Hydromagnetic diagnostics and geoelectric sounding

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A review is made of research on geomagnetic pulsations, which are hydromagnetic waves of natural origin in the frequency range 10^{-3}–5 Hz. Methods for diagnostics of the earth's magnetosphere and methods for electromagnetic sounding of the earth's crust on the basis of data from the observation of pulsations are described. It is believed that the electrical properties of the earth's crust are pertinent to the diagnostics of the magnetosphere, while consideration of the spatial structure of the inducing field is useful in geosounding. The fluctuation and critical properties of magnetic storms and pulsations are discussed on the basis of phenomenological models. Estimates of the properties of the magnetosphere and of the interplanetary medium at the earth's orbit can be made more accurate by taking account of the fluctuations in the diagnostic approach of a black box without an input.

1. INTRODUCTION

1.1. Geomagnetic pulsations

"Geomagnetic pulsations" are oscillations of the earth's electromagnetic field in the upper part of the overall range of geomagnetic variations. The pulsation range stretches from millihertz to several hertz.[1] Below this range is the range of storms, bays, and other aperiodic variations of the magnetic field; above it is the radio range.

The boundaries on this range are somewhat arbitrary, i.e., set by convention. The range could be extended to 10^{-3}–10^3 Hz, for example, if one worked from the definition of geomagnetic pulsations as hydromagnetic and ion cyclotron waves in the magnetosphere.[2]

For more than a century now geomagnetic pulsations have attracted research interest because of the beauty of their shapes and also because of their complex and puzzling behavior. They show us an example of a self-consistent interaction of waves and particles in a plasma, furnish information about remote regions of the space environment of the earth, and influence the course of magnetospheric processes, to the point that they even determine individual elements of the large-scale structure of the magnetosphere. These pulsations exhibit a great diversity of properties and themselves constitute an important part of the world accessible to us.

The classification of pulsations which is presently used was adopted at the Thirteenth General Assembly of the International Union of Geodesy and Geophysics.[1-3] Various types of pulsations have been assigned special abbreviations (Table I). The various types are put in two classes: Pc ("pulsations, continuous") and Pi ("pulsations, irregular"). Morphologically, the Pc oscillations are generally characterized by a quasisinusoidal nature and a prolonged duration, while the Pi oscillations are generally short trains of oscillations, noisy bursts, or wide-band radiation with a time-varying spectrum.

It has been established that the pulsations are excited as a result of plasma instabilities in the magnetosphere and also in the ionosphere and the solar wind.[2] This circumstance is the primary reason for the general scientific importance of research on pulsations. An instability and the associated nonlinearity are the most important properties of a plasma. The observation of pulsations makes it possible to study these properties in detail at cosmic scales.

Since the review which appeared in Usp. Fiz. Nauk twenty years ago,[3] much has been learned about the morphology and physics of pulsations. However, even a brief description of the progress in this field would take us far off our topic. We will therefore content ourselves with citing only two new results, which bear directly on hydromagnetic diagnostics.

a) It has been established that the pulsations of the most common type, the Pc3 pulsations, are excited not in the magnetosphere, as was previously believed,[3,4] but in the solar wind, more precisely, ahead of the earth's bow shock.[5,6] The discovery has resulted in the suggestion of a method for diagnostics of the interplanetary magnetic field on the basis of ground-based observations of Pc3 pulsations.[7,8]

b) Regardless of whether the pulsations are excited inside or outside the magnetosphere, their properties on the earth's surface depend strongly on how close the frequency of the pulsations is to the frequency of Alfvén oscillations of the magnetic shell which passes through the observation point. If these frequencies are equal, there will be a resonance. The structure of the field of pulsations near a resonance was established in some fundamental studies.[9,10] An understanding of this structure makes it possible to suggest an effective method for diagnostics of the magnetosphere (as discussed below).

1.2. Method of hydromagnetic diagnostics

Hydromagnetic diagnostics is a scientific method which furnishes the geophysicist or other interested user qualitative and quantitative information for drawing conclusions about the state and possible evolution directions of the medium near the earth on the basis of observations of geomagnetic pulsations. Where necessary, this diagnostic method also provides a basis for deciding which course to follow. Research on diagnostics first arose as an officially recognized activity in order to meet the needs of geomagnetism itself, for the most part. Although the methods and apparatus for studying such questions had been known previously, hydromagnetic diagnostics began to find systematic and widespread use in work by researchers at the Borok Geophysical Observatory of the Institute of Earth Physics, Academy of Sciences of the USSR (Ref. 11; see also the monographs of Refs. 1, 2 and the bibliographic index of Ref. 13).

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TABLE I. Classification of geomagnetic pulsations.

Type	Range of periods, s
Pc1	0.2—5
Pc2	5—10
Pc3	10—45
Pc4	45—150
Pc5	150—600
Pi 1	1—40
Pi 2	40—150

The physcial basis of hydromagnetic diagnostics is an understanding of the ways in which MHD waves are excited and propagate in the plasma environment of the earth. Part of what we know here has come from theoretical analysis of simplified models, and part has come from experiments. By no means is our understanding of the matter complete. To explain how the problem of hydromagnetic diagnostics should be understood in the face of this uncertainty, let us recall the overall classification of problems which are solved in electrodynamics.

The first class is that of the direct problems or problems of analysis. These are the familiar internal and external problems in which sources are given, and then these sources must be used to find the structure of a wave field in a medium whose properties are assumed to be known. The second class of problems is that of synthesis problems. These are more specialized problems, in which it is necessary to find the sources which excite a given field in a medium. The third and final class is that of so-called inverse problems. In these problems, one works from known sources and fields to find the structure of the medium.

It might seem that the problem of hydromagnetic diagnostics would fall in the class of inverse problems. However, as we stated with special emphasis above, we do not have reliable knowledge of either the sources or the fields. In most cases, we know something about the medium and something about the field and its sources. The problem of hydromagnetic diagnostics should thus be classified as a mixed problem. A picture of problems of this sort and of the approach to their solution was drawn by Krasnushkin in a theory for the propagation of very-low-frequency (VLF) radio waves. A quote from Krasnushkin and Yablochkin's monograph[14] explains the essence of the matter: "In our case the properties of the medium are given incompletely, so it is necessary to solve a so-called mixed problem, in which one works from known and reliable, but incomplete, data on the medium and also from additional known data about the wave field to determine unknown data about the medium and the remaining field." We believe that the VLF theory provides us with an example of the level which we should strive to reach in developing hydromagnetic diagnostic methods.

We will use another quote from Ref. 14 to express in a definite way our skepticism regarding the outlook for the use of various "empirical laws" regarding the behavior of geomagnetic pulsations for diagnostic purposes: "If a theory is to be useful in practical work, the functional relationships between the field and the medium must be derived from the field equations and constitutive equations of the medium, not directly from experimental data."

The desired level has not, however, been reached at this point. At this point the diagnostic situation is of the nature of an art, and a fairly subtle one at that. There are no written recipes for this art. Nevertheless, this art is a rational one, based on the methodical execution of several extremely important operations. More briefly, it has its own internal logic, which can be demonstrated best with specific examples. In selecting examples we were interested in simplicity as well as novelty.

1.3. Magnetotelluric sounding

Geomagnetic pulsations are used not only in the diagnostics of the magnetosphere but also in geoelectricity, to study the earth's crust by the method of magnetotelluric sounding. Magnetotelluric sounding is essentially the estimation of the vertical distribution of the electrical conductivity of the crust on the basis of the frequency dependence of the surface impedance.[15,16] The surface impedance is found from observations of geomagnetic pulsations.[17,18]

An idea which runs through this review is that hydromagnetic diagnostics and geoelectric sounding are united by the method of magnetotelluric sounding. In the past, these directions developed independently: In diagnostics of the magnetosphere, no use was made of information about the electrical conductivity of the earth's crust, and the work on magnetotelluric sounding made virtually no use of the wave structure of the inducing field. Exceptions to this rule were a series of studies which are generalized in Chetaev's monograph.[19] The earth's crust was modeled in Ref. 19 as a horizontally homogeneous conducting half-space. In the present review, in contrast, we emphasize an approximate account of the horizontal inhomogeneity of the earth's crust.

The asymptotic theory of the skin effect[20] will be of assistance in developing this idea of unity. Until recently, the theory of Ref. 20 was unknown in geoelectromagnetism. Attention was called to it in Ref. 21, where it was emphasized that the results of Ref. 20 could be used to advantage in geoelectric sounding by the method of magnetotelluric sounding. It was subsequently found that one of the equations of the theory of Ref. 20 (Rytov's formula) could find a variety of applications not only in geoelectricity[22,23] but also in hydromagnetic diagnostics.[24–26]

Let us briefly outline the rest of this review. In §2 we present the information about oscillations and waves in the magnetosphere which we will need. In §3 and §4 we discuss applications of Rytov's theory[20] to the diagnostics of the magnetosphere and geosounding. In §5 and §6 we discuss questions concerning the phenomenological modeling of geomagnetic disturbances, with an emphasis on the analysis of fluctuating and critical phenomena.

2. OSCILLATIONS AND WAVES IN THE MAGNETOSPHERE

2.1. Ray theory

The ray theory of the propagation of MHD waves, i.e., geometric optics for MHD waves or, more precisely, geometric magnetohydrodynamics, is based on a corresponding local dispersion relation which is understood as a Hamilton-Jacobi equation. In wave theory it is called on "eikonal equation." The characteristics of the eikonal equation satisfy Hamilton's canonical equations and are rays (trajectories) along which the energy of MHD perturbations propagates if the fairly stringent conditions for the applicability of the ray approximation hold in the vicinity of each point of the family of rays.

We restrict the present analysis to monochromatic waves. We are accordingly assigning the perturbations a time dependence $\exp(-i\omega t)$. The medium is assumed to be inhomogeneous but in a steady state. In other words, $\omega(\mathbf{k},\mathbf{x})$ depends on the coordinate $\mathbf{x}$ but not on the time. The wave vector $\mathbf{k}$ then varies in such a way along array that we have $\omega(\mathbf{k}(t),\mathbf{x}(t)) = \text{const}$. The ray equations are[27]

$$\frac{d\mathbf{x}}{dt}=\frac{\partial\omega}{\partial\mathbf{k}},\quad \frac{d\mathbf{k}}{dt}=-\frac{\partial\omega}{\partial\mathbf{x}}. \tag{2.1}$$

The goals and method of geometric MHD and those of geometric optics (or acoustics, seismics, etc) are the same; the only distinction is the subject matter, which is determined by the particular dispersion relation and, of course, the applications.

The simplest dispersion relations are

$$\omega=Ak \tag{2.2}$$

for magnetosonic waves and

$$\omega=Ak_{\parallel} \tag{2.3}$$

for Alfvén waves. They are derived from the linearized MHD equations when the pressure and dissipative processes are ignored.[28] Here $A = B/(4\pi\rho)^{1/2}$ is the Alfvén velocity, $k_{\parallel} = |\mathbf{kB}|/B$, $\mathbf{B}$ is the external magnetic field, and ρ is the plasma density.

Formally, relation (2.2) is the same as the dispersion relation for sound waves or for light in an isotropic medium. Consequently, all the well-known results from optics and acoustics concerning ray calculations[27,29-32] can be applied without changes of any sort to the case at hand. For example, we can write

$$R=-A(\mathbf{N}\nabla A)^{-1}, \tag{2.4}$$

where R is the radius of curvature of the ray, and the unit vectot $\mathbf{N}$ is the principal normal to the ray. In other works, a magnetosonic ray bends in the direction of decreasing Alfvén velocity, as a light ray bends in the direction of an increase in the refractive index.

One might say that formally relation (2.2) has nothing which is specifically "magnetohydrodynamic." The dispersion relation for Alfvén waves (2.3), in contrast, is extremely specific. It follows from (2.3) that the group velocity $\mathbf{v} = \partial\omega/\partial\mathbf{k}$ is always parallel or antiparallel to $\mathbf{B}$ (Ref. 28). This result means that the rays of Alfvén waves coincide with magnetic field lines, i.e., that the shape of the rays is determined completely once we specify the field $\mathbf{B}(\mathbf{x})$. We are left with calculating the refraction, i.e., the variation of $\mathbf{k}$ along the ray, by means of the second equation in (2.1). This problem can be solved in quadrature.[2]

The simplicity of this description, however, has been achieved at the cost of a far-ranging simplification. As a result (for example), we conclude from (2.3) that there are no simple caustics for Alfvén waves. Let us instead assume that a simple caustic exists. Near it, two rays will then pass through each point of the illuminated volume: One ray which has already touched the caustic and one which has not yet done so. This situation would be impossible, however, since magnetic field lines cannot intersect each other.

