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A scientific session of the Division of General Physics and Astronomy of the Academy of Sciences of the USSR was held on June 28, 1989 at the S. I. Vavilov Institute of Physical Problems of the Academy of Sciences of the USSR. The following reports were presented at the session:

1. Yu. V. Kopaev. Interference of dielectric and superconducting correlations in high-T_c superconductors (HTSCs).

2. N. N. Gor'kavyĭ and A. M. Fridman. Collective processes and structures in planetary rings.

Summaries of the reports are presented below.

Yu. V. Kopaev. *Interference of dielectric and superconducting correlations in high-T_c superconductors (HTSCs).*

1. *Introduction.* Different phase transitions, including the metal–insulator (semiconductor) phase transition, occur in HTSCs as the composition or temperature is varied. Thus the insulator $BaBiO_3$ doped with Pb to replace bismuth or potassium to replace barium becomes a strongly degenerate semiconductor and at some temperature T_c it transforms into the superconducting state. The same thing happens in La_2CuO_4 doped with strontium to replace La, in $YBa_2Cu_3O_6$ when the oxygen concentration is increased up to $YBa_2Cu_3O_{7-y}$, and in the bismuth systems BiSrCaCuO when Ca is replaced by Y.

Two classes can be distinguished based on the nature of the insulating state. In the first class, to which $BaBiO_3$ belongs, the insulating state arises owing to doubling of the period with respect to the charge distribution [a charge-density wave (CDW)] and the electron-phonon interaction, which apparently is responsible for the superconducting pairing arising with doping, is the leading interaction.

As pointed out above, when these insulators are doped they become conductors and superconductors. According to one point of view all vestiges of dielectric correlations (CDW or SDW) vanish at superconducting concentrations and the system is an ordinary metal, superconductivity in which can be described by the simple BCS scheme[1] or a variant of it with strong electron-phonon coupling.[2] The fact that all nonsuperconducting properties depend smoothly on the doping right up to T_c does not agree very well with this viewpoint, since if according to some concept the insulating gap were to vanish (this is unlikely to occur smoothly), then the carrier concentration would increase in a jump-like fashion up to the atomic concentration and would not be determined by the doping n.

In addition, when doping over a wide range is possible the dependence of $T_c(n)$ on the doping is observed to have a maximum, i.e., for large metal concentrations T_c is low or is equal to zero.

The existence of a CDW under conditions of superconducting doping is observed directly in $Ba_{1-x}K_xBiO_3$ in the diffraction of electrons.

The manifestation of antiferromagnetic correlations at superconducting compositions, however, is observed in $La_{2-x}Sr_xCuO_4$ and $YBa_2Cu_3O_{7-y}$ in the scattering of polarized neutrons and in the μSR method.

As the doping is increased dielectric correlations of a new type, which are not observed without doping, can appear. Thus incommensurate CDWs arise in $YBa_2Cu_3O_{7-y}$ and in bismuth and thallium HTSCs.

2. There exist two, at first glance apparently alternative, approaches for describing the effect of dielectric correlations on superconductivity: the band (Bloch) approach (I) and the localized-electron (Mott–Hubbard) approach (II).

The approach I has been intensively developed for the last 25 years, starting with the work of Little,[3] who proposed searching for HTSCs among quasi-one-dimensional (organic) systems, which, as is well known, as the temperature T is lowered cannot remain metals but rather must become insulators by undergoing either a structural (CDW) or antiferromagnetic (SDW) phase transition (PT). These dielectric and superconducting correlations must, strictly speaking, be studied together, and in the strictly one-dimensional case this makes it necessary to sum the so-called parquet diagrams.[4] It has been shown that in this case superconductivity is possible even with a purely Coulomb interaction within this one-dimensional electron subsystem without the presence of easily polarizable side radicals, as proposed in Ref. 3.

Unfortunately in the parquet approximation the physical quantities are determined to within a preexponential factor and the doping necessary for the appearance of superconductivity cannot be included in the scheme. Many characteristics for superconductivity in organic systems, in systems with so-called heavy fermions, a well as in $BaPb_xBi_{1-x}O_3$ can be understood on the basis of the band approach taking into account dielectric correlations.

