Synchrotron-type radiation processes in crystals and polarization phenomena accompanying them

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The justification for using the homogeneous-field approximation for the local description of the interaction of electrons (e^-), positrons (e^+), and high-energy γ-quanta with oriented crystals is discussed. Synchrotron-type dichroism and double refraction of γ-quanta, radiative self-polarization of $e^\pm$ and creation of transversely polarized $e^\pm$ in bent crystals are discussed. These phenomena can serve as the basis of methods of obtaining polarized beams of $e^\pm$ and γ on the most powerful existing proton accelerators and those under construction. The review also discusses the effect of spin rotation of particles in bent crystals, which enables one to measure the magnetic moments of short-lived particles, and also to observe the fundamental quantum electrodynamical effect of variation of the anomalous magnetic moment of $e^\pm$ in an intense crystal field.

1. INTRODUCTION

In summarizing an extensive set of studies of the effect of the medium on electromagnetic processes at high energies, M. L. Ter-Mikaélyan wrote[1]: "At first glance, the posing of the question of the influence of the medium on electromagnetic processes at high and superhigh energies (at which the wavelength of all the particles participating in the process is smaller by a factor of 10^4–10^6 than the interatomic distances) seems paradoxical .... However, a number of studies has shown that, even at high energies, when the wavelengths of all the particles are negligibly small in comparison with the interatomic distances, taking account of the medium in certain processes is necessary." In particular, it was found[1,2] that an ordered distribution of the atoms in a crystal leads to a substantial change in the cross sections for bremsstrahlung and pair production as compared with those in an amorphous medium. For a long time in the theoretical analysis of reactions excited by high-energy particles in crystals, the influence of the periodic arrangement of the atoms on the motion of the particles was taken into account within the framework of first-order perturbation theory. However, it was shown at the beginning of the 70s that the condition for applicability of this approximation breaks down with decreasing angle between the incident beam and a crystallographic axis or plane.[3]

Further study of the problem has shown that in this case the theoretical description is not simply altered, but qualitatively new physical phenomena arise. As the first of them, the emission of γ-quanta by channelled electrons (e^-) and positrons (e^+) began to be studied. The results of studying it in the energy region of e^+, $\varepsilon_\pm \lesssim 10$ GeV, are discussed, e.g., in the monographs of Refs. 4–6 and reviews of Refs. 7 and 8. Throughout the course of development of quantum electrodynamics (QED), the sharp parallel has been traced in the study of processes of emission and pair production (PP), which are the cross-channels of a single reaction. However, the study of emission during channeling was not accompanied by study of the related process of PP. Search for the latter led to rather unexpected effects. A PP effect was predicted[9-11] (see also Refs. 12 and 13) that was closely associated with the analogous process in an intense homogeneous field, and which is manifested in the same way, both under conditions of channeling of the motion of created particles and with considerable deviation from them. In the cross-channel this process is associated with quantum synchrotron radiation[5,14-17] (radiation in a homogeneous field for which recoil is substantial), by analogy with which the process of PP in crystals was also termed magnetic-braking-type (synchrotron-type). In line with the predictions of Refs. 15 and 16,[1)] the considerable magnitude of the averaged field intensities of the crystal axes (tens and hundreds of GV/cm) enabled observing processes of quantum synchrotron radiation[20-26] and PP[27-30] at energies of $e^\pm$ and γ exceeding several tens of GeV.

Synchrotron radiation processes and PP are accompanied by a number of polarization phenomena, which are the fundamental theme of our review. The effects of dichroism and double refraction of γ-quanta in crystals in the hard range ($\hbar\omega \gtrsim 1$ GeV) were found already in the study of PP processes in the Born approximation.[31,32] The manifestation of the quantum synchrotron nature of the processes of emission and PP in the region of even higher energies not only altered the character of these phenomena, but also led to the appearance of fundamentally new phenomena with participation of polarized $e^\pm$,[5,9] such as effects of radiative self-polarization of $e^\pm$ in crystals[33-35] and production of transversely polarized e^+e^- pairs.[11,36] These effects open up broad potentialities for obtaining polarized $e^\pm$ and γ-beams in the most powerful existing proton accelerators and those under construction. This review also discusses the effect of spin rotation in bent crystals.[33,37] Besides the measurement of the magnetic moments of short-lived particles,[9] it enables observing the manifestation of the very important quantum-electrodynamic effect of the anomalous magnetic moment of $e^\pm$ in an intense crystal field.[38,39]

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2. OPTICAL EFFECTS IN THE HARD γ-RANGE ACCOMPANYING THE PROCESS OF MAGNETIC SYNCHROTRON PAIR FORMATION IN CRYSTALS

The possibility of studying QED effects of a homogeneous intense field[40–45] in the passage of $e^{\pm}$ or γ through crystals was first discussed in Ref. 38, where the possibility was pointed out of observing a change in the anomalous magnetic moment of $e^{\pm}$, and also of synchrotron-type dichroism and double refraction of γ-quanta by crystals. The substantiation of using the QED of a homogeneous intense field to describe the PP process, which leads to the two latter effects, was performed in Ref. 10. A theory of the PP process that does not rest on perturbation theory[10,46] for the interaction with the crystal was constructed by analogy with the theory of emission,[47] in which the interaction of $e^{\pm}$ with the crystal is taken into account exactly. It was first shown on this basis that, at small angles of incidence of the γ-quanta on crystal planes, the formula for the probability of PP in the crystal is reduced to the probability of PP in an intense homogeneous field integrated over its volume (see the Appendix). The formulas of the theory[10,46,47] take account of quantum effects associated with the motion of $e^{\pm}$. At the same time it is well known[4–8] that, in the region of interest to us with $e^{\pm}$ and γ energies of tens of GeV and more, we can consider the motion of $e^{\pm}$ to be fully classical.

The quasiclassical method developed by Baĭer and Katkov[44,48] explicitly takes account of the classical character of the motion of $e^{\pm}$ and the quantum effect of recoil in emission, which enables us to proceed easily to describing the PP process, and also to include spin effects in the treatment. Despite the fact that this method was developed to describe processes that occur in macroscopic external fields, it is applicable also for describing processes in the microscopic fields of crystals.[49] By using the quasiclassical nature of the motion of $e^{\pm}$, we can establish the most important features of the PP process in crystals at high energies by starting with simple physical considerations.

2.1. The local synchrotron-type nature of the process of pair production in crystals at high energies

We recall that the motion of charged particles at small angles to crystal planes (or axes) is described by the field of the planes (or axes) averaged over the coordinates of the atoms in the plane (or axis), as well as the thermal displacements of the atoms from the equilibrium positions. The methods of calculating the potentials of the averaged fields have been described in detail in Refs. 6, 50–52. The parameters of the averaged fields of the planes and axes of certain crystals that will be necessary below, and which are calculated using the approximation of the atomic potential of Moliere, are given in Table I.

In moving at a small angle to a plane (or axis), the particle senses the action of the averaged field $\vec{\mathscr{E}}$, which is practically perpendicular to its velocity. Following Ref. 1, we can determine the length l_f for formation of an e^+e^- pair in this field in self-consistent fashion from the relationship[2)]

$$\frac{1}{l_\Phi} \sim k - p_{+z}(l_\Phi) - p_{-z}(l_\Phi) \approx \frac{1}{l_\Phi}(1 + \theta_\pm^2(l_\Phi)\gamma_\pm^2). \quad (2.1)$$

This associates the uncertainty $\Delta z \sim l_f$ of the z coordinate of the location of PP with the magnitude of the z component of the moment imparted to the filed (the crystal) in the direction parallel to the momentum of the γ-quantum (the z axis). The $\mathbf{p}_\pm(l_f)$ are the momenta of $e^{\pm}$ taken for $z = l_f$ when they began motion at the point $z = 0$ in the direction of the z axis. Here ω and $\varepsilon_\pm = \gamma_\pm m$ are the energies of the γ-quanta and of $e^{\pm}$, and m is the mass of $e^{\pm}$. Further, $\theta_\pm(l) = w_\pm l$ are the angles of rotation of $e^{\pm}$ over the length l under the action of the electric field, which imparts to them the transverse accelerations $w_\pm = \pm e\vec{\mathscr{E}}/\varepsilon_\pm$, where $e = |e|$ is the charge of e^+, and $l_0 \sim \varepsilon_+\varepsilon_-/m^2\omega$ is the coherent length introduced in Ref. 1. Upon using (2.1) in the case of a strong enough field $\theta_\pm(l_0) = 1/\gamma\xi^{1/2} > 1/\gamma_\pm$, we obtain the following expressions for the formation length and the angles of deviation of $e^{\pm}$ over it:

$$l_\Phi = l_0\xi = \frac{m}{e\mathscr{E}\xi^{1/2}},$$
$$\theta_\pm(l_\Phi) = w_\pm l_\Phi = \frac{1}{\gamma_\pm\xi^{1/2}}, \quad \xi < 1. \quad (2.2)$$

Here we have introduced the parameter

TABLE I. Certain parameters of the averaged potentials of the principal axes and planes of a number of crystals.

Element	Z	(Plane) ⟨Axis⟩	d_{pl} (d_{ax}), Å	T, K	u_1, A.	V_{max}, eV	$\mathscr{E}_{max}$, GV/cm	$\omega_{\chi=1}$ $=\varepsilon_{\chi=1}$, GeV
Diamond	6	(110)	1.26	293	0.04	20.8	7.7	890
		⟨110⟩	2.52	293	0.04	137	68	100
Si	14	(110)	1.92	293	0.075	21.5	5.7	1193
		⟨110⟩	3.84	293	0.075	133	46	145
Ge	32	(110)	2.00	293	0.085	37.7	9.9	684
		(110)	2.00	0	0.036	44.0	14.9	454
		⟨110⟩	4.00	293	0.085	229	78	87
		⟨110⟩	4.00	100	0.054	309	144	47
W	74	(110)	2.24	293	0.05	127	43	158
		⟨110⟩	2.24	0	0.025	142	57	119
		⟨111⟩	2.74	293	0.05	931	500	13.6
		⟨111⟩	2.74	0	0.025	1367	1160	5.8

Z—atomic number, d_{pl}—interplanar spacing, d_{ax}—interatomic distance along an axis, T—temperature of the crystal, u_1—rms amplitude of the thermal oscillations of the atomic nuclei, V_{max}—amplitude of the variation of the averaged potential of an axis or the total potential of planes, $\mathscr{E}_{max}$—maximum field intensity of an axis (total field of planes), $\varepsilon_{\chi=1} = \omega_{\chi=1} = m^3/e\mathscr{E}_{max}$ is the energy of γ, $e^{\pm}$ at which the quantum-electrodynamic parameters $\chi = e\mathscr{E}\varepsilon/m^3$, $\varkappa = e\mathscr{E}\omega/m^3$ attain a value of unity in a field of intensity $\mathscr{E} = \mathscr{E}_{max}$.

$$\xi = \left(\frac{m^2\omega}{e\mathscr{E}\varepsilon_+\varepsilon_-}\right)^{2/3} = (\theta_\pm(l_0)\gamma_\pm)^{-2/3}. \quad (2.3)$$

The latter plays a substantial role in determining the probability of PP in a transverse electric field[40-45,48,53]

$$W = \frac{\alpha m^2}{\sqrt{\pi}\omega^2}\int_0^\omega d\varepsilon_+\left[\int_\xi^\infty \Phi(y)\,dy + \left(2-\frac{\omega^2}{\varepsilon_+\varepsilon_-}\right)\frac{\Phi'(\xi)}{\xi}\right]. \quad (2.4)$$

Here

$$\Phi(\xi) = \frac{1}{\sqrt{\pi}}\int_0^\infty \cos\left(\xi t + \frac{t^3}{3}\right)dt \quad (2.5)$$

is the Airy function. Since it falls off proportionally to $\exp(-2/3\xi^{3/2})\xi^{1/4}$ when $\xi > 1$, the PP process in this case proves to be suppressed. Using (2.2), we obtain $\Delta\rho_\pm \sim l_f\theta(l_f)/2 \approx \gamma_\pm\xi^{1/2}/\omega \lesssim \lambda_e = 3.86\times10^{-11}$ cm as the transverse displacement of $e^\pm$ over the length l_f when $\xi \lesssim 1$. On the other hand, the averaged fields of the planes (or axes) are substantially altered upon transverse displacements $\Delta\rho > u$, where u is the root-mean-square amplitude of the thermal oscillations of the nuclei (see Table I). Since $u \sim 0.1$ Å $\gg \lambda_e$, when a γ-quantum is incident parallel to the planes (or axes) in the process of pair production, the $e^\pm$ "sense" the action of a practically constant field. This allows one to employ the probability of PP in a homogeneous field of (2.4) for the local description of the PP process in the crystals.

As was established in Ref. 40, many characteristics of emission processes in a homogeneous field, e.g., the probability of (2.4), are determined solely by the magnitude $w_\pm$ of the transverse acceleration of $e^\pm$ and do not depend on the nature of the field (electric or magnetic). Therefore we shall not only employ the results first obtained for the magnetic field to describe the processes of emission, PP, and the polarization phenomena in the electric fields of crystals, but also we shall call them processes (or phenomena) of synchrotron type.

Since the fundamental contribution to the probability of (2.4) comes from the energy of $e^\pm$, $\varepsilon_+ \sim \varepsilon_- \sim \omega$, we can rewrite the condition for intense synchrotron-type PP $\xi \lesssim 1$ in the form

$$\varkappa = \frac{\omega\mathscr{E}}{\mathscr{E}_0 m} \gtrsim 1, \quad (2.6)$$

The latter does not depend on the energy distribution in the pair ($\mathscr{E}_0 = 1.32\times10^{16}$ V/cm—the characteristic field intensity capable of intensely producing e^+e^- pairs from the vacuum.[41,48] The possibility of satisfying the condition (2.6) owing to the very substantial magnitudes of the intensities of the averaged fields of crystal axes at currently accessible energies of γ-quanta of tens of GeV was precisely the circumstance that predetermined the interest in studying the QED effects of an intense field in passage of $e^\pm$ and γ through crystals. Let us estimate the maximum magnitude of the intensity of the averaged electric field of an axis that is attained at a distance $\rho \approx u$ from it. For graphic description, let us study for this purpose the action of the nuclei of the atoms on a probe particle. In passing at a distance from the nucleus of $\rho \approx u$, it experiences the action of the field $\mathscr{E}_u \approx Ze/u^2$ over the distance $\Delta z \approx 2u$. Upon averaging it over the positions of the atoms of the axis, we obtain $\mathscr{E}_{max} \approx (\Delta z/d_{ax})\mathscr{E}_u \approx 2Ze/ud_{ax} \approx 3\times10^{10}Z$ (V/cm) $\lesssim 10^{12}$ V/cm, where d_{ax} is the interatomic spacing on the axis. In the case of planes, we obtain analogously an estimate smaller by about an order of magnitude, $\mathscr{E}_{max} \approx \pi Zen_0 d_{pl}$, where d_{pl} is the interplanar spacing. The magnitudes of $\mathscr{E}_{max}$ calculated by using the Moliere approximation for the atomic potential are given in Table I. In their ability to deflect $e^\pm$, the averaged crystal fields are equivalent to magnetic fields with intensities exceeding those obtained in explosive magnetic generators and at the focus of a laser beam,[54] and which reach 0.2–3 GG for axes and 20–200 MG for planes. Here the field intensity $\mathscr{E}' = \gamma\mathscr{E}$ in the intrinsic system of $e^\pm$ exceeds the critical intensity $\mathscr{E}_0 = 1.32\times10^{16}$ V/cm, even at energies $\varepsilon_\pm \approx 10$ GeV. As the characteristic energy of γ-quanta at which the process of synchrotron-type PP begins to be manifested intensively in the crystal, it is natural to choose the energy $\omega_{\varkappa=1} = m\mathscr{E}_0/\mathscr{E}_{max}$ at which the condition $\varkappa = 1$ is satisfied in the field of maximum intensity of the plane (or axis). These energies are also given in Table I. We can easily see that for axes we have $\omega_{\varkappa=1} \sim 10$–100 GeV, while for planes $\omega_{\varkappa=1} \sim 100$–1000 GeV. Since the energy of the γ-beams at CERN does not exceed 150 GeV, studies of the process of synchrotron-type PP are conducted there in the fields of crystal axes. The local synchrotron-type character of the PP process enables one to obtain its probability per unit length of crystal by a simple averaging of the probability of (2.4) over its cross section (see the Appendix).