Let us discuss another view, which corresponds to a different formulation of the problem. We assume that we have a point source of Alfvén waves. All rays emerging from this source coincide with the same magnetic field line. This line is a caustic (the envelope of a family of rays). There are no rays other than the caustic ray; i.e., the entire space except for the field passing through the source is in the caustic shadow.

This unusual picture is unacceptable for many reasons; in particular, it does not have structural stability. If we incorporate in (2.3) an arbitrarily weak dependence of the transverse component of the wave vector we find a complete change in the nature of the propagation: A deviation appears. In other words, a ray deviates from the field line, and a normal structure of caustics is partially restored.

In a cold plasma a weak $k_{\perp}$ dependence of ω (a transverse dispersion) arises for Alfvén waves because of the gyrotropic nature of the medium and/or the electron inertia. The gyrotropic effect is dominant at small values of $k_{\perp}$, and the inertial effect at large values. In a hot plasma a $k_{\perp}$ dependence of ω also arises because of spatial dispersion, which is manifested in this case as an ion-Larmor-radius effect.

Interestingly, despite the deviation of Alfvén rays associated with the transverse dispersion, these rays never turn back [Fig. 1(a)]. In other works, Alfvén caustics do not touch surfaces orthogonal to magnetic field lines at any point in space. This property of "never rolling up" is very simple. It does not disappear when gyrotropy or any other modifications of the dispersion relation are taken into account.

It would appear at first glance that atmospheric whistlers would have the same property. According to Story's formula

$$\omega=\alpha k_{\parallel}k \tag{2.5}$$

the direction of the group velocity of these waves deviates by no more than 19° from the direction of the external magnetic field.[33] (Here $\alpha = cB/4\pi eN$, where N is the electron density.) Actually, however, (2.5) should be replaced by (2.2) in the limit $k_{\parallel}\to 0$. As a result, the ray of an atospheric whistler can turn back, as shown in Fig. 1(b), in contrast with the ray of an Alfvén wave.

2.2. MHD waveguides

Let us use the ray theory to describe waveguides in the plasma environment of the earth.

Figure 2 shows a refractive waveguide which directs magnetosonic waves across the field lines of the geomagnetic

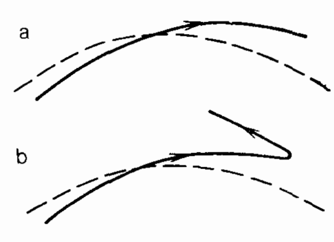

FIG. 1. Ray trajectories of (a) Alfvén wave and (b) an atmospheric whistler. The dashed lines are magnetic field lines.

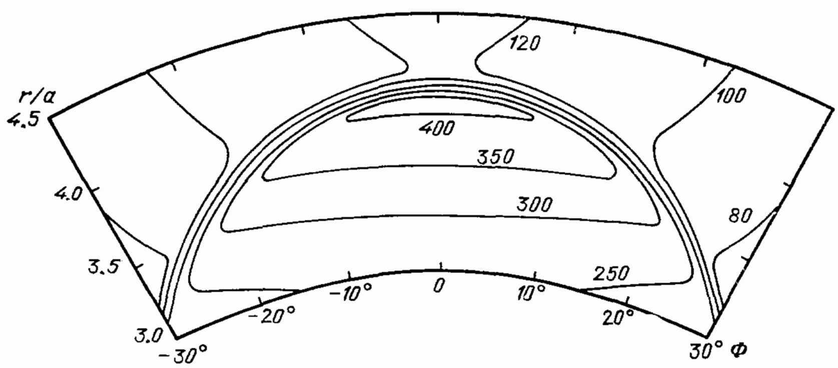

FIG. 2. Contour curves of the refractive index $n = c/A$ in the plane of the geomagnetic meridian. The plasmapause is at magnetic shell $L = 4$; a is the earth's radius.

field.[34] If we ignore the axial asymmetry of the magnetosphere, we can say that the axis of the waveguide coincides with the equator of the magnetic shell under the plasmapause. Here A increases with distance from the axis in any direction—northward, southward, and toward the earth—because of the increase in B, while it increases in the direction away from the earth because of the sharp decrease in ρ in the plasmapause.[1)] According to (2.4), magnetosonic rays bend in the direction of decreasing A, and as a result they are channeled along the axis of the waveguide.

In the waveguide, there is a self-excitation of waves at higher-index harmonics of the ion gyrofrequency.[35] For such waves, relation (2.2) holds only with $k_{\parallel} = 0$. Even a slight deviation of the propagation direction from the transverse direction causes the dispersion relation to take the form in (2.5). (We recall that these two relations approximate different parts of the same disperson curve.[33]) There is a general formula which describes the magnetosonic branch, but it is extremely complicated and inconvenient for analyzing ray trajectories. For this reason, it is customary to use the interpolation formula[36]

$$\omega = \alpha k (k_{01}^2 + k_{\parallel}^2)^{1/2}, \tag{2.6}$$

which reduces to (2.2) and (2.5) in the corresponding limits. Here $k_{01} = \omega_{01}/c$, where ω_{01} is the ion plasma frequency.

Near the plasmapause we introduce orthogonal coordinates (s,x,y) such that the coordinate lines $x = \text{const}$ and $y = \text{const}$ coincide with field lines in the geomagnetic field. We assume that the y axis runs along the magnetic shells, and the x axis across them. We adopt y as a course variable. Using the first integral $\omega(\mathbf{k}(t),\mathbf{x}(t))$, we then find from (2.1) and (2.6) the canonical equations

$$\frac{dp}{dy} = \frac{\partial H}{\partial q}, \quad \frac{dq}{dy} = -\frac{\partial H}{\partial p} \tag{2.7}$$

with the Hamiltonian

$$H = [\omega^2 (a + bk_s^2)^{-1} - fk_s^2 - hk_x^2]^{1/2},$$

where $p = (k_s, k_x)$, $q = (s,x)$, $a = A^2/g_y$, $b = \alpha^2/g_s g_y$, $f = g_y/g_s$, $h = g_y/g_x$, and g_i are the nonzero (diagonal) components of the metric tensor.

At this point we ignore the weak y dependence of H, and we transform to action-angle variables. Adopting a model of the medium (i.e., a functional dependence of a, b, f, and h on s and x), we can then analyze (2.7) by the methods of the theory of autonomous dynamic systems. We restrict the present discussion to pointing out the nature of the sole singular point of the system, which is found by equating the right sides of (2.7) to zero. In a dipole magnetosphere with a "gyrofrequency" distribution of the plasma along the field lines $[N(s) \propto B(s)]$, the coordinates of the singular point are $k_s = 0$, $k_x = 0$, $x = 0$, $x = x_0$, where x_0 is found as the solution of the equation

$$N^{-1} \frac{\partial N}{\partial x} = \frac{8}{L_p}. \tag{2.8}$$

Here s is reckoned from the equatorial plane, x is reckoned from the plasmapause (toward the earth), and L_p is the distance from the center of the earth to the equator of the plasmapause. This singular point will be attractive if the following condition holds at it:

$$\frac{\partial^2 N}{\partial x^2} < 0. \tag{2.9}$$

Conditions (2.8) and (2.9) hold near the plasmapause (closer to the earth).

A similar wave duct with an axis at the geomagnetic equator exists in the ionosphere near the maximum of the F2 layer. The refractive index for magnetosonic waves decreases with distance northward or southward from the axis because of the increase in the magnetic field, while it increases upward and downward because of the decrease in plasma density.

We turn now to a description of longitudinal waveguides. Their role in the propagation of MHD waves of the Pc1 type from one hemisphere to the other was first pointed out in Refs. 37 and 38. Among recent studies we will mention Refs. 39–41.

One occasionally hears the idea that Alfvén waves are ducted "better" than magnetosonic waves are (Ref. 42, for example). In the case of quasilongitudinal propagation, that idea is incorrect. It turns out that the conditions for longitudinal ducting are identical for the two types of waves. Furthermore, the equations which describe the shape of the rays of both magnetosonic and Alfvén waves reduce to the ray equation for waves in an isotropic medium.[39]

In the quasilongitudinal approximation, the rays of Alfvén waves thus have no distinguishing features. They do not differ from magnetosonic rays or, in general, from rays of any waves in an isotropic medium. This conclusion is of methodological importance since it allows us to make direct use of the existing results on the geometric optics of isotropic media.

It is convenient to replace Hamilton's equations (2.1) by the eikonal equation

$$(\nabla\varphi)^2=n^2. \tag{2.10}$$

Here φ is an eikonal, and n is the refractive index found from the local dispersion relation. This index generally depends on the frequency, the local wave vector $\mathbf{k} = (\omega/c)\nabla\varphi$, and the coordinates.[30]

In the case of quasilongitudinal propagation in a cold plasma we have the following expression for Alfvén waves (the upper sign) and magnetosonic waves (the lower sign):

$$n^2=\frac{1}{4}[\mathscr{L}(1\pm|\sec\theta|)^2+\mathscr{R}(1\mp|\sec\theta|)^2], \tag{2.11}$$

where $\mathscr{L} = \varepsilon + g$, $\mathscr{R} = \varepsilon - g$; θ is the angle between $\mathbf{B}$ and $\mathbf{k}$, and ε and g are components of the dielectric permittivity. The component g incorporates the gyrotropy of the medium.[1,43] In a two-component plasma we would have

$$\varepsilon=\frac{\omega_{0i}^2}{\Omega_i^2-\omega^2},\quad g=\frac{\omega}{\Omega_i}\varepsilon,$$

where Ω_i is the ion gyrofrequency. The applicability of (2.11) is limited by the inequalities

$$\sin^4\theta \ll \left(\frac{2\omega}{\Omega_i}\right)^2\cos^2\theta,\quad \omega^2\ll\Omega_i\Omega_e. \tag{2.12}$$

If $\omega^2 \ll \Omega_i^2$, we find $\theta^2 \ll 1$ from (2.12), and we can rewrite (2.11) as

$$n^2=(\varepsilon\pm g)\left(1+\frac{\theta^2}{2}\right). \tag{2.13}$$

This expression also holds over a wider frequency range if the condition $\theta^2 \ll 1$ is assumed to be independent.

We now use $\theta \approx \mathbf{k}_\perp/\mathbf{k}_\parallel$, introduce the notation $\varepsilon \pm g = n_\pm^2$, and rewrite (2.13) in the following way:

$$\frac{c^2}{\omega^2}\left(k_\parallel^2+\frac{k_\perp^2}{2}\right)=n_\pm^2. \tag{2.14}$$

We introduce the scale transformation $\mathbf{x}_\perp \to \mathbf{x}_\perp/\sqrt{2}$, $\mathbf{k}_\perp \to \sqrt{2}\mathbf{k}_\perp$, where $\mathbf{x}_\perp$ are the coordinates on the surfaces orthogonal to the field lines of the external magnetic field. In place of (2.14) we then have

$$n^2=n_\pm^2(\omega,\ \mathbf{x}). \tag{2.15}$$

In this form the refractive index (as in an isotropic medium) depends on the coordinates and the frequency but not the orientation of the wavefront.

