Complex differential geometry and supermanifolds in theories of strings and fields

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Complex Differential Geometry and Supermanifolds in Strings and Fields: Proceedings of the Seventh Scheveningen Conference, Scheveningen, the Netherlands, August 23–28, 1987, Eds. P. J. Bongaarts and R. Martini. Springer-Verlag, Berlin; Heidelberg, 1988. 252 pp. (Lecture Notes in Physics. V. 310).

The book contains a record of lectures presented at the seventh Scheveningen Conference held on 23–28 August 1987. The last several conferences in Scheveningen (including the seventh one) were devoted to differential-geometry aspects of mathematical physics. At the 1987 conference the principal attention was devoted to applications of complex differential geometry and the geometry of supermanifolds to supersymmetry, field theory, and strong theory. The contents of the book can be arbitrarily divided into three parts; Kähler manifolds in field theory (the lectures by Forger and Wess), supersymmetry and supermanifolds (the lectures by Batchelor, Rogers, Bryant and Jarvis), and string theory (the lectures by Caldi and Sanchez).

Forger discusses a number of subjects connected with supersymmetric sigma models ranging from possible applications in particle physics to mathematical aspects concerning Kähler and hyperkähler geometry. The review is concluded by a classification of homogeneous Kähler manifolds. In Wess's lectures "Nonlinear Realizations, Kähler Manifolds and the Virasoro Manifold" after recalling briefly the main concepts of differential geometry the calculation of the Ricci tensor is presented for the two-parameter family of Kähler metrics on the Virasoro manifold—the factor manifold of the group of the diffeomorphisms of a circumference according to the rotations subgroup.

The lectures of the next four authors are essentially of a mathematical nature. Batchelor reviews the algebraic approach to manifold theory. He explains what is the appearance in terms of function algebra of Lagrangian densities, differential operators, etc. Further a description is given of the generalization of this approach to supermanifolds with applications to the Berezin integral and to the construction of a supermanifold whose even part parametrizes the mappings from one given supermanifold into another one. As an example of a discussion is given of the group of automorphisms of a graded Riemann sphere. Rogers discusses integration along paths in superspace. She gives rigorous definitions of the Grassman generalization of the Wiener measure, of the Grassman Brownian motion, and proves the Feynman–Kac formula for fermions, investigates the Fourier transformation of the path integral. The analysis developed in infinite-dimensional spaces with fermions is applied for a rigorous proof (based on supersymmetric quantum mechanics) of the Gauss–Bonnet–Chern formula. The lectures by Bryant and Jarvis contain an introduction to the theory of supermanifolds and supersymmetry with examples of applications to different physical theories.

Caldi's lectures are of a review nature. They deal with a number of topics which underlie superstring theory, in particular the critical dimensionality, the cancellation of anomalies, the choice of the gauge group, the compactification from ten dimensions and the connection with the standard model in four dimensions, the connection of the number of generations with the index of the Dirac operator, the compactification to Calabi–Yau manifolds with simple examples and, finally, the formulation of string theory based on a Kähler geometry of loop space. In the lectures by Sanchez string theory in curved space-time is discussed. She analyzes the first quantization of strings in Rindler space-time and the Hawking-Unruh effect in string theory.

On the whole, in spite of its fragmentary nature, the book is of interest for those studying supersymmetry by providing an accessible introduction to some of the currently popular directions of research.

Translated by G. M. Volkoff