The approach II for explaining the insulating properties of La_2CuO_4 arose and has been under intensive development in connection with the observation of superconductivity in $La_{2-x}Sr_xCuO_4$.[5] This was connected, first, with the fact that in pure La_2CuO_4 the number of electrons per unit cell is odd, as a result of which it appeared impossible to understand the insulating properties from the band viewpoint. Second, experiments on photoemission in doped samples also did not show Fermi steps in the carrier distribution function; this indicated that the Fermi-liquid model is not applicable. This approach was popularized by Ref. 6, where

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it was suggested that the high value of T_c in HTSCs could be due to a new type of insulating state—a state with resonating valence bonds (RVB), in which Copper pairs appear to exist, but not in a coherent state.

Experimental results which rehabilitated the band approach soon appeared: 1) it turned out that experiments on photoemission on a freshly cleaved surface show a Fermi step in samples with superconducting doping[7]; 2) antiferromagnetism, i.e., period doubling, was observed in La_1CuO_4 and $YBa_2Cu_3O_6$ (Ref. 8); the number of electrons per doubled unit cell is then even, and there is no problem in explaining the insulating state.

The RVB state can replace the observed antiferromagnetic state only with finite doping.

3. In what follows the effect of dielectric correlations on superconductivity and on the normal properties will be described and an attempt will be made to demonstrate an analogy in this question in the approaches I and II.

In the case when the Fermi surface is nearly spherical (or circular in the two-dimensional case), which happens in the band approximation with almost free electrons, i.e., when the atomic wave functions of the electrons overlap strongly not only between nearest neighbors, interelectron correlations can bring about a significant change in the state of the electrons only when the interaction energy is greater than the kinetic energy, i.e., the width of the allowed band W.

The possibility of Pts such as the PT into the state of Wigner crystallization (analog of a CDW), into the Stoner ferromagnet state, etc., have been discussed in this situation. However a systematic description of such PTs has not been constructed because there is no small parameter; moreover, even the critical magnitude of the interaction with which a PT occurs and the specific final state of the PT cannot be determined.

The opposite situation exists in the case when the atomic wave functions of the electrons overlap significantly only for nearest neighbors. Band calculations show that precisely this situation occurs in high-T_c compounds.

For most crystalline structures the shape of the Fermi surface when the (integer) number of electrons per unit cell is odd exhibits nesting, i.e., separate sections of the Fermi surface are embedded in other sections with a displacement by some vector $\mathbf{Q}$ (the nesting vector) different from a reciprocal-lattice vector. In this case the metallic Fermi-liquid state is unstable, even when the interaction is weak, relative to a transition into one of the insulating states at a temperature below some critical value T_g. When the electron-phonon interaction is stronger than the Coulomb interaction a structural PT of the doubling type (CDW) occurs—on one center (for example, in $BaBiO_3$ on one Bi atom) the electron charge density becomes higher than the average density (unity) while on a neighboring center it decreases by the same amount. As the strength of the interaction increases the amplitude of the CDW increases, and on one center the charge approaches two (bipolaron) while on the neighboring center it approaches zero.

In what follows primarily the situation when the Coulomb interaction U predominates is discussed. Regarding this interaction as short-ranged (interaction only on one site) and retaining hopping t between nearest neighbors only we write the Hamiltonian in the following form (the Hubbard Hamiltonian):

$$H = -t \sum_{\langle ij \rangle} a_{i\sigma}^{+} a_{j\sigma} + U \sum_{i} \hat{n}_{i\sigma} \hat{n}_{i-\sigma}, \tag{1}$$

where $a_{i\sigma}^{+}$ ($a_{i\sigma}$) is the operator creating (annihilating) an electron with spin σ at the site i, $n_{i\sigma} = a_{i\sigma}^{+} a_{i\sigma}$, and the symbol $\langle ij \rangle$ indicates summation over nearest neighbors. In the momentum representation the Hamiltonian (1) for a square lattice has the form

$$H = \sum_{\mathbf{k},\sigma} \varepsilon(\mathbf{k})\, a_{\mathbf{k}\sigma}^{+} a_{\mathbf{k}\sigma} + \frac{U}{2} \sum_{\substack{\mathbf{k},\mathbf{k}' \\ \mathbf{q},\sigma}} a_{\mathbf{k}',\sigma}^{+} a_{-\mathbf{k}'+\mathbf{q},-\sigma}^{+} a_{-\mathbf{k}+\mathbf{q},-\sigma} a_{\mathbf{k}\sigma}, \tag{2}$$