This approach makes possible a consistent and concise description of a fairly wide range of phenomena involving the propagation of MHD waves in longitudinal waveguides in terms of the geometric optics of isotropic media. One can essentially make use of nearly all the results of that well-developed theory by identifying n with n_+ or n_-, by making a scaling transformation, and by following the conditions for the applicability of (2.11). For example, for plane paraxial rays the equation describing the deviation of the rays from the geomagnetic field line is[39]

$$\frac{d^2x}{ds^2}=-\frac{1}{2}\frac{\partial U}{\partial x},\quad U=\varkappa(s)\,x-\ln n_\pm(x,\ s); \tag{2.16}$$

where $\varkappa$ is the curvature of the axial line. The only distinctive feature of MHD rays is that the potential U is half that in the isotropic case.[44]

For a space plasma, the structure is typically layered and filamentary, with layers and filaments running along the magnetic field lines.[45] In the magnetosphere this structure forms a system of refraction waveguides, along which electromagnetic waves propagate from one hemisphere to the other. Of particular interest for an analysis of MHD waves is a waveguide which runs along the field lines under the arch of the magnetosphere. In it, waves are excited at frequencies ~ 1 Hz as the result of a cyclotron instability of the distribution of radiation-belt protons.[2] The growth rate reaches a maximum at $\theta = 0$; i.e., the conditions for the applicability of (2.16) hold, at least in the excitation region (in the equatorial zone). Using the analogy mentioned above, we can apply to (2.16) the adiabatic-invariant method,[46] in order to find, in a comparatively simple way, the period of the oscillations of a ray along s, the conditions for capture in a channel, the conditions for escape from a channel, etc.[39]

A ray trapped in a longitudinal waveguide moves toward the earth and reaches ionospheric heights. Here the wave is partially absorbed, while it is partially reflected back into the magnetosphere. Judging from observations, this reflected wave can go into the same waveguide, be amplified in the radiation belt again, and reach the ionosphere in the opposite hemisphere.[1] However, the circumstance of importance here is that some of the wave energy is incident on the so-called ionospheric waveguide and propagates horizontally along the earth's surface over a large distance (up to 10 000 km) from the point at which the ray entered the ionosphere. Although this effect is favorable for the observation of MHD signals from distant sources, it requires that we determine the directions of signals in the course of hydromagnetic diagnostics.

The idea of an ionispheric MHD waveguide was introduced by Tepley and Landshoff[47] on the basis of Pc1 observations and model-based calculations. The subsequent development of the theory has been based primarily on numerical solutions of the equations for low-frequency waves in ionospheric layers (see Ref. 12 and the bibliography there).

2.3. Oscillations of magnetic shells

In the range Pc3–5 (2–100 mHz) the lengths of MHD waves are comparable to the dimensions of the magnetosphere, so geometric optics is generally inapplicable. However, there is an approach along which the eikonal depends on two coordinates, rather than three, as was assumed above. In other words, the ray pattern is used along two spatial directions, while the wave (mode) structure of the field is retained along the third. In underwater acoustics, one speaks in terms of horizontal rays and vertical modes.[48] For a reason which will become clear below, we will speak here in terms of transverse rays and longitudinal modes.

Let us examine the oscillations of magnetic shells. We use Maxwell's equations

$$\mathrm{rot}\mathbf{E}=i\left(\frac{\omega}{c}\right)\mathbf{b},\quad \mathrm{rot}\,\mathbf{b}=-i\left(\frac{\omega}{c}\right)\hat{\varepsilon}\mathbf{E} \tag{2.17}$$

with a permittivity tensor $\varepsilon = \mathrm{diag}(\eta,\varepsilon,\varepsilon)$. We introduce the curvilinear coordinates (x^1,x^2,x^3) defined in such a way that the anisotropy axis coincides at each point with the tangent to the x^1 coordinate line. We assume for simplicity that $\hat{\varepsilon}$ does not depend on x^3. For the toroidal mode $\mathbf{b} = (0,0,b_3)$, $\mathbf{E} = (E_1,E_2,0)$ we then find from (2.17) the equation[49]

$$\frac{\partial}{\partial x^1}\frac{g_{22}}{\varepsilon g^{1/2}}\frac{\partial b_3}{\partial x^1}+\frac{\partial}{\partial x^2}\frac{g_{11}}{\eta g^{1/2}}\frac{\partial b_3}{\partial x^2}+\frac{\omega^2}{c^2}\frac{g^{1/2}}{g_{33}}b_3=0, \tag{2.18}$$

where g_{ik} is the metric tensor, and $g = \det g_{ik}$.

In a cold plasma in the low-frequency limit ($\omega \ll \Omega_i$) we have[50] $\varepsilon = \omega_{0i}^2/\Omega_i^2, \eta = -\omega_{0e}^2/\omega^2$, and (2.18) correspondingly becomes

$$\frac{\partial}{\partial s} E \frac{\partial \psi}{\partial s} + \omega^2 \left(F\psi - \frac{\partial}{\partial x} G \frac{\partial \psi}{\partial x} \right) = 0; \tag{2.19}$$

here $s = x^1$, $x = x^2$, and $\psi = b_3$. The coefficients in (2.19) depend on x and s and are given by

$$E = \frac{A^2 g_{22}}{g^{1/2}}, \quad F = g_{33}^{1/2}, \quad G = \frac{g_{11}}{k_{0e}^2 g^{1/2}}.$$

The transverse dispersion which arises from the electron inertia is taken into account in (2.19). In the limit $k_{0e} \to \infty$, Eq. (2.19) becomes the Dungey equation.[51]

Equation (2.19) has an interesting feature: The small parameter is found only in the transverse operator. In geometric-optics terms, the motion of the ray is fast along s and slow along x. This circumstance suggests a way to solve Eq. (2.19). Since the small parameter appears in a nonuniform way in the higher derivatives, the switch to the short-wave asymptotic behavior here will generally not be accompanied by a switch to the high-frequency limit. It becomes possible to study low-frequency oscillations by making use of the computational advantages of the short-wave approximation.[49]

For greater clarity we consider the following equation instead of (2.19):

$$\frac{\partial}{\partial s} \frac{E}{\omega^2} \frac{\partial \psi}{\partial s} + \mu \frac{\partial^2 \psi}{\partial x^2} + \psi = 0. \tag{2.20}$$

This equation retains all the basic features and all the complexities of Eq. (2.19), but it is more convenient for analysis. In addition, Eq. (2.20) makes it possible to incorporate the thermal motion of the particles at a qualitative level. For this purpose we need to set $\mu = -1k_{0e}^2$ in the cold plasma and $\mu \approx r_i^2$ in the hot plasma and to assume $k_\perp^2|\mu| \ll 1$ for uniformity in the two cases (here r_i is the gyroradius of the thermal ions).

We supplement (2.20) with a simplified boundary condition at the ionosphere:

$$\left.\frac{\partial \psi}{\partial s}\right|_{s=\pm s_0} = 0. \tag{2.21}$$

Yet another limitation, which in this case completely determines the spectrum, is the condition that the field does not grow exponentially as $x \to \pm\infty$.

Wishing to retain the mode structure of the field along s and thus not to take the high-frequency limit, we will attempt to use a version of perturbation theory to find the eigenfrequencies.[49] This version of perturbation theory is based on a breakup of the system into fast and slow subsystems.[52]

We first find the solution of the Dungey problem

$$-\frac{\partial}{\partial s} E(s, x) \frac{\partial \varphi_n}{\partial s} = \lambda_n \varphi_n, \quad \left.\frac{\partial \varphi_n}{\partial s}\right|_{s=\pm s_0} = 0, \tag{2.22}$$

where $\varphi_n(s,x)$ and $\lambda_n(x)$ are the eigenfunctions and eigenvalues of the longitudinal operator, which depend parametrically on the slow variable x. We seek a solution of our original problem, (2.20), (2.21), as an expansion in φ_n. We then use a method of successive approximations. The zeroth-approximation equation for the coefficients of the expansion of ψ in φ_n is

$$\mu \frac{d^2 a_n}{dx^2} + \left(1 - \frac{\lambda_n}{\omega^2}\right) a_n = 0.$$

This is an equation of the Schrödinger type. Following Ref. 50, we find its semiclassical solutions, and we find the spectrum from the quantization condition[49]

$$\int_{x_1}^{x_2} \frac{dx}{\mu^{1/2}} \left(1 - \frac{\lambda_n(x)}{\omega^2}\right)^{1/2} = \pi\left(\nu + \frac{1}{2}\right). \tag{2.23}$$

The mirror points $x_{1,2}$ are found from the vanishing of the expression in the radical; between the mirror points, this expression must be positive. It follows that natural Alfvén oscillations exist and that in this case they have a discrete spectrum $\omega_{n\nu}$ ($n = 1,2,...$; $\nu = 0,1,...$) only if we have $\mu \neq 0$, and the Dungey spectrum λ_n has a maximum ($\mu < 0$) or a minimum ($\mu > 0$) as a function of x (Ref. 55).

If $\mu = 0$, then there are no natural oscillations according to Refs. 53, 54 (see also Ref. 1). In this context one sometimes runs into the assertion that natural oscillations with a continuous spectrum exist (Refs. 55, 56), but that assertion contradicts the representation of natural oscillations as oscillations for which the frequency is determined by the system itself, not by an external agent (Ref. 57). Clearly, this entire situation is related in a definite way to the pathological behavior of Alfvén rays with $\mu = 0$, as mentioned above.

There is another way to derive (2.23), by using the representation of rays and modes in a nearly layered medium.

By "nearly layered" we mean a medium whose properties vary rapidly along some single coordinate.[48] It might appear that in our case this would be the coordinate x, since the plasma easily spreads out along the magnetic field lines. Actually, it is the coordinate x. The sort of rotation through $\pi/2$ of the stratification direction stems from the effect of the small parameter μ in (2.20). All this becomes obvious when we introduce new coordinates, in which distances along x are stretched out by a factor of $\mu^{1/2}$: $\xi = x/\mu^{1/2}$. The Alfvén velocity $A(s, \mu^{1/2}\xi)$ will depend on the transverse coordinate only through the combination $\mu^{1/2}\xi$, and the small parameter in front of the transverse operator in (2.20) disappears.

Having established that fact, we can construct an asymptotic theory for transverse rays and longitudinal modes, repeating the corresponding construction from underwater acoustics nearly word for word.[48] In the zeroth approximation we find an eikonal equation, from which we in turn find (2.23).

Figure 3, borrowed from Ref. 48, shows the L dependence of the Dungey spectrum. The characteristic bending of the curves results from the transition from dense plasma to rarefied plasma as the plasmapause is crossed; in this case, the plasmapause is at $L = 4$ (Refs. 1 and 12). Near the plasmapause we presumably have $\mu > 0$, so that the natural oscillations are apparently concentrated near the minima of the curves, and the $L = 4$ shell.

Outside this region there are no natural oscillations, but there are so-called Alfvén resonances: Oscillations of magnetic shells which are not natural oscillations. Resonances are excited by bulk (magnetosonic) waves which penetrate into the magnetosphere from the interplanetary medium, by surface waves propagating along the magnetopause, and also by sources inside the magnetosphere.

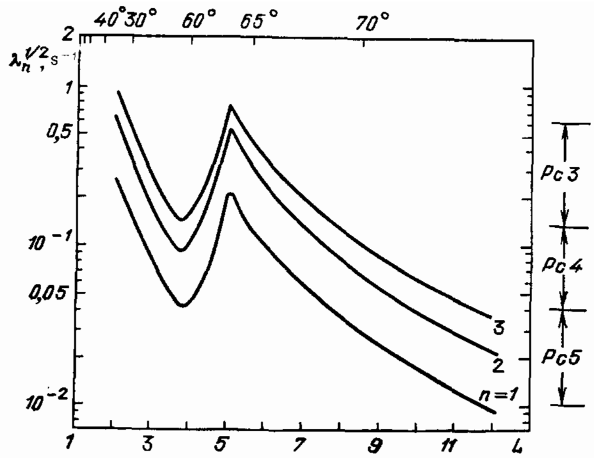

FIG. 3. The Dungey spectrum as a function of the magnetic-shell parameter L (Ref. 58). The upper scale shows the latitude at which the magnetic shell intersects the earth's surface.

The idea of Alfvén resonance was introduced by Hasegawa and Chen[9] and Southwood.[10] This idea is frequently used in hydromagnetic diagnostics. At the same time, this idea cannot be judged to be a simple one. It touches on conceptual questions in the theory of wave propagation. Not surprisingly, the widespread use of this idea of a resonance of field lines sometimes leads to errors and misunderstandings.

As a result, one would like to find some methodological tools for bringing some clarity to the picture of field-line resonances. Here we will work by analogy with Fösterling's problem of the oblique incidence of an electromagnetic wave on a slab of an isotropic dielectric. Analysis of the common and distinctive features of the two problems makes it possible to eliminate the slightest uncertainty regarding the question of a resonance of field lines.[59] The analogy is of heuristic value since Fösterling's problem is rich in physical content and has been studied thoroughly.[29,33]

Let us examine MHD waves in a plane-layer medium. We introduce Cartesian coordinates as shown in Fig. 4. The field lines of the external magnetic field run perpendicular to the plane of the figure. The plasma is inhomogeneous along x; $A(x)$ is a monotonic function, which smoothly converts into a constant as $x \to -\infty$. There are then no natural oscillations, but there can be a special forced oscillation of field lines, at a frequency set by an external force, while the localization along x is determined by the position of the Alfvén resonance.

