Self – oscillatory systems with high-frequency energy sources

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Three types of systems in which the excitation of oscillations due to high-frequency energy sources is possible have been considered. Despite widely held ideas, one can classify such systems as self-oscillatory. Systems with times of interaction with the energy source that are short in comparison with the period of the oscillations which arise are the first type. Systems of the second type are those having two degrees of freedom, one of which is high-frequency and the other is low-frequency. Transfer of the energy of the high-frequency source to the energy of low-frequency oscillations is achieved by the formation of combination frequencies. Thermomechanical systems are the third type. The role of the high-frequency energy source is to maintain the necessary temperature of the element being heated. Self-modulation of the parameters is the cause of oscillation excitation in such systems.

It was assumed until recently that self-oscillatory systems are systems which convert the energy of a steady, nonoscillating source into oscillatory energy.[1–7] Actually, such systems satisfy the characteristic traits of self-oscillatory systems which, in modern language, can be formulated in the following manner:

1) independence of the amplitude of steady-state oscillations from the systems initial state over a broad range, i.e., the existence of at least one attractor[1)] in phase space, and

2) independence of the spectrum of oscillations from the spectrum of the source.

However, the presence of a specifically steady energy source is not necessary to realize the indicated traits of self-oscillatory systems. The discovery of the possibility for chaotic oscillations in passive nonlinear oscillators that are under the influence of a periodic external force was the stimulus for revising the definition of self-oscillatory systems.[8,9] Such oscillations satisfy the traits listed above, and therefore they can be considered as self-oscillatory.

Looking back, one may discover that a number of systems which convert the energy of a high-frequency source[2)] into low-frequency oscillations whose frequencies are practically unrelated to the frequency of the source has been known for a long time in physics. These systems also satisfy the two indicated traits, and therefore, one can assign them to the category of self-oscillatory systems. Proper attention has not been paid until now to the occurrence of such systems; as a consequency of this, they are found in none of the well-known textbooks and study guides on the theory of oscillations.[5,8,10] At the same time, such systems not only extend our ideas about free oscillations; they also have extensive technical applications.

Three types of self-oscillatory systems with high-frequency energy sources are considered in the present review; the excitation of their oscillations has three different causes.

Systems with times of interaction with the energy source that are short in comparison with the periods of the free oscillations which arise are the first type. Here the system controls the energy input so that it receives over the time of interaction stimuli of the necessary magnitude and in the necessary phase.

Systems of the second type are nonlinear systems having two degrees of freedom, one of which responds to external high-frequency effects (this degree of freedom, in particular, may be degenerate), and the other one responds to low-frequency (internal) effects. Combination frequencies arise due to the nonlinear interaction between the dynamic variables, so that the oscillations become quasiperiodic. As a result of the interaction of these oscillations with the oscillations of the source, a transfer of the energy of the high-frequency source to the energy of the low-frequency oscillations occurs.

The third type of systems, thermomechanical systems, essentially coincides with the classical type. The role of the high-frequency energy source here is only to maintain the necessary temperature of the system element being heated. Unlike the first two types, the equations describing the free oscillations in systems of the third type are reduced to self-contained equations, i.e., they do not explicitly contain the time. These systems are included in the present review mainly because they are little known.

1. Let us consider a pendulum interacting with the high-frequency field of a current-carrying coil in some small interval of coordinate space near its equilibrium position (see Fig. 1). The coil power is supplied from an alternating current circuit or from an acoustic oscillation generator. The direction of the ponderomotive force from the coil is collinear with the direction of the pendulum's motion. Such a pendulum was suggested by the authors of Ref. 11. A description of the experimental arrangement and the results of observing the oscillations of this pendulum are given in Refs. 12 through 18.

For small initial departures from its equilibrium position, the pendulum executes very small forced oscillations at the frequency of the external force. Upon increasing the initial departure, the occurrence of steady-state oscillations at a frequency close to the pendulum's natural oscillation frequency is possible. Several stable oscillation regimes with different amplitude and phase values exist here. The selection of one regime or another depends on the initial conditions. The oscillation frequency depends slightly on the frequency of the voltage supplied to the electromagnet.

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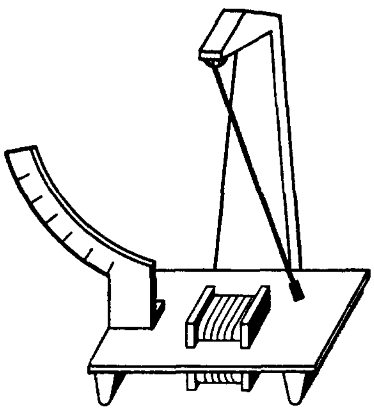

FIG. 1.

References 16 through 25 are devoted to attempts at a theoretical explanation of the observed effects. However, the theory that is expounded in them is either incomplete or contains errors.

Below we shall rely on the theory developed in Refs. 21 and 26. Unlike Ref. 26, here we shall allow for the nonlinearity of the restoring force acting on the pendulum.

