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L. N. Bulaevskiĭ, V. L. Ginzburg, A. A. Sobyanin, and A. A. Stratonnikov

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A scientific session of the Division of General Physics and Astronomy of the USSR Academy of Sciences was held on September 21, 1988, at the S. I. Vavilov Institute of Physics Problems of the USSR Academy of Sciences. The following reports were presented at the session:

1. *L. P. Gor'kov*. Ongoing research in the HTSC field (review).

2. *L. N. Bulaevskiĭ, V. L. Ginzburg, A. A. Sobyanin, and A. A. Stratonnikov*. Macroscopic theory of defect-free and defective superconductors with a small coherence length.

A brief summary of the second report is presented below.

L. N. Bulaevskiĭ, V. L. Ginzburg, A. A. Sobyanin, and A. A. Stratonnikov. *Macroscopic theory of defect-free and defective superconductors with a small coherence length.* When the coherence length ξ of "ordinary" superconductors with critical temperature $T_c < 25$ K, known prior to 1986, is extrapolated to zero temperature, $\xi(0) \equiv \xi_0$ is significantly larger than the characteristic interatomic or interelectronic spacing $d \sim 10^{-8}$–10^{-7} cm. Consequently, the critical region near T_c where fluctuations are significant turns out to be small. For this reason, mean-field theories and, concretely, the macroscopic Ginzburg–Landau (GL) or Ψ-theory of superconductivity apply practically everywhere near T_c.[1] On the other hand, in the high-temperature superconductors (HTSC) discovered in 1986, the coherence length ξ_0 is small and the ratio ξ_0/d cannot, in general, be taken as large. Thus, in addition to the most pressing current theoretical problem of elucidating the physical nature and mechanism of HTSC there exists a completely independent problem of developing a macroscopic theory of superconductivity that takes into account fluctuation effects and is valid in the critical region.

The smallness of ξ_0 leads to another distinguishing feature of HTSC: their physical properties are anomalously strongly influenced by various boundaries (including the metal–vacuum interface) and sample defects, including twins, grains, dislocations, and even individual "point" defects (impurities, interstitial atoms, etc.). Certainly defects also affect the properties of "ordinary" superconductors, but there the influence of defects is generally averaged out (since the mean separation between defects is $L \ll \xi_0$) or has an effect very close to T_c (for example, in the case of a metal–vacuum interface at $|t| = |T - T_c|/T_c \lesssim (d/\xi_0)^2$—see Ref. 2). In HTSC, on the other hand, the conditions $L \ll \xi_0$ and $(d/\xi_0)^2 \ll 1$ are often violated, making it impossible to ignore boundary and defect effects or to treat them as averaged quantities.

Finally, nearly all HTSC of which we are aware are characterized by a fairly strong anisotropy of critical magnetic fields and other parameters. These anisotropies should, of course, be included in the development of a microscopic theory.

In this report we will address only a few central questions; a more complete discussion of the problem is available in Refs. 3, 4.

Foundations. In the theory developed in Refs. 1 and 5, the order parameter is a scalar function $\Psi = \eta e^{i\varphi}$ (one then speaks of S-pairing and a two-component order parameter). Generally, one cannot exclude the existence of HTSC with more complicated order parameters.[6,7] Although a general theory that accommodates such cases is of interest, here we shall consider S-pairing only. Accordingly, our fundamental free energy equation[5] has the form:

$$F = F_n + \int \left[\frac{\mathbf{B}^2}{8\pi} + a|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \frac{1}{4m_l^*} \left| \left(-i\hbar\nabla_l - \frac{2e}{c} A_l \right) \Psi \right|^2 \right] dV; \qquad (1)$$

where $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic induction vector; F_n is the free energy of the normal state; $a = \alpha t$; $t = (T - T_c)/T_c$ is the relative separation from T_c; $2m_l^* = \{2m_x^*, 2m_y^*, 2m_z^*\}$ are the principal values of the effective mass tensor for superconducting electron pairs (of charge $2e$). Clearly, if $m_x^* = m_y^* = m_z^*$ we are dealing with the Ψ-theory of an isotropic superconductor.[1] Further, in (1) α and b are some positive constant; $\hbar$ is the Planck constant and c is the speed of light. Finally, in equation (1) and other expressions below, the coordinate axes are taken to lie along the major crystal symmetry directions, and the recurring subscript $l = \{x,y,z\}$ indicates summation over the axes.

