The physics of magnetic domains

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The physical principles and basic theses of the theory, enabling a unified description of the magnetic properties of all thermodynamically stable domain structures in magnets, are presented. The theory is based on the following proposition proved by the authors: a necessary condition for the formation of all thermodynamically stable domain structures in magnets is the existence of a first-order phase transition induced by an external magnetic field. The approach developed makes it possible to regard the physics of domain structures as a part of thermodynamics, to derive the conditions for the existence of domains with different number of phases, to study the structure of the region of existence of domains, and to formulate for magnets with a domain structure an analogue of Gibbs rule. The general assertions of the theory are amply illustrated with experimental results. The properties of domain structures are analyzed in detail, and an interpretation is given for many experimental results for the most studied spin-reorientational transitions.

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1. INTRODUCTION

Domain structures in ferromagnets have been studied for a long time in the physics of magnetic domains. In Ref. 1 it is proved that the formation of such domain structures is thermodynamically favored. According to Ref. 1 partitioning a finite ferromagnet into regions with different orientations of the magnetic moment **M** (domains) lowers the energy of the magnetic fields generated by the body. By the mid-1960s experimental studies as well as theoretical works, developing ideas and defining them concretely,[1] largely completed the construction of the methods and models of the traditional physics of magnetic domains. The achievements of this period are described in monographs,[2–4] collected works,[5,6] and reviews.[7,8]

Over the last two decades fundamental research on magnetic domains has been reborn and continues to grow at an accelerated rate. This situation is primarily dictated by the needs of technology and applied science. The appearance of high-quality epitaxial magnetic films in the mid-1960s opened up wide possibilities for the development of fundamentally new technological devices based on magnetic domains, realized in such films. The most promising development (and one which has already come to practical fruition) is the development of new devices for recording and storing information based on films with magnetic bubbles.[9–11] Such devices can obviously be built only based on detailed knowledge about the properties of domain structures in epitaxial films.

Another factor that stimulates interest in the physics of domains is the discovery of different types of domain structures in magnetically ordered crystals many properties of which cannot be understood within the framework of the traditional ideas. Here we should first mention domains, discovered in association with first-order phase transitions (PTI) induced by an external magnetic field **H**. In Refs. 12 and 13 the formation of a thermodynamically stable domain structure accompanying a spin-reorientational PTI induced by an external field was proved theoretically for the example of a spin-flop transition in an easy-axis antiferromagnet. By analogy to a superconductor[14–16] such domain structures were termed an intermediate state (IS) of the magnet. The first IS accompanying a spin-flop transition was observed in Refs. 17–21. The publications of Bar'yakhtar *et al.*,[12,13,17,18] Dudko *et al.*,[19,20] and King,[21] stimulated theoretical and experimental studies of IS at different PTI.[22–63,131–135]

Domains have also been discovered in different multi-sublattice magnets in noncollinear phases. Such domains

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have been observed in ferrites (so-called high-field domains)[64–67] and in orthoferrites in the region of spontaneous smooth spin-reorientation.[68–71] Such domains can exist in antiferromagnets with smooth spin orientation induced by an external field.[20,72] In Refs. 73 and 74 domain structures were observed in many-axis magnets and termed "multiplet" structures. Finally, domain structures near second-order phase transitions (PTII) have been studied theoretically and experimentally.[75–80]

The types of domain structures listed above[68–80] are characterized by properties that at first glance distinguish them strongly from one another as well as from the domains of a demagnetized ferromagnet. It was gradually determined that these domain structures are "special," and their properties were often opposite to those of the "usual" (Weiss) domains. For this reason, up to now, each type of domain structure listed above has been studied practically independently of the others. In this connection, two questions arise:

1. What are the physical reasons for the formation of different types of domains?

2. Is it possible to find the general laws governing the formation of domain structures in magnets?

The analysis in Refs. 81 and 82 of the conditions required for realization of thermodynamically stable domain structures gives the following answers to these questions:

1. The thermodynamic stability of any domain structure in a magnetically ordered body is determined, as in a ferromagnet, by the decrease in the energy of the magnetic fields generated by this magnet.

2. The equilibrium states realized in separate domains represent coexisting phases of an external-field-induced PTI. Moreover, it can be shown that the existence in a magnet of an external magnetic-field-induced PTI is a necessary condition for the formation of all thermodynamically stable domain structures.

In this connection we recall that in a uniaxial ferromagnet in a field $\mathbf{H} = 0$ a PTI occurs between states with $\mathbf{M}_1$ parallel to the axis of easy magnetization and $\mathbf{M}_2 = -\mathbf{M}_1$, which form the domain structure.[83] In a ferromagnet with higher symmetry the degenerate states, forming a multiphase domain structure, are also the competing phases of a PTI occurring in the field $\mathbf{H} = 0$ (this is discussed in greater detail below).

It follows from the foregoing discussion that there is no fundamental difference between the types of domain structures listed above and the "usual" domains of a ferromagnet with regard to either the conditions leading to their realization or the factors responsible for their thermodynamic stability. The difference in the physical properties of such domain structures is linked only with the character of the external-field-induced PTI, the corresponding equilibrium states, and the dependence of these quantities on the external parameters. The physical generality of all thermodynamically stable domain structures makes it possible to construct theory, in which the general laws of the behavior of a magnet with a domain structure can be studied without specifying the type of PTI with which its formation is linked. This approach makes it possible to regard the physics of domain structures as a subfield of thermodynamics, to derive the conditions for the existence of domains with a different number of phases, to study the structure of the region of existence of the IS, and to formulate for a magnet with domain structure an analogue of Gibbs phase rule.

This review is devoted to the systematic exposition of the principles and methods of the physics of magnetic domains. The basic assertions of the theory are amply illustrated with experimental results. The properties of domain structures are analyzed in detail and an interpretation is given for many experimental results for the most studied spin-reorientational transitions: spin-flop transition in easy-axis antiferromagnets and spontaneous transitions in orthoferrites.

2. PHENOMENOLOGICAL DESCRIPTION OF DOMAIN STRUCTURES

On the basis of the phenomenological theory (later termed micromagnetism; see Ref. 4) developed in Ref. 1 the problem of determining the equilibrium spin configurations in a magnet with a domain structure reduces to the solution of the equations specifying a minimum of the nonequilibrium thermodynamic potential

$$F=\int \Phi\, dV=\int \Big[\varphi\left(\mathbf{M}(\mathbf{r}),\ L_\nu(\mathbf{r}),\ \frac{\partial \mathbf{M}}{\partial x_i},\ \frac{\partial L_\nu}{\partial x_i}\right) - \mathbf{M}(\mathbf{r})\,\mathbf{H}-\frac{1}{2}\mathbf{M}(\mathbf{r})\,\mathbf{H}_{\mathbf{M}}(\mathbf{r})\Big] \quad (2.1)$$

together with the equations of magnetostatics

$$\mathrm{rot}\,\mathbf{H_M}=0,\quad \mathrm{div}\,\mathbf{H_M}=4\pi\rho_\mathbf{M},\quad \rho_\mathbf{M}=-\mathrm{div}\,\mathbf{M}, \quad (2.2)$$

where $\mathbf{M}$ is the total magnetization; L_ν are other internal parameters of the magnet, for example, the components of the antiferromagnetism vectors—definite linear combinations of the magnetization vectors of the sublattices $\mathbf{M}_\alpha$; $\mathbf{H_M}(\mathbf{r})$ is the magnetostatic field generated by magnetostatic charges ρ_M; and, φ is the part of the internal energy associated with the short-range interactions in a magnet (exchange, anisotropic, Dzyaloshinskiĭ interaction, etc.).

In its general form this problem leads to a system of integrodifferential equations. The "superproblem" of micromagnetism is to construct the solutions of these equations in the form of some nonuniform distributions $\mathbf{M}(\mathbf{r})$, $L_\nu(\mathbf{r})$. Because of the complexity of the equations this program is far from being completed.

In the experimentally observed domain structures, however, the distributions $\mathbf{M}(\mathbf{r})$ and $L_\nu(\mathbf{r})$ are almost always alternating, quite large regions of uniform distributions $\mathbf{M}(r)$ and $L_\nu(\mathbf{r})$ (domains), separated by narrow transitional regions with a strongly nonuniform distribution of internal parameters (domain boundaries). Thus the characteristic thicknesses of the domain boundaries x_0 are much smaller than the characteristic dimensions of the domains D. In addition, if the sample is not anomalously small (the exact criterion will be formulated in Sec. 13), then x_0, D, and its characteristic size L satisfy the inequality

$$x_0 \ll D \ll L. \quad (2.3)$$

This hierarchy of sizes makes it possible to simplify substantially the solution of problems in micromagnetism.

The inequality $x_0 \ll D$ made it possible in Ref. 1 to develop the "thin-wall approximation," which forms the basis for most modern methods for calculating the equilibrium domain structure: in the calculation of the characteristic parameters of domain boundaries the dimensions of the domains are assumed to be infinite, while in the calculation of the equilibrium parameters of a domain structure it is assumed that the domain boundaries are infinitely thin and are characterized by an integral parameter—the surface energy density.

The equilibrium parameters of a domain structure include the parameters characterizing the internal state of the magnet in separate domains $\mathbf{M}^{(k)}$, $L_v^{(k)}$ (k enumerates the domains with different internal states) on the one hand and the geometric parameters fixing the shape and dimensions of the domains on the other. The internal states of a magnet in the domains $\mathbf{M}^{(k)}$, $L_v^{(k)}$ are determined by the strengths of the exchange, anisotropic interactions, the Dzyaloshinskiĭ interaction, etc., as well as by the strength of the internal field

$$\mathbf{H}^{(i)}(\mathbf{r}) = \mathbf{H} + \mathbf{H}_M(\mathbf{r}). \tag{2.4}$$

The physical nature of the formation of a domain structure as well as its basic properties are most easily studied both theoretically and experimentally under conditions when uniform states are realized in the domains. It is obvious that the uniformity of the internal states in domains is predicated on the uniformity of the internal field $\mathbf{H}^{(i)}$ in them. Such uniformity of the internal field obviously cannot be achieved, in samples with a domain structure, because of the nonuniformity of the magnetostatic field $\mathbf{H}_M(\mathbf{r})$. This field is generated by magnetostatic charges ρ_M (2.2), present on the surface of the sample or in its volume.

We shall first study the field generated by the surface charges. It can be made uniform in the bulk of the sample if the sample is ellipsoidal while the domain structure realized is regular. By a regular domain structure we mean[81,82] a structure for which the magnetization $\langle\mathbf{M}(\mathbf{r})\rangle$, averaged over the dimensions x such that $L \gg x \gg D$, is uniform over the sample. Of course, the concept of a regular domain structure can be introduced only if the inequality (2.3) holds and is not predicated on strict periodicity of the distribution $\mathbf{M}(\mathbf{r})$. In this case the magnetostatic field in the bulk of the sample will be uniform and given by

$$\mathbf{H}_M = -4\pi\hat{N}\langle\mathbf{M}\rangle, \tag{2.5}$$

where $\hat{N}$ is the tensor of demagnetizing coefficients; $\mathbf{H}_M$ is nonuniform only in a layer of thickness of the order of D near the surface of the sample.

If the state of the domains is uniform, then the intravolume magnetostatic charges can be concentrated only on the domain walls. Of course, total vanishing of ρ_M in the domain boundaries, generally speaking, cannot be achieved (the Néel domain walls in ferromagnets are an example[84]); however, only distributions $\rho_M(\mathbf{r})$ that lead to the existence of an uncompensated magnetostatic charge on a domain wall, associated with a jump in the component of the magnetization vector normal to the domain wall

$$m_n = M_n^{(k)} - M_n^{(k')}; \tag{2.6}$$

where $\mathbf{M}^{(k)}$,$\mathbf{M}^{(k')}$ are the equilibrium magnetizations in neighboring domains, lead to an increase in the volume part of the magnetostatic energy. Thus the requirement that the thermodynamic potential be minimum leads to the condition

$$m_n = 0. \tag{2.7}$$

It follows from what was said above that in an ellipsoidal sample the regular domain structure is energetically favored since its formation prevents the appearance of an additional magnetostatic field $\delta\mathbf{H}_M$ in the bulk of the sample and the associated increase in energy

$$\Delta E \sim \frac{(\delta\mathbf{H}_M)^2}{8\pi} V,$$

where V is the volume of the sample.

The condition (2.7), together with (2.5), means that the internal field $\mathbf{H}^{(i)}$ in the bulk of a sample with a regular domain structure is uniform and is given by

$$\mathbf{H}^{(i)} = \mathbf{H} - 4\pi\hat{N}\langle\mathbf{M}\rangle. \tag{2.8}$$

We note that a domain wall in the form of an arbitrary cylindrical surface, whose generatrix is parallel to the vector[81,82]

$$\mathbf{m}_{kk'} = \frac{1}{2}(\mathbf{M}^{(k)} - \mathbf{M}^{(k')}). \tag{2.9}$$

satisfies the relation (2.7). For the reasons enumerated above, in what follows we shall study domain structure only in ellipsoidal samples, including also limiting forms of an ellipsoid: plane-parallel plates and elongated cylinders. The condition for realization of a domain structure in nonellipsoidal samples are discussed in Sec. 13.

Thus in an ellipsoidal magnet with a regular domain structure, because of the conditions (2.3) and (2.8), only the terms associated with uniform interactions make the main contribution to the total energy (2.1) (proportional to the volume of the ellipsoid):

$$\Phi = \sum_{k=1}^{n} \xi_k \varphi(\mathbf{M}^{(k)}, L_v^{(k)}, 0, 0) - \langle\mathbf{M}\rangle\mathbf{H} + 2\pi\langle\mathbf{M}\rangle\hat{N}\langle\mathbf{M}\rangle, \tag{2.10}$$

$$\langle\mathbf{M}\rangle = \sum_{k=1}^{n} \xi_k \mathbf{M}^{(k)}, \quad \sum_{k=1}^{n} \xi_k = 1, \tag{2.11}$$

where n is the number of different types of domains (n-phase domain structure) and ξ_k is the volume fraction of domains of the k th type. As regards the energy associated with nonuniformities (the nonuniform part of the magnetostatic energy $\Delta\Phi_{MS}$ and the energy of the domain walls Φ_{DW}), it is significantly smaller than the volume terms in the energy (2.10) and its contribution to the energy (2.1) is proportional to v^α ($\alpha < 1$; for example, $\alpha = \frac{1}{2}$ for a striped structure in a plane-parallel plate; $\alpha = \frac{2}{3}$ for domains branching at the surface), so that it can be neglected in the leading order approximation in the parameter D/L. In the limit $V \to \infty$, the main contribution $\Delta\Phi_{MS}/\Phi_{DW} \sim V^\alpha/V$ approaches zero. In the leading order approximation in D/L the energy of the ellipsoid with a regular domain structure is independent of the domain "microstructure" (the distribution of the magnetization in the domain walls, the shape and dimensions of the domains), so that in Refs. 81 and 82 this approximation was termed *thermodynamic*.

An important result follows from what was said above. In a magnet with a regular domain structure three groups of parameters characterizing the structure can be separated; each group is associated with different, with respect to the power of D/L, contributions to the energy. They form the following hierarchy: 1) structure of domain walls; 2) shape and size of domains; and, 3) internal states of the domains

$\mathbf{M}_{(k)}$, $L_\nu^{(k)}$. The problem of micromagnetism for regular domain structures thus reduces to the solution of three largely independent problems.

1) The theory of domain walls is concerned with the smallest scale in a magent with domains—nonuniform states in transitional regions. Here the structure, thickness, energy, and their dependence on the external parameters are calculated for isolated domain walls. This part of the theory of domain structures is most developed. Many results obtained in this field are described in the monographs of Refs. 84 and 85, and the total symmetric classification of domain walls in magnets is developed in Refs. 124 and 125.

2) The calculation of equilibrium geometric parameters of model domain structures. This includes problems in which certain domain configurations are modeled for magnets with a given shape and in the "thin-wall" approximation the equilibrium values of these quantities are determined by minimizing the energy with respect to their geometric parameters. The nonuniform part of the magnetostatic energy $\Delta\phi_{MS}$ plays the determining role in the formation of equilibrium geometric parameters of domains and the energy of the domain walls. For this reason the calculation of the magnetostatic energy for model domain structures is the chief part of the solution of this group of problems. Thus far such problems have been solved primarily for ferromagnetic plates with striped and cylindrical domains.[9-11,86-89] The equilibrium geometric parameters have also been calculated for domain structures with some spin-reorientational transitions.[12,13,22-25,35,90,91]

3) The thermodynamic theory of magnetic domains is a macroscopic theory, and in this theory the behavior of the domain structure is studied irrespectively of the shape and size of the domains. On the basis of this approximation it is possible to determine the equilibrium values of the internal states in the domains $\mathbf{M}^{(k)}$,$L_\nu^{(k)}$ and their dependence on the external parameters, the magnetization of the sample partitioned into domains, the region of existence of the domain structure, and a number of other quantities.

The first such approximation was apparently employed in Refs. 14 and 15 in a phenomenological description of the IS of a superconductor. Later analogous approximations were employed many times to solve different problems for both magnetic domains[92-94,127-130] and for other domain structures.[30,35,95] Until recently, however, the thermodynamic approximation was used for particular problems, while the range of problems in the traditional theory of magnetic domains was limited to the simplest models.

For the domain structures currently studied in experimental and applied physics (primarily for domains in the region of spin-reorientational transitions) the question of the magnitudes of the internal parameters in domains and their dependence on the external parameters is one of the most important questions. For this reason the thermodynamic approximation, on the basis of which this problem is solved, forms the basis for the theoretical analysis of domain structures. The values of $\mathbf{M}^{(k)}$,$L_\nu^{(k)}$ obtained in the thermodynamic approximation in the calculation of the characteristic parameters of solitary domain walls play the role of boundary conditions, and in determining the equilibrium values of the geometric parameters of model domain structures they form the parameters of the theory together with ϕ_{DW}. Figure 1 shows the quantities determined in each of the three groups of problems of the theory of magnetic domains listed above, and their interrelationship is indicated schematically.

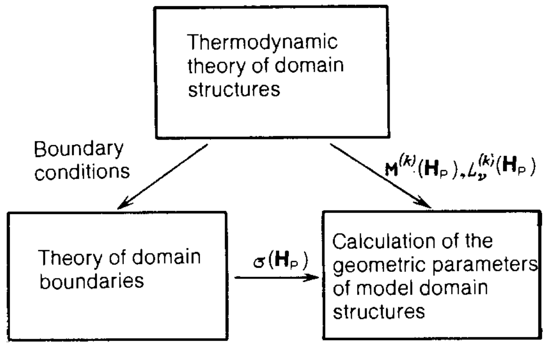

FIG. 1. Basic parts of the theory of domain structures and their interrelationships.

3. THERMODYNAMICALLY STABLE DOMAIN STRUCTURE AS AN INTERMEDIATE STATE OF A MAGNET

Thus in the thermodynamic approximation the nonequilibrium potential (2.1) of a magnet with a regular n-phase domain structure has the form (2.10). Minimizing the potential

$$\Phi_\lambda = \Phi - \lambda V\left(\sum_{k=1}^{n} \xi_k - 1\right), \tag{3.1}$$

where λ is an undetermined Lagrange multiplier, we obtain the following system of equations with respect to $\mathbf{M}^{(k)}$,$L_\nu^{(k)}$, and ξ_k, determining the equilibrium values of these parameters:

$$\xi_k \frac{\partial\varphi_k}{\partial L_\nu^{(k)}} = 0, \quad \xi_k\left(\frac{\partial\varphi_k}{\partial \mathbf{M}_k} - \mathbf{H}^{(i)}\right) = 0, \tag{3.2}$$

$$\varphi_k^0(\mathbf{H}^i) \equiv \varphi_k - \mathbf{M}\mathbf{H}^{(i)} = \lambda \equiv \varphi_0 \qquad (k = 1,\ 2,\ \ldots,\ n), \tag{3.3}$$

where $\mathbf{H}$ is given by the relation (2.8), and $\varphi_k \equiv \varphi(\mathbf{M}, L_\nu^{(k)}, 0, 0)$.

