Radiation transport in spectral lines

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We review a number of the important results of the theory of radiation transport of resonance radiation obtained mainly during the past decade. Situations are stressed in which the traditional hypothesis of complete frequency redistribution of quanta is not fulfilled in the process of reemission. A quite general approach to the problem of radiation transport is presented that assumes small frequency shifts in the scattering event. Results are presented pertaining to the rate of deexcitation in Doppler, Stark, and natural mechanisms of line broadening. The transport of polarized radiation is discussed, including the application to experiments on interference of atomic states (the Hanle effect, etc.). The problem is discussed of transport of high-intensity radiation, in particular as applied to the problem of radiation trapping in lines of hydrogen-like ions. The connection is discussed of the physical mechanisms of radiation transport in spectral lines and in a recombination continuum.

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1. INTRODUCTION

Interest in the problem of transport of resonance radiation in gases and plasmas first appeared in astrophysics in connection with the interpretation of spectra of celestial bodies. The astrophysical approach in transport theory is being developed actively even now, due to the nontransient importance of the original problem. In the physics literature the theory of radiation transport has been actively discussed since the beginning of the 1940s. The detailed monographs[1–3] and the thorough review of D. I. Nagirner,[4] which have been devoted to the astrophysical approach in transport theory, offer us the opportunity of treating the corresponding problems only very cursorily. We shall pay fundamental attention to the physical features of radiation-transport processes that are manifested under laboratory conditions. At the focus of attention will be the situation in which both the matter and the radiation are far from equilibrium, while the transport occurs in line spectra of atoms and ions. The first studies of radiation transport[5,6] assumed conservation of frequency in the event of light scattering by an atom. The theory thus constructed greatly resembled the ordinary theory of diffusion of matter and often led to conclusions that strongly differed from the experimental results.[7] In the 1940s different authors independently of one another assumed complete frequency redistribution (CFR). A reference to the fact that the CFR model was first proposed by Hautgast is found in Ref. 11. The essence of the CFR model consists in the idea that the atom emits a photon with a frequency that does not depend on the frequency of the absorbed quantum. Here one usually assumes that the frequency dependences of the emission and absorption coefficients are the same. The founding studies of Biberman[8,9] and Holstein[10] found an integral equation in the steady-state case, and an integro-differential equation in the non-steady-state case, that describe the behavior of the occupancies of atoms in the excited state. Many important results that agree well with experiment[12,13] have been obtained by using the CFR approximation. The transport theory based on the CFR approximation has been developed by devising methods of solving the appropriate integral equations using a rather complicated mathematical technique: this is the cycle of studies of Yu. Yu. Abramov, A. M. Dykhne, and A. P. Napartovich, the studies of the Leningrad astrophysicists of the school of V. V. Sobolev, the studies of the Dutch theoretician van Trigt, etc. As was noted,[4] transport theory in the CFR approximation for solving linear two-level problems can be considered to a certain degree complete. This theory is reflected in the monographs cited above, in the review of Ref. 4, and the monograph of Ref. 64. Hence we shall practically not treat the CFR in this review.

We shall focus our major attention on the situation in

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which an incomplete frequency redistribution (IFR) takes place. The essence of IFR consists in the idea that the frequencies of the absorbed and reemitted quanta are somehow correlated with one another. The character of the frequency change in the scattering event depends on the concrete mechanism of line broadening, and as a rule, one cannot find universal solutions in the IFR case. We shall describe the results that have already become generally known (Hearn, Hummer),[5] as well as problems that have arisen very recently. The theoretical studies will constitute the bases of the review, but wherever possible, we shall make a comparison with experiment.

Moreover, we briefly treat some other nontraditional problems of transport theory: propagation of polarized radiation, application of the theory to laser problems, transport of high-intensity radiation, and radiation transport in a spectral continuum.

2. FUNDAMENTAL EQUATIONS

The modern outlook on radiation transport with consistent account taken of the mechanism of frequency redistribution has mainly been developed in the studies devoted to astrophysical problems.[1-4] Although we shall follow the ideology of this approach in our presentation, the need for treating non-steady-state problems puts its imprint even on the system of symbols and writting of equations.[14,15]

Resonance radiation is characterized by a distribution function of photons $I(\mathbf{r}, t, \xi)$, or in other words, by the number of photons at the point $\mathbf{r}$ at the instant t of time having the frequency ω and direction of propagation $\mathbf{\Omega}$ (for breavity we further combine the latter variables into one: ξ). Restricting the treatment for now to a two-level formulation of the problem, we shall characterize the excited atoms by the population $N(\mathbf{r}, t)$. We can easily write an equation of balance for this quantity:

$$\frac{\partial N(\mathbf{r}, t)}{\partial t} = -(\gamma+\alpha) N(\mathbf{r}, t) + \int k(\omega) I(\mathbf{r}, t, \xi)\, d\xi + F(\mathbf{r}, t). \tag{2.1}$$

Here γ is the rate of spontaneous emission, α is the rate of quenching of the excited state in inelastic collisions, $F(\mathbf{r}, t)$ is the rate of nonradiative excitation of atoms, and $k(\omega)$ is the spectral absorption coefficient. Hereinafter the velocity of light is $c = 1$. For the treatment below it is useful to define the quantity $\varepsilon(\omega)$ such that:

$$k(\omega) = k_\Sigma \varepsilon(\omega), \qquad \int_{-\infty}^{\infty} \varepsilon(\omega)\, d\omega = 1.$$

Here k_Σ is the frequency-integrated absorption coefficient. We shall measure the frequency from the center of the line. Since the width of the spectral line is much smaller than the central frequency ω_0, the integration is performed over an infinite range. Moreover, we assume in (2.1) that the gas of unexcited atoms is unpolarized and is homogeneous throughout the volume.

The equation for the photons is substantially more complicated. The point is that the frequency of the reradiated quantum depends on the prehistory of the process. If the atom was excited as a result of an inelastic impact, we can assume that it will isotropically emit a quantum with a frequency distributed according to $\varepsilon(\omega)$. In this case, when a quantum has been absorbed with the parameter ξ', the probability that, after a time τ, a quantum will be emitted with the parameter ξ is $W(\xi,\xi',\tau)$. We can call the quantity $W(\xi,\xi',\tau)$ the time redistribution function; it is analogous to the transition probability in the classical kinetic theory of gases. We shall discuss the concrete form of the function $W(\xi,\xi',\tau)$ below. Taking into account what we have said, let us write the equation for $I(\mathbf{r},t,\xi)$:

$$(\mathbf{\Omega}\nabla I) = -k(\omega) I(\mathbf{r}, t, \xi) + \gamma \int_0^\infty \Big\{\exp[-(\gamma+\alpha)\tau] \times \Big[\int W(\xi, \xi', \tau) k(\omega') I(\mathbf{r}, t-\tau, \xi')\, d\xi' + \varepsilon(\omega) F(\mathbf{r}, t-\tau)\Big]\Big\}\, d\tau. \tag{2.2}$$

The first term on the right-hand side describes the absorption of photons (if nonresonance absorption is substantial, the absorption coefficient must be correspondingly increased), and the second term describes emission. Equation (2.2) takes no account of retardation, since under laboratory conditions it plays no appreciable role. The essential point is that Eq. (2.2) is integral with respect to time, and in general cannot be reduced to a differential equation. The concrete boundary conditions for (2.2) are imposed with account taken of the presence of external sources.

In the steady-state case, upon integrating with respect to τ, we arrive at the ordinary transport equation[3]:

$$(\mathbf{\Omega}\nabla I) = -k(\omega) I(\mathbf{r}, t, \xi) + \frac{\gamma}{\gamma+\alpha} \times \Big[\int W(\xi, \xi') k(\omega') I(\mathbf{r}, \xi')\, d\xi' + \varepsilon(\omega) F(\mathbf{r})\Big]. \tag{2.3}$$

Here

$$W(\xi, \xi') = (\gamma+\alpha) \int_0^\infty W(\xi, \xi', \tau) \exp[-(\gamma+\alpha)\tau]\, d\tau \tag{2.4}$$

is the steady-state redistribution function.

In the astrophysical literature the redistribution function is defined as

$$R(\xi, \xi') = W(\xi, \xi') k(\omega').$$

Although in this form the function is symmetric with respect to frequency, the quantity $W(\xi,\xi')$ is preferable in a number of cases, since it has the simple physical meaning of the conditional probability of emission of a photon with the characteristics ξ'. As we shall show below, taking account of nonisotropic scattering does not affect substantially the rate of escape of radiation. Upon averaging $W(\xi,\xi',\tau)$ over the angular variables, we can integrate Eq. (2.3) over the coordinates:

$$I(\mathbf{r}, t, \omega) = \gamma \int_V d\mathbf{r}' H(\mathbf{r}-\mathbf{r}', \omega) \int_0^\infty d\tau \exp[-(\gamma+\alpha)\tau] \times \Big[\int_{-\infty}^{+\infty} d\omega' W(\omega, \omega', \tau) k(\omega') I(\mathbf{r}, t-\tau, \omega') + \varepsilon(\omega) F(\mathbf{r}, t-\tau)\Big], \tag{2.5}$$

Here we have

$$I(\mathbf{r},\ t,\ \omega)=\int I(\mathbf{r},\ t,\ \xi)\,d\Omega, \tag{2.6}$$

$$H(\mathbf{r},\ \omega)=\frac{\exp[-k(\omega)\,|\mathbf{r}|]}{4\pi\,|\mathbf{r}|^2}.$$

Equation (2.5) is an analog of the Chapman-Kolmogorov equation for photons.