The probability thus calculated agrees well with the results of two experiments[28,29] to observe synchrotron-type PP (the first experiment[27] was unsuccessful). Figure 1 shows the results of experimental study of the increase in rate of PP with increasing energy of the γ-quanta, which agree with the predictions[55,56] and calculational results of the authors of Ref. 29. The experiment[28] observed approximately a 10% greater increase in the rate of PP, which agrees better with the predictions.[57] The orientational dependence of the probability of PP observed in both experiments[28,29] also agrees well with the predictions[55-57] (see below). The above said allows us to state that the correctness of the local description of the PP process based on the probability of PP in a homogeneous field is a firmly established fact.

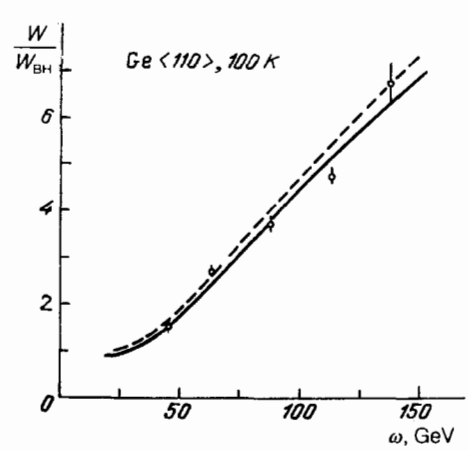

FIG. 1. Energy dependence of the probability of PP in the field of the ⟨110⟩ axis of a Ge crystal at $T = 200$ K expressed in units of the Bethe–Heitler probability W_{BH}. Dots—experimental data. The solid and dotted curves are respectively calculated with and without taking account of the energy dependence of the probability of incoherent PP.[55,56]

2.2. Synchrotron-type dichroism of crystals

An understanding of the local nature of the emission processes and PP has enabled prediction, by using the results of the QED of a homogeneous field, of a number of polarization phenomena that accompany the passage of $e^{\pm}$ or γ through crystals. First let us take up the effects of double refraction and dichroism.[9,10,38] According to Ref. 38 one can use an expression for the permittivity that describes the analogous effects in a macroscopic external field to describe these phenomena. As is known,[40–45,58] a vacuum region occupied by a transverse homogeneous electric field $\vec{\mathscr{E}} = n_x \mathscr{E}$ is characterized by the transverse permittivity tensor

$$\varepsilon_{ij} = \delta_{ij} + \varepsilon_x \delta_{xi}\delta_{xj} + \varepsilon_y \delta_{yi}\delta_{yj}. \tag{2.7}$$

Here $i, j = x, y$, and the x (or y) axis is parallel (or perpendicular) to the field. Also, both the imaginary and real components of the principal values of the tensor of (2.7) differ. As we know, this leads to the effects of dichroism and double refraction. In the usual situation the interaction of γ-quanta with the crystal is described by the permittivity of (2.7) averaged over the cross section of the crystal (However, see Ref. 59). Since the field at a crystal axis of most crystals has cylindrical symmetry (apart from certain crystals, e.g., ferroelectrics), averaging (2.7) leads to almost complete disappearance of these effects. They can be partially retained only by the breakdown of cylindrical symmetry owing to the influence of adjacent axes. At the same time, the geometry of the averaged field of crystal planes (Fig. 2) enables the polarization properties of a homogeneous field to be manifested fully, since the directions of the principal axes of the two-dimensional tensor of (2.7) throughout the crystal and at each point of it are the same here. One of them is parallel and the other perpendicular to the planes.

Let us proceed to a quantitative treatment. We shall start with the dichroism effect, which is described by the imaginary components of the permittivity. The latter are determined by the probabilities of PP in crystals by linearly polarized γ-quanta. According to Refs. 10 and 38, one can calculate them by starting from the PP probabilities for γ-quanta polarized parallel and perpendicular to the homogeneous transverse electric field $\vec{\mathscr{E}} = n_x \mathscr{E}$[40–45]:

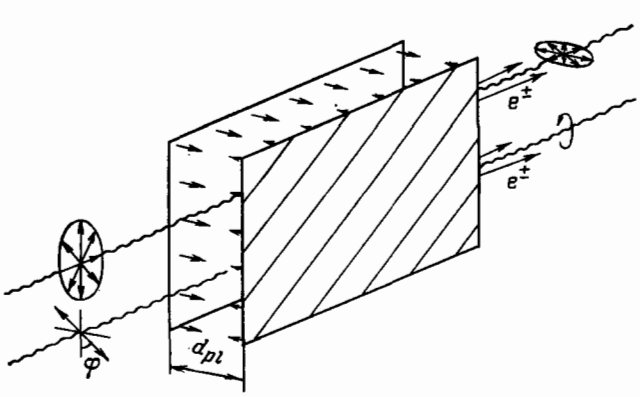

FIG. 2. The averaged field of crystal planes throughout the crystal parallel to the normal to them, whereby the crystal, similarly to a region of space occupied by a homogeneous electric field, possesses the properties of dichroism and double refraction of γ-quanta. φ is the angle formed by the linear polarized and the crystal planes.

$$W_{x,y}(\varkappa) = \frac{\alpha m^2}{\sqrt{\pi}\omega^2} \times \int_0^{\omega} d\varepsilon_+ \left[\int_{\xi}^{\infty} \Phi(y)\, dy + \left(2 \pm 1 - \frac{\omega^2}{\varepsilon_+\varepsilon_-}\right) \frac{\Phi'(\xi)}{\xi} \right]. \tag{2.8}$$

Here we have $\xi = (\omega^2/\varepsilon_+ \varepsilon_- \varkappa)^{2/3}$, and the upper (or lower) sign corresponds to polarization parallel to the x (or y) axis. Equation (2.8) indicates the very strong polarization dependence of the probability of PP in a homogeneous field: the ratio W_y/W_x varies from two for $\varkappa = \omega\mathscr{E}/\mathscr{E}_0 m \ll 1$ to 1.5 when $\varkappa \gg 1$. At not too small angles of incidence of γ-quanta on the crystal planes, $\psi > 0.1$ microradian, for which the probability of absorption of a γ-quantum in the field of a single plane is small and does not lead to redistribution of their flux (for more details, see Ref. 59), the probability of PP per unit length of crystal by γ-quanta polarized parallel ($\|$) and perpendicular ($\perp$) to the planes is obtained by averaging (2.8) over the interplanar interval $0 < x < d_{pl}$:

$$W_{\|,\perp}^{\text{oh}} = \int_0^{d_{pl}} W_{y,x}(\varkappa(\mathscr{E}(x)))\, \frac{dx}{d_{pl}}. \tag{2.9}$$

However, the expression (2.9) describes only the processes of PP in the averaged potential. By analogy with the theory of coherent PP based on the Born approximation,[1,7] we can naturally call the processes in the averaged potential of the planes (or axes) coherent. Strictly speaking, the processes with discrete momentum transfer in directions parallel to the planes (or axes) also belong to the coherent processes. However, we can neglect them at the energies being discussed.

It has already been established in the treatment of the process of PP in the Born approximation that, besides the coherent processes in crystals, incoherent processes exist, accompanied by momentum transfer to individual nuclei (and electrons) of the material. The probability of incoherent PP in this approximation proved not to depend on the energies and angles of incidence of the γ-quanta, while in magnitude they are 10–20% smaller than the Bethe–Heitler probability W_{BH}. Study of PP processes under conditions of manifestation of the synchrotron-type have shown[56,60] that, in the logarithmic approximation, they can also be classified naturally into the coherent processes described by (2.9) and incoherent processes. The latter are described by the local probability calculated for the density of nuclei averaged along the planes (or axes) using the cross section of the process of PP by an individual nucleus lying in a homogeneous field, as derived in Refs. 61 and 62 and refined in Ref. 56. As the intensity of the latter field we should naturally take the intensity of the averaged field at the given point. The probability of incoherent PP per unit length of crystal is obtained by averaging the local probability over the cross section. Like the Bethe–Heitler probability, it is proportional to Z^2. At the same time, in the region of intense PP, $\omega \sim (1-10)\omega_{\varkappa=1}$, the local probability of coherent PP is proportional to $1/\omega \propto 1/\omega_{\varkappa=1} \propto \mathscr{E}_{\max} \propto Z$. This leads to a proportionality of Z and the probability of coherent PP throughout the crystal. Therefore the relative magnitude of the contribution of incoherent processes and the total probability of PP in the crystal is proportional to Z. Hence it is

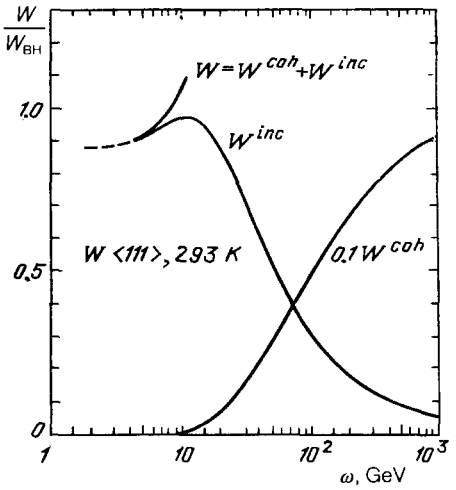

FIG. 3. Energy dependence of the coherent ($W^{\rm coh}$) and incoherent ($W^{\rm inc}$) contributions, expressed in units of the Bethe–Heitler probability $W_{\rm BH} = 2.35\,{\rm cm}^{-1}$, to the probability of PP by γ-quanta propagating parallel to the $\langle 111\rangle$ axis of a tungsten crystal at $T = 293$ K.

maximal in crystals of the heavy elements. For this reason, to illustrate the role of incoherent processes, we shall address ourselves in Secs. 2.2 and 2.3 to the case of a tungsten crystal, for which $Z = 74$. The main difference between incoherent PP processes under conditions of applicability of the synchrotron-type approximation and the analogous processes in the region of action of the Born approximation is the decline proportional to $\omega^{-2/3}$ of the probability $W^{\rm inc}$ of these processes when $\omega > \omega_{\varkappa=1}$ (Fig. 3). The difference between the total probability of PP, $W = W^{\rm coh} + W^{\rm inc}$, and the probability $W^{\rm coh} + W_{\rm BH}$ that was used prior to the construction of the theory of incoherent processes reaches 13–15% in tungsten for $\omega = 50$–100 GeV, and 6–7% in germanium for $\omega = 150$ GeV (see Fig. 1), which, e.g., exceeds the uncertainty of calculating the total PP probability involved with using the different approximations for the averaged potential of an axis.

To describe the dichroism of crystals, we must also take account of the contributions of the incoherent processes to the probability of PP by γ-quanta polarized parallel and perpendicular to the crystal planes. In the case of propagation of γ-quanta along the latter, these contributions are calculated following Ref. 56; they have the form (cf. Refs. 61, 62)

$$W^{\rm inc}_{\parallel(\perp)} = \frac{1}{15L_{\rm рад}\omega^3} \times \int_0^{d_{\rm pl}} \frac{n(x)}{n_0}\frac{dx}{d_{\rm pl}} \int_0^{\omega} d\varepsilon_+ \left[1 - \theta(1-\xi)\frac{\ln\xi}{2\ln(183Z^{-1/3})}\right] \times \{\omega^2[(1\pm 1)\xi^4\Upsilon \mp 6\xi\Upsilon - (3\pm 4)\xi^2\Upsilon' - (1\pm 1)\xi^3] + (\varepsilon_+^2+\varepsilon_-^2)[(1\mp 1)\xi^4\Upsilon + (3\pm 6) \times \xi\Upsilon - (5\mp 4)\xi^2\Upsilon' - (1\mp 1)\xi^3]\}. \qquad (2.10)$$

Here

$$\Upsilon = \Upsilon(\xi) = \int_0^{\infty} \sin\left(\xi t + \frac{t^3}{3}\right) dt \qquad (2.11)$$

is the upsilon function, $\Upsilon' = d\Upsilon(\xi)/d\xi$, $\xi = \xi(x) = [\omega^2/\varkappa(x)\varepsilon_+\varepsilon_-]^{2/3}$, $L_{\rm rad}$ is the radiation length, which is given by the relationship

$$\frac{1}{L_{\rm рад}} = 4\alpha n_0 \left(\frac{Z\alpha}{m}\right)^2 \ln(183Z^{-1/3}). \qquad (2.12)$$

Here $\alpha = 1/137$, and n_0 is the mean concentration of atoms in the crystal, and

$$n(x) = \frac{n_0 d_{\rm pl}}{\sqrt{2\pi}u} \exp\left(-\frac{x^2}{2u^2}\right) \qquad (2.13)$$

is the local concentration of nuclei averaged over the plane. The first integral in (2.10) describes the procedure of averaging over the interplanar interval of the local probability of PP at the nuclei distributed with the density of (2.13). To avoid extra complications, we have somewhat simplified Eq. (2.10). Namely, by using the certain degree of freedom inherent in the logarithmic approximation in choosing the parameters acting as arguments of the logarithm, we wrote (2.10) in a form that enables going over to $W_{\rm BH}$ for $\omega \ll \omega_{\varkappa=1}$ and neglected the insignificant asymmetry of incoherent scattering discussed in Ref. 63. Figure 4 enables one to compare the coherent and incoherent contributions to the probability of PP by linearly polarized γ-quanta propagating parallel to the (110) plane of the W crystal.

Now we can write the imaginary components of the principal values of the transverse permittivity tensor:

$$\operatorname{Im}\varepsilon_{\parallel(\perp)} = \frac{W_{\parallel(\perp)}}{\omega} = \frac{W^{\rm coh}_{\parallel(\perp)} + W^{\rm inc}_{\parallel(\perp)}}{\omega}. \qquad (2.14)$$

The latter is diagonalized in a system of coordinates, one axis of which is parallel and another perpendicular to the planes of the crystal. As is known, the difference between $\operatorname{Im}\varepsilon_\parallel$ and $\operatorname{Im}\varepsilon_\perp$ leads to the appearance of linear polarization of an initially unpolarized beam of γ-quanta. Actually, the latter can be represented as a mixture of two beams polarized parallel and perpendicular to the planes. The preferential absorption of the former of them leads to the appearance in the entire beam at the depth L of the linear polarization $P_l = \tanh[(W_\parallel - W_\perp)L/2]$ perpendicular to the crystal planes. By choosing the thickness L, one can make the degree of polarization very high. However, experimentally not only the magnitude of the obtained polarization is important, but also the intensity, which must also be high enough

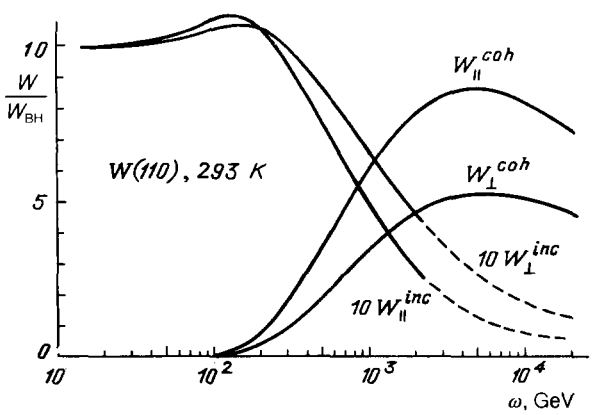

FIG. 4. Energy dependence of the coherent ($W^{\rm coh}_{\parallel,\perp}$) and incoherent ($W^{\rm inc}_{\parallel,\perp}$) contributions expressed in units of $W_{\rm BH}$ to the probability of PP by γ-quanta polarized parallel ($\parallel$) and perpendicular ($\perp$) to the (110) planes of tungsten, along which the γ-quanta are propagating.

