The Einstein-Podolsky-Rosen paradox for energy-time variables

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A new variant of the Einstein-Podolsky-Rosen experiment is discussed which illustrates the complementarity principle and the indeterminancy relations for the energy and the time of creation of photons emitted as correlated pairs in the decay of a metastable state of an atom or in parametric scattering of light. It is shown that it is not possible *a priori* to ascribe to such photons a definite temporal structure; it acquires an operational meaning only after one of the photons of the pair is recorded by a detector with a definite frequency characteristic. A simple interpretation of the effect is possible by means of an advanced wave emitted by one of the detectors at the instant of the photon being recorded.

INTRODUCTION

About 50 years ago Einstein, Podolsky, and Rosen (EPR) analyzed a thought experiment[1] in which the coordinate and momentum of a particle were measured without any disturbance by the measuring device—in apparent contradiction with the quantum mechanical uncertainty relation. Such experiments were later performed only in Bohm's variant,[2] when dichotomous variables are observed—the projections of the spin of protons or photons.[4–7] There is still great interest in such experiments because it is observed in them that Bell's inequalities are violated,[3] which precludes the possibility of describing them in terms of local hidden variables. In addition, the possibility of performing EPR experiments for observables with a continuous spectrum is also obviously of interest. In Ref. 10 it was shown that such an experiment, in which the transverse components of the momentum and coordinate of a photon are measured, can be performed with the help of parametric scattering of light.[6–16]

In this paper an analogous possible experiment is discussed for energy-time variables (a brief description is given in Ref. 11). The main feature of this variant is that in quantum theory time is not an operator quantity (the uncertainty relation for energy and time is discussed in Ref. 17). Here the object being measured is a wide-band optical field containing two photons. Such a field can be obtained with the help of two-photon, noncascade (without a real intermediate level) transitions in atomic beams[5] or with the help of parametric scattering (Fig. 1). Although the latter method is much more efficient and simpler than the first method, in what follows the more familiar atomic source will be studied (the theoretical description of this experiment, given in Appendix A, is essentially identical to the phenomenological theory of parametric scattering[10]). We note that several other instructive paradoxes are associated with noncascade, two-photon transitions[14]: the possibility of quantum amplification in an equilibrium medium, breakdown of the fluctuation-dissipation theorem, and violtaion of Kirchhoff's law.

The best studied (as far back as the 1930s) example of a source of correlated pairs of photons ("biphotons") with a wide spectrum is the transition of a hydrogen atom from the metastable $2S$ state into the ground $1S$ state ($\lambda_0 \sim 0.12\,\mu$m, $\tau_0 \sim 0.12$ sec), which gives a continuous spectrum from radio frequencies to the UV range (see Ref. 5). In the qualitative description of such transitions it is usually pointed out that both photons are emitted simultaneously and that they have energies $\hbar\omega$ and $\hbar\tilde{\omega}$, where ω is an arbitrary frequency in the band $0 - \omega_0$ and $\tilde{\omega} = \omega_0 - \omega$. It is obvious at the outset, however, that these two properties—simultaneity of creation and definiteness of the energy—are incompatible. It will be shown below that the quantities actually measured must satisfy the uncertainty relation in the form $\Delta\omega \cdot \Delta t \gtrsim 1$, where $\Delta\omega$ is the resolution of the spectral instrument, measuring the frequency and thereby the energy of the photons, and Δt is the average difference of the reduced moments at which the photons are recorded in the two detectors.

1. Experimental procedure. In the proposed experiment the parameters of the photons are measured with the help of two photodetectors, in front of which resonance filters with regulatable central frequencies ω_n ($n = 1$ and 2 is the number of the detector) and transmission bands $2\gamma_n \ll \omega_n$ are placed (Fig. 2). The detectors are assumed to have zero time constants (for this their characteristic band must be much greater than γ_n, while the duration and fluctuations of the delay of the output current pulse must be much les than $1/\gamma_n$).

The experiment consists of repeatedly preparing an atom in the metastable state at the time $t^{(i)} \equiv 0$ and recording the moments at which the pulses $t_1^{(i)}$ and $t_2^{(i)}$ appear (i is the number of the test). The cases when only one detector is

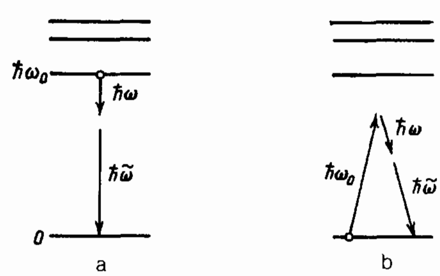

FIG. 1. The two basic methods for preparing a two-photon field with a wide spectrum: with the help of two-quantum transitions in atoms ($\tilde{\omega} = \omega_0 - \omega$) (a) and with the help of three-photon parametric processes in piezoelectric crystals (b).

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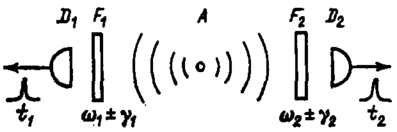

FIG. 2. Layout of an experiment demonstrating the EPR paradox. The atom A emits two photons in opposite directions; the photons are recorded at times t_n by zero-time-constant detectors D_n, in front of which resonance filters F_n with frequencies ω_n and transmission bands $2\gamma_n$ ($n = 1$ and 2) are placed.

triggered or no detector is triggered are ignored. This procedure (with fixed ω_n and γ_n) gives some set of pairs of numbers $t_n^{(i)}$ forming two random variables with values between zero and infinity. Most "reduced" values ($t_n^{(i)} \to t_n^{(i)} - r_n/c$, where r_n is the distance to the detectors) will not, however, exceed the lifetime τ_0 of the metastable state. By performing such series of experiments with different values of ω_n and γ_n it is possible to determine the dependence of the distribution $p(t_1, t_2)$ on the parameters of the filters (we note that the operational meaning of the symbols ω_n, γ_n, and t_n is substantially different: the parameters of the filters are established by the experimenter arbitrarily, while the moments of the reading are random quantities).

2. The quantum theory. The calculation of the correlation function of the intensities for a two-photon field, performed in Appendix A, shows that the joint distribution of the reduced moments of detection of the photons in some approximations depends only on the relative delay $\tau \equiv t_1 - t_2$:

$$p(\tau) = (2\pi\gamma_1\gamma_2)^2 \frac{\theta(\tau)e^{-2\gamma_1\tau} + \theta(-\tau)e^{2\gamma_2\tau}}{\Omega^2 + (\gamma_1+\gamma_2)^2}, \tag{1}$$

where $\Omega \equiv \omega_1 + \omega_2 - \omega_0$ (this distribution is not normalized).

We shall study two asymptotic forms of the function (1). In the case of nonselective detection ($\gamma_n \to \infty$)

$$p(\tau) \to \delta(\tau), \tag{2}$$

(insignificant constants are dropped). This result agrees with the customary assertion that in noncascade two-photon transitions the photons are emitted simultaneously. Of course, in a more accurate calculation the δ function will have a finite width of the order of the characteristic atomic time $1/\omega_0$. In addition, in a real experiment the observed "simultaneity" will be limited by the resolution time of the detectors, which is now of the order of 1 nsec (see, however, Ref. 9).