It follows from the relations (3.2) and (3.3) that the necessary condition of the existence of a thermodynamically equilibrium regular domain structure consisting of n phases is that their thermodynamic potentials $\varphi_k^0(\mathbf{H}^{(i)})$, corresponding to the equilibrium states (see (3.2)) in the internal field $\mathbf{H}^{(i)}$, must be energetically degenerate. We emphasize that in the system of equations (3.2) and (3.3) the internal magnetic field $\mathbf{H}^{(i)}$ plays the role of a fixed external parameter. This is associated with the fact that it is precisely $\mathbf{H}^{(i)}$ that acts on the magnetic moments of the atoms in the sample.

Thus the problem of determining the possible phases, comprising the domain structure, actually reduces to minimizing the nonequilibrium thermodynamic potential of a uniform system in a fixed field $\mathbf{H}^{(i)}$

$$\Phi_0 = \varphi(\mathbf{M},\ L_\nu,\ 0,\ 0) - \mathbf{H}^{(i)}\mathbf{M} \tag{3.4}$$

with respect to the parameters $\mathbf{M}$ and L_ν and determining the range of the internal field $\mathbf{H}^{(i)}$ in which the energy of the equilibrium uniform states φ_k^0 (3.3) is degenerate.

The degeneracy of the energy of the ground state could be determined by the symmetry of the magnet (1) or it could be accidental (2).

1) Such degeneracy of the energy in a magnet arises with spontaneous symmetry breaking. If the space group of the low-symmetry state is a subgroup with index n with respect to the group of the symmetric state, then the symmetric state can transform into a low-symmetry state by n mechanisms. In the process, n different states (phases), in each of which the transition is realized by one of n mechanisms, can be realized in the magnet. These n states transform into one another by means of those symmetry operations that the crystal loses in the transition into the low-symmetry state.[96]

As is well known, the phase transitions occurring with a lowering of symmetry include, for example, the transition of a ferromagnet into the ordered state.[83] In addition, such phase transitions include transitions into corner phases or into the region of smooth spin reorientation.[97] In this case a continuous restructuring of spin configurations from one symmetric state to another occurs in a definite range of change of external parameters.

We shall show that if spin configurations with different values of the vector **M** are realized in the transition into the low-symmetry state, then their formation can be linked with the existence of an external-field-induced PTI in the system.[81,82] It is obvious that states with different values of the vector **M** can be energetically equivalent only if there is no magnetic field **H** or the field is oriented along distinguished crystallographic directions. The tilting of **H** away from these symmetric directions removes the energy degeneracy: the direction of **M** that makes the smallest angle with **H** becomes energetically favored. Therefore it can be assumed that the magnetic field in this case induces the PTI. For example, in an easy-axis ferromagnet with no magnetic field two states are realized: $\mathbf{M}_1 \| Oz$ (Oz is the axis of easy magnetization) and $\mathbf{M}_2 = -\mathbf{M}_1$. The magnetic field $\mathbf{H}^{(i)} \| Oz$ removes this degeneracy, i.e., in the field $\mathbf{H}^{(i)} = 0$ a PTI occurs between phases with $\mathbf{M}_1$ and $\mathbf{M}_2$. The nontriviality of the situation here lies in the fact that the magnetic field **H** is an external vector parameter. Other phase transitions (both first and second order), associated with other components of **H** or some nonmagnetic external parameter, for example, the temperature, can occur, together with PTI induced by one of the components of **H**, in the phase diagram containing the field **H**. In the investigation of phase transitions in a magnet, occurring with spontaneous symmetry breaking, phase diagrams in the variables giving rise to the given transition were usually studied. In the process the fact that the component (or components) of the magnetic field that removes (remove) the energy equivalence of the degenerate states in the low-symmetry phase actually induces (induce) a PTI in the system was ignored.

2) We shall term energy degeneracy that is not associated with a transition of the magnet into a low-symmetry state accidental. In this case the energies of the different states assume the same value owing to a unique balance between the strength of the internal and external interactions. Thus, for example, in an easy-axis antiferromagnet for a certain value of the magnetic field, parallel to the axis of easy magnetization, the energies of the collinear phase, whose formation is associated with an advantage in the anisotropy energy, and of the spin-flop phase, corresponding to the minimum energy of interaction with an external field become equal.

It follows from what was said above that the degeneracy of the energy (3.4) with respect to **M** can always be linked with the existence of an external-field-induced PTI in the magnet. Thus all thermodynamically stable domain structures consist of domains of the competing phases of a PTI.

We call attention to the following fact. The invariance of the energy of the system with respect to the time-reversal operation leads to the conclusion that every state with nonzero **M** and L_ν are at least doubly degenerate in the absence of an external magnetic field: $(\mathbf{M}, L_\nu)$ and $(-\mathbf{M}, -L_\nu)$. This means that in a field $H = 0$ only an even number of phases can coexist under the conditions of phase transitions. The exception is the PTI from the magnetically ordered state into the paramagnetic state, where $\mathbf{M} = L_\nu = 0$.

In an external magnetic field with arbitrary orientation the degeneracy owing to symmetry, generally speaking, is removed and the accidental degeneracy remains. If the magnetic field is oriented along the symmetry axes (planes), then the degeneracy due to symmetry is removed only partially.

As is well known, the term "intermediate state," introduced in Ref. 12, has now been solidified in the literature for domains formed with external-magnetic-field-induced PTI. The results of this section lead to the following conclusion: *all thermodynamically stable domain structures are an intermediate state*. For this reason, it appears to be advantageous to employ the term "intermediate state" to denote thermodynamically stable domain structures. In this connection it is of interest to trace the evolution of the term "intermediate state." In Ref. 14 R. Peierls, who was studying the thermodynamics of a superconductor in the region of an external-field-induced PTI into the normal state, showed that in finite samples a transitional (buffer) region, which he termed "intermediate state," forms in a definite range of magnetic fields. The structure of the IS, however, was not studied in Ref. 14. The structure of the IS of a superconductor was clarified by L. D. Landau,[16] who showed that this transitional region (TR) is a thermodynamically stable domain structure consisting of regions of the metal in normal and superconducting states.

It was shown in Ref. 12 that the arguments analogous to those employed by L. D. Landau in Ref. 16 can be used to prove the formation of a thermodynamically stable domain structure with external-field-induced spin-reorientational PTI. By analogy to a superconductor such domain structures were termed intermediate states of the magnet. Thus the term "intermediate state," introduced by R. Peierls for conditionally denoting the transitional region in superconductors, as a result of Refs. 12, 13, and 17–63 has now acquired a wider meaning and is generally accepted for the thermodynamically stable domain structure arising with first-order phase transitions induced by an external magnetic field. It appears in the monograph and reviews of Refs. 84 and 97–99. Finally, the general conditions for the existence of IS have been clarified in Refs. 81 and 92.

4. PHASE DIAGRAMS OF MAGNETS. PHASE RULE

We shall derive some general relations for phase diagrams, containing d components of the magnetic field ($d = 0,1,2,3$) and λ components of other parameters of the system τ_i (the temperature, pressure, ..., $i = 1,2,..., \lambda$).

In a thermodynamically equilibrium domain structure states corresponding to coexisting phases accompanying PTI are realized in separate domains. For this reason, we must set $\mathbf{H}^{(i)} = \mathbf{H}_P$ in Eqs. (3.2) and (3.3), determining the equilibrium values of the internal parameters. For a domain structure consisting of n phases Eqs. (3.2) assume the form

$$\frac{\partial \varphi_k}{\partial L_\nu^k} = 0, \quad \frac{\partial \varphi}{\partial \mathbf{M}^{(k)}} - \mathbf{H}_P = 0. \tag{4.1}$$

Solving (4.1) with respect to the variables $L_\nu^{(k)}$ and $\mathbf{M}^{(k)}$, we find them as functions of $\mathbf{H}_P$ and τ_{iP}. Substituting the quantities found into Eqs. (3.3) and (2.8), we arrive finally at the following system:

$$\Phi_1(\mathbf{H}_P,\tau_{iP}) = \Phi_2(\mathbf{H}_P,\tau_{iP}) = \ldots = \Phi_n(\mathbf{H}_P,\tau_{iP}), \tag{4.2}$$

$$\mathbf{H} = \mathbf{H}_P + 4\pi\hat{N}\sum_{k=1}^{n}\xi_k\mathbf{M}^{(k)}(\mathbf{H}_P,\tau_{iP}), \tag{4.3}$$

$$\sum_{k=1}^{n}\xi_k = 1, \tag{4.4}$$

where

$$\Phi_k(\mathbf{H}_P,\tau_{iP}) = \varphi[\mathbf{M}^{(k)}(\mathbf{H}_P,\tau_{iP}), L_\nu^{(k)}(\mathbf{H}_P,\tau_{iP}), 0, 0] - \mathbf{M}^{(k)}\mathbf{H}_P \tag{4.5}$$

is the equilibrium thermodynamic potential of a uniform k-th phase for an internal field equal to the field of the phase equilibrium $\mathbf{H}_P$ and the value of the parameters τ_i, also belonging to the region of the PTI—τ_{iP}. The system of $n-1$ equations (4.2) gives in the phase diagram in the components of the internal field—$\mathbf{H}^{(i)}$, τ diagram—the region of phase equilibrium ($\mathbf{H}_P,\tau_{iP}$). The system (4.2)–(4.4) determines the region of existence of the IS in the phase diagram in the components of the external field—the $\mathbf{H}$, τ_i diagram, and it also gives the equilibrium parameters of the IS $\xi_k(\mathbf{H},\tau_{iP})$, $\mathbf{H}_P(\mathbf{H},\tau_{iP})$.

For systems whose phase equilibrium is determined by the accidental degeneracy in the $(d+\lambda)$-dimensional $\mathbf{H}^{(i)}$, τ diagram the region of the PTI is determined by Eqs. (4.2). For n coexisting phases Eqs. (4.2) have a solution, if their number $n-1$, in any case, does not exceed the number of variables $d+\lambda$. From here we obtain the well-known Gibbs rule[83]

$$n \leqslant d + \lambda + 1. \tag{4.6}$$

Introducing the so-called number of thermodynamic degrees of freedom—d_P, we obtain

$$d_P = d + \lambda - n + 1. \tag{4.7}$$

In the $(d+\lambda)$-dimensional space of the $\mathbf{H}^{(i)}$, τ phase diagram the system of equations (4.2) defines a hypersurface (region of PTI) with dimension d_P (4.7).

We shall now examine the $\mathbf{H},\tau$ phase diagram. The region of existence of the IS in it is described by the system (4.2), (4.3), and (4.4). Eliminating ξ_n from Eq. (4.3) with the help of (4.4), we rewrite it in the form

$$\mathbf{H} = \mathbf{H}_P + 4\pi\hat{N}\mathbf{M}_n + 4\pi\hat{N}\sum_{k=1}^{n-1}\xi_k(\mathbf{M}_k - \mathbf{M}_n). \tag{4.8}$$

In the case of accidental degeneracy the vectors $\mathbf{M}_k - \mathbf{M}_n$ are linearly independent, if their number $n-1$ is less than (or equal to) the number of equations (4.3) d, i.e.,

$$n - 1 \leqslant d. \tag{4.9}$$

In this case (4.8) maps each point on the hypersurface of the PTI $\mathbf{H}_P$, τ_i into a linear region with dimension $n-1$. For the dimension of the IS d_{IS} on the $(d+\lambda)$-dimensional $\mathbf{H},\tau$ phase diagram we obtain

$$d_{IS} = d_P + (n-1) = d + \lambda, \tag{4.10}$$

i.e., the dimension of the IS equals the dimension of the phase diagram.

If $n-1>d$, then the number of linearly independent vectors $\mathbf{M}_k - \mathbf{M}_n$ in (4.8) equals d. Now, among the $n-1$ vectors $\mathbf{M}_k - \mathbf{M}_n$

$$\gamma = n - 1 - d \tag{4.11}$$

vectors will be linearly dependent. It follows from here that every point in the region of the PTI will be mapped, as ξ_k is varied, into a d-dimensional linear region, and the dimension of the IS will equal

$$d_{IS} = d_P + d = 2d + \lambda - n + 1. \tag{4.12}$$

The relations (4.10) and (4.12) are the analogue of the Gibbs rule for the region of existence of domain structures. When the inequality (4.9) holds, in Eqs. (4.8) there exists a single-valued relation between ξ_k and $\mathbf{H}$. This means that in a fixed external field $\mathbf{H}$ from the region of the IS all ξ_k are determined uniquely. If, however, $n-1>d$, then at each point in the region of existence of the IS a γ-parametric family of ξ_k will satisfy Eq. (4.8), i.e., for fixed external parameters $\mathbf{H}$ and τ_i domain structures with different relative percentages of the phases can be realized in the IS.

If the energy degeneracy is of a purely symmetric nature, then the number of coexisting phases equals the index of the subgroup of the low-symmetry state (n_0) with respect to the symmetry group of the paraphase—the symmetry group of the magnet in the paramagnetic state in the same magnetic field. (The exceptions from the formulated criterion are the rare cases when the paramagnetic and magnetically ordered phases are not related by a subgroup relationship.) Thus in a cubic ferromagnet with $\mathbf{H}^{(i)} = 0$ six or eight phases can coexist. In an easy-plane tetragonal ferromagnet four phases can coexist on the line $\mathbf{H}^i \| Oz$, $H_z^{(i)} < H_c$) (Oz is the difficult axis and H_c is a critical field.

It is obvious from what was said above that in the presence of both symmetric and accidental degeneracy a solution of the system of equations (4.2) can exist even when the number of equations (4.2) $n-1$ exceeds the number of variables $d+\lambda$ in them. In this case the number of coexisting phases can be greater than permitted by the Gibbs phase rule (4.6). Breakdown of the inequality (4.6) means that the number of vectors $\mathbf{M}_k - \mathbf{M}_n$ in (4.8) exceeds the number of equations. For this reason, here, as above, for fixed external parameters $\mathbf{H}$ and τ_i a γ-parametric family ξ_k, where γ is given by the relation

$$\gamma = n - 1 - d^*, \tag{4.13}$$

d^* is the number of linearly independent vectors $\mathbf{M}_k - \mathbf{M}_n$ in Eqs. (4.8), will satisfy Eq. (4.8). Here it should be kept in mind that because of symmetric degeneracy d^* can be less than d.

As pointed out above, the equilibrium parameters of the n-phase IS are determined by the solution of the system of equations (4.2)–(4.4).

When $\mathbf{H}$ varies arbitrarily both the relative fraction of coexisting phases ξ_k and the value of $\mathbf{H}_P$ (and together with it $\mathbf{M}^{(k)}(\mathbf{H}_P)$, $L_\nu(\mathbf{H}_P)$ will vary.

Transferring now to the study of the region of the IS with arbitrary orientation of the external field, we distinguish the region of values of $\mathbf{H}$ for which the internal field has some fixed value $\mathbf{H}_P$ (we denote this region by $\{\mathbf{H}|\mathbf{H}_P\}$). It is also possible to distinguish a continuous set of values of $\mathbf{H}$, for which the quantities ξ_k are fixed: $\{\mathbf{H}|...\xi_k...\}$. Every point of the phase diagram of the magnet in the region of existence of domains is characterized by definite values of $\mathbf{H}_P$ and ξ_k, i.e., it is the intersection of the regions $\{\mathbf{H}|\mathbf{H}_P\}$, $\{\mathbf{H}|...\xi_k...\}$.

As $\mathbf{H}$ varies in one of the regions $\{\mathbf{H}|\mathbf{H}_P\}$ the internal states in the domains $\mathbf{M}^{(k)}(\mathbf{H}_P)$, $L_\nu^{(k)}(\mathbf{H}_P)$ do not change, and the system evolves only owing to the redistribution of the relative percentages of the phases, i.e., a pure process of displacement of domain boundaries occurs. Since the internal state in domains, just as the structure of the domain boundaries, is determined by the value of $\mathbf{H}_P$, varying $\mathbf{H}$ over the domain $\{\mathbf{H}|\mathbf{H}_P\}$ does not destroy the conditions for the occurrence of processes that depend on the values of the internal field and magnetic state of the system (for example, the conditions for the existence of magnetic resonance).

As $\mathbf{H}$ varies over the domain $\{\mathbf{H}|...\xi_k...\}$ the system will evolve only owing to the variation of $\mathbf{H}_P$ and the associated changes in $\mathbf{M}^{(k)}(\mathbf{H}_P)$ and $L_\nu^{(k)}(\mathbf{H}_P)$, i.e., via changes in the internal state in the domains. In the region $\{\mathbf{H}|...\xi_k...\}$ the quantities related with ξ_k remain constant. The sets of regions $\{\mathbf{H}|\xi_1\xi_2...\xi_n\}$ will also contain regions in which t of the n quantities ξ_k equal zero ($t = 1, 2,..., n-1$). It is easy to prove that such regions describe boundaries between the n-phase and $(n-t)$-phase domain structures. In particular, for $t = n-1$ the region $\{\mathbf{H}_P|...\xi_k...\}$ is the boundary between the n-phase domain structure and one of the uniform states.

In an easy-axis ferromagnet when $\mathbf{H}$ tilts away from the axis of easy magnetization in the region of PTI $|\mathbf{M}^{(k)}(\mathbf{H}_P)| = M_0 = \text{const}$ $(T=0)$,[83] i.e., in the region $\{\mathbf{H}|...\xi_k...\}$, only the orientation of $\mathbf{M}^{(k)}(\mathbf{H}_P)$ changes. In the traditional theory of magnetization of a ferromagnet this process is called rotation of magnetization in domains.[3] In an arbitrary magnet in the region $\{\mathbf{H}|...\xi_k...\}$ the moduli of $\mathbf{M}^{(k)}$ $(\mathbf{H}_P)$ can also change. In addition, in $\{\mathbf{H}|...\xi_k...\}$ as $\mathbf{M}^{(k)}$ $(\mathbf{H}_P)$ changes the vector $\mathbf{m}_{kk'}$ (2.9)—forming the surface of the domain boundary—also changes. This means that in the region $\{\mathbf{H}|...\xi_k...\}$ the domain boundaries can turn. In an easy-axis ferromagnet, however, the vector $\mathbf{m}_{kk'}$ is always parallel to the direction of easy magnetization, so that here there is no turning of the domain boundaries. An example of an IS, where $\mathbf{m}_{kk'}$ changes direction, is the domain structure of the orthorhombic antiferromagnetic in the vicinity of Morin temperature.[62] It follows from what was said above that for an arbitrary change in the region of existence of IS three basic processes will occur in the magnet:

1) diplacement of the domain boundary;

2) change, associated with a change in the internal field $\mathbf{H}_P$, in the internal state in separate domains; and,

3) turning of the domain boundaries along the vector $\mathbf{m}_{kk'}$ (2.8).

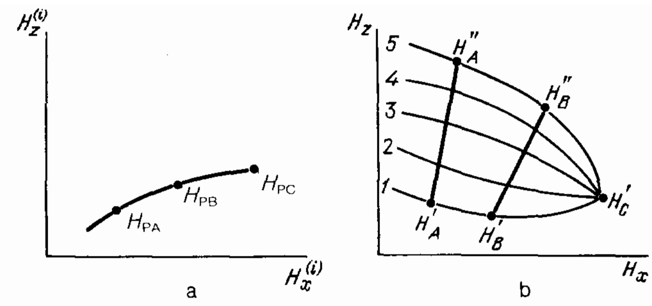

FIG. 2. Lines of PTI $\mathbf{H}_{PA}...\mathbf{H}_{PC}$ between two phases (a) and the corresponding region of the intermediate state (b). On the H_x, H_z diagram the thick lines denote lines of constant internal field: $H_A'H_A''$ – $\{\mathbf{H}|\mathbf{H}_{PA}\}$, $H_B'H_B''$ – $\{\mathbf{H}|\mathbf{H}_{PB}\}$, the thin lines denote regions of constant relative percentages of phases: 1—$\{\mathbf{H}|1;0\}$, 2—$\{\mathbf{H}|\frac{3}{4};\frac{1}{4}\}$, 3—$\{\mathbf{H}|\frac{1}{2};\frac{1}{2}$, 4—$\{\mathbf{H}|\frac{1}{4};\frac{3}{4}\}$, 5—$\{\mathbf{H}|0;1\}$ (the lines 1 and 5 denote the boundaries of the intermediate state).