In (2.5) we have not taken account of the polarization of the radiation, as well as of the influence of stimulated-emission processes. That is, we are studying a situation in which the population of atoms in the excited state is much smaller than in the ground state. The influence of stimulated scattering, as well as transport with allowance for a large number of excited levels, will be briefly treated below.

As was noted in the Introduction, the case of radiation transport with CFR has been studied in greatest detail up to the present. The latter assumes that the function $W(\omega,\omega',\tau)=\varepsilon(\omega)$. Here the starting equations are markedly simplified, and after simple calculations, we can derive from (2.1) and (2.2) [or (2.5)] the equation

$$\frac{\partial N(\mathbf{r},\ t)}{\partial t}=-(\gamma+\alpha)\,N(\mathbf{r},\ t)+\int G(\mathbf{r},\ \mathbf{r}')\,N(\mathbf{r}',\ t)\,d^3r'+F(\mathbf{r},\ t). \tag{2.7}$$

In the literature this is usually called the Biberman-Holstein equation.[64] It has a very perspicuous physical meaning, namely, the evolution of the population of excited atoms $N(\mathbf{r},t)$ is determined by several factors: radiative and collisional decay of the level [the first term on the right-hand side of (2.7)], the process of absorption at the point $\mathbf{r}$ of a quantum previously emitted at the point $\mathbf{r}'$ (second term), and population of the level owing to collisions (third term). The probability of reabsorption (trapping) of a quantum $G(\mathbf{r},\mathbf{r}')$ in the case in which the spectral absorption coefficient $k(\omega)$ is considered to be independent of the coordinates has the form

$$G(\mathbf{r},\ \mathbf{r}')=\frac{\gamma}{4\pi}\int_{-\infty}^{\infty}\varepsilon(\omega)\,k(\omega)\,\frac{\exp[-k(\omega)\,|\mathbf{r}-\mathbf{r}'|]}{|\mathbf{r}-\mathbf{r}'|}\,d\omega. \tag{2.8}$$

In closing this section we note that the described pattern of successive absorption and emission of photons, although rather graphic, is not self-evident. Nevertheless, thorough quantum-mechanical treatment of the problem performed in Refs. 16–19 with allowance for the various conditions confirms the correctness of the derived equations. The fundamental restriction is the smallness of the wavelength λ of the radiation as compared with the mean free path of a photon

$$k(\omega)\,\lambda\ll 1.$$

Although this restriction can break down in the center of a line, actual transport occurs in the outer tails, where this condition is always satisfied.

When the stated inequality holds we can assume that only one atom participates in the process of absorption and emission of a quantum. In the converse case one cannot treat the atoms as a set of independent emitters.[64,80] That is, we must take account of collective effects. One of these effects is the known selective reflection of resonance radiation from sodium vapor of high enough density. One can find a more detailed discussion of this phenomenon and references to the experimental studies in Ref. 81.

3. EFFECTIVE DEEXCITATION TIME AND THERMALIZATION LENGTH

In every physical theory, along with the exact relationships between the fundamental parameters of the problem, it is desirable to gain the possibility of qualitative estimates of the most important quantities. In the theory of transport of resonance radiation the effective deexcitation time $\tau_{\rm eff}$ plays the role of one of such quantities. If we create in a volume a certain number of excited atoms, then in the absence of further excitation and quenching, the number of excited atoms will begin to diminish owing to the escape of radiation. The characteristic time scale of this process according to Holstein[10] is $\tau_{\rm eff}$. We may approach the estimation of this quantity in a somewhat different way. If a stationary excitation source of power f exists, then as a result of competition of the processes of excitation and escape of radiation, the volume will constantly contain a certain number of excited atoms:

$$N=f\tau_{\rm eff}.$$

This definition was given by L. M. Biberman.[64] We should bear in mind that these two definitions, although close in meaning and magnitude, still are not identical and differ by a numerical coefficient that depends on the geometry of the problem.

Another important characteristic is the thermalization length $L_{\rm eff}$. In the medium, besides the processes of reradiation of quanta, "loss" of excitation can occur owing to quenching or nonresonance absorption of photons. Consequently an excited state that has arisen at any point can migrate from it for the distance $L_{\rm eff}$, whereupon the energy of the photon goes over into the thermal energy of the medium. If the characteristic dimension of the volume is $L\gg L_{\rm eff}$, then escape of radiation exerts no substantial influence on points remote from the boundary of the volume. In this case we can solve the problem of radiation transport for a thin layer along the surface. That is, here the approximation of a semiinfinite space is considered applicable. In the converse case $L\ll L_{\rm eff}$ only the escape of radiation plays a role, and we can neglect other processes. The thermalization length is connected to the effective lifetime. If as before α is the rate of quenching of excited atoms, and $\tau_{\rm eff}$ is the time for escape of radiation from a volume of characteristic dimension $L_{\rm eff}$, then we can find $L_{\rm eff}$ from the equation

$$\alpha\tau_{\rm eff}=1. \tag{3.1}$$

Under the assumption of completely coherent radiation, by analogy with the diffusion of matter we have

$$\tau_{\rm eff}\sim\frac{L^2}{\Lambda}. \tag{3.2}$$

Here Λ is the diffusion coefficient. In (3.2) we have assumed that transport occurs at a single central frequency. Under the hypothesis of CFR the result depends on the type of broadening. Table I presents the results of calculating $\tau_{\rm eff}$ for two ptypes of broadening: Doppler and dispersion broadening, and two very simple geometries: a plane parallel layer and a cylinder. The fundamental feature of transport for CFR consists in the fact that the effective transport occurs in

TABLE I.

Geometry	Doppler contour, $\tau_{\text{eff}}=\frac{k_0L(\pi \ln k_0L)^{1/2}}{g_D}$	Dispersion contour, $\tau=\frac{(\pi k_0L)^{1/2}}{g_L}$
Cylinder	$g_D=1{,}6$	$g_L=1{,}115$
Plane layer	$g_D=0{,}94$	$g_L=0{,}8$

the tail of the absorption line at frequencies at which $k(\omega)L\sim 1$. The transport process is represented as a large number of random walks of the photon at frequencies close to the central one, with a small displacement in space and rare shifts into the region of larger frequencies, which correspond to displacements comparable with the dimensions of the region.

4. ALMOST COHERENT SCATTERING

As we have already noted above, the transport of resonance radiation depends on the change in frequency in the scattering event. However, if the frequency change is small we can draw general conclusions without stipulating the details of the scattering process. For completely coherent radiation in the case of great optical thickness, we have the following from (2.5):

$$\frac{\partial I}{\partial t}=\frac{\gamma}{3k^2(\omega)}\Delta I-\alpha I+\frac{\gamma}{k_\Sigma}F(\mathbf{r},t). \tag{4.1}$$

The diffusion approximation is inapplicable near the boundary of the volume, where the boundary condition must be imposed that[3]

$$I=-\frac{2}{3}\frac{1}{k(\omega)}\frac{\partial I}{\partial z}. \tag{4.2}$$

Here z is the coordinate normal to the surface of the volume. Radiation at different frequencies propagates independently, the escape time of the radiation is $\tau_{\text{eff}}\sim(k(\omega)L)^2$ and it strongly depends on the frequency of the radiation.

Actually the applicability of Eq. (4.1) is restricted, even in the case of almost coherent radiation. As we shall see below, the change in frequency upon scattering has a tendency to accumulate and grow, and this substantially affects the rate of escape of radiation. Therefore the diffusion approximation (4.1) holds only for not very great thicknesses, at which the stated accumulation still hardly occurs in the escape time of the radiation.

