Wigner crystallization on the surface of liquid helium

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This report discusses the latest advances in research on the two-dimensional Wigner crystal on the surface of liquid helium.

The ensemble of electrons localized on the surface of liquid helium is a promising two-dimensional system of charged particles that has attracted much intensive research in recent years.

1. The system of electrons on a liquid helium surface has attracted particular interest because there one may observe various collective phenomena of Coulombic origin. These phenomena include general two-dimensional plasmons, particular varieties of two-dimensional plasmons known as edge or perimeter plasma waves, and Coulombic crystallization. The most fundamental of these collective phenomena is the Coulombic crystallization predicted for three-dimensional systems with a strong Coulomb interaction by Wigner[1-2] some 50 years ago. Wigner's ideas have benefited from extensive theoretical development, for this field holds the promise of describing various collective phenomena in strongly interacting systems. Experimentally, however, such systems became accessible only in recent years, largely due to advances in the fabrication of charged two-dimensional systems with a mobile subsystem spatially separated from the screening background. An example of such a system, characterized by very high electron mobility, is an ensemble of electrons on a helium surface. In this system Grimes and Adams[3] were the first to observe electronic crystallization.

Naturally the success of Grimes and Adams[3] stimulated further research into the properties of the electronic lattice. In this report we shall discuss the latest theoretical and experimental advances in this field.

2. In the original work of Grimes and Adams[3] the existence of the phase transition was ascertained by the appearance of specific electron-ripplon modes in the excitation spectrum of the electronic lattice on a liquid substrate. The nature of the additional branches in the spectrum, as well as the details of experiment,[3] have already been extensively analyzed in various reviews.[4,5] Thus we shall omit the discussion of this interesting subject. The next fundamental question following from the research of Grimes and Adams concerns the propagation of transverse sound in the electronic lattice. A finite shear modulus is an essential attribute of the crystalline state and consequently the existence of transverse sound in the electronic lattice is of great interest. An experimental approach to this problem, i.e. the development of a technique for exciting transverse sound and a detailed investigation of the properties of the two-dimensional lattice shear modulus, was developed in a series of experiments by French scientists.[6-8]

A schematic diagram of the cell used by Deville and coworkers[7] to observe transverse sound is illustrated in Fig. 1. The electronic system is localized on a helium film of finite thickness d which lies on top of a dielectric substrate containing a meandering delay line. The period of the line is of the same order as the helium film thickness. When an alternating current flows through the delay line in the presence of a magnetic field $\mathbf{H}$ normal to the helium surface, shear-type electric forces are produced in the electronic system. If the electronic system has a non-zero transverse rigidity and the excitation frequency coincides with the transverse sound frequency and the wavenumber corresponding to the delay line period, the interaction between the delay line and the electronic system is resonantly enhanced. This resonant enhancement is then manifested by increased losses in the delay line.

A specific feature of the transverse excitation spectrum of an electronic crystal on a soft substrate is the existence of a finite deformation-induced threshold ω_ξ^2:

$$\omega^2(k)=\omega_\xi^2+v_\perp^2k^2,\quad v_\perp^2=\frac{0{,}138e^2(\pi n)^{1/2}}{m},\quad \omega_\xi^2=\frac{e^2E_\perp^2}{2\pi\alpha m\langle u^2\rangle}; \tag{1}$$

where n is the average electron density; m is the electron mass; $E_\perp$ is the intensity of the clamping field; ω and k are the frequency and wavenumber of the oscillations; α is the surface tension coefficient of liquid helium; and $\langle u^2\rangle$ is the mean-square displacement of the electron from its equilibrium position, with $\langle u^2\rangle \ll n^{-1}$. The physical origin of the threshold ω_ξ in the excitation spectrum $\omega(k)$ is quite simple. The electronic lattice is confined to the free helium surface by the external field $E_\perp$. Recall that $\langle u^2\rangle \ll n^{-1}$ and, clearly, the effective pressure exerted by the lattice on the helium surface contains, in addition to the average value that is uniform over the entire surface, additional Fourier components that characterize the local deformation of the helium surface under each localized electron. This self-consistent deformation $\xi(r)$ leads to an additional energy $eE_\perp\xi(r)$. In the high-frequency limit this energy produces a deformation-induced threshold frequency ω_ξ in the $\omega(k)$ spectrum (1). This feature in the collective excitation spectrum of the electronic crystal on a liquid substrate was first predicted in Ref. 9.