For fairly small departures from the equilibrium position, the equation for the oscillations of the pendulum under consideration has the form

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2(1-\gamma x^2)x = f(x,t),^{*)} \qquad (1)$$

where $f(x,t)$ is the force of interaction between the pendulum and the electromagnet. Linear and cubic terms have been retained in the restoring force in Eq. (1). Allowance for the higher order nonlinear terms does not introduce anything fundamentally new to the results.

For simplicity, let us assume that the interaction force has the form

$$f(x,t) = A\cos\omega t \quad \text{for} \quad |x|\leqslant b,$$
$$= 0 \quad \text{for} \quad |x|>b,$$

i.e.,

$$f(x,t) = \vartheta(b+x)\vartheta(b-x)A\cos\omega t,$$

where $\vartheta(z)$ is the Heaviside function.

We shall also consider oscillation regimes of the pendulum for which the amplitude a is much larger than the interaction interval b. For this condition, one can consider the motion of the pendulum over the interaction interval as uniform with a velocity $\pm a\Omega$, where Ω is the oscillation frequency. Therefore, one can assume the force $f(x,t)$ to be dependent only on time and the oscillation amplitude of the pendulum, and can set

$$f(x,t) = F(a,t)$$
$$= \vartheta\left(t - t_n + \frac{b}{a\Omega}\right)\vartheta\left(t_n + \frac{b}{a\Omega} - t\right)A\cos\omega t, \qquad (2)$$

where t_n are the times of the passage of the pendulum through its equilibrium position.

Let us notice that, with allowance for Eq. (2), Eq. (1) is nonlinear because of the dependence of F on a, and moreover, such a nonlinearity belongs to the class of inertial nonlinearities.[4,6]

We shall seek a solution of Eq. (1) with allowance for Eq. (2) in the form

$$x = a(t)\cos(\Omega t + \varphi(t)),$$

where $a(t)$ and $\varphi(t)$ are slowly varying amplitudes and phases of the oscillations. Let us notice that the discrete time t_n can play the role of the "slow" time on which a and φ depend.

To a first approximation by an averaging method,[27] we obtain the equations for a and φ in the form

$$\dot{a} = -\delta a - \frac{2A}{\pi\omega}(-1)^n \sin\frac{\omega\pi}{2\Omega}\cdot\sin\frac{\omega b}{\Omega a}\cdot\sin\frac{\omega[(n+1)\pi-\varphi]}{\Omega},$$
$$\dot{\varphi} = -\Delta(a)$$
$$-\frac{Ab}{\pi\omega a^2}(-1)^n\sin\frac{\omega\pi}{2\Omega}\left(\frac{\Omega a}{\omega b}\sin\frac{\omega b}{\Omega a} - \cos\frac{\omega b}{\Omega a}\right)$$
$$\times\cos\frac{\omega[(n+1)\pi-\varphi]}{\Omega}, \qquad (3)$$

where $\Delta(a) = \Omega - \omega_0(a) \ll \Omega$, $\omega_0(a) = \omega_0[1-(3\gamma a^2/8]$ (the fact that $\omega \gg \Omega$ has been allowed for in doing the averaging).

In order that Eqs. (3) have a steady-state solution, it is necessary that their right-hand sides do not depend on the slow time t_n, i.e., on the number n. For this, it is sufficient to set $\Omega = \omega/(2m+1)$, where m is an integer.

The dependences of the stable (solid curves) and unstable (dashed curves) values of the steady-state amplitudes on the relative detuning $(\omega - M\omega_0)/\delta$, where $M = 2m_0+1$ is some odd number, and for

$$B_{m_0} = \frac{2A}{\pi b\delta\omega_0 M^2} = 1, \quad \frac{3\omega_0 b^2 M^3}{8\delta}\gamma = 20, \quad \frac{\omega_0}{\delta} = 10$$

are shown in Fig. 2. These dependences have the form of nearly periodic (with a "period" $2\omega_0/\delta$) sequences of a series of lobes which correspond to different values of possible oscillation amplitudes.[4)] For different values of external force amplitudes that are characterized by the quantity B_m, the lobes turn out to be enclosed one within the other, and moreover, the lobes with the smaller areas correspond to the smaller B_m values. The upper sequences of lobes gradually disappear for decreasing B_m values. Thus, the uppermost sequence disappears for $B_m = 2/\pi$.

Thus, as follows from the results obtained, for each fixed set of external force parameters A and ω, there exists a series of stable discrete values of the pendulum's oscillation amplitudes. The pendulum's exit to one oscillation regime or another is determined by the initial conditions. The values of possible oscillation amplitudes, starting from some number k, are approximately equal to[26,28]

$$a_{m,k} \approx \frac{2(2m+1)}{\pi(2k+1)}b,$$

where k is an integer. It follows from this and from Eq. (3) that the pendulum's interaction times with an external force during steady-state oscillations with these amplitude values are equal to

$$\tau_k = \frac{2b}{a_{mk}\Omega} = \frac{2b(2m+1)}{a_{mk}\omega} = \frac{b(2m+1)}{\pi a_{mk}}T \approx \left(k+\frac{1}{2}\right)T,$$

