A new book on the mechanics of turbulence

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Usp. Fiz. Nauk **155**, 547–549 (July 1988)

M. Lesieur. *Turbulence in Fluids*, M. Nijhoff, Dordrecht, Holland; Boston; Lancaster (1987) pp. XII + 286.

The book by the well-known French scientist M. Lesieur is basically devoted to the study of homogeneous and isotropic turbulence, with much attention being paid to the examination of conclusions following from the approximate "two-point closures" of the equations of the mechanics of turbulence. These closures are based on certain hypotheses which make it possible to close the equations for two-point correlation functions of turbulent pulsations (or, which frequently turns out to be more convenient, for the Fourier transforms of these functions, i.e., of the turbulence spectra). From this point of view the book under discussion is closely related to the well-known book by Leslie on the mechanics of turbulence,[1] which has the same special features. However, in comparing these two books one is immediately struck by the fact that they differ sharply from one another. Leslie's book is considerably greater in volume, and the greatest amount of space in it is occupied by a very detailed and accurate presentation of the two very first "two-point closures" which in 1973 appeared to be fundamental ones—the approximation of the direct interactions due to R. Kraichnan that appeared in 1959, and the modified theory developed somewhat later by the same author (which in contrast to the initial approximation already agreed with the well-known general results of A. N. Kolmogorov and A. M. Obukhov), in which the approximation of direct interactions is applied not to the Euler but to the Lagrangian turbulence characteristics (both these "two-point closures" are briefly discussed also in the book of Ref. 2). Lesieur's book, on the other hand, is written much less systematically: many awkward calculations have been omitted or merely indicated (with references to articles where they are presented in complete detail). For this reason the new book (in contrast to Leslie's book) cannot at all be used as a textbook; at the same time the brevity of presentation of complicated derivations has enabled Lesieur to examine very many problems which are not mentioned in Leslie's book (including a number of results obtained after the appearance of the latter book, and of conclusions of French scientists which have not been adequately dealth with in almost all the books by English and American authors). Lesieur has devoted very little space to the approximation of direct interactions and its modifications, while from a number of "two-point closures" he has discussed in greatest detail the so-called eddy-damped quasi-normal Markovian approximation—EDQNM—proposed by Orszag[3] which is utilized in the book under review in discussing a number of problems. We note in this connection that already after the appearance of Lesieur's book it was shown that by applying the method of the renormalization group it is possible to make the EDQNM approximation more accurate and to calculate theoretically the values of the fitting parameters appearing in it, which Lesieur recommends should be determined by comparing with experimental data the results of calculations based on EDQNM (cf., Ref. 4).