The ideas introduced above regarding the region of constant internal field $\{\mathbf{H}|\mathbf{H}_P\}$ permit constructing the region of existence of the domain structure, employing the system of equations (4.2), specifying the region of $\mathbf{H}_P$, the equilibrium values of magnetization in the competing phases (4.2) and the equations (4.3). For fixed values of $\mathbf{H}_P$ from the region of PTI with n phases the function $\mathbf{H}(\xi_k)$ has $n-1$ degrees of freedom (the ξ_k are coupled by the normalization relation (4.4)). This means that Eq. (4.3) maps every point in the region $\mathbf{H}_P$ onto an $(n-1)$-dimensional surface, which is the region of constant internal field $\{\mathbf{H}|\mathbf{H}_P\}$.

We shall construct the region of existence of the domain structure first for the simplest model of a two-phase domain structure. Let the region of PTI $\mathbf{H}_P$ be a line in the plane $H_x^{(i)}H_y^{(i)}$, and let the equilibrium magnetizations in the competing phases $\mathbf{M}^{(1)}$ and $\mathbf{M}^{(2)}$ also lie in the x, y plane. It follows from (4.3) that in this case the region of existence of the domain structure for an ellipsoid, one of whose principal axes coincides with the Oz axis, also belongs to the phase plane H_x, H_y. To preserve generality we shall study the segment of the line of PTI $\mathbf{H}_{PA}\mathbf{H}_{PC}$, containing the point of termination of PTI (the point $\mathbf{H}_C$) (Fig. 2). To study the domain structure around the point at which the phase transition terminates there is no need to specify the type of point. It is important that at this point the difference between spin configurations in the competing phases is lost, i.e., $\mathbf{M}^{(1)}(\mathbf{H}_C) = \mathbf{M}^{(2)}(\mathbf{H}_C)$, $L_\nu^{(1)}(\mathbf{H}_C) = L_\nu^{(2)}(\mathbf{H}_C)$. For some point $\mathbf{H}_{PA}$ (see Fig. 2) Eqs. (4.3) assume the form

$$\mathbf{H} = \mathbf{H}_{PA} + 4\pi\hat{N}\,(\xi_1\mathbf{M}^{(1)}(\mathbf{H}_{PA}) + (1-\xi_1)\,\mathbf{M}^{(2)}(\mathbf{H}_{PA})). \tag{5.1}$$

The vector equation (5.1) is a parametric definition (the parameter is ξ_1) of the segment of the straight line connecting the points (see Fig. 2)

$$\begin{aligned} \mathbf{H}_{1A} &= \mathbf{H}_{PA} + 4\pi\hat{N}\,\mathbf{M}^{(1)}(\mathbf{H}_{PA}); \\ \mathbf{H}_{2A} &= \mathbf{H}_{PA} + 4\pi\hat{N}\mathbf{M}^{(2)}(\mathbf{H}_{PA}), \end{aligned} \tag{5.2}$$

which lie on the boundaries of the region of existence of the domain structure. In an analogous manner Eq. (5.1) maps each point of the region of the phase transition $\mathbf{H}_P$ into a definite segment of the straight line. This construction is made in Fig. 2. For the point $\mathbf{H}_C$, at which the difference between the phases vanishes, the fields $\mathbf{H}_{1C}$ and $\mathbf{H}_{2C}$ are equal, i.e., Eq. (5.1) transfers the point $\mathbf{H}_C$ into the point $\mathbf{H}_{C}$, with the coordinates

$$\mathbf{H}_{C'} = \mathbf{H}_{PC} + 4\pi\hat{N}\mathbf{M}(\mathbf{H}_{PC}). \quad (5.3)$$

On each segment of straight lines of the type (5.1) ξ_1 runs through a continuous series of values from zero ($\mathbf{H} = \mathbf{H}_2$) to one ($\mathbf{H} = \mathbf{H}_1$). Connecting with lines the points with equal values of ξ_1 we obtain the region $\{\mathbf{H}|\xi_1,\xi_2\}$. In particular, among them, there will be the lines $\{\mathbf{H}|0;1\}$, $\{\mathbf{H}|1;0\}$, which form the boundaries of the region of existence of the domain structure. It is obvious that all lines $\{\mathbf{H}|\xi_1;\xi_2\}$ meet in the point $\mathbf{H}_{C'}$ (see Fig. 2).

We note that as the point of termination of the PTI is approached the difference between the magnetization and other internal parameters in separate phases decreased without limit. It turns out that in a narrow region around the critical point the inequality $x_0 \ll D$ is violated, i.e., the distribution of the magnetization is strongly nonuniform over the entire volume of the sample. The theory of such nonuniform states was developed in Refs. 75 and 76 (see Sec. 13).

In Secs. 9–11 we shall calculate the regions of existence of domain structures for concrete systems. Here we shall discuss in greater detail some general characteristics of the IS with an n-phase domain structure.

We start with an arbitrary two-phase domain structure. In this case the condition of phase equilibrium has the form

$$\Phi[\mathbf{M}^{(1)}(\mathbf{H}_P)\,\mathbf{H}_P] = \Phi[\mathbf{M}^{(2)}(\mathbf{H}_P)\,\mathbf{H}_P]. \quad (5.4)$$

This equation in the phase space of the components of the internal field $H_x^{(i)}, H_y^{(i)}, H_z^{(i)}$ defines a surface. Equation (4.3), determining the dependence of the parameters of the IS on the external field, has the following form for a two-phase system:

$$\mathbf{H} = \mathbf{H}_P + 4\pi\hat{N}\,[\xi_1\mathbf{M}^{(1)}(\mathbf{H}_P) - (1-\xi_1)\,\mathbf{M}^{(2)}(\mathbf{H}_P)]. \quad (5.5)$$

An important result follows from Eq. (5.5): for a two-phase domain structure every point in the region of the PTI $\mathbf{H}_P$ in the phase space of the external field H_x, H_y, H_z transforms into a segment of a straight line lying between the points (Fig. 3):

$$\mathbf{H}_1 = \mathbf{H}_P + 4\pi\hat{N}\mathbf{M}^{(1)}(\mathbf{H}_P), \quad (5.6)$$

$$\mathbf{H}_2 = \mathbf{H}_P + 4\pi\hat{N}\mathbf{M}^{(2)}(\mathbf{H}_P). \quad (5.7)$$

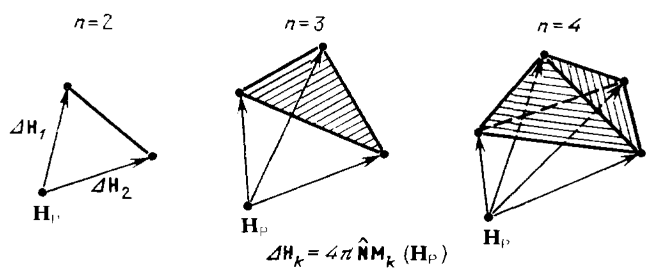

FIG. 3. Character of the mapping of the PTI point—$\mathbf{H}_P$ into the region of the intermediate state for an n-phase domain structure.

In other words, the relation (5.5) maps the surface of the PTI into the region of existence of the IS, representing in this case part of the space bounded by the surfaces $\mathbf{H}_1(\mathbf{H}_P)$ and $\mathbf{H}_2(\mathbf{H}_P)$ (5.7).

We shall now study the domain structure with $n > 2$ phases.

If under the conditions of PTI three phases coexist, then the condition that the potentials (4.2) are equal to one another leads to two independent equations, which in the phase space $\mathbf{H}^{(i)}$ give the line of phase equilibrium. For $n = 3$ Eqs. (4.3) and (4.4) have the form

$$\mathbf{H} = \mathbf{H}_P + 4\pi\hat{N}\,(\xi_1\mathbf{M}^{(1)}(\mathbf{H}_P) + \xi_2\mathbf{M}^{(2)}(\mathbf{H}_P) + \xi_3\mathbf{M}^{(3)}(\mathbf{H}_P)]). \quad (5.8)$$

The four equations (5.8), together with two equations giving the line $\mathbf{H}_P$, uniquely determine the external-field dependence of the components of $\mathbf{H}_P$ and the relative percentages of the phases ξ_1, ξ_2, ξ_3. Eliminating ξ_k from (5.8) it can be shown that a fixed value of $\mathbf{H}_P$ is preserved if the values of the external field on the H_x, H_y, H_z diagram fall within the plane triangle (Fig. 3) with the vertices

$$\mathbf{H}_k = \mathbf{H}_P + 4\pi\hat{N}\mathbf{M}^{(k)}(\mathbf{H}_P), \quad (5.9)$$

where $k = 1, 2$, and 3. On each side of the triangle one of the ξ_k vanishes, i.e., the sides of the triangles are the boundaries between two of the three regions of existence of the IS. At the vertices (5.9) two of three ξ_k vanish, i.e., at these points a transition into one of the uniform states is achieved.

Thus for a three-phase domain structure the line of phase equilibrium in the $\mathbf{H}^{(i)}$ phase diagram transforms in the phase space $\mathbf{H}$ into a definite region of existence of the IS.

The conditions for coexistence of the four phases lead to three equations, which, generally speaking, give in the $\mathbf{H}^{(i)}$ phase space an isolated point. Equation (4.5) in this case also determines uniquely the dependence of ξ_k on the external field ($\mathbf{H}_P$ is fixed). The point of the phase equilibrium of four phases in the $\mathbf{H}$ phase space corresponds to the region within the triangular pyramid with vertices (5.9) ($k = 1, 2$, 3, and 4). On each face one of the ξ_k vanishes, two ξ_k vanish on the edges, and three of four ξ_k vanish at the vertices. This means that the faces of the pyramid are the boundaries between the regions of coexistence of three- and four-phase domain structures, the edges of the pyramid are the boundaries between two- and four-phase regions, and a transition from a four-phase region into a uniform state occurs at the vertices (5.9) (see Fig. 3).

In the case of accidental degeneracy, as follows from (4.6), for $\lambda = 0$ ($\mathbf{H}$ diagram) the number of coexisting phases cannot exceed four. This restriction, as we have already mentioned, is removed in the case of symmetric degeneracy. Examples of the construction of phase diagrams are given in Secs. 9–11.

6. THERMODYNAMIC STABILITY OF DOMAIN STRUCTURES

It was shown above (see the system of equations (3.2) and (3.3)) that in the region of external-field-induced PTI the extremum of the function giving the energy of the magnet (2.1) corresponds either to uniform states or states with a domain structure. To determine the boundaries of thermodynamic stability of the domain structure, it is sufficient to

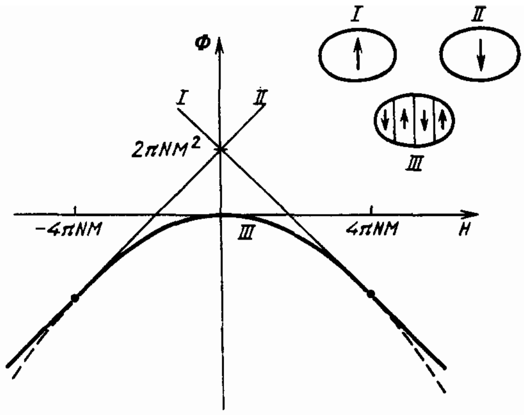

FIG. 4. The dependence of the equilibrium energies of the uniform states of an ellipsoidal ferromagnet (I, II) and energy of a ferromagnet in the intermediate state (III) on the external magnetic field.

find the domains of the external parameters in which the equilibrium energy Φ_{DW} of the magnet, partitioned into domains, is lower than the equilibrium energy in the uniform states Φ_0.

We start with the simplest model: an easy-access ferromagnet in a magnetic field, parallel to the axis of easy magnetization. The energy (3.4) of a such system has the following form[83]:

$$\Phi_0 = \frac{1}{2}\beta(M_x^2 + M_y^2) - \mathbf{H}^{(i)}\mathbf{M}, \tag{6.1}$$

where β is an anisotropy constant; for $\beta > 0$ and the Oz axis is the axis of easy magnetization. For $\mathbf{H}^{(i)} = 0$ a PTI occurs between the states with $\mathbf{M}^{(1)} \| Oz$(I) and $\mathbf{M}_2 = -\mathbf{M}_1$(II).[83] Let the ferromagnet be an ellipsoid, whose principal axes coincide with the magnetic axes ($N = N_{zz}$). The equilibrium energies of the ferromagnet in phases I and II equal

$$\begin{aligned} \Phi_{\rm I} &= -HM + 2\pi NM^2, \\ \Phi_{\rm II} &= \quad HM + 2\pi NM^2. \end{aligned} \tag{6.2}$$

The dependence of $\Phi_{\rm I}$ and $\Phi_{\rm II}$ on H is shown in Fig. 4.

The expression (2.10) for the nonequilibrium energy of a magnet with domain structure has the form

$$\begin{aligned} \Phi &= -\langle M\rangle H + 2\pi N\langle M\rangle^2, \\ \langle M\rangle &= (\xi_1 - \xi_2)M. \end{aligned} \tag{6.3}$$

In writing down (6.3) we took into account the fact that in this case the direction $\mathbf{H}\|Oz$ coincides with the line of constant field $\{\mathbf{H}|0\}$. Minimizing with respect to $\Delta\xi = \xi_1 - \xi_2$ gives the equilibrium value of ξ_1 and $\xi_2 = 1 - \xi_1$:

$$\Delta\xi = \frac{H}{4\pi NM}. \tag{6.4}$$

Substituting $\Delta\xi$ (6.4) into (6.3), we obtain for the equilibrium energy of a ferromagnet, partitioned into domains,[83]

$$\Phi_{\rm DW} = -\frac{H^2}{8\pi N}. \tag{6.5}$$

For all values of $\mathbf{H}$ the function $\Phi_{\rm DW}(\mathbf{H})$ does not exceed $\Phi_{\rm I}(\mathbf{H})$, $\Phi_{0\rm II}(\mathbf{H})$ (see Fig. 4). The values $0 \leqslant \xi_k < \leqslant 1$, are physically meaningful and this is achieved in the interval of fields $|\mathbf{H}| \leqslant 4\pi NM$. Thus in the entire region of fields where realization of a domain structure $0 \leqslant \xi_k \leqslant 1$ is physically possible, its energy is lower than the energy of uniform states; equality of the energies $\Phi_{\rm DW}$ and $\Phi_{\rm I}$, $\Phi_{\rm II}$ is achieved at the boundaries of the region of existence of domains ($H_{1,2} = \pm 4\pi NM$), where one of the competing phases is completely forced out.

What are the physical reasons for the thermodynamic instability of the domain structure? This question is most easily answered for a ferromagnet in the field $\mathbf{H} = 0$. According to Ref. 1, for $\mathbf{H} = 0$ partitioning into domains, without changing the internal energy of the magnet (we recall that $\mathbf{H} = 0$ is the field of PTI in which the energy of the phase I equals the energy of the phase II), lowers the magnetostatic energy. Comparing (6.2) and (6.5) we find that for $H = 0$ the gain in energy accompanying a transition of the ellipsoidal ferromagnet from the uniform state into a polydomain state equals the energy of the demagnetizing fields of a uniform magnetized ellipsoid ($2\pi NM^2V$). In the finite field $|\mathbf{H}| < 4\pi NM$ the equilibrium states of a polydomain ferromagnet are formed by the competition of two interactions: the minimum of the energies of interaction with an external field corresponds to a uniform state with magnetization parallel to the field, while the magnetostatic energy is minimum for a domain structure with $\xi_1 = \xi_2 = \frac{1}{2}$.

Why then for $\mathbf{H} \neq 0$ is the domain structure thermodynamically stable right up to complete forcing out of one of the competing phases? To answer this question we shall follow the evolution of the state of a ferromagnet as the external field is varied. It follows from (6.2) that in an ellipsoidal ferromagnet for $\mathbf{H} < 0$ the energy of the uniform state with $\mathbf{M}^{(2)}$ is lower than the energy of the uniform state with magnetization $\mathbf{M}^{(1)}$. As the field is increased from the region $H < -4\pi NM$ the energy of the phase II increases, since the direction of change of the field is antiparallel to the magnetization vector. For $H = -4\pi NM$ in a uniformly magnetized magnet with $\mathbf{M}^{(2)}$ the internal field vanishes, i.e., the conditions for a PTI are realized. With a further increase of the field there are two possible paths along which a magnetic state can be realized. One path consists of preserving the uniform state and the other consists of organizing a nonuniform state: in the field $H > -4\pi NM$ the state with $\mathbf{M}^{(2)}$ can be "diluted" by inclusions of regions with $\mathbf{M}^{(1)}$. In so doing the concentrations of the phases must be such that the condition for realization of a first-order phase transition $\mathbf{H}^i = 0$ must hold. (Otherwise the energy of one of the phases will be higher, and it will be forced out of the volume of the magnet by the motion of the interphase boundaries.) In the process of evolution of the domain structure the internal energy of the ferromagnet does not change (because of the condition $\mathbf{H}^i = 0$), and the external field performs work only against magnetic-dipole forces (redistribution of charges on the surface of the ferromagnet and formation of domain walls). In other words, the formation of domains prevents the internal states in a ferromagnet from changing as the field changes. Outwardly a ferromagnet with a domain structure behaves like a magnet with high susceptibility, determined solely by its shape:

$$\chi_{\rm IS} = \frac{d\langle M\rangle}{dH} = \frac{1}{4\pi N}, \tag{6.6}$$

while in the uniform state $\chi = 0$ (neglecting the paraprocess). This specific compliance of a ferromagnet with domains to the action of an external magnetic field is the reason that a domain structure is energetically favored over a uniform state.

We shall now go over to two-phase domain structures with arbitrary spin-reorientational transitions. We shall show that a two-phase domain structure in the IS with energy (2.10) can be described efficiently by a model of an easy-axis ferromagnet with a domain structure. Let in some magnet the range of variation of the internal field in which an external-field-induced PTI between two phases occurs, be $\mathbf{H}_P$, and let the equilibrium values of the internal parameters in the competing phases be $\mathbf{M}^{(k)}$ $(\mathbf{H}_P)$ $L_\nu^{(k)}(\mathbf{H}_P)$ $(k=1,2)$.

