

Classical mechanics of one-dimensional nonlinear systems

M. M. Tsypin

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M. Toda, *Theory of Nonlinear Lattices*. 2nd enlarged ed. Springer-Verlag, Berlin; Heidelberg; New York; London; Paris; Tokyo, 1989. 255 pp. (Springer Series in Solid-State Sciences. V. 20).

The book of M. Toda is devoted to the classical mechanics of a class of nonlinear dynamic systems, specifically one-dimensional lattices (chains) of particles interacting with nearest neighbors. The principal attention is paid to chains with exponential interaction (Toda chains) for which rigorous results are obtained.

Chapter 1 contains some basic facts concerning the theory of nonlinear dynamical systems. The Fermi-Pasta-Ulam problem and the Henon-Heiles model are discussed. The results of numerical simulation are given. Also the Korteweg-deVries equation and solitons are introduced.

In Ch. 2 a rigorous examination begins of the Toda chain for which exact periodic and soliton solutions are derived. Two-soliton solutions are also given. A chain of hard spheres and the long-wavelength approximation leading to the Korteweg-deVries equation are discussed. A realization of a nonlinear chain with the aid of an LC-circuit is examined.

Chapter 3 introduces the matrix formalism and with its aid solves the Cauchy problem for an infinite chain. It is shown how to obtain solutions of equations of motion with the aid of the Backlund transformation. Also exact methods for a chain with a finite number of particles are discussed, including the Moser method.

In Ch. 4 the Cauchy problem is discussed for the period-

ic Toda chain the solution of which is found with the aid of the inverse problem of spectral theory. In so doing the spectrum of the discrete Hill equation is utilized.

Action-angle variables for the Toda chain are constructed in Ch. 5.

Chapter 6 added in the second edition contains a brief review of the new results obtained since the publication of the first edition (1981). Problems are discussed associated with integrability, as well as a generalization of the Toda chain to a greater number of dimensions, calculation of the free energy of the Toda chain with the aid of the Bethe ansatz, the hierarchy of nonlinear equations and some numerical results.

The book is distinguished by the clarity of presentation; theoretical derivations are illustrated by explicit calculations using simple examples. The author turns repeatedly in the course of the whole book to two of them—the conoidal wave and the three-particle chain. An important appendix to the material are the 39 problems accompanied by notes and hints. Key problems are also provided with brief solutions placed at the end of the book.

The first edition was published in Russian translation by the "Mir" publishing house in 1984.

Translated by G. M. Volkoff