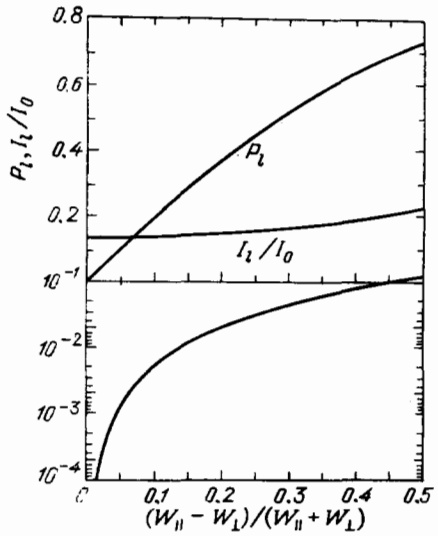

FIG. 5. Dependence on the asymmetry coefficient of PP in (2.15) of the optimal magnitudes of the linear polarization P_l and intensity I_l, together with the corresponding maximum value of the reduced intensity $P_l^2 I_l$ of an initially unpolarized beam passing through a dichroic crystal. The intensity and the reduced intensity are expressed in units of the intensity I_0 of the unpolarized beam. As the ordinate in the lower part of the diagram is plotted $P_l^2 I_l/I_0$.

to enable a reliable observation of the effects that accompany the interaction of the polarized beam. Therefore one usually decides[64] on the efficiency of the process of obtaining the latter from the magnitude of the reduced intensity of the polarized beam $P_l^2 I_l$, where $I_l = I_0[\exp(-W_\parallel L) + \exp(-W_\perp L)]/2$ is the final intensity of the beam obtained from the unpolarized beam of intensity I_0. Upon expressing the length of crystal in terms of the degree of polarization $L(P_l) = \ln[(1+P_l)/(1-P_l)]/(W_\parallel - W_\perp)$, one can represent the reduced intensity as a function of the polarization and the asymmetry coefficient of pair production

$$R = \frac{W_\parallel - W_\perp}{W_\parallel + W_\perp}. \tag{2.15}$$

The condition of maximizing the reduced intensity enables one to represent the optimal values of the polarization $P_l^{\rm opt}$ and the intensity $I_l^{\rm opt}$ as functions of this coefficient.[65] Graphs of them are shown in Fig. 5. The coefficient in (2.15) is very convenient for describing the polarizing properties of crystals, since it reflects both their absorptive properties and the polarization dependence of the probability of PP. The energy dependence of (2.15) for a number of crystals is shown in Fig. 6. It also shows graphs of the optimal thicknesses of the crystals $L_l^{\rm opt} = L(P_l^{\rm opt})$ that allow one to attain the optimal values of the degree of polarization as determined by the magnitude of the asymmetry coefficient. We can easily conclude from the data presented in Figs. 5 and 6 that, for the maximum energy of the CERN beam $\omega = 150$ GeV for a W crystal at $T = 293$ K, we have $r = 0.07$, $P_l^{\rm opt} = 14\%$, $L_l^{\rm opt} = 0.63$ mm, while for $T = 0$ K we have $R = 0.105$, $P_l^{\rm opt} = 20.5\%$, and $L_l^{\rm opt} = 0.53$ mm. In both cases we have $I_l^{\rm opt} \approx 0.14\, I_0$. The energy of the γ-beam of the Tevatron $\omega = 400$ GeV already lies at the boundary of the energy region optimal for using W crystals. For this energy and $T = 293$ K we have $R = 0.187$, $P_l^{\rm opt} = 35\%$, $I_l^{\rm opt} = 0.15\, I_0$, and $L_l^{\rm opt} = 0.28$ mm.

Using Si and Ge crystals at zero angle of incidence of the γ-quanta on the planes becomes more optimal than using W crystals at $\omega \gtrsim 1.05$–1.1 TeV. In the most favorable energy region $\omega = 3$–4 TeV for an Si crystal at $T = 293$ K, we have $R = 0.278$, $P_l^{\rm opt} = 49\%$, $I_l^{\rm opt} = 0.17\, I_0$, and $L_l^{\rm opt} = 1.5$–1.2 cm.

Figure 4 allows us to conclude that, in the energy region of greatest practical importance, $\omega = (1$–$10)\,\omega_{x=1}$, the contributions of the polarization and energy dependence of the probability of incoherent processes of (2.10) to the variation of the numerator and denominator of Eq. (2.15) compensate to a certain degree. This makes possible a good accuracy of the simplified description of incoherent PP processes in this region by the Bethe–Heitler probability. However, the accuracy of this approximation is impaired both by a decrease and by a considerable increase in the energy of the γ-quanta (Fig. 6). Owing to the vanishingly small probability

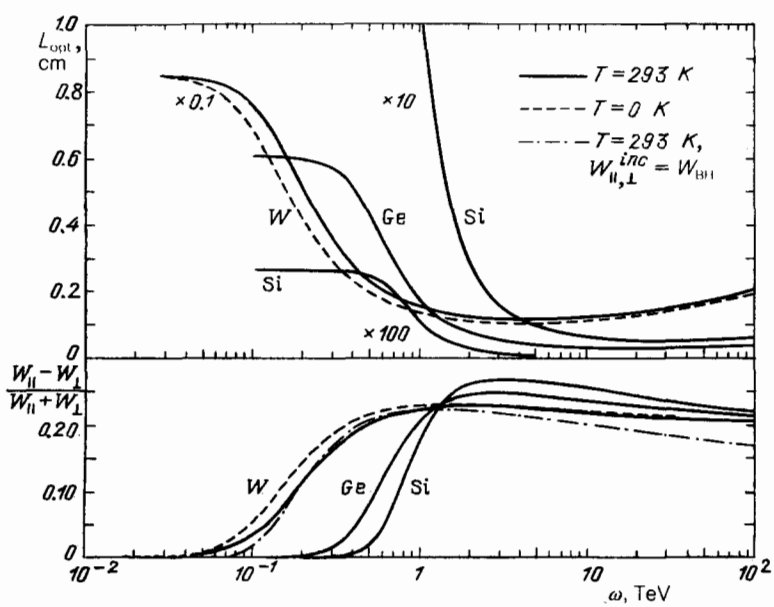

FIG. 6. Energy dependence of the thickness of crystals needed for achieving the optimal degree of polarization (above) (see Fig. 5) and the asymmetry coefficient of PP for the (110) planes of crystals of Si, Ge, and W at $T = 293$ K (below). Dotted curves—for W at $T = 0$ K, dot-dash—also for W, but neglecting the energy and polarization dependence of the incoherent contribution to the probability of PP at $T = 293$ K.

of incoherent PP for $\omega \gg \omega_{\varkappa=1}$,[56] the value of the coefficient in (2.15) approaches from above the limiting value, 0.2, of the analogous coefficient in the case of a homogeneous field for $\varkappa \gg 1$.

Since the theory of PP in a homogeneous field that we have used is applicable[40] for $\varkappa < \alpha^{-3/2} \approx 10^3$, the description given above of the effect of dichroism of crystals is valid up to energies of γ-quanta $\omega \approx 10^3\ \omega_{\varkappa=1}$, which are reached for the (110) plane of Si at 10^3 TeV. This quantity can be additionally increased by one or two orders of magnitude in going to crystals of lighter elements and using high-index crystal planes that have lower values of the averaged field intensity. Thus the dichroism phenomenon can be manifested in crystals at frequencies exceeding the frequencies of visible light by more than fifteen orders of magnitude.

2.3. Synchrotron-type double refraction of γ-quanta in crystals

The possibility of using the permittivity for a homogeneous field in (2.7) for local description of the interaction of γ-quanta with oriented crystals also has enabled the prediction[9,10,38] of the effect of synchrotron-type double refraction of γ-quanta propagating parallel to crystal planes. A quarter-wave plate designed on the basis of this effect enables one to convert linearly polarized beams of γ-quanta with energies of hundreds and thousands of GeV into circularly polarized ones. To describe the effect quantitatively, we shall use an expression[40-45,58] for the refractive index of γ-quanta linearly polarized parallel and perpendicular to the field $\vec{\mathscr{E}} = \mathbf{n}_x \mathscr{E}$:

$$n_{x(y)}(\varkappa) = \frac{1}{2}\operatorname{Re}\varepsilon_{x(y)}$$

$$= 1 - \frac{\alpha}{3\pi}\left(\frac{\mathscr{E}}{\varkappa\mathscr{E}_0}\right)^2 \int_0^{\omega} \frac{\Upsilon'(\xi)}{\xi}\left(\frac{\omega^2}{\varepsilon_+\varepsilon_-} + \frac{1 \mp 3}{2}\right)\frac{d\varepsilon_+}{\omega}. \tag{2.16}$$

When the angles of incidence of γ-quanta on the crystal planes are not too small, $\psi > 0.1$ microradian, at which the probability of PP by a γ-quantum in the field of a single plane is small (see Ref. 59), while its direction of motion intersects a large number of planes, to find the contribution of Eq. (2.16) to the refractive index of γ-quanta, it suffices to average it over the cross section of the crystal:

$$n^{\mathrm{coh}}_{\parallel(\perp)}(\omega) = \int_0^{d_{\mathrm{pl}}} n_{y(x)}(\varkappa(x))\,\frac{dx}{d_{\mathrm{pl}}}. \tag{2.17}$$

We note that one can derive Eqs. (2.16) and (2.17)[40-45] by using the dispersion relation associating the real and imaginary components of the principal values of the permittivity tensors of the field and the crystal. The latter are easily found owing to their connection (2.14) with the PP probabilities of (2.8) and (2.9). Since Eq. (2.9) describes the process of coherent PP, the contribution (2.17) associated with it by the dispersion relation to the refractive index for γ-quanta is also naturally termed coherent. By using the probability (2.10) of incoherent PP processes, we can use the same dispersion relation to find also the contributions of incoherent PP processes to the refractive index for linearly polarized γ-quanta:

$$n^{\mathrm{inc}}_{\parallel(\perp)} = n_{\omega \ll \omega_{\varkappa=1}} - \frac{\sqrt{\pi}}{30 L_{\mathrm{рад}}\omega^4}\int_0^{d_{\mathrm{pl}}} \frac{n(x)}{n_0}\frac{dx}{d_{\mathrm{pl}}} \times \int_0^{\omega} d\varepsilon_+ \{\omega^2[(1\pm1)\xi^1\Phi \mp 6\xi\Phi - (3\pm4)\xi^2\Phi'] + (\varepsilon_+^2 + \varepsilon_-^2)[(1\mp1)\xi^4\Phi + (3\pm6)\xi\Phi - (5\mp4)\xi^2\Phi']\}. \tag{2.18}$$

Here we have $\Phi = \Phi(\xi)$, the magnitudes of L_{rad} and $n(x)$ are determined by Eqs. (2.12) and (2.13), and $n_{\omega \ll \omega_{\varkappa=1}}$ is the value of the refractive index for $\omega \ll \omega_{\varkappa=1}$, which does not depend on the polarization of the γ-quanta and therefore does not contribute to the double-refraction effect. In substituting into the dispersion relation, we neglected in (2.10) the term containing logarithms, as is quite admissible in the most important energy region $\omega \sim \omega_{\varkappa=1}$ (see below). Owing to the polarization dependence of the probability of incoherent PP processes, they make different contributions to the refractive index for linearly polarized γ-quanta

$$n_{\parallel(\perp)} = n^{\mathrm{coh}}_{\parallel(\perp)} + n^{\mathrm{inc}}_{\parallel(\perp)}.$$

The difference between them leads to the existence of the effect of double refraction. The energy dependence of the coherent and incoherent contributions to the refractive index of polarized γ-quanta is shown in Fig. 7 for the case of the W crystal, which enables getting a picture of the maximum relative contribution of the incoherent processes.

A birefringent crystal will play the role of a quarter-wave plate if the phase difference acquired in it of the components of a γ-beam polarized parallel and perpendicular to the planes is $\pi/2$. The thickness $L_{\lambda/4}$ of such a crystal is determined by the known relationship

$$\omega(n_{\parallel} - n_{\perp})L_{\lambda/4} = \frac{\pi}{2}. \tag{2.19}$$

The energy dependence of the thicknesses of quarter-wave plates made of Si, Ge, and W is shown in Fig. 8. In order for a completely linearly polarized beam to be converted upon passing through a crystalline quarter-wave plate into a circularly polarized one, at its exit the intensities of the components of the beam linearly polarized parallel and perpendicu-

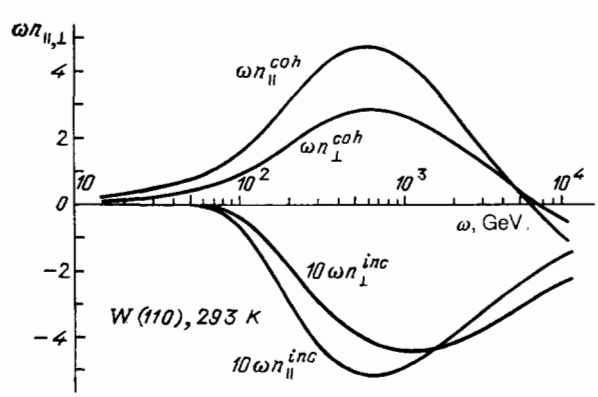

FIG. 7. The coherent $n^{\mathrm{coh}}_{\parallel,\perp}$ and incoherent $n^{\mathrm{inc}}_{\parallel,\perp}$ contributions, multiplied by the energy, to the refractive index of γ-quanta linearly polarized parallel and perpendicular to the (110) planes of a tungsten crystal at $T = 293$ K.

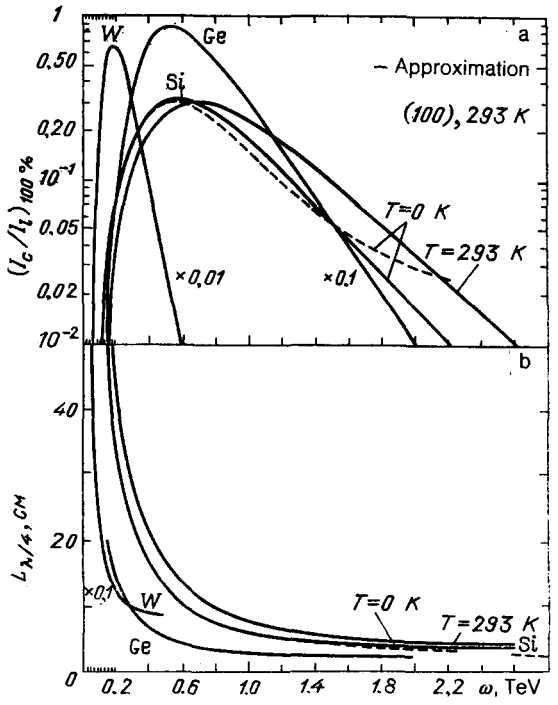

FIG. 8. Energy dependence of the attenuation coefficients of a completely linearly polarized beam in quarter-wave plates (a) and of the lengths of these plates, based on using the birefringent properties of the fields of the (110) planes of Ge and W crystals at $T = 293$ K, and also Si at 0 and 293 K (b). The dotted curves are calculated by using the simplified description of the refraction by Eq. (2.20) and the absorption by the Bethe–Heitler probability.

