The general theory of relativity is correct!

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In our paper "Gravitation, the general theory of relativity, and alternative theories,"[1] we considered the present state of the theory of gravitation and discussed ways of its further development. We also analyzed the criticism made of the general theory of relativity (GR) in the papers of Ref. 2 and examined the "relativistic theory of gravitation" (RTG) proposed there. The conclusion of Ref. 1 was that GR is correct and that the criticism of Ref. 2 is based on misunderstandings. There have recently appeared new studies[3,4] criticizing GR and constructing the RTG, which we have studied in detail.

Our final conclusion is the following: The most careful reading of the paper[3] and other studies[4] has not changed our opinions presented in Ref. 1.

As before, we believe that the general theory of relativity, which uses the idea of a curved space-time continuum, i.e., with metric relationships differing from the Minkowski metric, is a natural, correct, and consistent way of describing gravitation and the phenomena in which gravitation is important. In our view, GR does not have mathematical or physical contradictions or ambiguities, and is in excellent agreement with all the experiments at present available.

As we already said in Ref. 1, GR admits equivalent formulations: the ordinary geometrical formulation and a field formulation. In the geometrical formulation GR contains only the components of the metric tensor $g_{\mu\nu}$ of the curved space-time, and these are simultaneously the potentials of the gravitational field. In the field formulation, GR contains not only the components of the metric $\gamma_{\mu\nu}$ of an auxiliary (background) space-time, for example, Minkowski space, but also the components of a tensor gravitational field $h_{\mu\nu}$. A convenient connection between these quantities, proposed already in the well-known study of Deser,[5] is

$$(-g)^{1/2} g^{\mu\nu} = (-\gamma)^{1/2} (\gamma^{\mu\nu} + h^{\mu\nu}).$$

The field formulation of GR has the form of an exact and rigorous field theory on a given background. It possesses all the necessary attributes of such a theory—an action and equations of motion, an energy-momentum tensor of the gravitational field and conservation laws that reflect the symmetry of the background space-time, coordinate and gauge invariance, etc. The construction of the field formulation of GR was the subject of numerous investigations.[5–8] It has in the meanwhile been developed with exhaustive completeness. It is identical with the geometrical formulation of GR in all the experimentally verifiable conclusions and has the form of a traditional field theory and is convenient in theoretical investigations.[9–11]

We now consider the criticism of GR in Refs. 2–4. The criticism is concentrated around two assertions. According to one of them, GR gives ambiguous predictions for gravitational effects; according to the other, energy-momentum conservation laws do not hold in GR. We shall first consider these assertions briefly; technical details will be given at the end of the paper.

The authors of Refs. 2-4 believe that GR suffers from a serious practical ailment, namely, that its predictions for observational effects in the solar system are not unique. We completely disagree with this. For a brief explanation it is sufficient to recall that in 1985 a representative international conference was held in Leningrad specially devoted to relativistic effects in the solar system: "The Theory of Relativity in Celestial Mechanics and Astrometry." Excellent agreement between the theory (namely, the general theory of relativity!) and the observations was noted. The proceedings of the conference have been published.[12] A considerable number of other publications[13] has been devoted to these questions. General relativity long ago ceased to be merely a theory to be verified; today it is used in practice as a working theory for the compilation of astronomical annual ephemerides, to calculate the motion of the major planets, the moon, and space probes. The relativistic effects of GR have been reliably measured not only in the solar system, but also outside it through the motion of the radio pulsar PSR 1913 + 16, which belongs to a binary star system.[14] In our view the situation does not give rise to the slightest doubt concerning the correctness and uniqueness of the predictions of GR.

The consideration of the solar system and weak gravitational fields is particularly remarkable in clearly indicating the reason why the conclusions in the RTG to which we referred were obtained. It can be seen that the problem has nothing to do with complicated nonlinear gravitational fields, topological subtleties, or experiments that have not been made but that an incorrect interpretation has been made of the meaning of the coordinates and Minkowski metric that occurs in the calculations of the RTG (see below).