The equilibrium (most probable) value $\Psi = \Psi_e$ corresponds to the minimum of F, which is found by solving the equations:

$$\frac{1}{4m_l^*} \left(-i\hbar\nabla_l - \frac{2e}{c} A_l \right)^2 \Psi + a\Psi + b|\Psi|^2 \Psi = 0, \qquad (2)$$

$$\mathrm{rot}\ \mathbf{B} = \frac{4\pi}{c} \mathbf{j}, \qquad (3)$$

$$j_l = -\frac{ie\hbar}{2m_l^*} (\Psi^* \nabla_l \Psi - \Psi \nabla_l \Psi^*) - \frac{2e^2}{m_l^* c} |\Psi|^2 A_l; \qquad (4)$$

where $\mathbf{j}$ is the superconducting current density (the normal current density is taken to be zero).

The boundary conditions on equations (2)–(4) are that all components of the induction vector $\mathbf{B}$ are continuous at the superconductor boundary and another boundary condition on Ψ. Usually, at the superconductor–vacuum (dielectric) boundary, with $\mathbf{B} = 0$ for simplicity, this last condition is taken as

$$\left. \frac{d\Psi}{dn} \right|_S = 0, \qquad (5)$$

whereas at the superconductor–normal metal boundary one assumes a more general condition

$$n_l \Lambda_l \left. \frac{\partial \Psi}{\partial x_l} \right|_S = \Psi_S, \qquad (6)$$

where n_l are the components of the unit normal to the super-

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conductor boundary and Λ_l have the same dimensionality as length and are often labeled the extrapolation length parameters.

In HTSC boundary conditions similar to (6) probably should be applied at the vacuum boundary, as well as at the boundaries of all flat, linear, and point defects, in the same way as in the theory of superfluid[8] and other second-order phase transitions[9,10] where, generally, $\Lambda/\xi_0 \sim \xi_0/d \sim 1$.

Generalization to the limiting case of strong anisotropy. For very anisotropic (layered) superconductors the continuous medium approximation is invalid in the z direction perpendicular to the layers. In this case the free energy functional (1) is replaced by the differential–difference functional[11,12]

$$F = F_n + \sum_{j=1}^{N} \left\{ \int \left[\frac{\mathbf{B}^2}{8\pi} + a|\Psi_j|^2 + \frac{b}{2}|\Psi_j|^4 + \frac{1}{4m_l^*}\left|\left(-i\hbar\nabla_l - \frac{2e}{c}A_l\right)\Psi_j\right|^2 + r\alpha\left(|\Psi_j - \Psi_{j-1}|^2 + |\Psi_{j+1} - \Psi_j|^2\right)\right] d\rho \right\}. \quad (7)$$

Here the radius vector $\boldsymbol{\rho}$ lies in the (x,y) plane of the layers; j is the layer number; $l = \{x,y\}$; and the dimensionless positive constant r characterizes the "strength" of the Josephson interaction between the order parameters $\Psi_j(\rho)$ in neighboring layers.

When $r \gg |t|$ the functional (7) reduces to the three-dimensional isotropic functional (1), whereas the $r \to 0$ limit corresponds to purely two-dimensional superconductivity. In HTSC based on the yttrium and lanthanum group the experimental value is $r \sim 1$, hence near T_c they can be treated as ordinary three-dimensional superconductors.

Thermal fluctuations. A rigorous method of studying thermal fluctuations, which underpins the modern fluctuation theory of second-order phase transitions, consists of employing the free energy functional (1) or (7) as an effective Hamiltonian which determines the probability[13]

$$w \backsim \exp\left\{-\frac{1}{k_B T}\left(F[\Psi(\mathbf{r})] - F[\Psi_e(\mathbf{r})]\right)\right\} \quad (8)$$

of finding the system in a state described by a given function $\Psi(\mathbf{r})$ which differs from the equilibrium (most probable) state function $\Psi_e(\mathbf{r})$.

An approximate treatment, which has yielded good results near the superfluid λ-transition in ^{4}He (see Ref. 14), accounts for thermal fluctuations by introducing the temperture dependence of the coefficients into the free energy functional (1). Thus, in the region of small (Gaussian) fluctuations we have:[3,15]

$$a \to \alpha t\left[1 + \frac{3}{4}\left(\frac{t_G}{|t|}\right)^{1/2}\right], \quad b \to b_0\left[1 - \frac{9}{4}\left(\frac{t_G}{|t|}\right)^{1/2}\right], \quad \frac{1}{m_l^*} \to \frac{1}{m_{l,0}^*}\left[1 + \frac{3}{16}\left(\frac{t_G}{|t|}\right)^{1/2}\right], \quad (9)$$

whereas in the region of large (critical) fluctuations we have:[3,14]

$$a \to \alpha t\left(\frac{|t|}{t_G}\right)^{1/3}, \quad b \to b_0\left(\frac{|t|}{t_G}\right)^{2/3}, \quad \frac{1}{m_l^*} \approx \text{const.} \quad (10)$$

Moreover, since the coefficients a and b go to zero as $t \to 0$ in the critical region, equation (1) should be modified by an additional term $(1/3)g|\Psi|^6$ with a positive constant g.