We specify the incident field to be a plane magnetosonic wave which is propagating from bottom to top. Figure 4 shows a projection of the ray onto the x,y plane in the case in which $A(x)$ is an increasing function. We adopt the following y and z dependence in the incident wave: $\exp(ik_\parallel z + imy)$. By virtue of the homogeneity of the medium along y and z, the total field will have the same dependence; for example,

$$b_z = \psi(x)\exp(ik_\parallel z + imy).$$

The equation for ψ is

$$\varepsilon \frac{d}{dx}\left(\frac{1}{\varepsilon}\frac{d\psi}{dx}\right) + (\varepsilon - m^2)\psi = 0, \qquad (2.24)$$

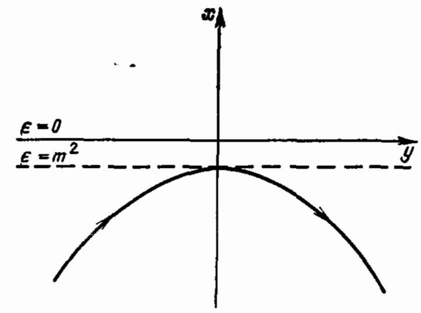

FIG. 4. Incidence of a magnetosonic wave on a slab of inhomogeneous plasma.

where $\varepsilon = (\omega/A)^2 - k_\parallel^2$. The transverse components of the magnetic field are expressed in terms of the longitudinal component by means of

$$b_x = i\frac{k_\parallel}{\varepsilon}\frac{\partial b_z}{\partial x}, \quad b_y = -\frac{k_\parallel m}{\varepsilon} b_z. \qquad (2.25)$$

Aside from changes in notation, Eq. (2.24) is the same as the corresponding equation of the Fösterling problem of the incidence of an H wave on a slab of an isotropic dielectric. We can thus immediately write a solution of (2.24).

In the region $\varepsilon(\chi) > m^2$ the field is described by the geometric-optics method; it is a superposition of the incident and reflected magnetosonic waves. The reflecting surface $\varepsilon(\chi) = m^2$ is a caustic. In the caustic shadow $[\varepsilon(\chi) < m^2]$ the field usually falls off exponentially with distance from the caustic. In the case at hand, a singularity arises at $\varepsilon(\chi) = 0$, against the background of an overall decrease. This singularity is an Alfvén resonance.

At resonance, energy of the incident wave is absorbed if the medium has any, arbitrarily small deviation from a conservative nature. In the Fösterling problem, this effect was found in Ref. 60. With some trivial changes, this effect can be transferred to the MHD problem.

The entire discussion below is taken from Ginzburg's monograph,[33] which used the analogy described above.

We assume that ω and $k_\parallel$ are fixed, while m varies.[2)] At $m = 0$, the singularity at the point $\varepsilon = 0$ disappears. This is obviously the case, since the value $m = 0$ corresponds to normal incidence on the slab. As m increases, the field singularity also disappears, sooner or later. The reasons are the increase in the distance between the mirror point $\varepsilon = m^2$ and the resonance points $\varepsilon = 0$, the exponential weakening of the field beyond the reflection point, and the presence of absorption—even if extremely slight—in a real system. It is thus natural to expect that the resonance would be manifested most obviously at a certain intermediate value of m, not very large, but also not very small.

To seek the n dependence of the Alfvén resonance we do not have to solve the MHD problem. A corresponding solution was found in Ref. 33; all that we need to do is make the necessary changes in notation in that solution.[59]

In the description of one of the methods of magnetospheric diagnostics we need an explicit expression for the field near the resonance. We set $\varepsilon = -ax$, where $a > 0$. The coordinate of the mirror point is $x_0 = -m^2/a$. As $x \to +0$, we then have

$$b_x = -ib_z \frac{k_\parallel m^2}{a} \ln(mx), \quad b_y = b_z \frac{k_\parallel m}{a} \frac{1}{x}, \qquad (2.26)$$

where b_z is taken at $x = 0$. If $x \to -0$, then we need to make the substitution $\ln x \to \ln|x| - i\pi$ in (2.26), in accordance with the limiting-absorption principle. The nonconservative nature of the situation can be dealt with by introducing an effective parameter Δ: $x \to x - i\Delta$. The parameter Δ is the distance over which $|b_y|^2$ falls off by a factor of two in moving away from the resonance.

2.4. Rytov's formula

In the solution of hydromagnetic-diagnostics problems, the boundary conditions at the earth's surface are usually adopted in the form $E_t = 0$, $b_n = 0$, where E_t (b_n) is the tangent (normal) projection of the electric (magnetic) field of the oscillations. In other words, the earth is treated as an ideal conductor.[1,3] In the present paper, in contrast, we take the finite conductivity of the earth into account explicitly. Furthermore, we take the horizontal inhomogeneity of the earth into account in an approximate way. For this purpose we use the theory of the skin effect,[20] one of whose equations is

$$b_n = -\frac{c}{i\omega} \operatorname{Div}(\zeta \mathbf{b}_t); \qquad (2.27)$$

where ζ is the surface impedance of the earth, and Div is a surface divergence.

The diversity of the applications of (2.27) in geoelectromagnetism is based on a simple idea. Let us assume that some of the quantities in (2.27) are known from experiments, while others are unknown, to be determined. Equation (2.27) then makes it possible either to calculate the unknown quantities immediately or to find certain limits on them. In other words, we will read Eq. (2.27) in various ways. In order to implement this program, of course, we must introduce some additional assumptions, as we will discuss below.

Let us give an elementary derivation of (2.27). We assume for simplicity that the earth's surface is planar, and we assume that the Leontovich approximate boundary conditions hold at the surface[29]:

$$E_x = \zeta b_y, \quad E_y = -\zeta b_x. \qquad (2.28)$$

(Here and below, the z axis is directed downward.) From the induction equation

$$\operatorname{rot} \mathbf{E} = i \frac{\omega}{c} \mathbf{b}$$

we find

$$b_z = \frac{c}{i\omega} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right).$$

Substituting (2.28) into this equation, we find (2.27) in the form

$$b_z = i \frac{c}{\omega} [(\mathbf{b}_t \nabla \zeta) + \zeta \operatorname{Div} \mathbf{b}_t]. \qquad (2.29)$$

A more general derivation of (2.29), from first principles, and with an explicit statement of the applicability conditions, is given in Ref. 20. In geoelectricity, Eq. (2.29) is widely used in the simplified form

$$b_z = i \frac{c}{\omega} \zeta \operatorname{Div} \mathbf{b}_t, \qquad (2.30)$$

i.e., without the first term on the right side of (2.29), which reflects the horizontal inhomogeneity of the earth (see Ref. 18 and the bibliography there). Interestingly, Eq. (2.30) is often used erroneously to study specifically the horizontal inhomogeneity; in the process, experimental facilities which permit the use of (2.29) are used.

In the present paper we are interested not in the subtle details associated with the conditions for the applicability of (2.29) (see the detailed analysis in Ref. 20) but simply in the fact that Eq. (2.29) is of broader applicability than (2.30). The range of applicability of (2.29) is even wider than that of the Leontovich boundary conditions. To demonstrate the point, we note that Eq. (2.29) does not change if we replace (2.28) by

$$E_x = \zeta b_y + \frac{\partial G}{\partial x}, \quad E_y = -\zeta b_x + \frac{\partial G}{\partial y}, \qquad (2.31)$$

where G is some function of x,y, and ω which is linear in the field and otherwise arbitrary. For example, if the impedance surface proper, $z = h > 0$, with an impedance ζ_0, is coated with a high-resistivity layer of variable thickness $h(x,y)$, then at the surface of the layer ($z = 0$) we would replace (2.28) by (2.31) with $G = hE_z$ and $\zeta = \zeta_0 - i\omega h/c$, where E_z is the vertical component of the electric field directly below the $z = 0$ surface. (See Ref. 22 for more details.)

3. DENSITY OF THE MAGNETOSPHERIC PLASMA

3.1. Diagnostics based on signal repetition period

We consider a wave packet with a carrier frequency ω in one of the longitudinal waveguides in the magnetosphere. In the geometric-optics approximation, the group-delay time (multiplied by two) for the delay of a packet of Alfvén waves between the ends of the waveguide is[2]

$$\begin{aligned} \tau(\omega) &= 8\sqrt{\pi} \frac{a}{B_e} L^4 \rho_0^{1/2} I(\omega), \\ I(\omega) &= \int_0^{x_0} \frac{1 - (\omega/2\Omega(x))}{[1 - (\omega/\Omega(x))]^{3/2}} (1 - x^2)^{3-(\nu/2)} \, dx, \\ \Omega(x) &= \Omega_0 \frac{(1+3x^2)^{1/2}}{(1-x^2)^3}, \quad x_0 = \left(1 - \frac{1}{L}\right)^{1/2}; \end{aligned} \qquad (3.1)$$

here $a \approx 6.37 \times 10^8$ cm is the earth's radius, $B_e \approx 0.315$ G is the magnetic field at the equator, and Ω_0 and ρ_0 are the proton gyrofrequency and the density of the plasma at the vertex of the field line which serves as the waveguide axis. The quantity L is the distance from this vertex to the center of the earth (in units of a). A dipole approximation of the geomagnetic field has been used. The plasma density distribution along the axis of the waveguide is taken to be $\rho(\chi) = \rho_0(1 - x^2)^{-\nu}$, where $x = \sin\phi$, and ϕ is the geomagnetic latitude.

A numerical calculation shows that the integral I is a "universal" function of the ratio ω/Ω_0 in the sense that it is only slightly sensitive to L and ν, i.e., to the latitude at which the waveguide axis intersects the surface of the earth and to the nature of the plasma distribution along the waveguide.[1] The physical meaning of this circumstance is that the dispersion of the signal is dominated by the equatorial region of the trajectory, where the difference $(\Omega - \omega)$ is at a minimum.

With the quantity τ we associate the repetition period of the Pc1 signals[31] (Refs. 1, 2, and 12). We thus assume that ω and τ are known from observations. To estimate ρ_0 from

(3.1) we need more information about the parameter L. Since we have $\rho_0 \propto L^{-8}\tau^2$ the estimate of L must meet some elevated accuracy requirements.

We have placed special emphasis on the circumstance that the Pc1 signals propagate horizontally in the ionospheric waveguide at large distances from the end of the longitudinal waveguide. The latitude of the observation point thus tells us nothing about L. (We will not go into a discussion of studies which ignore this obvious point.) One can attempt to find L indirectly, from the spreading of the signals, which in this case is manifested as a change in the slope with respect to the time axis of structural elements in the dynamic Pc1 spectrum. The corresponding technique, which is similar to the dispersion-analysis technique used for atmospheric whistlers, was proposed by Watanabe[61] and Dowden and Emery.[62] There are two points to be noted here, however, First, the width of the Pc1 band is small, in contrast with the whistler band. The accuracy of estimates of L is correspondingly low. Second, it has been found[63–65] that the very fact that the dynamic Pc1 spectrum is of a discrete nature is a result of a substantially nonlinear evolution of the wave field. If we can still count on the applicability of an equation like (3.1), we cannot interpret the dispersion of the signals by the linear theory (see also Refs. 1 and 2).

There is a direct method for determining the coordinate of the end of the longitudinal waveguide and thus the parameter L. The idea here is to measure two independent bearings (the directions of the horizontal propagation of the signal) and to determine the point at which they intersect. In general, one would still have to correct for the lateral refracton of the raise in the ionospheric waveguide, but we will assume that the waveguide is a plane-layer waveguide and focus on the Pc1 direction-finding method itself.

