## B-Meson oscillations

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A review is given of experimental data on $\mathbf{B}^{0} \overline{\mathrm{~B}}^{0}$ oscillations. Basic theoretical concepts are discussed and an analogy with $\mathrm{K}^{0} \overline{\mathrm{~K}}^{0}$ oscillations is noted. A detailed analysis is given of experimental searches for $\mathbf{B}^{0} \overline{\mathbf{B}}^{0}$ oscillations with particular attention devoted to the ARGUS data, which reveal $\mathbf{B}_{\mathrm{d}}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ mixing with $r=\Gamma\left(\mathrm{B}^{0} \rightarrow \overline{\mathbf{B}}^{0} \rightarrow \mathrm{X}^{\prime}\right) / \Gamma\left(\mathbf{B}^{0} \rightarrow \mathrm{X}\right)=0.21 \pm 0.08$. The consequences of large $\mathbf{B}^{0} \overline{\mathbf{B}}^{0}$ mixing are examined and future studies of the properties of the $b$ quark are discussed.

## I. INTRODUCTION

The last few years have been singularly successful in relation to studies of particles containing $b$ quarks. The discovery of B-meson oscillations in the UA1 ${ }^{1}$ and ARGUS ${ }^{2}$ experiments has attracted particular interest. Particle oscillations are defined as transitions of particles into antiparticles. This interesting quantum-mechanical effect has long been known in the case of neutral kaons. Thus, $\mathbf{K}^{0}$ and $\overline{\mathbf{K}}^{0}$ mesons differ from one another by having opposite strangeness, which is not a strictly conserved quantum number for weak interactions, so that transitions between $\mathrm{K}^{0}$ and $\overline{\mathbf{K}}^{0}$ are possible. Mixing of the two states in vacuum leads to split ting into $K_{1}$ and $K_{2}$, which have definite masses and widths. The $\mathbf{K}^{0}$ and $\bar{K}^{0}$ mesons are linear combinations of $K_{1}$ and $\mathrm{K}_{2}$ :

$$
K^{0}=\frac{1}{1 / 2}\left(K_{1}+K_{2}\right), \quad \bar{K} 0=\frac{1}{1 / 2}\left(K_{1}-K_{2}\right) .
$$

In the modern theory of weak interactions, which is commonly referred to as the standard model (SM), transitions involving a change of flavor by two units occur in the second order in the weak interaction. It follows that oscillations of neutral mesons constitute a sensitive instrument that could be used to search for new phenomena. For example, Glashow, Iliopoulos, and Maiani (GIM) showed that a small mass difference between $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ due to $\mathrm{K}^{0} \overline{\mathbf{K}}^{0}$ mixing could be naturally explained by a model involving four quark types, ${ }^{3}$ i.e., studies of neutral kaon oscillations have provided one of the basic arguments in favor of the existence of a new charmed c quark well before the discovery in 1974 of the $J / \psi$ particle, i.e., the ce bound state of quarks. Still greater hopes are invested in oscillations of mesons containing heavy quarks.

The $\Upsilon(4 S)$ resonance, which decays to the $B \bar{B}$ meson pair, i.e., particles with bare charm, was discovered in 1980 in the electron-positron storage ring (CESR) at Cornell University in the USA. It is the heaviest of known mesons that undergo weak decays. The mass of the charged $\mathrm{B}^{+}$meson ${ }^{1)}$ (quark composition $\bar{b} u$ ) is $5278.7 \pm 0.7 \pm 2.0 \mathrm{MeV} / c^{2}$ and the mass of the neutral $\mathrm{B}_{\mathrm{d}}^{0}(\overline{\mathrm{~b}} \mathrm{~d})$ meson is $5280.7 \pm 6.0 \pm 2.0 \mathrm{MeV} / c^{2}$ (Ref. 4), where the first indicated uncertainty is statistical and the second systematic. The $\mathbf{B}_{\mathrm{s}}^{0}(\overline{\mathrm{~b}} s)$ meson should be heavier still. Studies of B mesons provide information on the properties of the charmed $\mathbf{b}$ quark, i.e., the first and, so far, the only experimentally discovered representative of third generation quarks.

Figure 1 shows diagrams describing the B-meson oscil-
lations. According to the GIM mechanism, diagrams involving the exchange of the heaviest $t$ quark play the dominant role. However, it must not be forgotten that other, hypothetical, particles may contribute to the $\mathrm{B}^{0}$ oscillations in addition to the $t$ quark. Since, at present, we do not have a clear understanding of the nature of the different generations of leptons and quarks, the oscillations of charmed mesons are sensitive to the manifestations of new physics, such as the possible existence of fourth-generation fermions, right-handed $W$ bosons, supersymmetric particles, and charged Higgs bosons (see, for example, Refs. 5 and 6). Figure 2 shows examples of the corresponding diagrams.

A part from the search for new particles, another important task for B-meson research is the determination of the standard model parameters. It is well-known that the quark sector of the standard model involves ten adjustable parameters, namely, six quark masses, three mixing angles $\theta_{i j}$ between quarks belonging to different generations, and the phase $\delta$ that describe CP invariance violation in weak interactions. Mixing of quarks arises because the weak-interaction eigenstates are not identical with the eigenstates of the mass matrix. In the standard model, the connection between these states is determined by the Kobayashi-Maskawa matrix. ${ }^{7}$ There are different parametrizations of the KobayashiMaskawa matrix. We shall use the approximate parameterization ${ }^{8}$
$\left(\begin{array}{lll}V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & \Gamma_{\mathrm{tb}}\end{array}\right) \approx\left(\begin{array}{ccl}1 & s_{12} & s_{13} e^{-i \delta} \\ -s_{12}-s_{93} s_{13} e^{i \delta} & 1 & s_{23} \\ s_{12} s_{23}-s_{13} e^{i \delta} & -s_{23} & 1\end{array}\right)$,
where

$$
s_{i j} \equiv \sin \theta_{i j}, \quad \cos \theta_{i j} \approx 1
$$

The experimental determination of these and other adjustable parameters of the standard model is important as a means of confirming (or otherwise) the validity of this model , and thus pointing toward a new and more complete theory.

Five parameters, i.e., the masses of the $u, d, s$, and $c$ quarks and the Cabbibo angle $\theta_{12}$, are determined by study-


FIG. 1. Diagrams for the $\mathrm{B}_{\mathrm{d}}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{\mathrm{o}}$ oscillations in the standard model.


FIG. 2. Examples of diagrams defining the possible contribution of new particles to $\mathrm{B}^{\circ} \overline{\mathrm{B}}^{\circ}$ oscillations. Fourth generation quarks (a), right-handed W bosons (b), supersymmetric particles (c), and charged Higgs bosons (d).
ing the first two generations of quarks. In principle, the remaining five parameters can be obtained by investigating the properties of the $B$ mesons. The mass of the $b$ quark is equal to the mass of the $\mathbf{B}$ meson to within a few hundred MeV . The matrix elements $V_{\mathrm{ub}}$ and $V_{\mathrm{cb}}$ are found experimentally as a result of studies of $\mathbf{B}$-meson decays and lifetime.

Despite intensified searches for it, the $t$ quark has not as yet been found. We have already noted that $\mathrm{B} \overline{\mathrm{B}}$ oscillations, whose frequency is proportional to $m_{i}^{2}$, provide us with a unique possibility of exploring the influence of the $t$ quark well away from its production threshold. The $\mathbf{B}_{\mathrm{d}}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ and $\mathrm{B}_{\mathrm{s}}^{0} \overline{\mathbf{B}}_{\mathrm{s}}^{0}$ oscillations depend not only on $m_{\mathrm{t}}$, but also on $V_{\mathrm{td}}$ and $V_{\text {ts }}$. The requirement of unitarity imposes additional restrictions on these elements of the Kobayashi-Maskawa matrix (see, for example, Ref. 9). One hopes that further improvement in the precision of the matrix elements will result in reliable determinations of the phase responsible for CP invariance violation and of all the mixing angles.

Experimentally, the $\mathbf{B}^{0} \overline{\mathbf{B}}^{0}$ oscillations are particularly well-defined in the evolution of the $\mathbf{B}^{0} \overline{\mathbf{B}}^{0}$ system in time. The probability that the $B^{0}$ meson remains in this state or transforms to the antiparticle can be defined by analogy with the corresponding probabilities for $\mathrm{K}^{0}$ and $\overline{\mathrm{K}}^{0}$ (see, for example, Ref. 10):

$$
\begin{align*}
& w(t)=\frac{1}{4}\left(e^{-\Gamma_{2} t}+e^{-\Gamma_{2} t}+2 e^{-\Gamma t} \cos \Delta M t\right)  \tag{2}\\
& \bar{w}(t)=\frac{1}{4}\left(e^{-\Gamma_{1} t}+e^{-\Gamma_{2} t}-2 e^{-\Gamma t} \cos \Delta M t\right)
\end{align*}
$$

where $M_{1}, M_{2}, \Gamma_{1}, \Gamma_{2}$ are the masses and widths of the eigenstaes $B_{1}$ and $B_{2}$ with opposite CP parities, which are produced as a result of mixing: $\Gamma=\left(\Gamma_{1}+\Gamma_{2}\right) / 2$ and $\Delta M=M_{1}-M_{2}$. The functions of $\omega(t)$ and $\bar{\omega}(t)$ are shown in Fig. 3 for two values of the parameter $\Delta M / \Gamma$.