In the opposite case of highly selective detection, when $\gamma_n \to 0$, (1) gives the uniform distribution

$$p(\tau) \to \delta(\Omega), \tag{3}$$

i.e., now the energies of the detected photons are correlated. Thus the traditional ideas about two *a priori* properties of two-photon emission correspond to two different methods of observation: with $\gamma_n = 0$ and with $\gamma_n = \infty$. In the intermediate cases, however, according to (1) one cannot assign to the detected photons either a definite energy or a definite moment of creation.

We note that in the usual formulation of the experiment (see Ref. 5) the time resolution of the detectors is much greater than the inverse bandwidth of the detected radiation; in addition, the probability of a coincidence—the appearance of two counts in one experiment at any moment in time—is proportional to the integral of $p(\tau)$:

$$p \equiv \int_{-\infty}^{\infty} p(\tau)\,d\tau = \int_0^{\omega_0} |\eta_1(\omega)\,\eta_2(\tilde{\omega})|^2\,d\omega$$
$$\approx \frac{2\pi^2\gamma_1\gamma_2(\gamma_1+\gamma_2)}{\Omega^2+(\gamma_1+\gamma_2)^2} \xrightarrow{\Omega=0} \frac{2\pi^2}{\gamma_1^{-1}+\gamma_2^{-1}}, \tag{4}$$

(here, as in the derivation of (1), it was assumed that the frequency characteristic of the filters $\eta_n(\omega)$ has the form $\gamma_n/(\omega_n - \omega - i\gamma_n)$).

Next we shall study the possible interpretations of formula (1).

3. Is the quantum description of reality complete? This question was singled out in a heading of the famous paper by Einstein, Podolsky, and Rosen. We shall try to apply the logic of the EPR paper to our experiment.

In the limiting cases (2) and (3), studied above, one of the detectors is actually superfluous, since it does not give any new information. Indeed, by observing a pulse in one wide-band detector one can be sure, according to (2), that at the corresponding moment in time the atom also emitted the second photon, i.e., the nonselective detector *measures* indirectly, without any perturbation, the moment at which the second, unabsorbed, twin photon is created.

On the other hand the appearance of a count in one narrow-band detector, tuned to some frequency ω_1, means, according to (3), that a second photon with definite frequency $\tilde{\omega}_1$ was created at an indefinite moment in time in the same experiment, i.e., the selective detector performs an unperturbed measurement of the energy of the second photon.

Thus by varying the parameters of only the detectors one can choose to measure without perturbation either the energy or the creation time of photons created identically. This, *obviously*, means that the photons are characterized *a priori* by both these qualities simultaneously. But this conclusion contradicts the uncertainty relation of the quantum theory; therefore the quantum theory does not give a complete description of reality, and it can, in principle, be supplemented by some "hidden variables," which determine *a priori* all observed properties of the photons.

From the Copenhagen viewpoint this logical chain contains a weak link—the word *obviously* singled out above. The formal error made in Ref. 1 lies in assigning an individual wave function to a separate particle (in a pair of correlated particles), while it can strictly be characterized only by a "mixture" of wave functions, i.e., the density matrix. The uncertainty relation, however, pertains only to systems in the "pure" state, described by one wave function. The impossibility of describing EPR experiments (for polarization-correlated photons) on the basis of a wide class of theories with hidden parameters was later proved experimentally,[3–7] i.e., instead of supplementing quantum mechanics with hidden variables the EPR paper by an irony of history precluded this possibility.

4. Semiclassical model of photons. Bell's inequalities[3] are apparently not applicable to the experiment under discussion, and the experiment explicitly contradicts only one concrete model, which, however, is widely employed to study quantum optical effects.[1)] In this model the photons are described by classical wave packets or trains with integrated energy $\hbar\bar{\omega}$, where $\bar{\omega}$ is some average frequency (a quasimonochromatic field is usually studied). Stochasticity is introduced purely classically—it is assumed that there exists an ensemble of packets with different parameters: the time and direction of emission, the shape of the envelope, the polarization, etc. The packet propagates, is diffracted, and participates in interference (according to Dirac—only "with itself") according to classical electrodynamics (see, for example, Refs. 15 and 18).

The specific quantum properties are introduced essentially only in order to describe the process of detection in the form of a "reduction postulate": when a photon is observed at the point $\mathbf{r}, t$ the field in all space vanishes instantaneously (so that it cannot be observed at two points), and this event always occurs in a random, unpredictable manner with a probability proportional to the local energy of the field at $\mathbf{r}, t$ (averaged over the optical period of the oscillations). It should be noted that this postulate pertains only to existing "energy" photodetectors which annihilate the photon; in the last few years the possibility of "unperturbed" measurements of the field in which the energy or number of photons is conserved has been under discussion.[19] With the help of such methods it is in principle possible to observe even the "track" of one photon—analogously to the track of a charged particle in a Wilson cloud chamber (of course, the trajectory of the photon will be rectilinear only in the approximation of geometric optics: the detection of a photon by a detector with transverse size a "smears" the further path of the photon over the diffraction angle a/λ.

The longitudinal and transverse extent of a photon packet are only statistical quantities—the coherence length and the radius, measured in interferometers with the experiment repeated many times under macroscopically identical conditions. These parameters are determined theoretically in terms of the first-order correlation function of the field.[14,15,30] This model give a convenient and apparently adequate description of all known single-photon effects, observed with the help of energy photodetectors, including the famous double-slit experiment demonstrating the "wave-particle" duality.

In the case of two-photon experiments two packets with defininte parameters must obviously be studied in each test. We shall show that the numerator and denominator of formula (1) can be described *separately* in an elementary fashion in terms of photon packets. We first assume that an atom emits simultaneously two short δ pulses (Fig. 3). Passing through resonance filters they are transformed into quasimonochromatic packets with exponential envelopes. Assuming that the probabilities $p_n(t_n)$ that the photons are recorded by the detectors are proportional to the intensities $\theta(t_n)\exp(-2\gamma_n t_n)$, it is easy to show that the probability density for the difference of the detection times

$$p(\tau)=\int\limits_0^\infty p_1(t_1)\,p_2(t_1-\tau)\,dt_1, \tag{5}$$

has the form of the numerator in (1). If, however, it is assumed (as is usually done) that in each experiment the atom emits two monochromatic waves with random correlated frequencies ω and $\tilde{\omega}=\omega_0-\omega$, then the intensities $I_n(\omega)$ at the outputs of the resonance filters have the form $|\eta_1(\omega)|^2$ and $|\eta_2(\tilde{\omega})|^2$. Let the probability for the emission of a pair be

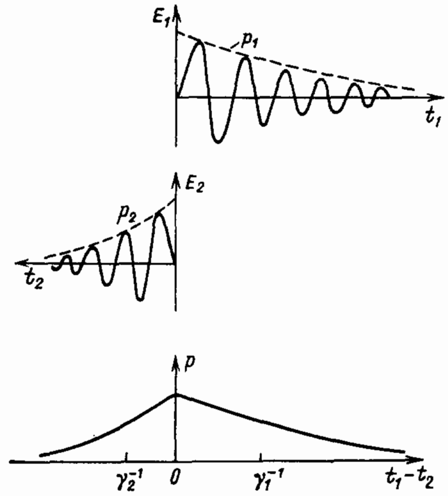

FIG. 3. Explanation of the dependence of the probability for observing a pair of photons p at the moment of detection t_n as a result of impact excitation of filters by photon particles.

constant in the interval $0-\omega_0$; the probability for detecting the pair is then proportional to the integral of the product $I_1(\omega)I_2(\omega)$ over this interval. Letting the limits of integration pass to $\pm\infty$ we obtain the denominator of the formula (1) [compare (4)].