Using Rytov's equation, (2.29), we can determine the bearing from Pc1 observations at only a single point.[24–26]

We first use geoelectric methods to measure ζ and $\nabla\zeta$ in the Pc1 range at the observation point. We then measure b_z and $\mathbf{b}_t$ in the same range. Since the signals arrive at the observation point after propagating through the ionospheric waveguide, we replace the surface operator ∇ in (2.29) by $i\mathbf{k}_t$, where $\mathbf{k}_t = (k_x, k_y)$ is the local wave vector of the horizontal propagation. We treat k_x and k_y as unknowns and solve the equation

$$Ak_x + Bk_y + C = 0 \qquad (3.2)$$

with the complex coefficients

$$A = \zeta b_x, \quad B = \zeta b_y, \quad C = \frac{\omega}{c} b_z - i\mathbf{b}_t \nabla \zeta,$$

which are known from experiment. The bearing, as the angle ϑ, between the meridian (the x axis) and the propagation direction, is $\vartheta = \arctan(k_y/k_x)$.

The idea of this method is similar to that of polarization Pc1 direction finding[1] and to directional analysis.[19] It differs from the former in that it does not require the additional hypotheses that the Hall electrical conductivity of the ionosphere is low, etc.; it differs from the latter in that it incorporates the horizontal inhomogeneity of the earth's crust.

Magnetohydrodynamic direction finding is useful not only for determining L in $\rho_0(L)$ diagnostics; knowing k_x and k_y, for example, one can also determine the phase velocity ω/k_t of the horizontal propagation of MHD waves in the ionosphere. The fluctuations in the arrival angle ϑ can then be used to draw conclusions about ionospheric inhomogeneities along the propagation path. Finally, the slow variations in the latitude and longitude of the end of the longitudinal waveguide can be used to draw conclusions about the large-scale electric field, which leads to the convection of magnetospheric plasma.[12,66]

3.2. Diagnostics based on oscillation spectra

The spectrum of magnetospheric MHD oscillations depends on the spatial distributions of the plasma and the magnetic field. Since the magnetic field structure is known, the data from spectral measurements can be used to evaluate the plasma density ρ. Of extreme interest in this regard are Alfvén resonances of geomagnetic field lines. Different parts of their spectrum are formed in different regions of the magnetosphere. It is thus possible to work from the known spectrum to reconstruct not only integral characteristics of the ρ distribution but also local characteristics. For example, from the latitude profile of the resonance frequency one can estimate the plasma density distribution across magnetic shells, $\rho_0(L)$, and from the unequal spacing of the harmonics at a fixed latitude one can evaluate the distribution of ρ along field lines (see Refs. 1, 3, 4, and 12 and the bibliographies there). For this purpose, data from a spectral analysis of geomagnetic pulsations in the Pc3–5 range are used.

The diagnostic procedure is quite simple. If f is the resonance frequency of the first harmonic of the oscillations of shell L, then we have

$$\rho_0(L) = \left(\frac{6,6}{L}\right)^8 \left(\frac{16,3}{f}\right)^2; \qquad (3.3)$$

here ρ_0 is expressed in units of the proton mass, and f in millihertz. The numerical coefficients in (3.3) correspond to the latitude zone near the "geostationary" shell ($L \sim 6.6$). Outside this zone, and for other harmonics, the coefficients will be slightly different. They are found through a numerical solution of the Dangey problem (2.22).

Here, as in diagnostics based on the signal repetition period, the primary difficulty is in estimating L. A wide-band external source excites shells over a wide L interval, and the spatial overlap of resonances causes the observer to detect a fairly wide oscillation spectrum. In this case the problem is to "sort out" the components of the detected spectrum with respect to L. If the source instead has a narrow band, the problem is to determine the value of L for that shell which is resonating at the frequency of the source. (Understandably, the position of the observer will not provide the information required, because of the finite spatial width of the resonance.)

The standard way to solve these problems is as follows: A chain of observatories is set up along a geomagnetic meridian in such a way that the interval of L of interest is spanned. An interpolation of the spectral components of the oscillations is then carried out with respect to L. Among recent studies in this direction we might cite Ref. 67.

We can describe an alternative approach, which starts from an analysis of Rytov's equation and which makes it possible to evaluate $\omega(L)$ over a finite L interval on the basis of observations at only a single point.[26]

We put the observation point in a region in which the electrical conductivity of the earth's crust is horizontally homogeneous.[4)] We detect the east-west component of the electric field, E_y, and the vertical component of the magnetic field, b_z, of the oscillations. We find the distance from the observation point ($x = 0$) to the shell which is resonating at the frequency ω from the expression

$$x_R(\omega) = |E_y|\,|b_z|^{-1}\lambda \sin\varphi. \tag{3.4}$$

Here $\varphi(\omega)$ is the phase difference between the spectral components $E_y(\omega)$ and $b_z(\omega)$, and $\lambda = c/\omega$. We find the function which is the inverse of $x_R(\omega)$, and from it we find $\rho_0(L)$, using (3.3). (The parameter L is related in a known way to x_R; Ref. 1.)

We find (3.4) from (2.29), taking into account the structure of the Alfvén resonances, (2.26). We denote by Δ the width of a resonance; we can then write

$$b_x(x) = b_x(x_R)\,[1 + i(x - x_R)\Delta^{-1}]^{-1}. \tag{3.5}$$

Here we have taken into account the rotation of the polarization through $\pi/2$ as the oscillations pass through the ionosphere.[12] From (3.5) we find

$$\frac{\partial b_x}{\partial x} = \frac{b_x(0)}{x_R + i\Delta} + i\Delta \tag{3.6}$$

at $x = 0$. On the other hand, we have

$$\frac{\partial b_x}{\partial x} = -i\frac{\omega}{c_z^2}\, b_z. \tag{3.7}$$

This equation follows from (2.29) in the case $\nabla\zeta = 0$ and when we note that near the resonance we have Div $\mathbf{b}_t \approx \partial b_x/\partial x$ within terms of the order of $(m\Delta)^2 \ln(m\Delta)$, where m is the azimuthal number, and $m\Delta \ll 1$. Combining (2.28), (3.6), and (3.7), we find (3.4).

In a series of studies which were recently published,[68–77] a pair of observatories separated by about 100 km along latitude and coupled by a telemetry link was used for ρ diagnostics. The idea in Refs. 68–77 was to work from the equality of the amplitudes of the spectral components, $|b_x(\omega)|$, at the two points to find the oscillation frequency of some fixed shell (that shell which runs strictly halfway between the observatories). The method of (3.4) has the following advantage over this method: The results of observations at a single observatory yield information about the resonance frequencies in a finite L interval, rather than at a fixed shell, as in Refs. 68–77, on the basis of observations at two observatories. There is also the obvious technical advantage (telemetry is not required). These advantages are achieved due to the implicit account in (3.4), by means of Rytov's equation, of the electrical conductivity of the earth's crust near the observation point.

4. ELECTRICAL CONDUCTIVITY OF THE EARTH'S CRUST

4.1. Gradient of the surface impedance

We have thus reached the conclusion that Rytov's equation is useful in hydromagnetic diagnostics. Another important field of application is geoelectric sounding. In this case, expression (2.27) is used (in particular) to measure the gradient of the surface impedance of the earth's crust.[22–26]

A trivial approach to the problem of measuring $\nabla\zeta$ is to carry out magnetotelluric sounding and to determing ζ at three or more points. In general, of course, it is not possible to avoid the procedure of multiple-point measurements of ζ. It is nevertheless interesting and useful to know that in certain special cases it is possible to measure $\nabla\zeta$ by observing and analyzing the components of the electromagnetic field at only a single point.

Let us assume that $\nabla\zeta$ in (2.29) is not known, while b_z and $\mathbf{b}_t$ are known quantities at the given point on the earth's surface. To calculate $\nabla\zeta$ we need more information about Div $\mathbf{b}_z$. The simplest soluton is to discard the second term on the right side of (2.29). The simplified equation,

$$b_z = i\frac{c}{\omega}\,\mathbf{b}_t \nabla\zeta, \tag{4.1}$$

can then be used to find[5)] $\nabla\zeta$.

In order to make the transition from (2.29) to (4.1), the length scale of the in homogeneity of ζ must be much smaller than the length scale of the inhomogeneity in $\mathbf{b}_t$. This condition is satisfied, for example, in seismically active regions near geological faults.[22] In this case, Eq. (4.1) has the advantages of simplicity and economy in terms of the number of measurements required. However, it is specifically in this case that violations of the conditions for the applicability of the more general equation, (2.29), are most likely.

For using (4.1) we would ideally have a field which we know at the outset to be transverse (Div $\mathbf{b}_t = 0$). The condition of transversality is satisfied for longitudinal and transverse resonances of the earth-ionosphere cavity resonator.[72]

Longitudinal resonances are excited in the cavity bounded from below by the earth's surface and from above by the lower boundary of the ionosphere. They are often called "Schumann resonances" after the investigator who pointed out their existence in the 1950s. Schumann was also the first to estimate their spectrum and quality factor, and he pointed out that lightning discharges were a possible source of oscillations. The resonance frequencies $f_n \sim nc/2\pi a$ are found from the condition that the circumference of the earth is equal to an integer number of wavelengths. An estimate yields $f_1 \sim 7.5$ Hz, $f_2 \sim 15$ Hz, and $f_3 \sim 22.5$ Hz; these figures are fairly close to the experimental values $f_1 \sim 8$ Hz, $f_2 \sim 14$ Hz, and $f_3 \sim 20$ Hz (Ref. 72).

For Schumann resonances, (4.1) becomes[23]

$$b_z^{(n)} = ia\mathbf{b}_t^{(n)}\,\nabla\zeta\,[n(n+1)]^{-1/2}, \tag{4.2}$$

where a is the earth's radius, and $n = 1,2,...$ is the index of the resonance. At a frequency of 8 Hz the field penetrates a depth of 530 m into the earth, if, for definiteness, we assume the conductivity of the rock to be $10^9\ \mathrm{s}^{-1}$. This result means that longitudinal resonances can be utilized to sound the upper layers of the earth. Since the oscillations are global, and storm sources are operating continuously, a measurement of $\nabla\zeta$ can be carried out at any point and at essentially any time. In discussing the applications of the method here we will limit ourselves to the general statement that it would be useful to have steady-state observations of variations in the horizontal inhomogeneity of the electrical conductivity of the earth near a given point in the region of interest. Observations of this type could provide information about the development of unfavorable geological processes (incipient stages of earthquakes, landslides, etc.).

The frequencies of the transverse resonances of the earth-ionosphere cavity resonator, $f_n \sim nc/2h$, are found from the condition that the vertical distance between the earth and the ionosphere is equal to an integer number of half-waves.[73] At night, the lower boundary of the ionosphere is at a height $h \sim 90$ km; hence $f_1 \sim 1.7$ kHz. At this frequency, the field would penetrate a depth of 37 m into the earth, if the conductivity of the earth is 10^9 s^{-1}. Transverse resonances can thus be utilized to sound the very upper layers of the earth.

Transverse resonances, like longitudinal resonances, are excited effectively by lightning discharges. In contrast with longitudinal resonances, the transverse resonances are not global; they are instead local or, more precisely, regional, since under natural conditions they are observed at distances up to 2000–3000 km from the source. They are known as "tweaks." The dynamic spectrum of a tweak is time-varying: As time elapses, the carrier frequency of the oscillations asymptotically approaches (from above), one of the transverse-resonance harmonics f_n. Correspondingly, the following expression holds asymptotically for tweaks:

$$b_z^{(n)} = \frac{ih}{\pi n} \mathbf{b}_t^{(n)} \nabla \zeta. \tag{4.3}$$

This expression is analogous to Eq. (4.2) for Schumann resonances.

A specialist in geoelectricity should note the analogy of the method for measuring $\nabla\zeta$ which we have justified here through an analysis of Rytov's equation to other, similar methods, known as the "Parkinson plane," the "Wilhelm ellipsoid," the "Wise-Smucker-Porash vector," and so forth (see the review in Ref. 74 and the bibliography there). One must be in agreement here, with one important stipulation: While the methods of Ref. 74 are based on empirical or, more precisely, heuristic considerations, the method at hand is based on the asymptotic theory of the skin effect.[20] In this sense it (first) allows generalizations and (second) does not take us out of the realm of phenomenological electrodynamics. The other methods which were listed above make use of additional geometric objects, which presumably reflect the internal structure of the earth; i.e., they implicitly assume a certain interpretation of the measurements. The use of the method under discussion here of course leads to an interpretation problem, but that problem can be dealt with as a problem independent from the measurements. The measurements, on the other hand, are carried out completely and in a unified way in terms of surface impedance.

