Stability of shock waves

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The progress already made in studies of the stability of shock waves and some tasks for the future are reviewed. The following aspects of the problem are discussed: 1) stability of shock waves as hydrodynamic discontinuities irrespective of the events which occur in the relaxation zone of a wave (this aspect includes also the problem of stability of a shock wave sustained by a piston and anomalies in the reaction of a wave to external perturbation, including reflection and refraction of perturbations); 2) stability of flow in the relaxation zone (structural stability) of a shock wave. The stress is on the theoretical side of the problem. However, potential practical realization of the shock-wave instability criteria are also considered. Schemes of decay of unstable shock-wave discontinuities are discussed.

INTRODUCTION

The problem of stability of shock waves has a number of aspects which are qualitatively different both from the point of view of the theory and in respect of possible practical applications. Therefore, each of these aspects should be discussed separately. In some cases these aspects are interrelated, but it is easier to understand them by investigating them separately. We shall therefore consider the following topics.

1. Stability of shock waves considered as hydrodynamic discontinuities irrespective of the events which occur in the relaxation zone of a wave. This formulation of the problem is of interest in studies of the stability of flow on a scale much greater than the length of the relaxation zone of a shock wave. We shall include here the problem of stability of a shock wave sustained by a piston, as well as anomalies in the reaction of a wave to external perturbations (reflection refraction of perturbations).

2. Stability of flow in the relaxation zone (structural stability) of a shock wave. This aspect of the problem is of interest mainly in studies of the flow and its characteristics on scales comparable with the length of the relaxation zone.

In an analysis of these or other deformations of the shock wave front it is necessary to distinguish the intrinsic structural instability from the boundary effects associated with the actual conditions during generation and propagation of a shock wave in a confined medium. This aspect of the problem will also be discussed briefly as an inherent part of the aspect 2.

3. Stability of shock waves propagating in a substance which is not in thermodynamic equilibrium. A separate section of the review is devoted to each of these three aspects of the problem. The conditions under which the aspects 1 and 2 are interrelated are discussed in Sec. 4. Potential practical realization of the criteria of instability of shock waves are discussed in Secs. 5 and 6. The Appendix gives schemes of decay of shock-wave discontinuities.

The review concentrates on the theoretical side of the problem and then only on key fundamental topics. Consequently, no attempt has been made to provide a comprehensive bibliography, particularly of experimental investigations.

1. HYDRODYNAMIC STABILITY OF SHOCK WAVES

1.1. General conditions of existence of a shock-wave discontinuity

In the problem of stability of shock waves, as in the theory of shock waves in general, we have to consider the wave front either as an infinitely thin surface of discontinuity or as a transition layer of finite width δ, which it in reality is, depending on the characteristic geometric scale of the problem. If we are interested in the flow of a scale much larger than δ, then the first of these approaches is acceptable and in this case we can assume formally that $\delta = 0$, and that all that occurs in the layer δ can be regarded as unimportant. This applies also to possible manifestations of the flow instability in this layer because the "memory" of such an instability (representing short-wavelength perturbations traveling downstream behind the shock wave front) decays because of the always-present viscosity, in a manner similar to dissipation of perturbations in a liquid at rest, which establishes a complete thermodynamic equilibrium.

The main conditions for the stability of a shock-wave discontinuity regarded as an interface between two constant-flow regions (the constancy is assumed to apply at least in a small part of a continuous medium) are well known from the fundamentals of the theory of shock waves.[1,2] They can be stated as follows.

1. The motion of the front of a shock wave should be supersonic relative to the matter ahead of the discontinuity and subsonic relative to the matter behind the discontinuity. These conditions can be expressed in terms of the Mach numbers Ma in the form

$$\mathrm{Ma}_1 > 1, \quad \mathrm{Ma}_2 < 1, \quad \mathrm{Ma}_i \equiv V_i/c_i, \tag{1}$$

where v_i is the velocity of matter relative to the front and c_i is the velocity of sound; the indices 1 and 2 apply to the regions ahead and behind the front, respectively.

In the case of oblique shock waves we have to replace the velocity V_i in Eq. (1) with its component V_{in} normal to the front and the Mach numbers have to be replaced with the quantities $\mathrm{Ma}_{in} \equiv V_{in}/c_i$. The stability conditions are then defined by

$$\mathrm{Ma}_{1n} > 1, \quad \mathrm{Ma}_{2n} < 1. \tag{2}$$

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If the first of the two conditions in Eq. (2) is disobeyed, perturbations travel forward from the wave front giving rise to a time-dependent (expanding) region of a transition from the state 1 to the state 2, i.e., a transition resulting in destruction of the shock wave front. When the second condition of Eq. (2) is disobeyed, the front loses its linkage with the "rear" and the intensity of the discontinuity becomes instantaneously (in the hydrodynamic sense) as small as we please, which again implies destruction of the shock wave front.

2. A necessary physical condition of the stability of a shock wave front is the requirement of an increase in the entropy when matter passes through a discontinuity:

$$s_2 > s_1. \tag{3}$$

All three conservation laws, relating to the mass, momentum, and energy, are symmetric relative to the indices 1 and 2 in the case of a shock wave. The inequality (3) is asymmetric and it thus imposes the direction of motion of a shock-wave discontinuity. As a rule, the condition (3) means that a shock wave compresses matter (Zemplen theorem[1,2]). However, in exceptional cases, corresponding to an inequality which is thermodynamically not forbidden but very rarely satisfied,

$$\left(\frac{\partial^2 v}{\partial p^2}\right)_s < 0 \ (v \text{ — is the specific volume}),$$

it follows from Eq. (3) that $v_2 > v_1$. In other words, in such cases there is a rarefaction shock wave. A compression shock wave is then unstable: its front expands without limit with time and in the asymptotic limit $t \to \infty$ it represents an isentropic compression wave.

The conditions (2) and (3) are necessary for the stability of a shock wave but they are insufficient, since they do not tell us anything of what will happen to the wave when its front is bent by perturbations, whether such perturbations decay or grow with time. Moreover, the states 1 and 2 satisfying the relationships applicable to one shock-wave discontinuity may sometimes be linked also by a sequence of other discontinuities and waves, such as shock waves of a different amplitude, contact discontinuities, and rarefaction waves. In the case of shock waves each of the theoretically possible linkage variants may satisfy the inequalities of Eqs. (2) and (3), but it is difficult to tell which variant occurs in practice.

1.2. Main results of a linear stability theory

The problem of stability of a shock wave against bending or corrugation of its front was first investigated by D'yakov.[3] He linearized the equations of hydrodynamics for perturbations and solved the characteristic equation for the complex frequency. (For brevity, this approach and its results will be called the linear stability theory.) The inequalities (2) and (3) were assumed to be satisfied. The main results of the linear stability theory were as follows. The nature of evolution of corrugated perturbations of the front of a wave, the intensity of which in the unperturbed state is governed by a pressure p_2, is described completely by the relationship between the following dimensionless parameters:

$$L \equiv J^2 \left(\frac{\partial v}{\partial p}\right)_H, \quad \mathrm{Ma}_2, \quad \theta \equiv \frac{v_1}{v_2}, \tag{4}$$

where $J \equiv [(p_2 - p_1)/(v_1 - v_2)]^{1/2}$ is the flow of matter across the shock wave front and $(\partial v/\partial p)_{H,p=p_2}$ is the derivative of the specific volume v with respect to the pressure p_2 along the shock adiabat. All the dimensionless parameters of Eq. (4) are known at any pressure p_2 if we know the shock adiabat $p_2 = f(v_2, p_1, v_1)$ and the velocity of sound behind the shock wave at the point (p_2, v_2).

There are three ranges of the values of the parameter L which differ qualitatively in respect of the evolution of corrugated perturbations. These ranges are limited by the inequalities[1]

$$-1 < L < L_0, \quad L_0 \equiv \frac{1 - \theta\,\mathrm{Ma}_2^2 - \mathrm{Ma}_2^2}{1 + \theta\,\mathrm{Ma}_2^2 - \mathrm{Ma}_2^2}, \tag{5}$$

$$L < -1 \quad \text{or} \quad L > 1 + 2\,\mathrm{Ma}_2, \tag{6}$$

$$L_0 < L < 1 + 2\,\mathrm{Ma}_2. \tag{7}$$

The inequalities of Eqs. (5)–(7) apply to both compression and rarefaction shock waves.[3]

In the case described by Eq. (5) we find that small corrugated perturbations of the shock wave front decay with time in accordance with a power law. In other words, a shock wave satisfying the conditions of Eq. (5) is stable against small perturbations.

In the cases described by Eq. (6) it follows from the linear stability theory that perturbations of the wave front increase exponentially with time. A nonlinear analysis yields possible asymptotic results of the evolution in time (Sec. 1.3).

The range defined by Eq. (7) is puzzling when considered on the basis of the linear stability theory.

It follows from the linear stability theory[3-5] that in a wider—compared with that defined by Eq. (7)—range of values of the parameter L

$$L^* < L < 1 + 2\,\mathrm{Ma}_2,$$

where L^* is a certain value of L lying within the range $-1 < L^* = L_0$, there are solutions with nondecaying perturbations of the shock wave front (which are stationary in a coordinate system gliding along the front) and only the acoustic waves arriving or departing at a certain angle are found just behind the front. In the case of these solutions the perturbations of all the quantities are proportional to a factor of the type

$$\exp[i(kx + ly - \omega t)] \tag{8}$$

with real values of k, l, and ω. (The x and y axes are directed along the shock wave front and along the normal to the front, respectively.) The ratio k/l, representing the orientation of an acoustic wave, depends on L. The values of L for the solutions with arriving acoustic waves are limited by the inequalities

$$L^* < L < L_0,$$

whereas for the solutions characterized by departing waves, these values are limited by the inequalities in Eq. (7).

When sound is reflected by shock wave[10-14] the above solutions with arriving waves correspond to vanishing of the reflection coefficient $\varkappa$, whereas the solutions with departing waves correspond to the infinite value of this coefficient ($\varkappa = p_r/p_f$, where p_f and p_r are the pressures in the incident and reflected waves).

In the case when waves intersect, the theoretical pattern of the reflection of sound is a configuration of four waves. The conditions under which $\varkappa = 0$ or $\varkappa = \infty$ are also the conditions of existence of a three-wave configuration formed by perturbed and unperturbed shock waves and an infinitesimally small arriving or departing wave.[11-15] The relationship between L and the angle γ in the three-wave configuration (Figs. 1–3) or in the solution given by Eq. (8) can be represented by[12]

$$\psi(\gamma) \equiv \left[1 - \frac{\theta}{A^2} + \frac{\Gamma 2S}{A} - L\left(1 + \frac{\theta}{A^2}\right)\right][2(1 + A^2) J^2 v]^{-1} = 0, \quad (9)$$

where

$$A \equiv \frac{(1+\Gamma^2)^{1/2}}{\mathrm{Ma}_2} - \Gamma, \quad S \equiv \mathrm{Ma}_2 (1 + \Gamma^2)^{1/2} - \Gamma, \quad \Gamma = \operatorname{ctg}\gamma.$$

Possible values of the angles γ for the arriving or departing waves[11-14] lie within the following limits (Figs. 1–3):

$$0 < \gamma < \gamma_0 \equiv \arccos \mathrm{Ma}_2, \quad (10)$$

$$\gamma_0 < \gamma < \pi. \quad (11)$$

The physical meaning of the solutions corresponding to Eq. (7) and characterized by departing waves and infinite $\varkappa$ (representing a resonance in accordance with the terminology of Refs. 11 and 13) can be found only by a special investigation. The possibility of an instability of a shock wave in the range defined by Eq. (7), where resonance reflection angles exist, was not excluded in Refs. 3, 11, and 13. However, the presence of resonances was treated much more positively in Ref. 14 (but a proof was not provided) and regarded as a direct manifestation of a shock wave instability. The principal interest is therefore the question whether reflection of an infinitesimally weak wave in the $\varkappa = \infty$ case leads to a finite perturbation of a shock wave and, therefore, to its instability.

The physical meaning of the solutions corresponding to Eq. (8) with departing waves and an infinite reflection coefficient had been identified relatively recently,[12,16a] so that it would be desirable to treat this topic in greater detail compared with the other aspects of the problem mentioned above. Such a treatment is given in Secs. 1.6 and 1.7.