Using Eq. (3.3) and introducing instead of $\mathbf{M}^{(1)}(\mathbf{H}_P)$ and $\mathbf{M}^{(2)}(\mathbf{H}_P)$ the quantities

$$\mathbf{m}=\frac{1}{2}(\mathbf{M}^{(1)}+\mathbf{M}^{(2)}) \text{ and } \mathbf{m}'=\frac{1}{2}(\mathbf{M}^{(1)}-\mathbf{M}^{(2)}), \tag{6.7}$$

we rewrite the internal energy (2.10) as

$$\varphi(\mathbf{M}^{(1)}, L_\nu^{(1)}, 0, 0)\xi_1+\varphi(\mathbf{M}^{(2)}, L_\nu^{(2)}, 0, 0)\xi_2$$
$$=\varphi_0(\mathbf{H}_P)+\mathbf{H}_P\mathbf{m}(\mathbf{H}_P)+\mathbf{H}_P\mathbf{m}'(\mathbf{H}_P)(\xi_1-\xi_2), \tag{6.8}$$

and we transform the magnetostatic energy into the following form:

$$2\pi\langle\mathbf{M}\rangle\hat{N}\langle\mathbf{M}\rangle=2\pi\mathbf{m}N\mathbf{m}+2\pi(\xi_1-\xi_2)^2\mathbf{m}'\hat{N}\mathbf{m}'+4\pi\mathbf{m}'N\mathbf{m}. \tag{6.9}$$

Taking into account (6.8) and (6.9) the energy of the magnet (2.10) acquires the following form:

$$\Phi=\varphi_0(\mathbf{H}_P)+\varphi^1+\Delta\varphi, \tag{6.10}$$

where

$$\varphi^I=2\pi\mathbf{m}\hat{N}\mathbf{m}-(\mathbf{H}-\mathbf{H}_P)\mathbf{m}(\mathbf{H}_P) \tag{6.11}$$

is the energy of a uniformly magnetized ellipsoid with magnetization $\mathbf{m}$ in the field $\mathbf{H}-\mathbf{H}_P$ neglecting its internal energy (3.3),

$$\Delta\varphi=-\tilde{\mathbf{H}}\mathbf{m}'(\xi_1-\xi_2)+2\pi(\xi_1-\xi_2)^2\mathbf{m}'\hat{N}\mathbf{m}' \tag{6.12}$$

and has the form of the energy of a ferromagnetic ellipsoid in a polydomain state with the magnetization $\pm\mathbf{m}(\mathbf{H}_P)$ in neighboring domains, located in an "external" field

$$\tilde{\mathbf{H}}=\mathbf{H}-\mathbf{H}_P-4\pi\hat{N}\mathbf{m}(\mathbf{H}_P). \tag{6.13}$$

The fact that $\Delta\varphi$ (6.12) is functionally identical to the energy of the ferromagnet, partitioned into domains, enables using for the analysis of domain structures in the region of spin-reorientational transitions the results obtained in the calculation of the corresponding domain structures of a ferromagnet. In Sec. 7 below this will be employed in concrete calculations. Here, however, it is important to note the physical essence of the energy transformation performed above. The linearity of the equations of magnetostatics permits regarding the sample consisting of domains with magnetizations $\mathbf{M}^{(1)}$, $\mathbf{M}^{(2)}$ as two magnets "inserted" into one another and having the same shape as the sample, one of which has the uniform magnetization $\mathbf{m}(\mathbf{H}_P)$ while the other is partitioned into domains with magnetizations $\pm\mathbf{m}'(\mathbf{H}_P)$ (6.7). The physical reason for the presence of the term $4\pi\hat{N}\mathbf{m}(\mathbf{H}_P)$ in the effective field (6.11) is also clear: this quantity is numerically equal to the demagnetization field, created by the first ellipsoid. For the second ellipsoid the field

$$\mathbf{H}'=4\pi\hat{N}\mathbf{m}(\mathbf{H}_P) \tag{6.14}$$

plays the role of an "external" field and is added to $\mathbf{H}$. As the external field is varied along one of the lines $\{\mathbf{H}|\mathbf{H}_P\}$, in the process of redistribution of the relative percentages of the phases the energy of the first ellipsoid φ^1 (6.13) does not depend on the state of the domain structure, while the second ellipsoid behaves as a ferromagnet with domains having the magnetizations $\pm\mathbf{m}'(\mathbf{H}_P)$. For it the idea presented above, that the formation of domains is energetically favored, as well as the calculations performed there, are valid. In particular, the maximum advantage of a polydomain state over a uniform state is achieved in the field $\tilde{\mathbf{H}}=0$ (where $\xi_1=\xi_2$), i.e., for

$$\mathbf{H}=\mathbf{H}_P+4\pi\hat{N}\mathbf{m}(\mathbf{H}_P), \tag{6.15}$$

and the advantage in energy constitutes $\Delta E=2\pi_{\mathbf{M}}\hat{N}_{\mathbf{M}}$. Thus the advantage in energy gained by the formation of domain structures in the region of spin-reorientational transitions is linked not with the full magnetizations in separate phases $\mathbf{M}^{(1)}$, $\mathbf{M}^{(2)}$, but rather with the magnitude of the jump in the magnetization $\Delta\mathbf{M}=\mathbf{M}^{(1)}-\mathbf{M}^{(2)}=2_{\mathbf{M}'}$ (6.7). In the above-examined model of a ferromagnet $\mathbf{H}_P=0$, $\mathbf{M}=0$, and the jump in the magnetization at a PTI has the maximum possible value $|\Delta\mathbf{M}|=2|M_0|$. Correspondingly, the advantage in energy gained at $\mathbf{H}=0$ is also maximum and equals $\Delta E=2\pi NM_0^2$.

The fact that a regular domain structure is energetically favored can be verified by comparing directly the energies of an ellipsoidal sample partitioned into domains and an ellipsoidal sample in the uniform state.[82]

The magnetization in the IS is determined from the relation (4.3):

$$4\pi\hat{N}\langle\mathbf{M}\rangle=\mathbf{H}-\mathbf{H}_P(\mathbf{H}). \tag{6.16}$$

The magnetization $\langle\mathbf{M}\rangle$ is a linear function of $\mathbf{H}$ only if the external field lies in the region $\{\mathbf{H}|\mathbf{H}_P\}$. For arbitrary variation of $\mathbf{H}$ in the region of the IS $\mathbf{H}_P$ and therefore $\mathbf{M}^{(k)}$ $(\mathbf{H}_P)$ also will vary, and this will cause the dependence $\langle\mathbf{M}\rangle(\mathbf{H})$ to be more complicated.

Differentiating (6.16) with respect to $\mathbf{H}$ we obtain for the tensor of the magnetic susceptibility in the intermediate state the expression

$$4\pi N_{\alpha\beta}\chi_{\beta\gamma}=\delta_{\alpha\gamma}-\frac{\partial H_{\Pi\alpha}}{\partial H_\gamma}. \tag{6.17}$$

The first term on the right side of (6.17) is associated with the displacement of the domain walls, while the second is associated with the change in the magnetization in the domains $\mathbf{M}^{(k)}$ $(\mathbf{H}_P)$. In the vicinity of PTI we shall write the equilibrium magnetization in the k-th phase as follows:

$$\mathbf{M}^{(k)}(\mathbf{H}^{(i)})=\mathbf{M}^{(k)}(\mathbf{H}_P)+\hat{\chi}_k(\mathbf{H}^{(i)}-\mathbf{H}_P), \tag{6.18}$$

where χ_k is the tensor of the internal susceptibility of the k-th phase.

It follows from (6.18) that the susceptibility of a magnet in the uniform state equals

$$\hat{\chi}=(\hat{\chi}_k^{-1}+4\pi\hat{N})^{-1}. \tag{6.19}$$

In the region of existence of the IS, as H varies along

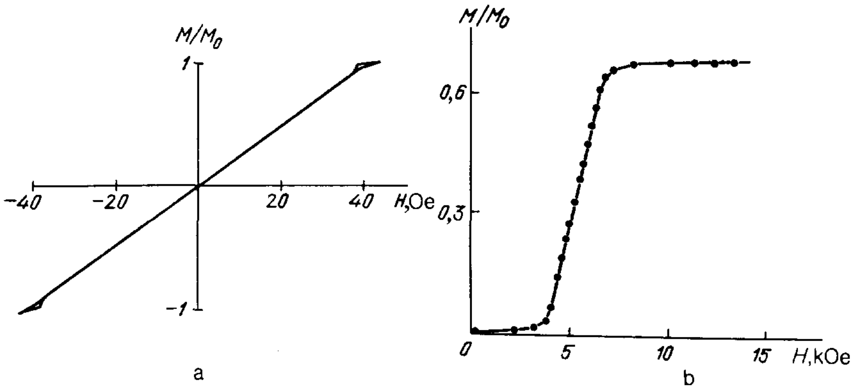

FIG. 5. Magnetization curves for magnets in the region of magnetic-field-induced first-order phase transitions.[114] a) for an epitaxial ferrite-garnet film[105]; b) for the metamagnet $Dy_3Al_5O_{17}$ in the region of the metamagnetic transition.

$\{\mathbf{H}|\mathbf{H}_P\}$, according to (6.17)

$$\hat{\chi} = (4\pi\hat{N})^{-1}. \qquad (6.20)$$

In Ref. 100 the tensor $\hat{\chi}$ is called the susceptibility of the body, ξ_k is the susceptibility of the material, and $(4\pi\hat{N})^{-1}$ is the shape susceptibility. Employing this terminology we can state that unlike the uniform state, in which the susceptibility of the magnet depends on both the susceptibility of the material and on the shape susceptibility (6.19), in the intermediate state (with **H** varying along $(\{\mathbf{H}|\mathbf{H}_P\})$ the susceptibility of the magnet is determined only by the shape susceptibility. For arbitrary variation of **H** in the region of existence of domains the susceptibility is determined by the expression (6.17). Here, aside from the shape susceptibility, a substantially new term appears, associated with the change in the field of the PTI ($\mathbf{H}_P$). Its magnitude is determined by the characteristic of the spin-reorientational transition, namely, the dependence $\mathbf{H}_P(H)$ and $\mathbf{M}^{(k)}(\mathbf{H}_P)$.

The process of magnetization of a magnet, studied here, is predicated on the realization of a thermodynamically equilibrium domain structure in it, i.e., no coercivity and hysteresis phenomena.

Figures 5 and 6 show the dependences $\langle\mathbf{M}\rangle(\mathbf{H})$ for a ferrite plate with the easy-axis parallel to the normal ($N=1$) and having a low coercivity H_c (0.5 Oe[101] (a), and for a spherical metamagnet $Dy_3Al_5O_{12}$ ($N=\frac{1}{3}$) in the region of the metamagnetic phase transition (b).[114] Figure 6 shows the curves $\chi(\mathbf{H})$ for $Dy_3Al_5O_{12}$ in the region of the metamagnetic transtion (a)[114] and for the easy-axis antiferromagnet $MnCl_2\cdot 4H_2O$ in the region of the spin-flop transition (b)[41] (both samples are spherical. In these magnets the external field is parallel to the axis of easy magnetization, i.e., **H** varies along the line $\{\mathbf{H}|\mathbf{H}_P$. It follows from (6.16) and (6.20) that in the IS

$$\langle\mathbf{M}\rangle = \frac{\mathbf{H}-\mathbf{H}_P}{4\pi N}, \qquad \chi_{IS} = \frac{1}{4\pi N}, \qquad (6.21)$$

which is in fact observed experimentally. The effect of the shape of the magnet on the character of the dependence $\langle\mathbf{M}\rangle(\mathbf{H})$ in the IS is illustrated in Fig. 7, where the dependence of the magnetization of MnF_2 in the region of the spin-flop transition is shown for cylindrical samples: with diameter 0.8 mm and height 2.5 mm (a) and with diameter 1.85 mm and height 0.2 mm (b) (the axis of the cylinder coincides with the easy axis of the magent).[19] The second sample is actually a plate ($N\approx 1$). For it the width of the IS is $\Delta H_{IS} = 4\pi\Delta M$, while $\chi_{IS} = \frac{1}{4}$. The first sample can be regarded as an ellipsoid of revolution with the ratio of the axes $a/b = 3$ and 1, and $N = 0.1$. For it $\Delta H_{IS} = 4\pi N\Delta M$ is significantly smaller, and $\chi_{IS} = 10/4\pi$.

The results obtained in this section show that the formation of all thermodynamically stable domain structures (including also in an easy-axis ferromagnet) is based on common physical processes:

When the conditions for the coexistence of phases, distinguished by the value of the magnetization vector **M** *(field-induced PTI), are realized in a magnet, an additional possibility for reducing the thermodynamic potential appears in a magnet with a finite shape: partitioning into domains of competing phases. As the external field varies the condition of*

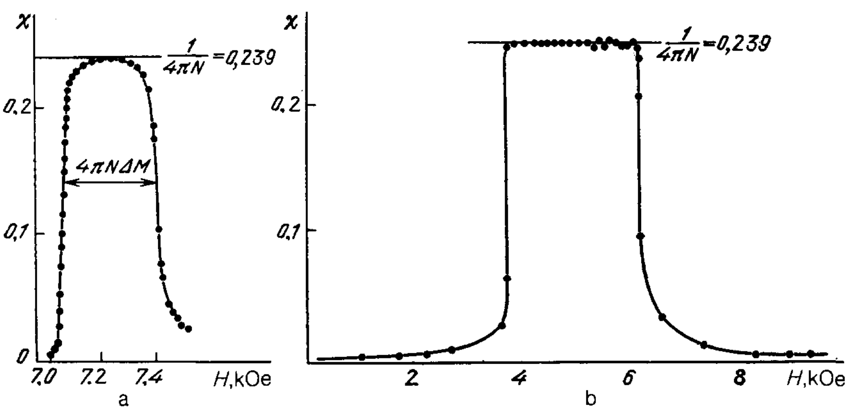

FIG. 6. The static susceptibility as a function of the external field. a) in the region of the spin-flop transition in $MnCo_2$, $4H_2O$[41]; b) in the region of the metamagnetic transition in $Dy_3Al_5P_{17}$[114] both samples are spherical.

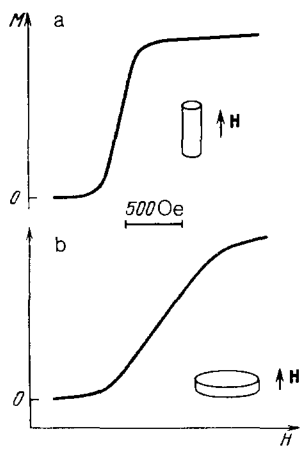

FIG. 7. Magnetization curves in the region of the spin-flop transition in MnF_2 ($H_P = 92$ kOe) for two cylindrical samples.[19] a) diameter 0.8 mm and height 2.5 mm. b) diameter 1.85 mm and height 0.2 mm.

coexistence of the phases—their internal energies are equal—is maintained in the magnet by means of redistribution of the relative fractions of the phases.

7. EQUILIBRIUM GEOMETRIC PARAMETERS OF MODEL DOMAIN STRUCTURES. STRIPED AND CYLINDRICAL DOMAINS IN PLATES

The equilibrium values of the geometric parameters are formed by balancing the terms in the energy (2.1) associated with the nonuniformities of the internal states—domain walls and the nonuniform part of the magnetostatic energy.

The shape and size of the domains of multiphase IS largely depend on the internal states in domains and the orientation of the vectors $\mathbf{M}^{(k)}$ relative to the axes of the ellipsoid. For this reason the calculation of the equilibrium structure of the IS is a quite complicated problem, and in each specific case it must be performed, generally speaking, separately. Virtually no such calculations have been performed for multiphase domain structures with $n > 2$.

The situation for two-phase IS is substantially different. As will be shown below, any two-phase domain structure can be effectively described by a model of a two-phase domain structure of a ferromagnet. This makes it possible to use many results of calculations of model domain structures of ferromagnets.

In the "thin" wall approximation the energy density of a magnet with a two-phase regular domain structure will consist of the energy of the uniform states in separate domains, the energy of the domain walls, and the magnetostatic energy:

$$\Phi = \varphi(\mathbf{M}^{(1)}, L_V^{(1)}, 0, 0)\xi_1 + \varphi(\mathbf{M}^{(2)}, L_V^{(2)}, 0, 0)\xi_2 + \frac{\sigma S}{V} - \frac{1}{2V}\int \mathbf{M}(\mathbf{r})\mathbf{H}_M(\mathbf{r})\,dV - (\mathbf{M}^{(1)}\xi_1 + \mathbf{M}^{(2)}\xi_2)\,\mathbf{H}, \tag{7.1}$$

where $\sigma(\mathbf{H}_P)$ is the energy density of the domain wall and S is the total area of these walls in the magnet.

If the period of the domain structure is comparable to the characteristic dimensions of the sample (the inequality $D \ll L$ is not satisfied), then the nonuniformity of the internal field, associated with the nonuniformity of the demagnetizing fields $\mathbf{H}_M(\mathbf{r})$, is now significant over the entire volume of the magnet and, generally speaking, leads to nonuniform distribution of $\mathbf{M}^{(k)}$, $\mathbf{L}_V^{(k)}$ in the domains. As a rule, however, within the region of existence of the domain structure the change in the magnetizations of the sublattices $\mathbf{M}_\alpha$ is insignificant, i.e., the components of $\mathbf{M}_\alpha$ in the IS satisfy the inequality

$$(\mathbf{M}_\alpha)_i \gg \chi_{ij}^{(\alpha)}(\Delta\mathbf{H}_{IS})_j, \tag{7.2}$$

where $\chi_{ij}^{(\alpha)}$ are the components of the static susceptibility tensor for separate sublattices of the magnet. For a ferromagnet $\Delta H_{IS} = 8\pi\hat{N}|\mathbf{M}|$ and the inequality (7.2) reduces to the condition $4\pi\chi_{ij} \ll 1$. Since the transverse components of $\hat{\chi}$ in a ferromagnet are inversely proportional to the anisotropy constant β, the last inequality is equivalent to the requirement that the quality factor $Q = \beta/4\pi$ be large. Under the conditions of spin-reorientational transitions it often happens that $|\Delta\mathbf{M}| \ll |\mathbf{M}|$ and the condition (7.2) turns out to be weaker. If the relation (7.2) holds, the nonuniformity of the spin states in the domains, even in the region $D \gtrsim L$, can be neglected and it can be assumed that, as before, states corresponding to competing phases of a field-induced PTI are realized in them. Thus, just as for a regular domain structure, the equilibrium values of the geometric parameters of the IS are determined by minimizing (7.1) for fixed values of $\mathbf{M}^{(k)}$, $L_V^{(k)}$.

We shall write with the help of the quantities $\mathbf{m}(\mathbf{H}_P)$ and $\mathbf{m}(\mathbf{H}_P)$ (6.7) the internal energy of an ellipsoidal magnet with a two-phase domain structure in the form (6.8) and the magnetostatic energy from (7.1) in the following form:

$$\frac{1}{2V}\int \mathbf{M}(\mathbf{r})\mathbf{H}_M(\mathbf{r})\,dV = 2\pi\mathbf{m}(\mathbf{H}_P)\hat{N}\mathbf{m}(\mathbf{H}_P) + \mathbf{H}'_M\mathbf{m}'(\mathbf{r}) + \frac{1}{2V}\int \mathbf{m}'(\mathbf{r})\mathbf{h}_M(\mathbf{r})\,dV, \tag{7.3}$$

where we introduced the distribution of the "ferromagnetic" moment $\mathbf{m}'(\mathbf{r})$ according to the relation

$$\mathbf{M}(\mathbf{r}) = \mathbf{m}(\mathbf{H}_P) + \mathbf{m}'(\mathbf{r}), \tag{7.4}$$

the magnetostatic fields $\mathbf{H}'_M$ (6.14) and $\mathbf{h}_M(\mathbf{r})$, generated by a uniform ellipsoid and an ellipsoid partitioned into domains, respectively, and we employed the reciprocity theorem[4]

$$\int \mathbf{H}'_M\mathbf{m}'(\mathbf{r})\,dV = \int \mathbf{h}_M(\mathbf{r})\,\mathbf{m}(\mathbf{H}_P)\,dV.$$

Using the idea of ellipsoids "inserted" into one another, the first term in (7.3) can be interpreted as the energy of the demagnetizing fields of uniformly magnetized ellipsoids with magnetization $\mathbf{m}(\mathbf{H}_P)$; the second term can be interpreted as the energy of an ellipsoid with the domains of magnetization $\pm\mathbf{m}'(\mathbf{H}_P)$ (6.7) in a magnetic field generated by the first ellipsoid $\mathbf{H}_M$; and, finally, the third term describes the characteristic energy of magnetostatic interactions of an ellipsoid with domain structure. Substituting (6.8) and (7.3) into (7.1), we represent the energy (7.1) in the form (6.10), and in addition $\Delta\varphi$ is functionally identical to the energy of a ferromagnet with antiparallel orientation of $\tilde{\mathbf{m}}$ in neighboring domains in an "external" field $\tilde{\mathbf{H}}$ (6.13), written in the "thin" wall approximation,

$$\Delta\varphi = \frac{1}{2V}\int \mathbf{h}_M\mathbf{m}'(\mathbf{r})\,dV + \frac{\sigma(\mathbf{H}_P)S}{V} - \tilde{\mathbf{H}}\mathbf{m}'(\mathbf{H}_P)(\xi_1 - \xi_2). \tag{7.5}$$

In the expression for $\Delta\varphi$ (7.5) the energies of the domain walls can be put into the following form:

$$\Phi_{DW}=\frac{\sigma(\mathbf{H}_P)S}{V}=2\pi\mathbf{M}'^{(2)}(\mathbf{H}_P)\tilde{l}(\mathbf{H}_P)g^{-1}, \qquad (7.6)$$

where

$$\tilde{l}(\mathbf{H}_P)=\frac{\sigma(\mathbf{H}_P)}{4\pi\mathbf{M}'^{(2)}(\mathbf{H}_P)}, \quad g=\frac{V}{2S}. \qquad (7.7)$$

The quantity g has the dimension of length and is determined solely by geometric factors—the shape and size of the sample and the domains. Conversely, the quantity $\tilde{l}(\mathbf{H}_P)$ is an internal characteristic of the domain-containing material and equals the ratio of the energy density of the domain walls to the energy of the demagnetizing fields; $\tilde{l}(\mathbf{H}_P)$ also has the dimension of length. For a ferromagnet $|\mathbf{m}'|=M_0$ and $l(0)=\sigma(0)/4\pi M_0^2$ is the so-called characteristic length. For this reason, by analogy to a ferromagnet we shall call $\tilde{l}(\mathbf{H}_P)$ (7.7) also the characteristic length. From the foregoing discussion it follows that the problem of determining the equilibrium geometric parameters of a model domain structure consisting of two phases with an arbitrary spin-reorientational transition reduces to the analysis of the energy of a polydomain ferromagnet in an effective displacement field

$$H^{(D)}=(\mathbf{H}-\mathbf{H}_P-4\pi\mathbf{M}(\mathbf{H}_P))\boldsymbol{\mu}, \qquad (7.8)$$

and $\boldsymbol{\mu}$ is a unit vector in the direction $\mathbf{m}$.