where $\varepsilon(k) = -2t(\cos k_x a + \cos k_y a) - \mu$, a is the lattice constant, the energy origin is chosen so that with one electron per center $n = 1$ (no doping) the Fermi level $\mu = 0$, and $\varepsilon(\mathbf{k}) = -\varepsilon(\mathbf{k}+\mathbf{Q})$, i.e., the nesting condition with $\mathbf{Q} = \pi/a(1,1)$ is satisfied. In the case of a weak interaction $U \ll t$ the instability noted above leads to pairing of an electron with momentum k with a hole with momentum $\mathbf{k}+\mathbf{Q}$, i.e., the wave function of the ground state ψ is constructed as a combination of Bloch states

$$\psi = u_{\mathbf{k}} \psi_{\mathbf{k}\sigma} + v_{\mathbf{k}} \psi_{\mathbf{k}+\mathbf{Q}\sigma'}. \tag{3}$$

The quantity $\Sigma \sim \Sigma_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}}$ plays the role of the mean field (the order parameter) and is determined at $T = 0$ from the following self-consistency equation:

$$\sum_{\mathbf{k}} \frac{1}{(\varepsilon(\mathbf{k})^2 + |\Sigma|^2)^{1/2}} = \frac{1}{U} \qquad (\Sigma \sim \langle a_{\mathbf{k}\sigma}^{+} a_{\mathbf{k}+\mathbf{Q}\sigma'} \rangle). \tag{4}$$

In the limit $U \ll t$ under study we obtain from (2)

$$\Sigma \sim t \exp\left[-2\pi \left(\frac{t}{U} \right)^{1/2} \right]. \tag{5}$$

Depending on the spin structure in (3) and the ratios of the phases of the coefficients $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ and therefore the phases of the order parameter Σ and its dependence on $\mathbf{k}$ one or another new property, whose spatial modulation is determined by the vector $\mathbf{Q}$, arises in the stable insulating state.

In the two-band model, when $\varepsilon_1(\mathbf{k}) = -\varepsilon_2(\mathbf{k}+\mathbf{Q})$, we have[9]:

1) the real singlet parameter Σ_{Re}^{s} corresponds to a CDW, i.e., a structural PT;

2) the real triplet parameter Σ_{Re}^{+} corresponds to an SDW, i.e., a spin antiferromagnet;

3) the imaginary singlet Σ_{Im}^{s} corresponds to a current-density wave (CrDW), i.e., an orbital antiferromagnet; and,

4) the imaginary triplet Σ_{Im}^{t} corresponds to a spin-current density wave (SCrDW).

In the one-band model under study the real Σ parameters correspond to a wave with the maxima falling at centers while the imaginary Σ parameters correspond to the maxima falling between centers.

The states 1)–2) correspond to the following dependence $\Sigma(\mathbf{k})$ of the s type: $\Sigma(\mathbf{k}) = \Sigma_s = \text{const}$, or $\Sigma(k) = \Sigma_s = (\sin k_x a + \sin k_y a)$. The current-carrying states 3)–4) correspond to a dependence $\Sigma(\mathbf{k})$ of the d type: $\Sigma(k) = \Sigma_{\mathrm{d}}^{s,t}(\cos k_x a - \cos k_y a)$, i.e., the parameter Σ has nodes on the Fermi surface.

The realization of one or another order parameter (or the coexistence of several order parameters) is determined

by interactions additional to (1), the degree of doping, and scattering by charged and magnetic impurities.[9] It is fundamentally difficult to observe modulation directly in the current state, especially in an SCrDW.