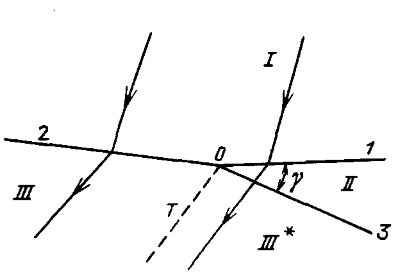

FIG. 1. Three-wave configuration with an arriving wave 3. Here, 1 denotes an unperturbed shock wave and 2 represents a perturbed shock wave; T is a tangential discontinuity. The arrows give the directions of the lines of flow in a coordinate system with a fixed point O.

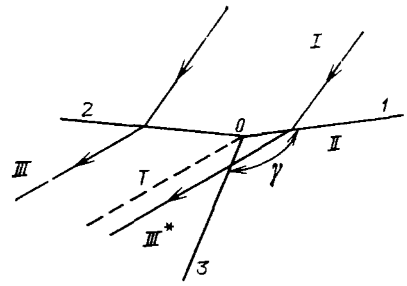

FIG. 2. Three-wave configuration with a departing wave 3: $\pi/2 < \gamma < \pi$ case. The notation is the same as in Fig. 1.

1.3. Decaying shock-wave discontinuities

Subsequent investigations of the shock wave stability problems, not solved by the linear theory, demonstrated that in the cases when $L > 1 + 2\mathrm{Ma}_2$ (Refs. 15, 17, and 18) or $L < 1 - 2\mathrm{Ma}_2$ (Ref. 18) a shock-wave discontinuity can be expanded into other elements (shock waves of different amplitude, rarefaction waves, contact discontinuities,[2)] as is true of any discontinuity). If $L > 1 + 2\mathrm{Ma}_2$ or $L < 1 - 2\mathrm{Ma}_2$ and $\mathrm{Ma}_2 = 1$, i.e., when any of the inequalities of Eq. (6) is satisfied, such elements or components move at different velocities without catching up and the decay of the initial wave is irreversible. In the cases defined by Eq. (6) a shock wave decays immediately to a configuration of other waves of finite amplitude.

A detailed analysis of the wave configurations formed as a result of decay of a shock-wave discontinuity in the cases when $L < -1$ and $L > 1 + 2\mathrm{Ma}_2$ was reported in Ref. 18 and the method used to calculate the configurations was given there. The shock adiabats (in pv and pu coordinates, where u is the velocity of matter behind the shock wave front in the laboratory coordinate system) which have sections with $L < -1$ and $L > 1 + 2\mathrm{Ma}_2$ are shown in Figs. 4 and 5, respectively. The Appendix gives a similar scheme of wave configurations formed as a result of decay of an original shock-wave discontinuity, as a function of the governing parameters. A similar scheme can be provided also in the case of rarefaction shock waves.[18]

All qualitative and quantitative data on the decay of a shock-wave discontinuity in the case when the inequalities of Eq. (6) are satisfied were obtained by a rigorous nonlinear analysis. This analysis was based on the application of the

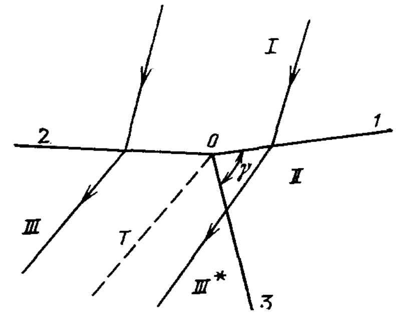

FIG. 3. Three-wave configuration with a departing wave 3: $\gamma_0 < \gamma < \pi/2$ case. The notation is the same as in Fig. 1.

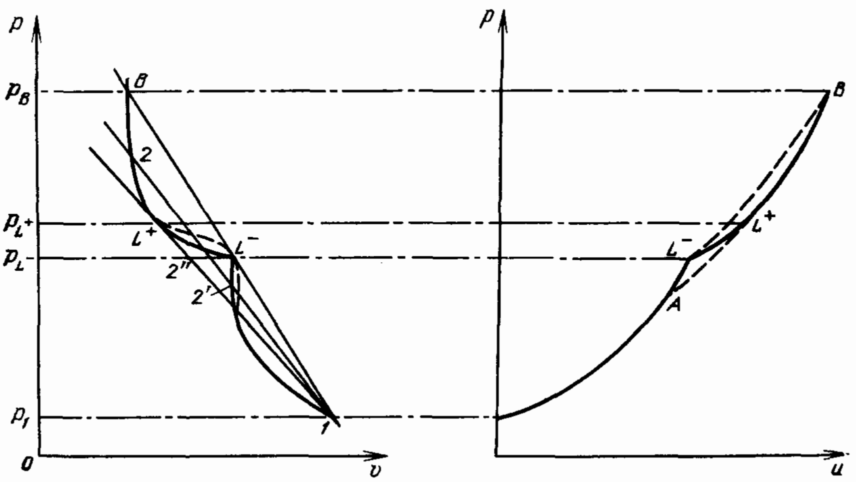

FIG. 4. Shock adiabat with a kink or a smooth inflection (dashed curve) at the point L . The inequality $L < -1$ is satisfied in the section L L^+ . The coordinates pu are used to show the following: an isentrope of the S^+ family (lower dashed curve), touching the shock adiabat at the point L^+ ; a shock adiabat of the H^+ family drawn from the point L (upper dashed curve).

general method of the theory of decay of discontinuities[21,22] without recourse to any expansions in powers of a small parameter.

It is important to stress (see Ref. 18) the following two circumstances. The range of values of the parameter L for shock-wave discontinuities which can be expanded into other elements (components) firstly includes completely both subregions defined by Eq. (6) and, secondly, an expansion into other elements is also possible for those shock-wave discontinuities for which small deformations of the surface do not increase with time according to the linear stability theory, i.e., which correspond to

$$-1<L<1+2\mathrm{Ma}_2. \qquad (12)$$

We shall use L^- and L^+ to denote the lower (characterized by a lower pressure) and upper limits of an interval L^- L^+ of the shock adiabat within which we have $L < -1$ or $L > 1 + 2\mathrm{Ma}_2$. If the varies monotonically along the shock adiabat,[3)] the values of L at the limits of the interval are -1 in the first case and $1 + 2\mathrm{Ma}_2$ in the second. We can show (Sec. 1.5) that in such cases the intervals L^- L^+ corresponding to a wave instability in accordance with the linear theory are distributed as shown in Figs. 6a and 6b relative to sections AB of the shock adiabat where a shock-wave discontinuity can be expanded into other elements (components).

The limits of the inequalities in Eq. (6), $L = -1$ and $L = 1 + 2\mathrm{Ma}_2$, are set mathematically by the condition that the imaginary part of the complex frequency ω should vanish in a solution of the type $\exp(-\omega t)$ representing the amplitude of a corrugated perturbation of the shock wave front.[3] We shall now give a clear physical interpretation of these limits and of the instability criteria described by Eq. (6).

1.4. Graphical interpretation of the instability criteria

1.4.1. $L<-1$ case. We can see from Fig. 4 that throughout the interval L L^+ the shock wave velocity D decreases on increase in the pressure; we recall that this velocity is given by $D = J^{-1}v_1 = v_1[(p_2 - p_1)(v_1 - v_2)^{-1}]^{1/2}$, i.e., that D^2 is proportional to the modulus of slope of the Rayleigh–Michaelson line relative to the v axis (Fig. 4). This means that after a random local reduction in the pressure the initially flat part of the shock wave becomes convex: cylindrical in the case of a two-dimensional perturbation and spherical for a three-dimensional geometry. The area of the

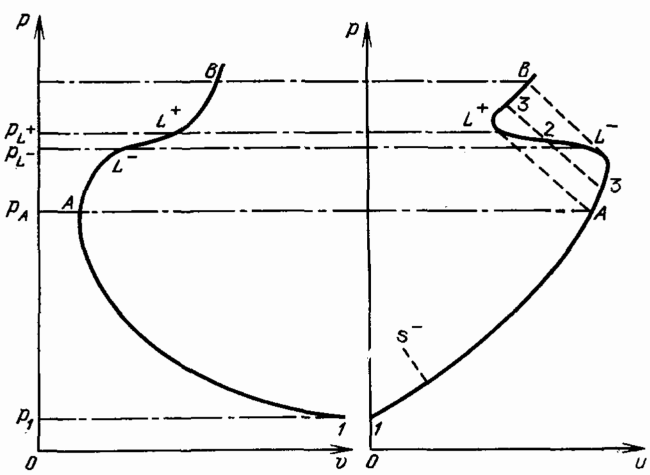

FIG. 5. Shock adiabat with a section L^- L^+ where $L > 1 + 2\mathrm{Ma}_2$. An isentrope of the S^- family is shown on the right using the pu coordinates (lowest dashed curve). The other dashed curves represent schematically isentropes of the S^- family (for downward transitions) or shock adiabats of the H^- family (for upward transitions); see the Appendix.

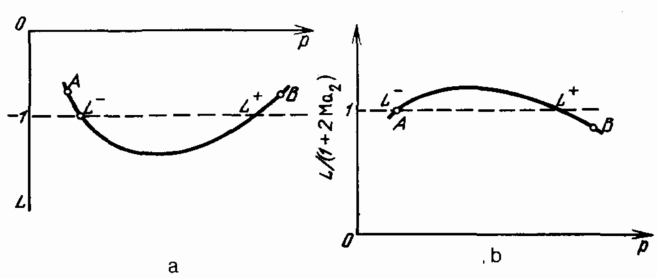

FIG. 6. Relative positions of the boundaries L^- and L^+ of an instability of a shock wave deduced using the linear theory and the boundaries A and B of a shock adiabat segment where representation of a shock-wave discontinuity is many-valued (point A lies below L^- in Fig. 6a).

convex front increases proportionally to time t or t^2 for a cylinder and a sphere, respectively, and this is known to reduce the front pressure compared with the pressure in a plane wave for identical parameters of the flow far from the front. This reduction in the front pressure increases even further the shock wave velocity at wave intensities corresponding to the shock adiabat interval $L^- L^+$, and so on, i.e., the plane surface of the front is unstable. The boundary of this "negative" dependence of the velocity of the shock wave front is given by

$$\frac{dJ}{dp} = 0, \tag{13}$$

which is identical to the equality $L = -1$. The instability region $L < -1$ established in this way is in full agreement with the results obtained from the liner theory of the shock wave stability. The simple proof of the instability given here can be regarded as a graphic interpretation of its mechanism in the case when $L < -1$.

1.4.2. $L > 1 + 2\mathrm{Ma}_2$ *case.* A corrugated instability of the shock wave front is in this case associated with a characteristic reaction of the wave front to the perturbations reaching it.

The coefficient $\varkappa$ representing the reflection of an acoustic perturbation by a shock wave front is given by the following expression which is valid in the case of normal incidence[23]

$$\varkappa = -\frac{L-(1-2\mathrm{Ma}_2)}{L-(1+2\mathrm{Ma}_2)}. \tag{14}$$

It follows from Eq. (14) that in the $L > 1 + 2\mathrm{Ma}_2$ case the reflection coefficient is negative and its absolute value is greater than unity. This means that weak compression (rarefaction) waves are reflected in the form of rarefaction (compression) waves with a larger absolute amplitude. This interaction of the shock wave front with the arriving behind-front perturbations gives rise to a characteristic wave instability mechanism.

Let us assume that reflection of a random local perturbation of finite size with the intensity of a shock wave increases in a small element of the surface of its front. In view of the finite size of a local perturbation (rarefaction wave), the process of its reflection takes a finite time during which the element of the shock wave front under investigation moves at a higher velocity than the unperturbed shock wave. The front of this wave therefore becomes convex. (This effect is of the same order of smallness as the initial random perturbation.) The behind-front region of flow reacts to such a deformation by "sending" a weak rarefaction wave in the direction of the convex part of the front. However, it follows from Eq. (14) that such waves further increase the pressure and the front propagation velocity,[4)] i.e., such a randomly initiated convexity of the front continues to grow.

The condition for this instability mechanism, expressed by the inequality $L > 1 + 2\mathrm{Ma}_2$, is again in full agreement with the linear theory of the shock wave stability.

This interpretation of the mechanism of the instability of a wave for a shock adiabat interval where $L > 1 + 2\mathrm{Ma}_2$ is not based on a rigorous mathematical analysis and should be regarded simply as a qualitative result.[5)]

In the case of the experimentally known anomalies of a shock adiabat which satisfy the necessary conditions for the decay of a wave (represented by expansion into other stable elements) and also the inequalities of Eq. (12) the wave decay is indeed observed: in the case of first-order phase transitions it is found that in a certain range of pressures (corresponding to the section AB of the adiabat in Fig. 4) the inequalities of Eq. (12) are obeyed, but instead of one wave a configuration of two shock waves traveling in the same direction is formed.[18,24-26] However, we cannot exclude the possibility that if the structure of the initial jump is regarded as stationary, we may find that in the region defined by Eq. (12) a shock wave satisfying the necessary decay conditions does not decay spontaneously. In such a case the parts of the adiabats where the conditions of Eq. (12) and the necessary decay conditions are satisfied simultaneously correspond to relatively stable shock waves which decay only as a result of a major modification of their structure under the influence of external perturbation sources. This problem requires further study. It must be stressed that the solution to this problem cannot be obtained using a conventional hydrodynamic analysis in which a transition region between the flow ahead and behind the front is replaced by a surface of discontinuity, but the structure of the surface is not investigated.