Thus far $\Delta\varphi$ (7.5) has been studied in detail for striped and cylindrical domains in plane-parallel plates whose normal $\mathbf{n}$ is parallel to the easy axis in a magnetic field $\mathbf{H}\|\mathbf{n}$.[9–11,86–89] In this case $\mathbf{H}$ varies along the line of constant internal field $\{\mathbf{H}|0\}$, so that the internal states in the domains $\mathbf{M}^{(1)}\|\mathbf{n}$, $\mathbf{M}^{(2)}=-\mathbf{M}^{(1)}$ will not change as $\mathbf{H}$ varies in the region of existence of the IS.

For a striped domain structure in a ferromagnetic plate the equations for the equilibrium values of the geometric parameters were derived in Refs. 86, 88, and 89. It is shown in Ref. 89 that the transition of the striped domain structure into a uniform state occurs by means of unlimited growth of the period and dimensions of the favored phase and is achieved in a field $\mathbf{H}^*$,[86] determined by the system of parametric equations

$$\begin{aligned}&H^*=4M\left[2\operatorname{arctg}\frac{1}{u}-u\ln\left(1+\frac{1}{u^2}\right)\right],\\&(1+u^2)\ln(1+u^2)-u^2\ln u^2=2\Lambda.\end{aligned} \qquad (7.9)$$

The dependence of the field $\mathbf{H}^*$ on the parameter $\Lambda=\tilde{l}/L$ (where L is the thickness of the plate) is shown in Fig. 8. The energy of the plate with single magnetic bubbles (MB) or a lattice of magnetic bubbles can be obtained by solving the corresponding magnetostatic problems.[9–11] Such calculations and analysis of the conditions of stability of bubbles are given in monographs and reviews of Refs. 9–11. We shall present the limiting fields in which single bubbles and a lattice of bubbles are stable as a function of Λ (see Fig. 8). H_{col} is the collapse field; H_2 is the field of an elliptical instability in a single magnetic bubble; H_c determines the field in which the period of the magnetic bubble lattice becomes infinite and the lattice transforms into a system of isolated MB; and H_{c2} is the field of the elliptical instability of the MB lattice.

For all values of the parameter Λ the limiting fields are related by the inequality

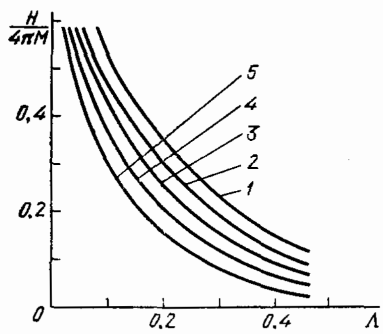

FIG. 8. Boundaries of existence of domain structures in a ferromagnetic plate.[11,89] 1) H_{col}, 2) H_c, 3) H^*, 4) H_2, 5) H_{c2}.

$$H_{c2}<H_2<H^*<H_c<H_{col}<4\pi M. \qquad (7.10)$$

In the limit $\Lambda\to 0$ all limiting fields approach the value $4\pi M$. In the region $\Lambda\geqslant 1$ the limiting fields are much less than $4\pi M$. Here explicit expressions can be obtained for them[89]:

$$\begin{aligned}&H_{col}=16Me^{-1/2}e^{-\Lambda},\\&H_2=16\cdot\frac{7}{3}e^{-11/6}e^{-\Lambda},\quad H^*=4Me^{1/2}e^{-\Lambda},\\&H_c=32e^{-3/2}e^{-\Lambda},\quad H_{c2}=16Me^{-11/6}e^{-\Lambda}.\end{aligned} \qquad (7.11)$$

To calculate the equilibrium geometric parameters of two-phase domain structures in a plate in the region of spin-reorientational transitions it is sufficient to substitute the values $\tilde{\mathbf{H}}$(6.13) and $\tilde{l}(\mathbf{H}_P)$ (7.7) into the corresponding equations obtained for a ferromagnetic plate. The problem of the limits of existence of different types of domains can be solved in an analogous manner, i.e., the limiting fields of a ferromagnet $\mathbf{H}_{lim}$ (7.10) are identical to the corresponding limiting fields for spin-reorientational transitions, expressed in units of the effective field $H^{(D)}$ (7.8). From here we obtain the following expression for the limiting fields under conditions of spin-reorientational transitions:

$$H'_{lim}=(\mathbf{H}_P+4\pi\mathbf{M}(\mathbf{H}_P))\boldsymbol{\mu}\pm H_{lim}; \qquad (7.12)$$

the lower sign describes $\mathbf{H}'_{lim}$ on the side of the low-field phase and the upper sign corresponds to the high-field phase. For $\mathbf{H}'_{lim}$ the inequality (7.10) remains valid, and all limiting fields lie in the region of the intermediate state, given in the thermodynamic approximation by the expressions

$$H'_1=(\mathbf{H}_P+4\pi\mathbf{M}^{(1)}(\mathbf{H}_P))\boldsymbol{\mu},\quad H'_2=(\mathbf{H}_P+4\pi\mathbf{M}^{(2)}(\mathbf{H}_P))\boldsymbol{\mu}. \qquad (7.13)$$

For a domain structure in the region of spin-reorientational transitions usually $\tilde{l}\ll L$. In this case $\mathbf{H}'_{lim}$ is virtually identical to H'_1 and $\mathbf{H}'_2$, respectively, and the striped domain structure is stable in virtually the entire region of existence of the intermediate state, while MDs can exist in a narrow neighborhood near the fields $\mathbf{H}'_1$ and $\mathbf{H}'_2$.

We shall examine in greater detail the question of the transformation of the plate from a uniform state into the intermediate state. For definiteness we shall study the transition for a low-field phase with magnetization $\mathbf{M}^{(1)}$ in the IS. Since the existence of domain walls (neglected in the thermodynamic approximation) increases the energy of the magnet with a domain structure, single magnetic bubbles will form not in the field $\mathbf{H}_1'$ (7.13), where $\mathbf{H}^{(i)} = \mathbf{H}_P$, but rather in some higher field H'_{1col} where already $H^{(i)} > \mathbf{H}_P$. Thus in the region of existence of single MBs of a high-field phase in a low-field matrix the internal field exceeds $\mathbf{H}_P$, and hence $\varphi_0(\mathbf{H})$ (6.10) can differ from $\varphi_0(\mathbf{H}_P)$ and can change as $\mathbf{H}$ changes. In an easy-axis ferromagnet with $Q \gg 1$, for which the limits of existence of MBs have been determined,[9–11] the longitudinal susceptibility equals zero, so that for a ferromagnetic plate in the field $\mathbf{H} \| \mathbf{n}$ $\varphi_0(H) = \varphi\partial(0)$ ($\mathbf{H}^{(i)} = 0$ is the PTI field). In an arbitrary magnet this is not the case, so that the change in $\varphi_0(\mathbf{H})$ with the field affects the values of the limiting fields in which MBs exist. In Ref. 90 it was proposed that the change in $\varphi_0(\mathbf{H})$ be taken into account by introducing into (7.5) an additional "displacement field," which for the model under study can be written as

$$H_A(\mathbf{H}) = \frac{\varphi_0(\mathbf{H}) - \varphi_0(\mathbf{H}_P)}{|\mathbf{M}'|}.$$

Because of the inequality (7.2), in this case H_A will change insignificantly the values of the limiting fields (7.12).

Generally speaking a PTI can occur also with respect to some nonmagnetic parameter ρ. As shown in Ref. 90, in this case, the existence of a displacement field H_A can lead to the formation of MB even without an external field.

We shall discuss the question of the domain structure in magnets which are hot plates. Since the demagnetizing fields are determined by the component of $\mathbf{m}(\mathbf{H}_P)$ normal to the surface, for magnets the radius of curvature of whose surface is significantly greater than D, the characteristic length will be different in different sections of the magnet. Denoting by $\mathbf{n}(\mathbf{r})$ the unit vector normal to the surface at the point $\mathbf{r}$, we obtain

$$\widetilde{l}(\mathbf{r}) = \frac{\sigma(\mathbf{H}_P)}{4\pi(\mathbf{M}'(\mathbf{H}_P)\mathbf{n}(\mathbf{r}))^2} = \widetilde{l}(\mathbf{H}_P)\frac{|\mathbf{M}(\mathbf{H}_P)|^2}{(\mathbf{M}'(\mathbf{H}_P)\mathbf{n}(\mathbf{r}))^2}.$$

It is obvious that now the equilibrium parameters of the domain structure, including also the period, will be different in different sections of the sample. In this connection, we recall that the condition for the domain structure to be regular (see Sec. 6) is not predicated on strict periodicity of the structure.

As is well known, in quite thick ferromagnets the domain structure at the surface is branched.[121] Models of ferromagnetic domain structures with different types of wedge-shaped domains have been studied in detail in Refs. 8 and 121–123. The results of these investigations can be employed, with the help of the regular procedure studied in this section, to analyze branched domain structures in the region of spin-reorientational transitions.

8. RESONANCE PROPERTIES OF A MAGNET WITH DOMAIN STRUCTURE

When a sample with domain structure is placed in a uniform alternating magnetic field $h(t) \sim \exp(-i\omega t)$, nonuniform, forced oscillations of the magnetization vectors of the sublattices $\mathbf{M}_\alpha(\mathbf{r})$ will appear in the sample. The effective field acting on $\mathbf{M}_\alpha(\mathbf{r})$ consists of the external field $\mathbf{H} + \mathbf{h}(t)$, short-range fields (exchange, anisotropic, etc.), as well as the long-range field of the magnetic-dipole interaction $\mathbf{H}_M + \mathbf{h}_M(t)$.

The alternating part of the magnetostatic field $\mathbf{h}_M(t)$ is generated by alternating magnetic charges on the surface of the sample and on domain walls. Their existence leads to three effects: the appearance of additional "rigidity" in the spectrum, nonuniform broadening of the lines, and a difference in the polarization fields $\mathbf{h}(t)$ and $\mathbf{h}_M(t)$.

Thus far homogeneous resonance in magnets with a domain structure has been studied theoretically in great detail for ferromagnets.[102] It has been shown that in ellipsoidal ferromagnets with a regular domain structure there are two "upper" resonance frequencies (ω_1, ω_2) as well as a "low" frequency (usually in the radio-frequency range) $\omega_3 \ll \omega_1, \omega_2$ associated with oscillations of the domain walls. The difference between the resonance frequencies ω_1 and ω_2 is determined by oscillations of the magnetic charges on the domain walls. Since in a ferromagnet $|\mathbf{H}_M|$ and the anisotropy field H_A are of the same order of magnitude, while the resonance frequency calculated neglecting the demagnetizing fields is $\omega_0 \sim H_A$, ω_1 and ω_2 can, generally speaking, differ substantially both from ω_0 and from one another. The same pattern will occur in the IS associated with the spin-reorientational transition, for which $\Delta\omega \sim |\mathbf{H}_M|$ is comparable to ω_0. If, however, $\Delta\omega \ll \omega_0$, then the oscillations of the magnetic charges on the domain walls do not appreciably affect the resonance frequencies. In this case there is virtually no coupling between the oscillations in different domains. For this reason the resonance spectrum of the IS will consist of the resonance frequencies of each of the coexisting phases ω_i in the field $\mathbf{H}_P$ as well as the spectrum of frequencies corresponding to oscillations localized on the domain walls. In addition, if a periodic striped domain structure with period D can be realized in the IS, then an additional possibility arises for exciting standing magnetostatic waves with wavelength $\lambda \sim D$ in separate domains. As the external field is varied arbitrarily the internal field in the IS will vary in accordance with the relation (30), assuming one of the values from the region of $\mathbf{H}_P$. The dependence of the resonance frequencies on $\mathbf{H}$ will be determined by the dependence $\mathbf{H}_P(\mathbf{H})$, while the ratio of their intensities is proportional to the ratio of the relative fractions of the phases ξ_k. The resonance frequencies are independent of the external field only when $\mathbf{H}$ lies in one of the regions of constant internal field $\{\mathbf{H}|\mathbf{H}_P\}$. As shown above, the position of the regions $\{\mathbf{H}|\mathbf{H}_P\}$ in the IS is determined by the demagnetizing fields, generated by the surface of the sample. For this reason, the dependence of the resonance frequencies on the external field is largely determined by the shape of the magnet. Since the NMR resonance frequency ω_{NMR} is determined by the internal field $\mathbf{H}^{(i)}$, everything said above is also true for the character of the excitation of NMR frequencies and the dependence $\omega_{NMR}(\mathbf{H})$ in the IS.

Magnetic resonance in the IS has been studied experimentally for the spin-flop transition in easy-axis antiferromagnets MnF_2,[21] $CuCl_2 \cdot 2H_2O$,[17,18,51,52] $NiNO_4$,[56], as well as in the orthoferrite $ErFeO_3$ in the region of the metamagnetic transition.[53,57,58]

In Refs. 52–54, 57, and 58, where the condition $\mathbf{H}_P = \text{const}$ was realized, it was found that the magnetic resonance frequencies are independent of the external field in the entire region of existence of the IS. Figure 9a shows the dependence of the antiferromagnetic resonance frequencies in the IS with a spin-flop transition for an MnF_2 plate.[54] In

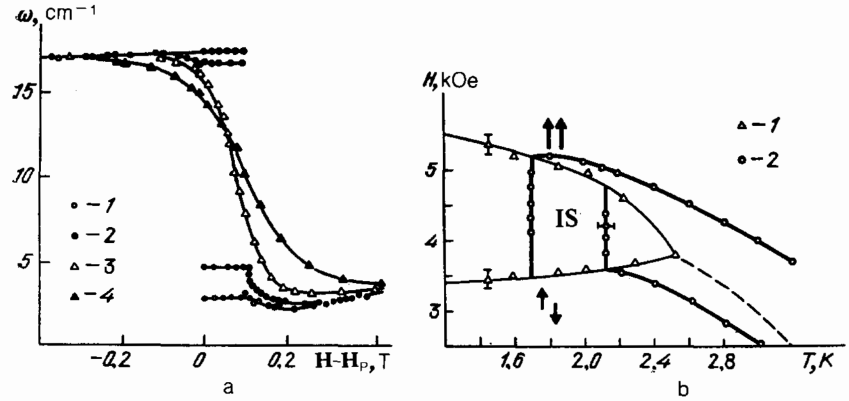

FIG. 9. Dependence of the magnetic resonance frequencies in the intermediate state. a) MnF_2 plate in the region of the spin-flop transition for different angles ψ[54]: 1) $0 \pm 5'$, 2) $10'$, 3) $20'$, 4) $40'$ (the first order phase transition occurs for $\psi \leqslant 18'$). b) spherical orthoferrite $ErFeO_3$ in the region of the metamagnetic transition[53]: 1) boundaries of the intermediate state; the broken line indicates the line of second-order phase transitions; 2) temperature dependence of the resonance fields at the frequency $\omega_0 = 50.0$ GHz.

the region of the IS ($\Psi \leqslant 18'$) independent excitation of the AFMR frequencies in separate phases of the domain structure is observed and ω_i does not depend on $\mathbf{H}$ ($\mathbf{H}$ varies along the line $\{\mathbf{H}|\mathbf{H}_P\}$). Magnetic resonance in the IS under conditions of a metamagnetic phase transition in $ErFeO_3$ was studied in Ref. 53. The fact that the resonance frequencies depend not only on $\mathbf{H}$ but also on other external parameters, for example, the temperature, was employed in Ref. 53. In the (ω, H, T) diagram the planes $\omega = \text{const}$ intersect the surfaces of the resonance frequencies $\omega_i(H,T)$ along some lines $H_{ri}(T)$. This fact makes it possible to employ a temperature scan instead of a frequency scan, i.e., to determine the lines $H_{ri}(T)$ at a fixed frequency ω_0. In Ref. 53 the frequency $\omega_0 = 50.0$ GHz, was employed. In $ErFeO_3$ this is a resonance frequency for both the antiferromagnetic $T = T_{AFM}$ and ferromagnetic $T = T_{FM} > T_{AFM}$ phases in the temperature range of PTI. The independence of the resonance frequency for each of the coexisting phases was observed in the region of the IS in the H–T diagram (Fig. 9b). Figure 10 shows the field dependence of the NMR frequencies of Fe^{57} nuclei in the vicinity of the metamagnetic phase transition in $ErFeO_3$.[57] In the region of the IS the NMR frequencies are independent of the external field.

In Refs. 17, 18, 52, and 55 nonellipsoidal samples were studied. Here the condition $\mathbf{H}_P = \text{const}$ is unachievable, so that the resonance frequencies did depend on the external field.

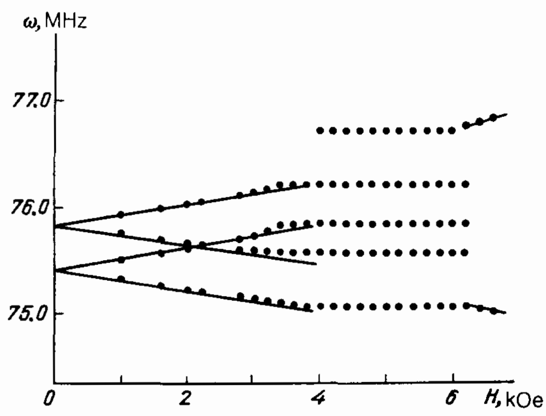

FIG. 10. Frequencies of NMR on Fe^{57} nuclei as functions of the external field in the region of the metamagnetic transition in $ErFeO_3$ at $T = 2$ K.[57]

9. DOMAIN STRUCTURE IN ORTHORHOMBIC FERROMAGNETS

Proceeding now to the analysis of the IS with a concrete external-magnetic-field-induced PTI, we shall present the regular procedure employed to investigate such domain structures theoretically:

1) The region of the PTI $\mathbf{H}_P$, τ_{iP} and the equilibrium states in the competing phases are determined by solving the system of equations (4.1) and (4.2). The region of coexistence of the domains and the structure of the IS are determined by solving simultaneously the equations (4.2), (4.3), and (4.4).

2) The characteristic parameters of the energy density of solitary domain walls with fixed boundary conditions $\mathbf{M}^{(k)}(\mathbf{H}_P)$, $L_\nu^{(k)}(\mathbf{H}_P)$ are calculated by the standard method.[1]

3) The magnetostatic energy of a sample with a given shape is calculated for a selected model distribution of the domains of separate phases, after which the equilibrium geometric parameters are determined by minimizing the energy (2.1).

4) The external-field dependences of $\mathbf{H}_P$ and of the equilibrium states in the domains, determined in the thermodynamic approximation (see Sec. 4), permit studying the dynamics and determining the resonance properties of a magnet with domain structure.

We shall first study two-phase domain structures for the example of domains in an orthorhombic ferromagnet and domains in the region of the spin-flop transition in an orthorhombic antiferromagnet (see Sec. 10).