Let us study the situation in which the frequency of the photon is shifted in the time of escape of the radiation from the volume by the amount ω_{eff}, which is substantially larger than the linewidth, but at the same time is such that

$$k(\omega_{\text{eff}})L\gg 1, \tag{4.3}$$

as is typical of almost coherent radiation. Here, in contrast to CFR, the photon does not succeed in entering the optically transparent tail of the line,[15] and the spectral line can be divided into two parts: the core $|\omega|\ll\omega_{\text{eff}}$, in which the photons practically do not participate in transport, and the tail $|\omega|\sim\omega_{\text{eff}}$, in which transport occurs. The escape of radiation from the core owing to frequency redistribution. In the equation (2.5) for frequencies of the order of ω_{eff} we can neglectthe term involving the external source and consider this equation in the quasi-steady-state approximation. Moreover, upon taking account of (4.3) we can go over to the diffusion approximation:

$$\frac{1}{3k(\omega)}\Delta I(\mathbf{r},t,\omega)-k(\omega)I(\mathbf{r},t,\omega)+\int_{-\infty}^{+\infty}W(\omega,\omega')k(\omega')I\,d\omega'=0. \tag{4.4}$$

The solution of Eq. (4.4) must be joined with the solution for the number of photons in the core of the line. These photons exist in equilibrium with the excited atoms:

$$I(\mathbf{r},t,\omega)=\frac{\gamma}{k_\Sigma}N(\mathbf{r},t),\quad |\omega|\ll\omega_{\text{eff}}. \tag{4.5}$$

The first term in (4.4) describes the spatial transport of radiation, and hence, the change in concentration of excited atoms at the point $\mathbf{r}$ is determined by its integral with respect to the frequency. Owing to (4.5), the radiation intensity is linearly related to $N(\mathbf{r},t)$, and we have

$$\frac{1}{3}\int_{-\infty}^{+\infty}\frac{1}{k(\omega)}\Delta I(\mathbf{r},t,\omega)\,d\omega=-\gamma\int_V G(\mathbf{r},\mathbf{r}')N(\mathbf{r}',t)\,d\mathbf{r}'. \tag{4.6}$$

Naturally, the form of the kernel $G(\mathbf{r},\mathbf{r}')$ differs from the CFR case. Taking (4.6) into account, we can write the equation of balance for the population $N(\mathbf{r},t)$ of excited atoms:

$$\frac{\partial N}{\partial t}=-\alpha N(\mathbf{r},t)+\gamma\int_V G(\mathbf{r},\mathbf{r}')N(\mathbf{r}',t)\,d\mathbf{r}'+F(\mathbf{r},t). \tag{4.7}$$

Thus the problem is reduced to an integro-differential equation for the population of excited atoms, provided that Eq. (4.4) has been solved.

One can distinguish two physically different types of almost coherent scattering. In the one case the probability of frequency change in scattering is small, but redistribution occurs within the limits of the entire spectral line (as in the case of rare broadening collisions). In the other case the frequency changes in each scattering event, but by a small amount (as from Doppler redistribution in the presence of considerable natural broadening). Here diffusion of the radiation over the frequency scale occurs in the region of the tail of the line, while Eq. (6) is substantially simplified. Upon introducing $D(\omega)$, the mean-square frequency change in scattering, we obtain

$$\frac{1}{3k(\omega)}\Delta I(\mathbf{r},t,\omega)+\frac{1}{2}\frac{\partial}{\partial\omega}D(\omega)k(\omega)\frac{\partial}{\partial\omega}I(\mathbf{r},t,\omega)=0. \tag{4.8}$$

In the approximation of almost coherent radiation we can easily solve the problem of calculating the effective rate of

deexcitation.[15] If the volume contains no extra excitation and quenching sources, then after a long enough time after the initial excitation, we can assume that

$$I(\mathbf{r},\ t,\ \omega) = I_0 f(\omega)\,\varphi_0(\mathbf{r})\ \exp\left(-\frac{t}{\tau_{\text{eff}}}\right). \tag{4.9}$$

Here $\varphi_0(\mathbf{r})$ is an eigenfunction of the Laplace operator. I_0 is a constant that depends on the initial conditions. The function $f(\omega)$ is determined from the solution of Eq. (4.4) or (4.8) and satisfies the equation

$$f(0) = 1. \tag{4.10}$$

If we know $f(\omega)$, we can find τ_{eff} by the formula

$$\tau_{\text{eff}}^{-1} = \frac{\gamma\lambda_0}{3L^3 k_\Sigma}\int_{-\infty}^{+\infty}\frac{f(\omega)}{k(\omega)}\,d\omega. \tag{4.11}$$

Here λ_0 is an eigenvalue of the Laplace operator, and L, as before, is the characteristic dimension of the region.

It is interesting to compare the time t in which a photon occurring in the tail of the line escapes the volume with the overall deexcitation time τ_{eff}. Evidently, $t \sim (k(\omega_{\text{eff}})L)^2/\gamma$. Thus we have

$$\frac{t}{\tau_{\text{eff}}} \sim \frac{k(\omega_{\text{eff}})\,\omega_{\text{eff}}}{k(0)\,\Delta\omega_0} \ll 1.$$

Consequently a quantum spends the major time in the core of the line, which justifies the neglect in (4.4) of the time derivative and the terms describing the rate of quenching and of external excitation.

In closing this section we note that, although the equations have been derived under the condition (4.3), which is known to break down in the CFR case, they yield a qualitatively correct result even in this case. Assuming in (4.4) that $W(\omega,\omega') = \varepsilon(\omega)$, we find by using (4.11) that

$$\tau_{\text{eff}}^{-1} = \gamma\int_{-\infty}^{+\infty}\frac{\varepsilon(\omega)\,d\omega}{1+(3k^2(\omega)\,L^2/\lambda_0)}. \tag{4.12}$$

Calculations[15] by Eq. (4.12) show that it yields a correct dependence on the optical thickness with a coefficient elevated by 10% for a sphere, by 30% for a cylinder, and by 50% for a plane layer. To avoid misunderstanding we note that this case cardinally differs from the case of completely coherent radiation for large optical thicknesses. We note that the equation for $N(\mathbf{r},t)$ has remained integral.

5. FREQUENCY REDISTRIBUTION OWING TO THE DOPPLER EFFECT

In the case of pure Doppler broadening the problem of the redistribution of the photon frequency in scattering is closely connected with the velocity distribution of excited atoms. Actually, in the absorption of a quantum of frequency ω' and direction of motion $\mathbf{\Omega}'$, an excited atom is formed whose velocity $\mathbf{v}$ satisfies the relationship

$$(\mathbf{v}\mathbf{\Omega}') = \frac{\omega'}{\omega_0}. \tag{5.1}$$

That is, only the component of the velocity perpendicular to $\mathbf{\Omega}'$ is uniformly distributed. If the excited atom has not changed in velocity during its time of existence, then in subsequent emission the frequency will be correlated with the

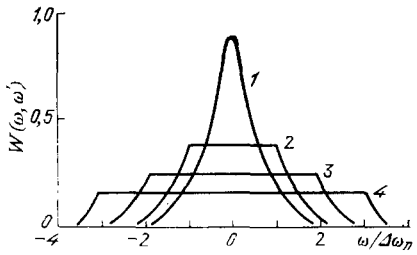

FIG. 1. Frequency redistribution function for Doppler broadening. *1*—$\omega'/\Delta\omega = 0$; *2*—$\omega'/\Delta\omega = 1$; *3*—$\omega'/\Delta\omega = 2$; *4*—$\omega'/\Delta\omega = 3$, $\Delta\omega_D = \omega_0 v_0$.

frequency ω' of the absorbed quantum. Thus in Doppler broadening complete frequency redistribution occurs only in the presence of collisions, which Maxwellize the distribution function within the time of spontaneous emission. Let us examine the opposite limiting case in which the role of collisions is negligibly small. Starting with the arguments presented above, we can calculate the redistribution function.[1-3,20] Without writing out the concrete expression, we shall present only the graphic results (Fig. 1).

The problem of radiation transport having this law of frequency change has been studied by different methods. In Refs. 21 and 22 the transport equation was solved numerically by finite-difference methods, in Ref. 23 by the Monte Carlo method, and in Refs. 24 and 25 by using an expansion of the velocity distribution function of the excited atoms using polynomials. In all the studies the authors concluded that the fact of incomplete frequency redistribution affects the escape velocity weakly. The effective rate of deexcitation and the fundamental part of the spectrum of the escaping radiation do not differ by more than 10% from the values calculated under the CFR assumption for any values of the optical thickness.

