METHODOLOGICAL NOTES

Physical mechanisms for the hydrodynamic beam-plasma instability

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A quantitative study is made of the physical mechanisms responsible for the development of a collective hydrodynamic instability of a charged-particle beam in an isotropic plasma. The corresponding growth rate is calculated through an analysis of the dynamics of the motion of the beam particles in the field of their radiation. The coherence of the beam particles is responsible for a substantial amplification of the collective field excited by these particles. This field forms coherent bunches by the Veksler-MacMillan self-phasing mechanism. This review is addressed to specialists in plasma physics and microwave electronics.

CONTENTS

1. INTRODUCTION

Theoretical research on collective beam-plasma interactions has its foundations in the classic studies by Akhiezer and Faĭnberg[1)] (Ref. 1) and Bohm and Gross[2] (see also Refs. 3 and 4). This research has now emerged as an independent direction in theoretical plasma physics: theoretical microwave plasma electronics. The fundamental results of this theory[2)] (see Refs. 4–34, for example) have served as the foundation for the development of some novel research methods and technical approaches and also for the development of new research installations, instruments, and technology. Nevertheless, the physical theory of beam-plasma instabilities cannot yet be regarded as complete, even for the linear stage of the instabilities. The reason is that in its initial stage of development this theory was dominated by a formalized approach, which limited the possibilities for identifying the physical mechanisms which are responsible for the exchange of energy between beam particles and the rf fields which they excite in a plasma. Consequently, these mechanisms have attracted research interest since the very beginning of the foundation of theoretical microwave electronics.[1,3–7] This interest subsequently was seen in an increase in the number of new publications on this subject and also in an improvement of their methodological level. Specifically, as time has elapsed, we have witnessed in these publications a gradual transition from the formulation of qualitative suggestions regarding the physical nature of these mechanisms to a quantitative reconstruction of the dynamics of the process by which the instability develops, on the basis of an analysis of the details of motion of and radiation from individual charges of the beam in the plasma. This tendency is seen most fully in Refs. 35–40, where the basic characteristics of hydrodynamic collective processes were established through an analysis of the field configuration and intensity of the spontaneous emission of moving charges[35–39] and also the dynamics of their motion in the resultant driving field[3)] (Refs. 35–40). The materials of the theoretical research in this direction have not previously been put in systematic form and generalized. One particular result of this situation is that even for the classical hydrodynamic instability of a monoenergetic beam in an isotropic plasma we still lack a comprehensive identification and interpretation of the reasons for the appearance of a threshold for this instability

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along the axis of perturbation wave numbers, the relationship between the thermal motion and the coherence of the elementary radiators, and the physical role played by collective effects in the frequency spectra of the excited waves. A detailed analysis of these questions shows that their resolution requires an expansion of the methodological base of the theory. The present review is basically an exposition of the methods and results of this analysis.

2. CLASSICAL LINEAR THEORY OF THE COLLECTIVE BEAM-PLASMA INTERACTION

2.1 Initial equations and analytic asymptotic expressions for the spectrum of spatially periodic one-dimensional perturbations

We assume that a charge- and current-neutralized beam of electrons with a spatially uniform equilibrium density N_b, which remains constant over time, is moving through a cold, homogeneous, isotropic, and weakly collisional plasma with a steady-state equilibrium density N_p. The time evolution of the amplitudes of small one-dimensional perturbations of this equilibrium state is described by a self-consistent linearized system of equations consisting of the Poisson equation for the field $(\tilde{E}_z)$ of the longitudinal plasma waves which corresponds to the perturbation of the beam and plasma charge density, $\tilde{\rho}(z,t)$,

$$\frac{\partial}{\partial z}\tilde{E}_z(z,\ t)=4\pi\tilde{\rho}(z,\ t)=4\pi e\int dv\tilde{f}(v,\ z,\ t), \qquad (2.1)$$

and a kinetic equation for the rf increment $[\tilde{f}(v,z,t)]$ induced by this field in the equilibrium velocity distribution of the beam and plasma particles, $f_0(v)$,

$$\frac{\partial\tilde{f}}{\partial t}+v\frac{\partial\tilde{f}}{\partial z}+\frac{e}{m}\tilde{E}_z\frac{\partial f_0(v)}{\partial v}=-\nu\tilde{f}, \qquad (2.1b)$$

where ν is the effective collision rate.

We restrict the analysis to spatially periodic, small-amplitude, one-dimensional perturbations characterized by a wave number $k=k_z$. The corresponding equation for the spectrum of natural waves of this system can be written as follows[8,11,13,18,24] for the simplest case of a cold plasma and a Maxwellian velocity distribution of the beam particles with a maximum at the point $v=V_b$:

$$1-\frac{\omega_p^2}{\omega(\omega+i\nu)}+\frac{1}{k^2a^2}\cdot\left(1+\frac{i\sqrt{\pi}\,\delta}{|k|\,v_T}\cdot W\left(\frac{\delta}{|kv_T|}\right)\right)=0, \qquad (2.2)$$

where

$$W(x)=e_0^{-x^2}\left(1+\frac{2i}{\sqrt{\pi}}\int_0^x dt\,e_0^{t^2}\right),$$

$$\delta=\omega-kV_b,\quad a=v_T/\omega_b,\quad e_0=2.71. \qquad (2.3)$$

Of primary interest here are the analytic asymptotic expressions for the solutions of this equation, $\omega(k)$, which correspond to the limiting case of beam densities which are relatively low (in comparison with the plasma density). In general, these asymptotic expressions do not depend directly on the reduced beam density N_b/N_p; instead, they depend on the dimensionless parameter $\mu=(N_bV_b^3/N_pv_T^3)$, which is determined by the relative intensity (N_b/N_p) and the thermal spread (v_T/V_b) of the beam particles. In particular, in the case $\mu\ll 1$ the complex beam-induced frequency shift $\delta=\delta'+i\delta''$ is proportional to the beam density and inversely proportional to the beam temperature:

$$\max\delta''_{\text{kin}}(k)=\frac{\sqrt{\pi}}{8e_0}\frac{N_b}{N_p}\frac{V_b^2}{v_T^2}\omega_p,$$
$$e_0=2.71\ \ldots. \qquad (2.4)$$

If the thermal spread is comparatively small, and the strong inequality $\mu\gg 1$ holds, the complex beam-induced frequency shift δ_{hd} is a nonlinear function of the beam current density and is substantially larger than (2.4) in this case:

$$\delta_{\text{hd}}(k)=\frac{\pm\omega_b}{[2|1-(kV_b/\omega_p)|]}\times 1,\quad 1\gg\frac{kV_b}{\omega_p}-1\gg\left(\frac{N_b}{N_p}\right)^{1/3},$$
$$\times i,\quad 1\gg 1-\frac{kV_b}{\omega_p}\gg\left(\frac{N_b}{N_p}\right)^{1/3}, \qquad (2.5a)$$

$$[\delta_{\text{hd}}(k)]_{\max}=\left(\frac{N_b}{2N_p}\right)^{1/3}e_0^{\pi i/3},\quad \left|1-\frac{kV_b}{\omega_p}\right|\ll\left(\frac{N_b}{N_p}\right)^{1/3}\ll 1. \qquad (2.5b)$$

The sketch in Fig. 1 shows the qualitative behavior of $\delta''_{\text{hd}}(k)$.

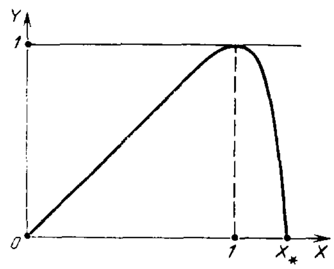

FIG. 1, Sketch of the dimensionless growth rate $Y\equiv\delta''_{\text{hd}}(k)/\text{Max}\ \delta''_{\text{hd}}(k)$ as a function of the dimensionless longitudinal wave number $X\equiv kV_b/\omega_p$.

2.2. Some features of the methodological apparatus of the classical theory

Expressions (2.4) and (2.5) reflect the essential features of the analytic results of the self-consistent linear theory of the spectra of the beam-plasma instability. The functional dependences of the growth rates of the beam-plasma instability on the external parameters of the beam and the plasma and also the perturbation wave number k which are described by these expressions are well known and have been presented in several places in the literature cited above on the theory of collective beam-plasma instabilities. Nevertheless, the physical nature of these relationships went unexplained for a long time. The essence of the questions of foremost interest from the scientific-methodological standpoint and from the applied standpoint can be reduced to two central questions:

Specifically how are the radiation conditions and the characteristics of the field of the spontaneous emission of individual charges related to the basic parameters of collective instabilities (the conditions under which they occur and their growth rates)?

What are the physical mechanisms which lead to a substantial increase in the energy loss of each beam particle in comparison with the energy loss of an individual radiator under the same conditions?