4.2. Impedance equation

We assumed above that the unknown quantities in (2.29) were first $\mathbf{k}_t$, then Div $\mathbf{b}_t$, and finally $\nabla\zeta$. We now assume that the surface impedance ζ is unknown, and we rewrite (2.29) in the form

$$A\frac{\partial\zeta}{\partial x} + B\frac{\partial\zeta}{\partial y} + C\zeta + D = 0. \tag{4.4}$$

Equation (4.4) may be thought of as a differential equation for the impedance $\zeta(\omega;x,y)$ under the condition that the coefficients

$$A = b_x,\quad B = b_y,\quad C = \mathrm{Div}\,\mathbf{b}_t,\quad D = i\frac{\omega}{c}b_z$$

are known from observation.[22] This approach generalizes the method of geoelectric sounding, which is widely used today, to the case of a horizontally inhomogeneous earth. Specifically, the algebraic relation[75,76]

$$\zeta = -\frac{D}{C} \tag{4.5}$$

is replaced by a differential equation for the impedance, (4.4).

In order to implement this approach, one clearly needs highly accurate synoptic observations from a fairly dense network of magnetometers. In contrast with the preceding section of this paper, this approach does not require *a priori* information about the structure of the inducing field, since everything that is necessary for the sounding is accumulated during the observation of the field by this network. Such networks exist. For example, in northern Scandinavia there are conveniently situated several tens of well-equipped observatories.[75-77]

In the limits $l_\zeta \ll l_b$ and $l_\zeta \gg l_b$ we can replace (4.4) by (4.1) and (4.5), respectively (here l_ζ and l_b are the length scales of the variations in the impedance and in the magnetic field). These two limiting cases, however, differ greatly in their importance to geosounding. Expression (4.1) provides nontrivial information about the structure of the earth's crust on the basis of observations at one point, while (4.5) requires observations at many points, so that, broadly speaking, the more general approach based on (4.4) can be taken.

4.3. Seismomagnetic waves

The motion of the conducting layers of the earth's crust in the magnetic field of the earth's core during the propagation of seismic waves induces an alternating electromagnetic field. These effects are included in the present review because seismic waves constitute a natural source of geomagnetic pulsations inside the earth. Furthermore, one can work from the results of synchronous detection of seismic and magnetic signals to form an interpretation parameter, whose frequency dependence would contain information about the structure of the geoelectric cut.[78-81]

The expected seismomagnetic effect is fairly large. At first glance this is a surprising result, since the sea-wave effect, which is analogous in many ways, is proportional to the amplitude of these waves and increases with increasing conductivity of the medium.[82] For moderate wave motion, the perturbation of the magnetic field is $\sim 10^{-10}$ T. The conductivity of the earth's crust is an order of magnitude lower than that of seawater, while the amplitude of seismic waves is at least two or three orders of magnitude smaller than the amplitude of sea waves. Nevertheless, the magnetic effects of seismic and sea waves are comparable. The reason is that the condition for freezing in for this circle of problems is

$$\gamma = 2\sigma T\left(\frac{v}{c}\right)^2 \gg 1,$$

where σ is the conductivity of the medium, and T and v are the period and phase velocity of the wave. This condition is not the same as the condition $R_m \gg 1$, where R_m is the magnetic Reynolds number. In both of these cases we have $R_m \ll 1$. However, while we have $\gamma \sim 1$ for seismic waves, we have $\gamma \ll 1$ for sea waves; i.e., in this sense the earth's crust is much more effective than a sea wave in entraining the geomagnetic field in its motion.

Let us treat the earth's crust as a conducting elastic object in an external magnetic field. We know that magnetoelastic waves can propagate in such an object; these waves are described by the self-consistent system of equations of the theory of elasticity with a ponderomotive force and the equations of quasisteady electrodynamics.[83,84] Since the inequality $B \ll (4\pi\alpha)^{1/2}$ holds by a wide margin in the earth's crust, however, the ponderomotive force can be ignored (here α is the shear modulus). In this approximation, magnetoelastic waves propagate at the same velocity as elastic waves but have a different polarization. Specifically, the propagation of a magnetoelastic wave is accompanied not by a deformation of the object but by oscillations of the electromagnetic field.

The deformation field $\xi(\mathbf{x},t)$ can thus be assumed given. In the far zone, we need consider only the surface Rayleigh wave, since bulk waves decay rapidly with distance from the focus of an earthquake, while a Love wave causes a negligible induced magnetic effect, because of features of the polarization.

Let us assume that a Rayleigh wave is propagating along the x axis along the surface of an object which fills the half-space $z \leqslant 0$: $\boldsymbol{\xi} = \mathbf{a}(z)\exp(ikx)$. There is no y dependence of $\boldsymbol{\xi}$, and we have $\xi_y = 0$. We assume a time dependence $\exp(-i\omega t)$, but we will not explicitly indicate it. The components $\mathbf{a}(z)$ are[85]

$$\begin{aligned} a_x &= \varkappa_t u \exp(\varkappa_t z) + kw \exp(\varkappa_l z), \\ a_y &= -iku \exp(\varkappa_t z) - i\varkappa_l w \exp(\varkappa_l z). \end{aligned} \tag{4.5'}$$

Here

$$\varkappa_t = \left(k^2 - \frac{\omega^2}{c_t^2}\right)^{1/2}, \quad \varkappa_l = \left(k^2 - \frac{\omega^2}{c_l^2}\right)^{1/2}$$

$$\omega = c_t k \nu, \quad \frac{u}{w} = -\left(1 - \frac{\nu^2}{2}\right)(1 - \nu^2)^{1/2};$$

ν increases monotonically from 0.874 to 0.955 as the Poisson ratio increases from 0 to 1/2.

In the conducting layers of the earth ($z < 0$) the quasisteady magnetic field $\mathbf{b}$ satisfies the equation[29]

$$(\Delta + p^2)\mathbf{b} = p^2[(\mathbf{B}\nabla)\boldsymbol{\xi} - \mathbf{B}(\nabla\boldsymbol{\xi})], \tag{4.6}$$

where $p = (1+i)(2\pi\sigma\omega/c^2)^{1/2}$. Let us assume that σ depends on only z and that the function $\sigma(z)$ is piecewise-constant. In air ($z > 0$), we can use the equation $\Delta\mathbf{b} = 0$. Furthermore, we have $(\nabla\mathbf{b}) = 0$ in both media. At the surface $z = 0$ and at the interfaces between conducting layers we have the condition that $\mathbf{b}$ is continuous. We substitute the field $\xi(\mathbf{x},t)$ into the right side of (4.6), use (4.5), and seek a solution which vanishes as $z \to \pm\infty$. The solution is determined uniquely by the conditions which we have imposed.

If σ does not depend on z, then at $z \geqslant 0$ we have

$$b_x = -ib_z = b_{x0}\exp[k(ix - z)], \tag{4.7}$$

where

$$b_{x0} = \left(\frac{uT}{q + \varkappa_t} + \frac{wL}{q + \varkappa_l}\right)\left(\frac{p^2 k}{q + k}\right), \tag{4.8}$$

and

$$T = ikB_x + \varkappa_t B_z, \quad L = i\varkappa_l B_x + kB_z,$$
$$q = (k^2 - p^2)^{1/2}, \quad \operatorname{Re} q > 0.$$

If displacement currents are ignored, we would have $b_y = 0$. When these currents are taken into account, it is found that b_y is smaller than b_x and b_z by a factor of about $(4\pi\sigma/\omega)$.

In this model, the magnetic signal $\mathbf{b}$ and the seismic signal $\mathbf{a}$ at the frequency ω, with known $\mathbf{B}$ and $v = \omega/k$ unambiguously determine σ. As the interpretation parameter we could use, for example, the ratio b_x/a_z at $z = 0$.

A more realistic model of the electrical conductivity of the earth should incorporate the stratification, including a sedimentary jacket, a crystalline foundation, and a well-conducting base. In the most general case of a multilayer medium, relation (4.7) remains unchanged, but the expression corresponding to (4.8) becomes extremely complicated. We will thus limit the discussion here to a limiting case which is in a sense opposite to that of a homogeneous half-space. Specifically, we assume that a stratum of sedimentary rock with a conductivity σ has a thickness h such that the relation $kh \ll 1$ holds. We furthermore assume that the nonconducting crystalline foundation is on an ideally conducting base, which lies at a depth $z = -H$, where $H \gg h$ and $kH \gg 1$. We then have[81]

$$b_{x0} = \frac{p^2 hk\,(uT + wL)}{(1 - i)\,k - p^2 h}. \tag{4.9}$$

Expressions (4.8) and (4.9) correspond to the high-frequency ($kh \gg 1$) and low-frequency ($kh \ll 1$) asymptotic cases. The first makes it possible to determine σ, and the second σh; together, they make it possible to determine both parameters of the sedimentary jacket—its conductivity and its thickness—if the frequency dependence of b_x/a_x is known over a sufficiently wide range.

For the earthquake of 28 March 1964, with an epicenter in Alaska, we have $\omega \approx 0.3\ \mathrm{s}^{-1}$, $k \approx 10^{-6}\ \mathrm{cm}^{-1}$, and $a \approx 1$ cm according to observations at Bergen Park (US).[86] We set $B \approx 5\times10^4$ nT, $\sigma = 10^9\ \mathrm{s}^{-1}$, $h \approx 2\times10^5$ cm, and $H \approx 5\times10^6$ cm. The conditions for the applicability of (4.9) are satisfied, and from that expression we find $b \approx 0.05$ nT. This is a small but observable quantity.

We take another example from the field of research on seismoelectromagnetic phenomena which accompany a powerful explosion on the earth. A few kilometers away from the industrial explosion in Khorezm Province on 25 July 1983, surface waves with $\omega \approx 6\ \mathrm{s}^{-1}$, $k \approx 2\times10^{-5}\ \mathrm{cm}^{-1}$, and $a \sim 1$ cm were observed.[87] The conditions for the applicability of (4.8) hold. The conductivity in the vicinity of the explosion is $\sigma \approx 2\times10^9\ \mathrm{s}^{-1}$. With $B \approx 5\times10^4$ nT we then find $b \approx 0.2$ nT.

Note that the induced seismomagnetic signal has a circular polarization in the vertical plane [see (4.7)]. This circumstance distinguishes this signal in a radical way from other magnetic effects of seismic origin, e.g., from the piezomagnetic signal or the magnetostatic signal associated with the motion of magnetic anomalies in the field of the seismic wave. Both these signals are linearly polarized. The specific polarization of the induced seismomagnetic signal can be utilized to discriminate this signal against noise.

Finally, we wish to call attention to the circumstance that the ratio of the horizontal components of the electric and magnetic fields gives us not the earth's surface impedance, as in the Tikhonov-Kan'yar method, but the velocity of a seismic wave[81]:

$$v = -i\,\frac{cE_y}{b_x},$$

where E_y is the coordinate system of the unperturbed surface of the earth.

5. PHENOMENOLOGY OF MAGNETIC STORMS

5.1. Predicting magnetic storms

An intense magnetic disturbance accompanied by an increase in the ring current in the radiation belt is called a "magnetic storm."[12] Diagnostic questions are of foremost importance for the prediction of storms, since in many cases the basic errors in a prediction stem from an incorrect assessment of the current state. With this point in mind, let us examine the prediction of magnetic storms of sudden commencement.

It is of course impossible to diagnose the preflare situation at the sun on the basis of the observations of geomagnetic pulsations alone. The opposite opinion has been expressed,[88] but we will assume that the flare has already occurred, and we take up the question of whether it is possible to work from data on pulsations to predict the time at which the storm will begin, i.e., the time at which the shock front of the flare-associated flux will make contact with the earth's magnetosphere.

Before the front of this flux touches the magnetosphere, seismic oscillations in the magnetic field due to the penetration of hydromagnetic waves into the magnetosphere may be observed at the earth's surface (these waves lead the flare-associated flux). According to the estimates of Ref. 89, the expected frequency of the hydromagnetic precursor is approximately 0.1 Hz, and the average lead time is close to 8 h. The front of the flux is also preceded by a charged-particle flux. The penetration of these particles into the magnetosphere and then into the ionosphere at high latitudes may form a precursor in the form of a riometric-absorption bay.[90]

Once the storm has begun, the prediction problem becomes one of estimating the strength and duration of the storm beforehand. For an accurate formulation of the problem we should choose a model for the evolution of the ring current, which is responsible for the main and recovery phases of a storm.