The situation differs in the case in which the excited atoms are formed with a strongly non-equilibrium velocity distribution function. Let us study the sensitized fluorescence in a mixture of mercury and thallium vapors.[26-27] Upon irradiating the cell containing the vapor mixture with the radiation from a mercury lamp, the 6^3P_1 state of mercury is excited. In the collision of these atoms with thallium atoms in the ground state, an excited thallium atom is formed in the $6^2D_{3/2}$ state. The 0.4-eV energy defect goes over into the kinetic energy of the colliding particles; the excited thallium atoms are formed with a strongly non-equilibrium velocity distribution. The $6^2D_{3/2}$ level is coupled by a radiative transition not only with the ground state (the 2768-Å line), but also with the metastable $6^2P_{3/2}$ level (the 3529-Å line). The emission of this line is also measured in the experiment. The line being measured is not reabsorbed. Hence the contour of the line resembles the distribution function of the atoms in the $6^2D_{3/2}$ state with respect to the projection of the velocity on the direction of observation. In the experiment the gas cell had the form of a parallelepiped of dimensions $25\times25\times20$ mm. The results of calculating the spectrum (the calculation was done by the Monte Carlo method) are shown in Fig. 2. Curve *1*—$N_{\text{Tl}} = 10^{10}$ cm^{-3} ($k_D L \sim 0.1$) corresponds to the absence of trapping and describes the velocity distribution of the atoms created as a result of collisions. Here k_D is the coefficient at the center of the line in Doppler broadening. Curve *2*—$N_{\text{Tl}} = 10^{12}$ cm^{-3} corre-

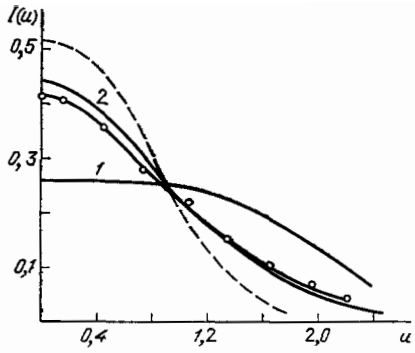

FIG. 2. $u = (\omega - \omega_0)/\Delta\omega_D$ (explanation in text).

sponds to $k_D L \sim 10$; the trapping in this case alters the spectrum toward equilibrium (the equilibrium distribution is shown by the dotted line). The dots show the experimental points.

6. FREQUENCY REDISTRIBUTION IN STARK BROADENING

The problem of radiation transport in Stark-broadened lines of hydrogenlike ions is essential for a plasma having the parameters characteristic of the problem of laser thermonuclear fusion ($N_i \sim 10^{21}$, $T \sim 1$ keV, $Z \sim 10$),[28] for which the Stark broadening exceeds other types of broadening by about an order of magnitude.[29]

The redistribution problem in Stark broadening requires analysis from several standpoints. In the time during which the ion exists in the excited state, the plasma field changes in value. This effect is described by the parameter

$$\delta = \frac{v_0}{\gamma r_0} = \frac{1}{\gamma}\left(\frac{2T}{M}\right)^{1/2}(2\pi N_i)^{1/3}. \tag{6.1}$$

Here v_0 is the thermal velocity of the ions, r_0 is the characteristic distance between them, and M is the reduced mass of the emitting and the perturbing ions. If $\delta \gg 1$, then the value of the plasma microfield (and the Stark shift) varies strongly within the time of emission, and correlation between the frequencies of the radiated and absorbed quanta is absent. That is, CFR occurs for each Stark component. In the opposite limiting case $\delta \ll 1$ the plasma field varies insignificantly. That is, almost coherent scattering takes place. Simple estimates show that the value of δ varies over a broad range, possibly including the case $\delta \ll 1$.[14]

A second essential factor can be the redistribution of an excited ion over the Stark sublevels owing to inelastic collisions. If the rate of this "mixing" substantially exceeds the rate of spontaneous emission, then complete redistribution over the Stark levels can occur within the lifetime of the excited state. In the opposite limiting case, which, as estimates show,[29] corresponds more to reality, the ion remains in the same Stark sublevel. And finally, the excited ion can go over upon emitting a quantum to an arbitrary Stark sublevel of the lower state. The effect of this circumstance on frequency redistribution has not yet been studied and we shall not treat it now. This effect is absent for lines of the Lyman series, to which we shall restrict the treatment.

We shall assume that we can neglect also collisional mixing over the Stark sublevels for lines with $L_\alpha \delta \ll 1$. In this case one can treat radiation transport separately in each of the side components. We shall adopt the "nearest-neighbor" model[30] for the microfield. That is, we shall assume that the microfield is created only by the nearest ion, which travels the distance $v_0/\gamma \ll r_0$ within the emission time. Calculation of the redistribution function yields the result

$$W(\omega, \omega') = \frac{1}{\delta\omega^2}\left(\frac{\omega'\Delta\omega_S}{\pi}\right)^{1/2}\int_0^\infty \frac{d\tau}{\tau}\exp\left[-\tau - \frac{\Delta\omega_S(\omega+\omega')}{\omega\omega'(\delta\tau)^2}\right] \operatorname{sh}\frac{2\Delta\omega_S}{(\delta\tau)^2(\omega\omega')^{1/2}}. \tag{6.2}$$

If the condition is satisfied that

$$\omega_{eff} \ll \frac{\Delta\omega_S}{\delta^2}, \tag{6.3}$$

where $\Delta\omega_S$ is the Stark width of the line, then the condition is applicable of almost coherent scattering, and $D(\omega) = 4\omega^3\delta^2/\Delta\omega_S$. In this case the solution of Eq. (4.8) is expressed in terms of the Macdonald function:

$$f(\omega) = \frac{1}{\delta\Gamma(7/8)}\left(\frac{\omega}{\omega_{eff}}\right)^{1/4} K_{1/8}\left(\left(\frac{\omega}{\omega_{eff}}\right)^2\right), \tag{6.4}$$

$$\omega_{eff} = \left(\frac{24}{\lambda_0}\right)^{1/4}(\delta k_S L)^{1/2}\Delta\omega_S, \qquad k_S = \frac{1}{8}\frac{\gamma N_i \lambda^2}{\Delta\omega_S}. \tag{6.5}$$

Calculation of the effective deexcitation rate by Eq. (4.11) yields the result

$$\tau_{eff}^{-1} = \left(\frac{\lambda_0}{24}\right)^{1/8}\frac{\delta^{7/4}}{(k_S L)^{1/4}}\gamma. \tag{6.6}$$

The obtained results hold under the condition

$$\frac{1}{\delta} \ll k_S L \ll \frac{2}{\delta^5}. \tag{6.7}$$

At smaller optical thicknesses frequency redistribution plays no role and transport is described by Eq. (4.1). At greater optical thicknesses conditions (4.3) and (6.3) simultaneously break down and the approximation of almost coherent scattering becomes inapplicable. Under the condition converse to (6.3), the redistribution function (6.2) is substantially simplified:

$$W(\omega, \omega') = \varepsilon(\omega)\frac{2}{\delta^3\sqrt{\pi}}\int_0^\infty \frac{d\tau}{\tau^3}\exp\left[-\tau - \frac{\Delta\omega_S}{\omega'(\delta\tau)^2}\right]. \tag{6.8}$$

We see from (6.8) that $W(\omega,\omega')$ differs from zero for $\omega' \sim \Delta\omega_S/\delta^2 \ll \omega_{eff}$, where the radiation is at equilibrium. Simple integration showed that the CFR condition is satisfied in this case within the limits of one Stark component. Accordingly the effective escape time of the radiation is

$$\tau_{eff} \sim (k_S L)^{0,6}. \tag{6.9}$$

The law (6.9) corresponds to the spectral dependence of $\varepsilon(\omega)$ in the tail of the line as $\varepsilon(\omega) \sim \omega^{-5/2}$.

Let us examine the rate of deexcitation found by the Monte Carlo method from a plane layer of thickness L for comparison with that determined by Eq. (6.6) under the condition that all the photons at the initial instant have the frequency $\omega = 2\Delta\omega_S$ (Fig. 3). We can see well three characteristic regimes with the example of the curve with $\delta = 0.05$: coherent scattering, diffusion in the tail of the spectral line, and CFR within the limits of a Stark component.

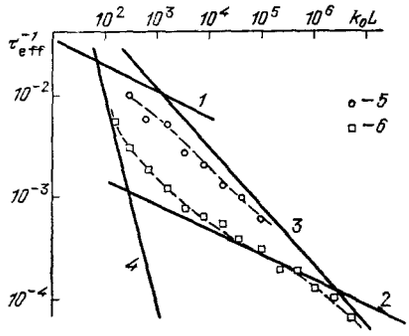

FIG. 3. Rate of deexcitation from a plane layer with Stark broadening. *1*, *2*—calculation by Eq. (6.6) respectively for $\delta = 0.2$ and 0.05; *3*—calculation by Eq. (6.9); *4*—coherent reemission; *5*—calculation with $\delta = 0.2$ by the Monte Carlo method; *6*—calculation with $\delta = 0.05$ by the Monte Carlo method.

Complete neglect of transitions between Stark components for the line L_α is physically not justified. The central component has a finite, albeit small, width caused by the Doppler effect and by collisions with electrons. Therefore reemissions in the center of the line will lead to effective remixing over the Stark sublevels. Thus only half of the excited ions will exist on the displaced sublevels, and the rate of deexcitation will prove to be half of that calculated by Eq. (6.9).

Now let us estimate under what conditions collisional broadening will have no effect on the frequency redistribution of radiation. Evidently, for this to happen the probability of such a collision must be small over the time during which the photon exists in the tail of the line. According to the estimates of Sec. 4, one can estimate this time as $[\gamma^5(\delta k_S L)^{1/3}]^{-1}$. If we take account of the fact that the rate of remixing collisions is $\sim Z^{-3}$,[31] while $\gamma \sim Z^4$, evidently this condition is satisfied for large Z.