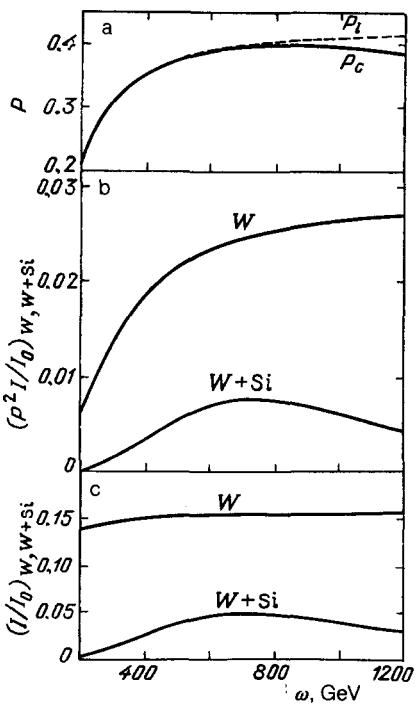

FIG. 9. Energy dependence of the linear polarization P_l acquired by an unpolarized beam of intensity I_0 and a W crystal of optimal thickness (see Fig. 6) and of the circular polarization P_c acquired by this beam upon subsequent passage through a silicon quarter-wave plate (a). The energy dependence of the reduced intensity (b) and the intensity (c) of the beam after passing through the W crystal, $(P^2I/I_0)_W = P_l^2 I_l I_0$, $(I/I_0)_W = I_l/I_0$ (upper curves), and also after its subsequent passage through a silicon quarter-waveplate $(P^2I/I_0)_{W+Si} = P_c^2 I_c/I_0$, $(I/I_0)_{W+Si} = I_c/I_0$ (lower curves).

lar to the planes must be equal. This condition fixes the angle $\varphi = \tan^{-1}\exp[-(W_\parallel - W_\perp)L_{\lambda/4}/2]$ formed by the plane of polarization of the initial beam and the crystal planes (see Fig. 2). In determining the efficiency of a quarter-wave plate, one naturally starts with the magnitude of the attenuation coefficient in it of a completely polarized beam $(I_c/I_l)_{100\%} = 2/[\exp(W_\parallel L_{\lambda/4}) + \exp(W_\perp L_{\lambda/4})]$. We can easily see that, at energies $\omega \sim \omega_{\varkappa=1}$, it has a maximum, whose origin we should discuss. We shall start with the fact that, when $\omega < \omega_{\varkappa=1}$, we have $\varkappa < 1$ and $\xi \gtrsim (4/\varkappa)^{2/3} > 1$ throughout the crystal. Thereby the incoherent contributions to the refractive index in (2.18) prove to be suppressed, while the expression for the coherent contributions is substantially simplified. Actually, since when $\xi > 1$ we have $\Upsilon(\xi) \approx 1/\xi$, (2.16) acquires the form

$$n_{x(y)} = 1 + \frac{\alpha}{180\pi}\left(\frac{\mathscr{E}}{\mathscr{E}_0}\right)^2 (11 \mp 3). \tag{2.20}$$

Upon substituting (2.20) into (2.17) and using the parabolic approximation of the averaged potential, for which $\mathscr{E}(x) = 2\mathscr{E}_{\max} x/d_{\rm pl}$, we obtain from (2.19) $L_{\lambda/4} \approx (45\pi^2/\alpha\omega)(\mathscr{E}_0/\mathscr{E}_{\max})^2$. Further, using the estimate $\mathscr{E}_{\max} \approx \pi Z e n_0 d_{\rm pl}$ and the fact that, when $\omega < \omega_{\varkappa=1}$, $W_\parallel \approx W_\perp \simeq W_{\rm BH} = 7/9\, L_{\rm rad}$ (see Fig. 4), we obtain the following estimate for the attenuation coefficient of a completely polarized beam in the quarter-wave plate (I_l and I_c are the intensities of the linearly and circularly polarized initial and final beams):

$$\left(\frac{I_c}{I_l}\right)_{100\%} \approx \exp(-W_{\rm БГ} L_{\lambda/4}) \approx \exp\left(-\frac{0{,}16Z}{d_{\rm pl}\,(\text{Å})}\,\frac{\omega_{\varkappa=1}}{\omega}\right), \quad \omega < \omega_{\varkappa=1}. \tag{2.21}$$

This confirms the decrease in the losses of γ-quanta as their energy increases up to $\omega \simeq \omega_{\varkappa=1}$. However, when $\omega \gtrsim \omega_{\varkappa=1}$, it is replaced by a sharp increase involving the increased intensity of coherent PP and retardation of the decline, and for $\omega \gg \omega_{\varkappa=1}$, also involving the increase in $L_{\lambda/4}$. Neglecting the contributions of incoherent processes to the absorption and refraction for $\omega \gg \omega_{\varkappa=1}$ (see Figs. 4 and 7) and employing the asymptotic behavior of the permittivity of a homogeneous field for $\varkappa \gg 1$,[40–45] we can easily convince ourselves that the attenuation coefficient of a completely polarized beam in the quarter-wave plate approaches 10^{-7} when $\omega \gg \omega_{\varkappa=1}$ and leaves no hopes of practical use. In practice the linearly polarized γ-beams used for conversion into circularly polarized beams will have only partial polarization (see Figs. 5 and 6). In this case the circular polarization and the attenuation coefficient of the beam will prove to be

$$P_c = \frac{(\sin^2 2\varphi)\,P_l}{1 - P_l\cos^2 2\varphi} < P_l, \quad \frac{I_c}{I_l} = \frac{P_l}{P_c}\left(\frac{I_c}{I_l}\right)_{100\%} > \left(\frac{I_c}{I_l}\right)_{100\%} \tag{2.22}$$

Here P_l is the linear polarization of the initial beam, φ is its angle of inclination (see Fig. 2), and $(I_c/I_l)_{100\%}$ is the attenuation coefficient in the quarter-wave plate for a fully polarized beam (see Fig. 8a). Upon using (2.22) and the results presented in Figs. 5, 6, and 8, we calculated the circular polarization and the attenuation coefficient (I_c/I_0) of a beam that had acquired a linear polarization in a W crystal of optimal thickness and then had passed through a silicon quarter-wave plate. The results show (Fig. 9) that obtaining

circularly polarized γ-beams from unpolarized beams at energies accessible in the Tevatron will be accompanied by 20–30-fold attenuation of the intensity. The degrees of circular and linear polarization (see Figs. 5 and 6) practically do not differ. The maximum values of the intensity of the final beam and of the reduced intensity $P_c^2 I_c/I_0$ that characterize the efficiency of obtaining it are reached at energies $\omega \simeq 0.7$ TeV $\sim \omega_{\varkappa=1}$, which are optimal for using a quarter-wave plate (see Fig. 8a).

The estimate (2.21) enables one to find the parameters of the crystals (Z, d_{pl}) and the corresponding energy regions in which the use of quarter-wave plates is accompanied by an acceptable attenuation of the beam. We can naturally consider these regions to be optimal and determine them by equating the argument of the exponential in (2.21) to unity. We can easily convince ourselves that, when $\omega \lesssim \omega_{\varkappa=1}$, the condition that it should be small can be satisfied only when $Z \lesssim Z^* \approx 6 d_{pl}$ (Å) = 10–20 (the rapid increase in attenuation of the beam when $Z > Z^*$ is illustrated in Fig. 8a). In this case an energy region optimal for using a quarter-wave plate exists for $0.16 Z/d_{pl}$ (Å) $\lesssim \omega/\omega_{\varkappa=1} \lesssim 1$, in which the attenuation of the beam is no greater than severalfold. The condition of smallness of the argument of the exponential in (2.21) enables one to diminish also the interplanar spacing d_{pl} along with the atomic number to a magnitude of the order of 0.15 Z Å. This is attained by going to the corresponding family of planes. Therefore in using crystals of the lighter elements the optimal energies $\omega \sim \omega_{\varkappa=1} \propto 1/\mathscr{E}_{max} \propto 1/Zd_{pl}$ can be increased proportionally to Z^{-2} (rather than to Z^{-1}). Thus, in going from the Si crystal with $Z = 14 \approx Z^*$, e.g., to the LiH crystal ($Z = 1; 3$), by choosing the planes, one can raise the energy $\omega_{\varkappa=1}$ from 1.2 TeV to $\omega_{\varkappa=1} \approx 1.2\,(Z^*/Z)^2 \approx 25$ TeV.

We note that the unpolarized component of a linearly polarized beam acquires the linear polarization $P_l' = \cos 2\varphi$ upon passing through a quarter-wave plate that possesses an appreciably marked dichroism at $\omega \sim \omega_{\varkappa=1}$. In a number of cases, as, e.g., in the conversion of circularly polarized γ-beams into longitudinally polarized $e^{\pm}$ beams (see below), the appearance of this polarization proves inessential. In the converse case we can eliminate it by assembling the quarter-wave plate from several different crystals.

One can easily convert a circularly polarized γ-beam into beams of longitudinally polarized $e^{\pm}$.[65–67] Actually in PP in an amorphous material the degree of longitudinal polarization of $e^{\pm}$ having the energy $\varepsilon_{\pm} = x\omega$ is associated with the degree of polarization of the γ-quanta by the relationship[68]

$$\zeta_{\pm}(\varepsilon_{\pm} = x\omega) = \frac{4x-1}{4x(x-1)+3} P_c, \tag{2.23}$$

We can easily establish from this relationship that, when $\varepsilon_{\pm} \gtrsim 0.7\omega$, the degree of polarization of the γ-quanta is almost completely transferred to $e^{\pm}$. This has the result that longitudinally polarized $e^{\pm}$ can be obtained without any energy selection.[66] Indeed, under conditions of conservation of the rapid decline of the spectrum with increasing energy of the γ-quanta characteristic of secondary beams of proton accelerators[69,70] upon passing through polarizing crystals, the fundamental contribution to the production of $e^{\pm}$ will come from γ-quanta whose energies slightly exceed the energy of $e^{\pm}$. In agreement with (2.23), this ensures effective transfer of polarization.

2.4. Regions of applicability of the Born and the synchrotron-type approximations

The birefringence and dichroism of crystals in the hard γ-range ($\omega \gtrsim 1$ GeV) were first studied[31,32] (see also Refs. 1 and 64) within the framework of a theory of coherent braking PP based on the Born approximation.[1,2,71] Studies in recent years have shown that the Born approximation and the synchrotron-type approximation that we have widely used (homogeneous-field approximation) are the two fundamental limits of the general theory of PP in crystals.[10,46,57] It turned out[72] that the entire set of energies and angles of incidence of γ-quanta on the planes (or axes) at which the crystal exerts a substantial influence on the PP process is naturally divided into two regions; far from their common boundary the PP process maintains the qualitative features described by one of these approximations. To find these regions we shall start with determining the angular width of the region of action of the synchrotron-type approximation. We shall start with the requirement of smallness of the relative change in the averaged field $\mathscr{E}$ that deflects $e^{\pm}$ in the length of pair production of (2.2) $l_f \approx m\varkappa^{1/3}$ (we shall assume that $\varkappa = e\mathscr{E}\omega/m^3 = \omega\mathscr{E}/\mathscr{E}_0 m \gtrsim 1$ and $\varepsilon_{\pm} \approx \omega/2$). For typical values of the field intensity we shall use the estimate $\mathscr{E} \sim V_{max}/\Delta 2\rho_{char}$, where $\Delta\rho_{char} \sim 0.1$–$0.3$ Å is the characteristic spatial scale of the variations of the averaged potential of an axis (or plane) in a direction normal to it. In the motion of $e^{\pm}$ at the angles $\psi > \theta_{\pm}$ (l_f) to the axis (or plane) [see (2.2)], the variation of the field in the distance l_f now determines not the deviation of $e^{\pm}$ from the line, but the translational motion along it. This enables us to consider their trajectories parallel to the momentum of the γ-quantum and to write the condition of applicability of the homogeneous-field approximation in the form of the inequality $l_f\psi \approx m\psi\varkappa^{1/3}\Delta\rho_{char}/V_{max} \ll \Delta\rho_{char}$, which imposes a restriction on the angle of incidence of the γ-quanta on the axis (or plane)[10,73]

$$\psi \ll \tilde{\psi} = \frac{V_{max}}{m\varkappa^{1/3}}. \tag{2.24}$$

When this condition is satisfied the PP process is locally described by the sum of the probability of PP in a homogeneous field in (2.4) and a correction to it proportional to $(l_f\psi/\Delta\rho_{char})^2 = (\psi/\tilde{\psi})^2$, which allows for the inhomogeneity of the field and the nonparallelism of the momentum of the γ-quanta to the crystal planes (or rows).[55,57] Since, in the regions of most intense averaged field $\mathscr{E} \sim \mathscr{E}_{max}$ that make the fundamental contribution to the probability of PP in the crystal, the relative magnitude of the contribution of the correction is comparable with unity for $\psi \approx \tilde{\psi}$, the characteristic features of the process of synchrotron-type PP must be conserved up to these angles.[10,73,74] Let us discuss in this regard the possible influence of channeled motion of the created $e^{\pm}$ on the process of synchrotron-type PP. It is pertinent to recall that the averaged-field model is applicable for describing processes of emission and PP up to angles of incidence $\psi \sim \psi_{max}$ that substantially exceed both ω_c and $\tilde{\psi}$ (see Fig. 10).[7,75,76] Indeed, the averaged potential of the axes (or planes) is obtained from the expansion in a Fourier series of the three-dimensional potential of the crystal averaged over

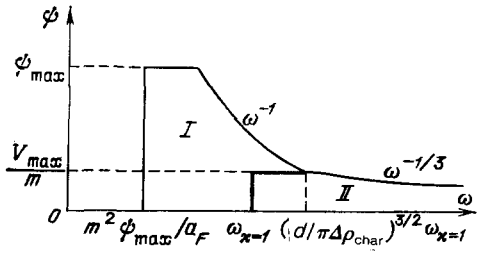

FIG. 10. Regions of energies and angles of incidence of γ-quanta in which the PP process in the first approximation is described by the Born (I) and synchrotron-type (II) limits. In incidence on an axis we have $\psi_{max} = a_F/d_0$, and on a plane, $\psi_{max} = (a_F/d_0)^2$, where d_0 is the characteristic interatomic distance, and a_F is the screening radius of the atom. See also Eq. (2.25).

the thermal oscillations by dropping all terms of the series that oscillate upon displacement along the axis (or plane). These terms describe the discrete transfer of momentum parallel to the axes (or planes), and which have a minimum value, e.g., in the axis case, of $2\pi/d_{ax}$. However, such a large momentum transfer is impossible in the motion of the particles at sufficiently small angles ψ to the axis (or plane). This justifies dropping the oscillating terms in the expansion of the potential. Actually in the studied energy region the magnitude of $2\pi/d_{ax}$ substantially exceeds the characteristic momentum $q_{\parallel}$ transferred to the crystal in the direction of motion of the γ-quantum that gives rise to the pair ($q_{\parallel} \sim m^2\omega/\varepsilon_+\varepsilon_-$) or $e^{\pm}$ that emit a γ-quantum ($q_{\parallel} \sim m^2\omega/\varepsilon\varepsilon'$). Yet, upon comparing the projection of the transverse momentum, which is equal to $q_z = \psi q_{\perp}$, that can be transferred to the axis with $2\pi/d_{ax}$, we convince ourselves that the momentum transfer along the rows in emission and PP processes will be suppressed, at least at angles $\psi < 2\pi/md_{ax} \approx 10$ milliradians. Actually the coherent emission and PP processes at large enough angles ψ are accompanied by transfers of transverse momentum appreciably smaller than m. Therefore the region of applicability of the averaged-potential model is characterized by even somewhat larger angles ψ_{max} (see Fig. 10). Without fearing for the applicability of the averaged-potential model, we can easily convince ourselves that, when $\varepsilon_{\pm} = \omega/2$, $\tilde{\omega}/\theta_c \simeq 0.5\varkappa^{1/6}$ $(m\Delta\rho_{char})^{1/2} \approx (2\text{–}5)$ $(\omega/\omega_{\varkappa=1})^{1/6} > 1$. That is, the process of synchrotron-type PP must be manifested over a broader angular range than the channeling of created $e^{\pm}$. The lack of a firm connection between these processes is explained by the smallness of the pair-production length in (2.2) as compared with the length of the characteristic period of the transverse oscillations of channelled $e^{\pm}$.[55] In this regard we recall that initially in Refs. 12 and 14 the synchrotron-type mechanism of emission and PP was associated with the effect of channeling of $e^{\pm}$. Moreover, at a certain stage of the studies it seemed to the authors of Ref. 27 that this connection had been experimentally confirmed. However, soon the error of the results of Ref. 27 was pointed out in Refs. 55, 57, and 77, whose predictions were quantitatively confirmed by subsequent experiments[28,29] (see Fig. 11).