The existence of the threshold ω_ξ is clearly visible in the inset of Fig. 1 which plots the transverse oscillation frequency against wavenumber k.

In regard to experiments concerned with the shear modulus, the existence of the threshold frequency in the transverse excitation spectrum means that the data should

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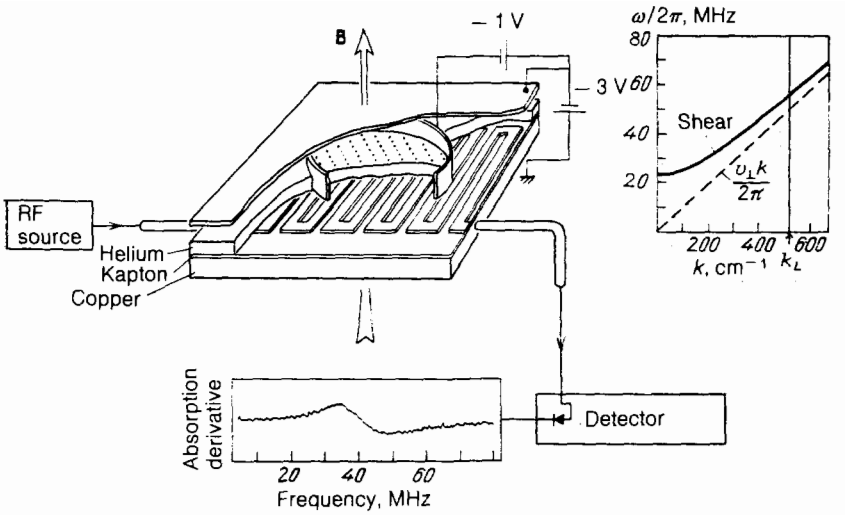

FIG. 1. Experimental cell of Ref. 7 employed to measure transverse rigidity of the Coulombic crystal. The inset shows a schematic dispersion relation for the transverse mode of the electronic lattice; the arrow marks the "operational" wavenumber k_L where one can measure the transverse speed of sound in the linear $\omega(k)$ regime.

be corrected for the additional factor of ω_ξ^2. More precisely, the transverse sound should be excited in the large wavenumber range where the dispersion relation (1) is linear, i.e. in the region where

$$\omega^2 \gg \omega_\xi^2. \tag{2}$$

The above requirement (2) places a stringent constraint on the design of a high-quality delay line of a sufficiently small period. In the case of an electronic crystal on a helium surface, the period λ of the delay line that satisfies condition (2) is such that $2\pi/\lambda \approx 500$ cm^{-1}. Nonetheless the French group at Saclay succeeded in building an experimental cell which made it possible to excite transverse sound in the linear dispersion regime and carry out the planned experiments.

The wavenumber k_L which corresponds to the excitation of transverse sound is marked in the inset of Fig. 1.

The final results of this research[6,7] are summarized in Figs. 2 and 3. Figure 2 illustrates the appearance of the resonant signal which indicates the formation of transverse rigidity in the 2D-electron system. The speed of transverse sound and, consequently, the shear modulus μ can be easily calculated from the resonant frequency and the period of the delay line. The corresponding experimental data on the temperature dependence of μ are collected in Fig. 3.

The temperature of the phase transition T_m that can be extracted from the temperature dependence of the shear modulus is in good agreement with the value of T_m obtained by Grimes and Adams,[3] as well as subsequent independent measurements.[10–12] Moreover, from the data in Fig. 3 one can proceed to theorize on the nature of the phase transition in the electronic system. To be more precise, can the value of this phase transition temperature be explained by the Kosterlitz–Thouless theory?[13,14] The French group obtained an affirmative result: the transition temperature is indeed determined by the Kosterlitz–Thouless relation (μ is the shear modulus; a_0 is Burger's vector; τ is the Poisson coefficient):

$$T_m = \frac{\mu a_0^2 (1+\tau)}{8\pi}. \tag{3}$$

as long as one lets $\tau \to 1$ (Ref. 14) and substitutes the temperature dependent value of the shear modulus $\mu(T)$, i.e., takes into account Morf 's theoretical computation.[15] Thus we find that the electronic lattice on the helium surface belongs to that interesting class of crystalline structures where melting occurs by the fluctuational multiplication of dislocations.