When $\delta \sim 1$, even if we restrict the treatment to the nearest-neighbor approximation, there are no grounds for assuming that the field is created by the same ion at the instant of absorption and at the moment of emission. Within the reemission time the nearest ion can change and the redistribution function of (6.2) will describe the process falsely. A complete calculation of this function in the nearest-neighbor approximation is given in Ref. 14. The time redistribution function $W(\omega,\omega',t)$ is calculated in the same reference. By using this function $W(\omega,\omega',t)$ the problem was solved by the Monte Carlo method of calculating the time τ_{eff} pertaining to emission from a spherical volume for different values of the parameter δ. The transition from the diffusion law for $\delta = 0$ to CFR for $\delta \gg 1$ occurs in a narrow interval of values of δ, which corresponds to the conclusions drawn above.

A numerical calculation of frequency redistribution with account taken of the action of many perturbing ions has been performed[32,33] by the method of molecular dynamics.

To treat radiation transport in other lines of the Lyman series within the framework of the two-level model is already incorrect. However, the problem of frequency redistribution is solved by averaging (6.2) over the Stark components.

7. TRANSPORT IN THE TAIL OF A LINE WITH NATURAL BROADENING

The problem is of substantial theoretical interest in which the characteristic frequency ω_{eff} at which the radiation escapes from the volume lies in a region of the spectrum determined by the natural broadening. Since the Doppler broadening usually considerably exceeds the natural broadening, the volume must have a rather great optical thickness. At the same time the pressure must be relatively small, so that collisions should not lead to complete loss of coherence in scattering. These conditions are realized in many astrophysical problems, for which low densities together with a considerable extent of the objects are characteristic. Another field is a high-temperature plasma containing ions of great multiplicity Z. The natural lifetime of excited states declines for them as Z^{-4}, and for large values of Z the natural linewidth can prove comparable with other broadening factors. Finally, many atomic lines from whose upper level an autoionization process occurs possess an anomalously large natural width.[65] An example is the copper line $\lambda = 453.9$ nm, whose natural width is ~ 1 Å.

In a single natural broadening event the quantum is scattered coherently by the atom. Other types of broadening (as before, we shall treat Doppler, collisional, and Stark broadening) lead to partial redistribution. If it is small, we apply the approach presented in Sec. 4.

Let us study the influence of Stark frequency redistribution. We can find the redistribution function in joint action of Stark and natural broadening from simple probability considerations. It is

$$W(\omega, \omega') = \int_{-\infty}^{+\infty} du\, W_S(\omega - \omega' + u, u) \frac{\varepsilon_S(u)\, \varepsilon_{sp}(\omega' - u)}{\varepsilon(\omega')}. \tag{7.1}$$

Here W_S and ε_S arise from a pure Stark mechanism, ε_{sp} is the natural broadening, and ε is the natural profile of the line as determined by the joint effect of both factors. Equation (7.1) in general form is very complicated. Beside, in the case of an ideal plasma, the mean-square variation of the frequency diverges. To estimate the escape time of the radiation we must solve the integral equation (4.4). However paradoxically, the case of a nonideal plasma is simpler, since the exponential cutoff of the tails of the Stark broadening owing to nonideality (see, e.g., Ref. 34) allows one to go over to the diffusion approximation throughout the frequency scale. As before, we shall consider transport in the L_α line. Under the conditions stated above the major role is played by the plasma fields, which lead to a frequency change of the order of $\Delta\omega_S/\varkappa^2 \gg \Delta\omega_S$, for which the nearest-neighbor approximation is applicable; here

$$\varkappa = (2\pi N_1)^{1/3} \frac{Z(Z-1)\, e^2}{T}$$

is the nonideality parameter of the plasma. We can no longer consider the motion of the perturbing ion to be rectilinear, and the expression for $D(\omega)$ becomes complicated. Let us examine this in two limiting cases. For $\delta \ll \varkappa$, when the change in the plasma microfield during the emission time is small, we have

$$D = \frac{16}{3} \frac{\Delta\omega_S^2}{\varkappa^3}. \tag{7.2}$$

In the converse case we have

$$D = \frac{2}{3} \left(\frac{\Delta\omega_S}{\varkappa}\right)^2. \tag{7.3}$$

The condition for applicability of both expressions is

$$\omega_{\rm eff} \gg \frac{\Delta\omega_{\rm S}}{\varkappa^2}. \tag{7.4}$$

Taking account of the fact that we can use the asymptotic value of the absorption coefficient[34] when $\omega \sim \omega_{\rm eff}$, we can easily solve Eq. (4.8):

$$f(\omega) = \exp\left[-\frac{1}{3}\left(\frac{\omega}{\omega_{\rm eff}}\right)^3\right], \tag{7.5}$$

$$\omega_{\rm eff} = \left(\frac{3D}{2\lambda_0}\right)^{1/6}\left(\frac{\gamma^2 k_\Sigma L}{2\pi}\right)^{1/3}. \tag{7.6}$$

Calculation of the effective deexcitation time yields the result

$$\tau_{\rm eff}^{-1} = \left(\frac{2}{3}\frac{D\lambda_0}{\gamma^2}\right)^{1/2}\frac{\gamma}{k_\Sigma L}. \tag{7.7}$$

Of the two conditions (4.3) and (7.4) necessary for applying the approximations that we have made, we see that the latter is the stricter one, and reduces to the form

$$\left(\frac{\gamma k_\Sigma L}{\Delta\omega_{\rm S}^2}\right)^{1/3} \gg \varkappa^{-3/2}, \quad \delta \ll \varkappa,$$
$$\gg \varkappa^{-11/6}, \quad \delta \gg \varkappa. \tag{7.8}$$

Evidently it can be satisfied.

What we have presented above on Stark redistribution when there is a substantial effect of natural broadening requires the following explanation. The fundamental assumption was the idea of the scattering process as a sequence of absorptions of a quantum with a change of the frequency of the Stark sublevel and emission of a quantum at another frequency. We stress that the time of variation of the frequency is actually the lifetime of the atom. Under the given conditions such an approximation is not obvious, and the problem requires further study.

Now let us study the Doppler frequency redistribution. The redistribution function corresponding to the case of joint action of Doppler and natural broadening can be calculated by using the pure Doppler redistribution function according to a formula analogous to (7.1). Although one cannot perform the analytic calculations in full, the properties of the corresponding redistribution function have been well studied.[3,20–21] An algorithm for fast calculation of this function is given in Ref. 35. The absorption coefficient for the joint effect of Doppler and natural broadening is described by the Voigt function.[31] It is expedient to introduce the Voigt parameter $a = \gamma/2\Delta\omega_{\rm D}$ and also the frequency $\omega_{\rm F}$ at which the frequency functions $\varepsilon(\omega)$ calculated for pure Doppler and pure natural types of broadening coincide. Figure 4 shows graphs of the redistribution functions for $a = 10^{-3}$. We see well from them that almost coherent scattering occurs when $\omega \gg \omega_{\rm F}$. If the condition is satisfied that

$$\omega_{\rm eff} \gg \omega_{\rm F}, \tag{7.9}$$

then again we return to Eq. (4.8), with

$$D = \Delta\omega_{\rm D}^2. \tag{7.10}$$

Different aspects of the problem in this approximation have been treated in Refs. 15, 36–40. The solution of Eq. (4.8)[15] is given by Eq. (7.5) with

$$\omega_{\rm eff} = \left(\frac{3}{2\pi\lambda_0}\right)^{1/6}(ak_{\rm D}L)^{1/3}\Delta\omega_{\rm D}, \tag{7.11}$$

$$k_{\rm D} = \frac{k_\Sigma}{\sqrt{\pi}\,\Delta\omega_{\rm D}}.$$

The effective rate of deexcitation according to (7.7) is

$$\tau_{\rm eff}^{-1} = \left(\frac{2\lambda_0}{3\pi}\right)^{1/2}\frac{\gamma}{k_{\rm D}L}. \tag{7.12}$$

The problem of the thermalization length is of interest.[38–39] If the deactivation in the medium is determined by quenching of excited atoms with the rate α, then, according to (7.12) and (3.11), we have

$$L_{\rm eff} = \left(\frac{a\gamma}{\alpha}\right)^{1/3}. \tag{7.13}$$

Yet if the deactivation is due to nonresonance absorption of photons in the medium, then the mean free path l of a photon is limited, rather than the lifetime of the excitation. We can easily show that in this case

$$L_{\rm eff} \sim \frac{e^{3/4}}{(ak_{\rm D})^{1/4}}. \tag{7.14}$$

The condition for applicability of the formulas derived for $\tau_{\rm eff}$ and $L_{\rm eff}$ acquires the form

$$(ak_{\rm D}L)^{1/3} \gg 1. \tag{7.15}$$

In the converse limiting case ($a \ll 1$), according to the results of Sec. 5, we must obtain the rate of deexcitation within the framework of the CFR for Doppler broadening.