Attempts to identify the physical nature of this hydrodynamic bema-plasma instability have been undertaken repeatedly, starting with the pioneering study by Akhiezer and

Faĭnberg[1] (Refs. 3–8, 11–13, 21, 25, 26, 31, 35, and 40). Because of shortcomings of the methodological apparatus of the theory and the specific features of its historical development, however, the corresponding final conclusions have fairly often turned out to disagree to some extent with the underlying assumptions. From the very beginning of its development, the self-consistent theory of beam-plasma instabilities has been based on the ideas and methods of the dispersion theory of linear perturbations in flows of liquids and cases. For this reason, the models which have been used most widely for the corresponding plasma systems are models of multicomponent fluids. In such models, each of the components is characterized by Eulerian variables: the density and velocity of the particles, their charge, and their mass. The choice of Eulerian variables is of substantial help in simplifying the problem of incorporating self-consistent fields; the Maxwell's equations (and Poisson equation) for these fields are also written in Eulerian variables. The primary advantage of this approach is that it becomes a relatively simple and quick matter to determine the conditions for the occurrence of an instability and the behavior of the growth rates as functions of the external parameters of the system. The contributions from the individual beam particles to the quantitative characteristics of these conditions and growth rates are hidden. Naturally, there are only limited possibilities here for monitoring the contributions of spontaneous and induced emission of the beam particles to the course of the instability and also the degree of coherence of this emission. The only exceptional case is the limiting case in which the beam current density tends toward zero, in which case the strong inequality $\mu \ll 1$ holds. In this limiting case, the kinetic growth rate, (2.4), is determined unambiguously by the energy losses of the elementary beam charges in the plasma and by their velocity distribution. This approach, which starts with the method of Einstein coefficients, was first taken in (the omitted) Ref. 5 [sic] for a theoretical modeling of the magnetobremsstrahlung instabilities of low-intensity beams of oscillators which are not in phase and which are revolving in an external magnetic field. That study served as the starting point for the development of an independent direction in theoretical plasma physics,[9,11,19] whose pursuit also yielded, in particular, kinetic growth rate (2.4) for the Cherenkov instability of a stream of free (not oscillating) charged particles in a cold, isotropic plasma.[42] In terms of the very physical nature of the initial assumptions underlying the method of Einstein coefficients, however, that growth rate is applicable only for describing a limited class of kinetic instabilities, which correspond to vanishing flux densities of the charged particles. It cannot be used to model hydrodynamic instabilities which correspond to nonzero beam intensities.[5]

Summarizing, we should say that even a qualitative analysis of the methodological apparatus of the classical theory of hydrodynamic beam-plasma instabilities reveals that this apparatus is not adequate for dealing with the problem of determining the physical nature of these instabilities. In principle, the most comprehensive information about these questions can be found by solving the three-dimensional kinetic equation for the beam particles by the method of characteristics (in terms of Lagrange variables) and the three-dimensional Poisson equation for the field which these particles produce in the plasma, by a Green's-function method. That approach, however, is very complicated and has rarely been taken (see, for example, Refs. 13, 35, and 43–45). With regard to the nonlinear stage of the evolution of hydrodynamic instabilities, on the other hand, in which there is no way to avoid using Lagrange variables, the theoretical modeling demands a computational apparatus and some additional and strong simplifying assumptions which impose substantial limitations on the possibility of monitoring the mechanisms by which energy is exchanged between the beam particles and the fields which they excite.[26,28,30]

Because of the complexity of the time evolution of the hydrodynamic beam-plasma instability and the particular way in which the methodological base of the corresponding classical theory was established, the basic physical mechanisms responsible for the occurrence of these instabilities have thus remained hidden and have been studied to a lesser extent than is required by the present scientific and methodological level of the theory and its applications.

2.3. Comparative analysis of the results and conclusions of the classical theory.

The assertion which we made above, that the methodological apparatus of the classical theory of hydrodynamic instabilities is not adequate for identifying the physical nature of these instabilities, finds convincing support in a comparative analysis of the basic results and conclusions of the earlier studies in this direction. For greater clarity, we will take a more-detailed look at the physical content of the corresponding specific questions.

2.3.1. Threshold wave number. That there is a threshold in the behavior of the collective hydrodynamic growth rate $\delta''_{\rm hd}$ as a function of the perturbation wave number k follows from the solution of the general equation for the spectrum: A beam instability can occur (the growth rate can be nonzero and positive) only if the perturbation wavelength is sufficiently large (see Ref. 13 and also Fig. 1 of the present paper):

$$k < k_* = \frac{\omega_{\rm p}}{V_{\rm b}} \left[1 + \left(\frac{N_{\rm b}}{N_{\rm p}}\right)^{1/3}\right]^{3/2}. \tag{2.6a}$$

That there is a threshold in the behavior of the growth rate for the hydrodynamic instability, $\delta''_{\rm hd}$ as a function of the wave number k is well known, but it has yet to be satisfactorily explained. Attempts have been made, in particular, to explain this effect on the basis of a change in the nature of the Coulomb interaction forces between the beam particles and the plasma as the sign of the dynamic permittivity $\varepsilon^1_{\rm p} = {\rm Re}\,\varepsilon_{\rm p} = 1 - (\omega^2_{\rm p}/\omega^2)$ of the plasma changes (e.g., Refs. 33 and 46). It can be shown, however, that this explanation does not reveal the essence of the mechanism by which the beam particles interact with the field which they excite. To see this, we note that, formally, the function $\varepsilon^1_{\rm p}(\omega)$ changes sign at the point $\omega = \omega_{\rm p}$. It is negative specifically in the region in which the beam is unstable. However, it absolutely does not follow from this formal coincidence that when the frequency crosses this valuė the sign of the Coulomb interaction force of the beam particles changes. Facts established previously[35] provide evidence that there is no substantial effect of the Coulomb fields of the beam particles in the plasma on the course of the hydrodynamic instability at relatively low beam current densities, at which the strong inequalities $N_{\rm b} \ll N_{\rm p}$ and $\mu \gg 1$ hold. In fact, the bunching of the beam particles into coherently emitting bunches, which is required for the onset of this instability, is provided not by the Coulomb field of a beam (which is zero in a plasma) but by the fields of the longitudinal plasma waves which are excited by the beam in the plasma (polarization oscillations of the plasma, which are synchronized in space by the beam; Ref. 35 and Subsection 3.2 of the present paper). Second, this wave field does not depend on the sign of $\varepsilon_p^1(\omega)$. In turn, the latter determines (in the one-dimensional case which we are treating here) not the sign of the field $\widetilde{E}_z$ but the sign of the longitudinal gradient of this field, which is excited by the corresponding Fourier component of the beam charge density, $\tilde{\rho}_b(z,\omega)$:

$$\frac{d}{dz}\widetilde{E}_z(z,\omega)=\frac{4\pi\widetilde{\rho}_b(z,\omega)}{\varepsilon_p(\omega)}. \tag{2.6b}$$

The explanation of the physical nature of the boundaries of the beam stability region along the axis of the perturbation wave numbers in terms of a change in the nature of the Coulomb interaction forces of the beam particles thus cannot be accepted as convincing. On the other hand, it follows from the results of Ref. 35 that the crossing of the threshold value of the perturbation wave number changes the sign of the feedback effect in the system, and it is this change which is the primary reason for the threshold in the dependence of the growth rate δ''_{hd} on the wavelength of the initial perturbation.

2.3.2. Dependence of the maximum growth rate on the beam density. A fact of particular importance to an identification of the physical nature of this instability is that the functional dependence of the maximum growth rate on the beam density changes substantially as this density is raised. This change is demonstrated, in particular, by a comparison of growth rates (2.4) and (2.5a), (2.5b), which describe the kinetic and hydrodynamic asymptotic behavior, respectively, of this instability. Specifically, while at low beam densities ($m\ll 1$) the maximum growth rate in (2.4) is proportional to a dimensionless parameter, the reduced beam intensity μ (i.e., it is linear in the beam density N_b), at sufficiently high densities ($m\gg 1$) this growth rate increases in proportion to the cube root of the beam density (see expression (2.5b) and Fig. 2]. This substantial change in the nature of this dependence is evidence that at $m\simeq 1$ there is a switch from one mechanism for a collective interaction of the beam with the plasma to another. The question of just how the interaction processes differ on the two sides of the point $m=1$ (i.e., at $m\ll 1$ and $m\gg 1$), however, has not yet been studied comprehensively in the literature on the theory

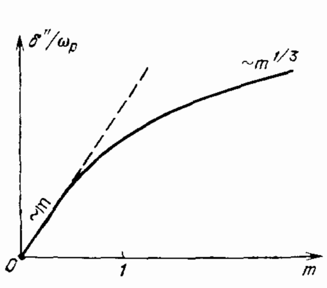

FIG. 2. Sketch of the growth rate $y\equiv\delta''_{max}/\omega_p$ as a function of the parameter $x=m$, the reduced beam intensity.