Unfortunately, because of the short lifetime of the $\mathrm{B}^{0}$ mesons, differential properties such as those defined by (2) cannot at present be measured. Modern experiments are sensitive only to the integrated quantities

$$
\begin{aligned}
& W=\int_{0}^{\infty} w(t) \mathrm{d} t=\frac{1}{4}\left[\frac{1}{\Gamma_{1}}+\frac{1}{\Gamma_{2}}+\frac{2 \Gamma}{\Gamma^{2}+(\Delta M)^{2}}\right], \\
& \bar{W}=\int_{0}^{\infty} \bar{w}(t) \mathrm{d} t=\frac{1}{4}\left[\frac{1}{\Gamma_{1}}+\frac{1}{\Gamma_{2}}-\frac{2 \Gamma}{\Gamma^{2}+(\Delta M)^{2}}\right] .
\end{aligned}
$$

The $\mathbf{B}$ and $\overline{\mathrm{B}}$ mesons are identified by examining their decays to final states X and $\mathrm{X}^{\prime}$, some of which may not be common to $B$ and $\bar{B}$.

The $\mathbf{B}^{0} \overline{\mathbf{B}}^{0}$ oscillations are usually characterized by the parameter ${ }^{11}$

$$
\chi=\frac{\bar{W}}{W+\bar{W}}=\frac{\Gamma\left(\mathrm{B}^{0} \rightarrow \overline{\mathrm{~B}}^{0} \rightarrow \mathrm{X}^{\prime}\right)}{\Gamma_{\operatorname{tot}}},
$$

i.e., the probability of finding $\overline{\mathbf{B}}^{0}$ when the initial particle is the $\mathrm{B}^{0}$ meson. Of course, searches for the $\mathrm{B}^{0} \overline{\mathrm{~B}}^{0}$ oscillations must make use of decays that are different for $\mathrm{B}^{0}$ and $\overline{\mathrm{B}}^{0}$.

Hence we readily obtain

$$
\chi=\frac{x^{2}+y^{2}}{2+2 x^{2}},
$$

where $x=\Delta M / T$ and $y=\Delta \Gamma / 2 \Gamma$. For $\mathbf{K}^{0} \overline{\mathbf{K}}^{0}$ oscillations, $x \sim y \sim 1$, so that $\chi \approx 0.5$, which corresponds to complete mixing. Since, for the $\mathrm{B}_{\mathrm{d}}^{0} \overline{\mathrm{~B}}_{\mathrm{d}}^{0}$ system, the ratio $\Delta \Gamma / 2 \Gamma$ should not exceed a few percent, ${ }^{12,13}$ we find that, for large $B^{0} \bar{B}^{0}$ mixing,

$$
\begin{equation*}
\chi \approx \frac{x^{2}}{2+2 x^{2}} \tag{3}
\end{equation*}
$$

Another oscillation parameter that is often found in the literature ${ }^{14}$ is

$$
\begin{equation*}
r=\frac{\Gamma\left(\mathrm{B}^{0} \rightarrow \overline{\mathrm{~B}}^{0} \rightarrow \mathrm{X}^{\prime}\right)}{\Gamma\left(\mathrm{B}^{0} \rightarrow \mathrm{X}\right)}=\frac{\chi}{1-\chi} . \tag{4}
\end{equation*}
$$

The magnitude of $\mathbf{B}^{0} \overline{\mathrm{~B}}^{0}$ mixing is not simple to estimate even in the standard model, and especially for the $\mathrm{B}_{\mathrm{d}}^{0}$ mesons. At one time, there were indications that the relatively light $t$ quark ( $m_{t} \approx 40 \mathrm{GeV} / c^{2}$ ) might exist, ${ }^{15}$ so that, in most theoretical papers, the parameter $\chi_{\mathrm{d}}$ was expected to be $0.1-1 \%$ for the $\mathbf{B}_{\mathrm{d}}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ oscillations. ${ }^{16}$ However, experiments have shown that this was not so, and that ${ }^{2} \chi_{\mathrm{d}}=0.17 \pm 0.05$. In the following sections, we shall discuss searches for, and the discovery of, the $\mathbf{B}^{0} \overline{\mathbf{B}}^{0}$ oscillations (Sec. 2), the consequences of large $\mathrm{B}_{\mathrm{d}}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ mixing (Sec. 3), and possible future studies of the b quark (Sec. 4).


FIG. 3. The probabilities $w(t)$ and $\bar{w}(t)$ for two values of the mixing parameter $x=\Delta M / \Gamma$ (Ref. 32). The dashed curves show the exponential decay of $\mathrm{B}^{0}$ and $\overline{\mathrm{B}}^{0}$ mesons. The shaded region corresponds to the probability of finding $\mathrm{B}^{0}$ in the $\overline{\mathrm{B}}^{\circ}$ state at the point of decay, and the cross-hatched region represents the corresponding probability for the $\overline{\mathbf{B}}^{\prime \prime}$ state.

## 2. THE SEARCH FOR AND THE DISCOVERY OF B-MESON OSCILLATIONS

At present, B mesons are being studied mostly in elec-tron-positron and proton-antiproton storage rings. There is a fundamental distinction between experiments at high energies and those near the threshold for $\mathrm{B} \overline{\mathrm{B}}$ pair production, especially in the region of the $\Gamma(4 \mathrm{~S})$ resonances.

### 2.1 Experiments at high energies

The $\mathbf{B}_{\mathrm{d}}^{0}$ and $\mathrm{B}^{ \pm}$mesons, as well as the $\mathrm{B}_{\mathrm{s}}^{0}$ mesons, are produced at high energies. As a rule, they are accompanied by a large number of light particles. The short lifetime of the B mesons has meant that their decay vertices have not yet been determined. Since the mean multiplicity of particles in the decay of the $\mathbf{B}$ mesons themselves is relatively high ( $\sim 8$ ), it is clear that it will be exceedingly difficult to reconstruct the complete picture of an event of this type. Indirect methods are therefore being used in searches for the $\mathbf{B}^{0} \overline{\mathbf{B}}^{0}$ oscillations at high energies. They rely on the fact that the decay of the $b$ and $\bar{b}$ quarks is accompanied by the appearance of negative and positive leptons, respectively (Fig. 4). Since the $b$ and $\bar{b}$ quarks are produced in pairs, leptons from their decays should have different signs. On the other hand, events involving leptons of the same sign arise when the $\mathbf{B}^{\mathbf{0}}$ meson transforms into the $\overline{\mathbf{B}}^{0}$ (or vice versa) as a result of oscillations.

In high energy experiments, the mixing parameter is determined by both $\mathrm{B}_{\mathrm{d}}^{0}$ and $\mathrm{B}_{\mathrm{s}}^{0}$ oscillations:

$$
\begin{equation*}
\langle\chi\rangle=\frac{\mathrm{Br}_{\mathrm{d}}}{\langle\mathrm{Br}\rangle} f_{\mathrm{d}} \chi_{\mathrm{d}}+\frac{\mathrm{Br}_{\mathrm{s}}}{\langle\mathrm{Br}\rangle} f_{\mathrm{s}} \chi_{\mathrm{s}}, \tag{5}
\end{equation*}
$$

where $f_{\mathrm{d}}$ and $f_{\mathrm{s}}$ are fractions of $\mathrm{B}_{\mathrm{d}}^{0}$ and $\mathrm{B}_{\mathrm{s}}^{0}$ mesons in the fragmentation of the $b$ quark, $\mathrm{Br}_{\mathrm{d}}$ and $\mathrm{Br}_{\mathrm{s}}$ are relative probabilities of the semileptonic decay of $\mathbf{B}_{d}^{0}$ and $B_{s}^{0}$, and

$$
\langle\mathrm{Br}\rangle=\sum_{i} f_{i} \mathrm{Br}_{i}
$$

is the mean relative probability of semileptonic decay of all the charmed particles.