It is interesting to trace how the transition for the effective rate occurs from (7.12) to the formulas of Sec. 3. One can obtain an exact solution of the problem only by numerical methods (e.g., the Monte Carlo method[41–44] and finite-difference methods[40,45–46]). However, if we assume that complete redistribution occurs when $\omega < \omega_{\rm F}$, while Eq. (4.8) holds when $\omega > \omega_{\rm F}$, we can propose an approximate formula[15]:

$$\tau_{\rm eff}^{-1} = \frac{2}{\pi}(\operatorname{arctg}\eta)\tau_{\rm H}^{-1} + \frac{1}{1+\eta}\tau_{\rm dif}^{-1}, \tag{7.16}$$

Here $\tau_{\rm H}$ is the deexcitation time according to Holstein for Doppler broadening, $\tau_{\rm dif}$ is defined by Eq. (7.12), and we have

$$\eta = \frac{\pi\lambda_0}{3}\left(\frac{\omega_{\rm F}}{\Delta\omega_{\rm D}}\right)^4\frac{1}{(ak_{\rm D}L)^2}. \tag{7.17}$$

Equation (7.16) yields an exact result in two limiting cases: CFR and almost coherent scattering. As is shown by

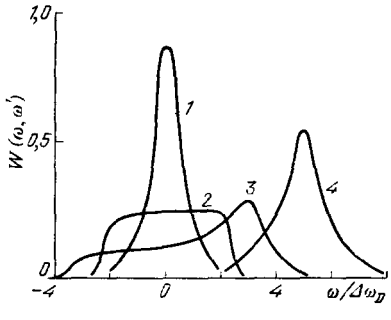

FIG. 4. Redistribution function for Doppler and for natural broadening. 1—$\omega'/\Delta\omega_{\rm D} = 0$; 2—$\omega'/\Delta\omega_{\rm D} = 2$; 3—$\omega'/\Delta\omega_{\rm D} = 3$; 4—$\omega'/\Delta\omega_{\rm D} = 5$.

comparison with numerical calculations, Eq. (7.17) holds throughout the range of the parameters with an accuracy of the order of 20%. The presented approach is in a certain sense a refinement and generalization of the approximate approach of Jeffries and White,[46] who generally neglected frequency diffusion of photons in the tail of the line. As we have already noted above, this can lead to considerable errors at great optical thicknesses.

Collisional frequency mixing leads to the redistribution function

$$W(\omega, \omega') = (1 - P_c)\,\delta(\omega - \omega') + P_c \varepsilon(\omega). \tag{7.18}$$

Here we have $P_c = \gamma_c/(\gamma_c + \gamma)$, where γ_c is the rate of broadening collisions, and $\varepsilon(\omega)$ is the dispersion profile. Physically Eq. (7.18) reflects the fact that, if no collision occurs within the time of existence of an excited atom, then the scattering is coherent. CFR occurs in the converse case. This approach is fully confirmed by quantum-mechanical calculations in the impact approximation.[47] The solution of Eq. (4.4) with this redistribution function is not complicated; it yields

$$f(\omega) = \frac{1}{1 + (\omega/\omega_{\text{eff}})^4}, \tag{7.19}$$

$$\omega_{\text{eff}} = \left(\frac{3P_c}{\lambda_0}\right)^{1/4} \left[\frac{(\gamma + \gamma_c)\, k_\Sigma L}{2\pi}\right]^{1/2}. \tag{7.20}$$

$$\tau_{\text{eff}}^{-1} = \gamma \left(\frac{\lambda_0}{3}\right)^{1/4} \frac{\sqrt{\pi}}{2} \left(\frac{\gamma + \gamma_c}{k_\Sigma L}\right)^{1/2} P_c^{4/3}. \tag{7.21}$$

The condition of applicability of diffusion theory in this case has the form

$$P_c \ll 1. \tag{7.22}$$

However, comparison of (7.21) with the results of Sec. 5 shows that (7.21) satisfactorily describes the situation almost throughout the range of the parameters.

In the joint action of Stark and Doppler redistribution mechanisms, their frequency-diffusion coefficients add. Taking account of collisional redistribution simultaneously with them requires numerical solution of Eq. (4.4). To an accuracy of ~20% the effective rate of deexcitation is given by the sum of (7.21) and (7.7):

$$\tau_{\text{eff}}^{-1} = \left(\frac{2}{3} D\lambda_0\right)^{1/2} \frac{\gamma}{k_\Sigma L} + \gamma \left(\frac{\lambda_0}{3}\right)^{1/4} P_c^{4/3} \frac{\sqrt{\pi}}{2} \left(\frac{\gamma + \gamma_c}{k_\Sigma L}\right)^{1/2}. \tag{7.23}$$

We see from (7.23) that the collisional and diffusional mechanisms of redistribution become comparable at

$$P_c \sim \left(\frac{D}{\gamma k_\Sigma L}\right)^{2/3} \ll 1. \tag{7.24}$$

To test the obtained results, a comparison was performed with the results of experimental measurement of the deexcitation rate of argon in the 1048-Å line[48] from a cylindrical volume of radius 1.1 cm in the pressure range 0.0015–10 Torr. The parameters were such that, at a pressure of the order of 0.1 Torr, the escaping radiation lies in the tail of the natural broadening, and considerable deviations were observed from the Holstein rate of deexcitation. At lower pressures the results are satisfactorily described within the framework of the CFR approximation with Doppler broadening. At higher pressures CFR is brought about by resonance collisions. We see in Fig. 5 that taking account of the joint action of the effect described by Eqs. (7.16) and (7.23) agrees well throughout the region with the experimental data. For perspicuity the effective rate of deexcitation calculated according to Holstein is given in the same place.[10] Results have recently been published of experiments[49–50] in which an analogous effect was observed in the 1849-Å line of mercury of the natural isotropic mixture. These data are also described rather well by frequency diffusion of photons with account taken of the hyperfine splitting of the lines.

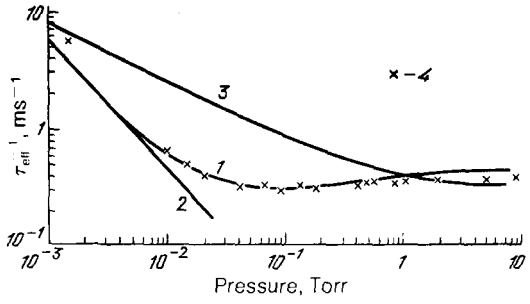

FIG. 5. Rate of deexcitation of the 1048-Å line of argon. *1*—result of Ref. 15; *2*—complete redistribution with Doppler broadening; *3*—complete redistribution with natural and collisional broadening; *4*—experimental data.[48]

8. TRANSPORT OF POLARIZED RADIATION

Up to now we have treated the radiation and the atoms of the medium without allowance for their polarization. The scattering process was considered isotropic. Actually polarization effects make the scattering process anisotropic, and one must elucidate to what extent this can alter the radiation yield. In the general formulation of the problem one must operate with the polarization density matrix of the photons and excited atoms, since in the scattering process an initial linearly polarized photon ceases to be such and goes over into a mixed state described by the density matrix. The equations of radiation kinetics in this form have been formulated in Ref. 16. Below we restrict the treatment to the case in which atoms have the total angular momentum $j_0 = 0$ in the ground state, and $j_1 = 1$ in the excited state. Upon scattering by such atoms a linearly polarized photon remains linearly polarized but changes its direction of polarization. The intensity of emission begins to depend on yet another argument—the direction **e** of polarization of the photons. The redistribution function of the polarizations and directions has the form:

$$W_{\text{pol}}(\mathbf{e}, \mathbf{\Omega}; \mathbf{e}', \mathbf{\Omega}') = \frac{3}{4\pi} (\mathbf{\Omega}\mathbf{e}')^2\, \delta\left(\mathbf{e} - \frac{\mathbf{e}' - (\mathbf{\Omega}\mathbf{e}')\,\mathbf{\Omega}}{[1 - (\mathbf{\Omega}\mathbf{e}')^2]^{1/2}}\right). \tag{8.1}$$

The first factor in (8.1) takes account of the angular dependence in dipole emission, and the second, the change in polarization of the radiation. We assume in (8.1) that, during the time of existence of an excited atom, it is not subject to collisions that can affect its polarization. Thus the density of the medium is considered to be low and the frequency redistribution function is determined by the Doppler effect.

The transport equations with account taken of polarization are very complex to solve. Some estimates of the characteristic quantities have been given in Ref. 51. In particular, it

was shown that the effective rate of deexcitation at large optical thicknesses is decreased in comparison with the Holstein value by the amount:

$$\Delta\tau_{\text{eff}} \sim \frac{\tau_{\text{eff}}}{\ln(k_0 L)} . \tag{8.2}$$

Numerical solution of the transport equations shows that the change in the effective deexcitation time does not exceed 10% at any optical thicknesses.