In the case $U \gg t$ the insulating state with $n = 1$ arises because it is impossible for two electrons to be localized on the same center and U plays the role of the insulating gap; in this case modulation (of the doubling type) does not appear for any parameter, unlike the modulation indicated above for $U \ll t$. In reality, however, a more detailed analysis shows that states of the type 1)–4) arise even in this case. Instead of electron-hole pairing (with total momentum $\mathbf{Q}$) in momentum space, as happens in the case $U \ll t$, in the case $U \gg t$ the coupling parameter $\chi_{ij} = \langle a_{i\sigma}^{+} a_{j\sigma'} \rangle$ is introduced which describes coupling at neighboring sites i and j and the spatial structure of the modulus and phase of which is once again determined by the vector $\mathbf{Q}$.[10,11] The mean-field approximation employed for χ_{ij} is constructed by formally introducing a large number N of states (colors) of an electron at a site; this suppresses bond fluctuations. In so doing it is hoped that the picture remains qualitatively the same on passing to the physical limit $N = 2$ (two spin projections). We note that for the system of strongly correlated electrons in copper for the CuO_2 plane in HTSCs the mean-field approximation for χ_{ij} can be constructed not by formally introducing $N \gg 1$, but rather by introducing weak hybridization, which suppresses fluctuations, with oxygen electrons, for which, unlike Emery's model[12] the dielectric correlations are significant owing to nesting of the Fermi surface, even with $U \ll t$, and for which, as mentioned above, the mean-field method is justified by the logarithmic singularity. It is important that the order parameters Σ and χ of both systems have the same symmetry. The experimental fact that electrons (holes) localized on copper are responsible for antiferromagnetism in a number of HTSCs can be explained in the model under study by the fact that of the two possible solutions (real and imaginary Σ) for the wave of spin density of oxygen electrons centered either on oxygen atoms or between them (i.e., on copper), the latter is realized.

The initially proposed variant of RVB[6] corresponds in the language of the parameter χ_{ij} to the situation when the moduli and phases of χ_{ij} are identical for all bonds. The spectrum of excitations corresponding to wandering (excitation) of the bonds does not have a gap in this case and the corresponding quasiparticles (spinons) form a Fermi surface. This was used as the basis for explaining the linear temperature dependence of the heat capacity below T_c observed in a number of experiments (it is now believed that this linearity is caused by an admixture in the sample of a different phase). As in the case of single-electron states, however, the Fermi surface satisfies the nesting condition and this RVB state is unstable[13] relative to period doubling (more accurately, modulation by the vector $\mathbf{Q}$). As in the case $U \ll t$, aside from charge and spin modulation, current modulation (flux-phase) is also possible[13]; this corresponds to an increase in phase by π in a circuit around a plaquette (square) of bonds. This increase in phase corresponds to equality of the hopping integral t and the factor Σ_d in the expression $\Sigma_d (\cos k_x a - \cos k_y a)$ for the Fourier component of the parameter χ_{ij}.

The electric current j over a plaquette of bonds is equal to

$$j \sim t\Sigma_d (\Sigma_d^2 - t^2), \tag{6}$$

i.e., for $j \neq 0$ the phase increment must be different from π. This corresponds to the current state Σ_{lm}^{s} in the two-band scheme or Σ_d^s in the one-band scheme.

The spin-current state corresponds to the same ratios of the moduli and phases for χ_{ij}, but with a triplet instead of a singlet spin structure of the bond.

Solutions analogous to the CDW and SDW states have also been found.[3]

Aside from a qualitative (symmetry) correspondence between the solutions for $U \gg t$ and $U \ll t$ we call attention to the surprising quantitative agreement between the results obtained formally from the equations valid for $U \ll t$ and the exact calculations.[15] From Eq. (4) with $U \gg t$ we obtain $2\Sigma_1 = U$, i.e., the excitation energy agrees with the Hubbard result. In accordance with this value of Σ the magnetization S per center is equal to μ_B. Moreover, taking into account fluctuations with respect to (4) gives already in the random-phase approximation (RPA) $S \sim 0.6\mu_B$, which agrees well with the value obtained in Heisenberg's model taking quantum fluctuations into account. In the same RPA the dispersion law and the velocity of spin waves for $U \gg t$ agree with Heisenberg's model. The results based on Eq. (4) with $U \approx t$ also agree well numerically with the exact calculations in this region for clusters of finite sizes.[16]

The experimentally observed suppression of antiferromagnetism in HTSCs with increasing doping is explained in the approach II by frustration owing to the absence of a carrier with localized ordered spin on some site. In the approach I with doping the part of the momentum space occupied by electrons in the conduction band (in the case of electron doping) or holes in the valence band (in the case of hole doping) drops out in the integration over $\mathbf{k}$ in (4), i.e., frustration in momentum space occurs. Whether changing the doping will simply change the carrier concentration in the semiconductor phase or the period of spatial modulation depends on whether the concentration of "excess" carriers (small reservoir) or the position of the Fermi level (large reservoir) is fixed.[17,18] Ba or Sr impurities in La_2CuO_4, the band of one-dimensional chains of CuO in $YBa_2Cu_3O_{7-y}$, and the BiO and TlO planes in bismuth and thallium systems play the role of this reservoir.