The following "structure" arguments are put forward in Ref. 26 to demonstrate an instability of a shock-wave discontinuity in the interval $L^+ L^-$ (Fig. 4). A straight line drawn from a point 1 to any point in the interval $L^+ L^-$ intersects the shock adiabat at two additional points. These intersections allow us to interpret the initial shock-wave discontinuity (Fig. 4) as a sequence of three shock waves separating respectively the states 1–2′, 2′–2″, and 2″–2 moving at the same velocity. In the case of a weak almost isentropic wave 1–2′ we can readily show [27] that the entropy decreases in a 2′–2″ wave, i.e., such a compression shock wave cannot exist. Hence, it is concluded that the overall shock-wave discontinuity 1–2 is unstable. However, such a proof is explicitly or implicitly based on the assumption that the whole phase path of a relaxing system is projected in the pv plane as continuous motion along the Rayleigh–Michaelson straight line, which is justified only in the case of weak almost isentropic waves solely in the case when the shear viscosity is unimportant.

An analysis of the existence and uniqueness of a shock-wave transition based on a qualitative study of stationary solutions of differential equations describing the motion of a viscous heat-conducting gas followed by reduction of the viscosity and thermal conductivity to zero and a consequent reduction of the finite thickness of the transition layer to a shock-wave discontinuity (see Ref. 28 and the literature cited there) essentially also applies only to weak waves. The equation of state, the equations of gas dynamics, and the linear relationships from the thermodynamics of irreversible processes which occur in them and are used in such analysis are known to be unsuitable for the description of the structure of strong shock waves characterized by a very small width of a density jump characteristic of strong shock waves. The very concept of temperature is inapplicable to the structure of such a jump and this is true even in the case of the subsystem of the translational degrees of freedom of molecules. Nevertheless, the results of such an analysis are of interest at least as a model description. Using this model we can show that a shock-wave transition from a point 1 to a point 2 is structurally possible only on condition that the whole section 1–2 of the shock adiabat between these points is located in the pv plane so that it does not lie to the right of the straight line joining these points. Consequently, intersection of this line with the shock adiabat section 1–2, apart from its boundaries, implies the impossibility (instability) of the $1 \to 2$ shock-wave transition (Fig. 4) at least within the framework of this model.

1.5. Relative positions of the boundaries L^- and L^+ of instability of a shock wave front deduced from the linear theory and of the boundaries A and B of a multivalued representation of a shock-wave discontinuity. Stability of a shock wave in the range of intensities corresponding to the sections AL^- and L^+B of a shock adiabat

1.5.1. $L > 1 + 2\mathrm{Ma}_2$ case. In this case a shock-wave discontinuity can be represented in two other ways using configurations $\overleftarrow{P}K\overrightarrow{Y}$ and $\overleftarrow{Y}K\overrightarrow{Y}$ (see the Appendix; the notation used here and later is as follows: Y is a shock wave, P is an isentropic rarefaction wave, and K is a contact discontinuity; the arrow identifies the direction of motion of the wave). An analysis of the conditions for a multivalued representation of a shock-wave discontinuity can be carried out conveniently using p (pressure) and u (velocity of matter in the laboratory system) as the coordinates, because these variables vary continuously across a contact discontinuity.

The ability to expand a shock-wave discontinuity at a certain pressure p_2 behind it into a $\overleftarrow{P}K\overrightarrow{Y}$ configuration corresponds to the condition of a second intersection of an isentrope drawn in the p–u plane from the point p_2 on the shock adiabat in the direction of lower pressures with the adiabat itself. The initial part of the shock adiabat (weak shock waves) plotted using the coordinates p and u coincides with that isentrope which corresponds to the motion of a behind-front isentropic perturbation in the direction of the wave front. Therefore, a family of isentropes for a perturbation in the opposite direction (we shall denote this family by S^-) *intersects* the initial part of the shock adiabat analogously to the specular reflection of the adiabat from constant-pressure lines and this happens only once–see Fig. 5 (the dependence of p on u in the initial part of the adiabat is nearly rectilinear, since in the case of a weak wave we have $p_2 - p_1 \approx \rho_1 c_1 u$).

The boundary of a region of multiple intersection of an isentrope with a shock adiabat corresponds to (but does not coincide with–as shown below) the point of tangency of these curves which is identified at L^+ in Fig. 5. The tangency condition is expressed in the form $L = 1 + 2\mathrm{Ma}_2$. (In order to avoid misunderstanding, it should be pointed out that intersection of a shock adiabat and an isentrope drawn from a point at a pressure p_2 to a point p_3 in the p–u plane does not imply equality of the entropy on the shock adiabat at the points p_2 and p_3. In the p–v plane such an isentrope does not intersect the shock adiabat at the point p_3.)

The ability to expand a shock-wave discontinuity with the pressure p_2 behind it into the $\overleftarrow{Y}K\overrightarrow{Y}$ configuration corresponds in the p–u plane to the condition of a further intersection of the initial shock adiabat with a shock adiabat for a wave traveling in the opposite direction (we shall denote a family of shock adiabats for waves traveling in the opposite direction by H^-, from the point p_2–see Fig. 5). The boundary of the region of multiple intersections corresponds to the point of tangency of the shock adiabats labeled L^- in Fig. 5. However, the adiabat H^- in the vicinity of the initial point p_2 coincides with the isentrope. Consequently, the point L^- is determined, like L^+, by the condition of tangency of the shock adiabat and an isentrope from the S^- family. However, the condition of tangency can be expressed in the form $L = 1 + 2\mathrm{Ma}_2$. This accounts for the coincidence of the boundaries L^- and L^+ and of the points of tangency mentioned above.

The points of tangency L^- and L^+ and the interval L^-L^+ between them exist if the shock adiabat is S-shaped (Fig. 5). The isentropes of the S^- family and the shock adiabats of the H^- family, which intersect the initial shock adiabat in the interval L^-L^+, intersect this adiabat at two more points, i.e., the number of intersections with this adiabat is three and not one as usual (Fig. 5). This corresponds to three different representations of a shock-wave discontinuity for the S-shaped region AB of the shock adiabat. One of these representations is a single shock wave and the other two represent a combination of several waves and a contact discontinuity. Depending on the position of the point p_2 (governing the intensity of a single shock wave) on one of the characteristic regions of an S-shaped curve, we can have various combinations listed in the Appendix. Three representations correspond to three pressures of a shock wave traveling forward. We shall label them in the order of increasing pressure by $p_3^{(1)}$, $p_3^{(2)}$, and $p_3^{(3)}$. If they are considered as a function of p_2, these three pressures are three branches of the solution, i.e., three roots of a certain equation $p_3 = f(p_2)$, such that one of the roots corresponding to a single shock wave is naturally $p_3 = p_3$. [The actual form of the function $f(p_2)$ can always be determined for any given equation of state,[18] but this is unimportant here.]

The lower and upper branches of the solution exist also far from the S-shaped region (each of which is a unique solution in its own range of values of p_2) and correspond to shock waves which are stable there. However, the middle branch exists only in the middle of the three characteristic sections of the S-shaped curve. This section is bounded by the points of tangency mentioned above, i.e., by the points L^- and L^+. However, according to the linear theory it is in this section that a shock-wave discontinuity is absolutely unstable. Thus, the middle branch of the solution corresponds to an absolutely unstable shock-wave discontinuity. This means that a discontinuity corresponding to any point p_2 in the interval L^-L^+ separating infinite regions of constant flow is specified artificially and immediately begins to decay going over asymptotically with time to new steady-state flow with a configuration of discontinuities which corresponds to the lower or upper branch of the solution, depending on the nature of the initial random (or deliberately specified) perturbation.

As pointed out already, these particular branches correspond to stable shock-wave discontinuities at points on the shock adiabat far from the S-shaped region and such that the solution of the equation $p_3 = f(p_2)$ is unique. In the triple-valued region the solutions naturally correspond to a relatively stable flow in the case of low perturbations and permit a transition from one to another as a result of sufficiently strong perturbations.

Obviously, we can expect here only those relationships which are well known from the physics of other phenomena (for example, in the case of an S-shaped current-voltage characteristic of an arc electric discharge, in the case of nonlinear processes of combustion, detonation with losses in a finite medium[29a] or without losses in an infinite medium but with nonmonotonic heat evolution[29b]) when for certain values of the parameters there are three solutions instead of one (bifurcation takes place) and these alternate in respect of the stability: there are two stable solutions corresponding to two different regimes of the process and one unstable solution between them.

1.5.2. $L < -1$ *case.* The shape of the shock adiabat with a monotonic rise of the pressure and with an interval $L^- L^+$ where $L < -1$ is shown in Fig. 4. The reasons for the multivalued representation of a shock-wave continuity in this case is basically the same as in $L > 1 + 2\mathrm{Ma}_2$ case discussed above. Once again we have two points of tangency L^- and L^+ of a shock adiabat with isentropes and these isentropes intersect the shock adiabat in the interval $L^- L^+$ and have two additional points of intersection with the adiabat. Such triple intersections correspond, as in the case $L > 1 + 2\mathrm{Ma}_2$ case described above, to three branches of the solution of the problem of variants of representation of a shock-wave discontinuity. The points labeled A, L^-, L^+, and B in Fig. 4 separate the section AB into three regions. The indeterminacy of the representation of a shock-wave discontinuity in these three parts and the nature of the stability of the wave configurations can be determined by repeating basically all the operations described above for the case when $L > 1 + 2\mathrm{Ma}_2$. However, there are differences. They follow primarily from the fact that the anomalous part of the shock adiabat is no longer S-shaped, but such as that shown in Fig. 4 between the points A and B.

Isentropes (and shock adiabats) with each of which the initial shock adiabat has three points of intersection belong in terms of the p-u coordinates to the families S^+ (and H^+). These families represent propagation of perturbations in the same direction as the initial shock wave. Wave configurations into which we can expand a single shock-wave discontinuity differ considerably from those in the case when $L > 1 + 2\mathrm{Ma}_2$, because they do not contain waves belonging to the families S^- and H^+. Waves composing a configuration move in the same direction. The existence of a configuration which can be used to describe a shock-wave discontinuity must not only ensure stability of the waves in this configuration. We must also make sure that the second wave does not catch up with the first.

The Appendix gives a scheme of wave configurations depending on the location of the point p_2, governing the intensity of a single shock wave, in one of the characteristic intervals AL^-, $L^- L^+$, and $L^+ B$.

In the case of a nonmonotonic variation of the pressure along the shock adiabat the above relationships are conserved and they describe the relative positions of the boundaries A and B for decay to the $\overleftarrow{P}\overrightarrow{K}Y$ configuration and of points at which $L = 1 + 2\mathrm{Ma}_2$, but between these points there are now sections of the shock adiabats where $L < -1$ (Ref. 18). A fuller analysis of the cases of nonmonotonic variation of the pressure will not be made because such shock adiabats are known to be of no practical interest.

1.6. Solutions of the type described by Eq. (8) with departing waves and the causality principle

In the hydrodynamic formulation of the problem (free of dispersion) weak perturbations travel behind the front of a shock wave at the velocity of sound c_2 which is independent of the frequency. The velocity v_t of perturbations along the shock wave front obtained for this solution given by Eq. (8) is described by

$$v_t = c_2 (1 - \mathrm{Ma}_2 \cos\gamma) (\sin\gamma)^{-1}. \tag{15}$$

A point O in a three-wave configuration travels at the same velocity v_t (Figs. 1–3). In the case of the solution described by Eq. (8) characterized by departing waves and also in the case of the corresponding three-wave configurations in the range of angles defined by Eq. (11) it follows from the identity (10) and the expression (15) that the inequality $v_t > c_2(1 - \mathrm{Ma}_2^2)^{1/2}$ is obeyed. However, the velocity of propagation of an acoustic signal along the surface of a shock wave is known to be $c_2(1 - \mathrm{Ma}_2^2)^{1/2}$. Therefore, for all the angles defined by Eq. (11) a weak perturbation (signal) should propagate, in accordance with the solution described by Eq. (8), at a supersonic velocity in contrast to the predictions based on the laws of linear hydrodynamics. It therefore follows that spontaneous propagation of "ripples" along the surface of the front of a shock wave, described by solutions of the (8) type, should not be in the form of signals within the angular range described by Eq. (11). However, a local initial perturbation of the surface, for example a three-wave configuration (Figs. 2 and 3), can be formed by a suitable superposition of solutions of the type (8) with a constant ratio k/l. In a dispersion-free medium such a perturbation would travel along the front of a shock wave at the velocity v_t (without a change in its profile) and it would form a signal. This applies to any other superposition of solutions of the (8) type with a constant value of the ratio k/l, limited initially along the coordinate x. The edge of such a perturbation also would propagate along the front of a shock wave at the supersonic velocity of Eq. (15).