The complex structure of the magnetosphere and the complex behavior of its constituent structural elements hinder a "microscopic" modeling of a magnetic storm, i.e., a systematic description on the basis of the equations of plasma physics. The microscopic approach does yield an understanding of parts of the overall picture, but if we work from first principles alone then we would be essentially unable to draw an overall picture of, for example, a Dst variation, which is the most important manifestation of a magnetic storm. As in other cases of this sort, we are thus justified in attempting phenomenological modeling.

Constructing a phenomenological model for describing a Dst variation means choosing an evolution equation as simple as possible. Ideally, this choice would be based on physical and geophysical considerations. The parameters of the equation must be found from observations. For example, the familiar RBM model is [91]

$$\dot{D}=q(t)-\alpha D, \tag{5.1}$$

where

$$D=D_0-\mathrm{Dst}+aU\rho^{1/2},$$
$$q=0,\quad E<E_0,$$
$$=\mu(E-E_0),\quad E\geqslant E_0,$$
$$E=-\frac{U}{c}B_z;$$

U and ρ are the velocity and density of the solar wind, B_z is a component of the interplanetary magnetic field, and D is related to Dst in such a way that it is equal in magnitude, and opposite in sign, to the magnetic disturbance due to the ring current. The right side of (5.1) models the sources and sinks which form the ring current of a magnetic storm. The parameters D_0, a, E_0, μ, and α of the model are sought from observations.

The idea underlying short-term predictions (a few hours in advance) can be summarized as follows: The Dst variation can be estimated beforehand by observing the state of the medium near the earth ahead of the front of the magnetosphere by means of a space vehicle.[6] Figure 5 shows an example of such a prediction for the storm of 23–24 February 1984 (Ref. 93). The solid line is the Dst variation, the crosses are predictions based on the RBM model,[91] and the circles are predictions based on one modification of that model.[94]

When the external sources feeding the ring current are turned off or greatly weakened, and the recovery phase (i.e., the final stage of the storm) begins, the researcher is faced with the question of how long this phase will continue. The RBM model estimates the decay time of the ring current to be $\sim 1/\alpha$. It turns out that this estimate can be improved substantially on the basis of the information about the current state of the magnetosphere which is embodied in geomagnetic pulsations. Clear predictive indications of the end of a storm have been found experimentally. For example, a reduced activity of Pc2 and Pi2 pulsations and an increased activity of Pc1 pulsations indicate a brief storm. In contrast, the absence of Pc1 pulsations and a high activity of Pc2 and Pi2 pulsations indicate a disturbance which is a long way from subsiding.[95]

5.2. Stochastic equivalent of the RBM model

In the RBM model, a Dst variation is treated as a signal from the output of some dynamic system. A prediction based

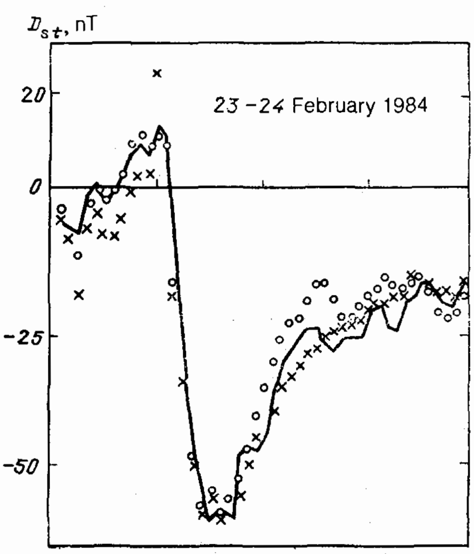

FIG. 5. Example of a Dst variation during the magnetic storm of 23–24 February 1984 (solid line). The crosses and circles show Dst predictions.[93]

on an analysis of this system is expressed by a point estimate, without an indication of confidence interval. However, since the RBM model performs a phenomenological reduction of an indefinitely large number of degrees of freedom of the magnetosphere, the degrees of freedom which have not been taken into account will experimentally create a scatter which converts the deterministic function $D(t)$ into a random function.

As in other, similar situations, it is useful here to simulate the degrees of freedom which have not been taken into account by a random force with a zero mathematical expectation and a δ-function correlation.[96] It is then possible to derive a Fokker–Planck equation for the distribution function $F(D,t)$:

$$\frac{\partial F}{\partial t}+\frac{\partial}{\partial D}[(q-\alpha D)F]=N\frac{\partial^2 F}{\partial D^2}. \quad (5.2)$$

For simplicity here we are assuming that the intensity of the random force, N, does not depend on the state of the system, D. Solutions of (5.2) make it possible to seek the confidence interval of the prediction [which is of the order of $(N/\alpha)^{1/2}$].

The additional (sixth) parameter of the model, N, simulates the effect exerted on the ring current by the rapidly varying processes which play out in the magnetosphere and/or the solar wind. The reader interested in how these processes are monitored experimentally and how numerical values are found for N is directed to Ref. 96. All that we will say here is that data on the intensities of geomagnetic pulsations are useful for estimating N.

5.3. Source and sinks

A modification of an equation of the type in (5.1) and the introduction of additional parameters in the model are widely used in the modeling of Dst variations.[92,94,97] In going from (5.1) to (5.2) we also introduced a new parameter, N. If we wish to go further in this direction, however, it is useful to choose some guiding principle. The theory of critical phenomena appears to be the most appropriate guidance here.[96] The dependence of the source q on the controlling parameter E in the RBM model suggests that we are dealing with a phase transition at a certain critical value $E=E_0$ (Fig. 6). Less obvious is the role played by phase transitions in the formation of sinks [the second term on the right side of (5.1)]. We believe that again in this case the theory of critical phenomena will provide a basis for some modification of the RBM model.

We consider the dynamic system

$$\dot{q}=-\frac{\partial W}{\partial q}, \quad (5.3)$$

so that we can reach an understanding, at the phenomenological level, of how a $q(E)$ dependence of the type in Fig. 6 arises. The question before us reduces to the choice of the form of the potential $W(q, E)$. The postulate

$$W \backsim -\mu(E_0-E)\frac{q^2}{2}+\frac{q^3}{3} \quad (5.4)$$

under the additional restriction $q\geqslant 0$ is sufficient to give us the RBM $q(E)$ model, since the stable critical points of (5.4) are $q=0$ at $E<E_0$ and $q=\mu(E-E_0)$ at $E\geqslant E_0$.

A dynamic treatment of q opens up the possibility of stochastic generalizations of (5.4), and this possibility in turn gives us a basis for a meaningful choice of N is (5.2).

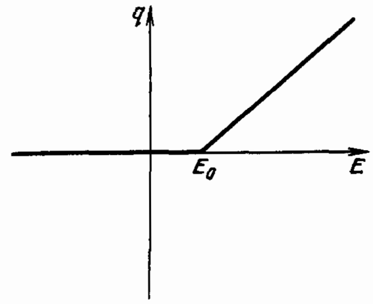

FIG. 6. The source of a Dst variation as a function of the azimuthal component of the electric field in the RBM model (schematic diagram).

However, what besides the empirical correspondence governs the choice of specific expression (5.4) for the potential W? In the pioneering study of Ref. 91, a dependence $q\sim E^2$ was tried as an alternative to $q\sim E$ at $E>E_0$. It was found that the effect was to make the approximation to the experimental points worse. However, the choice of $q\sim E^2$ for the comparison clearly was not motivated by anything. A dynamic approach indicates a more appropriate alternative.[96] For comparison with (5.4) we adopt the Ginzburg-Landau potential

$$W \backsim \eta(E_0-E)\frac{q^2}{2}+\frac{q^4}{4}. \quad (5.5)$$

Then $q=0$ for $E<E_0$ as before, but for $E>E_0$ we have

$$q=[\eta(E-E_0)]^{1/2}. \quad (5.6)$$

This dependence is shown by the solid line in Fig. 7 (Ref. 96). Also shown here are experimental points from Fig. 2 of Ref. 91. The fair agreement between theory and observations is evidence that expression (5.6) is at the least no worse than $q(E)$ of the RBM model.

The Ginzburg-Landau potential is usually chosen on the basis of the symmetry of the system. In the case at hand, we do not have such symmetry considerations to work with, and at this point it is not clear just how we are to seek the "actual" potential $W(q, E)$ unless we simply try various alternatives. At $E>E_0$ we must therefore set

$$q \backsim (E-E_0)^x$$

and attempt to determine accurately the critical index x from experiments. Until we have done this, we can assert no more than the following: The RBM model gives us $x=1$, which corresponds to potential (5.4), while the Ginzburg-

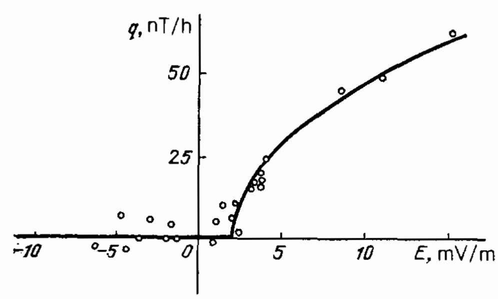

FIG. 7. Bifurcation diagram in a modified RBM model (see the text proper).

Landau model, (5.5), predicts $x = 1/2$, which does not contradict observations.[7]

We turn now to the sinks. The main question here is as follows: How are we to understand, and how are we to model, the well-known empirical fact that the duration of a storm decreases as its strength increases? Afanas'eva and Kalinin[100] state that "very large storms tend to be very brief." The same thought has been expressed in other ways in many papers, although opposite opinions have also been expressed on this matter.

This property is modeled by replacing $\alpha = \text{const}$ by $\alpha(D)$ in (5.1). In Ref. 94, $1/\alpha$ was represented as a piecewise-constant function of D. It is more convenient to make a smooth replacement and to expand $\alpha(D)$ around the origin:

$$\alpha(D) = \alpha_0 + \alpha_1 D + \dots, \tag{5.7}$$

where $\alpha_1 \geqslant 0$. The reason is that in the opposite case the model would describe an unnatural self-amplification of a Dst variation.

We link the meaning of the second term in (5.7) with the excitation of MHD waves as a result of an instability of the ring-current particles. (See Ref. 12, for example, regarding the meaning of α_0.) The particles are scattered by waves and escape from the ring current progressively more rapidly as the wave amplitude increases. The wave amplitude in turn becomes progressively larger as the current increases. Phenomena of this sort fall in the realm of critical phenomena. We do not have room here to describe comprehensively applications of the corresponding theory to the modeling of Dst variations. We will restrict the discussion to two recommendations which follow from an analysis of the self-excitation of waves in the magnetosphere.

First, it is useful to make the replacement $q \to qq_0/(q + q_0)$ in (5.1), where q_0 is one more phenomenological parameter of the system. The idea is that D should reach saturation $\sim q_0/\alpha$ [or $\sim q_0/\alpha_1)^{1/2}$ in the case of (5.7) as $q \to \infty$, in accordance with Kennel and Petchek's idea[102] regarding a "stability limit" of the radiation belt. Such a renormalization of the source is more effective than choosing a complicated D dependence of the sinks.

Second, the theory of critical phenomena makes it possible to reinterpret and possibly improve α_0 with allowance for the structural features of the magnetosphere. As in the RBM model, the source is time-varying, and it is useful to treat the sink as also being time-varying. However, while the time dependence of the sink was implicit in Ref. 94 and in Eq. (5.7), in the version which we are proposing here an explicit dependence would be introduced: α_0 increases as the plasmapause moves away from the earth more rapidly. The situation here is essentially that the critical flux of fast particles from the outer side of the plasmapause is higher than that from the inner side. For this reason, the outward motion of the plasmapause "eats away" particles of the ring current and weakens the Dst variation. The position of the plasmapause can be monitored from the ground by the methods described in $\oint$3.

To conclude this section of the paper we wish to stress that phenomenological modeling of Dst and other goemagnetic variations not only solves applied problems, e.g., the prediction problem, but also enriches and adds depth to the traditional set of problems. In other words, it raises some new questions and points some new directions for interpreting existing results. Finally, the phenomenology makes it possible to formalize some new ideas which follow from a morphological analysis of geomagnetic disturbances.