of beam-plasma instabilities. The most substantial initial information on this question comes from an analysis of the details of the procedure for calculating the kinetic growth rate from the energy loss of the beam particles in a plasma by the method of Einstein coefficients[5,9,11,19] and from an analysis of the physical content of the conditions for the transition from the kinetic asymptotic behavior to the hydrodynamic asymptotic behavior of the growth rate as the parameter m is increased. Specifically, one can clearly see in this procedure that the kinetic growth rate is proportional to the sum of the intensities of the induced emission of the individual beam charges[11]:

$$\delta''_{kin}(\mathbf{k})=\frac{\dot{N}_{\mathbf{k}}}{N_{\mathbf{k}}}=\frac{1}{(2\pi)^3}\int d_{\mathbf{p}}\,w_{\mathbf{p}}(\mathbf{k})\left(\hbar\mathbf{k}\cdot\frac{df_0}{d\mathbf{p}}\right); \tag{2.7}$$

$$w_{\mathbf{p}}(\mathbf{k})=\frac{1}{\hbar\omega(\mathbf{k})}\left(\mathbf{v},\ \frac{d\xi}{d\mathbf{r}}\right), \tag{2.7'}$$

$$N_{\mathbf{k}}=[8\pi\hbar\omega(\mathbf{k})]^{-1}\,|\,\mathbf{E}_{\mathbf{k}}|^2; \tag{2.7''}$$

where $\omega_p(\mathbf{k})$ is the probability for the spontaneous emission of a longitudinal photon with a momentum $\hbar\mathbf{k}$ by an electron with a momentum $\mathbf{p}$. This probability is equal to the ratio of the intensity for the spontaneous emission of this photon by an electron, on the one hand, to the energy of the photon, on the other. In addition, $N_{\mathbf{k}}$ is the number of photons which have a momentum $\hbar\mathbf{k}$; this number is proportional to the field intensity (the energy density). The function $f_0(\mathbf{p})$ is the distribution of beam electrons with respect to the momentum $\mathbf{p}$, and ε and $\mathbf{v}$ are the energy and velocity of an electron, respectively.

What is summed on the right side of (2.7) is not the fields but the intensities of the radiation from the beam particles. The radiation field itself is characterized in this case not by an amplitude and a phase but by the number of photons. It follows that under these conditions the induced emission of longitudinal plasma waves by beam particles is not coherent.[5,9] The reliability of the latter conclusion, which again does not follow directly from the known results (which we presented above) of a formal analysis of the two-fluid model, (2.4), is confirmed by an analysis of the physical content of the inequality $\delta''_{kin}(k)\ll kv_T$ which determines the range of values of the external parameters of the system in which the kinetic asymptotic behavior of growth rate (2.4) is applicable. The ratio $l_T=v_T/\delta''_{kin}$ thus gives us the distance over which beam particles whose initial coordinates differ by no more than half the length of the excited wave, $\lambda_p=V_b/\omega_p$, move apart from each other over the duration of the evolution of the instability $T_f=(\delta''_{kin})^{-1}$. It can thus be concluded that the inequality $\delta''_{kin}\ll kv_T$ is physically equivalent to the requirement that the thermal separation of the beam particles over this time ($l_T=v_T/\delta''$) must be substantially greater than the perturbation wavelength λ_p: $l_\Gamma\gg\lambda_p$ (Ref. 13). Since it is specifically this length which determines the maximum permissible dimensions of a coherently emitting bunch of beam particles (§3), it follows unambiguously from the inequality $\mu\ll 1$ that under the conditions under which the kinetic asymptotic behavior of the growth rate is applicable ($\delta''_{kin}\ll kv_T$) the beam particles cannot form coherently emitting bunches. It is this physical property of the radiation from a set of beam particles which explains the fact that kinetic growth rate (2.4) describes this collective interaction process only in the asymptotic limit in which the beam particle density approaches zero.

In light of the above arguments and estimates, we can formulate a suggestion regarding the role played by coherence effects in the onset of hydrodynamic instability of this system. Specifically, it follows from the condition for the applicability of the hydrodynamic asymptotic behavior of the spectrum $\omega(k)$ $(m = (D_\parallel / l_T)^3 \gg 1)$ that the thermal spreading of the beam particles, l_T, turns out to be small in comparison with the typical longitudinal dimensions of a bunch $(l_T \ll D_\parallel)$. This spreading is thus incapable of preventing coherence in the collective interaction of the beam with the plasma. If this coherence does indeed prevail, then the "turning on" of this effect as the beam intensity is raised, at $\mu \simeq 1$, might explain the change in the functional dependence of the growth rate on the beam current density (the parameter m). An explanation along this line, however, cannot be justified directly by the analysis which we presented above of the results of the formalized theory for the two-fluid model. The reason is that these results contain no information about the structure of the field which is excited by the individual beam particles in the plasma. A particular consequence of this situation is that one cannot determine directly from these results what the characteristic dimensions of a coherent bunch would be in the plane perpendicular to the velocity of the beam particles, and one cannot determine whether this coherence extends to a train of bunches (and what is the number of coherent bunches in such a case). In order to resolve all these questions, it is necessary to study the field configuration of the spontaneous radiation from a single moving charge and from a flow of such charges which is periodically modulated over space.

2.3.3. Mechanism for field intensification by a beam. The elementary effects of the emission of a field by beam particles are the only mechanisms by which kinetic energy of the beam is transferred to the field.[7,12,47] Accordingly, by promoting or obstructing the conditions under which these effects occur one can correspondingly intensify or weaken the processes of development of beam instabilities.[7,12] Of particular importance in solving the problem of controlling these instabilities is the circumstance that the number of these elementary effects is known to be limited.[7,12] Among these effects, in particular, are the (Vavilov-) Cherenkov effect, the normal and anomalous Doppler effects, transition radiation, and bremsstrahlung. In terms of the basic physics involved, the conditions required for the realization of the first three of these effects in the nonrelativistic classical (i.e., not quantum-mechanical) theory reduce to the requirement that the phase of the field remain constant in the proper frame of the moving charge. A differentiation of this phase with respect to time in the simplest case of one-dimensional motion yields the following relations among the frequency of the natural oscillations of the radiator (oscillator), Ω_0; its directed velocity $\mathbf{v}_0$; and the parameters (the frequency ω_0 and the wave vector $\mathbf{k}_0$) of the field which it radiates (Ref. 40):

$$\omega_0 - \mathbf{k}_0 \mathbf{V}_0 = s\Omega_0 \quad (s = 0, \pm 1). \tag{2.8}$$

The case $s = 0$ describes Cherenkov radiation for a free charge $(\Omega_0 = 0)$, while the cases $s = \pm 1$ correspond to the normal (+) and anomalous (−) Doppler effects.

The dispersion laws for the field radiated by the individual charges which are expressed by Eqs. (2.8) are the primary instruments for the diagnostics of the elementary mechanisms by which a field is intensified by beam particles in the classical theory of beam-plasma instabilities.[1,7,12,40] The characteristics of the fields radiated by the moving charges do not figure directly in the results of this theory (see Subsection 2.3 above). Accordingly, these particular mechanisms are identified in this theory solely on the basis of the dispersion law $\omega_0(k_0)$, which leads to a maximum of the growth rate of the corresponding instability at relatively low values of the beam current density. In particular, if the right side of the function $\Omega_0(k_0) = \omega_0(k_0) - k_0 V_0$ vanishes, an instability of this sort is regarded as a Cherenkov instability, while if the inequalities $\Omega_0(k_0) \gtrless 0$ hold the instabilities are regarded as those corresponding to the normal $(\Omega_0 > 0)$ and anomalous $(\Omega_0 < 0)$ Doppler effects. It is easy to show, however, that a criterion of this sort does not furnish an unambiguous answer to the question which was posed (identifying the mechanism for the intensification of a field by a beam) for beams of finite intensity. If we ignore the beam current density $(N_b \to 0)$ on the right side of (2.5b), we reach the conclusion that under these conditions there is a Cherenkov amplification of the field of longitudinal plasma waves by beam particles. For specifically this reason, and also because the conditions which would be required for the operation of alternative mechanisms for the spontaneous radiation from these particles are not satisfied here, this instability was identified as a Cherenkov instability a long time ago (Refs. 1 and 3; see also Refs. 4, 7, 8, and 12). On the other hand, when the thermal corrections are taken into account on the right side of (2.5b), the corresponding function $\Omega_0(k_0)$ goes negative:

$$\Omega_0(k_0) = \omega_0(k_0) - k_0 V_0 = -\omega_p \left(\frac{N_b}{16 N_p}\right)^{1/3}.$$

It was on this basis that it was concluded in Refs. 21, 31, and 40 that this instability results from an anomalous Doppler effect. We will show below (Subsection 3.1.3) that these two conclusions do not contradict each other, since they pertain to very different regions of values of the external parameters of the system.