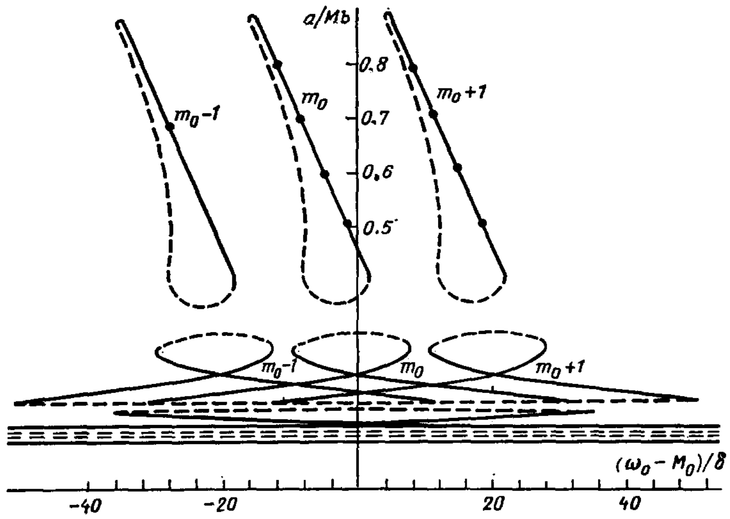

FIG. 2.

where $T = 2\pi/\omega$ is the period of external effect.

In connection with this, one can give the following physical explanation for the described effect. The average value of the force is non-zero over the interaction time and depends on the phase φ that is determined by the quantity B_m. The system selects the phase of the effect so that this force gives the pendulum a push. After half of the pendulum's period of oscillation, when the pendulum, moving in the opposie direction, will again enter the interaction zone, the direction of the average force will be changed to the opposite one, since an odd number of oscillations of the current in the coil occurs over the pendulum's period of oscillation. Therefore, energy which compensates for damping losses will be imparted to the pendulum during each interaction event.

The pendulum's oscillation amplitude and frequency may undergo sudden changes during a smooth change of the frequency ω of the effect. A "hysteresis" is possible here, i.e., the values of the pendulum's oscillation amplitude and frequency for increasing ω may not coincide with the same values for decreasing ω.

Let us notice that, although the pendulum's oscillation frequency must be an odd number of times less than the frequency of the effect, it always remains close to its natural frequency (this is achieved by the appropriate selection of the number m by the system). Therefore, one can consider the dependence of the oscillation frequency Ω on the frequency of the effect as weak and, consequently, one can assume that the necessary condition for self-oscillatory systems is present.

The so-called "gravitational machine"[5] also belongs to the type of systems that is considered here[29]; a model of it is depicted in Fig. 3. A small sphere is accelerated by falling onto an oscillating plate of infinite mass; as a result of this, steady-state oscillations may be set up. Such a model was practically constructed by Bragg and by other researchers. One of the models has been constructed by V. K. Astashev and used in a device for scanning a laser beam.[30] This model was a piezoelectric plate with glass glued onto it that was powered from an electric generator with a frequency of 200 kHz. A small elastic sphere fell onto the glass from above and, as a result, periodic oscillations of the small sphere at low frequency were set up.

Also the system which received the name "Andreev's Hammer"[2,31] is similar to the type of system under consideration.

2. The start of investigations of systems of the second type is found in M. J. Bethenod's paper[32]; he observed the undamped oscillations of a pendulum suspended above a solenoid switched into an alternating current circuit and whose frequency was considerably higher than the pendulum's natural frequency (see Fig. 4). The pendulum was a small iron sphere suspended on a thread. The attempts made by Bethenod and later on by Y. Rocard[33] to explain theoretically the observed phenomena did not meet with success. N. Minorsky,[34] without a completely valid basis, reduced the problem to the parametric excitation of the pendulum. An attempt to explain the Bethenod effect on the basis of the "hysteresis" phenomenon which arises in a non-linear oscil-

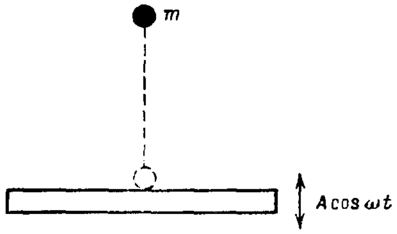

FIG. 3.

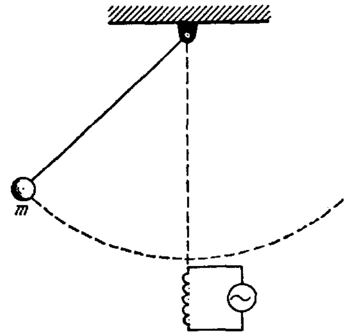

FIG. 4.

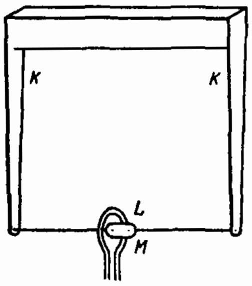

FIG. 5.

latory system has been made in Ref. 35[6)]; this is being discussed at present and is apparently not entirely devoid of any basis.