For an orthorhomic ferromagnet the energy (3.5) has the following form[83]:

$$\Phi = \frac{\beta}{2} M_x^2 + \frac{\beta'}{2} M_y^2 - \mathbf{HM}. \tag{9.1}$$

For $\beta' > \beta > 0$ the Oz axis is the axis of predominant magnetization, while the Ox axis is the central axis. If H lies in the x,z plane (let ψ be the angle between $\mathbf{H}$ and Oz), then the stable state of the system corresponds to $\mathbf{M}$ also lying in the x,z plane (we denote by θ the angle between $\mathbf{M}$ and Oz). Based on the foregoing discussion, we rewrite the potential (8.1) as follows:

$$\Phi = \frac{\beta M^2}{2} \sin^2\theta - HM\cos(\theta - \psi). \tag{9.2}$$

Analysis of the potential (9.2) gives the following results.[83]

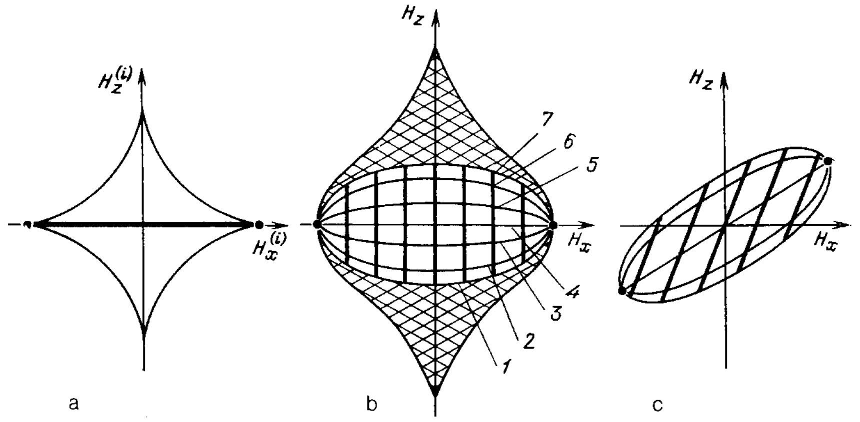

FIG. 11. Phase diagram of an orthohombic ellipsoidal ferromagnet. a) in components of the internal field; b, c) in components of the external field (b: $N_{xz} = 0$; c: $N_{xz} \neq 0$). In b) the double hatching denotes the region of restructuring of 360° domain walls (see Sec. 12); the thick lines are for constant internal field and the thin lines denote regions of constant relative fractions of the phases: 1—$\{\mathbf{H}|1; 0\}$, 2—$\{\mathbf{H}|\frac{7}{8}; \frac{1}{8}\}$, 3—$\{\mathbf{H}|\frac{5}{8}; \frac{3}{8}\}$, 4—$\{\mathbf{H}|\frac{1}{2}; \frac{1}{2}\}$, 5—$\{\mathbf{H}|\frac{3}{8}; \frac{5}{8}\}$, 6—$\{\mathbf{H}|\frac{1}{8}; \frac{7}{8}\}$, 7—$\{\mathbf{H}|0; 1\}$.

The PTI line is a segment of the straight line

$$H_{zP} = 0, \quad |H_{xP}| \leqslant H_{col}, \quad H_{col} = \beta M_0. \tag{9.3}$$

The points $\mathbf{H}(\pm H_{col}, 0)$ are the critical points of the PTI. The equilibrium states in the competing phases are given by the following equation:

$$\cos\theta_{1,2} = \pm\left[1 - \left(\frac{H_{xP}}{H_{col}}\right)^2\right]^{1/2}. \tag{9.4}$$

From here the magnetization on the phase-transition line equals

$$M_x^{(1)} = M_x^{(2)} = \frac{H_{xP}}{\beta}, \quad M_z^{(1)} = -M_z^{(2)} = M_0\left[1 - \left(\frac{H_{xP}}{H_{col}}\right)^2\right]^{1/2}. \tag{9.5}$$

The formation of the IS indicates that only thermodynamically equilibrium states are realized in the system, i.e., metastable states are not formed in the volume of the magnet. In the region of existence of metastable states, however, a characteristic restructuring of the domain walls occurs (see Sec. 12), and for this reason we shall present for the systems studied the equations for the lines of lability. In this case the boundary of existence of metastable states is determined from the system of equations $d\Phi/d\theta = 0$, $d^2\Phi/d\theta^2 = 0$, and in the phase plane $H_x^{(i)}$, $H_z^{(i)}$ (Fig. 11) it is an asteroid[83]

$$H_x^{2/3} + H_z^{2/3} = H_{col}^{2/3}. \tag{9.6}$$

Having determined the region of $\mathbf{H}_P$ (9.3) and the equilibrium states in the competing phases (9.5), we employ Eq. (4.3) to describe the domain structure. We assume that the magnet is an ellipsoid, one of whose principal axes is the Oy axis. In this case the existence of demagnetizing fields along the Oy axis is excluded, and therefore the region of existence of the intermediate state lies in the phase plane H_x, H_z. Equation (4.3) has the form

$$\begin{aligned} H_x &= H_{xP}(1 + Q^{-1}N_{xx}) + 4\pi N_{xz}M_0(2\xi_1 - 1)\left[1 - \left(\frac{H_{xP}}{H_{col}}\right)^2\right]^{1/2}, \\ H_z &= Q^{-1}N_{xz}H_{xP} + 4\pi N_{zz}M_0\left[1 - \left(\frac{H_{xP}}{H_{col}}\right)^2\right]^{1/2}; \end{aligned} \tag{9.7}$$

here we introduced the quality factor $Q = \beta/4\pi$. The system of equations (9.7) together with (9.3) and (0.5) uniquely determines ξ_1, ξ_2, $\mathbf{H}_P$, $\mathbf{M}_1(\mathbf{H}_P)$, $\mathbf{M}_2(\mathbf{H}_P)$ as a function of the external field.

The boundaries of the domain structure are determined by substituting the values $\psi_1 = 0$ and 1 into (9.7) and are described by the equation of an ellipse:

$$\begin{aligned} &[(Q + N_{xx})H_z - N_{xz}H_x]^2 + (N_{zz}H_x - N_{xz}H_z)^2 = A^2, \\ &A = (Q + N_{xx})Q^{-1}N_{zz} - Q^{-1}N_{xz}^2. \end{aligned} \tag{9.8}$$

Eliminating ξ_1 from the system (9.7) we obtain equations for the regions $\{\mathbf{H}|\mathbf{H}_P\}$, which in this case are families of parallel straight lines

$$N_{zz}H_x - N_{xz}H_z = H_{Px}[N_{zz} + Q^{-1}(N_{xx}N_{zz} - N_{xz}^2)]. \tag{9.9}$$

The straight lines (9.9) make with the H_z axis the angle $\varphi = \operatorname{arctg}(N_{xz}/N_{zz})$. The equations for the lines of constant relative percentages of the phases $\{\mathbf{H}|\xi_1, \xi_2\}$ form a family of ellipses

$$\begin{aligned} &[(Q + N_{xx})H_z - N_{xz}H_x]^2(2\xi_1 - 1)^{-2} \\ &\quad + (N_{xz}H_x - N_{zz}H_z)^2 = A^2. \end{aligned} \tag{9.10}$$

For $\xi_1 = \xi_2 = \frac{1}{2}$ the ellipses (9.10) degenerate into the straight line

$$(Q + N_{xx})H_z = N_{xz}H_x, \tag{9.11}$$

which makes with the H_z axis the angle

$$\varphi' = \operatorname{arctg}\frac{Q + N_{xx}}{N_{xz}}.$$

Equation (9.8), naturally, describes $\{\mathbf{H}|1; 0\}$ and $\{\mathbf{H}|0; 1\}$. All lines $\{\mathbf{H}|\xi_1, \xi_2\}$ (9.10) have two common points:

$$H_x = \pm H_{col}(1 + Q^{-1}N_{xx}), \quad H_z = \pm Q^{-1}N_{xz}H_{col} \tag{9.12}$$

and fix the values of the external field for which $H^{(i)}$ equals the critical value. This general property of the critical points of PTI was discussed above (see Sec. 5).

Figure 11 shows the region of existence of the IS for ellipsoids with $N_{xz} = 0$(b) and $N_{xz} \neq 0$(c). As pointed out above, for external fields in the regions $\{\mathbf{H}|\mathbf{H}_P\}$ only the process of displacement of the domain walls occurs in the intermediate state. It is obvious from Fig. 11 that in an ellipsoidal magnet whose principal axes are the magnetic axes $N_{xz} = 0$ for $\mathbf{H} \parallel$ easy axis in the region of existence of the domain structure, the evolution of the domain structure occurs owing to the displacement of the boundaries. If, however, the principal axes of the ellipsoid do not coincide with the magnetic axes ($N_{xz} \neq 0$) even in the field $\mathbf{H}\|$ easy axis, displacement of domain walls should be accompanied by rotation of $\mathbf{M}$ in the domains.

Everything said above is also true for ferromagnets in the form of a plate whose easy axis is tilted away from the normal by an angle α. In this case $N_{xx} = \sin^2\alpha$, $N_{zz} = \cos^2\alpha$, $N_{xz} = -\sin\alpha\cos\alpha$, and the lines of constant internal field make with the H_z axis the angle $\varphi = -\alpha$.

The phase diagrams of an orthorhombic ferromagnet with IS examined above contain only the components of the magnetic field.

As an illustration of the effect of a nonmagnetic parameter on the IS of a magnet we shall examine the IS of an orthorhombic ferromagnet at finite temperatures. As above, we shall assume that the vector $\mathbf{H}$ lies in the plane formed by the easy (Oz) and central (Ox) axes. In this case the equilibrium states of the magnet are determined by minimizing the free energy:

$$F=\frac{1}{2}\delta M_0^2\sigma^2 + \frac{1}{2}\beta M_0^2\sigma^2\sin^2\theta - HM_0\sigma\cos(\theta-\psi) - k_BT\eta(\sigma), \quad (9.13)$$

where δ is the exchange interaction constant, $\eta(\sigma)$ is the entropy, and k_B is Boltzmann's constant.

It can be shown[136] that for a fixed temperature $T < T_C$ (T_C is the Curie temperature) the straight-line segment

$$|H_x^{(i)}| < \beta M_0\sigma(T) = H_A\sigma(T), \quad H_z^{(i)} = 0 \quad (9.14)$$

in the $H_x^{(i)}$, $H_z^{(i)}$ phase diagram is a line of PTI, on which phases with $\theta_1 = \arcsin(H_x^{(i)}/H_{col}\sigma(T))$ and $\theta_2 = \pi - \theta_1$, coexist, while $\sigma(T)$ is a root of the equation

$$(\delta+\beta)\sigma = -k_BT\frac{d\eta}{d\sigma}(\sigma). \quad (9.15)$$

The region of the PTI in the H_x, H_z, and T phase diagram (Fig. 12a) is distinguished by cross hatching, and the line

$$H_x = \pm H_A\sigma(T) \quad (9.16)$$

is the line of critical points of PTI.

It follows from what was said above that to describe the IS at finite temperatures it is sufficient to make the substitution $M_0 \to M_0\sigma(T)$, $H_{col} \to H_{col}\sigma(T)$ in the relations (9.7)–(9.12). It is obvious that in so doing all features of the IS studied above (at $T=0$) are preserved. For an ellipsoidal sample the region of existence of the IS in the H_x, H_z, T phase diagram is bounded by a surface whose sections by the planes $T = \text{const}$ are ellipses (Fig. 12b). If the principal axes of the ellipsoid coincide with the magnetic axes, then in the H_z, T phase diagram we obtain for the boundary of the IS

$$H_{1,2} = \mp 4\pi N_{zz}M_0\sigma(T). \quad (9.17)$$

In the (H_x, T) phase diagram the boundary of the IS is described by the equation

$$H_{1,2} = \mp(\beta + 4\pi N_{xx})M_0\sigma(T). \quad (9.18)$$

In the (H_z, T) phase diagram the region of existence of the IS is the region of constant internal field with $\mathbf{H}_p = 0$—$\{\mathbf{H}, T|0\}$. In this case the evolution of the IS in a magnetic field occurs only owing to redistribution of the relative percentages of the phases (the process of displacement of DW), while the magnetic susceptibility is determined only by the shape of the sample (see (6.21)). In the (H_x, T) phase diagram the IS is a region of constant relative percentages of the phases with $\xi_1 = \xi_2 = \frac{1}{2}$. Here the evolution of the IS in a magnetic field occurs only owing to the change of state in the domains (rotation process), while the magnetic susceptibility is given by

$$\chi = \frac{1}{\beta + 4\pi N_{xx}} \quad (9.19)$$

and is determined not only by the shape of the sample, but also by the character of the change in the equilibrium states in the region of the PTI. In the field $\mathbf{H}\|Oz$ the phase transition into a uniform state on the line (9.17) occurs owing to complete expulsion of the unfavorable phase. For $\mathbf{H}\|Ox$ the transition into the uniform state is associated with a loss of difference between the states in separate domains. Figure 13 shows the experimental results for the boundaries of the IS in the field $\mathbf{H}$ parallel (a) and perpendicular (b) to the easy axis. As one can see in Fig. 12b the phase transition from the IS into the uniform state occurs in finite fields for any orientation of $\mathbf{H}$.

At the end of the 1960s interest arose in the question of the character of the (H,T) diagrams of ferromagnets in the vicinity of the Curie temperature. It is known from the self-consistent field theory that in an unbounded isotropic ferromagnet in the (H,T) phase diagram the Curie point is an isolated point of a second-order phase transition.[83] Indeed, in any finite field the ferro- and paramagnetic phases have the same symmetry. In this connection one talks about the

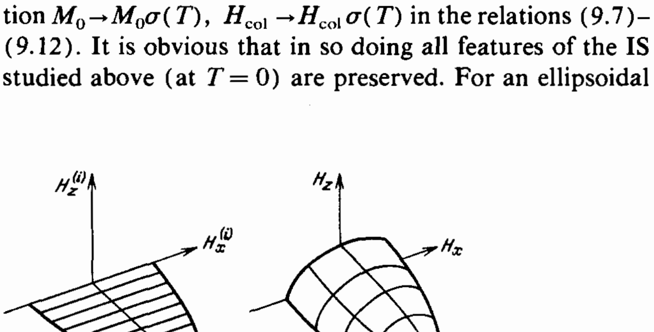

FIG. 12. H–T phase diagrams of an orthorhombic ellipsoidal ferromagnet. a) In components of the internal field; b) in components of the external field. In Fig. 1(a) the region of FTI is cross hatched.

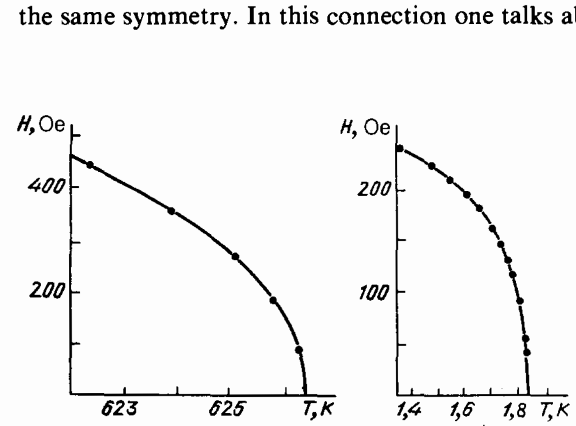

FIG. 13. H, T diagrams of spherical samples of nickel ($\mathbf{H}$ ∥ easy axis)[141] (a) and $Cu(MH_4)_2Br_4 \cdot 2H_2O$ ($\mathbf{H}\perp$ easy axis)[142] (b).

smearing of the PTII by a magnetic field. The singular behavior of a ferromagnet in a nonzero field in the vicinity of T_C was first observed in Ref. 137. The theoretical explanation of this phenomenon was given on the basis of the model of an isotropic ferromagnet. It was shown in Ref. 138 that the equilibrium energy of such a magnet, even taking into account the dipole-dipole interaction, in the thermodynamic limit is independent of the shape of the sample. On the other hand, the energy of demagnetization of a uniformly magnetized ferromagnet depends strongly on the shape of the magnet and the direction of **M** in it. It follows from this that the equilibrium state of an istropic ferromagnet with $\mathbf{H} = 0$ is not uniform,[139] and in addition the scale of the nonuniformity is much smaller than the characteristic dimensions of the sample. In Ref. 139 it was suggested that the PTII observed in the vicinity of T_C in a nonzero field is a phase transition from the nonuniform state into a uniform state. The hypothesis has been reliably confirmed in numerous experimental and theoretical studies.[75–77,140]

We note that in the transitions discussed above anisotropy plays the determining role. Unlike the infinite degeneracy of the ground state in the isotropic model, in an anisotropic ferromagnet with $\mathbf{H}^{(i)} = 0$ there is only a finite number of stable states (phases) between which a PTI occurs in a magnetic field. The $(\mathbf{H}^{(i)}, T)$ phase diagrams of a ferromagnet contain a region of PTI, bounded by lines of critical points, which are actually lines of PTII and terminate at T_c.[1] In finite ferromagnets the existence of field-induced PTI and the demagnetizing action of the surface of the magnet lead to the formation of an IS, which in its turn is responsible for the existence of phase transitions (in finite fields) from the IS into the uniform state, and in addition for arbitrary orientations of **H**.

It should be kept in mind that in weakly anisotropic ferromagnets (for example, in some cubic ferromagnets) the IS can have an irregular character: because of the smallness of the anisotropy the inequality $D \gg x_0$ may not hold, and then the distinction between a domain and its wall is lost. In this connection the nonuniform state of an isotropic ferromagnet can be regarded as the limiting case of an anisotropic ferromagnet with a vanishingly small anisotropy energy E_A: as $E_A \to 0$ broadening of the domain walls in the entire volume of the magnet will establish a complicated nonuniform distribution of $\mathbf{M}(r)$. In addition, the inequality $D \gg x_0$ is violated, this time for a different reason (see conclusions), in the vicinity of the lines of termination of PTI (lines of PTII). The characteristics of the IS in this region were studied in Refs. 75–77.

Summarizing the foregoing discussion we can assert that in a finite ferromagnet the formation of an IS leads to the existence of an IS in the vicinty of T_c in finite fields with arbitrary orientation of the external field. Exceptions are magnets with limiting shapes (thin plates and long cylinders). For some orientation of the crystallographic axes an IS may not form in such magnets; this leads to smearing of the phase transition by the external field.

10. DOMAIN STRUCTURE OF ORTHORHOMBIC ANTIFERROMAGNETS NEAR A SPIN-FLOP TRANSITION

We shall study a two-sublattice orthorhombic antiferromagnet without the Dzyaloshinskiĭ interaction (the systems $\bar{1}$ ($-$), t ($-$) according to E. A. Turov's classification[103]). The ground state of such an antiferromagnet in a magnetic field, tilted into the plane formed by the easy and central axes (the (x,z) plane), was studied in Refs. 104–112. As shown in Ref. 111, if the exchange interactions are much stronger than the relativistic interactions, then to describe the magnetic properties in the vicinity of the spin-flop transition it is sufficient to retain the following terms in the anisotropy energy Φ_A:

$$\frac{1}{2\lambda}\Phi_A = -B_1 l_z^2 - B_2 l_z^4 - (B_1 - \beta) m_z^2, \tag{10.1}$$

where $\mathbf{l} = (\mathbf{M}_1 - \mathbf{M}_2)/2M_0$ is the antiferromagnetism vector, $\mathbf{m} = (\mathbf{M}_1 + \mathbf{M}_2)/2M_0$ is the total magnetization vector, $\mathbf{M}_1$ and $\mathbf{M}_2$ are the magnetization vectors of the sublattices, $|\mathbf{M}_\alpha| = M_0$, and λ is the intersublattice exchange interaction constant.

For $B_1 + B_2 > 0$ Oz is the easy axis. With the exception of very particular ratios between the anisotropy constants of different orders (see, for example, Ref. 112 on this point) the stable state of the system in a magnetic field lying in the (x,z) plane corresponds to $\mathbf{M}_1$ and $\mathbf{M}_2$ lying in the same plane. Outside the vicinity of the Morin point usually $B_1 \gg B_2$. In this case after minimizing with respect to **m**, the energy of the antiferromagnet can be represented in the form

$$F = \frac{1}{H_p^2}\Phi = \frac{1}{2} a \sin^2 2\theta - (h_z - 1)\cos 2\theta - h_x \sin 2\theta,$$
$$\mathbf{h} = \frac{\mathbf{H}}{H_p}, \quad H_p = 2\lambda B_1^{1/2} M_0, \quad a = \frac{B_2}{B_1} + \frac{\beta}{\lambda}, \tag{10.2}$$

where θ is the angle between **l** and the Oz axis.

The fact that the potential (10.2) is functionally identical to the energy of an orthorhombic ferromagnet (9.2) makes it possible to employ the results presented in Sec. 9. For this, it is sufficient to make the substitution $\mathbf{H}_x \to h_x$, $\mathbf{H}_z \to h_z - 1, \beta \to a$ in the relations (9.2)–(9.4) and (9.6). It is obvious that the phase diagram of an orthorhombic antiferromagnet in the coordinates $(h_x, h_z - 1)$ is identical to the corresponding (H_x, H_z) diagram of the orthorhombic ferromagnet. In particular, the region of existence of metastable states is bounded by an astroid, the coordinates of whose cusps are

$$(-a;\ 1),\ (a;\ 1), \tag{10.3}$$
$$(0;\ h_\parallel),\ (0;\ h_\perp), \tag{10.4}$$

where $h_\parallel = 1 - a$ and $h_\perp = 1 + a$.