We thus see that the results of the classical theory of the two-fluid model do not by themselves give us an adequate basis for unambiguously identifying the elementary mechanism for the intensification of a field by a beam in the course of a hydrodynamic instability of a monoenergetic beam in a "cold" and isotropic plasma. Under these conditions, a study of the structure of the field of the spontaneous radiation from beam particles and of the mechanisms for the inverse effect of this field on the beam becomes the basic tool for studying the physical processes which are responsible for the onset of hydrodynamic beam instabilities.

3. LAGRANGIAN DYNAMICS OF THE SPONTANEOUS EXCITATION AND INDUCED ABSORPTION OF THE FIELD OF LONGITUDINAL PLASMA WAVES BY BEAM PARTICLES

3.1. Configuration of the field of a charge in uniform rectilinear motion in a homogeneous and isotropic plasma

An analysis of the picture of the field which is excited by a charge in uniform rectilinear motion in a homogeneous and isotropic plasma is of particular interest to research on the coherence of the radiation from beam particles and on the mechanisms for the inverse effect of this radiation on the dynamics of the particles. Specifically, it is the structure of the spontaneous-emission field of a moving charge which determines not only the typical dimensions of a bunch of coherently emitting particles but also the number of coherently emitting bunches.[48,49] In turn, these parameters determine the resultant field which acts on each beam particle and thus the efficiency with which the particles are grouped by this field into coherently emitting bunches. A quantitative analysis of the intensity and structure of the spontaneous-emission field of an isolated charge in a plasma is thus a crucial element of an analysis of the entire set of physical processes which are responsible for the development of a hydrodynamic beam-plasma instability.

We thus consider a particle of charge q and mass m which is in uniform rectilinear motion along a path

$$\mathbf{R}_s(t) = \mathbf{i}X_s + \mathbf{j}Y_s + \mathbf{k}(Z_s + V_0 t) = \mathbf{r}_s + \mathbf{V}_0 t,$$

in a homogeneous and isotropic plasma. Here s is the index (number) of the particle, $\mathbf{r}_s$ is its radius vector at the time $t=0$, and $\mathbf{V}_0$ is its directed velocity, which is oriented along the Z axis of the Cartesian coordinate system.

Let us find the total field $E_{z_1}(\mathbf{r},t;\mathbf{r}_s)$ which is excited by this charge in the plasma, and let us calculate the force exerted on this charge by the field which it produces:

$$F_z = \frac{q}{2}\lim_{\varepsilon\to 0}[E_{z_1}(\mathbf{R}_s+\mathbf{k}\varepsilon;\ t,\ \mathbf{r}_s) + E_{z_1}(\mathbf{R}_s-\mathbf{k}\varepsilon;\ t,\ \mathbf{r}_s)]. \tag{3.1}$$

The picture of the field excited by this charge in the plasma is determined by Maxwell's equations with a given charge current

$$\mathbf{j}_z^{(s)}(\mathbf{r},\ t) = q\mathbf{V}_0\delta[\mathbf{r}-\mathbf{R}_s(t)]$$

as the driving force.

The corresponding amplitude of the total induced field E_{z_1} at observation point $\mathbf{r}$ at time t is determined by the expression

$$E_{z_1}(\mathbf{r},\ t;\ \mathbf{r}_s) = \frac{iq}{\pi}\gamma\int_0^\infty dk_\perp J_0(k_\perp\rho_s)k_\perp\int_{-\infty}^{+\infty}\omega\, d\omega[\beta^2-\varepsilon_p^{-1}(\omega)] \times\frac{\exp[i\omega(z-Z_s(t))/V_0]}{\omega^2+\gamma^2[k_\perp^2V_0^2-\beta^2\omega^2(1-\varepsilon_p(\omega))]}, \tag{3.2}$$

$$\beta=\frac{V_0}{c},\quad \gamma=(1-\beta^2)^{-1/2},\ \varepsilon_p(\omega)=1-\frac{\omega_p^2}{\omega(\omega+i\nu)},$$
$$Z_s(t) = z_s + V_0 t,\quad \rho_s = [(x-x_s)^2+(y-y_s)^2]^{1/2}.$$

It is easy to see that at each fixed value of the transverse wave number $k_\perp$ the integrand in the integral over the frequency has as singularities only simple poles, at the points

$$\omega_\pm^{(\mathrm{Coul})} = \pm i\gamma(k_\perp^2V_0^2+\omega_p^2\beta^2)^{1/2},$$
$$\omega_\pm^{(\mathrm{pl})} = \pm\omega_p - \frac{i}{2}\nu,\ \nu\ll\omega_p. \tag{3.3}$$

The first pair of poles here gives the Coulomb field of the charge in the plasma, and the second gives the field of the longitudinal plasma waves excited by this charge. We will consider these fields separately.

3.1.1. Coulomb field. The configuration of this field is given by

$$E_{z1}^{(\mathrm{Coul})}(\mathbf{r},t;\mathbf{r}_s) = qV_0^2\int_0^\infty dk_\perp k_\perp^3 J_0(k_\perp\rho_s)[k_\perp^2V_0^2+\omega_p^2]^{-1} \times\left\{\exp\left[-|z-Z_s(t)|\gamma\left(k_\perp^2+\frac{\omega_p^2}{c^2}\right)^{1/2}\right]\right\} \times\ \mathrm{sgn}(z-Z_s(t)). \tag{3.4}$$

A point which will be important to the discussion below is that this expression is antisymmetric with respect to the plane $Z_s(t)$, in which the charge which excites this field is situated. This property means that the field acting on the charge, $E_{z_1}^{\mathrm{Coul}}$, vanishes at the position of the charge. This result is completely understandable from the physical standpoint: As in the case of vacuum ($\omega_p\to 0$), the Coulomb field of the charge itself can neither retard nor accelerate this charge. A second consequence of the asymmetry of the Coulomb field of the charge, and a consequence which is no less important than the first for the applications discussed below, is that the resultant field acting on each individual charge of an unmodulated and sufficiently dense beam vanishes: For each of the neighbors of a given charge, one can always find another neighbor which is positioned antisymmetrically (as a mirror image) with respect to the first neighbor, so the resultant field of the two neighbors vanishes.

3.1.2. Field of longitudinal plasma waves. **In contrast with the Coulomb field, this field is asymmetric with respect to the $z=Z_s(t)$ plane:**

$$E_{z1}^{(\mathrm{pl})}(\mathbf{r},t;\mathbf{r}_s) = 0,\quad z\geqslant Z_s(t),$$
$$= -\frac{2q\omega_p^2}{V_0^2}K_0\left(\frac{\omega_p\rho_s}{V_0}\right)\cos\Phi_s(z,\ t)\exp\left[-\frac{\nu}{2\omega_p}\Phi_s(z,\ t)\right],\quad z\leqslant Z_s(t), \tag{3.5}$$

where $K_0(x)$ is a modified Bessel function, and $\Phi_s(z,t)=\omega_p[z-Z_s(t)]/V_0$.

As would be expected on the basis of the physical meaning of this problem and the causality principle (which is reflected in the radiation conditions[4)]), this field vanishes in front of the charge, and at low collision rates ($\nu\ll\omega_p$) it takes the form of a quasimonochromatic wave which is traveling behind the charge at a phase velocity which is precisely equal to the velocity of the charge, v_0. This aspect of the structure of the longitudinal plasma waves excited by the charge has a transparent physical meaning: In the spatially homogeneous plasma which we are considering here, the natural frequencies of each point in the plasma are equal to each other. The field of these waves thus turns out to be monochromatic in frequency in the limit $\nu\to +0$. In the laboratory coordinate system, the times at which the wave excitation begins to move away from one plane, $z=Z_s(t)$, to another along with the charge, and the initial phases of the waves in each of these planes are fixed: They are equal to π for the field $E_{z1}^{(\mathrm{pl})}$ (a retarding field). It is thus the total field of the longitudinal waves which has the form of a wave which is traveling behind the charge at the velocity of the charge itself. As it excites longitudinal waves in the plasma, the moving charge of course loses kinetic energy. The resultant field of these waves thus differs from the Coulomb field in that it has a nonzero average value in the plane $z=Z_s(t)$, and it is this value which determines the retarding force which acts on the charge, (3.1):

$$F_z^{(\mathrm{pl})} = -\frac{q^2\omega_p^2}{V_0^2}\Lambda;\quad \Lambda = \ln\frac{V_0}{\omega_p\rho_{\min}}, \tag{3.5'}$$

where Λ is the Coulomb logarithm.