At the same time it is hardly possible to derive the entire quantum formula (1) with the help of solely the model of classical packets with random parameters. Thus if it is assumed that an atom emits packets of two types—short and long, then the distribution $p(t_1,t_2)$ will acquire a "pedestal" which contradicts (1). For example, in the case $\gamma_n=0$ short pulses will give at the output of the filters semiinfinite sinusoids; this will lead to a uniform distribution that is independent of t_n and ω_n. Long monochromatic pulses in the case $\gamma_n=\infty$ will give the same result. An analogous "pedestal" also appears in the classical description of interference of intensities[8,12,21] and polarization-correlation of intensities.[7] Thus it can be assumed that the distribution (1) reflects the duality of photons: the numerator corresponds to photons as particles and the denominator corresponds to photons as waves.

We shall study the correspondence between the packet model and quantum field theory. The single-photon state of a field with definite polarization and direction of propagation is described by the wave function

$$|t\rangle^{(1)}=\int\limits_0^\infty g(\omega)\,|1\rangle_\omega e^{-i\omega t}\,d\omega; \tag{6}$$

here $|1\rangle_\omega$ is the state of one mode with definite energy $\hbar\omega$. We note that (6) describes a state with a definite number of photons (equal to one) and with indefinite energy. (The reverse situation—the state with definite energy and indefinite number of photons—is also possible.[14]) In the semiclassical description this state corresponds to a packet with a definite time dependence, determined by the Fourier transform of the function $g(\omega)$. The function $g(\omega)=\delta(\omega-\omega_1)$ corresponds to a monochromatic wave, $g(\omega)=\text{const}$ corresponds to a pulse of the type $\delta(t)$, and the intermediate cases correspond to a quasimonochromatic wave having a spectrum of width $\Delta\omega$ and localized in time with uncertainty $\Delta t \sim 1/\Delta\omega$, following from the properties of the Fourier transform. When in (6) summation over different directions of propagation of plane waves is taken into account a definite spatial structure equivalent to a classical packet in the transverse direction is added.

In the case of a two-photon field a wave function of the type

$$|t\rangle^{(2)} = e^{-i\omega_0 t} \int_0^{\omega_0} f(\omega)|1\rangle_\omega |1\rangle_{\tilde{\omega}}\, d\omega, \quad (7)$$

must be employed instead of (6) (here we have in mind the limit $t \gg \tau_0$, when the atom is known to have passed into the ground state transferring energy $\hbar\omega_0$ to the field). Now one can no longer assign an individual time structure to the photons: the function $f(\omega)$ characterizes the general properties of both photons. In the general case it is analogously impossible to assign to the photons an individual polarization,[4–7] spatial structure,[10] and phase of oscillations.[12]

5. Copenhagen interpretation. It is still nonetheless possible to assign to a separate photon in some formal sense an individual wave function and a classical structure—but only after a count appears in one of the detectors, for example, in detector number 1. In so doing, according to the Copenhagen interpretation, partial *reduction* of the total wave function of the system occurs, i.e., the objective information about the possible results of other measurements changes: now it may be assumed that the atom is known to be in the ground state, while depending on γ_1 the second photon is in a state either with a definite time of creation or with a definite energy or in some intermediate state.

Thus in the Copenhagen interpretation the term *measurement* sometimes can also mean preparation of a system with a known wave function. In so doing we transfer (conceptually) detector 1 from the measurement part of the experimental apparatus into the preparatory part. Detectors with $\gamma_1 = 0$ and $\gamma_1 = \infty$ prepare photons with different wave functions, and therefore there is nothing surprising in the fact that $\Delta\omega_2' \cdot \Delta t_2'' \ll 1$ follows from (2) and (3). It may be assumed that a narrow-band detector prepares *photon waves* while a wide-bnad detector prepares *photon particles*. The initial paradox of EPR arises only if the uncertainties $\Delta\omega$ and Δt are referred to one and the same particle (or to particles in the same state).

At the same time for fixed γ_n the formula (1) satisfies the uncertainty relation, if $\Delta\omega$ and Δt are interpreted as the width of the maximum of (1) with respect to ω_2 and t_2 (or, equivalently, ω_1 and t_1), respectively. We shall determine this width at half maximum; then

$$\Delta\omega = 2\gamma_1 + 2\gamma_2,$$

and

$$\Delta t = \left(\frac{1}{2\gamma_2} + \frac{1}{2\gamma_2}\right) \ln 2,$$

so that

$$\Delta\omega \cdot \Delta t = \ln 2 \cdot (\varepsilon + \varepsilon^{-1})^2 \geqslant 4 \ln 2, \quad \varepsilon \equiv (\gamma_1/\gamma_2)^{1/2}. \quad (8)$$

We recall that we are talking only about the interpretation of the quantum formalism with the help of a system of some convenient terms and concepts. The distribution (1) is the only consequence of the rigorous quantum theory that can be checked experimentally (with the help of the procedure described above). For this reason when we talk about *reduction* (of a two-photon state into a one-photon state) or about *preparation* of photons, in an experiment this means only that the conditional probability $p(t_2|t_1)$ is measured, i.e., only a subensemble of tests with some fixed t_1, ω_1, and γ_1 is taken into account. In the theory, however, these terms mean that it is possible to determine some one-photon state which describes the same subensemble (see Ref. 10 for a discussion of an example close to our problem). This approach has the advantage that the one-photon state (unlike the two-photon state) admits a convenient "semiclassical" representation in the form of a packet.

In summarizing it can be asserted that the photon belonging to an n-photon field does not have *a priori* (prior to the detection of $n - 1$ photons) individual space-time structure and polarization (we exclude trivial cases described by a factorizable wave function or density matrix and the corresponding classical mixture of n single-photon states). In application to the spatial coordinates of the electrons (or any particles with finite rest mass) this approach appears trivial; after all it is emphasized already at the beginning of textbooks on quantum mechanics that an n-particle wave function is defined in an abstract $3n$-dimensional space. In the case of the electromagnetic field, however, the following three factors, which promote wide dissemination of the semiclasical "heresy," come into play: 1) there is a conviction, instilled in school, that optical or radio waves with a definite structure actually exist in the surrounding space; 2) it is possible to give a coordinate representation for the wave function of a field[22]; and, 3) two-photon effects have, until recently, been unfamiliar.