Most of these parameters have not as yet been measured. A number of different assumptions must therefore be employed to obtain information on $\chi_{\mathrm{d}}$ and $\chi_{\mathrm{s}}$.
(1) The MARK II group has carried out a search for lepton pairs of the same sign, using the electron-positron storage ring PEP at 29 GeV in the center of mass system. ${ }^{17}$ High-energy events have a well-defined double structure. The decay of the heavy B meson is accompanied by the appearance of leptons with high transverse momenta $p_{1}$ relative to the event axis, whereas leptons from $B$ mesons should be in different jets. On the other hand, decays of charmed primary particles should lead to lower transverse momenta. Events enriched with b or c quark decays are therefore iden-


FIG. 4. Diagrams showing the semileptonic decay of $B_{d}^{0}$ and $\bar{B}_{d}^{0}$ mesons.
tified by considering two kinematic regions, namely, those with transverse lepton momenta $p_{\mathrm{t}}>1 \mathrm{GeV} / c$ and those with $p_{\mathrm{t}}<1 \mathrm{GeV} / c$. The expected number of leptons from $b$ and $c$ decays in these two regions, and also the number of background effects, were determined by the Monte Carlo method. Allowance was made in these calculations for leptons from the primary $b$ quark, the primary c quark, the secondary c quark in the decay chain $\mathrm{b} \rightarrow \mathrm{c} \rightarrow l$, the decay of ordinary hadrons, and also the background due to particle identification errors.

In noncoherent B -meson production, the numbers of pairs of leptons of the same ( $\mathbf{N}_{++}+N_{--}$) and different ( $N_{+}-$) sign are given in terms of the mixing parameter $\langle\chi\rangle$ as follows:

$$
\begin{aligned}
& N_{+-}=\left[(1-\langle\chi\rangle)^{2}+\langle\chi\rangle^{2}\right]\left(N_{++}+N_{--}+N_{+-}\right), \\
& N_{++}+N_{--}=2\langle\chi\rangle(1-\langle\chi\rangle)\left(N_{++}+N_{--}+N_{+-}\right) .
\end{aligned}
$$

The recorded number of pairs of leptons of the same sign $N_{++}+N_{--}=9$ ) was found to be in agreement with the calculated number of background events $N_{\mathrm{b}}=12.6 \pm 3.2$ (in the region enriched with b-quark decays, there were four events corresponding to $N_{\mathrm{b}}=2.5 \pm 0.7$, which are included in this figure). This resulted in the upper limit $\langle\chi\rangle<0.12$ at the $90 \%$ confidence level.

As noted above, the probability of production of $\mathrm{B}_{\mathrm{d}}^{0}$ and $B_{s}^{0}$ mesons in jets containing the $b$ quark has to be known before information on the parameters $\chi_{\mathrm{d}}$ and $\chi_{\mathrm{s}}$ can be extracted. These paramters are still unknown, and there are no reliable theoretical predictions for them. If we consider that the relative probabilities of semileptonic decays of all the charmed particle are the same, and assume, as in Ref. 18, that the creation of baryons with a b quark amounts to $10 \%$, whereas the yields of $B_{d}^{0}, B^{ \pm}$, and $B_{s}^{0}$ are in the ratio 1:1:0.4, the MARK II results can be written in the form $0.375 \cdot \chi_{\mathrm{d}}+0.15 \cdot \chi_{\mathrm{s}}<0.12$ (at the $90 \%$ confidence level).
(2) An original method of establishing the upper limit for the mixing of the $\mathrm{B}^{0}$ and $\overline{\mathbf{B}}^{0}$ mesons was employed by the JADE group in an experiment ${ }^{19}$ performed on the PETRA electron-positron storage ring at 34.6 GeV in the center of mass system. Electron-positron annihilation is accompanied in the continuum by a charge-asymmetric angular distribution of primary leptons and quarks, due to interference between diagrams with photon and $Z^{0}$-boson exchange. This asymmetry has been reliably confirmed by experiment. ${ }^{20}$ The $\mathbf{B}^{0} \overline{\mathbf{B}}^{0}$ mixing should lead to a reduction in the asymmetry in b-quark production as compared with the standardmodel prediction ( $A_{\mathrm{SM}}$ ). The mixing parameter is related to the mesured asymmetry $A$ as follows:

$$
\frac{A_{\mathrm{SM}}-A}{A_{\mathrm{SM}}}=2\langle\chi\rangle
$$

It is important to note that, when the asymmetry in the production of $b$ quarks is measured, only one lepton need be recorded (and not two, as in the previous method).

The JADE group identified B mesons by using kinematic parameters such as the invariant jet mass, transverse muon momentum, and missing transverse momentum, and found that the asymmetry in the b-quark production was $A=(-22.8 \pm 6.0 \pm 2.5 \%)$. Standard-model calculations, without $\mathrm{B}^{0} \overline{\mathrm{~B}}^{0}$ mixing, predict $A_{\mathrm{SM}}=-25.2 \%$. The difference between $A$ and $A_{\mathrm{SM}}$ corresponds to $\langle\chi\rangle=0.05 \pm 0.13$.
(3) The first evidence for $\mathbf{B}^{0} \overline{\mathbf{B}}^{0}$ oscillations was ob-
tained in UA1 experiments on the proton-anitproton collider at CERN ${ }^{1}$ at 546 and 630 GeV in the center-of-mass system. The search for the $\mathrm{B}^{0} \overline{\mathbf{B}}^{0}$ oscillations was made by selecting events with two muons, in which both muon momenta at right angles to the beam axes were $p_{\mathrm{T}}>3 \mathrm{GeV} / c$. A total of 512 such dimuon events with invariant mass $m_{\mu \mu}>6$ $\mathrm{GeV} / c^{2}$ was found. Events from $\mathrm{Z}^{0} \rightarrow \mu^{+} \mu^{-}$decays were excluded.

In high-energy hadron collisions, the natural background process is the production of dimuon pairs by DrellYan mechanism via the virtual photon. ${ }^{21}$ The decay of the $\Upsilon$ resonance into the $\mu^{+} \mu^{-}$pair is also found to produce such isolated muons. All events were therefore divided into two groups, namely, those with two isolated muons and those in which at least one of the muons had a hadronic accompaniment. The first group was found to contain 98 events with muons of different sign and 15 events with muons of the same sign. The second group contained 257 and 142 events, respectively. Events with muons selected in this way were used to normalize the background due to the Drell-Yan process and the decays of $\Upsilon$ resonances. The background due to the erroneous identification of hadrons as muons was $132 \pm 21$ events, of which 8 corresponded to muons with different sign and a further 8 to muons of the same sign. The corresponding numbers for the second group were 58 and 58.

The result obtained after subtracting background events was

$$
R=\frac{N_{++}+N_{--}}{N_{+-}}=0.42 \pm 0.07 \pm 0.03
$$

where the first uncertainty is statistical and the second systematic. The background value of this ratio in the absence of $\mathrm{B}^{0} \overline{\mathrm{~B}}^{0}$ mixing was obtained by the Monte Carlo method, using different theoretical models of $b \bar{b}$ and $c \bar{c}$ quark-pair production and quark fragmentation into ordinary hadrons. The calculated background without mixing, $R=0.26 \pm 0.03$, was found to be less than the measured ratio, suggesting that there was an excess of events with dimuons of the same sign. This excess corresponds to $\langle\chi\rangle=0.121 \pm 0.047$. The statistical significance of the effect corresponds to 2.9 standard deviations. The authors of these calculations assumed that $\mathrm{B}_{\mathrm{d}}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ mixing was very small, and interpreted their results as $\mathbf{B}_{\mathrm{s}}^{0} \overline{\mathbf{B}}_{\mathrm{s}}^{0}$ mixing.
(4) The MARK group has recently presented an analysis of dimuon events in electron-positron annihilation ${ }^{22}$ for $s^{1 / 2}=29 \mathrm{GeV}$. They recorded five events with two muons of the same sign, and seven events with two muons of opposite sign. The background determined by the Monte Carlo method was $1.9 \pm 0.8$ and $8.6 \pm 1.5$ events, respectively. The excess of dimuons of the same sign was interpreted as the result of $B$-meson oscillations. The mixing parameter was found to be $\langle\chi\rangle=0.21 \pm{ }_{0.15}^{0.29}$, which gives a lower limit of $\langle\chi\rangle>0.02$ at the $90 \%$ confidence level. ${ }^{22}$

### 2.2 Experiments near the threshold ${ }^{81}$

The mixing parameter is determined in a different way in experiments involving searches for $\mathrm{B}^{0} \overline{\mathrm{~B}}^{0}$ oscillations in the region of the $\Upsilon(4 S)$ resonance. Thus, it may be considered that $\Upsilon(4 S)$ decays exclusively into $B^{+} B^{-}$or $B_{d}^{0} \overline{\mathbf{B}}_{d}^{0}$ meson pairs. ${ }^{22}$ The P -wave mixing parameter for the propagation of the $B \bar{B}$ pair is ${ }^{9}, 11,23$

$$
r=\frac{N\left(\mathrm{~B}^{0} \mathrm{~B}^{n}\right)+N \overline{\left(\overline{\mathrm{~B}}^{0} \overline{\mathrm{~B}}^{0}\right)}}{N\left(\mathrm{~B}^{0} \overline{\mathrm{~B}}^{0}\right)},
$$

which, for events with lepton pairs, is equivalent to

$$
\begin{equation*}
r=\frac{\left(N_{++}+N_{--}\right)(1+\lambda)}{N_{+-}-\left(N_{++}+N_{--}\right) \lambda} \tag{6}
\end{equation*}
$$

The parameter $\lambda$ in this expression represents the contribution of semileptonic decays of charged $B$ mesons to events with dileptons of different sign:

$$
\lambda=\frac{f^{+}}{f^{0}}\left(\frac{\mathrm{Br}}{\mathrm{~B}_{+}}\right)^{2},
$$

where $f^{+}\left(f^{0}\right)$ are the relative probabilities of the decay of $\gamma(4 \mathrm{~S})$ into a pair of charged (neutral) B mesons, and $\mathrm{Br}+\left(\mathrm{Br}_{0}\right)$ are the relative probabilities of semileptonic decays of charged (neutral) B mesons. The difference between $\mathrm{Br}_{+}$and $\mathrm{Br}_{0}$ is not expected to exceed a few percent. ${ }^{13}$ The $\mathrm{B}^{+}$mesons are not much lighter than the $\mathrm{B}^{0}$ : it is therefore usually assumed that, because of the large phase volume, $f_{+} /$ $f_{0} \approx 1.2-1.4$ (Refs. 2 and 25).