With increasing distance (ρ_s) from the path of the charge, the field of the longitudinal plasma waves, (3.5), falls off exponentially with an argument $\kappa_\perp = k_*$ which is exactly equal to the reciprocal of the longitudinal wave number: $\kappa_\perp = D_\parallel^{-1}$. These parameters determine the characteristic longitudinal ($\parallel$) and transverse ($\perp$) dimensions of the region in which the given charge can excite longitudinal plasma waves in a process which is coherent with the neighbors of the charge: $D_\parallel = D_\perp = \kappa_\perp^{-1} = \lambda_p = V_0/\omega_p$. For the field of longitudinal plasma waves, this coherence not only prevails in the vicinity of the moving charge but also propagates along its path, over the field damping length $L_\parallel = 2\pi V_0/\nu$: Each charge which is moving a distance l_n which is equal to the length of a longitudinal wave, $\lambda_p = 2\pi V_0/\omega_p$ ($l_n = n\lambda_p$, where n is an integer), behind the charge under consideration will excite in the plasma the same field as that which is excited by the preceding charge.[48,49]

By examining the picture of the field of the longitudinal waves which are excited by a moving charge in an isotropic "cold" plasma we have thus shown that the typical dimensions of the region in which this field is coherent (the typical dimensions of a coherently emitting bunch) are determined unambiguously by the wavelength of these waves and that the number ($\mathscr{M}$) of bunches which are exciting the plasma in a fashion coherent with the given bunch is determined by the ratio of the plasma frequency ω_p to the collision rate ν: $\max\mathscr{M} = 2\omega_p/\nu$.

3.1.3. Field intensification mechanism. This mechanism is essentially a spontaneous Cherenkov excitation of a field of natural longitudinal waves of the plasma. The governing role played by longitudinal plasma waves here can be seen in the fact that in the limit $\nu \to +0$ this field is represented by the residue at the point $\omega = \omega_p$ and is therefore monochromatic in frequency. That the collective interaction of the charge with the plasma is of a Cherenkov nature follows unambiguously from the circumstance that the phase velocity of any Fourier component of this field and of the total sum of these components is exactly equal to the velocity of the charge, V_0: $V_{\mathrm{ph}}^{(0)} = \omega_p(k)/k = V_0$. On the other hand, none of the other elementary mechanisms for the excitation of a field which we listed above could operate under these conditions. Specifically, transverse waves in the plasma in the transparency region of the plasma can not be excited by the Cherenkov mechanism, since they have phase velocities which are greater than the velocity of light, c ($V_\Phi^{(\mathrm{tr})} = c/\varepsilon_p'^{1/2}, \varepsilon_p' > 0$). For the normal and anomalous Doppler effects to occur, the particle would have to be an oscillator. Bremsstrahlung would require a nonuniform or nonrectilinear motion of the charge in external force fields. Transition radiation would require that the medium be inhomogeneous. We thus see that the Cherenkov excitation of longitudinal waves of the homogeneous plasma is, under the conditions assumed here, the only mechanism which could be responsible for the transfer of kinetic energy from the beam particles to the field.

We should stress that the conclusion which we have just formulated regarding the Cherenkov interaction of a low-density beam and a plasma under these conditions does not contradict a conclusion, which we mentioned above (Subsection 2.3.3), which was reached in Refs. 21, 31, and 40: that the anomalous Doppler effect plays a governing role in the collective interaction of intense electron beams with isotropic plasmas. The explanation is that the oscillatory motion of the beam particles in their self-field, which is characteristic of the latter effect, is important for beams of finite intensity in that range of their parameter values in which the resultant frequency of these oscillations, $\Omega_0(k)$, is high in comparison with the growth rate: $\Omega_0(k) \gg \delta''_{\mathrm{hd}}$. The latter requirement is satisfied not only in intense relativistic electron beams[40] but also in that finite neighborhood of the threshold wave number k_* in which this growth rate vanishes [see (2.6) and Fig. 1)]. It is this limiting case which was examined in Ref. 21 and in the monograph by Nezlin.[31] With regard to the resonant point $\omega_M = \omega_p$ at which the growth rate $\delta''_{\mathrm{hd}}(\omega_M)$ reaches its maximum, which is given by (2.5b), we note that the inequality $\Omega_0 \gg \delta''_{\mathrm{hd}}$ does not hold at this point: According to this formula, the maximum growth rate $\mathrm{Max}\,\delta''_{\mathrm{hd}}$ is larger by a factor of $\sqrt{3}$ than the beam frequency shift $\Omega_0(\omega_p)$. It follows that over the rise time of the instability near the resonant point $\omega_M = \omega_p$ there is not enough time for the oscillatory nature of the motion of the beam particles in their self-field, described by Eq. (2.8'), to be manifested.

In summary, the primary reason for the apparent discrepancy between the conclusions of Refs. 1, 3, 4, 7, 12, and 25, on the one hand, and Refs. 21, 31, and 40, on the other, is a difference in initial assumptions. Specifically, the authors in the first group of papers considered low-intensity beams and perturbation wavelengths corresponding to a neighborhood of the maximum of the growth rate, while the authors of Refs. 21, 31, and 40 considered the neighborhood of the threshold wave number k_* ($k_* - k \ll \mathrm{Max}[\delta''_{\mathrm{hd}}/V_0]$; see Refs. 21 and 31) and intense relativistic beams, for which the growth rates are small in comparison with the typical frequencies of collective longitudinal oscillations of the beam particles.[40]

3.2. Field of a sinusoidally modulated beam

Knowing the picture of the field of an individual charge, we can establish all the basic characteristics of the field which is excited in a plasma by a beam of moving charges whose density is sinusoidally modulated. For this purpose we consider a beam of identical charged particles which is of uniform density. Each particle has a charge q and a mass m and is moving in the plasma at a velocity V_0. The total field produced by this system of charges at a fixed point in the plasma with coordinates $\mathbf{r}$ at the time t is in general equal to the sum of the fields of the individual charges:

$$E_z^{(\mathrm{tot})}(\mathbf{r}, t) = \sum_s E_{zl}^{(\mathrm{pl})}(\mathbf{r}, t; \mathbf{r}_s). \tag{3.6}$$

If the beam particle density is sufficiently high, and the strong inequality $N_b\lambda_p^3 \gg 1$ holds (physically, this inequality means that the relative contribution of shot noise is small), we can switch from a summation to an integration on the right side of the latter equation, allowing for the nonuniform distribution of the charges along the coordinates $\mathbf{r}_s$ in their rest frame. For this purpose we express the number (dQ_s) of identical beam particles around the point $\mathbf{r}_s$ in

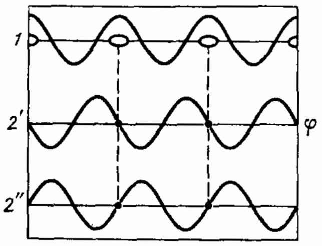

FIG. 3. Sketch of the mutual phasing of the perturbations of (1) the beam density $\tilde{n}(\varphi)$, and of the corresponding collective fields ${}^{\pm}\tilde{E}_z(\varphi)$, under the condition (2') $kV_0<\omega_p$ and $\pm\tilde{E}_z(\varphi)$ under the condition (2'') $kV_0<\omega_p$. The ovals at the maxima of the beam particle density are schematic representations of bunches.

terms of the density of these particles at the point $n_b(\mathbf{r}_s)$:

$$dQ_s(\mathbf{r}_s) = n_b(\mathbf{r}_s)\, d\mathbf{r}_s. \tag{3.6'}$$

We incorporate the sinusoidal nature of the modulation of the beam in velocity and density in our problem by setting (Fig. 3)

$$\begin{aligned} V_b(\mathbf{r}_s) &= V_0(1+g\cos kz_s),\\ n_b(\mathbf{r}_s) &= N_0(1+h\sin kz_s); \end{aligned} \tag{3.6''}$$

here h and g are the depths of the density and velocity modulation of the beam, respectively, $\lambda = 2\pi/k$ is the spatial period of the modulation, and N_0 and V_0 are the equilibrium (unperturbed) density and velocity of the beam particles.

Carrying out the integration on the right side of (3.6) over all the initial coordinates of the particles, using the field of an individual charge, (3.5), and the weight coefficients (3.6') and (3.6''), we finally find the following expression for the total field of one-dimensional longitudinal plasma waves which are excited by a slightly modulated beam of low density ($h \ll 1$; $N_0 \ll N_p$):

$$E_z^{(\mathrm{tot})}(z,\ t) = \frac{2\pi q h N_0 V_0}{\omega_p-\omega_M}\cos(kz-\omega_M t), \tag{3.7}$$
$$\nu \ll \delta''_{hd} \ll |\omega_p-\omega_M| \ll \omega_p;\quad \omega_M = kV_0.$$

Only the field of the longitudinal waves, (3.5), makes a nonvanishing contribution to the right side of the latter expression; the amplitude of the resultant Coulomb field of the beam in the plasma is zero as a result of the mutual coherent cancellation of the vacuum Coulomb field of the beam with its "image" in the plasma (in a plasma screen; see the Appendix).