Further, the authors of Refs. 2–4 see an incurable vice of GR in the fact that the energy-momentum density and stresses of the gravitational field are usually expressed by means of an energy-momentum pseudotensor[1)] and not a tensor, as in Maxwell's theory. From this a far reaching conclusion is drawn—that there is no energy conservation law in GR.

As an answer to this at the popular level, we may recall that historically the energy conservation law appeared as a generalization of a huge number of attempts to construct a perpetual motion machine. The argument of the authors of the RTG against GR would become serious if they were to show constructively how one can build a perpetual motion machine in the framework of GR. Nothing of the sort is done in the papers of these authors. In our view, the problem is not the absence of conservation laws in GR but the fact that the authors of the RTG regard only one form of expression of these laws as acceptable. We explain this in more detail.

In the question of energy and conservation laws the cen-

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tral point in the criticism is the most remarkable feature of the geometrical formulation of GR, namely, its property that the potentials $g_{\mu\nu}$ of the gravitational field are simultaneously the components $g_{\mu\nu}$ of the space-time metric tensor. In this the geometrical formulation of GR differs from the traditional field theories, in which there are metric components and field variables separately. For this reason, the energy-momentum tensor of the gravitational field calculated in the geometrical formulation of GR by variation with respect to the space-time metric $g_{\mu\nu}$ (and, therefore, with respect to the field variables) is identically equal to zero by virtue of the field equations. For this reason, the expression of covariant equations in a form containing a traditional ordinary (and not covariant) 4-divergence unavoidably leads to an energy-momentum psuedotensor of the gravitational field, and not to a tensor. For this reason doubts may arise concerning the integral conservation laws, since a curved space-time with metric $g_{\mu\nu}$ does not admit a group of motions in the general case.[21] For this reason, the analysis of the asymptotic behavior of the field of an isolated system is intertwined with analysis of the asymptotic behavior of the coordinate system determined by the components $g_{\mu\nu}$ and becomes technically complicated.

All these features of the geometrical formulation of GR have been well recognized and explained,[15–17] and they in no way prevent the solution of practical problems. For example, an isolated binary star "embedded" in an asymptotically flat space-time has all ten ordinary integrals of the motion in the approximation up to $(v/c)^4$ inclusively, for which the system is still conservative (v is the characteristic orbital velocity). In the $(v/c)^5$ approximation the emission of gravitational waves becomes important, the system loses its (Newtonian) energy, and the parameters of the Keplerian orbit change secularly. In recent years all these conclusions have been obtained[18] by solving the Einstein equations alone without any use of the energy-momentum pseudotensor. The energy loss is precisely equal to the loss described by Einstein's famous quadrupole formula. In the binary system with the pulsar PSR 1913 + 16 these conclusions are brilliantly confirmed by radio-astronomical methods with all the accuracy currently available—better than 4%.[14]

In the opinion of the authors of Refs. 2–4, the mentioned features of the geometrical formulation of GR are fundamental shortcomings of GR. It is asserted that they are completely eliminated in the RTG, which is based on concepts of a field defined in a flat Minkowski world with 10-parameter group of motions. In other words, the authors of the RTG see the solution to the problem of the energy and the conservation laws only in a return to field conceptions in a global Minkowski world.

In this connection we wish to recall that the representation of GR in the form of an ordinary field theory on the background of some auxiliary space-time and, in particular, on a Minkowski space background, is known with exhaustive completeness and is called the field formulation of GR. Careful study of Refs. 2–4 shows that the mathematical content of the RTG reduces entirely to the mathematical content of GR in a field formulation augmented by certain admissible (but not mandatory) additional conditions. The authors regard the solution of the problem of the energy and the conservation laws in the RTG as entirely satisfactory. Since in this question all the arguments and calculations of the RTG repeat what exists and has already been shown in the framework of the field of formulation of GR (see, for example, Ref. 8), we believe that the criticisms of GR by the authors of the RTG are automatically eliminated. (The technical details of the criticism of GR are analyzed below.)

Such is our basic point of view with regard to Einstein's general theory of relativity as set forth in more detail in Ref. 1, from which, as we have already said, we withdraw nothing.