A search for the $\mathrm{B}^{0} \overline{\mathrm{~B}}^{0}$ oscillations in the region of the $\gamma(4 \mathrm{~S})$ resonance was performed in the CLEO and ARGUS experiments.
(1) The CLEO experiment, performed on the Cornell electron-positron storage ring, established an upper limit for the $\mathbf{B}_{\mathrm{d}}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ oscillations. ${ }^{25}$ After the subtraction of the background, there were $5.1 \pm 5.9$ events with leptons of the same sign and $117.0 \pm 11.9$ events with leptons of different sign. Assuming that $\lambda=1.44$, the following upper limit was obtained for the mixing parameter at the $90 \%$ confidence level: $r<26 \%$. When this was combined with previously published data, ${ }^{26}$ the limit was found to be $r<24 \%$ at the $90 \%$ confidence level. It is important to note that the mixing parameter $r$ was calculated from the formula

$$
r=\frac{N_{++}+N_{-}}{N_{+-}}(1+\lambda)
$$

where, in contrast to (6), the contribution of semileptonic decays of charged $B$ mesons was not fully taken into account. This must be remembered when the results are compared.
(2) In 1987, the ARGUS group reported the discovery of a large $B_{d}^{0} \bar{B}_{d}^{0}$ mixing. ${ }^{2}$ This result was very unexpected because there was a large number of publications ${ }^{16}$ in which $\mathrm{B}_{\mathrm{d}}^{0} \overline{\mathrm{~B}}_{\mathrm{d}}^{0}$ mixing was predicted at the level $\chi_{\mathrm{d}} \approx 10^{-3}-10^{-2}$.

The experiment was carried out on the DORIS II elec-tron-positron ring at DESY. The result was obtained by analyzing 88000 decays of the $\gamma(4 \mathrm{~S})$ resonance.

The $B_{d}^{0} \bar{B}_{d}^{0}$ mixing was detected in three different ways. The first, and most direct, method was to search for completely reconstructed events from $\gamma(4 \mathrm{~S})$ decays into $\mathrm{B}_{\mathrm{d}}^{0} \overline{\mathrm{~B}}_{\mathrm{d}}^{0}$ or $\bar{B}_{\mathrm{d}}^{0} \overline{\mathrm{~B}}_{\mathrm{d}}^{0}$ pairs. Unfortunately, this requires much better statistics than the direct methods discussed above. The B mesons were reconstructed from decays to final states containing $\mathrm{D}^{*-}$ mesons, as follows:

$$
\begin{aligned}
\mathrm{B}_{\mathrm{d}}^{0} & \rightarrow \mathrm{D}^{*-} \pi^{+} \\
& \rightarrow \mathrm{D}^{*-\pi^{+} \pi^{0}} \\
& \rightarrow \mathrm{D}^{*-} \pi^{+} \pi^{+} \pi^{-}
\end{aligned}
$$

or

$$
\rightarrow \mathrm{D}^{*-l^{+} v} \quad\left(l^{+}=\mathrm{e}^{+}, \mu^{+}\right)
$$

The $\mathrm{D}^{*-}$ were reconstructed using the decay chains


FIG. 5. Reconstructed $\Upsilon(4 \mathrm{~S})_{2} \rightarrow \mathrm{~B}_{\mathrm{d}}^{0} \mathrm{~B}_{\mathrm{d}}^{0}$, decay recorded in the ARGUS experiment. ${ }^{2}$


Although the $\mathrm{B}_{\mathrm{d}}^{0} \rightarrow \mathrm{D}^{+-} \boldsymbol{l}^{+} \boldsymbol{v}$ decay, has the neutrino as a decay product, it can be partially reconstructed as follows. The $\Upsilon(4 S)$ mesons are created practically at rest in the case of the $B_{d}^{0}$ resonance. Hence, the square of the mixing mass in the $\mathrm{D}^{*-}{ }^{+}$system,

$$
M_{\mathrm{R}}^{2}=\left[\left(E_{0}-\left(E_{\mathrm{D}^{\star}}+E_{l^{+}}\right)\right]^{2}-\left(\mathbf{p}_{\mathrm{D}^{\star}}+\mathbf{p}_{l^{+}}\right)^{2},\right.
$$

must be close to zero ( $E_{0}$ is the beam energy) , and this was confirmed experimentally. ${ }^{2,37}$

The decay

$$
\mathrm{r}(4 \mathrm{~S}) \rightarrow \mathrm{Bd}_{\mathrm{d}} \mathrm{~B}_{\mathrm{d}}^{0},
$$

was found among the reconstructed events. It can occur only as a result of $\mathrm{B}^{0} \overline{\mathrm{~B}}^{0}$ oscillations. The event is illustrated in Fig. 5. The two $B_{d}^{o}$ mesons ( $B_{1}^{o}$ and $B_{2}^{0}$ ) decay along the following chains:


Both $\mathrm{D}^{*-}$ mesons have $\mathrm{K}^{+}$mesons among their decay products. They are unambiguously identified by measuring the
ionization loss in the drift chamber and the time of flight. Both positively-charged muons have small momenta in the event, and the ionization losses and times of flight are in agreement with the muon hypothesis. One of the muons, $\mu_{1}$, was recorded in the muon chambers. Although the second muon missed the region covered by the muon chambers, the kinematics of the event uniquely indicates that the $\mathbf{B}_{2}^{0}$ also underwent a semileptonic decay. Monte Carlo calculations were carried out to estimate the background. Among the $22000 \mathrm{~B}_{\mathrm{d}}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ pairs in which one of the $\mathrm{B}_{\mathrm{d}}^{0}$ muons was reconstructed in one of the above channels, and the multiplicity of the remaining charged and neutral particles was the same as in the recorded event, there was not one possible candidate simulating the $\mathbf{B}_{d}^{0} \mathbf{B}_{d}^{0}$ and $\overline{\mathbf{B}}_{\mathrm{d}}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ pairs. The probability that this event was due to the background is therefore negligible. The discovery of the $\Upsilon(4 S) \rightarrow B_{d}^{0} B_{d}^{0}$ decay suggests a large mixing of $\mathrm{B}^{0} \overline{\mathbf{B}}^{0}$ mesons, since, even for $r=0.2$, the expected number of reconstructed events is 0.3 .

The second method was based on the standard procedure used in searches for pairs of dileptons of the same sign. A total of 50 events with leptons of the same sign and 270 events with leptons of different sign, in which both leptons had momenta $p>1.4 \mathrm{GeV} / c$, was recorded. The background due to incorrectly identified hadrons, the $\mathrm{J} / \psi \rightarrow l^{+} l^{-}$decays, and the asymmetric conversion of gamma rays was estimated from the experimental data, and only the contribution due to secondary D-meson decays was obtained by the Monte Carlo method. After all the backgrounds were subtracted, the remaining events with leptons of the same sign amounted to $24.8 \pm 7.6 \pm 3.8$. A background fluctuation of this magnitude is statistically equivalent to four standard deviations of the signal. The number of events with leptons of different sign (after subtraction of the background) was found to be $270.3 \pm 19.0 \pm 5.0$. The mixing parameter for
$B_{d}^{0}$ and $\bar{B}_{d}^{0}$, obtained from (6) using the theoretical value $\lambda=1.2$, was $r=0.22 \pm 0.09 \pm 0.04$.