In the equations (9) the quantity t_G determines the width of the large fluctuation region and is related to the jump in the heat capacity ΔC_p at $T = T_c$ and the mean coherence length $\bar{\xi}_0 = (\xi_{0x}\xi_{0y}\xi_{0z})^{1/3}$ by the expression[3]

$$t_G = \frac{1}{32\pi^2}\left(\frac{k_B}{\Delta C_p \bar{\xi}_0^3}\right)^2. \quad (11)$$

The quantity t_G can also be determined directly by measuring the amplitude $C_0^{\pm}$ of the first (Gaussian) fluctuation correction $C_{fl} = C_0^{\pm}/|t|^{1/2}$ to the heat capacity at $t > 0$ and $t < 0$:

$$t_G = \left(\frac{C_0^-}{\Delta C_p}\right)^2 = 2\left(\frac{C_0^+}{\Delta C_p}\right)^2. \quad (12)$$

Concretely, the data in Ref. 16 indicate that in $YBa_2Cu_3O_{7-x}$ single crystals the parameter $t_G \sim 10^{-3}$, i.e. the width of the critical region is only of the order of 0.1 K. Nonetheless the contribution of fluctuation effects in C_p is quite significant, reaching 10% of ΔC_p at $|T - T_c| \sim 10$ K and $\sim 30\%$ at $|T - T_c| \sim 1$ K.

The most striking result of the fluctuation theory is that in the fluctuation region the ratio $\varkappa$ of the magnetic field penetration depth δ to the coherence length ξ should become smaller as $T \to T_c$ and, consequently, sufficiently close to T_c a type II superconductor should turn into a type I superconductor. Unfortunately, in the known HTSC the initial value of $\varkappa$ far from T_c is large and the effect is probably impossible to observe experimentally.

Effect of boundaries and defects. In the boundary condition (6), which remains valid in the critical region, the sign of parameter Λ is undetermined and, in principle, can be either positive or negative. The latter case is particularly interesting, as it leads to the appearance of superconductivity first at some temperature $T_c' > T_c$ in a narrow boundary layer (or in a small region surrounding a defect) with a charac-

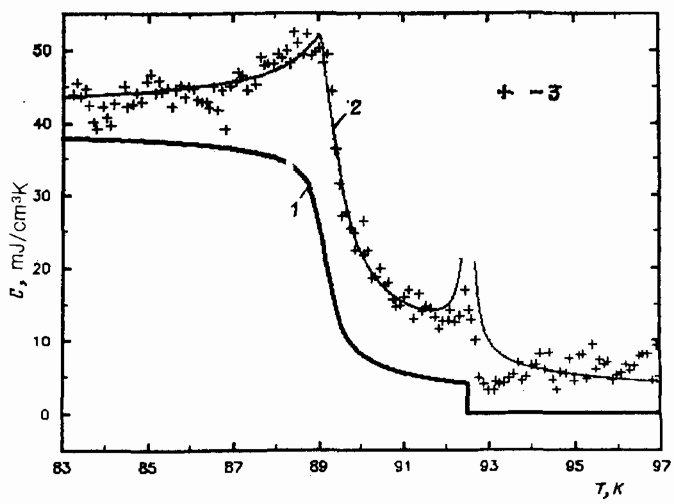

FIG. 1. Temperature dependence of the superconducting component in the heat capacity of the "Argonne" $YBa_2Co_3O_{7-x}$ crystal[16] in zero external magnetic field. 1—theoretical dependence[4] neglecting fluctuation effects with the following parameters: $\Delta C_p = 14$ mJ/cm^{3}K; $L/\xi_{0x} = 100$; $\Lambda/\xi_{0x} = -5$. 2—same calculation including thermal fluctuations with $\bar{\xi}_0 = 9$ Å. 3—experimental data.[16]