This contradiction leads to the following definite conclusion: There are no spontaneous perturbations of the front of a shock wave which can be described by solutions of the type given by Eq. (8) with waves departing at angles defined by Eq. (11). These solutions do not satisfy the causality principle.

Formally the solution (8) considered in the linear approximation satisfies the equations of hydrodynamics and the boundary conditions in the form of conservation laws on the front of a shock wave. However, within the range of angles defined by Eq. (11) this solution suffers from an implicit inconsistency. This is because in the range of angles defined by Eq. (11) the solution (8) and the corresponding three-wave configurations do not obey the stability condition of a shock wave given by Eq. (2). It was assumed in Refs. 3–5 that this condition is satisfied initially for an unperturbed shock wave. However, in the solution obtained this causal relationship is disobeyed in a special manner by a perturbed front within the range of angles defined by Eq. (11). The special manner means that "signals" from the perturbed region of the behind-front motion do not reach the point O (Fig. 3) and such regions should be related causally to the point O. The condition (2) for the point O is then satisfied only formally, in accordance with the "letter" but not "in the spirit" of the laws of hydrodynamics. In fact, when the angles are

$$\frac{\pi}{2} < \gamma < \pi \tag{16}$$

the components of the velocity of matter normal to the fronts 2 and 3 at the point O do not apply at all to the behind-front flow identified as a sector III in Fig. 2. In the remaining range of angles defined by Eq. (11),

$$\gamma_0 < \gamma < \frac{\pi}{2},$$

the normal to the front 3 drawn through the point O (Fig. 3) does reach the sector III, in contrast to the case described by Eq. (16), but nevertheless does not cross the region of behind-front flow for the front 3, i.e., the region between the front 3 and a tangential discontinuity (sector III* in Fig. 3). Perturbations from the sector III* do not reach the point O and, consequently, its motion and the state of matter at this point are not linked causally to the parameters of flow in the sector III*. (It should be noted that the normal to the perturbed shock-wave front 2, drawn through the point O, does not pass for any orientation of the front 3 through the region of the behind-front flow for the discontinuity 2, i.e., it does not pass through the region behind the front 2 and a tangential discontinuity. This is due to the fact that the wave 2 is always of the departing type. It appears as a result of the action of an external perturbation source on the shock wave 1.)

This violation of the causal linkage of the motion of the fronts 2 and 3 at the point O with behind-front flow results in immediate decay of the wave configuration first specified in the form shown in Figs. 2 and 3 or in the form of the solution (8): a rarefaction wave travels from the point O within the sector III, but the fronts 2 and 3 become bent, incident and reflected waves are formed by a shock-wave discontinuity, etc. This can be demonstrated by considering, for example, the problem of realization of the solutions of the type given by Eq. (8) or equivalent solutions for the configurations of three waves (Figs. 2 and 3) from the point of view of stability of such solutions. It is shown in Ref. 16b that a local perturbation of these solutions with departing waves, i.e., within the range of angles given by Eq. (11), occupies a region in the vicinity of the point O (Figs. 2 and 3) and this region increases with time so that the orientations of the fronts differ increasingly from the unperturbed orientation.

It is important that the intensity of a perturbation does not increase indefinitely with time and if the original rectilinear front 3 and the corresponding rectilinear part of the front 2 are of limited extent, then the rarefaction waves traveling from the region of behind-front unperturbed flow from left to right (Fig. 3) eventually result in complete decay of the wave 3, which represents a perturbation of the initial shock wave. However, in the case of a three-wave configuration with the arriving wave 3, i.e., when $0 < \gamma < \gamma_0$, a perturbation of the original configuration localized in the vicinity of the point of intersection of the fronts grows with time so that the configuration of waves with sufficiently extended rectilinear sections of the fronts approaches the initial one (in the vicinity of the point O). Variants of such perturbations and their qualitative evolution are shown in Fig. 7.

The results of a qualitative analysis of the stability of the solutions are in full agreement with the above conclusion that there cannot be any spontaneous waves which are not due to an external action or steady-state perturbations of the front of a shock wave with acoustic waves of the type (8) emerging from the front throughout the angular range given by Eq. (11).

The nonphysical nature of the solutions given by Eq. (8) in the case of spontaneous departing waves means the loss of validity of the conclusion of the linear theory of stability that corrugated perturbations of the front of a shock wave are steady-state ones in the range of the values of the parameter L described by Eq. (7).

The problems of the meaning of an infinite reflection coefficient and the meaning of the solution (8) with departing waves and whether it has physical application, as well as the stability of a shock wave in the range defined by Eq. (7), will all be discussed in the next subsection.

1.7. Nonlinear analysis

Limiting transition $p_f \to 0$ for solutions in the form of weak and strong families. Physical meaning of the solutions (8) with departing waves.

An analysis of the reflection of weak perturbations by a shock wave front carried out in the quadratic approximation in Ref. 12 demonstrated that in the case of a given sufficiently low amplitude of the pressure p_f of the incident wave in the vicinity of a resonance angle $\gamma = \gamma_{res}$ there are two solutions for the reflection coefficient $\varkappa$. We shall call them weak and strong families of solutions, by analogy with the familiar results for the reflection of a wave from a rigid wall.[1,2] In the case of the weak family solution when $p_f \to 0$ the pressure p_r of the reflected wave tends to zero as $p_f^{1/2}$ in the vicinity of the angle γ_{res}. As γ moves away from γ_{res}, the weak family solution approaches the corresponding result predicted by the linear theory. The square-root dependence of p_r on p_f demonstrates stability of a shock wave in the range defined by Eq. (7) against the fairly weak perturbations arriving at the front of a shock wave at angles close to the resonance value.

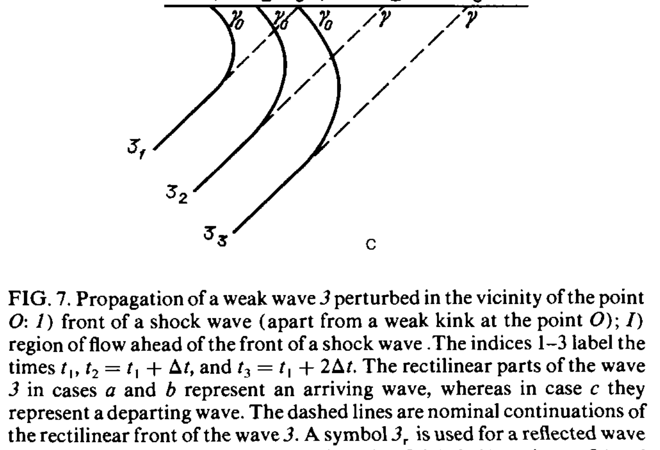

FIG. 7. Propagation of a weak wave 3 perturbed in the vicinity of the point O: 1) front of a shock wave (apart from a weak kink at the point O); I) region of flow ahead of the front of a shock wave. The indices 1–3 label the times t_1, $t_2 = t_1 + \Delta t$, and $t_3 = t_1 + 2\Delta t$. The rectilinear parts of the wave 3 in cases a and b represent an arriving wave, whereas in case c they represent a departing wave. The dashed lines are nominal continuations of the rectilinear front of the wave 3. A symbol 3_r is used for a reflected wave (shown in Fig. 7c on the right above the point O labeled by primes: O'_2 and O'_3).

Infinitesimally small perturbations reflected from the front of a shock wave do not result in a finite change of the wave intensity. The amplitude of a reflected wave becomes infinitesimally small and only the order of smallness is modified.[6] This result allows us also to understand the solutions (8) with departing waves. In the approximation linear in p_r the original cause of a perturbation may be a wave arriving at the resonance angle. Its intensity is proportional to p_r^2 and, consequently, it is outside the scope of this approximation, as are all other nonlinear effects. The causality effect is absent in the linear approximation, but it does exist in reality and if the cause disappears ($p_f \to 0$), so does the consequence ($p_r \to 0$).

Obviously, a weak external perturbation which is quadratic in terms of the pressure of the departing waves, may be also in the form of waves incident on the front of a shock wave at the appropriate resonance angle from the side of flow ahead of the front. In this case a departing wave is formed as a result of refraction of the incident wave (an analysis of the refraction of sound by the front of a shock wave is made in Ref. 11 using the linear approximation).

In the strong family solution the value of p_r is independent of p_f in the limit $p_f \to 0$ and it does not reduce to the results of the linear approximation for angles of incidence far from a resonance. If $p_f = 0$, this solution corresponds to a three-wave configuration with a weak departing wave. The dependence of the angle γ for this configuration on the pressure p of a weak departing wave 3 (Figs. 2 and 3) is described by the equation $\psi(\gamma) + ap = 0$, which reduces to Eq. (9) at $p = 0$ (the coefficient a in the above equation depends on the thermodynamic properties of matter[12]).

The solution (8) with departing acoustic waves can be regarded as the limiting case of the strong family solutions (for example, as an appropriate superposition of three-wave configurations in the limit $p_f = 0$). However, since in this limit there are no arriving waves, we face again the problem of causality of the solution. In the more general formulation this is the problem of the physical meaning of three-wave configurations not only in the limit $p_3 \to 0$, but also in the case of a finite intensity of a weak departing wave.

One can describe a thought experiment, causally fully justified, in which such three-wave configurations could be realized. We shall consider an infinitesimally thin flat piston sliding freely along the plane of a tangential discontinuity (sector III in Fig. 3). In the plane of the figure this piston is in the form of an infinitesimally thin needle (shown dashed) with one of its ends supported at the point O. The pressure on both sides of the needle is the same and the needle has no influence on the flow anywhere with the exception of the singular point O. The length of the needle is unimportant. The orientation of the needle in the system of coordinates in which the point O is at rest and, consequently, the velocity of the needle in the laboratory coordinate system are governed by the parameters of a three-wave configuration (calculations of a configuration carried out in the linear and quadratic approximations are reported in Refs. 12 and 18).

The solution with departing acoustic waves, obtained in the linear theory of stability of shock waves, can be represented by a superposition of such three-wave configurations with infinitely thin pistons (the piston thickness should then be of a higher order of smallness than the distance between neighboring pistons).

Thus, when the inequalities of Eq. (7) are satisfied, we can identify three types of process accompanied by steady-state [in a coordinate system traveling together with the front and with the velocity of Eq. (15) along the front] "ripples" on the surface of the front of a shock wave and by acoustic waves departing from these ripples. Each of these processes represents an external perturbation of the front of a shock wave which is not revealed explicitly in the linear approximation.

In connection with the meaning of the solutions of the (8) type discussed above we must mention another well-known example of how the solution of hydrodynamic equations with a shock-wave discontinuity is valid (i.e., has a physical meaning) only in the presence of an external cause which is not reflected in the equations themselves or in the boundary conditions. We have in mind the solution for what is known as weak or "undercompressed" detonation.[29a,30] A spontaneous detonation is known to travel at the Jouget velocity $D = D_J$, but the solution of the equations of hydrodynamics and the conservation laws for a shock wave admit the possibility of other velocities: $D > D_J$ for the same boundary conditions imposed on the flow at $y = \pm\infty$, where y is the coordinate measured from the front of a shock wave along the lines of flow. The solution corresponding to a velocity $D > D_J$ becomes physically meaningful and describes a real process if matter is "ignited" not by a shock wave but by some other initiator such as an electric discharge or focused light traveling along matter at a given velocity D. The energy and momentum contributed by such initiators are negligible and are ignored in the equations of hydrodynamics. An initiator simply sets the velocity of propagation of the process which would have been impossible (noncausal) in its absence.

The waves arriving at angles close to the resonance value[7] may also be of noise (fluctuation) nature, but a perturbation of the front of a shock wave by such waves and the formation ("generation") of reflected waves ceases after the disappearance of the cause, i.e., after decay of the incident wave.

These results demonstrate stability of a shock wave against fluctuation-type perturbations in the range defined by Eq. (7).