It should also be noted that the assertion that "the general theory of relativity is correct" does not in itself rule out any of the following points: a) the large number of unsolved problems in GR; b) the possible existence of some quantitative limits of validity of GR; c) the existence of one or several other theories completely equivalent to GR in all observable predictions.

We begin with the first point. It is well known that by no means all problems are solved even in the framework of classical celestial mechanics, and this applies all the more to problems of the relativistic theory of gravitation. General relativity is a live and actively developing science. An idea of the unresolved problems of general relativity, of the vast number of theoretical and experimental studies that have been made, and of the practical applications of GR can be obtained, for example, from individual papers,[19] from collections published in connection with the centenary of the birth of Einstein,[20] and from the proceedings of various international conferences.

Now point b). At the present time there is a widely held opinion (which we also support) that in the near future there will be created a "theory of everything," abbreviated TOE, as it is called in the English literature. This theory will combine gravitation with the other forces of nature—electromagnetism, the weak interaction, and chromodynamics (the theory of quarks, gluons, and nuclear forces). In addition, the general opinion is that the TOE will predict new particles and fields not hitherto observed in laboratory experiments. These particles and fields may play an important part in cosmology. Further, a number of authors believe that the TOE will be based on a space of more than four dimensions (for example, $D = 10, 11$ or 26?), from which only four "survive" as time and space.

All these considerations limit the region of applicability of GR. In this sense, general relativity is the low-energy limit of a general theory. It is certainly invalid at the Planck scales of mass, length, and time, at which quantum-gravity effects are important.

We note particularly that GR is a relativistic theory and therefore in GR there are no restrictions on the velocity or the magnitude of the gravitational potential (which has the dimensions of the square of a velocity). In particular, neither the theory of large black holes nor the cosmology of the classical period of expansion requires modification. The more general theory (including GR as a limiting case) can be required fully only for mini black holes, with a mass of the order of the Planck mass, and for problems of the type of the spontaneous creation of the universe.

Finally, we consider the last point—the existence of theories equivalent to GR. As we have already said, the general theory of relativity, being a generalization of the special theory of relativity, undoubtedly admits an equivalent (field) formulation, which uses the concept of a tensor gravitational field defined on a background of a flat Minkowski world. In such an approach the Minkowski metric plays a purely auxiliary role. The artificial and formal nature of the Minkowski metric is already seen in the fact that the propagation of light in the presence of gravitation does not take place along the null geodesics of the flat Minkowski world but along the null geodesics of the metric of the curved world. In other words, the causality cone is determined by the metric of the curved world and not the flat one. Moreover the causality cone and the world lines of real bodies may be situated both within and without the light cone formally defined by the Minkowski metric. Under such circumstances the attempt to interpret the metric relationships of the flat world as observable would be tantamount to recognizing that in a real physical process cause and effect can change places. (Nevertheless, in Ref. 3 the authors speak of the "objectivity and observability of all the properties that are inherent in Minkowski space"; p. 393).

As we said above, the field formulation of GR was developed in detail and published before the appearance in 1984 of the papers of A. A. Logunov and his collaborators on the "relativistic theory of gravitation" (RTG). Since the mathematical content of the RTG reduces to the mathematical content of GR (in the field formulation), it follows that if the meaning of the Minkowski metric is correctly interpreted the conclusions of the RTG are identical to those of GR, but if it is incorrectly interpreted then they contradict the experiment. But then: "Why is there this street if it does not lead to the temple?" ("Confession," film by T. Abuladze).

We now consider some technical points relating to the criticism of GR and the content of the RTG.