Comparing the field of the modulated beam, (3.7), with the field of an individual charge, (3.5), we see the following: The frequency of the field excited by a beam of this sort is equal to the frequency at which it is modulated in the laboratory frame of reference; the amplitude of this field does not depend on the coordinate in the plane $z = \mathrm{const}$; and this amplitude is generally much greater than the amplitude of the field of an individual charge and increases as the beam modulation frequency approaches the plasma frequency. Physically, all these characteristics of the field of the spontaneous emission of a modulated beam in a plasma can be explained not only qualitatively but even quantitatively in terms of effects of the coherence of the radiation from the beam particles, both within the bunches formed by the modulating signal (3.6'') and among these bunches.

In the absence of beam modulation ($h \to 0$), the fields of the longitudinal waves of two elementary charges, one lagging behind the other by a distance equal to half the length of a plasma wave, add together coherently and out of phase, so their resultant field is zero. It is for this reason that the resultant field of the longitudinal waves excited by an unmodulated beam in a plasma is identically zero. The grouping of the particles into bunches which is described by (3.6'') disrupts this mutual interference cancellation of the fields of the elementary charges and produces a nonzero resultant field (3.7), which is proportional to the depth (h) of the density modulation of the beam, to the beam density N_0, and to the charge of the beam particle, q. On the other hand, this grouping into bunches imposes on this field a frequency ω_M which is determined unambiguously by the spatial period of the beam modulation. This is why the frequency of the beam field, (3.7), differs from the frequency of the field of an individual charge, (3.5). With regard to the relative amplitude of the beam field given by (3.7), we note that even if the perturbations of the beam density are small ($hN_0\lambda_p^3 \gg 1$) this amplitude will be substantially greater than the amplitude of the field of an individual charge, (3.5), specifically because of the coherent summation of the fields excited by the individual charges. Physically, the coherence effect is incorporated in and described by the procedure outlined above of integrating field (3.5) over the entire volume of the initial Lagrangian coordinates of the beam particles. This procedure might be thought of as being carried out in two steps: an integration over a cylindrical shell with a thickness equal to the spatial period of the beam modulation, $\lambda = 2\pi/k$, and a summation over the set of all these shells in succession ahead of the elementary charge under consideration. The first of these steps gives a quantitative description of the coherent summation of the fields excited by all the elementary charges which make up one of the bunches in the periodic train. The corresponding coherence coefficient C turns out to be equal to the ratio of the amplitude (A_{bch}) of the field produced by this bunch, (3.7), to the amplitude of the field of an individual charge, (3.5):

$$C = A_{bch}\left(\frac{q\omega_p^2}{V_0^2}\Lambda\right)^{-1} = (hN_0)\,\pi\lambda_p^3\Lambda^{-1}. \tag{3.7'}$$

The first factor on the right side of this expression describes the perturbation of the beam density (the bunch density), while the second describes the volume of this bunch; their product gives us the number of particles in a bunch. The factor Λ describes the Coulomb logarithm, which determines the energy loss and the field of an individual charge. The second step incorporates the coherent summation of the fields excited by the periodic train of beam bunches[47,48] and contributes the following:

$$\mathcal{M} = \frac{\omega_p}{|\omega_p-\omega_M|},\quad \nu \ll \delta''_{hd} \ll |\omega_p-\omega_M| \ll \omega_p. \tag{3.7''}$$

Physically, the magnitude of the right side of the last expression can be explained on the basis of a disruption of the phasing between the natural waves of the plasma (with the frequency ω_p) and the force driving them (the beam bunches, which are following periodically with a frequency ω_M): Under the condition $\omega_p \gg |\omega_p - \omega_M|$, over a time equal to $\mathcal{M}$ periods of these waves, their phase difference increases by an amount of the order of $\pi/2$ (in magnitude).

The resultant amplification of the field as a result of the coherent summation of the fields of the elementary radiators within each bunch (C) and among bunches ($\mathscr{M}$) is equal to the product of the intensities of these two effects: $K_{\rm tot} = C\mathscr{M}$.

We have thus shown that in the case under consideration here, of a nonresonant hydrodynamic instability, the coherent summation of the fields of elementary Cherenkov radiators, which the beam particles constitute, results in a substantial intensification of the resultant field produced by the beam of charged particles in the plasma (the intensification is with respect to the field of the longitudinal waves of an individual charge).

3.3. Inverse effect of the field on the beam

Up to this point we have been discussing only the field of the spontaneous emission[5)] from the beam particles for a given motion of these particles. We have essentially ignored the inverse effect of this field on the motion of the beam particles, i.e., the induced absorption of the energy of this field by the beam. In this subsection we consider the induced interaction of a modulated beam with the field which it excites, and we show that the resultant growth rate due to this effect is precisely equal to the growth rate calculated on the basis of a hydrodynamic analysis of the two-fluid model.

Since the field of the modulated beam described by (3.7) is determined in terms of Eulerian variables, we will use these variables to describe the motion of the beam particles below; i.e., we use the hydrodynamic velocity $v(z,t) = V_0 + \Delta v(z,t)$ and the corresponding density $n(z,t) = N_0 + \Delta n(z,t)$:

$$\begin{aligned} v(z,t) &= V_0[1 + g(t)\cos(kz - \omega_{\rm M}t)]; \\ n(z,t) &= N_0[1 + h(t)\sin(kz - \omega_{\rm M}t)]. \end{aligned} \tag{3.8}$$

Here we have introduced dimensionless depths of the density and velocity modulation of the beam, $h(t)$ and $g(t)$, respectively, which depend on the Eulerian time t.

Substituting (3.8) into the equation of motion of the beam particles in resultant field (3.7),

$$m\hat{D}\Delta v(z,t) = qE_z^{(\rm tot)}(z,t), \quad \hat{D} = \frac{\partial}{\partial t} + V_0\frac{\partial}{\partial z}, \tag{3.8'}$$

and into the continuity equation

$$\hat{D}\,\Delta n(z,t) + N_0\frac{\partial}{\partial z}\Delta v(z,t) = 0, \tag{3.8''}$$

we find the following system of two first-order, linear, ordinary differential equations for the dimensionless amplitudes $g(t)$ and $h(t)$:

$$2(\omega_{\rm p} - \omega_{\rm M})\dot{g} = \omega_{\rm b}^2 h, \tag{3.9}$$

$$\dot{h} = \omega_{\rm M} g, \quad \omega_{\rm b}^2 = \frac{4\pi N_0 q^2}{m}. \tag{3.9'}$$

From the condition that this system should have no trivial (vanishing) solutions we unambiguously find the analytic asymptotic expressions for the corresponding growth rates:

$$(\delta''_{\rm hd})_{\pm} = \frac{\pm\omega_{\rm b}}{\{2[1 - (\omega_{\rm M}/\omega_{\rm p})]\}^{1/2}}, \quad \begin{array}{c}\omega_{\rm M} < \omega_{\rm p}, \\ \omega_{\rm p} \gg (\omega_{\rm p} - \omega_{\rm M}) \gg \omega_{\rm p}\left(\dfrac{N_0}{N_{\rm p}}\right)^{1/3}.\end{array} \tag{3.9''}$$

It is easy to see that these asymptotic expressions are precisely the same as the corresponding results of an analysis of the hydrodynamic model in the limiting case $v \ll \delta''_{\rm hd} \ll (\omega_{\rm p} - \omega_{\rm M}) \ll \omega_{\rm p}$ [see (2.5a)]. Furthermore, incorporating the circumstance that the number of coherent bunches near the resonant point ($\omega_{\rm M} = \omega_{\rm p}$) is finite in (3.9″), we find a resonance growth rate (2.5b). The minimum value of the magnitude of the frequency difference ($\omega_{\rm p} - \omega_{\rm M}$) is determined by the magnitude of growth rate $\delta''_{\rm hd}$, which in turn determines the resonance number of coherent bunches, $\mathscr{M}_{\rm res}$: $\mathrm{Min}[|\omega_{\rm p} - \omega_{\rm M}|] = |\delta_{\rm res}| \approx \mathscr{M}_{\rm res}^{-1}\omega_{\rm p}$. Substituting these relations into the right side of (3.9″), we find an estimate of the resonance growth rate which is precisely the same as the corresponding classical result (2.5b), in terms of its dependence on the external parameters: $(\delta''_{\rm hd\,res}) = \sqrt{3}\,[\omega_{\rm b}^2\omega_{\rm p}/16]^{1/3}$.