Let us now proceed to a brief review of the contents of individual chapters of the book. Chapter I is devoted to the definition of turbulence and a discussion of its role in the mechanics of fluids; here also are briefly presented modern concepts important for the mechanics of turbulence concerning the possibility of "chaotic behavior" of dynamic systems with a small number of degrees of freedom and concerning the appearance in the flow of fluids of ordered ("coherent") structures, and also a number of effective photographs are presented which illustrate some interesting examples of natural and laboratory turbulent flows on very different scales. In Ch. II the basic equations of hydromechanics are presented and also some consequences of them; here is a clear manifestation, which is also reflected in many other chapters of the book, of the special interest of the author in geophysical turbulence (in particular, the equations of motion of a thermally stratified fluid in a rotating system of coordinates and the theory of internal waves in a stratified fluid, which are important for geophysics, are discussed). In a short Ch. III a general idea is given concerning the linear theory of hydrodynamic stability (with examples of its applications), different ways of turbulization of flow are discussed and the basic dimensionless criteria of turbulization are introduced—the Reynolds, Rayleigh, Rossby, and Frood, numbers. Fourier transforms of the equations of hydrodynamics are discussed in Ch. IV—and in this connection certain useful (but not well known) expansions of the velocity field of fluid flow are indicated. Chapter V is devoted to the kinematics of homogeneous and isotropic turbulence; in so doing, in contrast to the books of Refs. 1, 2, isotropy is here defined as invariance of statistical properties of turbulence only with respect to rotations (but not mirror reflections), so that now an already homogeneous and isotropic turbulence can have nonzero helicity $H = \langle \mathbf{u}\cdot\mathrm{curl}\mathbf{u}\rangle/2$ (where $\mathbf{u}$ is the velocity field, and the angle brackets denote averaging), which plays an important role in the study of many magnetohydrodynamic fluid flows conducting a current. Here also an indication is given of the general form of the correlation and spectral tensors of homogeneous and isotropic (in the sense explained above) turbulence, and spectra of energy, enstrophy (the square of vorticity), helicity and scalar fields (i.e., fields of temperature and concentration of impurity) corresponding to such turbulence are introduced. In the initial part of Ch. VI a general equation is derived which associates the energy spectrum $E(k,t)$ of isotropic turbulence with the transport of energy over the spectrum $T(k,t)$ (this equation is the Fourier transform of the well-known Karman-Hovarth equation, which contains correlation functions of the second and third orders, and whose initial form is not given in Lesieur's book); further here are also presented the foundations of the general theory of locally isotropic turbulence developed by A. N. Kolmogorov and A. M. Obukhov, the "4/3 law" due to L. Richardson for the relative turbulent diffusion, which determines the velocity of spreading of a cloud of impurity in a turbulent flow, is examined, certain useful formulas for the scales of length and certain other characteristics of isotropic turbulence are indicated, and also a brief discussion is given of the important topic of the effect of fluctuations of the rate of dissipation of energy on the characteristics of turbulence which are related to the iterational interval of scales. Chapter VII is devoted to the so-called "analytical theories of turbulence" (i.e., to the different "two-point closures") and to the methods of numerical solution of equations corresponding to these theories; the principal attention here is focused on the EDQNM approximation. In Ch. VIII the problem is examied concerning the mixing by turbulence of the field of temperature or concentration of some material impurity in the flow; here a derivation is given of the general results of A. M. Obukhov, S. Corrsin, and G. Batchelor on the structure of the field of temperature or concentration in an inertial interval of scales and in a smaller-scale region and also the application of the EDQNM approximation to the study of diffusion of a scalar impurity is investigated. Chapter IX investigates (again utilizing the EDQNM-theory) the homogeneous and isotropic two-dimensional turbulence and the quasigeostrophic turbulence related to it, which is often a very convenient model of the set of large-scale perturbations in the atmosphere or the ocean; in this connection a discussion is given here of a number of concrete problems of geophysical hydrodynamics. In Ch. X a discussion is given of turbulence in a cubic (or square) "box" and of the model finite-dimensional systems of equations corresponding to it that are obtained by retaining only a finite number of Fourier components of the velocity field or the current functions; the material of this chapter is closely related to the method of modeling the equations of hydrodynamics by systems of a finite number of ordinary differential equations that has become quite widespread recently (cf., Ref. 5). Chapter XI is devoted to the statistical problem of the predictability of turbulent flows, i.e., of estimating the rate of growth with time of the error of the calculated values of the velocity field of a three-dimensional or two-dimensional turbulence for a given error in specifying the initial field at the time $t = 0$; here again calculations using the EDQNM method are used as the basis. Chapter XII discusses "modeling of large vortices", i.e., the method of a numerical calculation of the evolution in time of large-scale turbulent perturbations in which the dynamics of perturbations on a "sublattice scale" (a smaller typical distance between neighboring nodes of the lattice, utilized in solving on an electronic computer exact hydrodynamic partial differential equations) is modeled by applying to such perturbations some sort of an approximate "two-point closure" (for example, EDQNM). Finally in the last Ch. XIII (entitled "Towards real turbulence") the problem is examined of generalizing the results related to the idealized homogeneous and isotropic turbulence to the case of two classes of nonisotropic turbulent flows important for geophysics—a flow which arises after the creation at the instant $t = 0$ in a stably stratified (and, consequently, nonisotropic) medium of a turbulent flow produced by the loss of stability of the layer of mixing of two plane parallel flows characterized by velocities differing in magnitude (in the latter case particular attention is devoted to the appearance in the layer of a mixing of coherent structures). The book concludes with a list of references containing 291 titles and a subject index.

Lesieur's book has the nature of a review monograph, where considerably more attention is paid to the physical aspects of the matter, than to mathematical fine points; in this respect it differs sharply from very frequently occurring books on hydrodynamics in which this discipline is in fact regarded as a part of applied mathematics. It is clear that the book being reviewed must be of considerable interest for physicists who wish, without spending much time, to become acquainted with the present-day situation in the theory of turbulence; it will be particularly useful for a wide circle of geophysicists, who encounter turbulent flows in their work.

[1]D. C. Leslie, *Developments in the Theory of Turbulence*, Clarendon Press, Oxford, 1973; 1983 (with corrections).
[2]A. S. Monin and A. M. Yaglom, *Statistical Fluid Mechanics*, MIT Press, Cambridge, MA, 1975 [Russ. original, Nauka, M., Part 2, 1967].
[3]S. A. Orszag, J. Fluid Mech. **41**, 363 (1970).
[4]W. P. Dannevik, V. Yakhot, and S. A. Orszag, Phys. Fluids **30**, 2021 (1987).
[5]E. B. Gledzer, F. V. Dolzhanskiĭ, and A. M. Obukhov, Systems of Hydrodynamic Type and Their Applications (In Russian), Nauka, M., 1981.

Translated by G. M. Volkoff