However, in a fuller analysis of the problem of stability of a shock wave against relatively small perturbations in the range of values of the parameter L given by Eq. (7) we must tackle also the following problem. A small perturbation of the behind-front flow which is of finite extent along the coordinate y and is characterized by a pressure p_r, which may be random or created deliberately by an external agency, catches up with the front of a shock wave at an angle close to the resonance value and is reflected from it in the form of acoustic and entropy waves of lower order of smallness ($\propto p_f^{1/2}$).[12] The interaction of these waves creates an arriving wave of higher order of smallness ($\propto p_f$). Its reflection can again reduce the order of smallness to $p_f^{1/2}$ and so on. The question is whether such a multiple reflection process is characterized by divergence of the perturbation amplitude. This question can be answered by considering two different types of an initial arriving perturbation.

1. A perturbation (at a pressure p_f) is a wave oriented at an angle so close to the resonance value that the reflected acoustic and entropy waves are characterized by a lower or-

der of smallness $p_f^{1/2}$. An arriving wave created by the interaction between these waves is characterized by an order of smallness p_f. However, this new wave is oriented along a direction which is far from the resonance. (The orientation is governed by the law of specular reflection of an acoustic wave from an almost plane entropy inhomogeneity and the equality of the angle γ to the resonance value γ_{res} would be an unlikely accident.) Reflection of such a wave from a shock-wave front does not alter the order of smallness of a perturbation and, consequently, this process of successive reflections of a perturbation decays with time.

2. An initial perturbation is not of "single-angle" nature and represents an integral superposition of plane waves with different orientations (i.e., it is a Fourier integral with respect to the wave vector **k**). The amplitude of the pressure of partial waves which have the moduli of the wave vectors lying within an infinitesimally small interval dk is in this case proportional to dk (i.e., it is also proportional to the differential $d\gamma$) and, consequently, this amplitude is infinitesimally low. The amplitude of a wave reflected from a shock wave front is characterized everywhere, with the exception of only the resonance point $\gamma = \gamma_{res}$, by the same order of smallness as the incident wave and can be described by[12]

$$d\,p_r = \frac{-\psi(\gamma_r) + [\psi^2(\gamma_r) - 4\psi(\gamma_f)\,a\,d\,p_f]^{1/2}}{2a} = -\frac{\psi(\gamma_f)}{\psi(\gamma_r)}\,d\,p_f, \tag{17}$$

where γ_f and γ_r are the values of the angle γ for the incident and reflected waves, respectively; a is a coefficient with a value which is unimportant in the subsequent discussion. The dependence of the function ψ on γ and on the parameters representing the thermodynamic properties of matter and the shock adiabat is given in Ref. 12. From our point of view the only important property of this function is that it vanishes only at the resonance reflection angle $\gamma_r = \gamma_{r,res}$. Then, in the vicinity of the point $\gamma_{r,res}$ we have

$$\psi(\gamma_r) = \mathrm{const}\cdot(\gamma_r - \gamma_{r,res}).$$

The pole $\gamma = \gamma_r$ in Eq. (17) does not result in divergence of the integral pressure p_r of the reflected waves because of their interference. Consequently, in calculation of p_r the integral should be the principal value:

$$p_r = -\fint_0^{\gamma_0} \frac{\psi(\gamma_f)}{\psi(\gamma_r)}\,\frac{d\,p_f}{d\gamma_f}\,d\gamma_f.$$

It therefore follows that in case 2 a perturbation reflected from the front of a shock wave does not alter the order of smallness and, consequently, the process of multiple reflection of perturbations decays with time.

According to Ref. 31, the arriving waves generated by a nonlinear interaction between departing waves may, for a certain form of the shock adiabat from the range defined by Eq. (7), result in unlimited amplification of perturbations of the shock wave front.[8)] However, this result applies to monoharmonic waves of finite amplitude incident at the resonance angle (and not to the "white noise" of the fluctuation type). A nonlinear analysis given in Ref. 12 shows that the interaction of weak waves of finite amplitude, alternating in sign in respect of p_f and incident at the resonance angle, with the front of a shock wave does not reduce to a simple configuration of four waves (incident and reflected weak waves, unperturbed and perturbed shock waves; see Footnote 6).

The above analysis completes the proof of impossibility of existence of spontaneous (not induced by an external agency) steady-state perturbations of the front of a shock wave with acoustic waves of the type described by Eq. (8) emerging from this front and traveling within the angular range $\gamma_0 < \gamma < \pi$.

However, it is important to note that, in spite of the stability of a shock wave in the region defined by Eq. (7) against thermodynamic types of noise, the range (7) is still special in the following respects. As pointed out already (see Footnote 6), a regular reflection of a weak perturbation by the front of a shock wave is not always possible. A weak compression or rarefaction [depending on the sign of the coefficient a in Eq. (17)] wave of finite amplitude p_f incident on the front of a shock wave at an angle γ is separated from the resonance value by a sufficiently small amount $\Delta\gamma = \gamma_{res} - \gamma$ (the higher the value of p_f, the greater can be the difference $\Delta\gamma$),[12] results in a qualitative modification of the shock wave: this wave decays into a wave of a very different intensity and into other elements (components), by analogy with the decay of an arbitrary discontinuity. In other words, a shock wave defined in the range (7) is unstable against small perturbations of special type in the form of monochromatic finite-amplitude waves incident at a near-resonance angle.

A second special property of shock waves in the range (7) is anomalously strong amplification of certain components of the noise background incident at angles close to resonance angles of reflection or refraction. Consequently, such components are special compared with other random perturbations.

1.8. Stability of a shock wave sustained by a piston

We shall conclude our analysis of the problem of the hydrodynamic stability of shock waves by considering the interaction of a shock wave with a piston. Strictly speaking, this is outside the scope of the present paper and it represents the problem of stability of flow characterized by shock waves (see, for example, the collection of papers listed as Ref. 32). However, under conditions typical of investigations of shock waves and in the case of different physicochemical processes occurring in shock waves some form of a piston is often used. For example, in shock tubes the role of a piston may be played by a dense driver gas; when a wave is generated as a result of impact of the investigated material against a membrane with a high acoustic rigidity the membrane itself acts as a piston. Finally, anybody moving in a medium at a supersonic velocity can be regarded as a piston of finite transverse dimensions for a departing shock wave. Therefore, the problem of stability of a shock wave sustained by a piston can be justifiably considered as belonging to the theory of stability of shock waves.

The problem can be formulated as follows. Let us assume that a given constant velocity u of a flat piston corresponds to a specific velocity of a shock wave generated by this piston. Let us also assume that the velocity of the piston changes instantaneously by a small amount δu. A weak planar perturbation generated in this way catches up with the shock wave, is reflected by it, is then reflected by the piston, and so on. We have to consider whether this process of multiple reflections is converging, i.e., whether it creates a new steady-state motion in which the shock wave velocity differs from the initial value also by a small amount. An analytic solution of this problem is given in Ref. 23, whereas a graphical variant of the solution is provided in Ref. 33. The answer to this question follows also from an analysis of multiple reflections of a perturbation by parallel fronts of two plane shock waves moving in opposite directions.[34] The solution is in the form of an inequality imposed on the parameter L [see Eq. (4)] and can be formulated as follows. The process of multiple reflections is convergent if $L < 1$, but divergent if

$$L \geqslant 1, \tag{18}$$

i.e., a wave sustained by a piston is stable when $L < 1$ and unstable when the opposite inequality is obeyed.[9)]

The condition (18) is no as difficult to satisfy in practice as the second inequality of Eq. (6), but this has not yet been done. The possibility of satisfying the criteria of instability is considered in Secs. 5 and 6.

Numerical modeling of the process of the interaction of a shock wave with a piston and the associated instability of flow in the case of a special theoretical model of the equation of state, leading to parts of a shock adiabat with $L > 1$ and also with $L > 1 + 2\mathrm{Ma}_2$, are given in Ref. 36.

2. POSSIBLE STRUCTURAL INSTABILITY OF A SHOCK WAVE

2.1. Introductory comments and a brief summary of the data on a structural instability of shock waves

As pointed out in Sec. 1.1., the problems of instability of the structure of shock waves are of interest basically in the solution of those problems in which we have to know the characteristics of flow on a small scale comparable with the size δ of a relaxation layer in a shock wave. In Sec. 4 we shall deal with those exceptions when the structural instability and the nature of flow behind the front are mutually related at distances large compared with δ.

The hydrodynamic conditions of instability of shock waves [Eq. (6)] or even the less stringent conditions [Eqs. (18) and (7)] for an unstable interaction of a shock wave with a piston and for resonant reflection and refraction of sound, respectively, are usually difficult to satisfy because the required thermodynamic properties of matter are difficult to establish (Sec. 5). Therefore, for a long time the problem of a possible instability of a shock wave has been regarded as largely academic from the practical point of view: shock waves employed in practical applications are as a rule stable.[10)] Moreover, even detonation waves characterized by a strong positive feedback between the intensity of a shock wave initiating heat evolution and the rate of heat evolution are stable on a large scale. Only very fine special investigations carried out in the fifties (see monographs listed as Refs. 30 and 38) have established that detonation waves stable on a macroscopic scale have often a nonlaminar "pulsating" structure. When the compression created by a detonation wave is sufficiently high, these structural features of its front disappear. These observations together with other extensive experimental evidence that shock waves are structurally stable have been used even in approximate estimates of the degree of compression of detonation waves at which the pulsating structure disappears.[39]

However, accumulation of experimental data on the propagation of strong shock waves in various media exhibiting complex relaxation processes (vibrational relaxation, chemical reactions, shock and radiative processes resulting in electron excitation and ionization) have revealed some deviations from a simple stationary structure of the waves characterized by a homogeneity along directions parallel to the wave front and by monotonic (and in some very rare cases weak nonmonotonic) changes in the density and pressure along the lines of flow. The nature of such deviations varies within wide limits ranging from one-dimensional sequences of maxima and minima of the density along the lines of flow[40–43] to a radical modification of the structure[41,42,44–49] and its "turbulization" manifested by a random distribution of dark and bright spots which are visible when the front is viewed from the end of a shock tube in very strong shock waves (as reported for rare gases argon and xenon[49]). These papers were only a few selected from a large number of experimental reports. Some original experimental data and a bibliography of the subject can be found in Ref. 48.

2.2. Theoretical considerations concerning structural instability of shock waves and some tasks for the future

The intensities of a shock wave required for the observation of qualitative changes in the pattern of behind-front flow, given in Sec. 2.1., vary within wide limits depending on the investigated gas and other experimental conditions. There is no doubt that we are dealing here with a variety of different phenomena and much experimental and theoretical work is needed to identify the causes and mechanisms underlying them.

First of all, it is important to establish in what cases can we expect an intrinsic instability of a wave structure and in which cases the sources of the observed perturbations are some boundary effects which simply imitate an instability. In view of technical difficulties such an analysis is frequently not carried out. However, there is some experimental evidence of the boundary effect. There are reports[50–53] of a strong chaotic perturbation of the front of a shock wave due to the familiar Rayleigh–Taylor instability[54,55] of a contact discontinuity representing a contact surface which separates the driver and driven gases in a shock tube[50] or which is created by exploding a condensed explosive material in a gaseous medium (a cylindrical charge is exploded in air argon, or xenon as reported in Refs. 51–53).[11)] As the front of a shock wave moves from the contact surface, the intensity of perturbation of the front should decrease. However, when a contact discontinuity becomes turbulent sufficiently rapidly, the perturbations arriving from this discontinuity may still deform significantly the front of a wave at the moment when it is recorded with measuring apparatus.

It is pointed out in Ref. 49 that the crisis phenomena observed through a side window in the case of propagation of strong shock waves in xenon and argon[41,42] may be due to the appearance of a precursor in the form of products of evaporation of the walls of a tube heated by radiation. The characteristic features of the structure of a shock wave included also complex wall wave configurations formed under certain conditions[56–58] and other effects of the interaction of a flowing gas with the wall of a shock tube. The formation of vortices in Freons behind a curvilinear front of a departing shock wave flowing around a blunt body and the likely transient perturbations of the front[59] must also be distinguished from a structural instability of the front.

In view of insufficient experimental data on the structural instability of shock waves, it would be highly desirable to investigate this topic from the general scientific point of view and also with a view to practical applications.

We shall now consider briefly the theoretical ideas on a possible structural instability of shock waves and we shall outline some tasks for the future.

