Aleksandr Aleksandrovich Fridman (Friedmann): a biographical essay

V. Ya. Frenkel'

A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR, Leningrad

Usp. Fiz. Nauk **155**, 481–516 (July 1988)

This essay describes the life of the outstanding Soviet scientist A. A. Fridman (1888–1925), known in the western literature as Friedmann, in connection with the centenary of his birth. The main stages in the life of Fridman are discussed in chronological order, beginning with his years at high school and university. Archive files are used in an account of the first steps of Fridman's scientific activity dealing with the problems in mathematical physics and aerodynamics. His participation in the First World War (1914–1917) is then described, followed by his professorship in Perm' University (1818–1920) and finally by the last very fruitful part of his life in Petrograd (1920–1925) where he taught at leading higher educational establishments and worked at the Main Geophysical Laboratory of which he became Director during the last year of his life. Particular attention is given to Fridman's work on relativistic cosmology (1922–1925) and dispute on the subject between Fridman and Einstein in connection with Fridman's theory of an expanding universe. The full bibliography of Fridman's work (73 items) is included.

1. A. A. Fridman (1888–1925), who was known in the West as Friedmann and the centenary of whose birth we celebrated in June 1988, made a contribution which can be justifiably classified as belonging to the highest category of "epoch-making." Modern relativistic cosmology originates from Fridman's theory of a non-steady-state expanding universe and was founded (more accurately, continues to grow) on his work.

The acknowledgement of Fridman's contribution to cosmology has been very dramatic. Initially, Einstein objected to Fridman's cosmology,[1)] but later acknowledged fully its importance and depth (as described below). However, in over six decades since these events, particularly in the last 25 years, Fridman's theory has gained universal acceptance. Books on relativistic cosmology and astrophysics, reviews and popular papers on the subject begin with an account of this theory (Refs. 2–7).[2)] There is therefore no need discuss the importance of the contribution made by A. A. Fridman to the development of cosmology or, even in summary fashion, to present his ideas.

Another important interest of Aleksandr Aleksandrovich, with which he was concerned for 12 years of his short 37-year life, was a range of problems in hydrodynamics and dynamic meteorology. This work was fully recognized, both in the Soviet Union and outside, during his lifetime and the subsequent development of these subjects in the Soviet Union is linked to the names of representatives of Fridman's school made famous by N. E. Kochin, P. Ya. Polubarinova-Kochina, I. A. Kibel', N. V. Roze, B. I. Izbekov, and other scientists in later (including postwar) generations who were indirect pupils ("grandchildren") of A. A. Fridman. Special papers have been written on this work and its origins, whereas the subsequent developments are described in a number of monographs on meteorology, theory of turbulence, hydrodynamics, and aerodynamics.[8–10]

This satisfactory position about our knowledge of the creative scientific activity of Aleksandr Aleksandrovich is in striking contrast to the paucity of purely biographical details of his life. It would seem that his life should have attracted long ago the attention of historians of science and of writers of documental or fictional biographies. In fact, surprisingly little, next to nothing has been written about Aleksandr Aleksandrovich: a tiny booklet about him was published in 1963 (Ref. 11), a relatively brief scientific biography appeared on the initiative of P. L. Kapitza in a volume of collected works of A. A. Fridman published in the series "Classics of Science"[12]; the collected works include the already mentioned important papers[6,7] on his work on cosmology. A "peak" but not an explosion of information on Fridman dates back to 1925–1927: about ten obituaries and papers[13,14] (partly reprinted in the "Classics of Science" volume) appeared under the fresh impact of his premature death. Later a few pages about her teacher were included in the books of P. Ya. Polubarinova-Kochina.[15,16] The position has improved because of the centenary of the birth of Aleksandr Aleksandrovich: books about him are being prepared for press, there will be conferences and symposia, which can be regarded as precursors of a large information explosion reflecting the interest in the life and work of this remarkable scientist and his unusual fate.