The interaction of the oscillations of the current in the circuit which supplies power to an electric motor with the angle of turning of the rotor has been considered in a paper by N. D. Papaleksi.[36] It has been shown that the non-synchronous rotation of the rotor[7)] which occurred in the experiment is caused by the excitation and interaction of multiple frequencies. This is apparently the first correct explanation of the phenomena that are observed in systems of a similar type. One must notice that Papaleksi indicated the similarity of the phenomenon that he considered to the excitation of the oscillations of Bethenod's pendulum.

The system depicted in Fig. 5 has been investigated by S. M. Rytov.[37] With the condition that the frequency of the voltage that is impressed on the loop L is considerably higher than the natural oscillation frequency of the small iron sphere, this system can operate as a Bethenod pendulum.

A system that is, to some degree, analogous to a Bethenod pendulum has also been suggested in later papers.[38,39] This is an electromechanical vibrator with a capacitor in its power supply circuit and which, together with the solenoid, forms an oscillator circuit (see Fig. 6). The correct qualitative explanation for the excitation of the vibrator's free oscillations on the basis of the multiple interaction of the oscillation frequencies of the current and of the vibrator's mechanical part is given in Refs. 38 and 39. However, some errors have been made in them, and the results have not been obtained in analytical form.[8)]

Before going on to other examples of systems of the class under consideration, let us examine the operation of the system depicted in Fig. 6. The equations of the system have

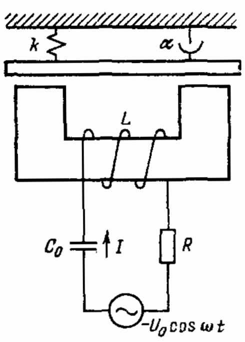

FIG. 6.

the form

$$\frac{d^2}{dt^2}(L(x)I) + R\frac{dI}{dt} + \frac{I}{C_0} = U_0\omega \sin \omega t,$$
$$m\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} + kx = F(x, I), \qquad (4)$$

where $L(x)$ is the inductance of the coil with a core, which depends on the size of the gap between the plate of mass m and the core, which is determined by the displacement of the x plate, $F(x,I) = (I/2)d\Phi/dx$ is the ponderomotive force acting on the plate,[40] and $\Phi = L(x)I$ is the magnetic flux.

It has been shown in Ref. 38 that, with sufficient accuracy, one can set $d\Phi/dx = I\,dL/dx$.

For small x values, one can represent the inductance $L(x)$ in the form of a polynomial

$$L(x) = L_0(1 + a_1x + a_2x^2 + a_3x^3).$$

Then, $F(x,I) = L_0((a_1/2) + a_2x + (3/2)a_3x^2)I^2$.

It is obvious that, because of the dependence of L on x, the oscillations of the current in the electrical oscillation circuit may be quasiperiodic with the fundamental frequency ω and the quasi-period $2\pi/\nu$, where ν is the oscillation frequency of the mechanical part of the system (with variable x). Therefore, it is expedient to make a substitution of variables,[26] after setting

$$I = A\sin(\omega t + \varphi) + y,$$

where A and φ are the amplitude and phase, respectively, of the forced current oscillations at $L = L_0$.

After writing the equations for the variables x and y and solving them by an averaging method, we shall obtain the following shortened equations for the amplitude B and phase φ_1 of the variable x:

$$\dot{B} = (\mu - \delta_2)B - \beta_1B^3,$$
$$\dot{\varphi}_1 = -\Delta + KA^2 - \beta_2B^2, \qquad (5)$$

where

$$\mu = \frac{a_1^2\omega^4[(\omega^2 - \Omega_0^2)(\omega^2 + 3\Omega_0^2) - 4\delta_1^2\omega^2]}{2mL_0[(\omega^2 - \Omega_0^2)^2 + 4\delta_1^2\omega^2]^3}U_0^2, \qquad (6)$$

$\Omega_0 = (L_0C_0)^{-1/2}$, $\delta_1 = R/2L_0$, $\delta_2 = \alpha/2m$, $\Delta = \nu - \nu_0$, $\nu_0 = (k/m)^{1/2}$, and K, β_1 and β_2 are coefficients which depend on the system's parameters.

All nonlinear terms containing powers of the amplitude B higher than three have been discarded in Eqs. (5).

From Eqs. (5) there follows the condition for the self-excitation of oscillations: $\mu \geqslant \delta_2$. This condition determines the critical value U_{cr} for the amplitude of the power source as a function of the frequency ω and of other system parameters. The dependence $U_{cr}(\omega)$ has a non-monotonic character and is depicted in Fig. 7 for two particular cases: a) $\Omega_0 \gg \delta_1$, and b) $\Omega_0 \ll \delta_1$.