The example of a combustion wave is used in Ref. 60 to show that weak waves which have[1] a very large (compared with the mean free path of molecules l) relaxation zone, due to macroscopic dissipative processes of viscous flow and conduction, are structurally stable. A viscous density jump in strong shock waves of size comparable with l cannot be described by the equations for the mechanics of continuous media. From the microscopic point of view such a structure is not stationary: Molecules in this structure undergo random motion and we can speak of the average stationary conditions. The scales of microscopic inhomogeneities in a density jump of a strong shock wave are comparable with l. Averaging over a macroscopic region containing an enormous number of molecules results in manifestation of microscopic inhomogeneities only in the form of fluctuations which are much larger in the structure of a jump than under thermodynamic equilibrium conditions,[61-64] but are nevertheless very small compared with thermodynamic and gasdynamic properties of a macroscopic region. The influence of such small fluctuations on the flow on a macroscopic scale are important only in those cases when the flow is unstable. This is possible if the hydrodynamic stability criteria are not satisfied (so that decay of a shock-wave jump into other stable elements or components is possible–see Sec. 1.3.). However, if according to the hydrodynamic criteria a shock wave is stable (which is usually true), an instability of the structure of a shock wave on the macroscopic scale is possible only in those cases when relatively slow relaxation processes occur in a shock wave and these are responsible for the macroscopic size or length δ of the relaxation zone (these processes include vibrational relaxation, chemical reactions, impact and radiative excitation of electrons, and ionization).

An instability can grow if there is a positive feedback between the kinetic factors or between hydrodynamic and kinetic factors. The mechanisms of a positive feedback (or at least some of them) are known for detonation waves.

One of the reasons for an instability of a plane front of a detonation wave is as follows. The hydrodynamic theory of one-dimensional steady-state adiabatic flow predicts that the pressure along the lines of flow should decrease (increase) if the flow is accompanied by exothermal (endothermal) reactions. Behind the front of a detonation wave the reactions are exothermal (heat is evolved) and the pressure decreases. This falling pressure curve, considered as a function of the distance y from the front of a shock-wave discontinuity, begins at the point $y = 0$ and terminates at the acoustic point (Jouget point)[1,29a,30,38]; this curve is frequently called a chemical peak.

A small random increase in the pressure in some section of a tube of flow adjoining the front of a shock wave of a detonation system shortens the heat evolution time in this flow tube (we shall call it the central tube) and results in a faster fall of the pressure in a chemical peak. Almost throughout the falling part of the chemical peak the pressure in the neighboring flow tubes is higher than the pressure in the central flow tube, which results in compression in this central tube and an increase in the energy carried by it. This may be sufficient for further acceleration of the exothermal reaction and reduction of the size of the chemical peak in the central flow tube, etc. A qualitative approximate criterion of the existence of such a positive feedback, sufficient for the development of an instability, was obtained by Shchelkin[65] (see also Ref. 66 for refinement of this criterion and Ref. 67 for a qualitative analysis of the mechanism of fluctuations of a detonation front). It should be stressed that the main "elements" of this instability mechanism are the existence of a chemical peak and a sufficiently strong reduction of the extent of this peak on increase in the shock wave intensity. Qualitatively similar criteria follow from a linear theory of stability of a detonation wave[68,69] and from an analysis of the stability of a detonation wave considered in the one-dimensional approximation.[70]

However, in contrast to a detonation wave, a feedback between the hydrodynamic and kinetic processes in a shock wave is usually negative: instead of a fall of pressure typical of a detonation-wave chemical peak, the pressure usually rises in the relaxation zone of a shock wave and this is due to the endothermal nature of the irreversible processes. The exception to this "rule" is represented by the cases of nonmonotonic absorption of heat which may occur in principle when several irreversible processes appear in a sequence. Then, in some parts of the relaxation zone of a shock wave instead of the usual reduction in the energy of the translational degrees of freedom of a gas there is an increase in this energy which creates a structure representing a detonation-wave chemical peak. However, the pressure at the maximum of such a chemical peak is known[71-73] to exceed the pressure in the equilibrium zone behind the front of a shock wave only by an amount of the order of 1%. Such a weak chemical peak is clearly insufficient for the appearance of a positive feedback of the detonation type and for an instability of the wave structure irrespective of the actual dependences of the rates of relaxation processes on the shock wave intensity.

A much higher (by about an order of magnitude) chemical peak was reported in Ref. 74 when calculations were made of the structure of a strong shock wave in xenon (for the Mach number range $\mathrm{Ma} \approx 20$–40). According to Ref. 74, this result is due to accumulation in the wave structure of high concentrations of diatomic ions Xe_2^+ in excess of the equilibrium value. However, the treatment in Ref. 74 suffers from two inaccuracies: at the investigated intensities of a shock wave

1) the data on the cross section of the two-channel reaction

$$Xe + Xe \longrightarrow \begin{cases} Xe_2^+ + e, \\ Xe^+ + Xe + e, \end{cases}$$

used in Ref. 74 to calculate the rate of formation of Xe_2^+ apply in reality mainly to the second reaction channel;

2) the enthalpy of the vibrations and rotation of the molecular ion Xe_2^+, assumed to be $(5/2)kT$ in Ref. 74, is strongly overestimated because at the investigated tempera-

tures kT, which are 2–2.5 times higher than the dissociation energy D of the Xe_2^+ ion, the vibrational and two rotational degrees of freedom of this ion have a total energy close to D, i.e., 5–6 times less than $(5/2)kT$. Introduction of appropriate corrections reduces a chemical peak to values of the same order of magnitude as in the cases mentioned above ($\sim 1\%$) or suppresses such a peak completely.

The following general conclusions can be drawn about this detonation-type structural instability. No structures with a sufficiently clear chemical peak of shock waves have been found so far. Such structures are theoretically possible, but their manifestation in a shock wave requires a large separation on the time scale between high-energy relaxation processes of which fast should be endothermal and slow exothermal. These requirements are almost mutually exclusive, suggesting that structures of this type can be only a rare exception.

We shall complete our discussion of the instability mechanism due to a chemical peak in the relaxation zone of a wave by noting that a positive feedback exists in this mechanism in the case of a stationary (or a quasi-stationary) structure of the perturbed region of flow. Therefore, the characteristic size of inhomogeneities which can make this instability mechanism effective should be sufficiently large to establish a quasistationary structure during the existence of a perturbed flow region. The experimentally observed cellular structure of the front of detonation waves corresponds to the required scaling conditions and this is in qualitative agreement with the above model of a positive feedback between the kinetics of chemical reactions and the hydrodynamics of flow in a detonation wave. However, we cannot exclude the possibility that there may be also other effective mechanisms of a positive feedback between the kinetics and hydrodynamics which can have a smaller scale compared with the characteristic scales in space and time needed to establish steady-state flow in the relaxation zone of a wave. Such an instability mechanism would clearly be limited by more stringent conditions imposed on the dependences of the rates of chemical reactions on the wave intensity, but it would not be related to the presence of a chemical peak and may act even if the alternative instability mechanism discussed above is absent. Investigations of the criteria of a relaxation instability of this type in shock waves would be of considerable interest.

Another structural instability may be due to the existence of singularities in a system of equations describing physicochemical relaxation processes. Changes in the pressure and volume along the Rayleigh–Michaelson straight line during relaxation then provide only a background representing an additional condition of the same type as, for example, the condition that a process should be isobaric or isochoric. We have in mind here a possible instability of the solutions of the relaxation equations manifested by a strong "divergence" of phase paths (including also the initial data of the integral curves) as a result of small perturbations. Such paths are located near separatrices of singular points of a system of equations.

Thermodynamic properties of all substances and the properties of all the relaxation equations describing the relatively slow transition of a system to an equilibrium state are such that integral curves emerging from a small region near one common initial point (at the initial moment in time $t=0$) converge again in the limit $t \to \infty$ in a small neighborhood of a certain point in the phase space corresponding to an equilibrium behind the front of a shock wave. This is a fairly stringent limitation for this type of "divergence" of paths. However, in principle, this is still possible.

An analysis of the singularities and the field of integral curves of a system of relaxation equations is very difficult because of the multidimensional nature of the phase space and because of the nonlinearity of the equations. The problem requires further study in the case of specific problems and a search should be made for general relationships.

It should be pointed out that the attempts (sometimes found in the scientific literature) to investigate the stability of the solution of a system of equations describing the structure of shock waves by linearization of perturbations and by solution of the secular equation for the eigenfrequencies using instantaneous (local) values of the coefficients in such equations (i.e., by a method which is valid in the case of linear equations with constant coefficients) are not self-consistent and may yield fundamentally wrong conclusions.

In classification of the singularities of the structure of a shock wave one should allow also for the possibility of alternation of several maxima and minima of the pressure, density, or other variables representing the regular structure of a wave. A regular quasisteady structure (with decay of oscillations on approach to a thermodynamic equilibrium) can be, unless it is due to the wall effects, the result of reproducible alternation of endothermal and exothermal relaxation processes from one experiment to another.

According to Ref. 43, a structure with several oscillations of the density and concentrations of the components was observed in the course of propagation of a shock wave in gaseous NO_2 (when the initial pressure was 0.25–4.5 Torr and the temperature was $T_2 = 2600$–3000 K). It was assumed that these oscillations were due to a positive feedback between the rate of dissociation of the NO_2 molecule and the vibrational energy E of the NO_2 and O_2 molecules influencing this rate. Accumulation of the energy E occurred as a result of a secondary exothermal reaction

$$NO_2 + O \rightarrow NO + O_2.$$

However, other authors (Zuev, Tarasenko, and Tkachenko[75]) failed to observe oscillations in the wave structure in analogous experiments. Further experimental and theoretical investigations are needed before a clear idea of the structure of a shock wave in NO_2 can be provided.

3. STABILITY OF A SHOCK WAVE PROPAGATING IN A PREVIOUSLY EXCITED SUBSTANCE

3.1. Thermodynamic-equilibrium excitation

A hydrodynamic structural instability of a shock wave may be due to the action of external energy sources resulting in thermodynamic-equilibrium or selective excitation of the degrees of freedom of molecules ahead of the front of a shock wave or behind the front. A thermodynamic-equilibrium excitation of a substance ahead of the front of a shock wave does not give rise to any fundamentally new aspects of the wave stability. Such excitation is simply one of the methods of establishing an initial equilibrium state which gives rise to a family of adiabats instead of one shock adiabat. The problems of stability associated with a possible form of such adiabats and with the characteristics of the structure of shock waves propagating in a substance under thermodynamic equilibrium conditions are discussed in Secs. 1 and 2.

3.2. Nonequilibrium excitation

Relatively new results on shock waves were provided by a series of investigations on the passage of shock waves through matter disturbed previously from its thermodynamic equilibrium state by some external energy source (electric discharge, infrared laser radiation, etc.) acting homogeneously along directions parallel to the plane of the front.[12] Nontrivial experimental results can then be obtained if a shock wave strongly accelerates the relaxation process. Otherwise either relaxation occurs before the arrival of a shock wave or the region of constant flow behind the front of a shock wave can be too large and, therefore, unattainable in the case of typical experimental scales. However, when this condition applies, a substance subjected to a preliminary excitation is similar to any other substances which are in a metastable state of thermodynamic nonequilibrium and, in principle, capable of detonation.[76] Strictly speaking we are then dealing with such an adiabat for equilibrium states in the front of a wave which is of detonation rather than shock type. In contrast to a shock adiabat, it does not (in particular) pass through the point representing the initial state of the investigated substance.[1]

An equilibrium detonation adiabat and a shock adiabat corresponding to the state of matter compressed at a density jump, but still "frozen" in respect of all the degrees of freedom (except translational and rotational), usually satisfies the hydrodynamic stability conditions of Eq. (5). However, a structural instability of such quasidetonation processes is very probable. In theoretical estimates of the range of the parameters representing a nonequilibrium state ahead of a wave and its intensity (degree of compression), in which the wave is structurally unstable, we can use criteria derived in the theory of stability of detonation waves. However, in each specific case it is difficult to find the limits of such a range of parameters because of the difference between the real relaxation processes and those simple models of the kinetics of heat evolution which are used to obtain the stability criteria for a detonation wave.

4. INTERRELATIONSHIP BETWEEN HYDRODYNAMIC AND STRUCTURAL STABILITIES OF SHOCK WAVES

It follows from Secs. 1 and 2 that in an investigation of the stability of a shock wave as a whole it is usually sufficient to consider separately the hydrodynamic and structural aspects. Thus, if a wave is unstable in accordance with the hydrodynamic criteria of the linear theory, it is pointless to consider the stability of its structure (because there is no wave). However, if a shock wave is stable in accordance with the hydrodynamic criteria, the structural "pulsations" do not disturb this stability because on a large scale such pulsations decay as a result of dissipative friction and heat transfer processes. However, strictly speaking, this last conclusion is not always true. There are at least two cases when a hydrodynamic instability develops from a structural instability or when they are closely related representing one unified problem.