Thus on the basis of the Copenhagen interpretation it is operationally meaningless to ask what the atom actually emits in a given experiment and what the structure and polarization of the field are. We know how (for the present?) to calculate and measure only the parameters and indications of some macroscopic instruments starting from the parameters and indications of other, "preparatory" devices. According to Bohr's *complementarity* principle, in our case we cannot assign *a priori*—prior to interaction with a classical measurement device—any attributes to a quantum object; thus the concepts of the coordinate and momentum of a particle characterize the device and the method of measurement and not the properties of the particle (Fok proposed a more apt term—*the principle of relativity with respect to observation devices*[23]).

6. The effective field and advanced waves. The classical interpretation of the distribution (1) in terms of photon packets is still nonetheless possible. For this it is "only" necessary to allow the photons to propagate backwards in time: from one of the detectors (number 1) back to the atom and then to the detector 2.

The intensity correlation function (1), according to (A12), can be represented as the squared modulus of some complex function:

$$p(t_1, t_2) = |F|^2, \quad F \sim \int_0^T D_{20} D_{10} e^{-i\omega_0 t_0}\, dt_0. \quad (9)$$

Here D_{n0} are functions describing the propagation of pho-

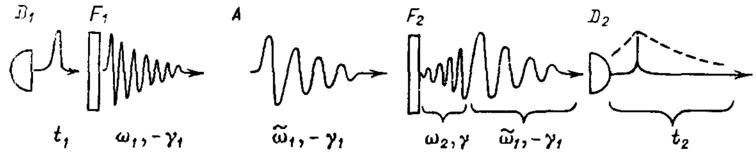

FIG. 4. Explanation of the dependence of the probability for observing a pair of photons on the moments of detection t_n and the parameters of the filters ω_n, γ_n with the help of the advanced field, growing with the time constant $-1/\gamma_1$, and the effective field, containing growing and decaying parts.

tons from the point of emission $\mathbf{r}_0$, t_0 to the point of detection $\mathbf{r}_n$, t_n (or vice versa) taking into account the action of the filters (see Appendix A).

Assume that the dependence of the conditional probability of a reading at detector 2 on t_2 for some fixed value of t_1 is being measured. According to (9) and the postulate of detection the function $F(t_2)$ plays the role of an effective field, whose intensity at the point $\mathbf{r}_2$, t_2 determines the probability for a reading at detector 2. We shall study the structure of this field. The function D_{10} describes both the propagation of the usual, *retarded* field from $\mathbf{r}_0$ to $\mathbf{r}_1$ and the propagation of the *advanced* field from $\mathbf{r}_1$ to $\mathbf{r}_0$, so that with the product $D_{20}D_{10}$ we can associate the path $\mathbf{r}_1 \to \mathbf{r}_0 \to \mathbf{r}_2$ (see Ref. 10 for a more detailed discussion). The integration in (9) in the approximation $T = \infty$ is a Fourier transform, as a result of which each component ω of the advanced field generates the component $\tilde{\omega} \equiv \omega_0 - \omega$ of the effective field, i.e., the *frequency subtraction* effect, well-known in nonlinear optics, occurs (the analogous transformation can be observed in reality if the atom and the field are in coherent states; unlike the well-known incoherent spontaneously induced emission[14,15] here the incident and scattered fields have definite phases).

Thus, according to (9), to determine the time structure of $F(t_2)$ of the photon 2 it can be assumed that at the time t_0 the detector 1 emits an advanced wave in the form of a short δ pulse (Fig. 4), which is converted by the filter 1 into a packet with an average frequency ω_1 and an envelope *growing* exponentially with the time constant $-1\gamma_1$. Under the action of this packet the atom emits a retarded packet of the same shape but with the "conjugate" carrying frequency $\tilde{\omega}_1$, which is what gives the function $\theta(\tau)\exp(-2\gamma_1\tau)$ in (1). On passing through the filter 2 the effective field undergoes minimum attenuatioin at resonance ($\tilde{\omega}_1 = \omega_2$), which is described by the denominator of (1). Finally, the function $\theta(-\tau)\exp(2\gamma_2\tau)$ in (1) is explained by impact excitation of the characteristic oscillations of the filter 2 accompanying the passage of the sharp trailing edge of the effective field. The complete structure of the effective field $F(t_2)$ taking into account the "carrying" frequencies is shown in Fig. 4.

Calculating $F(\tau)$ according to (A16) gives the following simple result for its Fourier transform:

$$F(\omega) = \eta_1(\omega)\,\eta_2(\tilde{\omega}) = -\frac{\gamma_1\gamma_2}{(\omega_1 - \omega - i\gamma_1)(\tilde{\omega}_2 - \omega + i\gamma_2)}, \tag{10}$$

(in comparing with Fig. 4 the indices 1 and 2 must be interchanged). This function characteristically has poles in both the upper and lower complex frequency half-planes, so that $F(\tau)$ does not satisfy the causality principle. If one filter is absent ($\gamma_2 = \infty$), then $F(\tau)$ is simply a function of the response of the other filter. From (10) we find (in the approximation ω_0, $T = \infty$):

$$F(\tau) = \int_{-\infty}^{\infty} F(\omega)\, e^{-i\omega\tau}\, d\omega$$

$$= 2\pi i \gamma_1\gamma_2 \frac{\theta(\tau)\exp[(-i\omega_1 - \gamma_1)\tau] + \theta(-\tau)\exp[(-i\tilde{\omega}_2 + \gamma_2)\tau]}{-\Omega + i(\gamma_1 + \gamma_2)}, \tag{11}$$

(here the factor $\exp(-i\omega_0 t_0)$, ensuring that F is symmetric in the indices 1 and 2, has been dropped). We note that the function $F(\tau)$, more precisely the convolution with itself, can actually be observed—with subpicosecond resolution—with the help of the interference method.[7,9,11]

Thus by admitting backward propagation of the signal from one of the detectors to the emitter it is possible to explain in an elementary fashion the distribution (1) by successive passage [see (10)] of *one* packet through *both* channels (taking into account the nonlinear scattering of the advanced field by the excited atom).

7. Action at a distance? In interpreting the formula (1) in terms of reduction or the effective field there naturally arises the question of whether or not superluminal transmission of information is possible [this problem was discussed extensively in connectin with polarization EPR experiments (see Refs. 24–26)].