Now let us establish the region in which the features of the process of coherent braking PP are conserved. The influence of the crystal lattice is manifested most substantially in the Born approximation when the condition is fulfilled of

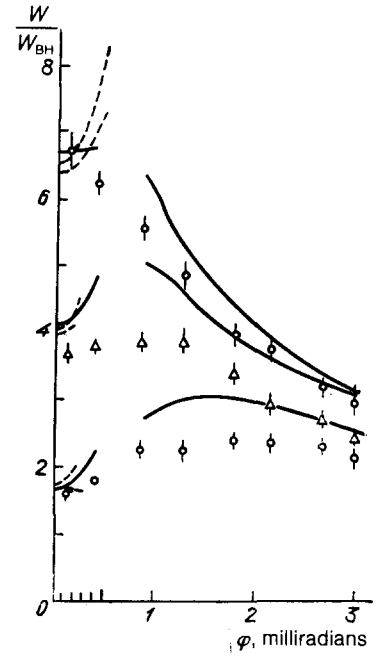

FIG. 11. Orientation dependence expressed in units of $W_{BH} \approx 0.33\ \text{cm}^{-1}$ of the probability of PP by γ-quanta in the field of ⟨110⟩ axes of Ge at $T = 100$ K. The experimental data[29] are averaged over the indicated energy intervals. The calculations were performed for energies $\omega = 47$, 90, and 138 GeV. The solid curves are taken from Ref. 29. The dotted curves calculated by the equations of Ref. 55 and 57 limit pairwise the regions of possible course of the orientation dependences of the probability of PP under the experimental conditions of Ref. 29. The lower open circles—for 40–50 GeV, the triangles—for 70–100 GeV, and the upper open circles—for 125–150 GeV.

constructive interference of the amplitudes of the processes initiated by different planes (or axes). In the case of PP this condition acquires the form[1,2] $m^2\omega d/2\varepsilon_+\varepsilon_-\psi_j = 2\pi j$ ($j = 1,2,\ldots$) or, when $\varepsilon_{\pm} = \omega/2$, or $\psi_j = m^2 d/\pi\omega j$, where d is the distance between the planes of axes forming the plane parallel to the momentum of the γ-quanta. The deflection of $e^{\pm}$ by the averaged field that is not taken into account in the Born approximation will substantially influence the PP process when $\psi < \tilde{\psi}$.[72] Therefore, to observe even one coherent peak, the condition $\psi_{j=1} > \tilde{\psi}$ must be fulfilled, or

$$\frac{\omega}{\omega_{\varkappa=1}} \lesssim \left(\frac{d}{\pi\Delta\rho_{xap}}\right)^{3/2}. \tag{2.25}$$

In the axis case the right-hand side of (2.25) can be as much as ten to twenty. In the planar case it can be severalfold. Thus the characteristic features of the process of coherent braking PP[1,2] can be manifested only at energies that do not exceed too substantially the energy of the pseudothreshold of the process of synchrotron-type PP, $\omega_{\varkappa=1}$. The regions of applicability of the Born and synchrotron-type approximations for describing the PP process and the effects of dichroism and double refraction associated with it are shown schematically in Fig. 10. We can easily see that the narrowing of the interval of angles of manifestation of these effects with increasing energy of γ-quanta is substantially retarded in going from the region of applicability of the Born approximation to that of the synchrotron-type approximation. This relaxes the requirements on the quality of the crystals and the accuracy of relative orientation of their planes and direc-

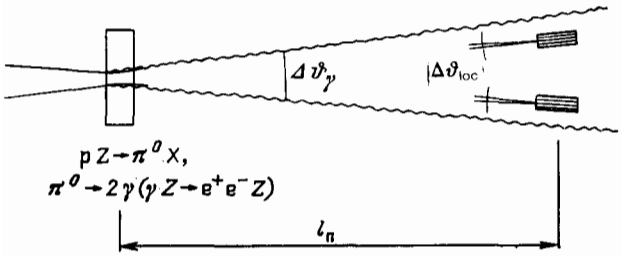

FIG. 12. Emergence of secondary γ-quanta (or $e^\pm$) from the site of production leads to a decrease in their local divergence $\Delta\vartheta_{loc}$. This enables one to ensure incidence of $e^\pm$ or γ at the required angles on the crystal planes as axes.

tions of incidence of γ-quanta in obtaining linearly polarized beams.

Although synchrotron-type dichroism and double refraction are manifested at angles of incidence of the γ-quanta on the plane $\psi < \tilde{\psi}$ and can be described by the method of Ref. 55, the existence of an orientational dependence of the polarizing properties of the crystal unavoidably affects the efficiency of conversion of a γ-beam having the divergence $\Delta\psi \sim \tilde{\psi}$. This can be avoided by narrowing the interval of angles of incidence of the γ-quanta on the plane. Since decreasing the divergence of beams by collimation is accompanied by a considerable weakening of their intensity, let us turn our attention in this regard to the possible use of selection of the γ-quanta and $e^\pm$ in direction of their momentum as they propagate from the production site (Fig. 12). We recall that the methodology of obtaining secondary $e^\pm$ or γ-beams in proton accelerators is based on the idea of Markov[69,70] of using γ-quanta from the decay of π^0 mesons formed upon collision of protons with nuclei. Upon decaying, they give rise to γ-beams with a broad spectrum and divergence $\Delta\vartheta_\gamma \gtrsim \vartheta_{\pi^0} = m_{\pi^0}/\omega$, where $m_{\pi^0} = 135$ MeV is the mass of the π^0 meson, and ω is the typical energy of the γ-quanta. By subsequent conversion of the γ-quanta in a material, $e^\pm$ are obtained, which can be used to obtain beams of "tagged" γ-quanta having a narrow spectrum. In moving away from the site of production of $e^\pm$ or γ, the local angular divergence of their beams at each point, in contrast to the divergence of the beams as a whole, declines in inverse proportion to the distance l_n (see Fig. 12). Very simple estimates show that, in the energy region $\varepsilon_\pm$, $\omega \sim 1$ TeV, the local divergence of the $e^\pm$ or γ-beams is decreased, say, to values $\Delta\theta_{loc} \sim \theta_c$ and $\Delta\theta_{loc} \sim 0.1\tilde{\psi}$, which enable correspondingly the entry of an appreciable fraction of the $e^\pm$ into a channeling regime (see Sec. 3.1) and a one-percent accuracy of the homogeneous-field approximation, even when the crystal is moved tens of meters away. The transverse dimensions of the beams here reach several centimeters. Here, since the mean local direction of motion of $e^\pm$ or γ varies over the cross section of the beam (see Fig. 12), to make it agree in direction at every point with the direction of the crystal planes (or rows) requires using a set of differently oriented crystals or appropriately bent single crystals.

In closing the discussion of the problems involving the orientational dependence of the process of synchrotron-type PP, we call attention to the closely associated effect of anisotropy of absorption of γ-quanta manifested at angles of incidence $\psi \lesssim 10$ microradian on the crystal axes.[59]

3. EFFECTS WITH PARTICIPATION OF POLARIZED $e^\pm$

The elucidation of the synchrotron-type nature of the interaction of $e^\pm$ and γ with oriented crystals also led to the prediction of the effects of radiative self-polarization of $e^\pm$,[33-35] spin rotation,[33,37] dependence of the anomalous magnetic moment of $e^\pm$ on the intensity of the intracrystalline electric field,[38,39] and the formation by γ-quanta of transversely polarized $e^\pm$.[11,36] These effects enable one to obtain beams of transversely polarized $e^\pm$, measure their polarization, rotate the direction of the latter, and to determine from the angle of rotation the anomalous magnetic moments of various particles, while determining in the case of $e^\pm$ also their dependence of the energy and the intensity of the averaged fields of the crystals. In contrast to synchrotron-type dichroism and double refraction, the stated effects have no analogs in the theory of coherent braking radiation and PP,[1,2] and are manifested only when $\psi < \theta_c$ in region II in Fig. 10.

As we have already noted, all the listed effects are described locally by the electrodynamics of a homogeneous field. The possibility in principle of existence in a homogeneous transverse electric field of effects involving the spin of $e^\pm$ is indicated by the existence of the pseudovector

$$\mathbf{v} = [\vec{\mathscr{E}}\mathbf{v}], \tag{3.1}$$

where $\mathbf{v}$ is the velocity of $e^\pm$. We shall describe the polarization states of $e^\pm$ by the classical spin vector $\boldsymbol{\zeta}$, which is equal to twice the mean value of the spin operator in the rest system of $e^\pm$. The dependence of the probabilities of emission and PP on the P-even product $\mathbf{v}\boldsymbol{\zeta}$ actually leads to the appearance of a transverse polarization of $e^\pm$ in emission or pair production in a homogeneous field.[40,43-45] Moreover, in the motion in the electric field the mean spin vector precesses about the vector of (3.1).[48] However, this still does not suffice for the appearance of the analogous polarization phenomena in crystals. The problem is that the electric field, and hence also the vector of (3.1), which determines the orientation of the polarization or the axis of its rotation, has opposite directions on the different sides of the crystal plane (or axis). Therefore the mean polarization of all the $e^\pm$ emitting and being produced in the crystal, just like the angle of rotation of the vector $\boldsymbol{\zeta}$, will be practically equal to zero. One can obtain polarized $e^\pm$ or achieve a rotation of the vector $\boldsymbol{\zeta}$ only by separating out the $e^\pm$ that are moving or being created in regions having a predominant direction of the field and of the vector of (3.1). In the case of thin crystals such a separation can be performed by selecting $e^\pm$ based on the direction of their emergence from the crystal.[11,35] The problem of separating the $e^\pm$ that have been created and/or were moving in regions having a predominant direction of the electric field in the case of thicker crystals can be solved by bending them. Actually, in the motion in a bent crystal and trajectories of channelled $e^\pm$ are displaced from the symmetrical positions in the channels, whereby the e^+ (or e^-) move for most of the time in an averaged field of the planes directed inward (or outward) with respect to the circle formed by the bent planes (Fig. 13).

Since the increase in permissible thickness of the crystals that becomes possible upon bending enables one to increase the probabilities of radiative processes and the angles of spin rotation, the use of bent crystals offers the broadest

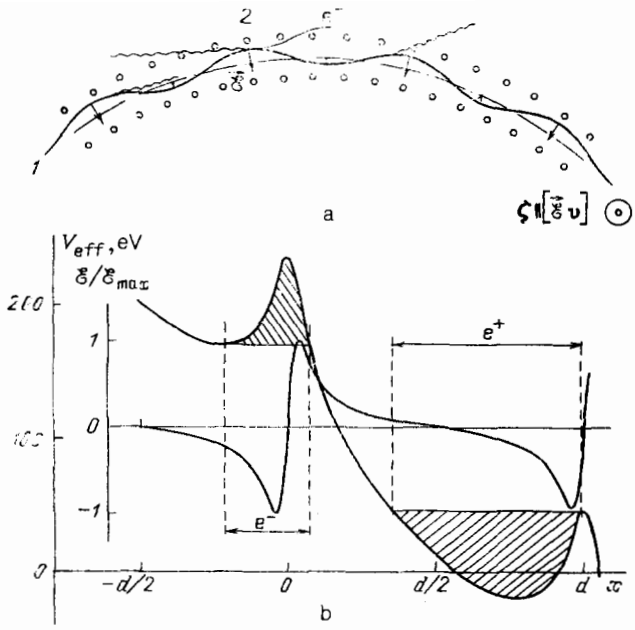

FIG. 13. In bent crystals the channelled e^+ (or e^-) move in regions have an electric field directed mainly or completely towards the inside (or outside) of the circle formed by the bent planes (a). This leads to appearance of polarization of the e^+ during emission of (*1*) or creation of (*2*) γ-quanta. The motion of e^+ in the bent crystal is described by the effective potential shown for the (110) plane of W at $T = 293$ K and $R = 5R_{min}$ (b). In this case, in the regions of bound motion of e^+ demarked with cross hatching and vertical lines, a field predominates that is directed towards the inside of the circle formed by the bent planes. At the same time the regions of bound motion of e^- contain fields in opposite directions that hinder obtaining highly polarized channelled e^-

potentialities for affecting the polarization of particles, to which this section is devoted. We note that the description of polarization phenomena involving e^+ requires knowing the dynamics of their motion under condition of intense synchrotron-type emission and multiple scattering. In contrast to the case of γ-quanta, its description is a complex and not finally solved problem, whose theoretical and experimental studies are being conducted with growing activity.[20-26,78-83] Nevertheless the fundamental characteristics of polarization phenomena involving e^+ can be established even within the framework of a rather simple analysis.

3.1. Radiative self-polarization of $e^\pm$ in the intense fields of crystals

One can naturally conduct the study of self-polarization in crystals in comparison with the known effects of self-polarization of e^+ in storage rings.[43,44,84] In describing the essence of this effect, we can consider the magnetic field **H** of the storage ring to be homogeneous and transverse. The component of the polarization of the $e^\pm$ beams parallel to it is determined by the distribution of $e^\pm$ between the polarization states $\zeta = 1$ and $\zeta = -1$ ("with" and "against" the field). An initially unpolarized beam will become polarized owing to the difference between the probabilities $w_{\uparrow\downarrow}$ and $w_{\downarrow\uparrow}$ of emission of a γ-quantum with reversal of the spin of $e^\pm$, which exist correspondingly in the states $\zeta = 1$ and $\zeta = -1$. The redistribution of $e^\pm$ between the polarization states caused by this difference occurs against the background of intense synchrotron-type radiation, to characterize which one usually employs the parameter $\chi = eH\varepsilon/m^2 \approx 0.444\times 10^{-7}\, H\,(\text{kOe})\,\varepsilon(\text{GeV})$. For typical energies of the emitted quanta, this allows us to write the estimate $\omega \sim \omega_{char} \approx \varepsilon\chi(1+\chi)$. At the characteristic values $H = 20$ kOe, $\varepsilon = 1$ GeV, we have $\chi \approx 10^{-6}$ and $\omega_{char} \approx 1$ keV. When $\chi \ll 1$, the estimates[43,44,84] hold that $w_{\uparrow\downarrow} \sim w_{\downarrow\uparrow} \sim \chi^2 w$, where w is the total emission probability, which weakly depends on the polarization of $e^\pm$. Since $w \sim I/\omega_{char}$, where I is the intensity of synchrotron-type emission, we obtain an estimate for the characteristic self-polarization time $\tau = 1/|w_{\uparrow\downarrow} - w_{\downarrow\uparrow}| \sim 1/w_{\uparrow\downarrow}$ of $\tau \sim 1/w\chi^2 \sim \varepsilon_\pm/I\chi$. This allows us to convince ourselves that the energy losses of $e^\pm$ during the self-polarization time amount to $I\tau \sim \varepsilon_\pm/\chi$. That is, they exceed the energy of $e^\pm$ by a factor of 10^6. These losses are continually replenished by the high-frequency accelerating field. The characteristic variation in the energy of $e^\pm$ is determined here by the number of synchrotron-type quanta[85] $N_\gamma \approx 2\pi\alpha\gamma \sim 100$ emitted by $e^\pm$ in one revolution in the storage ring. The relative magnitude of this variation $N_\gamma\omega_{char}/\varepsilon_\pm \approx 2\pi\alpha\gamma\chi \sim 10^{-4}$ allows us to assume that the process of self-polarization in a storage ring occurs at practically constant energy. It remains to add that the limiting value of the polarization amounts to $|w_{\uparrow\downarrow} - w_{\downarrow\uparrow}|/(w_{\uparrow\downarrow} + w_{\downarrow\uparrow}) \approx 0.924$ and is directed along the field for e^+ and against it for e^-. The self-polarization time τ usually is as much as several tens of minutes.