In the third method of analysis, which was a combination of the first two, one of the $B_{d}^{0}$ mesons, reconstructed by the above procedure, was examined together with a fast ( $p>1.4 \mathrm{GeV} / c$ ) lepton. This method is less sensitive to hadron identification errors than the last method. Moreover, the influence of charged $B$ mesons is elliminated. Five candidates for events with $B_{d}^{0}$ meson mixing and 23 candidates without mixing were found. The background was $0.9 \pm 0.3$ and $2.2 \pm 1.1$ events, respectively. This gives the following mixing parameter:

$$
r=\frac{N\left(\mathrm{Bd}_{\mathrm{d}}^{\mathrm{o}} l^{+}\right)+N\left(\overline{\mathrm{~B}} \mathrm{~B}^{0} l^{-}\right)}{N\left(\mathrm{~B}_{\mathrm{d}} l^{-}\right)+N\left(\overline{\mathrm{~B}}^{\mathrm{d}} l^{+}\right)}=0.20 \pm 0.12 .
$$

There was partial overlap between events found by the second and third methods. Among them, there were two events with leptons of the same sign and eleven events with leptons of different sign. When this correlation was taken into account, the final results was found to be $r_{\mathrm{d}}=0.21 \pm 0.08$ (for $\lambda=1.2$ ). This corresponds to [see (3) and (4)]

$$
\chi_{d}=0.17 \pm 0.05 \text { and } x_{d}=0.73_{-0.19}^{+0.17}
$$

Figure 6 compares the results obtained in the above experiments. The CLEO data were recalculated using (6) and are now given for $\lambda=1.2$. This had practically no effect on the published result. The $\mathrm{B}_{\mathrm{d}}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ mixing found by the ARGUS group sets a lower limit for this parameter. Thus, $0.08<\chi_{\mathrm{d}}<0.19$ at the $90 \%$ confidence level. When the uncertainties in $\mathrm{B}_{\mathrm{d}}^{0}$ meson production are taken into account, the high-energy experiments do not lead to an improvement in these limits. The parameter $\chi_{\mathrm{s}}$ has not as yet been determined i.e., the $B_{s}^{0} \bar{B}_{s}^{o}$ mixing is uncertain.

## 3. CONSEQUENCES OF LARGE $B_{d}^{o} \bar{B}_{d}^{o}$ MIXING

In the standard model, the mixing parameter $x_{\mathrm{d}}=\Delta \boldsymbol{M} / \Gamma$ increases rapidly with increasing mass of the t quark and the matrix element $V_{\text {id }}$ of the KobayashiMaskawa matrix (1):

$$
\begin{equation*}
x_{\mathrm{d}} \approx \tau_{\mathrm{B}} \frac{G_{\mathrm{F}}^{2}}{6 \pi^{2}} \eta_{\mathrm{KX} Z} m_{\mathrm{B}}\left(B_{\mathrm{B}} f_{\mathrm{B}}^{2}\right) m_{\mathrm{t}}^{2 \mathrm{E}}\left(\frac{m_{\mathrm{t}}^{2}}{M_{\mathrm{W}}^{2}}\right)\left|V_{\mathrm{td}}^{*} V_{\mathrm{tb}}\right|^{2} \tag{7}
\end{equation*}
$$

where $\tau_{\mathrm{B}}$ is the lifetime of the B meson, $\eta_{\mathrm{QCD}} \sim 0.85$ in the QCD correction, ${ }^{12} B_{\mathrm{B}}$ and $f_{\mathrm{B}}$ are constants that parametrize the matrix element of the transition of $B^{0}$ to $\bar{B}^{0}$, and

$$
\xi(y)=1-\frac{3}{4} \frac{y(1+y)}{(1-y)^{2}}-\frac{3}{2} \frac{y^{2}}{(1-y)^{3}} \ln y
$$

is a slowly-varying decreasing function that takes into account the effect of the W-boson propagator ${ }^{28,29}: \xi(0)=1$, $\xi(1)=3 / 4 ; \xi(\infty)=1 / 4$.

The annihilation constant $f_{\mathrm{B}}$ is similar to the wellknown constant $f_{\pi}$ that determines the $\pi^{+} \rightarrow \mu^{+} v$ decay:

$$
\langle 0| \bar{d} \gamma_{\mu} \gamma_{5} b\left|\overline{\mathrm{~B}}_{\mathrm{d}}^{0}\right\rangle=i f_{\mathrm{B}} p_{\mu}
$$

The constant $B_{\mathrm{B}}$ calibrates the matrix element for the transition of $\overline{\mathrm{B}}^{0}$ to $\mathrm{B}^{0}$

$$
\left\langle\mathrm{B}_{\mathrm{d}}^{0}\right| \bar{d} \gamma_{\mu}\left(1+\gamma_{\mathrm{5}}\right) b \bar{d} \gamma_{\mu}\left(1+\gamma_{5}\right) b\left|\overline{\mathrm{~B}}_{\mathrm{a}}^{0}\right\rangle=\frac{8}{3} f_{\mathrm{B}}^{2} B_{\mathrm{B}} M_{\mathrm{B}}^{\mathrm{B}}
$$

When this matrix element is evaluated, it is usually assumed that the analysis can be confined to the intermediate vacuum state, which corresponds to $B_{B}=1$. All the parameters in


FIG. 6. Experiments on $\mathrm{B}^{0} \overline{\mathrm{~B}}^{0}$ oscillations. Dashed lines-central values, dash-dot and solid lines-1 and 2 standard deviations, respectively; shaded region-forbidden at the $95 \%$ confidence level. It was assumed for the high-energy experiments that $(\chi)=0.375 \chi_{\mathrm{d}}+0.15 \chi_{\mathrm{s}}$.
(7) except for $m_{1}$ and $V_{\text {td }}$ are either known or can be calculated.

Unfortunately, there is a considerable spread in the predicted values of the product $B_{\mathrm{B}} f_{\mathrm{B}}^{2}$. A discussion of the uncertainties in $B_{\mathrm{B}}$ can be found in the review literature. ${ }^{30,31}$ We note that recent calculations ${ }^{32}$ using the QCD sum rule gave $B_{\mathrm{B}}=0.9-1$. The estimates for $f_{\mathrm{B}}$ run from 50 MeV (Ref. 33) to 300 MeV (Ref. 34). Predictions giving $f_{\mathrm{B}}>200$ MeV begin to run into conflict with experimental data. The upper limit for the relative $\mathrm{D}^{+} \rightarrow \mu^{+} \boldsymbol{v}$ decay probability, established by the MARK III group, leads to the following limit for the annihilation constant of the $\mathrm{D}^{+}$meson: $f_{\mathrm{D}}<290 \mathrm{MeV}$ at the $90 \%$ confidence level. ${ }^{35}$ Since the annihilation constant should decrease with increasing meson mass, ${ }^{36-38}$

$$
\begin{equation*}
\frac{f_{\mathrm{B}}}{f_{\mathrm{D}}}=\left(\frac{M_{\mathrm{D}}}{M_{\mathrm{B}}}\right)^{1 / 2}\left(\frac{\alpha_{\mathrm{s}}\left(m_{\mathrm{c}}\right)}{\alpha_{\mathrm{s}}\left(m_{\mathrm{h}}\right)}\right)^{2 / 9} \tag{8}
\end{equation*}
$$

it follows from the above limit for $f_{\mathrm{D}}$ that $f_{\mathrm{B}} \leqslant 200 \mathrm{MeV}$. A detailed analysis of the theoretical situation, reported by Shifman, ${ }^{31}$ shows that the most reliable calculations are those given in Refs. 39 and 40, which use the QCD sum rules. They predict $f_{\mathrm{B}}=100-130 \mathrm{MeV}$.

Since $V_{\mathrm{id}}$ depends on $\theta_{13}$ and the CP-odd phase $\delta$, the measurement of $x_{\mathrm{d}}$ leads to a relationship between three as yet unknown SM parameters of the quark sector, namely, $m_{1}, \theta_{13}$, and $\delta$. The quantity $\varepsilon_{\mathrm{K}}$ that characterizes CP violation in kaon decays depends on the same parameters. The complete expression for $\varepsilon_{\mathrm{K}}$ can be found, for example, in Ref. 29. It is usually considered ${ }^{31}$ that the $B_{\mathrm{K}}$-analog of $B_{\mathrm{B}}$


FIG. 7. The mass of the $t$ quark as a function of $\left(s_{13} / s_{23}\right)^{2}$ for two values of $x_{d}$.
lies in the range $1 / 3 \leq B_{\mathrm{K}} \leq 1$ and is definitely closer to its upper limit. The experimental result is ${ }^{41}$ $\left|\varepsilon_{\mathrm{K}}\right|=(2.27 \pm 0.02) \cdot 10^{-3}$.