By varying the parameters of, for example, filter 1—the frequency ω_1 or the bandwidth γ_1—it is obviously possible to modulate the frequency or duration of the pulses of the effective field $F(t_2)$. If it is acknowledged, following the Copenhagen interpretation, that on emission photons do not have a definite structure and photon 2 acquires structure only at the moment of detection in the detector 1, then an instantaneous change of, for example, γ_1 should instantaneously affect the longitudinal extent of the photon 2, no matter how far away it is at the time. The interval between the points of detection of the photons can be easily made space-like (even taking into account the delay in the filters) by increasing the distance between the detectors. (We note that in the case of a parametric source the photons are strongly correlated with respect to the directions of propagaton, so that the detector 2 can certainly overlap each photon whose twin is detected by the detector 1.) Thus information about the change in γ_1 should propagate with superluminal velocity, and this contradicts the special theory of relativity.

This paradox is resolved by the fact that no method exists for measuring the extent (just like the *a priori* polarization[25]) of one photon (this assertion pertains only to detectors that annihilate photons; in principle, it is possible to detect a photon without perturbing its energy,[19] but in this case the formula (1) is not applicable and a special analysis is required). It is tempting to employ a quantum amplifier to "clone" photons,[26] but the characteristic noise of the amplifier makes this method useless.[25]

If, on the other hand, the moment of emission of a given pair t_0 and the distance $\mathbf{r}_2$ are known, then by observing at some moment t the reading in the wide-band detector 2 it is possible to draw a conclusion with some degree of reliability about the extent of the photon 2, hardly much greater than $t - t_0 - (r_2/c)$. However t_0 is a random, unpredictable quantity, which is unknown to observer 2. It can be evaluated from t_1, $\mathbf{r}_1$, and γ_1, but transmission of this information to the observer 2 requires an auxiliary communication channel, which immediately undermines the idea of a superluminal telegraph. In other words, the starting formula (1) itself is predicated on comparison of the indications of both detectors with the help of standad methods of communication. Attempts are sometimes made to resolve such paradoxes in quantum mechanics using terms of the type "noninformative action at a distance," which obviously do not contain anything new compared with the term "reduction of the wave function."

Quantum correlation, however, can still be employed for communication, but not superluminal communication.[10] We shall employ frequency modulation. We place in the path of the photon 2 a dispersive prism and two detectors 2′ and 2″, recording photons with frequency ω_2' and ω_2''. But the appearance of a count, for example, in the detector 2′ still does not permit asserting that a definite filter with frequency $\omega_0 - \omega_2'$ has been placed in front of the detector 1, for after all the photon 1 with this frequency could have been absorbed by any filter 1. This difficulty can be overcome by placing in the path of the photon 2 an optical shutter which opens only when a count appears in the detector 1. The shutter eliminates the "superfluous" photons 2 (i.e., it replaces the coincidence scheme), but in the process the interval between the transmitter and the detector becomes time-like.

CONCLUSIONS

Thus wide-band two-photon radiation, formed when an atom in a metastable state decays (or when pumping photons in matter decay owing to macroscopic nonlinearity) enables the realization of a variant of the EPR experiment in which the field does not have *a priori* a definite temporal structure and there is a *complementarity* between the observed energy of the photon and its localization in time. The form (8) of the uncertainty relation, unlike many others, contains directly only the measured parameters. The duality of the photons is clearly manifested in the structure of the formula (1), whose numerator is characteristic for particles while the denominator is characteristic for waves. Unlike the traditional two-slit experiment, here the "naive" semiclassical model with photon packet and the detection postulate is rejected (the possibility of unifying these two characteristic quantum experiments—the two-slit and EPR type experiments—was examined in Ref. 12).

The foregoing graphic interpretation with the help of advanced and effective fields could also be useful for studying other two-photon experiments, including polarization[4–7] and interference,[7,8,12,27] as well as experiments associated with the spatial localization of photons.[10] This interpretation predicts, for example, the existence of quantum beats in noncascade two-photon transitions: when a metastable level is split by an amount $\hbar\Omega$ the distribution (1) in the case $\gamma_n \gg \Omega$ should have the form $\delta(\tau)(1 + \cos(\Omega t_1))$ (compare the beats owing to splitting of the intermediate level in cascade transitions, observed in Ref. 28).

The experiment discussed above, with verification of the distribution (1), is fully realizable. Of course, such an experiment will not lead to anything unexpected, and it is interesting only as a graphic illustration of the most paradoxical aspect of the quantum theory. We offer in connection with this the following statement by Jaynes[29]: "I am convinced that many who defend the Copenhagen interpretation most fervently do so only because they never thought deeply enough to realize its full implications."

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APPENDIX A. INTENSITY CORRELATION IN THE TWO-PHOTON FIELD TAKING FILTERING INTO ACCOUNT

The probability of detecting two photons at the points x_1 and x_2 is proportional to the *normal* (*normally ordered*) intensity correlation function[30]:

$$p_{12} = \langle 0 | E_{H1}^{(-)} E_{H2}^{(-)} E_{H2}^{(+)} E_{H1}^{(+)} | 0 \rangle; \tag{A1}$$

here $E_{Hn} \equiv E_H(\mathbf{r}_{tn}, t_n)$ is the field at the point x_n (for now we assume that $t_2 > t_1$), the index H corresponds to the Heisenberg representation, and the averaging is performed over the intial state $|0\rangle = |b\rangle|\text{vac}\rangle$ (b is the index of the metastable state of the atom). The transformation to the interaction representation is performed by the unitary operator S: $E_{\rm H}(t) = S^+(t)E(t)S(t)$ (see, for example, Ref. 14).

Let the interaction occur only in the bounded time interval $0 - T$ (T can be determined, for example, by the time of flight of the atom through the "field of view" of the detectors; in the case of parametric scattering T is the duration of the pumping pulse), and then for $t > T$ the operator S does not depend on the time. In this case $S(t_2)S^+(t_1) = 1$, so that

$$p_{12} = \langle 0 | S^+ E_1^{(-)} E_2^{(-)} E_2^{(+)} E_1^{(+)} S | 0 \rangle \quad (t_{1,2} > T). \tag{A2}$$

Here the operators $E_1^{(+)}$ and $E_2^{(+)}$ (as well as $E_1^{(-)}$, $E_2^{(-)}$) commute, so that $p_{12} = p_{21}$ and the restriction $t_2 > t_1$ can be dropped.