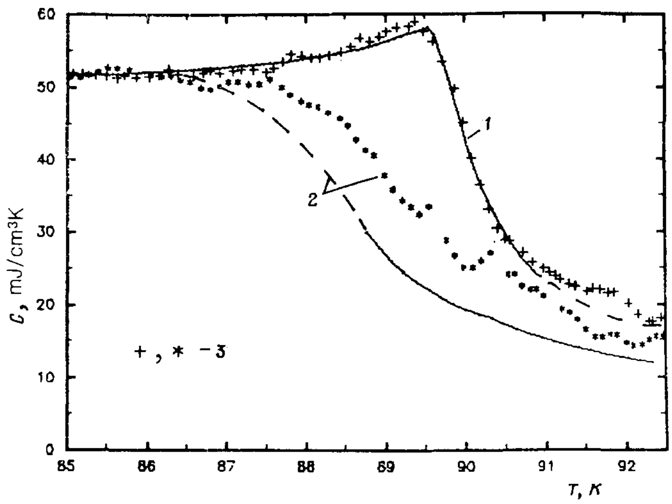

FIG. 2. Temperature dependence of the superconducting component in the heat capacity of the "Illinois" crystal: 1—$H = 0$; 2—$H = 30$ kOe. Solid curves are computed theoretically[4] using $\Delta C_p = 42$ mJ/cm^{3}K; $\xi_0 = 8$ Å; $L/\xi_{0x} = 87$; $\xi_{0x}/\xi_{0z} = 8$; $\Lambda/\xi_{0x} = -5$. Dashed regions indicate extrapolation of calculated curves to temperature regions where exact calculations are difficult. 3—experimental data.[16,21]

teristic thickness $\xi(t_c') = \xi_{0x}|t_c'|^{-1/2}$, where $t_c' = |T_c' - T_c|/T_c$. As the temperature is lowered the superconducting layer expands until, at $T < T_c$ the entire crystal becomes superconducting.

The appearance of this "local" superconductivity has been observed near the twin boundaries in many "ordinary" superconductors.[17] Thus the effect is quite likely in HTSC, where the twinning-planes (TP) are generally very numerous. This conclusion is supported by a large number of experimental facts,[18-20] particularly the data of the heat capacity in $YBa_2Cu_3O_{7-x}$ "single crystals" reported in Ref. 16, 21 and analyzed in detail in Ref. 4. In Figs. 1 and 2 we compare the calculated results of Ref. 4 (based on expressions (1)–(6), including Gaussian fluctuation effects) with experimental data[16,21] for external magnetic intensities of zero and 30 kOe.

The good agreement of calculated and experimental curves corroborates the initial assumption that superconductivity is somewhat enhanced near TP. Also, a comparison with experimental data[16,21] makes it possible to compute all superconductor parameters that enter into (1) and evaluate the values of the extrapolation length Λ and the mean twin separation L. The results of these calculations are as follows (m_e is the free electron mass): $\Lambda \approx -75$ Å, $L \approx 1500$ Å, $\bar{\xi}_0 = 8 \pm 0.5$ Å, $\xi_{0x} \approx \xi_{0y} = 15 \pm 3$ Å, $\xi_{0z} = 2 \pm 0.5$ Å, $a = 1.2 \cdot 10^{-14}$ erg, $b = 4.0 \cdot 10^{-36}$ erg·cm^3, $m_x \approx m_y \approx (10\text{–}14) m_e$, $m_z \approx (250\text{–}1500) m_e$.

In obtaining the last four quantities we have neglected the possible weak anisotropy of effective mass in the plane of the Cu–O layers and normalized the order parameter with respect to the concentration of superconducting pairs at zero temperature $n_s(0) = a/b = 3 \cdot 10^{21}$ cm^{-3}, corresponding to approximately half a pair per unit cell. Clearly this choice of Ψ normalization is arbitrary and does not effect the physical results of the theory.

Conclusions. We have discussed several features of the macroscopic theory of HTSC and demonstrated its validity by considering the experimental measurements on the critical heat capacity anomaly in $YBa_2Cu_3O_{7-x}$ crystals. The macroscopic theory is insensitive to the concrete microscopic superconductivity mechanism and is, therefore, simpler and more reliable in this regard than a microscopic theory. At the same time a number of unresolved questions still remains in the macroscopic theory of HTSC. One such question is related to the possibility of more complex superconducting phases in some HTSC, which may be described by vector or tensor order parameters analogous to the superfluid ^{3}He phase. The description of such phases[6,7] differs in some respects from the above macroscopic description of "scalar" superconductivity.

Another very important question concerns the recently suggested possibility[22] that the classical methods of accounting for fluctuations—based on expression (8)—which have successfully described all other second-order phase transitions may prove inapplicable in the case of superconductors with a strongly retarded electron-phonon interaction.

The clarification of these questions and, more importantly, the verification of the various predictions of the macroscopic theory are of great interest and await further experimental and theoretical research.

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