1) It is asserted in Ref. 3 that in the arguments of Einstein, and also Klein, there is hidden a simple but fundamental error. It is asserted that the quantity J_σ, which Einstein identified with the energy and momentum of an isolated system, "is found on more careful examination to be a quantity that vanishes identically." Similar assertions are found in other studies,[4] including popular scientific ones. Details of this assertion are contained in Ref. 21. There the authors begin by quoting Einstein[22]:

"The integral momentum and energy conservation laws are obtained from Eq. (1) [Eq. (1) in this paper is a consequence of the Einstein equations written in the form

$$\frac{\partial U_\sigma^\nu}{\partial x_\nu}=0. \quad \text{— Ya. Z., L.G.]}$$

by integrating this equation with respect to x_1, x_2, x_3 over the region B. Since on the boundaries of this region all the U_σ^ν vanish,

$$\frac{\mathrm{d}}{\mathrm{d}x_4}\int U_\sigma^4\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3=0. \tag{3}$$

These four equations express, in my opinion, conservation laws of the momentum ($\sigma=1, 2, 3$) and energy ($\sigma=4$). We denote the integral that occurs in Eq. (3) by J_σ. I now assert that the quantities J_σ do not depend on the choice of the coordinates for any system of coordinates identical outside the region B with one and the same Galilean system."

The authors of Ref. 21 then write:

"However, it is easy to show, that, following Einstein, we arrive at a zero value of the energy and momentum of any isolated system. To show this, we write the Hilbert-Einstein equations in the form

$$U_\sigma^\nu=T_\sigma^\nu+\tau_\sigma^\nu=\partial_\mu\sigma_\sigma^{\mu\nu}, \tag{6.4}$$

where $\sigma_\sigma^{\mu\nu}=-\sigma_\sigma^{\nu\mu}$ is the density of an antisymmetric (pseudo) tensor of third rank.

Substituting Eq. (6.4) in the expression for the 4-momentum of the isolated system, we obtain

$$J_\sigma=\int \mathrm{d}v\,U_\sigma^4=\int \mathrm{d}v\,\partial_n\sigma_\sigma^{n4}=\int \mathrm{d}S_n\,\sigma_\sigma^{n4}. \tag{6.5}$$

Since the surface of integration S is outside the region B, where all the components of the tensor $g_{\mu\nu}$ are constant and have Galilean values, the quantities σ_σ^{n4} are zero everywhere on the surface S. It therefore follows from the expression (6.5) that $J_\sigma=0$.

Since Einstein did not note that $J_\sigma=0$, he regarded the above definition as correct and as establishing the concepts of energy and momentum of a closed system as clearly as in classical mechanics. Similar incorrect assertions are repeated almost literally in a number of other books (see, for example, . . .)."

Comparison of these two excerpts shows that there is a confusion. On the transition from (1) to (3) Einstein omitted the volume integral of the three-dimensional divergence $\partial U_\sigma^n/\partial x_n$, which reduces to a surface integral:

$$\int \mathrm{d}v\,\frac{\partial U_\sigma^n}{\partial x_n}=\int \mathrm{d}S_n\,U_\sigma^n.$$

This can be done, since the asymptotic behavior of the metric of an isolated system is

$$g_{\mu\nu}=\eta_{\mu\nu}+O\left(\frac{1}{r}\right),\quad g_{\mu\nu,\sigma}=O\left(\frac{1}{r^2}\right).$$

It is such that the surface integral $\int dS_n\,U_\sigma^n$ tends to zero as $r\to\infty$. The authors of Ref. 21, evidently by analogy, also omit the surface integral on the right-hand side of (6.5) and, thus, conclude that $J_\sigma=0$. However, for the same isolated system, with the same asymptotic behavior of the metric, this surface integral, i.e., $\int \mathrm{d}S_n\,\sigma_\sigma^{n4}$, does not tend to zero at all and cannot be ignored. Thus, the identical vanishing of J_σ does not occur.

In a different place in Ref. 3 the quantity J_σ in (6.5) is actually calculated, but in this case it is not the identical vanishing that is discussed but nonuniqueness. On p. 373 the authors of the RTG write that they calculate P^0 "using the definition of GR for the inertial mass of a body (or its total energy),

$$P^0=\lim_{r\to\infty}\oint \mathrm{d}s_k h^{00k},$$

in which $\mathrm{d}s_k=$. . . ." But in Ref. 3 it is not noted that the given expression and its interpretation as mass presupposes the use of definite asymptotic values of the metric $g_{\mu\nu}$. In other words, the definition of the mass adopted in the geometrical formulation of GR includes the requirement that definite asymptotic coordinates be used. This circumstance is specially emphasized in Landau and Lifshitz's book *The Classical Theory of Fields*,[15] in Misner, Thorne, and Wheeler's book *Gravitation*,[16] and others, and also in Faddeev's review of Ref. 23. For example, in Ref. 16 the following is said:

"This requirement for far-away flatness is a remarkable feature of the flux integrals (20.9); it is also a decisive feature. [The reference is to formulas for P^0 and other integrated quantities expressed in the form of surface integrals.] Even the coordinates must be asymptotically Minkowskian; otherwise most formulas in this chapter fail or require modification." Further: "Summary: Attempts to use formulas (20.9) in ways that lose sight of the Minkowski boundary conditions (and especially simply adopting them unmodified in curvilinear coordinates) easily and unavoidably produce nonsense."

Thus, the ambiguous result obtained by the authors of RTG for the mass in arbitrary coordinates is explained by the use of the formula for P^0 outside the region of its validity.

This question has been treated formally and rigorously in a paper by Faddeev.[23] The conclusions contained in it and his analysis of the criticism addressed to GR are not discussed in Ref. 3. We should also mention studies in which explicit use is made of symmetries in a complete centered space-time or at its infinity and which lead to coordinate-independent expressions.[17,24]

For our part, we note the following. It can be seen from the discussion that the subject of the criticism made by the authors of the RTG is the transformation properties of quantities of the type P^0 and, in particular, the question of the possibility of using arbitrary spatial coordinates. To answer these questions, it happens that precisely the field formulation of GR is convenient. As was emphasized in Ref. 1, the field formulation of GR contains an energy-momentum tensor (and not pseudotensor) of the gravitational field that in no way can be "made" into a function of only $g_{\mu\nu}$ and essentially contains the background metric $\gamma_{\mu\nu}$. The explicit form of this tensor, a discussion of its properties, an analysis of the coordinate and gauge transformations, the proof of the existence of conservation laws reflecting the symmetry of the background space, and examples of the use of this tensor can be found in the paper "Exact theory of the (Einstein) gravitational field on the background of an arbitrary space-time."[8] In particular, for isolated systems the field formulation of GR automatically leads to expressions for P^0 and other quantities that are covariant with respect to spatial transformations (the limitation on the choice of the spatial coordinates is completely eliminated), and the numerical values of the conserved quantities are equal to the standard values (see Ref. 11). We must emphasize strongly that we are speaking here of GR and not some other theory. Since in this question all conclusions and arguments made in the framework of the RTG repeat what is known and done in GR (in the field formulation), the recognition that the solutions of these problems in the RTG is satisfactory amounts to recognition that their solution in GR is also satisfactory.

It may also be added that for the solution of practical problems, for example, for the description of the motion of gravitating bodies in the solar system, the questions under discussion have no significance at all. There are no dynamical equations in addition to the Einstein equations in which it would be necessary to substitute independently some particular definition or value of P^0. Among the arbitrary coordinate transformations (and, in the field formulation, arbitrary gauge transformations) there are some that change the terms which occur in P^0, but at the same time other terms in the equations are changed, so that the equations are still the same. It is not necessary to solve anything apart from Einstein's equations. And, as is well known, the final equations of motion of gravitating bodies can be obtained uniquely and in complete agreement with observations from Einstein's equations. As we have already said, GR long ago became a working theory in the relativistic celestial mechanics of the solar system and ephemeris astronomy.[28]