Thus, if we know $x_{d}$ and $\varepsilon_{\mathrm{K}}$ we can find the relationship between any parameters among the three unknown SM parameters of the quark sector. Figure 7 shows $m_{\mathrm{t}}$ as a function of $\left(s_{13} / s_{23}\right)^{2}$. It was obtained by Ural'tsev and Khoze ${ }^{42}$ for $x_{\mathrm{d}}=0.73$ and $x_{\mathrm{d}}=0.5$ is close to the experimental lower limit $^{2} x_{\mathrm{d}}>0.44$ at the $90 \%$ confidence level. Uncertainties in the parameters in (7) lead to an uncertainty in $x_{b}$. Of course, there are also considerable uncertainties in the theoretical predictions for $\varepsilon_{\mathrm{K}}$. However, when the ratio $s_{13} / s_{23}$ is not too small, $\sin \delta$ is found to be small, and the uncertainty in the parameters that determine $\varepsilon_{\mathrm{K}}$ have little effect on the mass of the t quark. A change in $\varepsilon_{\mathrm{K}}$ by a factor as large as 2 is then found to produce a change in $m$, by only a few $\mathrm{GeV} / c^{2}$ (Ref. 42)

The upper limit found in Ref. 4 for the ratio $s_{13} /$ $s_{23} \approx\left|V_{\mathrm{ub}}\right| /\left|V_{\mathrm{cb}}\right|$ is $\left|V_{\mathrm{ub}}\right| /\left|V_{\mathrm{cb}}\right|<0.2$ at the $90 \%$ confidence level. ${ }^{2)}$ This follows from an analysis of the momentum spectrum of leptons from the semileptonic decays of B mesons. It can be deduced if we know the shape of the lepton spectrum for the $\mathrm{b} \rightarrow u l v$ and $b \rightarrow c l v$ transitions (Fig. 8). There are no reliable theoretical predictions, and calculations have to rely on models. Different models lead to different limits for $\left|V_{\mathrm{ub}}\right| /\left|V_{\mathrm{cb}}\right|$. It is therefore common to use the nonrelativistic model of Grinstein et al. ${ }^{43}$ which gives the least stringent limit.

A more reliable limit for $\left|V_{\mathrm{ub}}\right| /\left|V_{\mathrm{cb}}\right|$ follows from measurements of the yield of charmed particles in B-meson decays. On average, each $B$ meson decay produces about one $c$ quark. ${ }^{4}$ Consequently, the $b \rightarrow c$ transition (see Figs. 8b, d, and $f$ ) is the dominant one. However, since charmed particles are also created in the $\mathrm{b} \rightarrow \mathrm{u}$ transition (see Fig. 8e), the limit is found ${ }^{44}$ to be relatively weak: $\left|V_{\mathrm{ub}}\right| /\left|V_{\mathrm{cb}}\right|<0.5$ at the $90 \%$ confidence level. Despite the theoretical uncertainties, the upper limit commonly employed is $\left|V_{\mathrm{ub}}\right| /\left|V_{\mathrm{cb}}\right|<0.2$, which follows from the analysis of leptonic spectra. Once we know the upper limit for $s_{13} / s_{23}$ and the dependence of $m_{1}$ on this ratio (see Fig. 7), we can calculate the lower limit for the mass of the $t$ quark. This mass is largely determined by $x_{d}$. The result is improved by a few $\mathrm{GeV} / \mathrm{c}^{2}$ when data on $\varepsilon_{\mathrm{K}}$ are employed. ${ }^{45.46}$ Most papers devoted to the analysis of the consequences of large $\mathbf{B}_{d}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ mixing, ${ }^{5,42.45-50}$ based on different assumptions about existing uncertainties, suggest that the minimum t-quark mass is approximately $45-60 \mathrm{GeV} / c^{2}$, and that the most "natural" value of this mass that corresponds to central values of the parameters $f_{\mathrm{B}}, x_{\mathrm{d}}$, and $s_{13} / s_{23}$


FIG. 8. Diagrams for Cabbibo-allowed $b \rightarrow u$ and $b \rightarrow c$ transitions.
lies in the region of $100 \mathrm{GeV} / c^{2}$. On the other hand, analyses of the radiative corrections used in the determination of $\sin ^{2} \theta_{\mathrm{w}}$ lead to the upper limit ${ }^{51} m_{\mathrm{t}}<180 \mathrm{GeV} / c^{2}$.

Unfortunately, it is hardly possible to assign a confidence level to the lower limit on the $t$-quark mass because it is subject to both experimental and very significant theoretical uncertainties. When a very large $B_{B} f_{B}^{2}$ is employed (for example, $0.06 \mathrm{GeV}^{2}$ ), the limit on $m_{1}$ can be reduced to 25 $\mathrm{GeV} / c^{2}$ (Ref. 52). This figure was obtained as a result of direct searches for the $t$ quark in the TRISTAN electronpositron ring. ${ }^{53}$ This large value of $B_{\mathrm{B}} f_{\mathrm{B}}^{2}$ is, of course, in conflict not only with most theoretical estimates, but also with the limit on $f_{\mathrm{B}}$ that follows from the experimental limit on $f_{\mathrm{D}}$, and seems very reliable.

One further argument in favor of a large $t$-quark mass has recently been advanced by the UA1 group ${ }^{54}$ which has reported that $m_{\mathrm{i}}>44 \mathrm{GeV} / c^{2}$. On the other hand, this result is based on a number of assumptions that might be regarded as doubtful. ${ }^{52}$

From now on, we shall adopt the generally held view that the observed large $B_{d}^{o} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ mixing can be explained in terms of the standard model with three generations of quarks, but only if the $t$-quark mass exceeds $45-50 \mathrm{GeV} / c^{2}$ and the most probable value of $m_{t}$ lies in the region of 100 $\mathrm{GeV} / c^{2}$. This large mass ensures that all the experimental data that are sensitive to it are satisfactorily described.

Apart from the lower limit on the $t$-quark mass, the large $\mathbf{B}_{d}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ mixing leads to a whole series of consequences within the standard model. One of the most interesting is the prediction of large $B_{s}^{0} \overline{\mathbf{B}}_{\mathrm{s}}^{0}$ mixing. Although, as noted above, there are considerable uncertainties in the calculated product $B_{\mathrm{B}} f_{\mathrm{B}}^{2}$, the ratio of these factors for $\mathrm{B}_{\mathrm{s}}^{0}$ and $\mathrm{B}_{\mathrm{d}}^{0}$ should be close to unity. ${ }^{3)}$ Their lifetimes should not differ by more than $10 \%$ (Refs. 12 and 13). Hence, using (7), we obtain

$$
\begin{equation*}
\frac{x_{\mathrm{g}}}{x_{\mathrm{d}}}=\frac{\left|V_{\mathrm{tg}}\right|^{2}}{\left|V_{\mathrm{td}^{2}}\right|^{2}} \frac{f_{\mathrm{B}_{\mathrm{g}}}^{2} B_{\mathrm{B}_{\mathrm{g}}}}{f_{\mathrm{B}_{\mathrm{d}}} B_{\mathrm{B}_{\mathrm{d}}}} \frac{\tau_{\mathrm{B}_{\mathrm{g}}}}{\tau_{\mathrm{B}_{\mathrm{d}}}} \approx \frac{\left|V_{\mathrm{tg}}\right|^{2}}{\left|V_{\mathrm{td}_{\mathrm{d}}}\right|^{2}} . \tag{9}
\end{equation*}
$$

In the parametrization of the Kobayashi-Maskawa matrix (1) that we have chosen, we have

$$
\begin{equation*}
\frac{\left|V_{t s}\right|^{8}}{\left|V_{t d}\right|^{2}}=\frac{s_{23}^{2}}{\left|s_{12} s_{23}-s_{19} e^{i \delta}\right|^{2}}=\frac{1}{\left|s_{12}-\left(s_{19} / s_{23}\right) e^{i \delta}\right|^{2}} . \tag{10}
\end{equation*}
$$

This expression has a maximum for $\delta=180^{\circ}$ (actually, $\delta$ cannot be exactly equal to $180^{\circ}$, since then $\varepsilon_{\mathrm{k}}=0$ ). Hence

$$
\begin{equation*}
\frac{x_{\mathrm{E}}}{x_{\mathrm{d}}}>\frac{1}{\left[s_{12}+\left(s_{13} / s_{23}\right)\right]^{2}} \geqslant 5.6 \tag{11}
\end{equation*}
$$

where we have used the limit $s_{12} / s_{23}<0.2$ to obtain the second inequality (even when the conservative estimate $s_{12} /$ $s_{23}<0.5$ is employed, $x_{\mathrm{s}}$ is found to be greater than $x_{\mathrm{d}}$ by a factor of 2). Since $x_{d}>0.44$ at the $90 \%$ confidence level, ${ }^{2}$ $\mathbf{B}_{\mathrm{s}}^{0} \overline{\mathbf{B}}_{\mathrm{s}}^{0}$ mixing is expected to be largely independent of the mass of the $t$ quark, i.e., $x_{s}>2.5$. This conclusion remains in force for most simple extensions of the standard model, ${ }^{5}$ e.g., supersymmetric models, models with several Higgs doublets, and minimal left-right symmetric models. However, the relationship between $x_{d}$ and $x_{\mathrm{s}}$ can, of course, be quite different if there is a fourth generation of quarks. ${ }^{5,6,50.56} \mathrm{Un}$ doubtedly, the verification of (9) and (11) is one of the most critical tests of modern theoretical ideas.