The procedure described above for summing the fields of the spontaneous emission from the elementary radiators of a beam[35–39] and for dealing with the inverse effect of the resultant field on the beam (the induced absorption of the resultant field by the beam[6)]; Refs. 35–40) is thus successful in incorporating and describing quantitatively the basic physical processes which are responsible for the collective beam-plasma interaction under conditions such that the thermal motion of the beam particles is inconsequential.

3.4. Mechanism for the grouping of beam particles into coherent bunches

We have explained the elementary mechanism for the intensification of a field by a beam (Subsection 3.1), and we have established the coherence of the spontaneous emission from the bunches of beam particles which are formed by a modulating signal (Subsection 3.2). In the present subsection we wish to determine the physical nature of the appearance of a boundary on the instability region along the wave-number axis. We will show that the answer to this question comes from an analysis of the mechanism by which the beam particles are grouped into coherently emitting bunches. It is this grouping process which is, as we showed above, a necessary condition for the onset of hydrodynamic beam instability: It provides the intensification of the field emitted by each charge through a coherent summation of the fields of the particles forming the coherently emitting bunch. This grouping process is essentially the key element of the mechanism for the inverse effect of the field on the motion of the beam particles: With increasing depth of the density modulation of the beam (h), the amplitude of the field emitted by the beam (3.7), increases, and this field causes a more intense grouping of the beam particles into coherent bunches (i.e., increases the modulation depth h), according to (3.9″). The qualitative physical arguments presented above simply explain the governing role played by the mutual promotion of the spontaneous-emission process and the process of the induced absorption of the field by the beam particles—it does not reveal the essence of the mechanism for this inverse effect of the field on the beam. Accordingly, this explanation does not, in particular, answer the question which we posed above: Why does this grouping process occur (why is the feedback in the system positive) only at sufficiently large wavelengths ($\lambda > \lambda_{\rm p} = 2\pi V_0/\omega_{\rm p}$)? We will show below that this threshold in the dependence of growth rate (2.5) on the perturbation wavelength stems from the discontinuous dependence of the sign of the effective feedback in the system

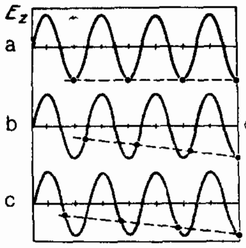

FIG. 4. Sketch of the relative orientation of (1) the exciting bunch, and (2,3,4,...) the bunches which it focuses. a—$k_M = \omega_p/V_0$; b—$k_M > \omega_p/V_0$; c—$k_M < \omega_p/V_0$.

on the ratio of this wavelength to the plasma wavelength[35] $\lambda_p = 2\pi V_0/\omega_p$.

This dependence can be explained qualitatively and quantitatively in terms of the Veksler-MacMillan self-phasing mechanism.[51-55]

For a qualitative illustration of this dependence, we consider the effect of the field of the spontaneous emission of an individual point bunch on the stability of the longitudinal (phase) motion of the following bunches. It can be seen from Fig. 4 that under the strict equality $\lambda = \lambda_p$ each following test bunch falls precisely at the boundary between the regions of focusing and defocusing phases of the field of longitudinal waves of the plasma which is excited by the emitting bunch: The squares of the local phase oscillation frequencies

$$\Omega_\Phi^2 = -\frac{q}{m}\frac{\partial E_z^{(\mathrm{tot})}}{\partial z} \tag{3.10}$$

vanish at this point.

If, on the other hand, the equality $\lambda = \lambda_p$ does not hold, each following test point bunch falls either in a region of defocusing phases of this field $\lambda < \lambda_p$, $(\Omega_\Phi^2 < 0)$ or in a region of focusing phases $(\lambda > \lambda_p, \Omega_\Phi^2 > 0)$. It is this distinction which is responsible for the particular dependence of the sign of the feedback on the sign of the difference $(k - k_p)$: The grouping of the beam particles into coherent bunches which is required for the onset of an instability is possible only in the case of long waves.[35]

For a more comprehensive and more rigorous quantitative explanation of the effect (the threshold in the dependence of the growth rate on the perturbation wave number), we first note that in a first approximation in the small parameter of the problem $(\delta_{\mathrm{hd}}^{\parallel}[\omega_M|\varepsilon_p'(\omega_M)|]^{-1} \ll 1)$ the phase velocity of the emission field is precisely equal to the equilibrium velocity of the beam particles:

$$V_\Phi = \frac{\mathrm{Re}\,\omega(k)}{k} = V_0 + \frac{\delta_{\mathrm{hd}}'(k)}{k} = V_0\left[1 + O\left(\left(\frac{\omega_b}{\omega_M\varepsilon_p'(\omega_M)}\right)^2\right)\right].$$

In this case the equilibrium (synchronized) phases of the field, φ_s, in which the charged particles can move in synchronism with the wave, are multiples of an odd number of quarter-wavelengths[53-55] (Fig. 5):

$$\varphi_{sp}^{(\pm)} = kZ_{sp}^{(\pm)}(t) - \omega(k)\,t = \frac{\pi}{2}(4p \pm 1)\ (p = 0, \pm 1, \pm 2, \ldots). \tag{3.10'}$$

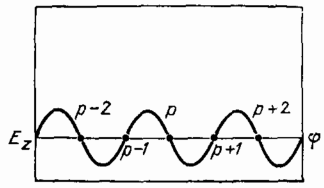

FIG. 5. Positions of synchronized phases under the condition $V_0 = V_{ph}$.

Physically, the points $\varphi_{sp}^{(+)}$ correspond to minima of the potential of wave field (3.7), while the points $\varphi_{sp}^{(-)}$ correspond to maxima. In general, only one of the two values of the synchronous phase on each field period (i.e., for a given p) is stable.[53-55] The stability condition is that the sign of (3.10), the square frequency of the phase oscillations of particles which deviate slightly from a synchronous particle, be positive.[26,53-55] Substituting field (3.7) into the right side of this expression, we easily see the following. The phases $\varphi_{sp}^{(+)}$ which correspond to those maxima of the beam density which are determined by the beam modulation law [see (3.8″)] are stable only for perturbations in the long-wavelength region $(\omega_M = E_z = kV_0 < \omega_p)$ (Ref. 35):

$$\Omega_\Phi^2(kz - \omega t = \psi_{sp}^{(\pm)}) = \pm\frac{\omega_b^2\omega_M}{2(\omega_p - \omega_M)}. \tag{3.10''}$$

We thus see that the existence of a boundary on the beam stability region along the axis of perturbation wave numbers is explained unambiguously by a specific feature of the Veksler-MacMillan self-phasing mechanism, which provides a grouping of the beam particles into coherent bunches by the field which they emit only in the case of long waves.

3.5. Mechanism for energy transfer from the beam to the field

The Veksler-MacMillan self-phasing mechanism, which forms the coherent bunches, does not by itself explain the transfer of kinetic energy from beam particles to the field which is required for the onset of instability. It is directly in the stable synchronous phase $\varphi_{sp}^{(1)}$ of the field (3.7) that the amplitude of the field which retards a bunch is zero. As they are grouped in this phase at strict synchronism of the wave with the beam $(V_{ph} = V_0)$, the particles leading the wave are slowed by the excited field, while the lagging particles are accelerated. As a result, in the linear stage (at small field amplitudes and at small displacements of the beam particles in this field) there is no transfer of energy from the beam to the field. For such a transfer to occur, the bunch would have to shift into the region of retarding phases of the field, leading the wave in the process, during the onset of instability.[26] It is easy to show that this phase slippage of a bunch is provided automatically as a result of a collective effect, a decrease in the phase velocity of the wave excited by the beam with respect to the beam velocity. Specifically, it follows from (3.7″′) and (2.5″) that a lowering of this type occurs throughout the region of unstable values of the perturbation wave numbers. The sign of the relative phase shift of the bunch, Δ, over the instability rise time determined by this effect, $T_f = (\delta_{\mathrm{hd}}^{\perp})^{-1}$, is positive:

$$\Delta = k(V_0 - V_\Phi(k))\,T_f = \frac{\Omega_0(k)}{\delta_{\mathrm{hd}}''(k)} > 0. \tag{3.11a}$$

This result means that the bunch leads the wave, mov-

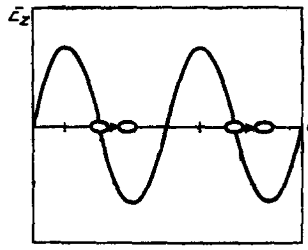

FIG. 6. Sketch of the phase slippage of bunches.

ing into the region of its retarding field (see Ref. 26 and Fig. 6 of the present paper). The amplitude of this displacement increases monotonically as the beam modulation frequency (ω_M) approaches the resonant frequency (ω_p) and as the equilibrium beam current density N_0 increases. At the resonant point itself, it reaches a maximum which does not depend on N_0:

$$\Delta(k) = \frac{\omega_b \cdot \omega_M^2}{(\omega_p^2 - \omega_M^2)^{3/2}}, \quad \frac{N_0}{N_p} \ll \left(\frac{\omega_p^2}{\omega_M^2} - 1\right)^3 \ll 1,$$
$$= \frac{1}{\sqrt{3}}, \quad \left(\frac{\omega_p^2}{\omega_M^2} - 1\right)^3 \ll \frac{N_0}{N_p} \ll 1. \tag{3.11b}$$

The effect of the Veksler-MacMillan self-phasing mechanism, which groups the beam particles into coherently emitting bunches in the case of long-wavelength perturbations ($kV_0 < \omega_p$), is thus intensified by the movement of the bunches formed in this manner into retarding phases of the field which they excite, as a result of a dependence of the phase velocity of this field on the beam particle density (a purely collective effect[7]). It is easy to see that under the condition $\omega < \omega_p [1 + (N_0/N_p)^{1/3}]^{3/2}$ the condition $V_\Phi = \omega/k < V_0$ (Ref. 26), which is a necessary condition for this motion to occur, and which imposes a lower limit on the region of unstable perturbation wave numbers k, does not contradict inequality (2.6), which places an upper limit on this region.