The effect of radiative self-polarization in storage rings long ago enabled obtaining polarized e^+ beams with energies reaching tens of GeV. However, owing to the rapid increase in the intensity of synchrotron-type radiation, the construction of $e^\pm$ storage rings at energies reaching several hundred GeV and more was acknowledged to be inexpedient. However, the production of polarized e^+ of such high energies will be possible in proton accelerators using polarization phenomena in crystals. The first of them predicted was the effect of radiative self-polarization.[33-35]

Despite the fact that $e^\pm$ in a crystal move in an electric, rather than a magnetic field, the reason for the existence in it of the self-polarization effect is also the dependence of the emission probability on the projection of the spin vector of $e^\pm$ on the direction of the pseudovector, whose role in this case is played by the vector product of (3.1), which determines the direction of possible polarization of e^+. Similarly to the storage-ring case, the character of the self-polarization process in crystals is also determined by the properties of the emission process in a homogeneous field, which substantially depends on the magnitude of the parameter $\chi = e\mathscr{E}\varepsilon/m^3$, which is the analog of the parameter $eH\varepsilon/m^3$ in the magnetic-field case and is equal to the ratio of the electric field intensity $\mathscr{E}' = \gamma\mathscr{E}$ in the intrinsic system of e^+ to the critical field intensity $\mathscr{E}_0 = m^2/e = 1.32\times 10^{16}$ V/cm. At the same time, the process itself of radiative self-polarization in crystals substantially differs from the analogous process in storage rings. This involves the lack of a real possibility of replenishment of the energy losses. Here, even if they emit all their energy, the $e^\pm$ acquire a polarization that does not substantially exceed the magnitude of the parameter χ (Fig. 14a). Under these conditions one can obtain an appreciable polarization only when $\chi \gtrsim 1$,[11,35] at which value the characteristic energies of the synchrotron-type quanta $\omega_{char} \sim \varepsilon_\pm\chi/(2+\chi)$ become comparable with the energies of $e^\pm$. This implies that the recoil effect is essential in the emission; it is closely associated with spin effects,

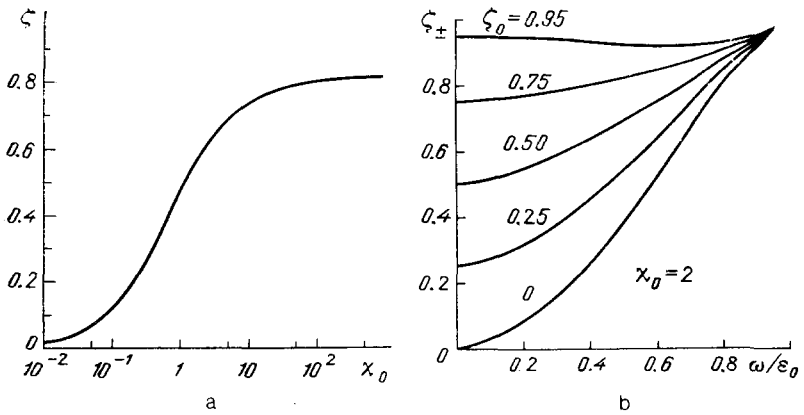

FIG. 14. Dependence on the initial magnitude of the parameter $\chi_0 = \varepsilon_0 \mathscr{E}/\mathscr{E}_0 m$ of the maximum polarization acquired by e^+ upon total loss of energy to radiation in a homogeneous field (a). Dependence of the magnitude of the final polarization $\zeta_\pm$ on the energy of the emitted γ-quantum for $\chi_0 = 2$ (b). Here ζ_0 is the magnitude of the initial polarization of e^+.

which are expressed in the fact that the probabilities of emission with spin reversal $w_{\uparrow\downarrow}$, $w_{\downarrow\uparrow}$, as well as their difference, become comparable with the total emission probability w. Consequently the emission of one quantum of energy $\omega \sim \omega_{\text{char}} \sim \varepsilon_\pm$ leads to an appreciable change in the polarization of $e^\pm$ (Fig. 14b). Thus, when $\chi \gtrsim 1$, e^+ can acquire a polarization that reaches several tens of percent without replenishment of the energy losses, which are comparable in magnitude with the energy of the e^+ themselves. Even in the case of a homogeneous field, the description of the process of self-polarization of e^+ for $\chi \gtrsim 1$ is rather complicated. The point is that the considerable magnitudes of the energies of the emitted γ-quanta lead to a large width of the energy distribution of the e^+ and a nontrivial distribution in space.[86] Without interesting ourselves in the latter, we have modeled the processes of energy losses and variation in the polarization of e^+ by the Monte–Carlo method with an initial value of the parameter $\chi = 2$. The results of the modeling (Fig. 15) demonstrate the considerable width of the spectrum and the high efficiency of the process of self-polarization of e^+ lying in the left-hand half of the spectrum owing to considerable energy losses. In addition, Fig. 15 illustrates a polarization in the opposite direction for e^+ that have undergone relatively small energy losses, and have remained in the right-hand, hard half of the energy spectrum. The appearance of this polarization does not involve the difference between the probabilities $w_{\uparrow\downarrow}$ and $w_{\downarrow\uparrow}$ of emission with spin reversal, but the difference, which becomes appreciable when $\chi \gtrsim 1$, between the probabilities $w_\uparrow$ and $w_\downarrow$ of emission from e^+ initially polarized with and against the vector of (3.1). The character of the spin dependence of these probabilities leads to the appearance for weakly emitting $e^\pm$ of a polarization in the opposite direction. Although the polarization of weakly emitting $e^\pm$ increases rather slowly (see Fig. 15), the small energy losses can make it appropriate to use them in experiments that require polarized $^\pm$ of the highest energies.

A systematic description of the process of self-polarization in crystals must be based on an understanding of the dynamics of motion of channelled $e^\pm$ accompanied by quantum synchrotron-type radiation. The corresponding experimental studies are being conducted as yet only in the axis case.[20–26] In the case of planes they require energies $\varepsilon_\pm \gtrsim \varepsilon_{\chi=1}$, which attain hundreds of GeV, which are accessible only in the Tevatron, where studies of the interaction of $e^\pm$ with crystals are not yet being conducted. Moreover, even in the region of not so high energies, the features of channeling of negative particles in bent crystals are not understood.[87] Under these conditions we must restrict the treatment of estimating the possibilities of radiative self-polarization in bent crystals based on the results obtained above for the case of a homogeneous field. Owing to the fact that the fundamental contribution, both to the energy losses and to the change in polarization of e^+, comes from the regions of most intense field $\mathscr{E} \sim \mathscr{E}_{\max}$, the fundamental characteristics of the process of self-polarization in a crystal will not differ strongly from its characteristics in a homogeneous field of intensity $\mathscr{E}_{\max}$ if the most intense field is only one of the opposite directions predominates in the regions of bound motion of e^+ (see Fig. 13). Since the maximum degree of polarization in the homogeneous-field case attains appreciable values when $\chi \gtrsim 1$ (see Fig. 14a), the process of radiative self-polarization in crystals will be effective at energies $\varepsilon_\pm \gtrsim \varepsilon_{\chi=1}$ (see Table I), for which we have $\chi \gtrsim 1$ in

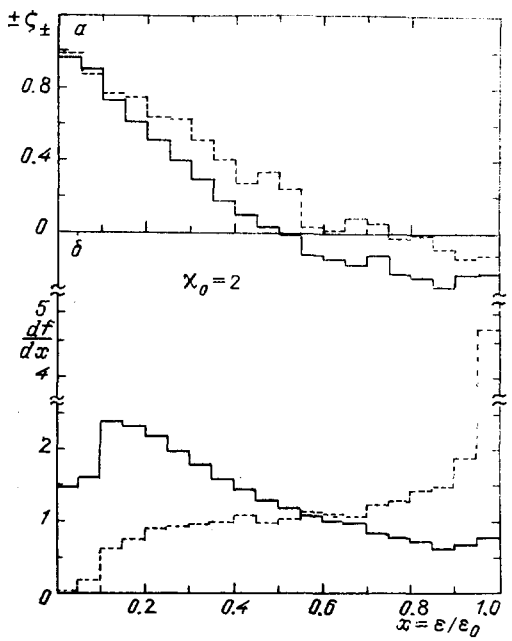

FIG. 15. The polarization (a) and energy distribution function (b) obtained by Monte-Carlo modeling of $e^\pm$ that had an initial value of the parameter $\chi_0 = 2$ and had passed through a path of length $L = 7L_0$ (solid lines) and $L = 3L_0$ (dotted lines) in a homogeneous field of intensity $\mathscr{E}$, where $L_0 = m/\alpha e\mathscr{E}$ [cf. (3.3)].

the regions of maximum field. We recall that the motion of charged particles in a bent crystal is described by the effective potential (see Fig. 13), which possesses local minima that make possible a channelled motion only at radii of bending $R > R_{\min}$, where

$$R_{\min} = \frac{p_{\pm} v_{\pm}}{e\mathscr{E}_{\max}} \approx \frac{\varepsilon_{\pm}}{e\mathscr{E}_{\max}}, \tag{3.2}$$

or $R_{\min}$ (cm) $\approx \varepsilon_{\pm}$ (GeV)/$\mathscr{E}_{\max}$ (GV/cm). One can always, as R approaches $R_{\min}$, gain the predominance, in the regions of bound transverse motion, of an intense field in a certain direction, that is necessary for all the polarization effects discussed in this section.

The optimal length of crystal used to impart polarization to $e^{\pm}$ need not differ greatly, from the characteristic length of their energy losses, whose magnitude we can easily estimate (see below). Actually, for a substantially greater distance the energy losses of e^{+} constitute a very significant amount and do not allow using the fundamental advantage of secondary e^{+} beams of proton accelerators over e^{+} beams of storage rings—their higher energy. At the same time, the probability of emission will be large over the characteristic length of the radiation losses, at least, of one rather hard synchrotron-type γ-quantum, which suffices for the e^{+} to acquire an appreciable polarization. To estimate the characteristic length of the energy losses, we shall use the fact that, in the region $1 \lesssim \chi \lesssim 10$ that will be accessible in the forthcoming decades for using planes, the intensity of synchrotron-type emission is described by the formula[88] $I = d\varepsilon/dz \approx 0.13\alpha e\mathscr{E}/m$. Upon averaging it over the trajectories of the channelled e^{+}, we obtain the following estimate for the optimal thickness of crystal to be used for radiative self-polarization (cf. Fig. 15):

$$l_{\text{РСП}} \approx \frac{15m}{\alpha e\mathscr{E}_{\max}} \tag{3.3}$$

or $l_{\text{RSP}} \approx 1/\mathscr{E}_{\max}$ (GV/cm), whence, using the data of Table I, we have $l_{\text{RSP}} \approx 0.2$–$2$ mm.

The entry of e^{+} into a channeling regime necessary for self-polarization can be made possible by local separation of the e^{+} in terms of direction of motion upon emerging from the site of generation of γ-quanta (see Fig. 12). Here the converter of the latter into e^{+} can lie at any site of the path of the γ-quanta. The entry into a channeling regime of an appreciable fraction of the e^{+} with energies of hundreds of GeV is achieved with distances of flight of several tens of meters. Here the transverse dimensions of the beams will reach several centimeters. To use all the e^{+} that are formed, the transverse dimensions of the polarizing crystals must be as large as the above values, substantially exceeding their thickness in (3.3). In this regard we recall that the problem of obtaining a bent crystal of large transverse cross section is solved by dividing it into thin enough plates. We note also that the inconveniences of working with crystals of thickness $l \sim l_{\text{RSP}} \sim 0.1$–$1$ mm can be avoided by growing them together with an amorphous material having a small atomic number and thickness along the beam of the order of a centimeter.

The intensity of polarized $e^{\pm}$ beams obtained by self-polarization in bent crystals is primarily restricted by the efficiency of capture of $e^{\pm}$ into a channeling regime. In the regions of channelled motion of e^{-}, the most intense fields in the opposite directions are separated by the very small distance ~ 0.15–0.20 Å (see Fig. 13). Under these conditions, to create the substantial predominance of the field in a certain direction in the regions of bound motion of e^{-} that is necessary for effective self-polarization (in the case of e^{-} it is parallel to the outer normal to the bent planes), the bending radius of the crystal should be close to (3.2). Along with the narrowness of the potential well, this has the result that only a small fraction of the e^{-} will enter a regime of bound transverse motion upon entering the crystal.

The width of the potential well describing the bound motion of e^{+} is appreciably larger—of the order of 1–2 Å (see Fig. 13). Even when R appreciably exceeds $R_{\min}$, this enables one to gain a predominance, sufficient for effective self-polarization of e^{+}, in the regions of their bound motion of the electric field directed toward the axis of bending of the crystal. Along with the appreciable width of the potential well, this opens up the possibility of entering a regime of bound transverse motion of several tens of percent of the e^{+} incident on the crystal.

Another factor that can influence the intensity of the polarized beams is multiple scattering of the e^{+}. At the energies and crystal thicknesses of interest to us, it is not risky for most of the e^{+}. At the same time, very simple estimates of the dechanneling length of e^{-} show that it can prove substantial for all bound e^{-}, in particular, for $R \approx R_{\min}$. The insufficient level of study of the process of dechanneling of negative particles in bent crystals does not allow us to treat this problem quantitatively at present. However, we should note that the quantum synchrotron-type emission of e^{+} in the fields of planes (or axes) can oppose the increase of the transverse energy of e^{+} in scattering.[35,79–81]

We note also that the coupling of the rotation of the momentum of the particles with the rotation of their spin (see Sec. 3.3) leads to depolarization of e^{+} beams. However, the relative decrease in the magnitude of radiative polarization in the characteristic length of (3.3) that it causes does not exceed 10^{-4}–10^{-5} (see Sec. 3.3).

In summarizing the treatment of the process of radiative self-polarization of e^{+} in bent crystals, we can state that a considerable fraction of an e^{+} beam (of the order of tens of percent), upon being captured into a channeling regime, will prove to be drawn into the process of radiative self-polarization. Here most of the e^{+} can receive a polarization $\gtrsim 50\%$ while decreasing in energy by severalfold. A smaller fraction of the e^{+} ($\lesssim 10\%$) will acquire a polarization of 10–20% in the opposite direction without substantial decrease in energy. A reliable estimate of the efficiency of the process of radiative self-polarization of e^{-} beams requires supplementary studies of the dynamics of motion of e^{-} with account taken of multiple scattering and radiation cooling. However, even in the last case the yield of polarized e^{-} will be, at a minimum, severalfold lower than the yield of polarized e^{+}.

3.2. Creation of γ-quanta by transversely polarized $e^{\pm}$

We recall that secondary $e^{\pm}$ are obtained in proton accelerators by conversion of γ-quanta in an amorphous material.[69,70] These $e^{\pm}$ possess no polarization. Yet the appearance of the latter during the process discussed in Sec. 3.1 of self-polarization in crystals is accompanied by decrease in the energy and intensity of the $e^{\pm}$ beams. The possibility of

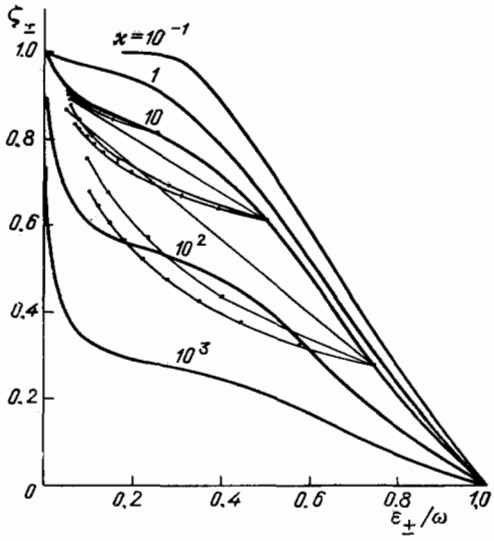

FIG. 16. Dependence of the magnitude of the polarization of $e^\pm$ on the fraction of the energy of the γ-quantum generating them that they bear away, as calculated for different values of the parameter $\varkappa = \omega\mathscr{E}/\mathscr{E}_0 m$. For the case $\varkappa = 10$ the thinner lines indicate the evolution of the energy and polarization of $e^\pm$ upon emission by them after creation of sequences of γ-quanta of characteristic energies.