Figure 9 shows the boundary that follows from inequality (11) and from the restrictions on mixing parameters $x_{\mathrm{d}}$ and $x_{\mathrm{s}}$ obtained in the UA1, MARK II, CLEO, and ARGUS experiments (Refs. 1, 17, 25, and 2, respectively). Condition (11) and the results of the ARGUS and CLEO groups define the small region of possible values of $x_{\mathrm{d}}$ and $x_{\mathrm{s}}$ shown shaded in the figure. The upper limit provided by the MARK II group could reduce the width of the allowed regions still further if we knew how frequently the $B_{s}^{0}$ and $\mathbf{B}_{d}^{0}$ mesons are created in electron-positron annihilation for $s^{1 / 2}=29 \mathrm{GeV}$. However, the $\mathbf{B}_{\mathrm{s}}^{0}$ and $\mathbf{B}_{\mathrm{d}}^{0}$ yields are difficult to predict reliably, especially in view of the fact that most of them arise, as in the case of charmed particles, ${ }^{57,58}$ from the excited states of $\mathbf{b s}$ (for example, $\mathbf{P}$-levels) which can decay ${ }^{4}$ ) to $\overline{\mathbf{B}}_{\mathrm{s}}^{0} \pi \pi$ rather than $\overline{\mathbf{B}}_{\mathrm{d}}^{0} \mathrm{~K}$ and $\overline{\mathbf{B}}_{\mathrm{d}}^{0}{ }^{*} \mathrm{~K}$ (Ref. 59). It is clear from Fig. 9 that $\chi_{\mathrm{s}}$ turns out to be close to its maximum value in the allowed range.


FIG. 9. Limits for the mixing parameters $\chi_{\mathrm{d}}$ and $\chi_{\mathrm{s}}$ at the $95 \%$ confidence level. Dashed curve shows limits that follow from the standard model. The range of parameter values allowed in the standard model is shown shaded.

Although the $B_{s}^{0} \overline{\mathbf{B}}_{\mathrm{s}}^{0}$ mixing is predicted to be large, its detection is not a simple matter. When lepton pairs are investigated, the $\mathbf{B}_{\mathrm{s}}^{0} \overline{\mathbf{B}}_{\mathrm{s}}^{0}$ mixing effect will have to be selected against the background of a large $B_{d}^{0}$ signal. Even the detection of correlations between leptons and strange particles will not unambiguously confirm that leptons from $\mathrm{B}_{\mathrm{s}}^{0}$ decays are present, because of the possible decay of the excited states of $b \bar{s}$ to $\overline{\mathbf{B}}_{\mathrm{d}}^{0} \mathrm{~K}$ and $\overline{\mathrm{B}}_{\mathrm{d}}^{* 0} \mathrm{~K}$. At the same time, the reconstruction of a sufficient number of exclusive $B_{s}^{0}$ decays will hardly be possible in the near future.

The determination of the precise value of $x_{5}$ will be an even more difficult task because the integrated parameters become insensitive to the magnitude of $x$ in the case of large mixing. For example, as $x$ is varied from 5 to 25 , the quantity $\chi$ changes from 0.48 to 0.499 . This is readily understood, since large $x=\Delta M / \Gamma$ signifies that the $B$ meson has executed many oscillations before decay, and has "forgotten" whether it was initially a particle or an antiparticle, i.e., we have complete mixing ( $\chi \approx 0.5$ ). As in the case of kaons, the time dependence of oscillations has to be investigated before $x_{\mathrm{s}}$ can be determined. Figure 3 shows the time dependence of the number of $B$ and $\bar{B}$ mesons during the decay of tagged $B_{d}^{0}$ mesons $\left(\Delta M / \Gamma=0.73\right.$ ) and $B_{\mathrm{s}}^{0}$ mesons for which we have used the typical prediction $\Delta M / \Gamma=10$. A reliable reconstruction of secondary vertices in $B$-meson decays, which is essential for the investigation of the time dependence of the oscillations, should lead to a radical change in studies of the properties of both $B_{s}^{0}$ and $B_{d}^{0}$ mesons. In this respect, the new proton accelerators, namely, the TEVATRON, UNK, and SSC, in which the B-meson range will reach some tens of centimeters, are particularly promising. A detailed discussion of this question can be found in the literature. ${ }^{6,60,61}$

For a large t-quark mass, the predicted relative probabilities of many rare decays will become accessible to experimental verification in the near future. The relative probability of the $\mathrm{K}^{+} \rightarrow \pi^{+} v \bar{v}$ decay (Fig. 10a) is now predicted to be more than $10^{-10}$ (Refs. 42 and 46 ). It is very dependent of the t-quark mass and is proportional to the number of types of neutrino. The observation of this decay would serve as an important verification of the standard model, and would impose more stringent restrictions on the range of its parameter values.

The predictions for the $\mathbf{B} \rightarrow \mathrm{K}^{*} \gamma$ decay, in which the photon is emitted as a result of the $\mathrm{b} \rightarrow \mathrm{s} \gamma$, transition (Fig. 10b), are also close to existing experimental limits. It is expected $^{62}$ that, for a t-quark mass in the region of $100 \mathrm{GeV} / c^{2}$, $\mathrm{b} \rightarrow \mathrm{s} \gamma$, whereas the present upper limit is $4.2 \times 10^{-4}$ at the $90 \%$ confidence level. Decays due to the $b \rightarrow s \gamma$ transition are very sensitive (like the $\mathrm{K}^{+} \rightarrow \pi^{+} v \overline{\mathrm{v}}$ decay) to new physics outside the range of the standard model. For exampale, the existence of the fourth generation of quarks could increase or reduce the relative $\mathbf{B} \rightarrow \mathbf{K}^{*} \gamma$ decay probability by an order of magnitude. ${ }^{63}$ This means that searches for the above decays will be even more interesting.

If the $t$-quark mass is found to be greater than $M_{w}$, we shall have to re-examine the methods used to search for new phenomena in future accelerators because it will not be $\mathbf{W}^{+}$ that will decay into tb, but the t-quark into $W^{+} b$. The properties of toponium will change radically if $m_{\mathrm{t}} \gtrsim 150 \mathrm{GeV} / \mathrm{c}^{2}$. These questions are discussed in detail in Ref. 42. In the standard model, the angles and phases of the Kobayashi-

a

$b$

Maskawa matrix and the masses of the quarks are adjustable parameters. This can hardly be regarded as satisfactory from the theoretical point of view. Moreover, there are well known simple phenomenological relationships which suggest that there is a connection between the quark masses and the angles in the Kobayashi-Maskawa matrix:

$$
\begin{align*}
& \theta_{12}=0.22\left(\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}\right)^{1 / 2} \approx 0.22\left(\frac{m_{\mathrm{s}}}{m_{\mathrm{b}}}\right)^{1 / 2} \approx 0.17, \\
& \theta_{23} \approx 0.05\left(\frac{m_{\mathrm{u}}}{m_{\mathrm{c}}}\right)^{1 / 2} \approx 0.06\left(\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}}\right)^{1 / 2} \approx 0.04,  \tag{12}\\
& \theta_{13}<0.01\left(\frac{m_{\mathrm{u}}}{m_{\mathrm{t}}}\right)^{1 / 2}<0.01
\end{align*}
$$

where the quark masses are taken from Ref. 64. Theoretical attempts are therefore continuing, if not to fix the masses and the angles, then at least to reproduce the relationship between them. Special forms of the $u$ and $d$ quark mass matrices with the number of parameters $N<10$ (see Introduction) have been selected on the basis of different generalizations of the standard model, or simply phenomenologically. After the diagonalization of mass matrices, this results in $10-N$ relationships between the parameters of the Kobaya-shi-Maskawa matrix and the quark masses.