1. As pointed out in Sec. 1.5., an instability boundary deduced from the linear theory is adjoined by parts of a shock adiabat where the criteria of Eq. (5) deduced from the linear theory are satisfied, but a shock-wave discontinuity can be expanded into other elements. In such cases the problem of the stability of a shock-wave discontinuity against decay into these elements should be solved by an analysis of the structural stability of the discontinuity.

2. There is a possible unified mechanism of the hydrodynamic and structural instability. Propagation of a shock wave may be accompanied by energy losses depending on the wave intensity and on the relaxation processes in its structure. In the case of an unbounded medium such losses do occur as a result of, for example, escape of electromagnetic radiation ahead of a wave penetrating an unperturbed gas. Such radiation has a finite range at any frequency. Therefore, if the scale of the problem is sufficiently large, there are no losses whatever and the role of the radiation reduces to just an additional factor of the formation of the shock-wave structure. However, usually in the case of realistic scales of the problem (for example, in experiments on shock tubes or in the case of supersonic motion of a blunt body) a cold gas ahead of a wave is practically transparent to a large part of the radiation spectrum and such radiation escapes forward to "infinity," so that losses do occur.

The form of an equilibrium shock adiabat depends on the relative magnitude of the losses and if this magnitude changes as a result of a change in the intensity of a shock wave, then this shock adiabat may in principle have regions that do not satisfy the hydrodynamic criteria of stability, the criteria of stability of motion of a wave driven by a piston [Eq. (16)] or the conditions of resonant reflection and refraction of weak perturbations [Eq. (7)] for specific (resonance) angles of incidence.

Calculations of a shock adiabat in the specific case of hydrogen and rare gases[77] have shown that radiative losses (including part of the radiation emitted by a black body from the equilibrium zone behind the front of a shock wave, which corresponds to frequencies lower than the transition frequency of an atom to the first excited state) do not result in conditions under which resonant reflection or refraction of weak perturbations is possible [and particularly the conditions of instability given by Eq. (6) are not satisfied]. However, this is only a small proportion of the necessary future investigations. The radiative losses generally depend on the structure of a shock wave front, which itself forms with the aide of radiation. The limiting case of a strong influence of radiation on the structure of a shock wave is encountered for waves with what is known as the critical or supercritical intensity.[78] However, in the case of mutual relationships of relaxation processes in the structure of a shock wave and the losses due to the emission of radiation which could result in a structural or a hydrodynamic instability, it is of interest to consider also intensities which are much lower than critical. (It is reported in Ref. 49 that the radiation emitted by the front of a strong shock wave in rare gases has a spotted structure.)

One should mention also that a characteristic interaction of structural processes with a shock-wave discontinuity as a whole occurs when a shock wave satisfies the conditions of the hydrodynamic stability "at the limit," i.e., when a small perturbation is sufficient to disturb this stability. If at the same time the wave is structurally unstable, then these perturbations can be structural "pulsations" that have not been smoothed out completely outside the relaxation zone. In view of the chaotic nature of such pulsations, their dissipation, and interference effects, the investigated interaction of the instability mechanisms can apparently result only in some lengthening of the region of decay of pulsations but not in an instability of flow on a large scale. A more detailed analysis of these processes is not essential because of the very low probability of such a combination of the initial conditions.

5. WAYS OF SATISFYING THE CRITERIA OF THE HYDRODYNAMIC INSTABILITY OF SHOCK WAVES IN PRACTICE

The criteria of the instability of shock waves [Eq. (6)] obtained over 30 years ago are still basically theoretical predictions. These criteria are not in conflict with the thermodynamic inequalities, $(\partial p/\partial v)_T < 0$ and $c_v > 0$, but they are nevertheless so "anomalous" from the thermodynamic point of view, that substances have not yet been found with the equations of state satisfying the inequalities of Eq. (6), with the exception of those cases when shock adiabats have a kink or a continuous inflection shown in Fig. 4. [In a linear analysis of the stability in Ref. 3 it is assumed that a shock adiabat has no inflection points, such as the point L (Fig. 4).] The limiting case (with a kink of the shock adiabat shown in Fig. 4) of the inequality $L < -1$ may be satisfied by first-order phase transitions or in the case of plasticity. It is known that in this case instead of one wave, we can expect—under certain conditions—configurations with the main elements in the form of two shock waves traveling in the same direction (see Fig. 21 in the Appendix). The inequality $L < -1$ is also realized in very rare cases of a negative second isentropic derivative[1]: $(\partial^2 v/\partial p^2)_S < 0$, provided this inequality is satisfied in the vicinity of the initial point of a shock adiabat.

Another criterion of the instability—represented by the second inequality in Eq. (6)—is in practice much more difficult to satisfy and this has not yet been done.

It is important to note also that the smaller values of the parameter L within the limits set by Eq. (7) have so far been established reliably only in exceptional cases: in a certain range of states of a two-phase liquid–vapor system (such as copper[79] or water [80]; the stability of shock waves in gases is discussed below in Sec. 6.1.). We recall that the values of L satisfying the inequalities in Eq. (7) are remarkable because for each value there is a specific angle of incidence which ensures resonant reflection or refraction of sound by the front of a shock wave. Moreover, the wave is then unstable against one of the stages (compression or rarefaction) of a resonant perturbation of finite amplitude (Sec. 1.7.). The general nature of the thermodynamic properties of two-phase liquid–vapor systems (see Refs. 81 and 82) allows us to assume that the range defined by Eq. (7) exists also in the case of other two-phase systems, such as Freons. The possibility of experimental observation of resonant reflection and other characteristics of the interaction of the front of a shock wave with small perturbations in systems of this kind is limited because of gravitational convection (floating up of gas bubbles) and long relaxation times (in the case of heat and mass transfer). However, there are certain characteristic possibilities of realization of the instability conditions [Eqs. (6) and (16)] or the conditions of resonant reflection of sound [Eq. (7)] which are not related to anomalies of thermodynamic properties, but are due to changes in the equations of conservation for a shock wave which are due to some energy losses, for example, radiative losses. According to the calculations[77,83] carried out for hydrogen and rare gases (specifically, xenon) an increase in radiative losses on increase in the wave intensity does not give rise to regions of shock adiabats described by Eqs. (6) and (7). However, in view of the complex dependences of the spectral coefficients of the absorption of light on the temperature of a substance and its density, regions defined by Eqs. (6) and (7) may exist and the problem requires further investigation.

6. STABILITY OF SHOCK WAVES IN DISSOCIATING AND IONIZABLE IDEAL GASES

6.1. Hydrodynamic instability

The shock adiabats of dissociating and ionizable gases are characterized (see, for example, Ref. 83) by a nonmonotonic temperature dependence of the specific volume with alternation of regions with negative (such as that, for example, for an ideal gas with a constant Poisson exponent of the adiabat) and positive derivatives $(\partial v/\partial p)_H$. This form of shock adiabats is due to alternate contributions (on increase in temperature) to the thermodynamic functions of the processes of dissociation, and of single, double, and multiple ionization, and it is manifested increasingly strongly on reduction in the initial gas density.[13)] It would seem that in this manifold of the values of $(\partial v/\partial p)_H$ there is a real possibility of satisfying the various instability criteria by varying the parameters of the initial state of a gas and of the shock wave intensity. However, sampling numerical calculations demonstrate[77,84] that in all the cases discussed the shock adiabats of dissociating and ionizable gases satisfy the inequalities of Eq. (5), which correspond to the stability of shock waves and the absence of resonant reflection and refraction of sound by the front of a shock wave. However, in many cases the values of L and L_0 are close. Moreover, in these calculations the initial gas is assumed to be so cold that both dissociation and ionization in this gas can be ignored. Therefore, on the basis of these calculations we cannot say that the inequalities of Eq. (5) are satisfied in the case of dissociating and ionizable ideal gases in general.

Additional analytic and numerical investigations are needed on this topic. It should be mentioned that the inequality $L > L_0$ may be obeyed without the need to satisfy $(\partial v/\partial p)_H > 0$ (see Ref. 80). Therefore, in such investigations we cannot confine our attention to just those parts of the shock adiabat where $(\partial v/\partial p)_H > 0$.[14]

6.2. Structural stability

Almost all the relaxation processes in shock waves traveling along gases initially in thermodynamic equilibrium are definitely endothermal. The exception to this rule is represented by some secondary reactions such as the formation of ozone (when the molecular oxygen O_2 is the initial gas), and of H_2 and Cl_2 molecules (if the initial gas is HCl), and so on, as well as the formation of negative ions. However, the concentrations of such "secondary" components in gases with normal or low density do not exceed a few percent (and are even less for negative ions) for any intensity of a shock wave, and the characteristic constants of endothermal and exoth-

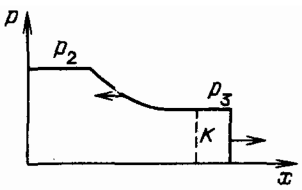

FIG. 8. Configuration $\overleftarrow{P}K\overrightarrow{Y}$: $p_{A} < p_3 < p_{L^-}$. (Transition from the middle branch and the section $L^- L^+$ to the lower branch and the section AL^-.) The configuration is stable against small perturbations.

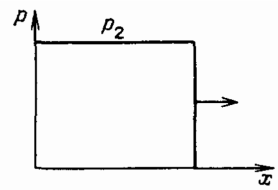

FIG. 10. Single shock wave. Absolutely unstable shock-wave discontinuity. Decays to any one of two configurations (Figs. 8 and 9).

ermal processes do not satisfy those conditions under which we could obtain a structure with a clear chemical peak characterized by a density tens of percent higher than in the equilibrium zone behind the shock wave front. Usually, the stronger the endothermal nature of the process, the longer the characteristic time τ_i of this process. However, for a considerable degree of supersaturation of any high-energy internal degrees of freedom of particles, leading to the formation of a chemical peak, the dependence of τ_i on the endothermal nature of the process should be opposite. The published calculations of the structure of shock waves[71,73,87] are in agreement with these qualitative ideas.

A structural instability of a shock wave in gases may probably be associated with radiative transport and its influence on the length of the relaxation zone (see Sec. 2.2.). In view of the scarcity of data on the spectral coefficient of absorption of electromagnetic radiation by "hot" components of a gas in the nonequilibrium zone of a shock wave, a theoretical prediction of such a structural instability requires solution of a number of auxiliary problems.

6.3. Shock wave in a gas which is not in thermodynamic equilibrium

All that we have said in Sec. 3.2. on the passage of a shock wave through a previously excited gas which is not in thermodynamic equilibrium applies fully to a dissociating and ionizable gas. Since a molecular gas is characterized by several slowly relaxing high-energy subsystems (internal molecular vibrations, chemically reacting components, etc.), there are some interesting problems of the stability of a shock wave in a gas in which any one of these subsystems is excited selectively ahead of the wave. Papers on these topics have been usually concerned with one-component diatomic gases. The propagation of a plane shock wave in vibrationally excited nitrogen and in other gases, and attainment of self-maintained detonation are discussed in Refs. 88–91. Calculations of a detonation adiabat and of the Jouget point parameters in the case of a gas which is not in vibrational equilibrium are reported in Refs. 90–92. The structure of a detonation wave in such a gas is investigated in Ref. 93. Amplification of an acoustic wave traveling through a nonequilibrium diatomic gas and its conversion into a shock wave is discussed in Refs. 94–97. The problems of the interaction of a shock wave with inhomogeneities present initially in a gas are treated more generally in Ref. 98.

Experimental data on the passage of a shock wave through a previously excited gas have been obtained mainly by excitation in an electric discharge and are reported, for example, in Refs. 74 and 99–103.

APPENDIX. CASES OF A MANY-VALUED REPRESENTATION OF A SHOCK-WAVE DISCONTINUITY

I. A shock adiabat with a monotonic increase in the pressure exhibiting a section where $L > 1 + 2\mathrm{Ma}_2$

A shock adiabat of this type is shown in Fig. 5 using the pv and pu coordinates. At the points L^- and L^+ this shock adiabat and isentropes of the S family have common tangents.

A shock-wave discontinuity has a triple-valued representation in the form of a single shock wave and two more complex wave configurations in a section $L^- L^+$ and in sections AL^- and $L^+ B$ adjoining it from below and above. An isentrope of the S family which touches the shock adiabat at the point L^+ intersects this adiabat at the point A. This gives the position of the point A. A shock adiabat of the H family, drawn from L^- as the starting point, intersects the initial shock adiabat at the point B.