2) The assertion of the authors of the RTG of a nonuniqueness in the predictions of GR "for gravitational effects" is based on a misunderstanding. If by "gravitational effects" one understands quantities such as the energy and momentum of a moving test particle, then they, of course, are defined only with respect to a particular coordinate system and depend on it, but in this respect GR in no way differs from the special theory of relativity or other theories. But if nonuniqueness refers to predictions for "gravitational effects" that by their nature do not depend on the coordinate mesh used to identify events, such effects must obviously be independent of the coordinate system, and the formalism of GR ensures this independence. For example, the time of propagation of a signal between planets in the gravitational field of the sun, measured on one of them, does not depend on the particular coordinate mesh used to cover the solar system. The contradictory result obtained by the authors of the RTG for this example (and for the thought experiment in which a "massive body is placed on a needle") is explained by the fact that they insist on the same numerical value of the letter r, the radial coordinate that describes the positions r_e and r_p of the planets in the different coordinate meshes. But then, on the transition to a different coordinate mesh, the planets are on spheres of different area, i.e., on different orbits, at physically different distances. The invariants of the curvature tensor at the points of transmission and reception (or reflection) of the signal now also take on different values. Essentially, the authors of the RTG propose that one should calculate the signal propagation time between a different pair of planets. Naturally, in this case there is, and indeed must be, a difference between the signal propagation times. But if the source and receiver are not moved by design, the numerical values of r (the radial coordinate) are different in the different meshes, but the values of the invariants of the curvature tensor and the propagation time naturally remain the same. (Specifically, for a given pair of planets the values of R in Eqs. (1.12a) and (1.13a) of Ref. 3 are different, and not the same, as the authors of the RTG assume.) As we have already said in Ref. 1, one establishes whether the positions of bodies in different coordinate meshes are the same by means of the invariant, operationally defined characteristics and not the numerical values of the coordinate r, even if it is conceived as "the primary variable in the previously stipulated arithmetization of space." It can be seen from this that there is no ambiguity in the observational predictions of GR or contradictions with GR. In particular, they do not occur in the signal delay effect, as has been explained in detail in numerous studies.[25]

3) The complete set of RTG equations can be reduced

in terms of the metric $g_{\mu\nu}$ of the curved space-time to Einstein's equations together with the harmonic coordinate condition that was so successfully used by Fock.[26]

Indeed, as was emphasized in Ref. 1, the field variables $h^{\mu\nu}$ of the field formulation of GR (in Ref. 3 they are denoted by $\Phi^{\mu\nu}$) can, by virtue of the gauge freedom, be made to satisfy the subsidiary condition

$$h^{\mu\nu}_{;\nu}=0,$$

where the semicolon denotes the covariant derivative with respect to the background metric (in Ref. 3 this derivative is denoted by the symbol D_μ, and the corresponding equation is Eq. (2.1)). By virtue of the connection between $h^{\mu\nu}$, $\gamma^{\mu\nu}$, and $g^{\mu\nu}$, given at the very beginning of this paper,

$$(-g)^{1/2}\,g^{\mu\nu}=(-\gamma)^{1/2}\,(\gamma^{\mu\nu}+h^{\mu\nu})$$

(in Ref. 3, this connection is called the "geometrization principle" and is introduced by Eqs. (2.2) and (2.3)), this condition can be rewritten in the form

$$((-g)^{1/2}\,g^{\mu\nu})_{;\nu}=0$$

(in Ref. 3 this is Eq. (2.40), which is supplementary to the Einstein equations (2.39)). It is most convenient to take the background Minkowski metric expressed in Lorentz coordinates, i.e., in the form

$$d\sigma^2=c^2dt^2-dx^2-dy^2-dz^2.$$

Then the subsidiary condition takes the form

$$((-g)^{1/2}\,g^{\mu\nu})_{,\nu}=0,$$

where the comma denotes the ordinary derivative. The obtained condition is precisely the so-called harmonic condition[26] on the components $g^{\mu\nu}$. Thus, Einstein's equations are augmented by the harmonic condition.

All that we have said can be rephrased as follows: In terms of the field variables $h^{\mu\nu}$ the equations of the RTG reduce to the equations of the field formulation of GR together with the necessary fulfillment of certain subsidiary conditions, the possibility of the choice of which is foreseen in the field formulation of GR in the form of a gauge symmetry. (In breaking the gauge symmetry, the subsidiary conditions will naturally contain the metric of the background space-time. On this basis it is said in Ref. 3 that this "makes the metric $\gamma^{\mu\nu}$ of Minkowski space uneliminable from the theory" and that "by virtue of the field equation (2.1) the Minkowski space metric occurs organically in the theory. This is the fundamental difference between the RTG and GR.")