The question of the degree of polarization of the excited atoms is of interest. Results will be given below of a numerical experiment performed by the Monte Carlo method. A plane layer of thickness L is considered in which unpolarized excited atoms are formed homogeneously throughout the volume at the rate F (this corresponds, e.g., to excitation by thermal electrons). The number of excited atoms at each point will be determined by excitation by inelastic impacts and absorption of the radiation emitted at other points of the volume. Since the radiation is nonisotropic, this leads to formation of polarized atoms. As the measure of polarization we can take the quantity

$$p = \frac{n_0 - 0{,}5\,(n_1 + n_{-1})}{\sum_i n_i} . \tag{8.3}$$

Here the subscripts 0 and ± 1 denote the projection of the angular momentum of the excited atom on an axis perpendicular to the planes bounding the plane layer. When all the atoms have a zero projection of the angular momentum, then we have $p = 1$; if there are no such atoms at all, then we have $p = -0.5$. The value of the degree of polarization differs by a numerical coefficient from the alignment factor as defined as the second moment of the density matrix of the excited atoms.[52] Figure 6 shows the results of calculating the quantity p for a plane layer of differing optical thickness. We see that the strongest alignment is observed in a layer of optical thickness of the order of unity. With increasing optical thickness the alignment as a whole declines, which corresponds to the estimates of Ref. 52, where it was shown that, at the center of the layer with $k_D L \gg 1$, we have

$$p \sim \frac{1}{k_D L \ln^{1/2}(k_D L)} . \tag{8.4}$$

In agreement with the estimates of Ref. 52 the alignment changes sign in a narrow region at the boundary. This is

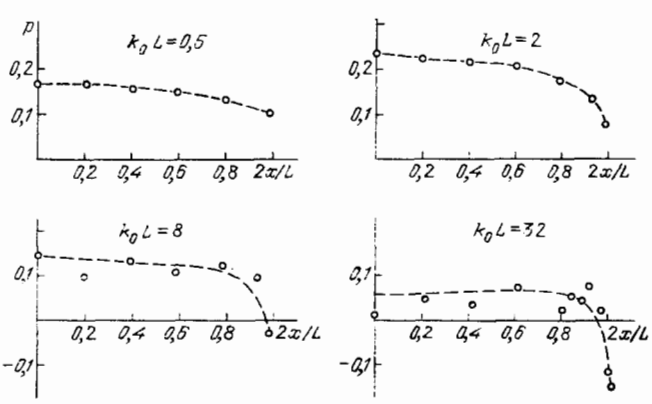

FIG. 6. Alignment of excited atoms in a plane layer of thickness L.

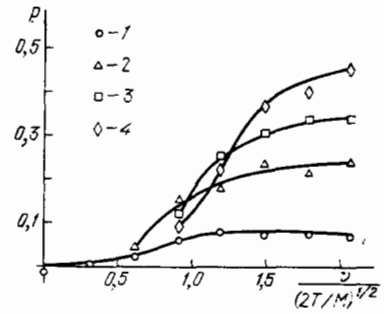

FIG. 7. Latent alignment at the center of a sphere. 1—$k_0R = 0.5$; 2—$k_0R = 2$; 3—$k_0R = 4$; 4—$k_0R = 8$.

explained by the fact that in the center of the layer the atoms are excited mainly by radiation propagating along the layer, but at the boundary by that escaping the layer.

In the situation examined above the alignment was created by nonisotropic radiation at the given point of the volume. However, even if the radiation is isotropic (as, e.g., at the center of a spherical volume), it has a spectral finite width, an atom possessing the velocity **v** will receive this radiation anisotropically owing to the Doppler effect. Radiation distributed perpendicularly to the motion of the atom will be absorbed more efficiently than that propagating collinearly. This has the result that an ensemble of atoms with the velocity **v** will have an alignment with the axis of **v**, although as a whole the atoms are unpolarized.

In the terminology of M. P. Chaĭka,[52] this alignment is called latent. The rate of alignment calculated by the authors by the Monte Carlo method at the center of the sphere is shown in Fig. 7. We see that, with increasing velocity the alignment can become considerable, but there are extremely few atoms having such a velocity.

In closing this section, we can conclude that the effects of polarization affect the radiation yield weakly. At the same time, the alignment varies in the magnetic field and an observation of a dependence of the alignment on the field strength (or a depolarization of the radiation) can serve as an instrument for determining atomic parameters, relaxation constants, magnetic characteristics, etc.[52,53] However, a discussion of these problems lies outside the scope of this review.

9. NONLINEAR EFFECTS IN THE PROBLEM OF RADIATION TRAPPING

The process of transport of high-intensity radiation is very complicated. Even in the case that a monochromatic wave of considerable amplitude is interacting with a two-level isolated atom, we must bear in mind a number of effects difficult to take into account: hole-burning, change in the spectral composition of the scattered radiation, etc.[54] Extensive literature has been devoted solely to discussing this problem. Yet there are practically no results in the problem of the interaction with a two-level system of several waves having different frequencies and directions. Therefore CFR has been assumed in the small number of studies on transport of high-intensity radiation. In Ref. 55 the problem was solved in a two-level formulation with conservation of the number of atoms in the two levels. In transforming from the coordinates to a new variable that is actually the optical path length, the authors[55] obtained a transport equation, now lin-

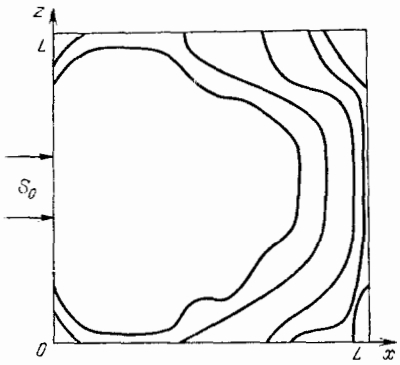

FIG. 8. Contours of the population of the excited state for a considerable radiation intensity. The arrows indicate the position of the exciting beam.

ear, but on an unknown scale. They obtained an estimate of the solution in a semiinfinite space and calculated the line contour of the escaping radiation. In Ref. 56 a solution of the problem of the population distribution in a plane layer was found on the basis of an asymptotic theory. The qualitative conclusion from Refs. 55 and 56 is rather lucid: stimulated scattering diminishes the optical thickness of the transition and decreases the extent of variation of the population. In Ref. 57 the problem of the interaction of high-intensity radiation with a gas of two-level atoms with Doppler broadening was studied by the Monte Carlo method. The transmission and reflection coefficients were calculated for a plane layer as functions of the intensity of the radiation incident on the layer. It was shown that the increase in the coefficients of reflection owing to the gradient of the absorption coefficient somewhat compensates the effect of hole-burning. They noted that, in scattering of a spatially restricted beam, the hole-burning effect alters the indicatrix of the backscattered radiation. The intensity becomes more extended in the "backward" direction—the layer begins to operate as a mirror. Figure 8 shows the contours of the populations, and Fig. 9 the indicatrix of the scattered radiation. In Ref. 58 the problem was solved of calculating τ_{eff} for a cylinder in the presence of stimulated emission under conditions of Doppler broadening, also by the Monte Carlo method. At the initial instant the populations of the lower and upper states were taken equal. We can formulate the result of the modeling as: with $k_0R < 1$, τ_{eff} coincides with the radiative lifetime, and when $k_0R > 1$, τ_{eff} rapidly goes over to the Holstein asymptotic. Here k_0 is taken to mean the "generalized" absorption coefficient (with account taken of stimulated emission) at the center of the line.

We should note that the assumption of CFR is not ob-

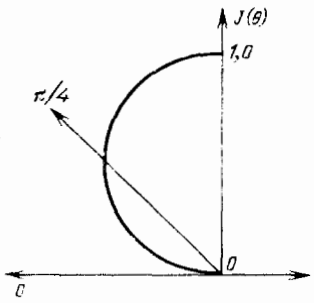

FIG. 9. Indicatrix of the backscattered radiation.

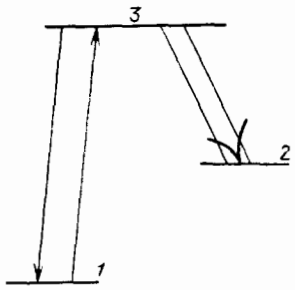

FIG. 10. Diagram of levels for the problem of trapping in a three-level system. *1*—ground state; *2*—lower laser level; *3*—upper level.

vious when the excitation of the medium is appreciable, and it is known to become false in the presence of inversion. In a medium with inversion the frequency distribution is narrowed, and strong anisotropy of the radiation occurs, etc. In all cases (both absorption and amplification) one must take account of the distortion of the emission and absorption contours by the radiation.