The quantity U_{cr} reaches a minimum in both cases for some value of the frequency $\omega = \omega_m$. If $\Omega_0 \ll \delta_1$, then $\omega_m = \sqrt{8}\delta_1$ and, in the opposite case, when $\Omega_0 \gg \delta_1$, we have $\omega_m = \Omega_0 + (\delta_1/\sqrt{5})$. It follows from (6) that the self-excitation of oscillations is only possible for $a_1 \neq 0$, i.e., in Bethenod's experiment, the pendulum must be suspended asymmetrically.

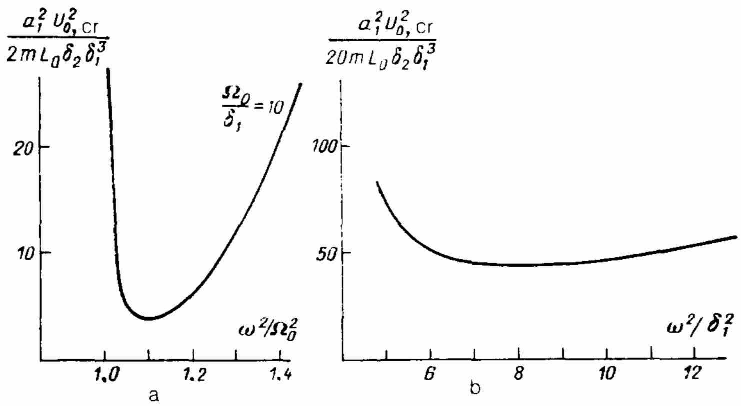

FIG. 7.

The sign of the coefficient β_1 at the excitation threshold determines whether the excitation of oscillations is soft ($\beta_1 > 0$) or hard ($\beta_1 < 0$). The quantity β_1 is proportional to U_0^2 and depends in a fairly complicated way on the frequency ω and the coefficients a_1, a_2 and a_3.

An analysis of the expression for β_1 shows that the coefficient β_1 is positive for $a_2 < a_1^2/4$ and $a_1 a_3 > 0$ near the left boundary in frequency of the self-excitation region, i.e., oscillations are excited softly, and $\beta_1 < 0$ near the right boundary; there, the excitation of oscillations is hard. It is interesting to notice that $\mu = 0$ in the case of symmetric suspension of the pendulum in Bethenod's experiment, i.e., there cannot be self-excitation of the pendulum, but $\beta_1 < 0$ for $\omega > 2\delta_1$ and, consequently, hard excitation is possible.

The oscillation frequency ν is close to the natural frequency ν_0. The correction Δ to the natural frequency in the steady-state regime is determined from the second of Eqs. (5) at $\dot{\varphi}_1 = 0$.

The instability of a small-displacement capacitance sensor that is used in so-called experiments with test bodies is considered in Refs. 41, 42, and 43. Such a sensor is a capacitor, one of whose plates is connected with a body whose displacement is to be measured. The capacitor is part of an electrical oscillator circuit which contains a source of alternating voltage. One can represent the schematic diagram of the sensor in the form depicted in Fig. 8. It is well known that mechanical free oscillations of a body of mass m arise in a certain range of frequencies ω when the voltage U_0 exceeds a critical value.

To explain this effect, a lag of the restoring force acting on the oscillator was introduced artificially in Refs. 41, 42, and 43 (this force is equal to the force of attraction between the capacitor's plates). Such a procedure causes objections, if only because the system with the lag has an infinite number of degrees of freedom, whereas the initial system has only two degrees of freedom [see Eqs. (7) that are shown below, which are also found in Ref. 43]. It is stated in Ref. 43 that the results obtained agree with an approximate solution of Eqs. (7). However, this solution is not shown. Nothing is said about the physical mechanism itself for exciting free oscillations due to the generation and interaction of multiple frequencies. This mechanism will be discussed specifically below.

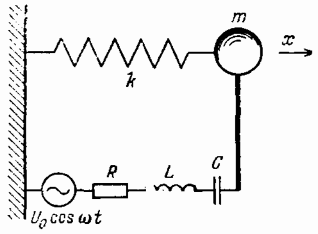

FIG. 8.

The equations of the system have the form

$$\ddot{q} + 2\delta_1\dot{q} + \frac{C_0}{C(x)}\Omega_0^2 q = \frac{U_0}{L}\cos\omega t,$$
$$\ddot{x} + 2\delta_2\dot{x} + \nu_0^2 x = -\frac{1}{m}F(x, q), \tag{7}$$

where q is the charge on the capacitor, $C(x) = C_0[1 + (x/d_0)]^{-1}$ is the capacitor's capacitance, $\Omega_0 = (LC_0)^{-1/2}$ is the natural frequency of the electrical oscillation circuit for $x = 0$, d_0 is the distance between the capacitor's plates with the spring undeformed,

$$F(x, q) = \frac{Sq^2\varepsilon_0\varepsilon}{2C^2(x)(d_0 + x)^2} = \frac{q^2}{2C_0 d_0}$$

is the force of attraction between the capacitor's plates, and m is the mass of the small sphere.[9)]