We do not see any grounds for calling the general theory of relativity augmented by admissible (but not necessary) subsidiary conditions, either in the field or the geometrical formulation, by a new name and even less for asserting that now it "leads to physical consequences qualitatively different from GR" (p. 386). Since the authors of the RTG insist that the RTG "leads to a number of conclusions fundamentally different from those that follow from GR,"[2-4] the question arises of the extent to which these conclusions have been correctly obtained.

4) One of these consequences is the assertion according to which by virtue of the RTG equations "a Friedmann homogeneous and isotropic universe can only be infinite and flat."[3] To show that this is not the case, it is sufficient to look at §94 of Fock's book,[26] in which there is, for example, a discussion in explicit form of the metric of a spatially open (and not spatially flat) homogeneous and isotropic Friedmann universe in harmonic coordinates, i.e., a metric that satisfies the complete set of RTG equations.

5) In the opinion of the authors of the RTG, a further fundamental difference between it and GR is that "the RTG radically changes the picture of the evolution of gravitational collapse." This assertion is also given in the popular literature. We quote from the journal *Priroda*[4]: "In the RTG ... the proper time for a falling test body is slowed down without limit as the so-called Schwarzschild radius is approached. Thus, in accordance with the RTG it is in principle impossible ... for black holes to occur in nature. All this can be illustrated by the example of a spherically symmetric non-steady-state problem for dust ... The proper time interval $d\tau$ for the falling body is related to the Minkowski space-time interval dt by the simple relation

$$d\tau=dt\left(\frac{\rho-GM}{\rho+GM}\right),$$

where ρ is the radial variable in Minkowski space.

It can be seen directly from this formula that as ρ approaches the value GM the proper time differential $d\tau$ tends to zero. This means that all physical processes in the falling body are slowed down without limit."

This excerpt gives a very specific exposition of the well-known fact that the proper time τ of a falling body remains finite as the Schwarzschild radius is approached, i.e., $\tau\to\tau_c$ as $t\to\infty$. The behavior of test particles in the Schwarzschild metric expressed especially in harmonic coordinates was investigated in Ref. 26. The corresponding formulas of the RTG are, of course, identical to those of GR. The physics of the phenomena that occur has also been well clarified (see, for example, Refs. 7 and 15–17). However, in the RTG the parameter t is conceived as the primary variable of Minkowski space, and therefore as $t\to\infty$ and $\tau\to\tau_c$ the complete analysis is stopped. The RTG dispenses with consideration of the further fate of the falling body. One can hardly be satisfied by the giving up of the study of the complete evolution of a falling body simply for the reason that for the description of part of this evolution it is necessary to use up an infinity of values of some parameter letter t that the authors of the RTG conceive as a primary variable. It is clear that nothing happens to the falling observer and his time does not stop simply because the variable $t\to\infty$.

In fact, in Ref. 3 two assertions are already found together. For the considered case, in which there is discussion of a falling test particle or the collapse of a dust sphere (and not the situation in which the collapse is stopped by pressure or the size of the sphere is so small that quantum processes affect its fate), these two assertions cannot be reconciled. On the one hand, it is stated in Ref. 3 (footnote 24) that: "Thus, in accordance with the RTG the solutions (2.47)–(2.49) are physically invalid when $\tau\geqslant\tau_c$." But two pages later we read: "Therefore, according to the RTG the time evolution of a collapsing object is not terminated by the ending of its contraction (during a finite proper time and a finite time of the external observer) but goes over to a new stage with normal subsequent flow of both the proper time and the time of the external observer."

Completing this note, we should like to remark on the confirmation of the strength and beauty of GR, which are also manifested in the fact that determined attempts to find a substitute for it come back to GR, as can be seen from the considered example of the comparison of the RTG and GR.

[1)]The term "pseudotensor" in the given context means that this quantity does not behave as a tensor under all arbitrary coordinate transformations but only with respect to a certain fairly large class of transformations, which includes the Lorentz transformations.

[2)]The presence of a group of motions indicates a symmetry of space-time or of a surface. For example, a plane or sphere can "slide" over itself—it admits a group of motions—but, say, the crumpled fender of an automobile does not admit a group of motions.

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