6. PHENOMENOLOGY OF MAGNETIC PULSATIONS

By analogy with wave phenomena in optics, acoustics, and radiophysics,[102–104] we would naturally expect that fluctuations of the field of geomagnetic pulsations would contain information about the excitation and propagation mechanisms and—a particularly important point for our purposes—about the structure of the magnetosphere. One can cite only a few papers which have allowed for the circumstance that the magnetosphere is a randomly inhomogeneous medium. To a large extent, this situation is due to the difficulty of experimentally studying effects of the fluctuations of the medium and of the wave field in the range of geomagnetic pulsations. With the measurement apparatus available today, it is not possible to take up many interesting problems, e.g., that of fluctuations in the angle of arrival of the radiation.[8)]

There is yet another, and not unimportant, reason for the delay in stochastic studies of pulsations. This is the inadequate level of development of the deterministic theory. Dynamic problems, if they are posed more or less appropriately, i.e., if the nonlinearity of the pulsations and the complex structure of the magnetosphere are taken into account, are not amenable to solution. For the time being we are thus blocked from taking the customary path to analyzing stochastic systems, which is essentially one of replacing the numerical functions in the corresponding deterministic model by random functions and evaluating the probability for some state or other of the system.

With these comments in mind, we should refer to the study of fluctuation and critical phenomena which was begun in Refs. 105–109. Only the "coarse" parameters of the fluctuations, i.e., parameters which could be measured reliably (amplitude fluctuations, group delay, etc.) were selected for study. In an effort to avoid the second difficulty listed above, the deterministic models of the pulsations were replaced by extremely simplified phenomenological models, which were used to formulate some simple problems for choosing between alternative possibilities.

6.1. Self-oscillator or filter?

Geomagnetic pulsations are quite frequently quasimonochromatic oscillations. Two types of models are being discussed in the geophysical literature in an effort to explain these oscillations. We will call these the "self-oscillation" and "filtration" models. In the models of the first type it is suggested that the pulsations arise from a plasma instability, i.e., upon a bifurcation from an equilibrium state of the focus type, with a transition to a nonlinear regime and the formation of a limiting cycle. In theories based on models of the second type it is assumed that the magnetosphere contains selective filters (or amplifiers) which pass narrow bands of the spectrum of noise which penetrates into the magnetosphere from the solar wind.

The carrier frequency and other spectral properties of the pulsations are simulated equally well by the models of the two types. Nevertheless, a choice between the the two types of models can be made by studying flucuations of the amplitude of pulsations. In the case of a self-oscillator, a Gaussian amplitude distribution will be observed at the output, while in the case of a selective filter there will instead be a Rayleigh distribution.

Experimentally, one constructs an empirical distribution of the fluctuations of the pulsation amplitude and compares them with Rayleigh and Gaussian distributions. In the case of a reliable approximate agreement, one can draw a conclusion about the type of oscillatory system which generated the pulsations.

6.2. Black box without an input

The black-box idea always arises where the subject under study is inaccessible to direct observation. In dealing with such entities we advance hypotheses regarding the internal structure; i.e., we construct models as structural and functional approximations of the object. The hypotheses are usually tested against experimental results in an input-output approach. The object is approximated by a purely dynamic model; i.e., the fluctuation phenomena which occur in any real system are totally ignored. The incorporation of fluctuations in the model and the use of the methods of the statistical theory of oscillations open up the possibility of obtaining information from the output signal alone. The diagnostic model of a black box without an input is based on the idea that the distributions of the flucutations in the amplitude and phase in oscillatory systems of various types may be quite different from each other. It is thus possible to work from the output signal to draw certain conclusions about the internal structure and operation of the system. The method pointed out in the preceding section of this paper provides a very simple realization of this idea. At this point we will discuss a more complex example, in which the correlation properties of the pulsations are used to seek dynamic equations describing certain aspects of the oscillation process. In other words, the inverse problem of the statistical theory of oscillations is solved.

The correlation method for studying an uncontrolled self-oscillation system on the basis of its signal was proposed by Gudzenko.[104] This method is being used successfully to analyze the mechanisms which form the cyclic activity of the sun.[110] The ideas of Refs. 104 and 110 were used in Ref. 106 to expand the range of application of the correlation method: It was used to study the oscillation properties of the magnetosphere on a simple empirical basis involving observations of geomagnetic pulsations.

Dynamic equations for modeling pulsations are sought in a class of models with a single degree of freedom. The disturbance of the geomagnetic field is taken to be the signal at the output, and it is assumed that the instantaneous state of the system is characterized by a point in a phase plane. The motion of the imaging point is described by a system of two second-order differential equations.

In addition to the dynamic characteristics (which are to be sought) the system contains fluctuation δ-correlated terms. It is assumed that if fluctuations are ignored the system has a limiting cycle, i.e., an asymptotically stable closed orbit in the phase plane. Fluctuations lead to normal and tangential excursions from the limiting cycle. The dynamic parameters of the system are determined through a correlation analysis of the excursions of the trajectory from the average limiting cycle.

By taking this route one can find the form of the limiting cycle which simulates the pulsations which originate inside the magnetosphere, the rigidity of the system, the deviation of the system from isotropy, the nonlinear distortion factor, the amplitude dispersion, and the phase diffusion. Consequently, the output signal by itself, which has the form of a fragment of a sine wave, contains much nontrivial information about the magnetosphere.

As an alternative to the self-oscillation model, the following model is being studied: an oscillator with friction which is subjected to a resonant external force in the presence of a Langevin source. This formal model corresponds to the idea that the pulsations are of extramagnetospheric origin, with an additional idea about local Dungey-Hasegawa resonances. Waves coming from behind the shock front penetrate into the magnetosphere and act on the resonator in the manner of a periodic force. The additional Langevin force simulates the noise. The corresponding Fokker-Planck equation for the distribution functions of the amplitude and phase of the output signal is used to find an equilibrium solution which makes it possible to formulate a criterion for testing the model.

6.3. Diagnostic applications

Diagnostics of the interplanetary medium ahead of the front of the magnetosphere on the basis of data from ground-based observations rest to a large extent on the idea that geomagnetic pulsations of one type are of extramagnetospheric origin.[5] For the most part, these are pulsations in the Pc3 range (20–100 mHz). However, with anomalously large ($B > 15$ nT) or anomalously small ($B < 3$ nT) values of the interplanetary magnetic field ahead of the magnetospheric front, waves are excited in the Pc2 range (0.1–0.2 Hz) or the Pc4 range (7–20 mHz). These waves can penetrate into the magnetosphere and contribute to the pulsation spectrum observed on earth.[111,112] On the other hand, sources inside the magnetosphere may be activated in the Pc3 range. The net result is that we are faced with the problem of separating the intramagnetospheric and extramagnetospheric pulsations not on the basis of the range to which they belong but on the basis of some independent characteristic.

The diagnostic model of a black box without an input appears to be the most suitable one for solving this problem. According to Ref. 106, the empirical distribution of the amplitude of "typical" Pc4 pulsations is, with a high probability, approximately Gaussian, while a Rayleigh distribution is a poor approximation of the observations. Such oscillations arise more probably inside the magnetosphere than outside it. A further analysis of the phase portrait of the oscillations confirms this conclusion. Incidentally, in the course of this study it was found that there is a weakly defined sawtooth nature in the form of the Pc4 pulsations; this point is of importance for diagnostics of the magnetospheric plasma on the basis of the pulsation spectrum.

In contrast, the fluctuation properties of "typical" Pc3 pulsations correspond to the model of an oscillator subjected to external forces.[107] Pulsations with such properties can be utilized to evaluate the strength of the interplanetary magnetic field B (Ref. 1) and the velocity of the solar wind, U (Ref. 113). In the B diagnostics one takes the frequency, and in U diagnostics the amplitude of the pulsations in combina-

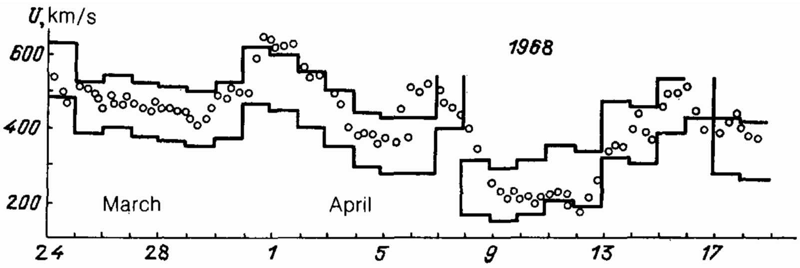

FIG. 8. Broken line—Interval estimate of the velocity of the solar wind on the basis of data on geomagnetic pulsations; points—results of direction observations.[113]

tion with geomagnetic-activity indices. Figure 8, borrowed from Ref. 113, illustrates the situation. The interval estimate of U has been made on the basis of observations of geomagnetic pulsations near Irkutsk. The points show the results of direct measurements on a satellite.

7. CONCLUDING REMARKS

The simplicity and richness of Rytov's formula make possible some nontrivial applications in magnetospheric physics and geology. This is not, however, an exact formula. We have avoided discussing the applicability conditions, but even so it is clear that (2.27) and applications based on it do not work near a shoreline, near a geological fault, or in other such places.

Is it possible to suggest an alternative approach which would retain the idea in general terms but which would not be based directly on (2.27)? With respect to geological applications, the answer is definitely no. Applications of that type are phrased in terms of a surface impedance. They lean heavily on the structure of (2.27) and break down if that structure is violated. In such cases it is necessary to resort to other methods. These other methods are described in detail in the specialized literature.[17–19,114–117]

For magnetospheric applications, in contrast, there is the hope of retaining the general idea of our approach in cases in which Rytov's formula cannot be used. For this purpose we need to study the conductivity distribution in the lower half-space near the observation point by geoelectric methods and then jointly solve the internal problem (for the earth's crust) and the external problem (for the magnetosphere). The solutions are to be joined at the interface. In other words, we should examine the problem of hydromagnetic diagnostics as a mixed problem in the sense stated in the Introduction. The emphasis here is on the preliminary study of the electrical conductivity of the lower half-space. Once this indefiniteness has been removed, it becomes possible to use additional relationships between the components of the electromagnetic field in order to improve the accuracy of magnetospheric diagnostics. Diagnostic applications of (2.27) may be thought of as a simplified model of procedures of this type.

The second set of questions which we have covered in this review concerns fluctuation and critical phenomena. Some interesting possibilities in this direction have been pointed out. In addition, we would like to call attention to Ref. 108, as we have already mentioned. Kalisher and Polyakov have raised the question of diagnostics of inhomogeneities of the magnetospheric plasma on the basis of data on fluctuations of the Pc1 repetition period.

The phenomenological modeling to which we have restricted the present review is sometimes contrasted with (on the one hand) a microscopic description and (on the other) a search for empirical relationships by regression analysis. We have already discussed the difficulties in a microscopic description of global processes of the Dst-variation type. With regard to the regression method, we note that it is capable of solving applied problems, but since it is not oriented toward interpretation it suffers from a semantic vagueness and does not by itself enrich our understanding of geomagnetic phenomena. Even if we take into account the increased accuracy of regression analysis, as discussed in Ref. 97 (among other places), the phenomenology still has the advantage that it gives us a general picture of magnetospheric processes, at the cost of discarding details. Our purposes in these concluding remarks has been to answer the criticism of phenomenological models undertaken in Ref. 97.

1) If the thickness of the plasmapause is ignored, the result is something akin to a whispering gallery.

2) If the problem is reformulated in terms of magnetospheric physics, m becomes the azimuthal number.

3) Here we are ignoring the circumstance that a Pc1 trajectory may be "composite,"[1] i.e., may consist of regions in which energy is carried by magnetosonic waves. (See the preceding section regarding the longitudinal ducting of magnetosonic waves.)

4) In other words, we have $\nabla\zeta = 0$ in (2.29). The generalization to the case $\nabla\zeta \neq 0$ is obvious.

5) In order to calculate both components of the complex vector $\nabla\zeta$, one needs two independent measurements of the magnetic field components for different polarizations of $\mathbf{b}_t$.

6) An estimate of q on the basis of observations of the activity of geomagnetic pulsations on the earth was made in Ref. 92, and a Dst variation was successfully reproduced.

7) A preliminary analysis[99] of 15 magnetic storms yields x values in the interval 0.33–0.75.

8) This problem, at least, can apparently be solved in the near future by means of the "MHD direction finder" described in §3.

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