Comparing Eqs. (7) and (4), we see that they differ only in the character of the nonlinearity. Making suitable calculations, we obtain equations for the amplitude B and phase φ_1 of the same form as Eqs. (5) with

$$\mu = \frac{\Omega_0^4\delta_1[(\omega^2 - \Omega_0^2)(3\omega^2 + \Omega_0^2) + 4\delta_1^2\omega^2]}{2mLd_0^2[(\omega^2 - \Omega_0^2)^2 + 4\delta_1^2\omega^2]^3}U_0^2. \tag{8}$$

From this, it is evident that the excitation of oscillations is possible mainly at $\omega > \Omega_0$, i.e., on the right slope of the resonance curve. This agrees with the results of Refs. 41, 42, and 43, and is confirmed experimentally. It follows from Eq. (8) that the condition for exciting free oscillations is the easiest of all, i.e., for the lowest U_0 value, it is fulfilled for the frequency $\omega \approx \Omega_0 + (\delta_1/\sqrt{5})$. This lowest voltage value is

$$U_{0\,\min} = \frac{24 d_0 \delta_1^2}{5}\left(\frac{6mL\delta_2}{\Omega_0\sqrt{5}}\right)^{1/2}$$

To this value of the source voltage, there corresponds the voltage on the capacitor

$$U_c = 12d_0\delta_1\left(\frac{m\delta_3 V\overline{5}}{\Omega_0 C_0}\right)^{1/2} \qquad (9)$$

Equation (9) differs from the one shown in Refs. 41, 42, and 43 only by its numerical factor, although the mechanism for exciting the oscillations turns out to be different.

An analysis of the expression for the nonlinearity coefficient β_1 shows that the excitation of vibrations must be soft.

An analogous effect for the self-excitation of mechanical oscillations must also occur in small-displacement optical sensors, where free oscillations arise due to light pressure.[43] It was observed experimentally during the action of an ultra-high-frequency field on a torsion pendulum.[44]

The self-excitation mechanism described forms the basis for the occurrence of the mechanical free oscillations of resonators filled with some kind of powerful radiation, for example, electromagnetic radiation. Thus, oscillations of the walls were observed in resonators that are used in powerful colliding beam accelerators.[45] The occurrence of elastic wave generation in dielectric resonators that are pumped up by a high-frequency electromagnetic field is described in Refs. 46, 47, and 48. Questions that are essentially similar about the appearance of negative friction for mechanical structures in electromagnetic fields are discussed in Ref. 49.

Mechanical and electrical systems with two degrees of freedom that are powered by energy sources of comparatively high frequency are considered in Refs. 50, 51, 52, and 53. We consider the system depicted in Fig. 9,[54–57] the equations for which can be written in the form

$$\begin{aligned} &m_1\ddot{x}_1 + \mu\dot{x} + kx + F(x, \dot{x}) = U_0\cos\omega t, \\ &m_2\ddot{x}_2 + \mu_2\dot{x}_2 - \mu\dot{x} - kx - F(x, \dot{x}) + k_2x_2 = -U_0\cos\omega t, \end{aligned} \qquad (10)$$

where $x = x_1 - x_2$ is the relative displacement of the masses m_1 and m_2, and $F(x,\dot{x})$ is the nonlinear part of the elastic and dissipative forces between the masses.

The nonlinear function $F(x,\dot{x})$ in Refs. 54, 55, 56, and 57 was selected to be of two types:

$$\begin{aligned} F(x, \dot{x}) &= \alpha x^2, \\ F(x, \dot{x}) &= 0 \qquad \text{for} \quad x \geqslant 0, \\ &= k_1x + \mu_1\dot{x} \quad \text{for} \quad x < 0. \end{aligned}$$

As follows from Eqs. (10), the oscillations of the variables x_1 and x_2 in both cases must be quasiperiodic with the fundamental frequencies ω and ν, where ν is the lower frequency of the free oscillations. As an example, let us consider the case $F = \alpha x^2$.

Proceeding in a way similar to the previous case, we obtain equations of the form of Eqs. (5) for the amplitude and phase of the oscillations of the variable x_2 at the frequency ν.

The analysis of the condition of excitation in this case is more cumbersome than in the previous ones, but the qualitative behavior of the dependence of the critical value of the voltage U_{cr} on the frequency ω is the same as before. The frequency range in which the self-excitation of oscillations is possible for a given U is also determined in a similar way.

We note that the so-called decay instability phenomena of wave processes,[58] in which the energy of a high-frequency wave is effectively converted into a significantly lower frequency wave, are also among the effects of the type under consideration.

3. One can assign certain of the large group of so-called thermomechanical systems to the third type of self-oscillatory systems with high-frequency energy sources.[10] We shall pause for the systems considered in Refs. 59 and 60.

Consider a weightless metallic wire with a load at its center, which is included in an alternating current circuit of frequency ω (see Fig. 10). Under definite conditions, such a wire can execute free oscillations both in the plane of the diagram and also perpendicular to this plane around the O_1O_2 axis. Let us first of all consider the case when the motion of the wire occurs in the plane of the diagram.[59]

The equation for the oscillations of the load in this case has the form

$$m\ddot{x} = mg - 2F\sin\beta - h\dot{x}, \qquad (11)$$

where F is the tension in the wire, $\sin\beta = x/l = x(x^2 + L^2)^{-1/2}$, $2L$ is the distance between the supports, and h is the coefficient of friction.