The two-photon emission is described by second-order perturbation theory (we assume that $T \ll \tau$), so that in (A2)

$$S = S^{(2)} = (i\hbar)^{-2} \int_0^T dt_0 V(t_0) \int_0^{t_0} dt V(t). \tag{A3}$$

The operator S must transform the state $|b\rangle|\text{vac}\rangle$ into $|a\rangle|2\rangle$ (where $|2\rangle$ is a two-photon state), so that in (A3) in the dipole approximation

$$\begin{aligned} V(t_0) &= -\mathbf{d}(t_0)\,\mathbf{E}_0^{(-)} = -\sum_m \sigma_{am}(t_0)\,\mathbf{d}_{am}\mathbf{E}_0^{(-)}, \\ V(t) &= -\int_0^\infty d\omega \sigma_{mb} \mathbf{d}_{mb} \mathbf{E}_{-\omega} \exp[i(\omega_{mb} + \omega)t], \end{aligned} \tag{A4}$$

where $\sigma_{mn} = |m\rangle\langle n|$, $\sigma_{mn}(t) = \sigma_{mn} e^{i\omega_{mn}t}$, $\sigma_{am}\sigma_{nb} = \sigma_{ab}\delta_{mn}$, and E_ω is the Fourier transform of E_0. Now (A3) assumes the form

$$S^{(2)} = i\hbar^{-2} \int_0^T dt_0 \sigma_{ab}(t_0) \sum_m \mathbf{d}_{am} \mathbf{E}_0^{(-)} \times \int_0^\infty d\omega\, \mathbf{d}_{mb} \mathbf{E}_{-\omega} e^{i\omega t} (\omega_{mb} + \omega)^{-1}. \tag{A5}$$

Let us assume that the spectrum of the detected field is restricted to the region $\omega_0/2 \pm \Delta$, and $\Delta \ll |\omega_{mb}|$; then in the denominator of (A5)

$$\omega_{mb} + \omega \approx \omega_{mb} + \frac{\omega_0}{2} = \omega_{ma} - \frac{\omega_0}{2}. \qquad (A6)$$

In addition

$$S^{(2)} = (2i\hbar)^{-1}\sigma_{ab} \int_0^T dt_0 e^{-i\omega_0 t_0} \mathbf{E}_0^{(-)} \varkappa \mathbf{E}_0^{(-)}, \qquad (A7)$$

where we have introduced the tensor

$$\varkappa_{\alpha\beta} \equiv -2\hbar^{-1} \sum_m d_{am}^{(\alpha)} d_{mb}^{(\beta)} \left(\omega_{mb} + \frac{\omega_0}{2}\right)^{-1},$$

which we shall assume is real ($\alpha,\beta = x,y,z$). We note that the formula (A7) is equivalent to using the following effective "dispersion-free" Hamiltonian[11]

$$V_{\text{eff}} \equiv \frac{1}{2} \sigma_{ab} \mathbf{E}_0^{(-)} \varkappa \mathbf{E}_0^{(-)} + \text{h.c.}, \qquad (A8)$$

determining $S^{(2)}$ to first order in $\varkappa$.

We substitute (A7) into (A2) (to simplify the problem we ignore the tensor nature of $\varkappa$):

$$p_{12} = \left(\frac{\varkappa}{2\hbar}\right)^2 \iint_0^T dt_0' \, dt_0 e^{i\omega_0(t_0' - t_0)} \times \langle \text{vac} | E_{0'}^{(+)2} E_1^{(-)} E_2^{(-)} E_2^{(+)} E_1^{(+)} E_0^{(-)2} | \text{vac} \rangle. \qquad (A9)$$

It is easy to verify that the correlation function in (A9) and therefore p_{12} can be factored: $p_{12} = |F_{12}|^2$. Here the function

$$F_{21} = F_{12} \equiv \frac{\varkappa}{2\hbar} \int_0^T dt_0 e^{-i\omega_0 t_0} \langle \text{vac} | E_2^{(+)} E_1^{(+)} E_0^{(-)2} | \text{vac} \rangle, \qquad (A10)$$

was introduced; it is the "probability amplitude" or the "effective field."

We reorder the operators in (A10) into normal order with the help of the commutator

$$[E_n^{(+)}, E_0^{(-)}] \equiv -i\hbar D_{n0} = i\hbar D_{0n}^*. \qquad (A11)$$

Since $E_n^{(+)}|\text{vac}\rangle = 0$, we can set in (A10)

$$E_1^{(+)} E_0^{(-)2} = [E_1^{(+)}, E_0^{(-)2}] = -2i\hbar D_{10} E_0^{(-)};$$

multiplying this equality on the left by E_2 gives $-2\hbar^2 D_{20} D_{10}$, so that (A10) assumes the form

$$F_{12} = -\hbar\varkappa \int_0^T dt_0 e^{-i\omega_0 t_0} D_{20} D_{10}. \qquad (A12)$$

Thus the effective field is essentially the Fourier transform (at the frequency ω_0) of the function $D_{20}D_{10}$, truncated for $t_0 < 0$ and $t_0 > T$.

Representing $E^{(\pm)}$ in (A11) as a sum of photon creation and annihilation operators gives the spectral distribution of the function D_{10}:

$$D_{10} = i(2\pi)^{-2} \int d^3k \omega_k \exp[i(\mathbf{k}\mathbf{r}_{10} - \omega_k t_{10})], \qquad (A13)$$

where $\omega_k = c|\mathbf{k}|$, $\mathbf{r}_{10} = \mathbf{r}_1 - \mathbf{r}_0$, $t_{10} = t_1 - t_0$ (the analogous expression for D_{20} is obtained by interchanging the indices 1 and 2). Integrating over directions gives

$$D_{10} = A_1 \int_0^{\omega_0} d\omega \eta_1(\omega) \exp\left[-i\omega\left(t_{10} - \frac{r_{10}}{c}\right)\right], \qquad (A14)$$

where $A_1 = \omega_1^2/2\pi c^2 r_{10}$, $r_{10} = |\mathbf{r}_{10}|$. Here the term with the phase $\omega[t_{10} + (r_{10}/c)]$ was dropped (since it describes the propagation of the retarded field from $\mathbf{r}_1$ to $\mathbf{r}_0$, which contradicts the condition $t_1 > T \geqslant t_0$) and the spectrum has been limited by the transition frequency ω_0. In addition, the substitution $\omega^2 \to \omega_1^2 \eta_1(\omega)$ was made; here $\eta_1(\omega)$ is the transfer factor of the filter in channel 1 and ω_1 is the central frequency of this filter. This phenomenological description of the filtering process is admissible for calculating the normal correlation functions of the field (see Appendix B).

Setting the limits of integration in (A12) equal to infinity gives, substituting (A14),

$$F_{12} = \hbar\varkappa A_1 A_2 e^{-i\omega_0 t_2} F(\tau), \quad \tau = t_1 - t_2, \qquad (A15)$$

$$F(\tau) \equiv \int_0^{\omega_0} d\omega \eta_1(\omega) \eta_2(\omega_0 - \omega) e^{-i\omega\tau}, \qquad (A16)$$

(the substitution $t_n r_n/c \to t_n$ was made). Let $\eta_n = \gamma_n(\omega_n - \omega - i\gamma_n)^{-1}$, where $\omega_0 > \omega_n \gg \gamma_n > 0$, and then the formulas (10) and (11) follow from (A16). Finally, from (11) we obtain the expression (1) for $p(\tau) \equiv |F(\tau)|^2$ (it was assumed that $\theta(x)\theta(x) = \theta(x)$ and $\theta(x)\theta(-x) = 0$).