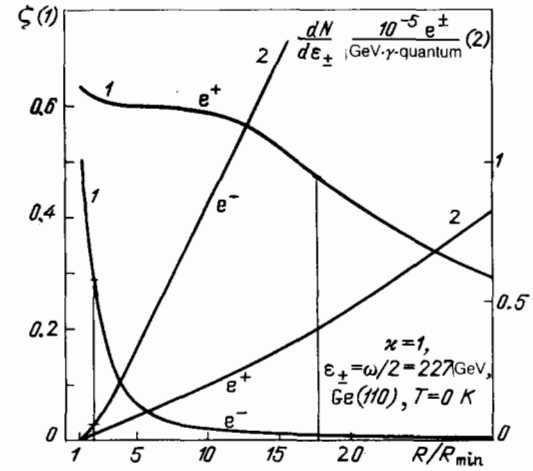

FIG. 17. Dependence on the bending radius of a Ge crystal [(110) plane, $T = 0$ K] of the differential number and degree of polarization of $e^\pm$ created with energy $\varepsilon_\pm = \omega/2 = \omega_{\varkappa=1}/2 = 227$ GeV. The vertical lines mark the radii at which the maxima are reached in the reduced differential numbers $\zeta_\pm{}^2 dN/d\varepsilon_\pm$ of e^+ and e^-.

acquiring polarization by an electron (or a positron) at the instant of pair production by a γ-quanta would enable not only merging the stages of producing $e^\pm$ and polarizing them, but also avoiding the considerable energy losses in the latter stage. Such a possibility is realized upon using the process of synchrotron-type PP in thin[11] or thicker bent crystals.[36]

The existence in a homogeneous electric field of the pseudovector of (3.1) indicates the possible appearance upon $e^\pm$ production of a polarization whose considerable magnitude is confirmed by an elementary calculation[36] performed for different values of the parameter $\varkappa = \omega\mathscr{E}/\mathscr{E}_0 m$ (Fig. 16). Here the polarization of e^+ (or e^-) is parallel (or antiparallel) to the vector of (3.1). As we have already noted, bending the crystal leads to appearance of a predominant strong electric field directed along the outer (or inner) normal to the bent planes in the region of motion of the bound e^- (or e^+). Owing to the local nature of the process of synchrotron-type PP, the regions of bound motion coincide with the regions of formation by γ-quanta of the bound $e^\pm$, which will be created in the polarized state owing to the predominance in these regions of electric fields in certain directions. The procedure of calculating the probabilities of creation and polarization of channelled $e^\pm$ reduces to averaging the local probability of formation of polarized $e^\pm$ over the regions of creation of bound $e^\pm$. It is described in detail in Ref. 36, where graphs are presented of the number and degree of polarization of the $e^\pm$ created in a W crystal over a broad range of energies of γ-quanta. Figure 17 presents analogous graphs for the (110) plane of Ge at $T = 0$ K and the energy $\omega = \omega_{\varkappa=1} = 454$ GeV, which enables one to formulate the corresponding experiment with the γ-beam of the Tevatron. Let us take up some characteristic features of the process of formation of bound $e^\pm$ that are manifested in these graphs. The considerably slower decline in the polarization of positrons with increasing radius of bending of the crystal involves the predominance in the regions of their bound motion of an electric field directed toward the bending axis of the crystal (see Sec. 3.1 and Fig. 13b). Since exactly as many electrons as positrons are created, while the regions of intense PP lie near the minima of the potential energy of e^-, which expands the range of angles of incidence of γ-quanta that yield channelled e^-, the number of the latter exceeds the number of e^+ (for more details see Ref. 36). This still proves insufficient to obtain a considerable yield of highly polarized e^- owing to too rapid a decline in the polarization of e^- with increasing bending radius of the crystal. We can convince ourselves of this by comparing the differential numbers of e^+ and e^- created at bending radii of the crystal $R = 17.6\ R_{min}$ and $R = 2\ R_{min}$ (they are marked in Fig. 17 by the vertical lines, with $R_{min} \approx 15$ cm) that make possible the maximal reduced numbers of $e^\pm$, $\zeta_\pm{}^2\, dN/d\varepsilon_\pm$. Moreover, the described method of separating the polarized e^- will be complicated by their dechanneling. In this regard we should call attention to the alternative possibilities of isolating polarized e^- that are opened up by the use of fast-action electronics. Thus, e^- beams having the same intensity and polarization as the beams of channelled e^+ can be obtained by selecting the e^-, no longer obligatorily channelled, by the signal of simultaneous creation of channelled e^+, which can be easily identified, either from the direction of emergence from the crystal or from the magnitude of the ionization losses.[89] Coincidence of the degrees of polarization of the e^- and e^+ beams follows here from the equality of the magnitudes of the oppositely directed polarization of e^- and e^+ created by a single γ-quantum. The use of electronic selection methods is desirable also for another reason. The point is that the $e^\pm$, which bear away a considerable fraction of the energy of the γ-quantum ($\varepsilon_\pm \approx \omega$), have a low polarization (see Fig. 16). However, under conditions of an exponentially declining spectrum of secondary γ-quanta, such $e^\pm$ play a substantial role in forming the entire spectrum of $e^\pm$ (see Sec. 2.4). Under these conditions the selection of the highly polarized $e^\pm$ bearing away not too great a fraction of the energy of the γ-quanta ($\varepsilon_\pm \lesssim \omega/2$) can be easily performed with fast-action electronics.

Upon entering a state of bound transverse motion upon creation, the $e^{\pm}$ naturally will radiate and change their degree of polarization. The authors of Ref. 90 took this into account, however, while neglecting the polarization acquired by the $e^{\pm}$ immediately at creation. However, the graphs of Fig. 16 convince one that the latter source of polarization of $e^{\pm}$ is the fundamental one. Actually the curves describing the variation of the polarization and energy of $e^{\pm}$ in emission of a sequence of γ-quanta of characteristic energies, while originating on the graph of the energy dependence of the polarization of the created $e^{\pm}$, lie below it. This indicates that the $e^{\pm}$ created with lower energies have a higher polarization than the $e^{\pm}$ created with higher energies and then subjected to the effect of homopolarization. Therefore we should hope to increase the polarizing efficiency of the process of formation by γ-quanta of polarized $e^{\pm}$ by additionally applying the effect of self-polarization of the produced $e^{\pm}$. Consequently the fundamental factor in the influence of the radiation from $e^{\pm}$ in the process of formation of transversely polarized $e^{\pm}$ in bent crystals proves to be the energy losses of $e^{\pm}$, which restrict the length in (3.3) and the bending angle of the crystal $\varphi \simeq l_{\mathrm{SRP}}/R$. By estimating the ratio of the latter to the angle $2\vartheta_{\pi^0}$ that characterizes the divergence of the beam of uncollimated decay γ-quanta, we obtain the following expression for $\varepsilon_{\pm} = \omega/2$:

$$\varphi/2\vartheta_{\pi^0} = \frac{15m}{\alpha m_{\pi^0}} \frac{R_{\min}}{R} \approx \frac{7.8R_{\min}}{R}, \tag{3.4}$$

Hence we see that the angles of bending of the crystals can exceed the total divergence of the beam of γ-quanta. Since the channelled $e^{\pm}$ are created only in regions where the crystal planes form angles with the momenta of the γ-quanta $\theta < \theta_c = (2V_{\max}/\varepsilon_{\pm})^{1/2} \ll \theta_{\pi^0}$, this excess implies the possibility of creation by each decay γ-quantum of channelled $e^{\pm}$ in the corresponding region of the volume of the crystal. Moreover, when $\varphi > 2\vartheta_{\pi^0}$, by appropriately selecting the dependence of the thickness of the bent crystals on the transverse coordinate, one can control the flux of all the polarized $e^{\pm}$ beyond the crystal (focus it or make the trajectories of $e^{\pm}$ almost parallel, which the condition $\vartheta_c \ll \phi$ allows).

However, when $\varphi \sim \vartheta_{\pi^0} \gg \theta_c$ the γ-quanta can produce channelled and polarized $e^{\pm}$ only over a small fraction $(\theta_c/\vartheta_{\pi^0})\,L$ of the entire length L of the crystal. Pair production over the remaining length of the crystal leads only to useless losses of the intensity of the γ-beam. For this reason the use of crystals having a bending angle $\varphi \approx \vartheta_{\pi^0}$ allows one to obtain only 10^{-3}–10^{-2} polarized $e^{\pm}$ per γ-quantum. One can substantially decrease the unproductive losses of γ-quanta by selecting a bending angle of the crystal comparable with θ_c. Here an appreciable fraction of all the created $e^{\pm}$ will enter a channeling regime if the directions of the momenta of most of the γ-quanta will be tangent to the bent planes inside the crystal. Here the divergence of the beams of γ-quanta should not substantially exceed $2\theta_c$. The requirements on the length of the flight distance and the cross section of the crystal coincide with those discussed in Sec. 3.1.

Pair production in bent crystals remains an effective method of obtaining transversely polarized $e^{\pm}$ beams over a broad range of energies of γ-quanta $\omega = (1\text{–}100)\ \omega_{\varkappa=1}$.[36] As increase in the latter leads to increasing the number and accelerating the decay of polarization of the channelized e^{+} and of the e^{-} created with them in the pair as the ratio $R/R_{\min}$ is increased.[36]

Thus the effect of creation in bent crystals of transversely polarized $e^{\pm}$ will enable obtaining an amount of the order of 10^{-3}–10^{-2}, and possibly even more, of polarized $e^{\pm}$ per γ-quantum. The energies of the highly polarized $e^{\pm}$ cannot be too close to the energy of the γ-quanta. When one uses electronic selection, the possibilities of obtaining polarized e^{-} are not inferior to the analogous possibilities for the e^{+} case.

3.3 Effects involving spin rotation

In addition to the processes of synchrotron-type radiation and PP, the possibilities have been actively discussed in recent years of manifestation of the more exotic effects of QED of an intense field.[38,39,91–93] We shall take up the effect of variation of the anomalous magnetic moment of $e^{\pm}$.[38,39] In our view it combines in the greatest degree a fundamental character with a real possibility of observing it. We recall[40–45] that, in the region where quantum synchrotron-type radiation is manifested, $\chi = \gamma\mathscr{E}/\mathscr{E}_0 \gtrsim 1 (\gamma = \varepsilon_{\pm}/m)$, the anomalous component of the magnetic moment μ' begins to decline, while in the absence of a field it is equal to the known Schwinger value $\mu_{\mathrm{Sch}} = \alpha\mu_{\mathrm{B}}/2\pi$, where μ_{B} is the Bohr magneton. When $\chi = 0.5$, 1, and 2, respectively, we have $\mu' = 0.66\mu_{\mathrm{Sch}}$, $0.47\mu_{\mathrm{Sch}}$, and $0.30\mu_{\mathrm{Sch}}$.[44] Observation of the variation of μ' in the averaged crystal field becomes possible when one uses the effect of spin rotation of channelled e^{+} in a bent crystal.[33,37] We recall[48] that relativistic effects lead to precession of the mean spin vector around the vector of (3.1). Since also the velocity vector of the charged particles rotates in an electric field, we can expect the existence of a coupling of the angles of rotation ϕ_ζ and ϕ of their spin and velocity. Such a coupling actually exists,[94] and in the case of plane trajectories, such as the unperturbed trajectories of particles moving in the field of the planes of a bent crystal, it has the form

$$\varphi_\zeta = \varphi_\zeta(\varphi) = \left(\frac{\mu'}{\mu_{\mathrm{B}}}\frac{\gamma^2-1}{\gamma} + \frac{\gamma-1}{\gamma}\right)\varphi. \tag{3.5}$$

Both for ultrarelativistic hyperons ($\mu' \sim \mu_{\mathrm{B}}$) and for $e^{\pm}$ in the energy region in which we are interested, $\varepsilon_{\pm} \gtrsim \varepsilon_{\chi=1}$, where $\gamma \gg \mu_{\mathrm{B}}/\mu_{\mathrm{Sch}} = 2\pi/\alpha \approx 861$, the fundamental contribution to the quantity in parentheses comes from the first term, whereby we have $\varphi_\zeta \approx \gamma(\mu'/\mu_{\mathrm{B}})\,\varphi$. The increase in the angle φ_ζ with increasing energy has the result that, e.g., at energies of several hundred GeV, the magnitude of this angle over the decay length, which also increases with the energy, of charmed or beautiful hyperons reaches a radian. Owing to the proportionality of the angle of rotation of the spin to the magnetic moment, this opens up a unique possibility of measuring the latter for particles having a lifetime $\tau \gtrsim 10^{-13}$ s.[9,95,97] It is proposed to start the preliminary experiments with measuring μ' of strange particles.[98]

Since at energies of $e^{\pm}$ $\varepsilon_{\pm} \gtrsim \varepsilon_{\chi=1}$ the radiation losses substantially complicate the experimentation using a crystal whose thickness exceeds the length in (3.3), its bending angle $\varphi = 1/R$ will not exceed $l_{\mathrm{RSP}}/R_{\min}$. Substituting in (3.5) $\mu' = \mu_{\mathrm{Sch}}$ and $\varphi = l_{\mathrm{RSP}}/2R_{\min}$, we obtain $\varphi_\zeta \approx 15/4\pi \approx 1.2$ radian. Therefore, since $\varphi_\zeta \propto \mu'$, the decrease in μ' in an intense crystal field by tens of percent leads to a change in the angle φ_ζ by an amount of the order of a radian, which will

create the optimal conditions for detecting it. The possibility is most real of observing a change in μ' in the motion of e^+ in the field of the planes of a bent crystal for $\varepsilon_+ \approx \varepsilon_{\gamma=1} \gtrsim 100$ GeV.

We must touch on yet another aspect of the manifestation of the coupling of the evolution of the spin of particles with their motion. Since in a material, and in particular in a crystal, charged particles undergo random deflections, this coupling leads to rotation of their spin, which causes a decline in the degree of polarization of the particle beam (for more details see Refs. 5 and 94). In the motion of channelled particles in the field of planes bent at the angle φ_0, the angles of their rotation in the bending plane lie in the interval $\varphi_0 - \theta_c < \varphi < \varphi_0 + \theta_c$. The differences in the angle of deflection can diminish the component of the polarization lying in the bending plane, while a measurement of its angle of rotation enables us to measure μ'. By using (3.5), we obtain for the characteristic angle of deflection of this component from the mean angle of its rotation $|\Delta\varphi_\zeta| = |\varphi_\zeta(\varphi) - \varphi_\zeta(\varphi_0)| \lesssim (\mu/\mu_B)\gamma\theta_c$. For beautiful hyperons of energy 1 TeV we have $|\Delta\varphi_\zeta| \lesssim 3$ milliradians. For e^+ we have $|\Delta\varphi_\zeta| \approx 0.01\ (\varepsilon_+/\varepsilon_\gamma)^{1/2}$. Since in all cases $|\Delta\varphi_\zeta| \ll 1$ radian, we can neglect the relative variation of the projection of the polarization of the particles on the bending plane of the crystal caused by the difference in angles of rotation of the spin of the particles of the beam, which is equal[5,94] to $|\Delta\varphi_\zeta|^2/2$.

The transverse polarization of e^+ parallel to the crystal planes that arises in self-polarization (Sec. 3.1) and PP (Sec. 3.2) undergoes the action of only the deflections of e^+, which are also parallel to the planes and are caused by incoherent scattering. When $|\varphi_\zeta|, |\varphi| \ll 1$, this action can also be described by Eq. (3.5). We recall that the intensities of multiple scattering in planes—parallel and perpendicular to the crystal planes—differ insignificantly.[63] Therefore, in the dechanneling length, over which the angle of incoherent scattering in a plane perpendicular to the crystal plane does not exceed θ_c, the scattering angle in a plane parallel to the crystal planes is also close to θ_c. Since this same angle determines the depolarization of e^+ polarized perpendicular to the crystal planes, as discussed above, the degree of depolarization of e^+ transversely polarized parallel to the planes will also be equal to $|\Delta\varphi_\zeta|^2/2 \approx (\gamma\theta_c\mu_\parallel/\mu_B)^2/2 \approx 10^{-4}\ \varepsilon/\varepsilon_{\gamma=1}$. Therefore, in the energy region in which we are interested, $\varepsilon \sim (1\text{–}10)\ \varepsilon_{\gamma=1}$, we can also neglect the influence of depolarization. Thus depolarization does not threaten the manifestation of any of the polarization phenomena that we have discussed above. However, we note that in certain cases one can gain an advantage from the depolarization effect, which opens up another possibility for measuring the magnetic moments of short-lived particles.[99]

Incoherent scattering can also narrow the possibilities of using polarization phenomena in bent crystals by restricting the time of motion of the particles in a channel. Very recent experimental studies of dechanneling of protons of the energies $\varepsilon \gtrsim 100$ GeV in which we are interested, moving in a bent crystal of constant curvature, have shown that for positive particles one need not fear such a narrowing of the possibilities. Yet the insufficient level of study of the motion of negatively charged particles in bent crystals as yet does not allow us to estimate the prospects of using polarization phenomena in this case.