A different dependence $\Sigma(\mathbf{k})$ is obtained for different types of insulating states. For this reason the suppression of the parameter Σ by doping will be different for these different states, a PT from one state, for example, the antiferromagnetic state, into a different state, for example, a current-carrying state (flux-phase), can occur as the doping is increased. The comparatively sharp suppression of antiferromagnetism in HTSCs could be an indication of such a PT.

The analogy in the description of the insulating phases in the approaches I nd II was traced above by introducing the parameters $\Sigma(\mathbf{k})$ and χ_{ij}, respectively. In the approach II, in addition, a description based on the introduction of so-called auxiliary bosons or on the separation of the charge (holons) and spin (spinons) electronic degrees of freedom, is employed as an alternative. We note that such soliton solutions were first constructed in the approach I for the one-dimensional case[19] and are an example of the above-noted nonuniform solution arising in the ground state with a definite level of doping and reservoir size.

The nonuniform solutions in the form of holons and spinons can apparently be stable in two- and three-dimensional cases. Thus the idea of the separation of charge and spin degrees of freedom as a consequence of the Coulomb interaction is, first, adequate in both approaches I and II and, second, necessarily arises under definite physical conditions and not simply as a formal competing method of description (slave boson).

4. We shall now discuss the conditions under which superconductivity appears against the background of one (or several) of the insulating states studied above and we shall answer the main question of whether or not superconductivity owing only to the Coulomb interaction can arise against the background of an insulating state.

We note immediately that if long-range dielectric order is absent, but the size of the region of short-range order exceeds the superconducting correlation length ξ_0, then for the question in which we are interested the situation remains virtually identical to that in the case of long-range order.

An indication that the answer to the question posed above is affirmative is that in the limit $U\to\infty$ and in the absence of doping[20] the Hamiltonian (1) exhibits SU(2) symmetry (local symmetry): the description of the insulating state in terms of electron-hole averages $\chi_{ij}=\{\langle a^+_{i\sigma}a_{j\sigma'}\rangle$ is identical to the description in terms of the averages $b_{ij}\sim\langle a_{i\uparrow}a_{j\downarrow}-a_{i\downarrow}a_{j\uparrow}$.[14] The latter corresponds to the formation of an electron pair on neighboring sites. This symmetry arises because in this limit there is only one electron, instead of two, per center (without doping), i.e., a hole is actually always present with an electron, and for this reason the correlation of an electron on a neighboring center with this electron or hole is identical.

Superconductivity requires phasing (bose condensation) of such pairs, i.e., the phase and not the number of particles per center must be fixed. As the doping is increased the degree of fixation of the number of particles becomes less stable, but in the process the SU(2) symmetry is destroyed. It must be kept in mind that the operators b_{ij} are not purely Bose operators.[21] For this reason there still does not exist a systematic description of the appearance of superconductivity in the limit $U\gg t$.

The coupling of electron-hole and electron-electron scattering processes in a parquet in the absence of doping is the analog of SU(2) symmetry in the limit $U\ll t$.[4,22] On the basis of the Hamiltonian (1)and SDW state arises[22] and electron pairs with different total momenta exist (bose condensation does not occur).

Increasing the doping and also even small overlapping of the wave functions on non-nearest neighbors and the effect of the crystal field[23] uncouple the parquet diagrams, and the problem of the existence of dielectric and superconducting correlations can be solved exactly for $U\ll t$.

When the electron-phonon interaction is the determining one this problem reduces to the coexistence of CDWs and superconductivity ($BaBiO_3$ with Pb and K).

In this case dielectric correlations are manifested as a change in the electron spectrum, i.e., the electron density of states $N(E)$, and in the wave functions, which leads to renormalization of the matrix elements of the electron-phonon interaction λ through the so-called coherence factors. The change in the electron spectrum can substantially alter the dependence of T_c on the coupling constant λ from the traditional BCS dependence $\exp(-1/\lambda)$ with $N(E)=$ const to λ^2 (Ref. 24) with $N(E)\sim 1/E^{1/2}$, arising with dielectrification near the edge of the allowed band.