3. The next important series of experiments, also carried out by French physicists,[16,17] focussed on the heat capacity of the electronic lattice and the behavior of this heat capacity during the crystal–liquid transition. At first sight this problem appears insoluble because the heat capacity of the low-density two-dimensional system should be swamped by the much greater heat capacity of the liquid substrate. An ingenious experimental approach described in Refs. 16, 17 solved this problem and made it possible to measure reliably the heat capacity of the electronic subsystem.

In order to determine the heat capacity of the lattice one must measure the change in temperature ΔT_e of the crystal as it absorbs a given quantity of heat ΔQ. The measurement

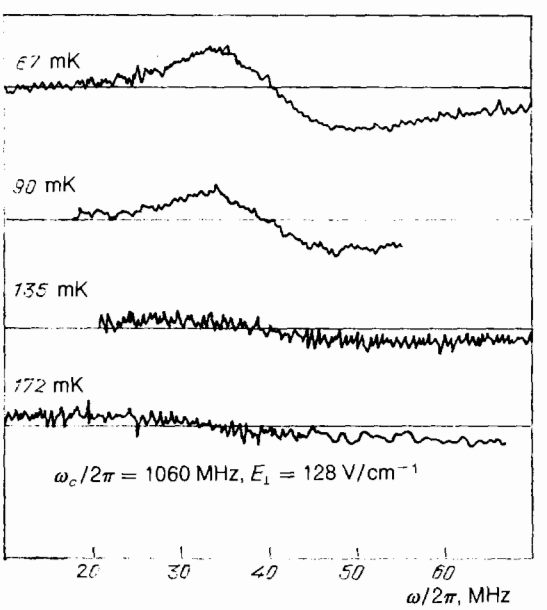

FIG. 2. Resonance appears as the temperature is lowered, indicating the formation of a finite shear modulus μ in the electronic lattice. Density of the 2D-electron system $n = 6 \cdot 10^7$ cm^{-2}, other parameters shown in the figure (from Ref. 7).

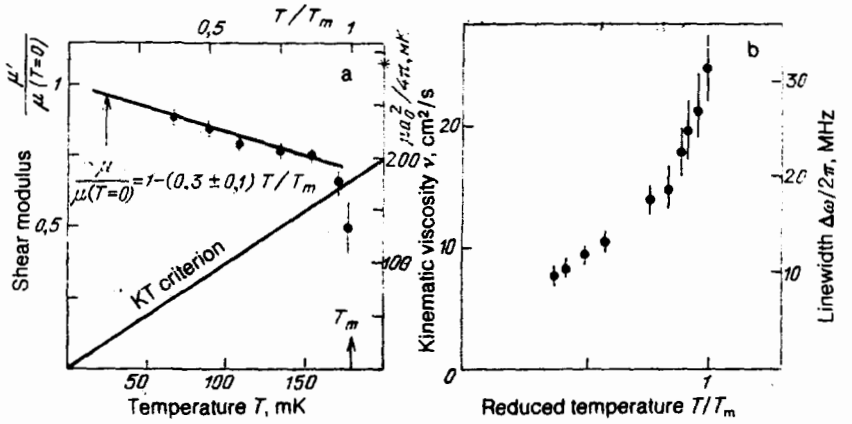

FIG. 3. (a) The elastic shear modulus μ normalized to its extrapolated $T=0$ value, plotted as a function of temperature T. The solid line plots the temperature dependence $\mu(T)$ calculated by Morf[15]; experimental points fall onto this line with good accuracy. The right-hand scale permits a direct comparison with the Kosterlitz–Thouless (KT) stability criterion in the $\mu a_0^2/4\pi > T$ regime; T_m is the melting temperature of the electronic crystal at the experimental density $n = 6\cdot 10^7\ \mathrm{cm}^{-2}$ (Ref. 7). (b) Kinematic viscosity of the electronic crystal vs T for transverse oscillations at the frequency $\omega/2\pi = 1060$ MHz. The characteristically sharp rise in the viscosity near the melting temperature can be attributed to the appearance of a large number of fluctuational dislocations in this temperature range (Ref. 7).