In the scheme proposed by Fritzsch, ${ }^{65}$

$$
M_{\mathrm{u}}=\left(\begin{array}{lll}
0 & a_{\mathrm{u}} & 0 \\
a_{\mathrm{u}} & 0 & b_{\mathrm{u}} \\
0 & b_{\mathrm{u}} & d_{\mathrm{u}}
\end{array}\right), \quad M_{\mathrm{d}}=\left(\begin{array}{ccc}
0 & a_{\mathrm{d}} e^{i \Phi_{1}} & 0 \\
a_{\mathrm{d}} e^{-i \Phi_{\mathbf{1}}} & 0 & b_{\mathrm{d}} e^{i \Phi_{\mathbf{1}}} \\
0 & b_{\mathrm{d}} e^{-i \Phi_{\mathbf{1}}} & c_{\mathrm{d}}
\end{array}\right),
$$

i.e., there are eight parameters and therefore two relationships between the measured quantities. In particular

$$
\begin{equation*}
\theta_{23} \geqslant\left(\frac{m_{\mathrm{S}}}{m_{\mathrm{b}}}\right)^{1 / 2}-\left(\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\right)^{1 / 2} \tag{13}
\end{equation*}
$$

In another scheme, due to Berezhianai and Chkareuli, ${ }^{66}$ the assumption of horizontal symmetry is used to show that the diagonal elements of the mass matrices for the $u$ and $d$ quarks are proportional to one another. Among matrices of this type, the best known are the mass matrices of Stech. ${ }^{67}$ In this case,

$$
\begin{equation*}
\theta_{23} \approx\left(\frac{m_{\mathbf{s}}}{m_{\mathrm{b}}}-\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\right)^{1 / 2} \tag{14}
\end{equation*}
$$

There are also other models of mass matrices. It is readily seen that (13) and, especially, (14) are not satisfied when the $t$-quark mass is large. Careful analysis of all these relationships, performed by Harari and $\mathrm{Nir}^{47}$ and by Ellis et al., ${ }^{45,46}$ has shown that the observed values of $x_{\mathrm{d}}$ and $\varepsilon_{\mathrm{k}}$ are not consistent with the above quark matrices (more precisely, Harari and Nir, and also Albright et al. ${ }^{68}$ assumed somewhat higher uncertainties and found narrow regions in
which the Fritzsch scheme was still in agreement with experiment, but at the very limit of the possible parameter values).

Other variants of the above quark mass matrices, ${ }^{69}$ i.e., different versions of the schemes discussed above, are also in good agreement with large $x_{\mathrm{d}}$.

We thus see that the observation of large $B_{d}^{0} \overline{\mathbf{B}}_{\mathrm{d}}^{0}$ mixing imposes very stringent restrictions on the standard model parameters and leads to new possibilities for its critical verification. If we find that $B_{s}^{0} \bar{B}_{s}^{0}$ mixing is small $\chi_{\mathrm{s}}<2$, or if a relatively light t -quark is discovered, e.g., in the TRISTAN electron-positron ring, an explanation will have to be sought outside the framework of the standard model (provided, of course, that the result of the ARGUS group will stand the test of time). ${ }^{5)}$

Many of the new hypothetical particles can contribute to $\mathrm{B}^{0} \overline{\mathbf{B}}^{0}$ mixing (see Fig. 2) and can modify the predictions of the standard model. So far, their existence is not required for the explanation of experimental data, and we shall not discuss this very interesting problem here. A detailed presentation of the current state of the subject can be found, for example, in Ref. 5.

## 4. CONCLUSION. PROSPECTS FOR INVESTIGATING THE b QUARK

We have seen that b-quark physics involves a wide spectrum of questions, ranging from the basic parameters of the standard model to the limits of validity of this model. There is particular interest in searches for rare processes in which the predictions of different models can be decisively verified. A typical situation is that of a small CP invariance violation in weak interactions for which there are new possibilities of observation in $\mathbf{B}$-meson decays, following the confirmation of large $\mathrm{B}^{0} \overline{\mathrm{~B}}^{0}$ mixing. Although the first CP parity violation was observed more than 20 years ago, the mechanism for this phenomenon is still not understood. Until recently, all experimental data were in agreement with the conclusion that neutral-kaon mixing was the only parameter of CP invariance violation. However, evidence ${ }^{70}$ became available in 1987 that the parameter $\varepsilon_{\mathrm{K}}^{\prime}$, which describes CP violation directly in kaon decays, is also different from zero: $\varepsilon_{\mathrm{K}}^{\prime} / \varepsilon_{\mathrm{K}}=(3.5 \pm 1.4) \cdot 10^{-3}$. This is in good agreement with standard model predictions in the case of large $t$-quark mass (see, for example, Refs. 42, 45, and 46). Unfortunately, the observation of CP noninvariant effects in kaon decays is difficult because the corresponding asymmetries are small.

Large $\mathbf{B}^{0} \overline{\mathbf{B}}^{0}$ mixing should lead in the standard model to large CP violation directly in B-meson decays ${ }^{9,24,71}$ ( CP violating effects that arise only in $\mathrm{B}^{0} \overline{\mathrm{~B}}^{0}$ mixing are very
small ${ }^{72}$ ). In some decay channels, CP-odd asymmetries can reach values of the order of $10 \%$ or more (see, for example, Ref. 73). On the other hand, the relatively low probability of these decays means that about $10^{8} \mathrm{~B} \overline{\mathrm{~B}}$ pairs are needed to ensure that the effect will be recorded. A comparable number of $B$ mesons will be produced in future accelerators. At the same time, it must be remembered that, in experiments near the $\Upsilon(4 S)$ resonance, the integrated parameters cannot be used because CP-odd effects vanish after integration in $\mathbf{P}$-wave $\mathbf{B} \overline{\mathbf{B}}$ pair production. ${ }^{24}$ Before the time-dependent CP -odd asymmetries can be measured, we have to have data on B-meson decay vertices. Asymmetric electron-positron rings that combine the advantages of studying $B$ mesons in the region of the $\Upsilon(4 \mathrm{~S})$ resonance with the possibility of reconstructing the vertices appear to be particularly promising from this point of view. At high energies, at which the Bmeson range is greater, it is realistic to consider detailed experimental investigations of the angular distribution of $B$ and $\overline{\mathrm{B}}^{0}$ mesons. ${ }^{6,60}$ Studies of the time dependence of charm oscillations (2) can lead to information on CP invariance violation.

Intensive studies of B -meson physics are continuing in the ARGOS and CLEO experiments. At the same time, attempts are being made to increase the luminosity of the DORIS II, CESR, and VEPP-4M storage rings. For example, it is expected that the CESR luminosity will reach ${ }^{74}$ $\sim 2 \times 10^{32} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$. New experimental installations, namely, CLEO II $^{75}$ and KEDR ${ }^{76}$ are under development. Moreover, b-quark physics is regarded as a promising component of programs developed for practically all the highenergy accelerators that are being built or are planned at present (see, for example, Ref. 61). It is important to note, however, that the number of B mesons created in SLC, LEP, and HERA accelerators will be insufficient for studies of CP invariance violation.

Interest in b-quark physics has led to numerous proposals for new-generation $\mathrm{e}^{+} \mathrm{e}^{-}$colliders, i.e., the so-called Bmeson factories working near the threshold and designed for the investigation of B mesons. These projects can be divided into three classes, namely, (a) symmetric, high-luminosity $\mathrm{e}^{+} \mathrm{e}^{-}$storage rings ( $\sim 5 \times 10^{32}-10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$, Refs. 77 and 78), (b) high-luminosity linear colliders ( $\sim 10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$; Refs. 79 and 80 ), and (c) asymmetric $\mathrm{e}^{+} \mathrm{e}^{-}$storage rings based on the principle that a moving system, e.g., the $\Upsilon(4 S)$ meson, is created in collisions between electrons and positrons of different energy. The B-meson decay vertices could then be recorded, so that the complete picture of the event would be reconstructed more effectively and B-meson oscillations investigated along the time axis. The replacement of the electron ring with a linear accelerator in this scheme should then result in an increase in luminosity to $10^{34}$ $\mathrm{cm}^{-2} \mathrm{~s}^{-1}$ of even higher. ${ }^{81}$

Estimates of the expected number of $B \bar{B}$ pairs produced in B-meson factories suggest that, after a few years of operation, it will be possible to reach the level necessary for the detection of $\mathbf{C P}$ invariance violation.

The diversity of physical problems and experimental programs involved in the investigation of $\mathbf{B}$ mesons should lead in the near future to new results in this very interesting branch of physics.

It is our pleasant duty to thank our colleagues in the ARGUS group, especially W. Schmidt-Parzefall, H.

Schroder, and D. B. Macfarlane. We are greatly indebted to Z. G. Berezhiani, I. A. Golutvin, N. G. Ural'tsev, V. A. Khoze, and Dzh. L. Chkareuli for numerous discussions relating to the questions touched upon in this review. We are particularly grateful to M. B. Voloshin, V. A. Lyubimov, L. B. Okun', and M. A. Shifman who took on the difficult task of reading this manuscript and made many important and constructive suggestions.
${ }^{1 /}$ The charge-conjugate state will also be usually implied.
${ }^{2 /}$ We have rounded off the exact value because of the large theoretical uncertainties.
${ }^{3}$ It is natural to expect that $f_{\mathbf{B}_{r}}$ is only slightly greater than $f_{\mathrm{B}_{4}}$ (Refs. 36 and 55), just as in the case of $D$ mesons.
${ }^{4}$ We are indebted to A. B. Blinov and A. B. Kairdalov for discussions of this possibility.
${ }^{5)}$ At the 24th International Conference on High-Energy Physics, held in Munich in August 1988, this result was confirmed by the CLEO group, who reported that the mixing parameter was $r=0.18 \pm 0.08$.
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Translated by S. Chomet.