The tension F in a wire heated by the passage of an electric current equals[61]

$$F = ES\frac{l - l_0(1 + \alpha T)}{l_0},$$

where E is Young's modulus ($\sim 2\cdot 10^9$ g/cm·sec^2), S is the cross section area of the wire, T is the temperature difference between the wire and its environment, α is the linear thermal expansion coefficient ($\sim 10^{-5}$ deg^{-1}), $l_0 = (x_0^2 + L^2)^{1/2}$ is half the length of the wire in the unstretched state at $T = 0$, and x_0 is the sag of the wire for $T = 0$ and $m = 0$.

Assuming $(l - l_0) \ll l_0$, we obtain the following expression for $F\sin\beta$:

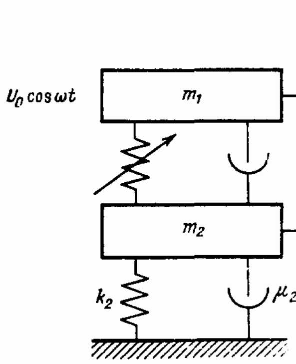

FIG. 9.

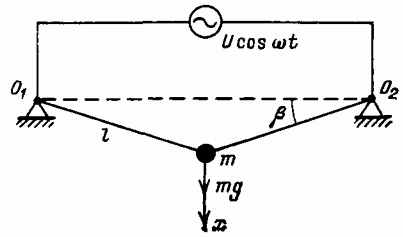

FIG. 10.

$$F\sin\beta = ES\left(\frac{x^2 - x_0^2}{2l_0^2} - \alpha T\right)\frac{x}{l_0}. \quad (12)$$

We write an equation for the temperature T assuming that the cooling of the wire occurs according to Newton's Law with a heat transfer coefficient q which depends on the velocity modulus.[11] In accordance with Refs. 60 and 62, we set $q = q_0 + q_l|\dot{x}|^{1/2}$, where $q_0 \approx 0.0013 S_{\rm sid}$ W/deg, and $S_{\rm sid}$ is the area of the side surface of the wire.

Then the equation for T will have the form

$$mc\dot{T} = \frac{U^2}{2R} - q_0 T - q_1|\dot{x}|^{1/2}T, \quad (13)$$

where c is the specific heat of the material of the small sphere ($c \approx 0.5$ J/g·deg), and R is the resistance of the wire. The resistance R increases as the wire is heated and lengthens,[12] so that

$$R = R_0\left(1 + \beta_1 T + \beta_2\frac{l - l_0}{l_0}\right), \quad (14)$$

where $\beta_1 \approx 0.006$ deg^{-1} and $\beta_2 \approx 0.2$ (the value of β_2 is determined by the Poisson coefficient).

In order to investigate the stability of the steady-state solution of Eqs. (11) and (13), we write linearized equations for the departures from the steady-state solution $\xi = (x - x_{ss})/l_0$, and $\vartheta = (T - T_{ss})/T_{ss}$. These equations have the form

$$\ddot{\xi} + 2\delta\dot{\xi} + \omega_0^2\xi = K\vartheta, \quad \dot{\vartheta} + \gamma\vartheta = -a\dot{\xi}, \quad (15)$$

where

$$2\delta = \frac{h}{m}, \quad \omega_0^2 = \frac{ES}{ml_0}\left(3\frac{x_{ss}^2}{l_0^2} - \frac{x_0^2}{l_0^2} - 2\alpha T_{ss}\right),$$

$$K = 2ES\frac{\alpha T_{ss}x_{ss}}{ml_0^2},$$

$$\gamma = \frac{q_0}{mc}\left(1 + \frac{\beta_1 T_{ss}}{1 + \beta_1 T_{ss}}\right), \quad a = \frac{q_0}{mc}\frac{\beta_2}{1 + \beta_1 T_{ss}}\frac{x_{ss}}{l_0}.$$

The system of Eqs. (15) describes self-oscillatory systems with inertial excitation in a linear approximation.[64–67] The condition for the self-excitation of oscillations in such systems has the form

$$Ka \geqslant 2\delta(\omega_0^2 + 2\delta\gamma + \gamma^2). \quad (16)$$

It follows from this that the dependence of the wire's resistance on its deformation ($a \neq 0$) and sufficient inertia for the wire's temperature change ($\gamma < \gamma_{\rm cr} = [\delta^2 + (Ka/2\delta) - \omega_0^2]^{1/2} - \delta$) are the causes of the possible self-excitation of oscillations.

In order to check whether the excitation of oscillations is soft or hard, one must retain the most significant nonlinear terms in the initial equations. Solving these equations approximately by the Krylov-Bogolyubov asymptotic method, we obtain to a first approximation for a small parameter the following equation for the amplitude A of the oscillations:

$$\dot{A} = (\mu_1 - \delta + \mu_2\sqrt{A} - \mu_3 A^2)A, \quad (17)$$

where $\mu_1 = Ka/2(\gamma^2 + \omega_0^2)$, and μ_2 and μ_3 are some positive coefficients which depend on the parameters of the system. It follows from the fact that $\mu_2 > 0$ that the excitation of oscillations is hard. Therefore a steady-state regime of free oscillations is possible even for $\mu_1 < \delta$.