4. CONCLUSION

Our aim has been to depict the possibilities of observing and using varied polarization phenomena that arise in oriented crystals at energies of particles reaching hundreds of GeV and more. All the discussed phenomena are closely associated with the local nature of emission and pair production, which enables using the quantum electrodynamics of a homogeneous field to describe them. The polarization phenomena inherent in the latter are best manifested in the fields of crystal planes, the averaged intensities of which determine the necessary particle energies that enable one to observe and apply these phenomena in the secondary e^+ and γ-beams of the Tevatron already at present, as well as in any more powerful proton accelerator being built—the Accelerator-Storage Ring Complex (UNK), the Large Hadron Collider (LHC), and the Superconducting Supercollider (SSC)—with extraction of the beam in the latter two. Here the energies of the obtained e^+ and γ-beams will exceed the energies attainable with electron accelerators. The effects of dichroism and double refraction will enable obtaining linearly and circularly polarized γ-beams, the latter of which can also be converted into longitudinally polarized e^+ beams. The effects of radiative self-polarization and formation by γ-quanta of transversely polarized e^+ in bent crystals will enable obtaining transversely polarized e^+ beams. Thus the secondary e^+ and γ-beams (or only γ-beams alone) of proton accelerators can be converted into e^+ and γ-beams having a needed type of polarization. The production of polarized beams does not exhaust all the possibilities of application of polarization effects in crystals. The effect of spin rotation of particles in bent crystals enables one to observe the change in the anomalous magnetic moment of e^+ predicted by the quantum electrodynamics of intense fields and also to measure the magnetic moments of short-lived hyperons.

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APPENDIX. THE RELATION BETWEEN THE THEORY OF PAIR FORMATION IN CRYSTALS AND THE QUANTUM ELECTRODYNAMICS OF A HOMOGENEOUS INTENSE FIELD

The possibility of using the quantum electrodynamics of a homogeneous field to describe the process of PP in crystals was substantiated within the framework of the theory of the PP process,[10,46] which does not rely on perturbation theory in the interaction of e^+ with the crystal, and which is constructed by analogy with the theory of radiation.[47] The PP cross section (and the emission cross section) in the field of an individual nucleus without using perturbation theory was found by Olsen and Maximon on the basis of the Sommerfeld–Maue approximation.[48,68] This study of Baryshevskiĭ and Dubovskaya[47] showed that this approximation is applicable also for calculating the probabilities of radiation processes in crystals in the presence of channeling. According to Refs. 10 and 46 (see Ref. 48; below we shall assume that $\varkappa = c = 1$) we have

$$dw = 2\pi\delta(\omega - \varepsilon_+ - \varepsilon_-)\left|e\frac{\sqrt{4\pi}}{\sqrt{2}\,\omega^{1/2}}M_{fi}\right|^2\frac{d^3p_+\,d^3p_-}{(2\pi)^6}, \qquad (A.1)$$

where

$$M_{fi} = \int d^3r \Psi^{(-)*}_{\varepsilon_- p_-}(\mathbf{r})\, \alpha e \exp(i\,\mathbf{kr})\, \Psi^{(+)}_{-\varepsilon_+ -p_+}(\mathbf{r}), \qquad (A.2)$$

α are the Dirac matrices, ω is the energy of the γ-quantum, and e is the charge of e^+. Further,

$$\Psi^{(\pm)}_{\mp\varepsilon_\pm \mp p_\pm}(\mathbf{r}) = \frac{1}{\sqrt{2\varepsilon_\pm}} \exp(\mp i\,\mathbf{p}_\pm \mathbf{r}) \left(1 \pm \frac{i\alpha\nabla}{2\varepsilon_\pm}\right) u^\pm \varphi^\pm(\mathbf{r}) \qquad (A.3)$$

are the wave functions of e^+ and e^- found on the basis of the Sommerfeld–Maue approximation in the averaged potential of the planes or axes. In their asymptotic behavior they contain respectively converging and diverging spherical waves, along with a plane wave. Here and below, the upper (or lower) sign pertains to e^+ (or e^-), the $u^\pm$ are bispinors, and $\mathbf{p}_\pm$ and $\varepsilon_\pm$ are the momenta and energies of the $e^\pm$ that have left the crystal.

We shall study PP in the averaged potential of planes, which depends only on the x coordinate measured along the normal to the planes. In this case we can expand the functions, $\varphi^\pm$ in the eigenfunctions of one-dimensional Schrödinger equations,

$$\left[-\frac{\Delta}{2\varepsilon_\pm} \pm V(x)\right] \psi^\pm_n(x) = \varepsilon^\pm_{\perp n} \psi^\pm_n(x), \qquad (A.4)$$

which describe the transverse motion of $e^\pm$ having the energy $\varepsilon^\pm_{\perp n}$. At the energies in which we are interested, the latter is quasiclassical, which permits us to treat only the motion of $e^\pm$ within the limits of one potential well. The bound transverse motion of $e^\pm$ in this case has a discrete spectrum of levels numbered by the integers $n = 0,1,\ldots$ To establish the relation of the discussed theory of the QED of a homogeneous intense field,[40-45] it suffices to treat the case of zero angle of incidence of γ-quanta on the planes of a crystal of thickness $L \gg 1/|\varepsilon^+_{\perp n} - \varepsilon^-_{\perp n'}|$. Upon averaging (A.1) over the polarizations of the γ-quanta and summing over the polarization of $e^\pm$, we obtain in this case,[10] after integrating over the momenta of e^-:

$$\frac{dw}{d^3p_+} = \frac{\alpha L}{\pi\omega} \sum_{i,f} Q_i(-p_x)\, \delta\left[\varepsilon^+_{\perp i} + \varepsilon^-_{\perp f} + \frac{\omega m^2}{2\varepsilon_+\varepsilon_-}\left(1 + \frac{p_y^2}{m^2}\right)\right]$$
$$\times \left\{\frac{m^2}{2\varepsilon_+\varepsilon_-} |J_{if}|^2 + \frac{(\varepsilon_+^2 + \varepsilon_-^2)}{4\varepsilon_+^2\varepsilon_-^2} \left[|I_{if}|^2 + (m^2 + p_y^2)|J_{if}|^2\right]\right\}, \qquad (A.5)$$

where we have

$$\begin{Bmatrix} J_{if} \\ I_{if} \end{Bmatrix} = \int \psi^{-*}_f(x) \begin{Bmatrix} 1 \\ i\, d/dx \end{Bmatrix} \psi^+_i(x)\, dx. \qquad (A.6)$$

The Q_i are the occupancy coefficients, and the indices i and $f = 0,1,\ldots$ respectively number the levels of the transverse energy of e^+ and e^-. Let us analyze the equation

$$\varepsilon^+_{\perp i} + \varepsilon^-_{\perp f} = -\frac{\omega m^2}{2\varepsilon_+\varepsilon_-}\left(1 + \frac{p_y^2}{m^2}\right), \qquad (A.7)$$

which reflects the conservation of the projection of the momentum on the z axis parallel to the planes. In referring the potential and the energy $\varepsilon^\pm_\perp$ to the middle of the interplanar interval (Fig. 18), we have $\varepsilon^+_{\perp i} > 0$ and $\varepsilon^-_{\perp f} > -V_{\max}$, where $V_{\max} = V(0)$. Therefore Eq. (A.7) can be satisfied only when $\varepsilon^+_{\perp i} < V_{\max}$ and $\varepsilon^-_{\perp f} < 0$, i.e., only for subbarrier states. Equation (A.7) and the inequality $\varepsilon^+_{\perp i} + \varepsilon^-_{\perp f} > -V_{\max}$ imply that $\omega > \tilde{\omega} = 2m^2/V_{\max}$. However, the extremely small

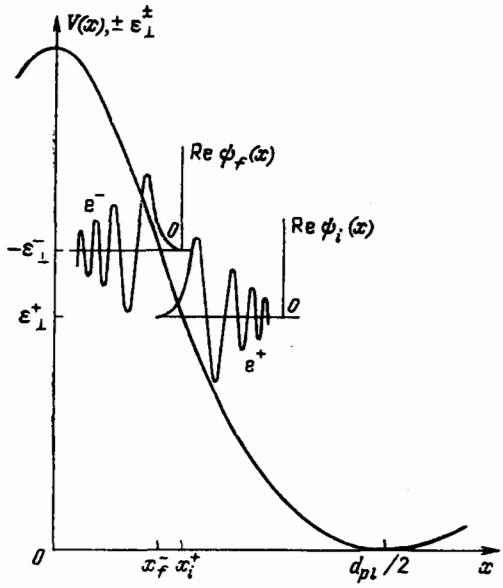

FIG. 18. Wave functions of the transverse motion of e^+ and e^- in the region of their overlap. Here x_i^+ and x_f^- are their turning points of classical motion. At zero angle of incidence of the γ-quanta on the plane (or axis), the regions of classical motion do not overlap. In this case in the quasiclassical description of the PP process, the region surrounding the points x_i^+ and x_f^- and having the dimensions $\Delta x \sim x_i^+ - x_f^- \simeq \lambdabar_c = 3.861 \times 10^{-11}$ cm is the projection on the x axis of the region of pair production (see Sec. 2.1).

probability of PP renders illusory the probability of observing threshold phenomena at energies $\omega \approx \tilde{\omega}$. Further, the condition $\varepsilon^+_{\perp i} + \varepsilon^-_{\perp f} < 0$ [see (A.7)] implies that the regions of classical motion of e^+, $x_i^+ < x < d_{pl} - x_i^+$, and of e^-, $0 < x < x_f^-$, $d_{pl} - x_f^- < x < d_{pl}$ (see Fig. 18), do not overlap. Therefore we can expect a considerable increase in the probability of PP as the boundaries of the regions of classical motion of e^+ and e^- approach each other as their energy increases [see (A.7)]. Here the fundamental contribution to the integrals of (A.6) will come from the region surrounding the segment (x_f^-, x_i^+) of the interplanar distance of width $\Delta x \sim \lambdabar_c = 3.861 \times 10^{-3}$ Å $\ll d_{pl}$ in which we can consider the electric field to be constant: $\mathscr{E} \simeq \mathscr{E}(x_i^+) \simeq \mathscr{E}(x_f^-)$. In the quasiclassical description of the PP process, the segment $\Delta x \sim \lambdabar_c$ corresponds to the transverse displacement of e^+ by the pair-production length (see Sec. 2.1). The homogeneous-field approximation thus introduced[10-13] (see also Refs. 14–17) enables one to use the wave functions

$$\psi^\pm_n(x) = \frac{c^\pm_n}{(2e\mathscr{E}\varepsilon_\pm)^{1/6}} \Phi[\pm(2e\mathscr{E}\varepsilon_\pm)^{1/3}(x^\pm_n - x)]. \qquad (A.8)$$

Here $x^\pm_n$ is the boundary of the regions of classical motion of $e^\pm$ lying in the region $0 < x^\pm_n < d_{pl}/2$ [$\pm V(x^\pm_n) = \varepsilon^\pm_{\perp n}$], and $\Phi(x)$ is the Airy function [see (2.5)]. We shall not discuss the finding of the normalization coefficients $c^\pm_n$. Further, using the known formula for the convolution of the Airy function and assuming the potential $V(x)$ to be symmetric, we easily obtain from (A.6)

$$\begin{Bmatrix} |J_{if}| \\ |I_{if+1}| \end{Bmatrix} = \frac{\sqrt{\pi} c^+_i [1 \pm (-1)^{i+f}]}{(2e\mathscr{E})^{2/3} (\varepsilon_+\varepsilon_-)^{1/6} \omega^{1/3}} \begin{Bmatrix} c^-_f \Phi(y) \\ -\dfrac{(2e\mathscr{E}\varepsilon_+\varepsilon_-)^{1/3}}{\omega^{1/3}} c^-_{f+1} \Phi'(y) \end{Bmatrix}, \qquad (A.9)$$

$$y=\left(\frac{2e\mathscr{E}\varepsilon_+\varepsilon_-}{\omega}\right)^{1/3}(x_i^+-x_f^-)$$

$$=\frac{\xi}{2^{2/3}}\left(1+\frac{p_y^2}{m^2}\right),\quad \xi=\left(\frac{m^3\omega}{e\mathscr{E}\varepsilon_+\varepsilon_-}\right)^{2/3}. \qquad (A.10)$$

Owing to the quasiclassical character of the transverse motion, we can go over to (A.5) from summation to integration over the continuous variables $\varepsilon_\perp^\pm$. The integration over one of them is easily performed using the delta-function. By using the relationship $\varepsilon_\perp^\pm = \pm V(x^\pm)$, we shall represent the integration over the second of them in the form of integration over the interplanar spacing. Then, upon substituting (A.9) into (A.5) and integrating over p_x, we obtain the expression

$$\frac{d^2w}{d\varepsilon_+\,dp_y}=\frac{L}{d_{pl}}\int_0^{d_{pl}}dx\,\frac{2^{2/3}\alpha m^2\xi^{1/2}(x)}{\pi\omega^2}$$
$$\times\left[\Phi^2(\xi(x))+\frac{\varepsilon_+^2+\varepsilon_-^2}{2^{1/3}\xi(x)\,\varepsilon_+\varepsilon_-}(\Phi'^2(\xi(x))+\xi(x)\,\Phi^2(\xi(x)))\right]. \qquad (A.11)$$

Finally this allows us to see the connection with the QED of an intense homogeneous field. Namely, the integral over the interplanar spacing in (A.11) contains an expression that coincides, apart from notation, with the doubly differential expression for the probability of PP in a homogeneous field [see Eq. (56) on p. 82 of Ref. 40]. Upon applying the standard transformations (see pp. 82–83 of Ref. 40), we obtain the following from (A.11):

$$w=WL=L\int_0^{d_{pl}}W(\mathscr{E}(x))\frac{dx}{d_{pl}}. \qquad (A.12)$$

Here $W[\mathscr{E}(x)]$ is the probability of PP given by (2.4) in a transverse homogeneous field (see Refs. 53 and 40–45, 48). Since in deriving Eq. (A.12) we used the wave function of a photon

$$\exp(i\mathbf{kr})/\sqrt{2}\,\omega^{1/2},$$

which does not take account of absorption, it is valid only when $w \ll 1$. Here the total probability w of PP in the crystal equals the product of the thickness L of the crystal and probability of PP per unit length as obtained from (2.4) by averaging over the interplanar distance (cf. Ref. 59). The main result of the treatment that we have conducted in the demonstration that the theory of PP in crystals[10,46] in the case of zero angle of incidence of γ-quanta on crystal planes (or axes) leads to results that agree with the formulas of QED of an intense homogeneous field.

The local character of the PP process discovered in deriving (A.12) allows us easily to write also the expression for the probability of PP per unit length of crystal by γ-quanta incident at zero angle on a crystal axis (all the chains of atoms are considered equivalent):

$$W=\int W(\mathscr{E}(\rho))\frac{d^2\rho}{S_{ax}}. \qquad (A.13)$$

The integration in (A.13) is performed over a unit cell lying in a plane perpendicular to the axis and having the area $S_{ax} = 1n_0d_{ax}$, where d_{ax} is the interatomic distance on the axis, and n_0 is the number of atoms of the crystal per cubic centimeter. Equation (P.13) was first used in Ref. 100 [see Eq. (7)]. Later it was applied in Ref. 74.

We note that the first attempts to escape the framework of the Born approximation in treating the process of PP in crystals was undertaken in Refs. 101 and 102. The treatment of the PP process in Refs. 12 and 13 was based on an approach that essentially does not differ from that described here. The authors of Refs. 12 and 13 also discovered the possibility of a substantial simplification of their formulas based on the homogeneous-field approximation. However, they are not able to derive an analog of Eq. (P.13) agreeing with the results of QED of a homogeneous field , as was pointed out in Ref. 74.

[1)] Pronounced overestimates of the energy of $e^\pm$ and γ were also given in Refs. 18, 19.

[2)] The system of units employed is with $\hbar = c = 1$.

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