of ΔQ presents little difficulty because it arises from the active losses from the RF circuit of the electronic system. As for measurement of ΔT_e (the delicate part of this technique), according to Glattli[16] one can make use of the threshold frequency ω_ξ in (1). Indeed, direct measurements[6,18] near the formation threshold of the Coulomb lattice point to a marked dependence of this frequency on temperature (Fig.4). Assuming that as the electronic lattice becomes hot with respect to the liquid substrate the frequency ω_ξ reacts to the electronic temperature, we can employ the data of Fig. 4 on the temperature dependence of ω_ξ for calibrating the frequency $\omega_\xi(T_e)$ and use the latter as a thermometer for the electronic system. The validity of this assumption can be experimentally verified. For example, if we fix the temperature of the liquid substrate at some "separation" from the melting temperature T_m of the crystal and then raise the lattice temperature adiabatically while keeping track of the threshold frequency $\omega_\xi(T_e)$, this frequency clearly should diminish and eventually go to zero at the melting point of the crystal, i.e. when $T_e = T_m$. In other words, the necessary conditions for calibrating the electronic thermometer do exist.

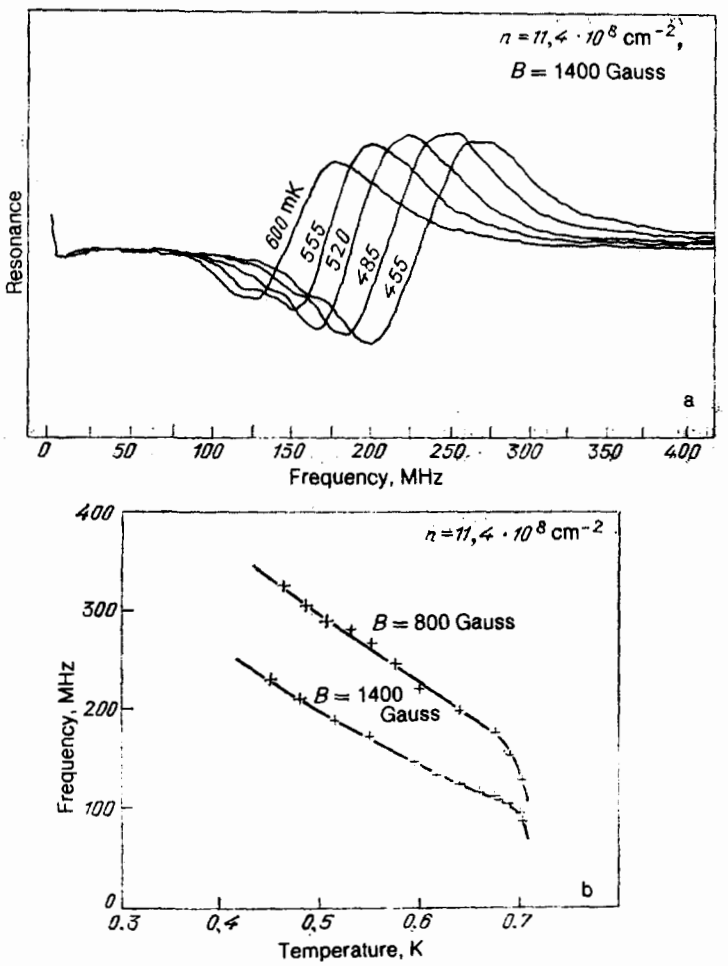

FIG. 4. (a) Appearance of a resonance at the frequency ω_ξ (1) as the temperature is lowered in the $T < T_m$ range (Ref. 6). (b) Temperature dependence of ω_ξ (Ref. 18). Note the fairly strong dependence of ω_ξ on the magnetic field, arising from the effect of H on $\langle u^2\rangle$. The influence of H on the melting temperature of the crystal is much weaker, however, in agreement with the Kosterlitz–Thouless theory.[13]

