N. N. Gor'kavyĭ and A. M. Fridman. *Collective processes and the structures in planetary rings.* Among the four planets in the solar system which are now known to have rings (Jupiter, Saturn, Uranus, and Neptune) Saturn's rings have the most diverse structure. The so-called hierarchical structure of Saturn's rings (similar to the Russian nesting dolls) is most interesting: the densest ring—the 25.5 thousand km wide *B* ring—consists of a collection of thousand-kilometer rings, which in their turn consist of hundred-kilometer rings, etc. The narrowest ringlets are ~ 100 m wide (Saturn's rings are $\simeq 30$ m thick). The mass distribution of the particles is close to the so-called Salpeter law $n(m)/n(m_0) \sim (m_0/m)^{1.5}$, where $n(m)$ is the number of particles with mass ranging from m to ∞. However a sharp "drop" occurs in the spectrum for particle sizes $a \gtrsim 5$ m: $n(a) \sim a^{-(5\div 6)}$.[1]

Saturn's rings consist of ice particles and for this reason the collisions of small particles are substantially inelastic. As the particles become larger the particle density drops sharply and already for particles with $a \gtrsim 1$ m only gravitational collisions occur (no contact). The basic equations describing the dynamics of such a system are the kinetic equation in an accelerated coordinate system

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + (\mathbf{e} + [\mathbf{v}, \mathbf{h}])\frac{\partial f}{\partial \mathbf{v}} = C_G + C_* \quad (1)$$

and Poisson's equation

$$\Delta\Phi = 4\pi G \int f \, d^3 v. \quad (2)$$

Here we introduced the following notation:

$$\mathbf{e} \equiv -\frac{\partial \mathbf{W}}{\partial t} - \nabla\left(\Phi - \frac{\mathbf{W}^2}{2}\right), \quad \mathbf{h} \equiv \operatorname{rot} \mathbf{W}, \quad (3)$$

where $\overline{\mathbf{W}}(\mathbf{r},t)$ is the relative velocity of the accelerated and inertial coordinate systems, $\mathbf{v}$ is the velocity of a particle in the accelerated coordinate system, C_G and C_* are collision terms describing elastic (gravitational) and inelastic (contact) collisions, respectively, and Φ and G are the gravitational potential constant, respectively.

If C_* is eliminated from the right side of Eq. (1), then Eq. (1) becomes identical to the kinetic equation for a charged particle in an electromagnetic field. In this case Eq. (1) contains Chandrasekhar's stoβ-term C_G,[2] which is identical, to within transformations, to Landau's stoβ-term.[3] Since we are interested in dynamic processes in Saturn's protorings that lead to the observed ring structure we note that large particles for which $C_G \gg C_*$ make the main contribution to the mass of the rings. This means that the theory of plasma transport in a magnetic field[4] can be used to construct a theory of transport in Saturn's rings, taking into account the fact that the transport coefficients in planetary rings are not constants as well as the fact that different conditions are realized for particles with different sizes: $\Omega\tau_G \ll 1$, $\Omega\tau_G \gg 1$, and $\Omega\tau_G \sim 1$, where τ_G is the free-flight time of the particles and Ω is the angular rotation velocity of the rings. In the first case the Chapman-Enskog method[5] can be employed, the second inequality corresponds to the case of a magnetized plasma,[4] and the last case is the most difficult case. The problem is greatly complicated both by the differential nature of the rotation $\Omega(r)$ and by the existence of dust flow from space onto the planet along the plane of the rings. The latter situation is a consequence of the Poynting-Robertson effect: the loss of angular momentum by the dust particles in collisions with particles in the solar wind. As a result the first moment equation—the continuity equation—is inhomogeneous. A new feature of the heat-balance equation is that this equation also contains a term owing to inelastic collisions.

The complete system of transport equations is written out in Refs. 6–8 and 10. The determination of the eigenvalues of this system of equations led to the discovery of a series of instabilities of the rings[6-10]; these instabilities generate the observed structures.

The physics of the *Jeans* (*gravitational*) instability consists of the fact that after modulation of the surface density of the initially uniform ring any test particle will be attracted to the nearest hump in the density and the modulation will be intensified even more.

The physics of the *dissipative instability of negative-energy waves* consists of the fact that the total energy of a density wave in a ring can be negative, if the gravitational energy (being negative) exceeds in modulus the sum of the kinetic energy of the particles (in a rotating coordinate system) and the energy of their chaotic motion. In this case the dissipation of the positive energy of the wave can be viewed as an increase in its negative energy—the absolute magnitude of the energy of the wave increases.

The physics of the *negative-diffusion instability* is determined by the inelastic character of the collisions. As a result, in the region of dense rings, where collisions occur more often than in a neighboring sparse ring, the losses of kinetic energy owing to inelastic collisions are also larger, which slows down the particles—particles accumulate in the dense ring. An example of the negative-diffusion instability in everyday life is the growth of a crowd at a narrow passage.

The three instabilities of planetary disks studied above lead to the formation of *narrow* ringlets. *Wide* ringlets are formed when the *accretion* instability discovered by the authors develops; this instability is associated with the flow of cosmic dust from space toward the planet along the plane of the rings. The nature of this instability is similar to the mechanism of dune formation in a desert by a flow of sand. Thus the wide ringlets are dunes of cosmic dust in Saturn's rings.

The rings of Uranus are fundamentally different from the rings of Saturn. They are narrow and dense, and the distance between them is several orders of magnitude greater than their width. Our preceding paper in Uspekhi Fizicheskikh Nauk (Ref. 11) is devoted to the clarification of the resonance nature of Uranus' rings, which makes it possible to predict new satellites. The high correlation coefficient[12] (0.78) between the rings of Uranus and the satellites discovered by Voyager-2 is an additional proof of the fact that the rings of Uranus are genetically related with the new satellites of Uranus.

The physics of planetary rings will be published in detail in our review prepared for Uspekhi Fizicheskikh Nauk.

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