We note that the condition for the self-excitation of oscillations $\mu_1 \geqslant \delta$, which follows from Eq. (17), agrees with inequality (16) if $\delta\gamma \ll \omega_0^2 + \gamma^2$. This inequality is the condition for the applicability of the asymptotic method used.

Let us now consider the general case, when the wire can, in addition to the vertical displacements, execute oscillations around the O_1O_2 axis. Introducing the turning angle φ, let us write the equations for the motion of the load and for the change of temperature T:

$$\begin{aligned} m\ddot{x} &= mg\cos\varphi - 2F\sin\beta - h\dot{x} + mx\dot{\varphi}^2, \\ mx\ddot{\varphi} &+ 2m\dot{x}\dot{\varphi} + H\dot{\varphi} + mg\sin\varphi = 0, \quad (18) \\ mc\dot{T} &= \frac{U^2}{2R} - q_0 T - q_1(\dot{x}^2 + x^2\dot{\varphi}^2)^{1/4}, \end{aligned}$$

where F is determined by Eq. (12) and R by Eq. (14). The equations for the departures $\xi = x - x_{ss}$ and $\vartheta = T - T_{ss}$ are independent of φ in a linear approximation. Therefore, everything that has been said previously is valid for them.

If the frequency of the free oscillations of the variable x lies in one of the parametric resonance regions for the variable φ,[3] then the excitation of vertical oscillations of the small sphere will necessarily lead to oscillations of the turning angle φ. But if this condition is not fulfilled, then the excitation of oscillations of φ is possible only in a hard manner due to the fact that the temperature oscillations will contain the second harmonic of the oscillations of the angle φ, which will cause modulation of the variable x at this frequency. The latter, in turn, will lead to the "parametric" pumping of energy into the oscillations of the variable φ. Eqs. (18) allow one to calculate the steady-state amplitude of the oscillations for such a process.

The examples considered in this paper show convincingly that certain nonautonomous sytems, and especially systems containing high-frequency energy sources, can behave like self-oscillatory systems. Here the mechanisms for exciting free oscillations can be diverse. Three such mechanisms have been considered in this paper, but is it possible that other ones also exist. The understanding of this type of phenomena helps the correct approach to analyzing the processes in similar systems. We note that examples of all three types of systems were achieved experimentally and have been demonstrated at a number of conferences and seminars.

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[1]The set of points in the phase space of the system towards which all the neighboring phase trajectories converge is called an attractor. In particular, a stable singular point and a stable limiting cycle are attractors.

[2]A source whose oscillation frequency is much higher than the frequencies of the free oscillations that are excited is called a high-frequency source both here and below.

[3]The case where the function defining the coordinate dependence of the periodic force is odd has been considered in Refs. 68 through 71.

[4]The sequences are not strictly periodic because of the change of the number m.

[5]Such a name appears to us to be not entirely fortunate, since here the energy is not drawn from a gravitational field but from the high-frequen-

cy source which causes the oscillations of the plate.

[6)]In patent depositions[72,73] and in other papers, M. I. Kozadov, D. B. Duboshinskii, and Ya. B. Duboshinskii gave the following explanation of the mechanism for exciting oscillations in systems similar to Bethenod's pendulum: The asynchronous conversion of high-frequency energy to low-frequency energy during the interaction of two physically homogeneous or heterogeneous oscillatory systems occurs because of the fact that at least one of the parameters of the high-frequency oscillatory system is modulated by the motion of the passive low-frequency system, so that the high-frequency system passes through the values of the direct and reverse resonances an even number of times over the period of the low-frequency oscillations; as a result of this, the force acting on the low-frequency oscillatory system is transformed from a symmetric (quadratic) to a dynamically double-valued one.

[7)]Rotation whose frequency is not a multiple of the power source frequency is called non-synchronous.

[8)]We note that the system suggested in Refs. 38 and 39 apparently corresponds most closely to the type of oscillations described in the present paper. For in the Bethenod and Rytov systems, the excitation of oscillations can be caused both by the mechanism described here, and also by the action of a periodic quadratic force that is non-linear in the coordinate.

[9)]All expressions are written in the plane capacitor approximation.

[10)]Thermomechanical oscillations of conductors in a vertical plane were demonstrated in 1924 by N. I. Dobronravov and A. I. Shal'nikov; the intense free oscillations of a heated conductor around the O_1O_2 axis (see Fig. 10), or the so-called horizontal thermomechanical oscillations were apparently first experimentally detected in 1971 by Ya. B. Duboshinskii.

[11)]An error has been made in Ref. 59: it is assumed that $q = q_0 + q_1\dot{x}$, which cannot be.

[12)]The change of resistance during deformation is called the tensoresistive effect.[63]

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