For $a > 0$ the line of PTI is a segment of the straight line $h_z = 1$, $|h_x| \le a$ (Fig. 14a), on which phases with different values of the components m_z coexist[111]:

$$m_z^{(1),(2)} = m_{0z} \pm m_{1z} = (B_1 + B_2)^{1/2}[1 \pm (1 - \nu^2)]^{1/2},$$
$$m_x^{(1)} = m_x^{(2)} = m_{0x} = (B_1 + B_2)^{1/2}\nu, \quad \nu = \frac{h_x}{a}. \tag{10.5}$$

For $a < 0$ the line of PTI is a segment of the straight line $h_x = 0$, $|h_z - 1| \le |a|$ (Fig. 14b), on which the states with l tilted away from the easy axis by angles of $\pm\theta$ coexist. For these states the quantity m_x assumes opposite values:

$$m_x^{(1)} = -m_x^{(2)} = \left[\frac{(h_z - h_\parallel)(h_\perp - h_z)}{h_\perp - h_\parallel}\right]^{1/2},$$

$$m_z^{(1)} = m_z^{(2)} = \frac{h_z - h_\parallel}{h_\perp - h_\parallel}. \tag{10.6}$$

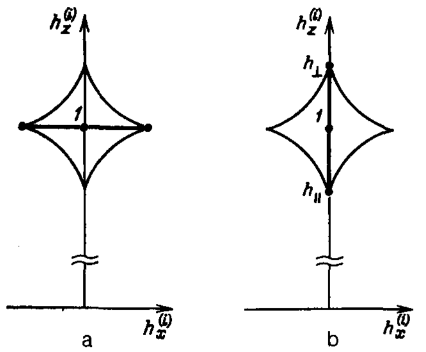

FIG. 14. Phase diagrams of an orthohombic ferromagnet in components of the internal field. a) for $a>0$; b) for $a<0$.

The existence of states with different values of the vector **m** on the lines of PTI leads to the formation of an intermediate state in the region of the spin-flop transition.

For an ellipsoidal sample with $a>0$ Eq. (4.3) can be written as

$$h_x = (a + q^{-1}N_{xx})\,\nu + q^{-1}N_x\ (\xi_1 - \xi_2)\,(1-\nu^2)^{1/2} + q^{-1}N_{xz}. \qquad (10.7)$$

$$h_z = q^{-1}N_{xz}\nu + q^{-1}N_{zz}\,(\xi_1-\xi_2)\,(1-\nu^2)^{1/2} + 1 + q^{-1}N_{zz}\ \ q=\frac{\lambda}{\pi}. \qquad (10.8)$$

The relations determining the structure of the IS of an antiferromagnet are analogous to the relations (9.7)–(9.10), derived for an orthorhombic ferromagnet. Figure 15 shows the region of existence of the IS of an ellipsoidal orthorhombic antiferromagnet ($a>0$) with $N_{xz}=0$ (a) and $N_{xz}\neq 0$ (b). Unlike a ferromagnet, here the region of existence of the IS is shifted along the h_z axis ($N_{xz}=0$), while for $N_{xz}\neq 0$ it is also shifted along the h_x axis. To understand this we turn to the above-introduced representation of ellipsoids "inserted" into one another. An orthorhombic antiferromagnet with a domain structure in the region of a spin-flop transition can be regarded as a uniformly magnetized magnet with magnetization $\mathbf{m}_0$ (10.5) and another one of the same shape with "ferromagnetic" domains that have a magnetization $\pm\mathbf{m}_1$ (10.5). We can say that an ellipsoid with "ferromagnetic" domains is subjected to an additional "external" field, $\mathbf{h}_0 = 4\pi\hat{N}\mathbf{m}_0$, generated by the magnetization $\mathbf{m}_0$ (10.5). This is what leads to the displacement of the region of the IS.

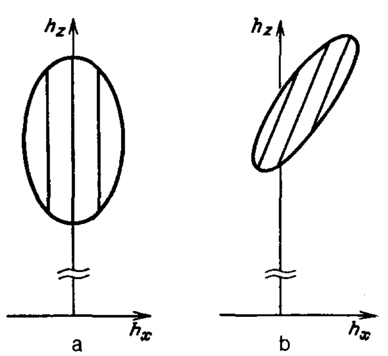

FIG. 15. Phase diagrams of an orthorhombic ellipsoidal antiferromagnet with $a>0$. a) $N_{xz}=0$; b) $N_{xz}\neq 0$.

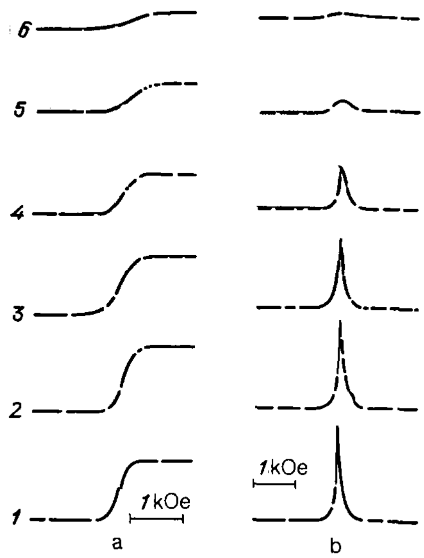

FIG. 16. Oscillograms of the longitudinal component of the magnetization (a) and the magnetic susceptibility (b) in $NiWO_4$ in the region of the spin-flop transition for different angles ψ[42]: 10′ (1), 32′ (2), 54′ (3), 74′ (4), 98′ (5), 120′ (6) (for $\psi\leqslant 72'$ a first-order phase transition occurs).

Experimental studies of the IS in the region of a spin-flop transition were performed in MnF_2,[19–21,52] CoF_2,[38] $CuCl_2\cdot 2H_2O$,[17,18,51] $CdAlO_3$,[39,40] $MnCl_2\cdot 4H_2O$,[41] $NiWO_4$,[42] $(C_2H_5NH_3)_2\,CuCl_4$.[43] Figures 6 and 7 show the dependence of the magnetization of $MnCo_2\cdot 4H_2O$[11] and MnF_2[15] in the region of the spin-flop transition in a magnetic field oriented parallel to the easy axis. The parameters of the IS accompanying a spin-flop transition in an oblique field were studied in the easy-axis antiferromagnet $NiWO_4$ (Fig. 16).[42] In complete agreement with the theory in the interval $|\psi|\leq 72'$ $\chi_{IS}=\frac{1}{4}\pi N$ remains constant, and the jump in the

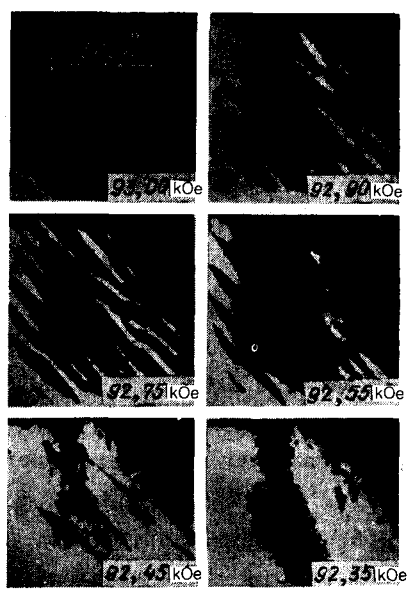

FIG. 17. Photographs of the domain structure in the region of the spin-flop transition in an MnF_2 plate.[21] The dark regions indicate the spin-flop phase and the light-colored regions indicate the antiferromagnetic phase.

magnetization drops monotonically as ψ increases (see the formulas (6.19) and (10.5)).

Figure 17 shows photographs of the domain structure in the region of the spin-flop transition for an MnF_2 plate.[21]

11. DOMAIN STRUCTURE OF ORTHOFERRITES NEAR SPONTANEOUS PHASE TRANSITIONS

We shall study the properties of an IS with a multiphase domain structure ($n>2$) for the example of an orthoferrite in the region of smooth spin reorientation (a,c reorientation). The energy density (3.4) for an orthoferrite in this region equals[97,113,116]

$$\Phi = K_2 \sin^4\theta + (K_1 - K_2)\sin^2\theta - \mathbf{Hm}, \qquad (11.1)$$

K_1 and K_2 are effective anisotropy constants, expressed in terms of the anisotropy, the Dzyaloshinskiĭ interaction, and the exchange interaction constants, and θ is the angle between the spontaneous magnetization vector $\mathbf{m}$ and the c axis.

In the rare-earth orthoferrites studied the a,c reorientation occurs smoothly in a definite temperature range.[97] This process is described by the potential (11.1) with $K_2>0$. In the absence of a field the region of smooth spin reorientation is realized in a temperature range where $-K_2<K_1(T)<K_2$, and in addition in the equilibrium state

$$\cos 2\theta = \frac{K_2}{K_1}. \qquad (11.2)$$

If $\mathbf{H}\|c$, then the angular phase exists for $K_1<K_2$ and $|\mathbf{H}|<|H_c^{(k)}|$, where

$$H_c^{(k)} = \pm 2(K_1 - K_2)m, \qquad (11.3)$$

while the equilibrium states θ in the angular phase are given by the equation

$$\cos^3\theta - \frac{1}{2}\left(1+\frac{K_1}{K_2}\right)\cos\theta - \frac{Hm}{4K_2} = 0. \qquad (11.4)$$

In an analogous manner the following results can be obtained for $\mathbf{H}\|a$. The region of existence of the angular phase $K_1> -K_2$, $|\mathbf{H}|<|H_a^{(k)}|$, where

$$H_a^{(k)} = \pm 2(K_1 + K_2)m, \qquad (11.5)$$

while θ is determined by the equation

$$\sin^3\theta - \frac{1}{2}\left(1-\frac{K_1}{K_2}\right)\sin\theta - \frac{Hm}{4K_2} = 0. \qquad (11.6)$$

In accordance with the general result proved above, the regions of existence of angular phases studied here are surfaces of external-field-induced PTI. Indeed, for example, for $\mathbf{H}\|a$, in the region bounded by the straight lines (11.5) the states of the orthoferrite in the angular phase, given by Eq. (11.6), are doubly degenerate: the solution of (11.6) is θ and $\pi-\theta$. A magnetic field $\mathbf{H}\|c$ removes this degeneracy: the state with $\mathbf{m}$, making a smaller angle with $\mathbf{H}$, becomes energetically favored. This means that the region of existence of the angular phase with $\mathbf{H}\|a$ is a region of PTI induced by the component of the field parallel to the c axis (H_z), and the PTI occurs at $H_z=0$. Analogous arguments can also be given for the region $K_1<K_2$, $H_x=0$, $|H_z|<|H_c^{(k)}|$. Thus the regions of existence of angular phases in the phase space H_x, H_z, K_1 are surfaces of PTI, while the straight lines (11.3) and (11.5) are lines of termination of PTI—critical lines.

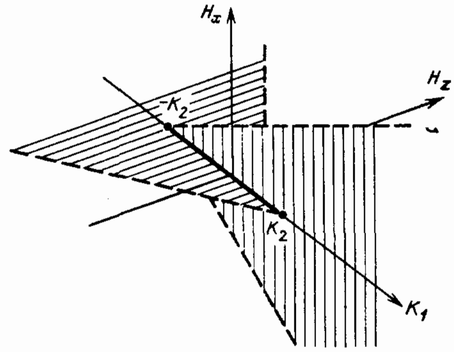

FIG. 18. $H_x^{(i)}, H_z^{(i)}$, K_1 phase diagram of an orthoferrite in the region of smooth spin reorientation. The cross hatching denotes regions of first-order phase transitions, the broken lines denote lines of termination of first-order phase transitions.

Figure 18 shows the H_x, H_z, and K_1 phase diagram of an orthoferrite. The regions of PTI (angular phases) are cross hatched, and the broken lines are the critical lines of PTI (11.3) and (11.5). The region $\mathbf{H}=0$, $|K_1|<K_2$ (the region of smooth spin reorientation) is the intersection of two surfaces of PTI, i.e., it is the region of coexistence of four phases (from (11.4)):

$$\theta_1 = \frac{1}{2}\arccos\frac{K_1}{K_2}, \quad \theta_2 = -\theta_1, \quad \theta_3 = \theta_1+\pi, \quad \theta_4 = \theta_2+\pi. \qquad (11.7)$$

In other words, the region of smooth spin reorientation is a line of PTI between the four phases (11.7). Figure 19 shows the (H_x, H_z) diagrams of orthoferrite, obtained by the section of the H_x, H_z, and K_1 diagram by the plane $K_1(T)$ = const (so as not to clutter the figure, the lines of lability of separate phases are not drawn). In the region $|K_1|>K_2$ $H_x^{(i)}$, $H_z^{(i)}$ qualitatively corresponds to the phase diagram of an easy-axis ferromagnet (Fig. 11), while for $|K_1|\gg K_2$ the fourth-order anisotropy in (11.4) can be neglected, and (11.1) transforms into (9.2).

Since in the region of smooth spin reorientation a PTI occurs between two and four phases, they will correspond to two- and four-phase domain structures.

The regions of four-phase degeneracy ($H^{(i)}=0$, $|K_1|<K_2$) are given by Eq. (11.7). For $\mathbf{H}$ lying in the a, c plane,

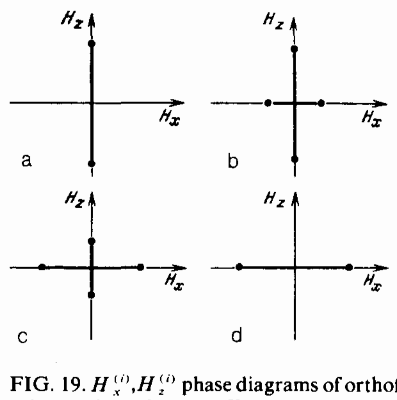

FIG. 19. $H_x^{(i)}, H_z^{(i)}$ phase diagrams of orthoferrite in the region of smooth spin reorientation. a) $K_1<-K_2$; b) $K_2<K_1<0$; c) $0<K_1<K_2$; d) $K_1>K_2$.

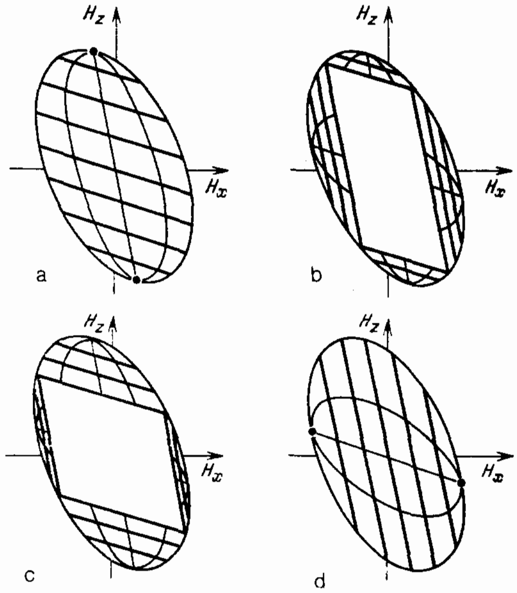

FIG. 20. H_x, H_z phase diagrams of an ellipsoidal orthoferrite. a) $K_1 < -K_2$; b) $K_2 < K_1 < 0$; c) $0 < K_1 < K_2$; d) $K_1 > K_2$. The interior of the parallelogram in Figs. b and c is the region of existence of the four-phase domain structure ($\mathbf{H}^{(i)} = 0$); four regions with a two-phase domain structure are contiguous to it. In the regions of existence of the two-phase domain structures lines of constant internal field (segments of straight lines and lines of constant relative fractions of the phases) are drawn.

Eqs. (4.3) have the following form:

$$\begin{aligned} h_x &= 4\pi m N_{xx} (\xi_1 - \xi_2 - \xi_3 + \xi_4) \cos\theta \\ &\quad + 4\pi m N_{xz} (\xi_1 + \xi_2 - \xi_3 - \xi_4) \sin\theta, \\ h_z &= 4\pi m N_{xz} (\xi_1 - \xi_2 - \xi_3 + \xi_4) \cos\theta \\ &\quad + 4\pi m N_{zz} (\xi_1 + \xi_2 - \xi_3 - \xi_4) \sin\theta, \\ \xi_1 &+ \xi_2 + \xi_3 + \xi_4 = 1. \end{aligned} \tag{11.8}$$

Together with Eq. (11.7) the system (11.8) determines ξ_k as a function of $\mathbf{H}$ and $K_1(T)$. In the (H_x, H_z) diagram the region of existence of the four-phase domain structure is a parallelogram (Fig. 20).

At the vertices of the parallelogram a transition into the uniform state occurs (here one of the ξ_k equals 1, and the others equal 0). On each side of the parallelogram two of the four quantities ξ_k vanish in pairs, i.e., these segments are boundaries of four- and two-phase domain structures. The system of equations (11.8) contains three equations for four unknowns ξ_k. This means that the solutions are not single-valued, as discussed above (see Sec. 5): in a given external field different sets of ξ_k will satisfy the system (11.8). In this case $n = 4$, $d = 2$ and, therefore, $\gamma = 1$ (see Eq. (4.13)), i.e., the one-parameter family ξ_k will satisfy the system (11.8).

We shall now describe the region with a two-phase domain structure. For example, for the PTI line $H_{x\text{P}} = 0$, $0 < H_{z\text{P}} < H_c^{(k)}$ the equations (4.3) assume the form

$$\begin{aligned} H_z &= H_{z\text{P}} + 4\pi N_{zz} (\xi_1 + \xi_2)\, m \cos\theta \\ &\quad + 4\pi N_{xz} (\xi_1 - \xi_2)\, m \sin\theta, \\ H_x &= 4\pi N_{xz} (\xi_1 + \xi_2)\, m \cos\theta \\ &\quad + 4\pi N_{xx} (\xi_1 - \xi_2)\, m \sin\theta, \\ \xi_1 &+ \xi_2 = 1. \end{aligned} \tag{11.9}$$

Together with Eq. (11.4) the system (11.9) also gives the dependence of the internal parameters on $\mathbf{H}$. Unlike the four-phase domain structure, where $\mathbf{H}^{(i)} = 0$ in the entire region of its existence, and the system evolves only owing to the redistribution of the relative percentages of the phases, in the two-phase domain structure the internal states in the domains will also change as $\mathbf{H}$ changes. Eliminating ξ_k from (11.9) we obtain the lines $\{\mathbf{H}|\mathbf{H}_\text{P}\}$, which form a family of straight lines

$$N_{xx}(H_{\Pi z} - H_z) - 4\pi m \cos\theta\, (N_{xx}N_{zz} - N_{xz}^2) = H_x N_{xz}, \tag{11.10}$$

parallel to the corresponding side of the parallelogram.

Eliminating $H\,\mathbf{p}_z$ and θ from the system (11.9) we obtain an equation for the regions $\{\mathbf{H}|\xi_1, \xi_2\}$ and, in particular, the boundaries between the region of existence of the two-phase domain structure and the uniform state—$\{\mathbf{H}|1;0\}$, $\{\mathbf{H}|0;1\}$. The structure of three other regions with a two-phase domain boundary is determined in an analogous manner. The (H_x, H_z) phase diagrams of an ellipsoidal sample were constructed from computational results (Fig. 20).

We shall trace the evolution of the (H_x, H_z) diagram of orthoferrites as $K_1(T)$ varies. For $|K_1(T)| < K_2$ the phase diagram contains a region with a four-phase domain structure (parallelogram) and four adjacent regions with a two-phase domain structure (see Fig. 20). As $K_1(T)$ approaches $\pm K_2$ the parallelogram is compressed, and for $K_1(T) = \pm K_2$ it constricts into a segment. For $|K_1(T)| > K_2$ only a region with a two-phase domain structure, analogous to the domain structure of an easy-axis ferromagnet (compare Fig. 11 with Fig. 20, a and d), exists.