In treating the transport of high-intensity radiation in a dense plasma of hydrogenlike ions, Refs. 32 and 33 have analyzed the frequency redistribution function with Stark broadening. In this case the redistribution function begins to depend on the radiation intensity. At low intensity the result is close to the linear theory, while at very strong intensity CFR sets in under the "field" broadening.[54] In the intermediate case the problem is extremely complicated, and the redistribution function can be found only numerically. Yet, insofar as we know, no solution of the problem of radiation transport with such a redistribution function exists in the literature. Thus the study of problems of transport of high-intensity radiation is in its beginning stage.

A three-level system of an atom is studied in Refs. 59 and 60 in which a laser transition occurs between levels 3 and 2, and radiation trapping in the $3 \to 1$ transition (Fig. 10). The problem arose in connection with the interpretation of experiments[61] that established a deviation of the data of measurements and calculations for laser action. Attempts to interpret the smoothing of the Lamb dip owing to collisins led to anomalously large collision cross sections. Therefore in Refs. 59 and 60 the "diffusion into the dip" was explained by a frequency change upon light scattering in the $3 \to 1$ transition. Qualitatively one can understand this phenomenon by studying the formation of "Bennet holes" in the velocity distribution function of excited atoms (in level 3) in the presence of radiation trapping. The existence of "Bennet holes," as they are known, involves the fact that, in Doppler broadening of lines, only a small fraction of the atoms interacts with the laser field. The induced $3 \to 2$ transitions for this group of atoms decreases the number of atoms in level 3. In the presence of trapping in the $3 \to 1$ transition, the atoms from other regions in state 3 can spontaneously go to level 1 and return owing to absorption in the interaction region. The reverse course is also possible. Evidently the rate of the former transitions prevails over the rate of the latter. Consequently a partial "smearing" of the dip occurs. The theory of this phenomenon proposed in Refs. 59 and 60 agrees quantitatively with the experimental data.

10. RADIATION TRAPPING IN THE CASE OF FREE-BOUND TRANSITIONS

Radiative free-bound transitions in a plasma can be formally correlated with bound-bound transitions. Actually, if

we assume that the atom has only two states—the ground and the ionized states—and also radiation is present that can ionize the atoms, then the processes of radiation transport in the lines and in the continuum formally coincide. On the basis of this analogy, the authors of Ref. 62 derived an equation for the concentration of ions of the type of the Biberman-Holstein equation:

$$\frac{\partial N_1}{\partial t} = -n_e N_1 k_r + \int_V G(\mathbf{r}, \mathbf{r}') n_e(\mathbf{r}') N_1(\mathbf{r}') d^3\mathbf{r}'. \quad (10.1)$$

Here k_r is the recombination coefficient.

If we assume that the electrons and the ions are formed only by photoionization, then we have $n_e(\mathbf{r}) = N_i(\mathbf{r})$, and Eq. (10.1) becomes nonlinear. If we assume that the ions of the given type are a small addition, while the electrons are supplied by an easily ionizable component, then $n_e(\mathbf{r})$ is a known quantity, and Eq. (10.1) remains linear. The authors of Ref. 62, while assuming CFR for the recombination continuum, went over in Eq. (10.1) to the τ_{eff} approximation, taking $N_i(\mathbf{r})$ outside the integral. In this case this method is not fully obvious. The assumption of CFR in this case arouses no doubts. Actually CFR is attained owing to free-electron collisions with the other components of the plasma. As a rule, the cross sections of collisions with velocity change are much larger than the corresponding cross sections for photorecombination. For the method to be justified it requires also the coincidence of the frequency characteristics of the emission and absorption coefficients.[63] In the given case this is specifically not so, and the accuracy of the τ_{eff} approximation can turn out to be low. As was shown in Ref. 63 with a model example, the deviation from the exact solution can be as great as an order of magnitude. In the case of a recombination continuum, the mean free path of a quantum is a finite quantity and equals in order of magnitude the mean free path at the boundary of the continuum. The diffusion approximation is more natural, but no solution of Eq. (10.1) has been performed in this approximation, so that no final conclusion has been drawn at present on the accuracy of the approximation.

The two-level approximation is less substantiated than for transport in lines. Actually an electron can recombine also into an excited level, which leads to "fragmentation" of the quantum.

The small number of studies on transport in a recombination continuum indirectly indicates that the formation of an ionized state by radiation is a relatively rare situation, in contrast to trapping of radiation in lines.

11. JOINT EXCITATION TRANSPORT

In treating a number of experimental situations, along with taking account of a pure radiative transport mechanism, one must estimate the influence of the intrinsic displacement of the excited atoms. Chaotic motion of the latter can be interpreted as a diffusion process. Here the diffusion coefficient of the excited atoms is determined not by the gas-kinetic mean free path l, but by the effective mean free path[64]:

$$l_{eff}^{-1} = l^{-1} + \frac{\gamma + \alpha}{\langle v \rangle}. \quad (11.1)$$

Here $\langle v \rangle$ is the mean velocity of an atom. The latter term in (11.1) takes account of the decrease in the mean free path owing to deexcitation and quenching. An analytical study of the influence of diffusion of atoms has been performed in Refs. 71–73; numerical calculations are described in Refs. 74 and 75. Some of the theoretical results have been tested experimentally in Ref. 76. For our purposes we must first bear in mind the results of the simple estimates as well as of the numerical and experimental studies cited above, which imply that, for an optically allowed transition, the intrinsic diffusion of atoms can compete with radiation transport only in a thin layer near the boundaries of the volume, while it does not substantially alter the overall pattern of the process. If the probability γ is small, the role of the motion of the atoms can become predominant. The criterion for absence of an influence of diffusion of excited atoms can be the relationship

$$\frac{\langle v \rangle l_{eff}}{L^2} \ll \frac{1}{\tau_{eff}}. \quad (11.2)$$

If the medium is moving at the velocity v, this affects the occupancies of excited atoms under the condition that

$$\frac{v}{L} \gtrsim \frac{1}{\tau_{eff}}. \quad (11.3)$$

This condition is often realized in calculating the density of excited atoms ahead of the front of a shock wave.[77–79]

12. CONCLUSION

In closing we shall present some results and cite some related and unsolved problems.

At present the theory of transport with CFR is close to completion. The determination of the region of applicability, the detailed comparison with experiment, the large number of problems solved in the asymptotic region, and also by approximate and numerical methods—all this allows us to draw the above conclusion. The situation is more complicated upon strong deviation from the simplified conditions of the linear two-level problem. Transport of high-intensity radiation and interaction of radiation with a large number of levels of the atom are problems far from fully worked out. The difficulty of theoretical analysis consists here in the nonlinear character of the equations. We should also note that radiation transport in these cases must be considered jointly with other kinetic processes and the hydrodynamic movement of the medium.

The theory of transport with IFR is also far from perfection. Although the frequency redistribution functions have been calculated for many cases of broadening, there are as yet very few examples of solving concrete problems for IFR. The appearance of new objects of study, e.g., a laser plasma, requires construction of a theory under conditions sharply differing from those traditionally studied. The studies in which both factors, IFR and nonlinearity, are manifested simultaneously are quite solitary. The theory is taking only the first steps along this line. As a rule the results of transport theory under the assumption of IFR adequately describe the physical processes observable in laboratory experiment. The individual parameters when CFR is not fulfilled are presented in Secs. 5–7; we see from them that deviation from CFR leads to considerable difference in the observable quantities. However, the greatest interest arises in transport theory with IFR in solving problems associated

with extremal states of the medium: in studying strong shock waves, a laser plasma, a plasma of high-current discharges and other objects in which ions of high multiplicity Z are formed. As the estimates show, under these conditions CFR ceases to be fulfilled for a number of reasons (mainly because of the increase in the rate of spontaneous transitions in proportion to Z^4).

However, radiation transport in these objects is a very important factor, but not the only one. In modeling one must also take into account the conditions of energy dissipation, hydrodynamics, kinetics of ionization, transport in the continuous spectrum, etc. It is very difficult to point out "pure" experiments that isolate the role of radiation trapping alone. The approaches described above for solving transport problems under IFR conditions are at the same time an integral part of the more general, complicated calculation.

This review has treated radiation transport under conditions in which the medium is far from an equilibrium state, including discussion of extreme nonequilibrium situations in which the rate of the radiative processes is much greater than the rates of the collisional processes. However, radiation transport in lines can also play an important role under conditions of LTR. Without altering the occupancies of the levels explicitly, radiation can act on them indirectly—owing to heat exchange. A large series of studies along this line has been performed by V. G. Sevast[3] yanenko and his associates (see, e.g., Ref. 66).

Having focused attention on transport in the spectra of atoms and ions, we have not touched upon problems of radiation transport in molecular media. A series of studies on radiation transport in spectra of diatomic molecules[67,68] has been constructed by analogy with transport in atomic spectra. In the "photon shoot-through" approximation, a number of problems of atmospheric optics has been solved.[69,70]

However, we should note that the analogy with atoms is not far-reaching, and the problem of transport in molecular spectra as before remains topical, both on the level of substantiating the starting equations and in solving applied problems.

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