Renormalization of λ owing to dielectric coherence factors depends strongly both on the type of insulating state (CDW, SDW, CrDW, SCrDW) and on the type of superconducting parameter: Δ_s (s type) or Δ_d (d type). For some combinations of the pairs Σ and Δ the coherence factors intensify the manifestation of singularities $\nabla(E)\sim 1/E^{1/2}$ in T_c and significantly increase T_c over the BCS value; for other combinations the coherence factors compensate the effect of $N(E)$ and reduce T_c below the BCS value.[25,26]

In particular, for a CDW an increase in T_c occurs for the parameter Δ_s (symmetric case in the two-band model), which is nodeless on the Fermi surface. In this case T_c has a sharp maximum as a function of both the degree of doping and the value of Σ.[25] The critical temperature T_c can increase by a factor of the order of 20 even with $\lambda\approx 1$.[27] The reverse situation occurs for the state Δ_d. Analogous relations occur for the spin-current state (Σ^t_{lm} in the two-band model and Σ^t_s in the one-band model).

For the SDW and the current-carrying state (CrDW) T_c increases for the state with Δ_d, i.e., with nodes on the Fermi surface.

We note that even in the case of the electron-phonon mechanism of superconductivity the isotope effect could be virtually absent owing to the singularity in $N(E)$.[27]

The kinetic properties of the superconducting state can differ significantly from those in the BCS model, since they are determined by the interference of the dielectric and superconducting coherence factors.[28] One result, for example, is that the peak in the temperature dependence of the spin-lattice relaxation rate is shifted toward T_c on the low-temperature side and the magnitude of the peak is reduced[28]; this is observed in SCrDW for oxygen. In the limit $U\gg t$ the peak vanishes altogether[29]; this is observed in SCrDW for copper.[30]

The same interference can also explain the characteristics of the temperature dependence $\Delta(T)$ and the large values of $2\Delta/T_c$ obtained from the current-voltage characteristics (IVCs) in tunneling experiments[31] and in the IR reflection spectra.[32] The large value of $2\Delta/T_c$ could be a result of an incorrect interpretation of the observed corresponding singularities, owing not to Δ, but rather the parameter Σ and the degree of doping.

The singularities observed in SCrDW in the temperature dependence of the velocity of sound and in the attenuation coefficient of sound near T_c agree qualitatively with the predictions.[28]

A number of observed features in the temperature dependence of the kinetic coefficients at temperature $T>T_c$, such as the magnetic susceptibility $\chi(T)$,[33] the Hall constant $R_H(T)$,[34a] the resistance $\rho(T)$,[34b] and the negative sign of the second derivative of T with respect to H_{c2},[35] can be explained just as simply on the basis of this scheme.

The above-noted possibility of doping-induced spatial modulation of the parameter Σ should lead to modulation of the superconductivity parameter; this could lead to a number of features.[26] For example, the degree of modulation of the parameter Δ should increase as the temperature is lowered; this could cause the critical current to decrease as the temperature is lowered.

5. We shall now discuss the possibility of superconductivity owing to the Coulomb interaction.

The self-consistency equation for Δ

$$\Delta(\mathbf{k}) = \sum_{\mathbf{k}'} V(\mathbf{k}-\mathbf{k}') \frac{\Delta(\mathbf{k}')}{(\varepsilon^2(\mathbf{k}') + \Delta^2(\mathbf{k}'))^{1/2}} \tag{7}$$

for the case Δ_d, i.e., changing sign along the Fermi surface, also relates the regions over $\mathbf{k}'$ in which the signs of Δ are different. If the scattering matrix element $V(\mathbf{k}-\mathbf{k}')$ predominates in these regions, then Eq. (7) can have a solution with positive $V(\mathbf{k}-k')$, i.e., with repulsion. Precisely this situation also occurs in the case of nesting of the Fermi surface,[36] and $\mathbf{k}-\mathbf{k}' \approx \mathbf{Q}$, plays the role of the distinguished region, since in this case the polarizability of the system is high, which also leads to a PT into the insulating state, which was ignored in Ref. 36. This mechanism of superconductivity is also possible in the case when, for example, doping completely suppresses the PT into the insulating state.[37] The quantity $\delta \equiv (1-n)$ (the degree of doping) must exceed some critical value δ_{cr}.