We shall explain the physical meaning of the function D_{10}. With the help of (A11) we find

$$\langle E_1 E_0 \rangle_{\text{vac}} = \langle E_1^{(+)} E_0^{(-)} \rangle_{\text{vac}} = -i\hbar D_{10}, \qquad (A17)$$

i.e., D_{10} determines the correlation function of the free field (antinormal function) in the vacuum state ("zero fluctuations"). Furthermore, the operators in the interaction representation $E^{(\pm)}$ satisfy the homogeneous wave equation $\Box E^{(\pm)} = 0$, so that D_{10} and therefore F_{12} (with $\eta_n = 1$) are also the solution of this equation (this follows easily from the spectral representation (A13). Now assume that a clasical source of the field with a given time dependence is present in the vicinity of the point $\mathbf{r}_0$. We shall use for simplicity the "dipole" interaction Hamiltonian $-d(t)E(\mathbf{r}_0,t)$ (this is admissible in the case of a quasimonochromatic source—see, for example, Ref. 15) and the scalar description; then to first order in d

$$S^{(1)}(t_1) = -(i\hbar)^{-1} \int_0^{t_1} d(t_0) E(\mathbf{r}_0, t_0) \, dt_0. \qquad (A18)$$

Setting $S = 1 + S^{(1)}$ gives the field in the Heisenberg representation (i.e., taking into account the source) at the time t_1 and at the point $\mathbf{r}_1$ of the wave zone:

$$E_{H1}^{(+)} = E_1^{(+)} + [E_1^{(+)}, S^{(1)}(t_1)] = E_1^{(+)} + \int_0^{t_1} d(t_0) D_{10} \, dt_0. \qquad (A19)$$

It is easy to see that the next corrections equal zero (since d and D are not operators). The hermitian conjugate equality expresses $E_{H1}^{(-)}$ in terms of $D_{10}^* = -D_{01}$ and therefore the total field $E_{H1} = E_{H1}^{(+)} + E_{H1}^{(-)}$ is determined analogously to (A19) in terms of $2\text{Re}\, D_{10} = i[E_1, E_0]/\hbar$. With the help of A13 it can be shown[22] that $2\text{Re}\, D_{10}$ can be expressed in terms of the second derivatives of the function

$$D_0(r,\ t)=\frac{1}{r}\left[\delta\left(t-\frac{r}{c}\right)-\delta\left(t+\frac{r}{c}\right)\right], \qquad \text{(A20)}$$

which is called the Jordan-Pauli propagator (here $r=r_{10}$, $t=t_{10}$). Thus the hermitian free-field operators at the points x_1, x_0 do not commute only if these points can be connected by a light signal.

We emphasize that the sign of the difference $t_1-t_0\equiv t$ in the formulas (A18)–(A20) can be arbitrary: they associate with $d(t_0)$ both the future ($t>0$) and the past ($t<0$) field. If, however, based on physical considerations the function D_{10} in (A19) is multiplied by $\theta(t)$, then the equivalence of the future and past is destroyed. In this case the upper limit of integration in (A19) can be replaced by $+\infty$, so that the product $D_{10}\theta(t)$ will be the Green's function ("retarded") for the positive-frequency field $E_H^{(+)}$. Thus the factor $\theta(t)$ as well as $\theta(-t)$ transforms the solution of the homogeneous wave equation into the solution of the inhomogeneous wave equation with the function $\delta^4(x)$ on the right side. We note that a combination of the form $D_{10}\theta(t)+D_{01}\theta(-t)$—the causal or Feynman Green's function (propagator)—is often employed.

APPENDIX B. PHENOMENOLOGICAL DESCRIPTION OF FILTERING IN QUANTUM OPTICS

From the classical viewpoint the description of the action of a linear frequency filter on the radiation is elementary—one must simply multiply the propagation function in the frequency representation $D(\omega)$ by the transfer factor of the filter $\eta(\omega)$. In the temporal representation the field is transformed by the filter according to the law $E'(t)=\eta E(t)$, where $\hat{\eta}$ is the integral operator corresponding to $\eta(\omega)$. In the quantum theory, however, such a transformation of the field operators destroys the commutation relations (i.e., it is not unitary), and it extinguishes the zero-point fluctuations. This transformation is nonetheless admissible, but with one stipulation—the free-field operators $E^{(\pm)}(t)$ must, in this case, form a normal correlation function.

We shall first study one mode of the field. If the mode is in a coherent state $|\alpha\rangle$, then as a result of a linear interaction with a cold thermostat the state remains coherent, and only the amplitude of the state changes: $|\alpha\rangle'=|\eta\alpha\rangle$.[15,31] In application to our problem η is the transfer factor of the filter at the frequency of the mode ω_k. A wide class of states of the field can be described with the help of a diagonal coherent representation of the Glauber-Sudarshan density matrix $P(\alpha)$.[30] In this case the normal observables $f_N(a^+,a)\equiv f_N(a)$ are calculated according to the classical averaging rule:

$$\langle f_N(a)\rangle=\int f_N(\alpha)\,P(\alpha)\,\mathrm{d}^2\alpha. \qquad \text{(B1)}$$

Thus $P(\alpha)$ plays the role of a two-dimensional probability density in the plane $\mathrm{Re}\,\alpha\times\mathrm{Im}\,\alpha$ (for some states, however, $P(\alpha)$ is singular and even assumes negative values, so that it is said to be a *quasiprobability*).

It is obvious that changing the scale $\alpha\to\eta\alpha$ results in the transformation

$$P'(\alpha)=CP\left(\frac{\alpha}{\eta}\right), \qquad \text{(B2)}$$

where $C=|\eta|^{-2}$. Therefore after the filter

$$\langle f_N(a)\rangle'=C\int f_N(\alpha)\,P\left(\frac{\alpha}{\eta}\right)\mathrm{d}^2\alpha=\langle f_N(\eta a)\rangle. \qquad \text{(B3)}$$

This rule can also be proved in an analogous manner with the help of a nondiagonal representation of the density matrix, so that itholds for arbitrary initial states of the mode. We note that according to (B2) the damping does not change the functional form of the P distribution; however the analogous conclusion for the distribution of other quantities, for example, the number of photons, is not always valid (this situation can be employed for absolute photometry[15,16]).

The rule (B3) can also be extended to a multimode field. When there is no mixing of modes

$$P'(\{\alpha_k\})=CP\left(\left\{\frac{\alpha_k}{\eta_k}\right\}\right),\quad C^{-1}=\prod_k|\eta_k|^2, \qquad \text{(B4)}$$

analogously to (B2). From here

$$\langle f_N(\{a_k\})\rangle'=\langle f_N(\{\eta_k a_k\})\rangle. \qquad \text{(B5)}$$

This relation justifies introducing the factor $\eta_1(\omega)$ in (A14). We shall find, for example, the two-point correlation function at the output of the filter:

$$\begin{aligned}\langle E_1^{(-)}E_2^{(+)}\rangle' &= \int \mathscr{E}_1^*(\{\alpha_k\})\,\mathscr{E}_2(\{\alpha_k\})\,P'(\{\alpha_k\})\prod_k d^2\alpha_k\\ &=\int \mathscr{E}_1^*(\{\eta_k\alpha_k\})\,\mathscr{E}_2(\{\eta_k\alpha_k\})\,P(\{\alpha_k\})\prod_k d^2\alpha_k\\ &\equiv\langle(\hat{\eta}^*E_1^{(-)})\,\hat{\eta}E_2^{(+)}\rangle,\end{aligned} \qquad \text{(B6)}$$

here

$$\mathscr{E}_n(\{\alpha_k\})\equiv\frac{i}{2\pi}\sum_k(\hbar\omega_k)^{1/2}\alpha_k\exp(i\mathbf{kr}_n-i\omega_k t_n). \qquad \text{(B7)}$$

The antinormal correlation function $\langle E_2^{(+)}E_1^{(-)}\rangle$ differs from (B6) by the "zero-point fluctuations" $-i\hbar D_{21}$, on which the filters do not operate. The phenomenological description of other linear transformations can be included in quantum optics in an anaolgous fashion[10,14]: diffraction, focusing, spatial and polarization filtering, etc.