We will complete this section of the paper by refining the scope of the concept of self-phasing which we used above. Veksler and MacMillan predicted and described a mechanism of phase grouping of nonisochronous oscillators: The charged particles which are circulating along closed circular equilibrium orbits in cyclic resonant accelerators are oscillators of precisely this type. In the adiabatic approximation, however (i.e., at small values of the ratio of the phase oscillation frequency to the frequency of the field of the accelerating resonant wave), the intensity of the phase grouping does not depend on the curvature of the equilibrium orbit of the charge. Evidence for this conclusion comes from the fact that in both cyclic and linear resonant accelerators the dynamics of the phase motion of a charge is described by the same mathematical-pendulum equation in this approximation.[53-55] For these reasons, the mechanism which we have discussed here for the longitudinal grouping of beam particles into coherent bunches by the field which they themselves emit is actually a particular case of the Veksler-MacMillan self-phasing effect.

4. CONCLUSION

Summarizing the results of this review, we can describe the key steps in the onset of a collective hydrodynamic instability of a nonequilibrium beam-plasma system in the following way. The initial-perturbation signal groups the beam particles into a periodic train of coherent bunches. The typical linear dimension of the region of coherent field of each charge is equal in order of magnitude to the plasma wavelength. Consequently, the number of particles in each coherent bunch is proportional to the amplitude of the perturbation of the beam particle density and to the cube of the plasma wavelength. This train of coherent bunches excites longitudinal waves in the plasma at the beam modulation frequency as the result of the spontaneous Cherenkov emission involving these waves. The induced Cherenkov absorption of the resultant collective field causes a growth of the initial depth of the beam density modulation (a growth in the number of coherent beam particles in each bunch). A necessary condition here is that the perturbation wavelength be greater than the plasma wavelength: Only under this condition will the bunches form in stable phases of the excited field. In this case the dynamics of this distributed system is, from the physical standpoint, completely analogous to the dynamics of a mathematical pendulum near the upper equilibrium point. Specifically, the angular displacement of the pendulum from its equilibrium position is analogous to the depth of the density modulation of the beam, and the deflecting force of gravity is analogous to collective field (3.7), which amplifies this modulation. Because of the linear increase in the force with increasing perturbation amplitude, the latter grows exponentially over time, with an argument which is proportional to the square root of the stiffness of the deflecting force (the acceleration due to gravity; the equilibrium density of beam particles). Since the phase velocity of the wave which is excited is always lower than the beam velocity, the bunches formed in this manner lead the wave, moving away from the zero phase into a retarding phase.

The scientific and methodological importance of the quantitative explanation offered above for the key steps in the onset of the beam-plasma instability is not restricted to the particular case of this instability, since all hydrodynamic beam instabilities develop essentially in accordance with this scheme: in a process involving the formation of coherently emitting bunches by the driving field of their collective emission.[35-40] The specific features of each instability are determined only by the particular type of spontaneous emission which is responsible for the intensification of the field. Our only reason for selecting this particular beam-plasma instability is that the explicit analytic expressions for the Green's function of the Poisson equation, which describes the field of an individual charge in a plasma, take the simplest form in this case.

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APPENDIX. COULOMB FIELD OF A SINUSOIDALLY MODULATED BEAM IN AN ISOTROPIC PLASMA

The resultant Coulomb field of a beam with a modulation (3.6) is described by an expression analogous to (3.7):

$$E_{z\,\mathrm{tot}}^{(\mathrm{Coul})}(\mathbf{r}, t) = \int E_{zi}^{(\mathrm{Coul})}(\mathbf{r}, t; r_s)\, dQ_s$$
$$= A_{\mathrm{tot}}^{(\mathrm{Coul})} \cos(k_M z - \omega_M t), \tag{A.1}$$

in which the configuration of the Coulomb field of an individual charge is determined by (3.4).

We integrate first over z_s and then over $k_\perp$. These operations give us the amplitude and phase of the Coulomb field of a sinusoidally modulated filament at a distance $\rho_s = \text{const}$ from it:

$$E_z^{(\text{Coul})}(\mathbf{r},\ t;\ \rho_s) = A^{(\text{Coul})}(\rho_s)\cos(k_{\text{M}}z - \omega_{\text{M}}t), \qquad (\text{A}.2)$$

where

$$A^{(\text{Coul})}(\rho_s) = -\frac{4\pi qhN_0V_0^3k_{\text{M}}}{(b^2-a^2)\gamma^2}(b^2K_0(b\rho_s) - a^2K_0(a\rho_s)), \qquad (\text{A}.3)$$

$$b = \left(\frac{\omega_{\text{p}}^2}{c^2} + \frac{k_{\text{M}}^2}{\gamma^2}\right)^{1/2}, \quad a = \frac{\omega_{\text{p}}}{V_0}.$$

The first term in square brackets on the right side of (A.3) is nonzero even in vacuum (in the limit $N_{\text{p}} \to 0$). It describes the Coulomb field, slightly modulated by the presence of a plasma, of a linear filament with a charge described by sinusoidal law (3.6b). It can be seen from (A.3) that this field falls off exponentially with increasing distance from the filament, ρ_s, with the index of the exponential term being b. We will call this field, which remains nonzero even at a vanishing plasma density ($N_{\text{p}} \to 0$), the "quasivacuum" field. Physically, the second term on the right side of (A.3), which vanishes in the absence of a plasma ($N_{\text{p}} = 0$), incorporates and describes the screening of the quasivacuum field by the plasma, i.e., the image field of the sinusoidally charged filament in an unbounded plasma. The sign of this field is opposite to that of the quasivacuum field. The configuration of this purely plasma field is similar to the configuration of the quasivacuum field: It also falls off exponentially with increasing distance from the filament, although the index of the exponential is different and is determined by the plasma density and the beam velocity ($a = \omega_{\text{p}}/V_0$). The amplitudes of the two components of the Coulomb field are of such a nature that their resultant amplitude is exactly zero:

$$A_{\text{tot}}^{(\text{Coul})} = 2\pi \int_0^\infty d\,\rho_s\rho_s A^{(\text{Coul})}(\rho_s) = 0. \qquad (\text{A}.4)$$

The physical meaning of this result is that the screening of the Coulomb field of this filament is complete, as it would be in an ideal superconductor.

[1]This work was carried out in 1948.

[2]Here and below, we are citing primarily the generalizing publications, which present the results of the theory of beam-plasma instabilities and which contain detailed bibliographies of the corresponding original papers.

[3]We distinguish between spontaneous and stimulated interactions of beam particles with a field on the basis of the functional dependence of their intensity on the charge of the particle and the field amplitude[41]: In the former case the intensity is proportional to the square of the charge and is independent of the field amplitude, while in the latter case it is quadratic in both of these parameters.

[4]These conditions were established in Ref. 50 for beam waves in an isotropic plasma.

[5]Under the conditions under consideration here, the field of the longitudinal waves of a "cold" plasma does not propagate in the plasma: The charge essentially "detaches" from the field which it excites, rather than vice versa. It is for this reason that we are using the term "emission" here and below to mean exclusively the effect in which a field of longitudinal plasma waves is excited and results in an intensification of the field by the beam; i.e., we are using this term in the sense opposite to "absorption."

[6]In the review by Kuzelev and Rukhadze,[40] the driving field was assumed to be given. Its interrelationship with the characteristics of the fields of the spontaneous emission of the beam particles was not considered.

[7]For a quantitative description of this effect, we would also need to incorporate the self-field of the beam (in addition to the field radiated by the beam particles, $E_z^{(\text{pl})}$) on the right side of (3.8') (Refs. 40 and 44, for example).

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