 2, but we must bear in mind that in this case we have $u_1 = 0$, $\varepsilon_1 = \mu_1 = 1$, $\mathbf{u}_2 = \mathbf{u} = u_x\mathbf{e}_x + u_y\mathbf{e}_y$, $\mu_2 = 1$, $\varepsilon_2 = \varepsilon$. We shall consider the case when the electric vector $\mathbf{E}_0$ of the incident wave is perpendicular to the plane of incidence (which is the x, z plane), i.e., $\mathbf{E}_0 = E_{0y}\mathbf{e}_y$. Then, using the boundary conditions of Eq. (1.3), the dispersion equation (2.11), and the relationships in Eq. (4.1), we obtain the following expressions for the components of the electric vectors of the reflected $(\mathbf{E}_1)$ and transmitted $(\mathbf{E}_2)$ waves[11]:

$$E_{1y} = \left\{\frac{k_{0z}-k_{2z}}{k_{0z}+k_{2z}} + \frac{\varkappa\beta_y^2}{1-\beta^2}\frac{k_{0z}}{(k_{0z}+k_{2z})}\frac{2k_{0z}}{k_{2z}+(\varkappa+1)k_{0z}}\right\}E_{0y},$$

$$E_{2y} = \frac{2k_{0z}}{k_{0z}+k_{2z}}\left\{1+\frac{\varkappa\beta_y^2}{1-\beta^2}\frac{k_{0z}}{k_{2z}+(\varkappa+1)k_{0z}}\right\}E_{0y},$$

$$E_{1x} = E_{2x} = \frac{\varkappa\beta_y[\beta_x-(ck_{0x}/\omega)]}{1-\beta^2}\frac{k_{0z}}{(k_{0z}+k_{2z})} \times\frac{2k_{0z}}{k_{2z}+(\varkappa+1)k_{0z}}E_{0y},$$

$$E_{0x} = 0, \quad (4.4)$$

where $\varkappa = (\varepsilon - 1)$, $\beta^2 = (\beta_x^2 + \beta_y^2)$, $\mathbf{u} = c\beta = c\beta_x\mathbf{e}_x + c\beta_y\mathbf{e}_y$, $k_{0x} = (\omega/c)\sin\vartheta_0$, $k_{0z} = (\omega/c)\cos\vartheta_0$ and the

expression for k_{2z} can be obtained from the expressions in Eq. (4.2). The components E_{1z} and E_{2z} are described by

$$(\mathbf{k}, \mathbf{E}) + \varkappa \frac{\omega - (\mathbf{k}, \mathbf{u})}{c(1-\beta^2)} (\mathbf{u}, \mathbf{E}) = 0, \tag{4.5}$$

which in a medium at rest is equivalent to the condition that the electric field vectors $\mathbf{E}$ are transverse. In the case of a reflected wave propagating in vacuum this condition is of the form $(\mathbf{k}_1, \mathbf{E}_1) = 0$.

We shall now analyze the expressions given above. It follows from them that if the incident wave $\mathbf{E}_0$ has only one component E_{0y}, then the reflected and refracted waves have all three components of the electric vector. This means that the electric vectors of the reflected ($\mathbf{E}_1$) and refracted ($\mathbf{E}_2$) waves are not perpendicular to the plane of incidence, i.e., they are rotated relative to the vector $\mathbf{E}_0$ in the incident wave. It follows from the expressions in Eq. (4.4) that the components $E_{1x} = E_{2x}$ governing the rotation of vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ relative to the vector $\mathbf{E}_0$, are proportional to the component β_y of the velocity of the medium in the direction of the vector $\mathbf{E}_0$. If $\beta_y = 0$, we have $E_{1x} = E_{2x} = 0$, i.e., the vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ are parallel to the vector $\mathbf{E}_0$ and the rotation of the plane of polarization no longer takes place. It should also be mentioned that $E_{1x} = E_{2x}$ vanishes for $\varkappa = 0$, because in this case there is no interface and it also vanishes when $\beta_x = ck_{0x}/\omega$. The details of the treatment are found in Refs. 9, 11, and 41.

We note here one further interesting possibility associated with the transformation of the waves by a tangential discontinuity. Let us assume that the $z = 0$ plane separates two media (Fig. 3), one of which is located in the region $z > 0$ and is moving at a velocity u along the x axis and the other located at $z < 0$ is at rest. We shall assume that a wave whose coordinate and time dependences are described by a factor $\exp[i(k_{0z}z + k_{0x}x - \omega t)]$ is incident from the medium at rest characterized by ε_1 and μ_1 on the interface. At $z = 0$ the field at the interface becomes $\exp[i(k_{0x}x - \omega t)]$. This means that an excitation created by the incident wave travels along the interface. The velocity of this excitation is ω/k_{0x}. If the refractive index of the medium at rest $n_1 = (\varepsilon_1\mu_1)^{1/2}$ is sufficiently large, then in a certain range of angles of incidence the excitation travels along the interface at a velocity much less than the velocity of light in vacuum. In this case (Fig. 3) the velocity of the moving medium may be higher than the velocity of the excitation $u > \omega/k_{0x} = c/(n_1 \sin \vartheta_0)$, i.e., the medium may overtake the excitation traveling along the interface. If the velocity of the medium relative to the excitation exceeds the phase velocity of light in this medium (in a reference system in which the medium is at rest), the reflected wave removes from the interface a greater energy than the energy delivered to the interface by the incident wave. Therefore, the presence of reflection amplifies the wave. The necessary energy is provided by the moving medium which is therefore slowed down. This effect was pointed out first by Lupanov[44] and it is analogous to the inverse Vavilov–Cherenkov effect.[45,46]

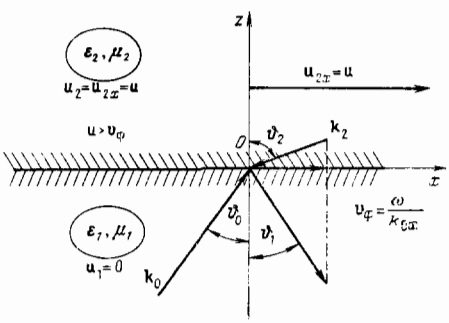

FIG. 3.

CONCLUSIONS

The following point should be made. Einstein proposed in 1905 a practically complete form of the special theory of relativity. In over 80 years from this time nothing was added to this work as far as fundamentals of the theory are concerned. This applies not only to the intellectual content of the theory, but also to the formulations used in this theory and the concepts introduced for the purpose. In many branches of physics (for example, in the design of charged-particle accelerators) the special theory of relativity is the foundation of engineering calculations. The examples of the transformation of waves by moving interfaces considered above essentially follow from one particular problem considered by Einstein in his well-known paper of 1905.

[1)] Allowance for the group velocity in the Fresnel problem is considered in Ref. 1.

[2)] A considerable proportion of the results given in Ref. 7 was obtained by the authors in 1955 and published in a report (No. 1021) of the Physico-technical Institute of the Ukrainian Academy of Sciences. The paper of M. A. Lampert[5] was published a year later.

[3)] It should be pointed out that there is an error in Eq. (3.6) in the translation of Einstein's paper[2] (Vol. 1, p. 29).

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