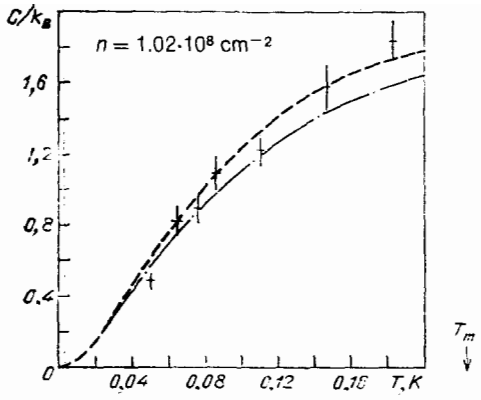

FIG. 5. Heat capacity of the electronic lattice normalized by the Boltzmann constant, C/k_B, measured for $n = 1.02 \cdot 10^8$ cm^{-2}. Dash-dotted line plots the calculated phonon heat capacity of a two-dimensional electronic lattice at this density using the $\mu(T=0)$ shear modulus; dashed line plots the same calculation using the temperature-dependent $\mu(T)$ shown in Fig. 3(a). The arrow marks the melting temperature T_m (experimental points and calculations from Refs. 16, 17).

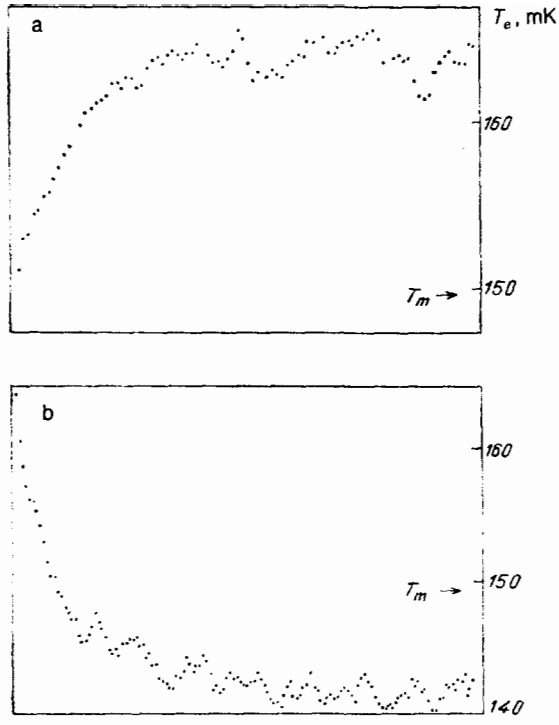

FIG. 6. Temperature of the electronic system as a function of time after the heating field has been (a) switched on and (b) switched off. The x-axis is time in μs, the y-axis is the electronic temperature T_e in mK. Evidently in the vicinity of T_m the electronic temperature changes smoothly and monotonically, indicating no appreciable heat of melting in the crystal–liquid phase transition. Helium temperature was 135.5 ± 0.3 mK, densities $n_1 = 0.44 \cdot 10^8$ cm^{-2}, $n_1 = 0.95 \cdot 10^8$ cm^{-2} (Refs. 16, 17).

The authors of Refs. 16 and 17 employed this technique to measure the temperature dependence of the heat capacity C, which is illustrated in Fig. 5. According to estimates of Ref. 16 this heat capacity corresponds to the phonon contribution calculated for the two-dimensional lattice. In Fig. 5 the theory is compared to the experiment: one of the theoretical curves uses $\mu = \text{const} = \mu(0)$, while the other takes into account the temperature dependence $\mu(T)$. The latter curve accounts for the experimental data better.

This outstanding series of experiments[6–8,16,17] was crowned by the experimental measurement of the heat of melting of the electronic lattice in the solid–liquid transition. This experiment was hindered by the additional complication that in the liquid phase the threshold frequency ω_ξ goes to zero (1) and hence the temperature of the electronic lattice is no longer directly measurable. The authors of Refs. 16, 17 overcame this difficulty by designing a cell to hold an electronic system with two regions of different electron densities n (actually, the cell was cylindrical in geometry, where the region $0 \leqslant r \leqslant R_1$ contained density n_1 and the region $R_1 \leqslant r \leqslant R_2$ contained a different density n_2). The two electronic subsystems were in good thermal contact so their electronic temperatures could be taken as equal. The experimental parameters were selected such that near the melting temperature of the subsystem 1, which had a lower density, subsystem 2 would remain in the crystalline state. Consequently the threshold frequency $\omega_\xi{}^2$ could be used as a thermometer for the whole system.

The experimental value of the heat of transition approached zero within experimental error (Fig. 6). This correlates with the predictions of the Kosterlitz–Thouless theory of phase transitions that are driven by fluctuational multiplication of dislocations.

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