In the case $\delta < \delta_{cr}$ a PT into one of the insulating states must be taken into account; this corresponds to a generalization of the BCS scheme to Éliashberg's scheme,[2] but a fundamental restructuring of the ground state and not simply a change in the effective mass occurs only under the action of the same Coulomb interaction U. As a result the change in polarization owing to the change in the ground state, i.e., owing to the soft collective mode, must be taken into account.[38] In Ref. 15 this procedure was performed analytically for SDW in the limit of weak doping. In the process the exchange of amplitude spin oscillations above the ground state in the SDW also leads to an attractive effective interaction for d-type superconductivity

$$U_{\text{eff}} = -\frac{U^2}{\hbar\omega_0}, \quad \lambda = \frac{U^2}{\hbar\omega_0 t}, \tag{8}$$

where ω_0 is the limiting frequency of these vibrations ($\hbar\omega_0 < t$). The sign of the interaction changes owing to the fact that the contribution from the region $\mathbf{k}-\mathbf{k}' \approx \mathbf{Q}$ decreases as a result of the dielectric coherence factors.

For s-type superconductivity the corresponding coherence factor is positive, and superconductivity of this type is impossible under the conditions of SDW.

Thus taking into account collective excitations in the insulating state, determined by the interelectronic Coulomb interaction, leads to an effective interelectronic attraction between quasiparticles even in the random-phase approximation, but taking the interference of dielectric and superconducting coherence factors into account. Therefore there is a qualitative agreement with the results obtained in the more "complicated" parquet situation. This pertains to both the condition for the coexistence of different types of insulating and superconducting phases and the conditions for the change in the sign of the effective interaction.[4,22] Moreover numerical calculations[22] show that the effective coupling constants obtained in these two approximations are also in good quantitative agreement.

As the degree of doping is increased the diagrams in which each polarization loop contains not one, as in the RPA, but two lines of the bare Coulomb interaction are singled out (we note that the parquet approximation corresponds to an infinite number of such lines).[40] The effective attraction in this case is determined by the high nonlinear (quadratic) polarizability.

The first steps in the solution of the problem of determining the effect of different types of dielectric orderings on superconductivity with $U \gg t$ were taken in Refs. 10, 13, and 41. For this, simultaneous solutions for the parameters χ_{ij} and Δ must be sought, as in the case $U \ll t$. The qualitative agreement between the results obtained for $U \ll t$ and numerical cluster calculations[16] with $U \approx t$ suggests that this agreement will also be preserved in the limit $U \gg t$.

A new result in the case $U \gg t$ is that the width of the allowed bands, which are split as a result of hybridization, decreases, and the degree of narrowing decreases as the doping is increased.[10,13] The characteristic singularities in the density of states $N(E)$, determined by the presence of nesting of the Fermi surface of the starting band, are also preserved in this case.[13,42] For this reason in this case there will be a variety of functional dependences of T_c on λ depending on the degree of doping, as happens in the case $U \ll t$.

The question of whether or not the above-studied d-type superconductivity, owing to the Coulomb interaction, is realized in SCrDW is still open both experimentally and theoretically. The results of different physical experiments which at first glance seem inconsistent—some of which suggesting s-type superconductivity and others suggesting d-type superconductivity—can be made consistent if it is assumed that in reality both types of superconductivity coexist.[43]

From the theoretical viewpoint the existence of d-type superconductivity should be inhibited by scattering not only by magnetic impurities (as for s-type) but also by any nonuniformities; this is very unfavorable for existing SCrDWs. Of course, if the effect of dielectric correlations on superconductivity is strong, the superconductivity will be sensitive to nonmagnetic scattering in the case of s-type superconductivity also,[17] since dielectric correlations are sensitive to nonmagnetic scattering. Moreover the suppression of d-type superconductivity under conditions of dielectric correlations can be substantially weakened (this question is under study).

Superconductivity of the s-type (Δ_s) owing to Coulomb interaction is possible, however, if a spin-current state SCrDW (Σ_d^s) is realized. The description of the coexistence of these states is analogous to that studied above for SDW and Δ_d.

Another variant of the existence of s-type superconductivity owing to the Coulomb interaction can be realized when, for example, aside for CrDW and the d-type superconductivity determined by it, as in SDW, a CDW arises. Then owing to the presence of the term

$$\delta F = \gamma \Sigma_s^s \Sigma_d^s \Delta_d \Delta_s \tag{9}$$

in the free energy there necessarily arises a parameter Δ_s at the same temperature T_c at which the parameter Δ_d arises.[44] This situation roughly corresponds to the scheme of Ref. 45, when the charged-boson state arises as a consequence of the formation of a bound state of a charged fermion (owing, in the case under study, to the CDW state) with a vortex current (owing to the CrDW state).

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