Representations of a shock-wave jump with an intensity (pressure p_2) corresponding to sections $L^- L^+$, AL^-, and $L^+ B$ of a shock adiabat:

1) $p_{L^-} < p_2 < p_{L^+}$ (Figs. 8—10);
2) $p_A < p_2 < p_{L^-}$ (Figs. 11, 12);
3) $p_{L^+} < p_2 < p_B$ (Figs. 13, 14).

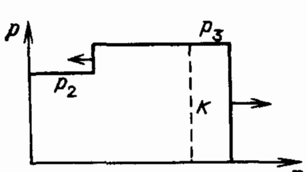

FIG. 9. Configuration $\overleftarrow{Y}K\overrightarrow{Y}$; $p_{L^+} < p_3 < p_B$. (Transition to the upper branch and the section $L^+ B$.) The configuration is stable against small perturbations.

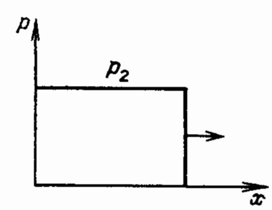

FIG. 11. Single shock wave. Shock-wave discontinuity stable against small perturbations.

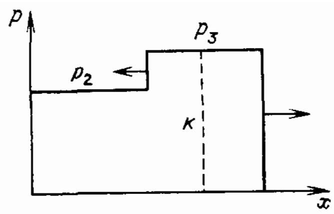

FIG. 12. Configuration $\overleftarrow{Y}K\overrightarrow{Y}$; $p_{L'} + < p_3 < p_B$ (transition to the upper branch and the section $L'B$) or $p_L < p_3 < p_{L'}$ (transition to the middle branch and the section $L\ L'$). In the first case the configuration is stable against small perturbations. In the second case the configuration is unstable since a shock-wave discontinuity at a pressure p_3, lying within the range $p_L < p_3 < p_{L'}$, is absolutely unstable (Fig. 10).

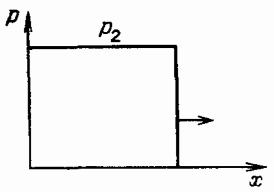

FIG. 13. Single shock wave. Shock-wave discontinuity unstable against small perturbations.

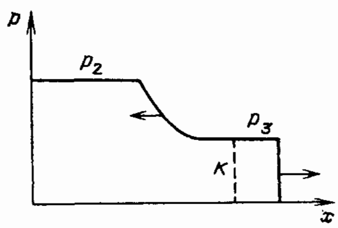

FIG. 14. Configuration $\overleftarrow{P}K\overrightarrow{Y}$; $p_A < p_3 < p_L$ (transition to lower branch and the section AL) or $p_L < p_3 < p_{L'}$ (transition to the middle branch and the section $L\ L'$). In the first case the configuration is stable against small perturbations. In the second case the configuration is unstable since a shock-wave discontinuity at a pressure p_3, lying in the range $p_L < p_3 < p_{L'}$, is absolutely unstable.

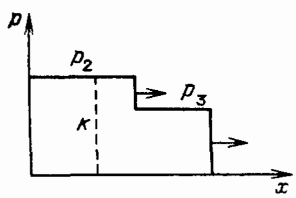

FIG. 15. Configuration $K\overrightarrow{Y}\overrightarrow{Y}$; $p_A < p_3 < p_L$. (Transition to the lower branch and the section AL.) Stable configuration.

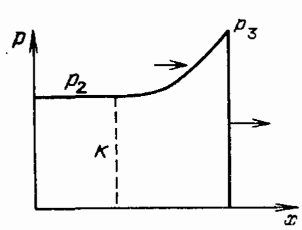

FIG. 16. Configuration $K\overrightarrow{P}\overrightarrow{Y}$; $p_{L'} < p_3 < p_B$. (Transition to the upper branch and the $L^+ B$ section.) This representation of a discontinuity is only formal; in practice the configuration cannot appear because the second wave ($\overrightarrow{P}$) catches up with the first ($\overrightarrow{Y}$).

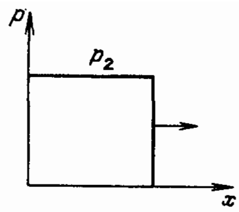

FIG. 17. Single shock wave. Shock-wave discontinuity absolutely unstable. It decays going over to the $K\overrightarrow{Y}\overrightarrow{Y}$ configuration (Fig. 15).

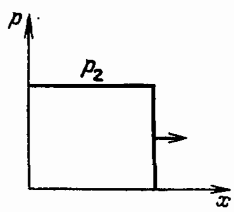

FIG. 18. Single shock wave. The shock-wave discontinuity is stable.

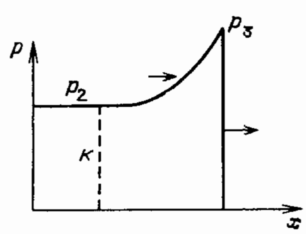

FIG. 19. Configuration $K\overrightarrow{P}\overrightarrow{Y}$; $p_{L'} < p_3 < p_B$ (transition to the upper branch and the section $L'B$) or $p_L < p_3 < p_{L'}$ (transition to the middle branch and the section $L\ L'$). Both representations are only formal for the same reason as in Fig. 16. Moreover, the shock-wave discontinuity at a pressure p_3 lying within the range $p_L < p_3 < p_{L'}$ is absolutely unstable (Fig. 17).

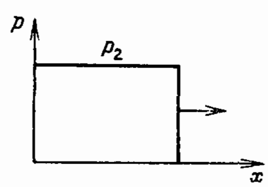

FIG. 20. Single shock wave. The discontinuity satisfies the stability criteria of the liner theory, but can be represented in the form of a stable configuration characterized by $p_A < p_3 < p_L$ (Fig. 21).

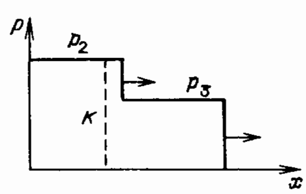

FIG. 21. Configuration $K\overrightarrow{Y}\overrightarrow{Y}$; $p_A < p_3 < p_L$ (transition to the lower branch and the section AL^-) or $p_L < p_3 < p_{L'}$ (transition to the middle branch and the section $L^- L^+$). In the first case the configuration is stable, whereas in the second case it is unstable since a shock-wave discontinuity at a pressure p_3, within the limits $p_L < p_3 < p_{L'}$, is absolutely unstable (Fig. 17).

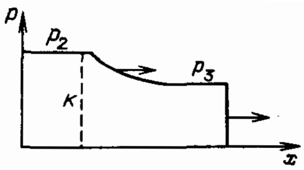

FIG. 22.

II. Shock adiabat with a monotonic rise of the pressure and a section L^-L^+ where $L < -1$

Such a shock adiabat is shown in Fig. 4 using the pv and pu coordinates. At the points L^- and L^+ this shock adiabat and isentropes of the S^+ family have shared tangents. A shock-wave discontinuity may have a triple-valued representation in the form of a single shock wave and two more complex wave configurations in the section $L^- L^+$ and in the sections AL^- and L^+B adjoining it from below and above. An isentrope of the S^+ family which touches the shock adiabat at the point L^+ intersects this adiabat[15)] at the point A. A shock adiabat of the H^+ family drawn from the point L^- regarded as the starting point intersects the original shock adiabat at the point B.

Representations of a shock-wave jump with an intensity (pressure p_2) corresponding to the sections $L^- L^+$, AL^-, and L^+B of the shock adiabat:

4) $p_{L^-} < p_2 < p_{L^+}$ (Figs. 15–17);
5) $p_A < p_2 < p_{L^-}$ (Figs. 18,19);
6) $p_{L^+} < p_2 < p_B$ (Figs. 20,21).

In the case of a shock adiabat with a smooth inflection (shown in the figures with a dashed section of the curve) a shock-wave discontinuity is represented basically as shown in Figs. 15–21. However, then instead of a $K\vec{Y}\vec{Y}$ configurations (Figs. 15 and 21), in a certain range of pressures[16)] defined by $p_L < p_2 < p^*$ we can expect formation of a $K\vec{C}\vec{Y}$ configuration with an isentropic compression wave $\vec{C}$ (Fig. 22).

At higher pressures p_2 lying in a certain range $p^* < p_2 < p^{**}$ a $K\vec{Y}\vec{C}\vec{Y}$ configuration is formed (Fig. 23). Finally, if $p^{**} < p_2 < p_B$, the wave configuration is the same as in the case of a shock adiabat with a kink ($K\vec{Y}\vec{Y}$; Figs. 15 and 21).

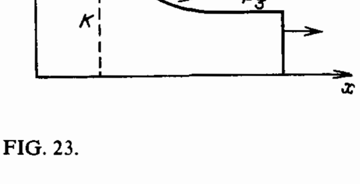

FIG. 23.

1) The expression for L_0 given in Ref. 3 is in error because in finding the angular boundary between waves arriving and departing from the front no allowance was made for the motion of matter behind the shock wave front. According to Ref. 3, we have $L_0 = (1 - \theta M_2 + 2M_2^2)/(1 + \theta M_2^2)$. The above correct expression for L_0, i.e., Eq. (5), was obtained by Kontorovich[4] and Iordanskiĭ[5]; see also Refs. 6–9.

2) A wave adiabat linking the initial and final states in a system of waves without contact discontinuities (including an isentropic compression wave) and propagating in the same direction was considered in Refs. 19 and 20.

3) The case of nonmonotonic variation of the pressure is discussed at the end of Sec. 1.6.

4) We can show that in the case when $L > 1 + 2\mathrm{Ma}_2$ the amplification of the perturbation amplitude and phase reversal may occur, by analogy with Eq. (14), also in the case of oblique incidence of a perturbation on the front of a shock wave.[12]

5) An interpretation of the instability criteria of Eq. (8) in connection with an analysis of the motion of a shock wave along a tube of variable cross section can be found in Ref. 1.

6) For a fixed difference $\gamma - \gamma_{\rm res}$ and a sufficiently large amplitude of the incident wave a regular reflection is impossible for one of the phases (compression or rarefaction) of the wave; see Ref. 12.

7) The energy of a fluctuation wave is proportional to $\Delta\gamma$ and it vanishes for the waves which are emitted at an exactly defined angle γ.

8) It was assumed in Ref. 31 that the primary perturbation is a departing wave. Independent motion of such a wave along a shock-wave front does not satisfy the principle of causality. A physically correct formulation of the problem now requires specifying the primary perturbation in the form of arriving waves. However, mathematically speaking, the direction of travel of the initial perturbation (toward the front or away from it) is of no fundamental importance in studies of the nature of a feedback between the front of a shock wave and the flow behind it.

9) The derivative du/dp_2 vanishes at the point $L = 1$. If $L > 1$ the inequality $du/dp_2 < 0$ is obeyed. Hence, and also from the observation that the region of the anomalous form of the shock adiabat (in particular, the region where $L > 1$, i.e., where $du/dp_2 < 0$) is very limited, it follows that any one value of u, taken in the section of the adiabat where $L > 1$, corresponds to at least three values of p_2 (Ref. 35). Two of these values (minimal and maximal) correspond to stable flow, and the third to unstable flow. It is this value of p_2 which is located in the section of the adiabat with $L > 1$.

10) Special experiments on reflection of light by the front of a shock wave[37] have demonstrated that the front is free of at least those inhomogeneities which are of the order of, or greater than, the optical wavelength.

11) It should be pointed out that at the high shock wave intensities which were attained in the experiments reported in Refs. 51–53, "turbulization" of the front may be influenced by perturbations of the contact surface and by intrinsic relaxation processes associated with the radiative transfer.

12) We shall ignore here the shock waves with spatially inhomogeneous initial states.

13) In the case of shock adiabats of condensed media it is usually found that $(\partial v/\partial p)_H > 0$; the part of the adiabat with a positive value of $(\partial v/\partial p)_H$ is obtained by specially preparing the initial material in the form of a porous substance or a foam. A natural substance of this kind is, for example, snow which is a porous variety of ice. The nonmonotonic pressure dependence of the compression, associated with the successive collapse of electron shells of the atoms, is achieved also on compression of nonporous monolithic substances. However, then one needs enormous pressures of the orders of tens of megabars.[85]

14) The proof that the inequality $L > -1$ cannot be disobeyed by an ideal gas subject to single ionization is provided in Ref. 86 without allowance for bound excited states of atoms or ions for all reasonably realistic parameters of matter (temperature and density) ahead of a wave.

15) Usually kinks of a shock adiabat are observed in the case of relatively weak compression when a shock adiabat differs little from an isentrope. The point A is then close to a kink.

16) Calculations of the pressures p^* and p^{**} are reported in Ref. 18.

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