Since the components of the tensor $\hat{N}$ are determined by the ratio between the axes of the ellipsoid and their arrangement relative to the magnetic axes, the shape of the magnet strongly affects the dimensions of the region of existence of the domain structures. Thus, for example, if the sample studied is a plate whose normal is parallel to one of the crystallographic axes (a or c), then one should expect a domain structure related only with the demagnetizing action of the projections $\mathbf{m}^{(k)}$ on the axis parallel to the normal. In this case the region of four-phase domain structure will not appear at all. In practice, however, because of the finiteness of the transverse dimensions of the plate, owing to the demagnetizing action of its faces the four-phase domain structure can be observed.[71] Since magnetooptical studies of domains are performed on samples in the form of plates, plates cut out at an angle to the magnetic axes a and c are most useful for observing the four-phase domain structures of an orthoferrite. Thus plates with $\alpha = 0$–$10°$ (α is the angle between $\mathbf{n}$ and the c axis) were employed. The results of experimental studies, as well as qualitative description of the evolution of a four-phase domain structure as a function of T, undertaken in Ref. 70, are in complete agreement with the results presented. It follows from (11.8) that for $\mathbf{H} = 0$ in the region of smooth spin reorientation $\xi_1 = \xi_3$, $\xi_2 = \xi_4$, and the relative percentages of the phases ξ_1, ξ_3, and ξ_2, ξ_4 can be arbitrary. This arbitrariness in the distribution of the relative fractions of the phases is explained by the shape "hysteresis," discovered in Ref. 70, of the domains (Fig. 21). The behavior of the domain structures of orthoferrites, observed in Ref. 70, in the fields $\mathbf{H}\|\mathbf{n}$ and $\mathbf{H}\perp\mathbf{n}$ can be interpreted as a successive transition from a four-phase domain structure to a two-phase structure, and then into a uniform state. The existing

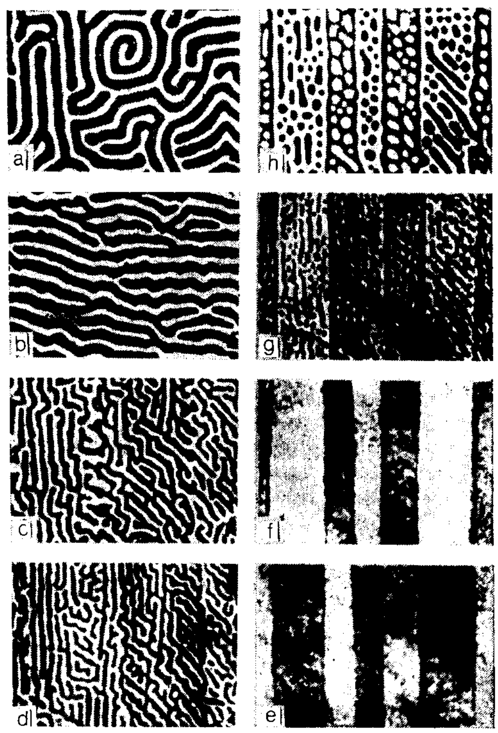

FIG. 21. Domain structure of the orthoferrite $Sm_{0.55}Tb_{0.45}FeO_3$ in the region of smooth spin reorientation (210–300 K) at different temperatures (a–h) in the field $H = 0^{70}$: 295 K (a); 270 K (b); 251 K (c); 245 K (d); 217 K (e); 228 K (f); 255 K (g); 295 K (h). The states in a–e were obtained by cooling the sample with simultaneous magnetic "shaking"; the states in f–g were obtained while heating the sample.

data, however, are not sufficient for quantitative comparison with theory. It would be useful to study experimentally the evolution of domain structures in the region of smooth spin reorientation with an arbitrary orientation of the field in the *ac* plane, including using plates with $\alpha \approx \pi/4$, when the demagnetizing fields along the *a* and *c* axes are of comparable magnitudes.

The region of existence of an angular phase of ferrites,[97] is also the region of external-field-induced PTI,[61] and therefore an IS is realized here. Such domain structures (so-called high-field domains) were observed in Refs. 65 and 66, and the theory was constructed in Ref. 61.

12. EVOLUTION OF DOMAIN WALLS IN THE REGION OF SPIN-REORIENTATIONAL TRANSITIONS

In the foregoing discussion we examined thermodynamically stable domain structures. However, domain walls can exist even outside the region of external-field-induced PTI. In this case the formation of domain structures is not thermodynamically advantageous, since the increase in the energy of the system owing to the formation of nonuniform states (domain walls) is not compensated by a gain in the energy of other interactions. Such domains nonetheless appear in the process of formation of an ordered state and are termed kinetic domains.[3] Examples are domains in a collinear antiferromagnet with antiparallel orientation,[115] separated by 180° domain walls. In an easy access ferromagnet 360° domain walls, separating sections of the ferromagnet with collinear orientation of **M**, can exist outside the region of PTI.

If the kinetic domain walls contain spin configurations corresponding to metastable states of the spin-reorientational PTI, then the structure of domain walls undergoes characteristic changes in the vicinity of such a transition.[32,97,108,117–119] We shall study this phenomenon within the framework of the simplest model: for a flat domain wall, whose magnetic states are fixed by one configurational variable θ. In the "thin" wall approximation the potential (2.1) for this case reduces to the following[83]:

$$\Phi = \int_{-\infty}^{\infty} \left[\alpha \left(\frac{d\theta}{dx} \right)^2 + \Phi(\theta) \right] dx, \qquad (12.1)$$

where x is a spatial coordinate, oriented along the normal to the domain wall; α is the nonuniform exchange interaction constant; $\Phi(\theta)$ is the uniform part of the energy density. The structure of the domain wall $\theta(x)$ is determined by varying the functional (12.1) with the standard boundary conditions[1]

$$\frac{d\theta}{dx}_{x=\pm\infty} = 0, \quad \theta_{x=+\infty} = \theta_1^0, \quad \theta_{x=-\infty} = \theta_2^0,$$

where θ_1^0 and θ_2^0 are the equilibrium values of Θ in neighboring domains. The Euler equation for (12.1)

$$\alpha \frac{d^2\theta}{dx^2} - \frac{d\Phi}{d\theta} = 0 \qquad (12.2)$$

has the first integral

$$\alpha \left(\frac{d\theta}{dx} \right)^2 = \Phi(\theta) - \Phi_{min}, \qquad \Phi_{min} = \Phi(\theta_1^0) = \Phi(\theta_2^0). \qquad (12.3)$$

The distribution $\Theta(x)$ and the energy of the domain wall Φ_{DW} are determined by direct integration of (12.3)[84]:

$$x = \int_{\theta_1^0}^{\theta_2^0} \left(\frac{\alpha}{\Phi(\theta) - \Phi_{min}} \right)^{1/2} d\theta, \qquad (12.4)$$

$$\Phi_{DW} = \int_{\theta_1^0}^{\theta_2^0} (\Phi(\theta) - \Phi_{min})^{1/2} d\theta. \qquad (12.5)$$

It follows from Euler's equation (12.2) that the points of inflection of the function $\theta(x)$ are found from the equations $d\Phi/d\theta = 0$ (the roots corresponding to the stable state of the system (θ_1^0, θ_2^0) which are realized in the domains, naturally, must be excluded from these solutions). If the values of θ corresponding to the new phase belong to the starting domain wall, then on transition into the region of metastable states they, being solutions of the equation $d\Phi/d\theta = 0$, will lead to additional inflections in the function $\theta(x)$. Thus the character of the functions $\theta(x)$ is qualitatively different inside and outside the region of the metastable states. Outside the region of metastable states in the interval $[\theta_1^0, \theta_2^0]$ the equation $d\Phi/d\theta = 0$ has one root, corresponding to the maximum of $\Phi(\theta)$. This value of θ determines, in this case, the only inflection point of the function $\theta(x)$ (Fig. 22). The region of existence of metastable states, in the interval $[\theta_1^0, \theta_2^0]$, already contains at least three extremal values of the function $\Phi(\theta)$: one (θ_3^0) corresponds to a local minimum (metastable state), while the other two correspond to a maximum of $\Phi(\theta)$ (see Fig. 20). As the field of the PTI is approached the energy of the metastable state

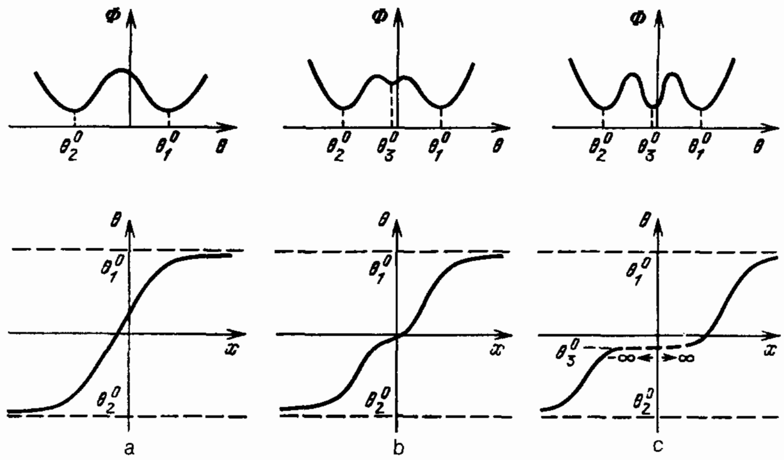

FIG. 22. Schematic view of the one-dimensional potential $\phi(\theta)$ and the corresponding distribution $\theta(x)$ in a solitary plane domain wall separating the states θ_1^0 and θ_2^0. a) Outside the region of metastable states associated with a first-order phase transition into the phase θ_3^0; b, c) inside the region of metastability (b: $\mathbf{H}^{(i)} \neq \mathbf{H}_P$; c: $\mathbf{H}^{(i)} = \mathbf{H}_P$).

decreases $(\Phi(\theta_3^0) - \Phi_{\min}) \to 0$. It follows from Eq. (12.3) that in this case $(d\Phi/dx)_{\theta_3^0} \to 0$, i.e., the change in $\theta(x)$ in the vicinity of θ_3^0 slows down (Fig. 22). Finally, in the field of PTI $\Phi(\theta_3^0) = \Phi_{\min}$, and according to (12.4) the region $\theta = \theta_3^0$ expands in an unbounded fashion. To avoid misunderstandings, we recall that in the model of isolated domain walls, studied here, the stable state of the system (the domains themselves) is assigned an infinite width. For this reason, the process of unbounded expansion of the region $\theta = \theta_3^0$ in reality merely means that at $\mathbf{H}^{(i)} = \mathbf{H}_P$ a domain of a "new" phase forms.

The change in the form of the distribution $\theta(x)$ accompanying a transition into the region of metastable states as well as the character of its further evolution have a simple physical interpretation. In the domain wall all states θ have a higher energy than the equilibrium states. The greater the difference $(\Phi(\theta) - \Phi_{\min})$ the more "quickly" the system strives to leave this state. Equation (12.3) is actually a mathematical formulation of this assertion. For this reason, the strongest change occurs in the region of values corresponding to maximum $\Phi(\theta)$. Conversely, at the point corresponding to a metastable state the energy of the system is lower than that of neighboring states. For this reason it is energetically advantageous for a large part of the wall to be in states close to the metastable state. As the phase-transition field is approached this tendency intensifies, a domain of a new phase forms in the field $\mathbf{H}_P$ and the remaining parts of the wall transform into new domain walls, separating domains consisting of the "old" and "new" phases. This means that the domain walls in the "old" phase serve as centers of nucleation of the "new" phase. The formation of domains from domain walls of the "old" phase was observed in Refs. 33, 34, and 48. Reference 126 is devoted to the detailed experimental study of this question.

We shall examine this question on the example of an orthorhombic, ellipsoidal ferromagnet, whose H_x, H_z diagram is shown in Fig. 11. In a magnetic field $\mathbf{H}\|$ (easy axis) for $H_z > H_0 = \beta M + 4\pi N_{zz} M$ 360°domain walls can exist in the sample. In the field $H_z = H_0$ the internal field becomes equal to the field of lability ($H_z^{(i)} = \beta M$) and expansion around the value $\theta = \pi$ (metastable state) starts in the 360° domain wall. As the field decreases this process develops right up to values of the external field $H_z = 4\pi N_{zz} M$, when the field in the sample becomes equal to the PTI field $H_z^{(i)} = 0$. Here the region with $\theta = \pi$ transforms into a domain, and the sections of the 360° domain wall $[0,\pi]$, $[\pi,2\pi]$ transform into two 180° domain walls, separating domains with the intermediate state $\theta = 0$, $\theta = \pi$. An analogous process also occurs in an inclined external field in the entire range of $\mathbf{H}$, for which $\mathbf{H}^{(i)}$ assumes values from the region of metastable states. In the $H_x H_z$ phase diagram the boundary separating the region of fields where the described evolution of 360° domain walls occurs is determined by the simultaneous solution of Eqs. (4.3) and Eq. (9.6), which gives the lability boundary. In Fig. 11 the region, where restructuring of 360° domain walls occurs, is marked by oblique cross hatching.

Thus we can draw the following conclusion: if spin configurations corresponding to a "new" phase of a spin reorientational transition are realized in the domain walls of a magnet, then in the range of the external field when $\mathbf{H}^{(i)}$ corresponds to the region of metastable states the restructuring examined here occurs in the domain walls. This mechanism of restructuring of domain walls was formulated in Refs. 32 and 117. The change in the 180° domain wall in the region of metastable states accompanying a spin-flop transition in an easy-axis antiferromagnet was studied in Refs. 108 and 118. The evolution of domain walls in the region of spontaneous spin-reorientational transitions in orthoferrites was studied in Refs. 97 and 119.

In Refs. 108 and 118 the unfortunate term "transitional domain structure" was employed to denote the region of metastable states in which restructuring of the domain walls occurs; this region was actually identified with the region of the IS, the idea of which for magnets was introduced in Ref. 12. This error was repeated in a number of other publications, including the monographs of Refs. 98 and 120. We must therefore make the following clarifying remarks.

In the region of metastability under the conditions stipulated above, the structure of a magnet changes in an insignificant part of the magnet (in the domain walls); in the process the magnetic state does not change over the main volume (in the domains). The formation of a thermodynamically stable domain structure (IS) is associated not with the existence of metastable states, but rather with the presence of energy degeneracy accompanying realization of the conditions for PTI in the magnet. A domain structure exists in the range of external fields where the screening action of the demagnetizing fields generated by the surface of the sample permits preserving within the sample the conditions for PTI $-\mathbf{H}^{(i)}=\mathbf{H}_P$. For this reason the region of existence of IS is determined by the shape of the magnet as well as by the condition for realization of PTI, and is completely independent of the character of the metastable states.

A characteristic feature of external-magnetic-field-induced PTI in magnets is that the region of coexistence of several phases is contiguous to regions of coexistence of a larger number of phases. For example, in an orthoferrite with smooth spin reorientation (see Sec. 11) the regions of PTI between two phases are contiguous to the region of PTI with four coexisting phases. If the IS of the low-phase transition in domain walls contains spin configurations corresponding to one of the coexisting phases of a multiphase PTI, then in the region of metastable states the structure of the domain walls undergoes changes analogous to those described above for kinetic domain walls.

In spite of the fact that the restructuring of domain walls preceding the formation of IS occurs in an insignificant volume of the magnet, this process can be studied, for example, based on the change in the character of resonance in the domain walls. As far as we know, such experiments have not yet been performed.

13. CONCLUSIONS

In this review the physical processes and basic assertions of the theory enabling a unified description of the magnetic properties of thermodynamically stable domain structures in magnets were presented. This theory is based on the following thesis: *a necessary condition for the formation of all thermodynamically stable domain structures in magnets is the existence of an external-field-induced PTI in the system.* States corresponding to competing phases of a given transition are realized in the domains.

The generality of the physical nature of domain structures opens up the possibility of using the well-developed methods of the theory of ferromagnetic domains to study the magnetic properties in the neighborhood of spin-reorientational transitions. On the other hand, the results of investigations in domains near spin-reorientational phase transitions can be employed to gain a deeper understanding of the behavior of domain structures in ferromagnets and magnets with spontanious magnetization.

A transition is now occurring from the period of discoveries of magnetic domains near spin-reorientational transitions to the systematic study of their properties. It is not surprising that at the first stage investigators were interested in those aspects of the behavior of new domains that distinguish the letter from the usual domains. As is clear from this paper, the unusual nature of the behavior of such domain structures lies not in their special nature, but rather in the specifics of phase transitions, with which the formation of such domains is linked. We hope that the theory developed in this paper enables purposeful studies of magnets with domain structure.

Finally, we want to indicate the limits of applicability of the theoretical methods, employed in this review, for describing domain structures.

It was assumed in the analysis of domain structures that thermodynamically equilibrium states are realized in the system, i.e., there is no coercivity and there are no hysteresis phenomena. To construct a theory of regular domain structures it was important that the inequalities $x_0 \ll D \ll L$ (2.3) hold. As calculations for a plate show, the inequality $D \ll L$ is always violated as the region of saturation is approached (see Sec. 7). Moreover, in sufficiently thin plates even at the center of the IS ($\tilde{\mathbf{H}}=0, \xi_1=\xi_2$) the equilibrium period can be comparable to the characteristic dimensions of the sample and even much larger than the sample. For $D \gtrsim L$ the nonuniform part of the magnetostatic energy is already significant in the entire volume of the magnet. This means that the condition $\mathbf{H}^{(i)}=\mathbf{H}_P$ no longer holds, and nonuniform states are realized in the domains. If in the region of existence of the IS the inequality (7.2) holds, then the effect of the nonuniformity on the formation of the internal states in domains can be neglected, and it can be assumed that $\mathbf{M}^{(k)}$, $L_\nu^{(k)}$ are determined, as before, from the system of equations (4.1). In any case the domain structure with these states can be regarded as a model structure, whose energy is higher than the true energy. For this reason the calculations of the transition fields on the basis of such an approximation can be regarded as a first step of a perturbation theory.

As the critical point of the PTI is approached the inequality $x_0 \ll D$ is violated, since the height of the potential barrier separating the equilibrium states as well as the difference of the magnetizations in separate phases approach zero. In the process the relative contribution of the magnetostatic energy to the energy of the system decreases without limit, and finally becomes comparable to the energies associated with the nonuniformities. In this region the "thin" wall approximation is inapplicable: the distribution of $\mathbf{M}^{(k)}$, $L_\nu^{(k)}$ is strongly nonuniform over the entire volume of the sample. The characteristics of the domain structure in the region of the critical points are studied by special theoretical methods.[75–80]

The systematic solution of the problem of domain structure with non-180° walls requires taking into account in the thermodynamic potential the elastic and strictional interaction energies.[121] Their contribution to the energy of the system equals, in order of magnitude,

$$\frac{\lambda^2 M_0^4}{c},$$

where c is the elastic modulus and λ is the magnetostriction constant. If

$$\frac{\lambda^2 M_0^2}{c\pi} \ll \left(\frac{\Delta M}{M_0}\right)^2,$$

then the magnetoelastic interactions can be neglected compared with magnetic-dipole interactions. Usually $\lambda \sim 1$, $\lambda^2 M_0^2/c\pi \sim 10^{-15}$. For this reason, the magnetostrictional interactions can play an appreciable role only for $\Delta M/M_0 \lesssim 10^{-2}$. The effect of the magnetostriction interaction on the region of the intermediate state of a magnet was studied in Ref. 63. It was shown that the magnetostrictional interaction leads to narrowing of the region of the IS, and under certain conditions can even lead to total blocking of the IS.

We shall briefly discuss the question of the character of the domain structure in nonellipsoidal samples. In an ellipsoid, everywhere where $D \ll R$, the uniformity of the internal field $\mathbf{H}^{(i)}=\mathbf{H}_P$ ensures that uniform internal states will be realized in separate domains $\mathbf{M}^{(k)}(\mathbf{H}_P)$, $L_\nu^{(k)}(\mathbf{H}_P)$. In samples of a different shape the internal field is nonuniform, so that as $\mathbf{H}_P$ varies over the volume of the magnet the internal states in the domains, and hence the equilibrium period and other geometric parameters, will also vary. In those sections of the magnet where $\mathbf{H}^{(i)} \neq \mathbf{H}_P$, a uniform state will be realized. Thus in nonellipsoidal samples the domain structure will have a complicated character: the regions of uniform magnetization can be contiguous to regions with different types of domain structures.

In spite of the fact that everywhere in this review we discussed a domain structure in magnets, all the chief fundamental results are also applicable to materials in which first-order phase transitions induced by an external electric field E occur as the electric polarization P varies (ferroelectrics, etc.). It should be noted that strictional effects are more important here than in magnets. In addition, unlike magnets, in the study of domain structures in ferroelectrics the effect of free electric charges must be taken into account.

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