[1)]Aside from the breakdown of Bell's inequalities other, less universal, criteria for the quantum effects to be nonclassical are known; for example, breakdown of the Cauchy-Schwartz inequality, the nonpositive nature of the Glauber-Sudarshan P distribution, etc. (see Refs. 13 and 20). Here we confine our attention to a discussion of the nonclassical nature of the distribution (1) only at the qualitative level.

[1]A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[2]D. Bohm, *Quantum Theory*, Prentice-Hall, Englewood Cliffs, NJ, (1951) [Russ. transl., Fizmatgiz, M., 1961].
[3]D. J. Bell, Physics **1**, 195 (1964); J. F. Clauser, M. A. Horne, A. Shimoni, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969); J. F. Clauser and A. Shimoni, Rep. Prog. Phys. 41, 1881 (1978); A. A. Grib, Usp. Fiz. Nauk **142**, 619 (1984) [Sov. Phys. Usp. **27**, 284 (1984)].
[4]C. A. Coher and E. D. Commins, Phys. Rev. Lett. **18**, 575 (1967); S. J. Freedman and J. F. Clauser, *ibid.* **28**, 938 (1972); E. S. Fry and R. C. Thomson, **37**, 465 (1976); A. Aspect, J. Dalibard, and G. Roger, *ibid.* **49**, 1804 (1982).
[5]T. Haji-Hassan, A. S. Dunca, W. Perrie, H. J. Beyer, and H. Kleinpoppen, Phys. Lett. A **123**, 110 (1987); A. J. Duncan, *Microphysical Reality and Quantum Formalism*, edited by A. van der Merve *et al.*, Academie Publ., Kluwer, Holland, 1988, p. 345.
[6]C. O. Alley and Y. H. Shih, *15th International Quantum Electrodynamics Conference*, Baltimore (1987).
[7]Z. Y. Ou and L. Mandel, Phys. Rev. Lett. **61**, 50 (1988).
[8]R. Gosh and L. Mandel, *ibid.* **59**, 1903 (1987).
[9]C. K. Hong, Z. Y. Ou, and L. Mandel, *ibid.*, 2044.
[10]D. N. Klyshko, Usp. Fiz. Nauk **154**, 133 (1988) [Sov. Phys. Usp. **31**, 74

(1988)]; Zh. Eksp. Teor. Fiz. **94**, 82 (1988) [Sov. Phys. JETP **67**, 1131 (1988)].
[11]D. N. Klyshko, Phys. Lett. A **128**, 133 (1988).
[12]D. N. Klyshko, *ibid.* **132**, 299.
[13]B. Ya. Zel'dovich and D. N. Klyshko, Pis'ma Zh. Eksp. Teor. Fiz. **9**, 69 (1969) [JETP Lett. **9**, 40 (1969)].
[14]D. N. Klyshko, *Photons and Nonlinear Optics* (in Russian), Nauka, Moscow (1980).
[15]D. N. Klyshko, *Physical Foundations of Quantum Electronics* (in Russian), Nauka, Moscow (1986).
[16]D. N. Klyshko and A. N. Penin, Usp. Fiz. Nauk **152**, 653 (1987) [Sov. Phys. Usp. **30**, 716 (1987)].
[17]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, Pergamon Press, N.Y., 1965 [Russ. original, Fizmatgiz, M., 1963]; Yu. I. Vorontsov, Usp. Fiz. Nauk **133**, 351 (1981) [Sov. Phys. Usp. **24**, 150 (1981)]; V. V. Dodonov and V. I. Man'ko, Tr. FIAN SSSR **183**, 5 (1987).
[18]E. H. Wichmann, Quantum Physics, V. 4 of Berkeley Physics Course, McGraw-Hill, N.Y., 1970. [Russ. transl., Nauka, M., 1977, p. 174].
[19]R. M. Shelby and M. D. Levenson, Opt. Commun. **64**, 553 (1987); V. B. Braginsky and S. P. Vyatchanin, Phys. Lett. A. **132**, 206 (1988); G. Bjork and Y. Yamamoto, Phys. Rev. A **37**, 125 (1988).
[20]M. D. Reid and D. F. Walls, *ibid.* **34**, 1260 (1986); K. Wodkiewicz, Phys. Lett. A **129**, 1 (1988).
[21]H. Paul, Rev. Mod. Phys. **58**, 209 (1986).
[22]A. I. Akhiezer and V. B. Berestetskiĭ, Quantum Electrodynamics, Interscience, N.Y., 1965 [Russ. original, Nauka, M., 1964 and 1981].
[23]V. A. Fok, Voprosy Filosofii, No. 3, 46 (1971).
[24]G. C. Ghirardi, R. Grassi, A. Rimini, and T. Weber, Europhys. Lett. **6**, 95 (1988).
[25]R. J. Glauber, *Frontiers in Quantum Optics*, (Eds) E. R. Pike and S. Sarkar, Hilger, Bristol, 1986, p. 534.
[26]N. Herbert, Found. Phys. **12**, 1171 (1982).
[27]M. Zucowski and J. Pykicz, Phys. Lett. A **127**, 1 (1988); M. A. Horne and A. Zeilinger, New Techniques and Ideas in Quantum Measurement Theory, (Ed) D. Greenberger, New York Academy of Sciences, N.Y., 1986, p. 469.
[28]A. Aspect, J. Dalibard, P. Grangier, and G. Roger, Opt. Commun. **49**, 429 (1984).
[29]E. T. Jaynes, in: *Foundations of Radiation Theory and Quantum Electrodynamics*, (Ed) A. Barut, Plenum Press, N.Y., 1980, p. 39.
[30]R. J. Glauber, Optical Coherence and Photon Statistics in: Quantum Optics and Electronics, Lectures at the 1964 Les Houches Summer School of Theoretical Physics, (Eds) C. de Witt, A. Blandin, and C. Cohen-Tannoudji, Gordon and Breach, N.Y., 1965, pp. 63–185. [Russ. transl. Mir, M., 1966, p. 91].
[31]R. J. Glauber, Phys. Lett. **21**, 650 (1966).

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