In the pages that follow the names of Einstein and Fridman will be often juxtaposed (because of the closeness of their interest and the importance of the work of Aleksandr Aleksandrovich). Therefore, it is appropriate to mention a remark of Einstein which he made in a conversation with science historian B. Cohen. He told Cohen that "he was as interested in the biographies of scientists as in their ideas. He liked to know about the lives of those who created major theories and carried out important experiments; he liked to know what type of people they were, how they worked, and

 0038-5670/88/070645-21$01.80

what were their relationships with contemporaries."[17] Following this comment, we shall concentrate on the details of the life of our remarkable countryman as a citizen and a man.

2. Practically the whole life of A. A. Fridman was associated with St. Petersburg/Petrograd/Leningrad. His genealogy, especially on his mother's side, can be followed quite far back. However, we shall confine ourselves to the "short-range order." His father, also Aleksandr Aleksandrovich [1866–1908(9)?] was a graduate of St. Petersburg Ballet School. He had become an artist of the Court Ballet of the Imperial Theatre in St. Petersburg, he finished St. Petersburg Conservatory in a class of N. A. Rimskiĭ-Korsakov, and was author of the music of ballets presented in various St. Petersburg theatres. Fridman's mother, Lyudmila Ignat'evna Voyachek (1869–1953) was also a pupil of St. Petersburg Conservatory and she studied piano. She did not become known in her chosen profession, but one could say that she was triply famous: as the mother of a great scientist, as the daughter of the well-known Russian composer and conductor, Ignatiĭ Kasparovich Voyachek (1825–1916), who was professor at St. Petersburg Conservatory, and as the sister of the founder of the Soviet otolaryngology Lieutenant-General of Medical Services Academician V. I. Voyachek (1876–1971).

We shall describe later the active political life of young Fridman. It is remarkable that his nearest relatives—both grandfathers and fathers—served in departments close to the imperial court: his grandfather on the father's side, Aleksandr Ivanovich Fridman (1839–1910) was one of the reservists and a doctor's assistant of the Court Medical Circle, I. K. Voyachek was for many years an organist and conductor of the Imperial Mariinsky Theatre, and his father A. A. Fridman was bandmaster of the Preobrazhensky Regiment of the Life Guards, also attached to the imperial court.

A. A. Fridman was thus born in an artistic St. Petersburg family in June 1888. For biographers of important people it is an interesting task to unravel the confusion of dates of birth given in various sources. In the case of Fridman the date usually quoted is June 17, without indication whether the old or new style calendar is used. The starting date should be that published back in 1927 in Fridman's autobiography (curriculum vitae) and written by him in 1925. Surprisingly, Aleksandr Aleksandrovich made a mistake, as demonstrated by the preserved birth certificate, which says:

"In consistory records of the Vvedensky Church of the Semenov Life Guard Regiment for 1888 in the first part on births under item 182 for males there is an entry:

"Son named Aleksandr was born to Aleksandr Aleksandrovich Fridman, artist of the ballet company of the Imperial St. Petersburg Theatres, and his wife Lyudmila Ignat'evna, both of Orthodox faith, on 4th of June of the year 1888 and was christened on June 9. The godparents were Aleksandr Aleksandrovich Oblakov, artist of the ballet company of the Imperial St. Petersburg Theatres and Miss Maria Aleksandrovna Fridman, daughter of governor's secretary.

This is certified and confirmed by the church seal of the Semenov Regiment

Archpriest Sergeĭ Bogoyavlenskiĭ and psalm reader Ivan Fedorov".[3)]

It follows that Fridman was born on the 4th of June and if we correct the date to the new style (4 + 12 = 16), then

A. A. FRIDMAN
(1888–1925)

FIG. 1

the curriculum vitae is inaccurate (we shall see later that this is not the only inaccuracy) and 100 years from the date of birth of Fridman passed on June 16, 1988.

The marriage of Fridman's parents did not last; it was dissolved (in 1897) and the young boy was raised by his father and relatives on his father's side: his grandfather A. I. Fridman and the latter's daughter M. A. Fridman (also a pianist). He became reconciled with his mother only after the Revolution and the echos of the family unhappiness were sensed by me in 1971 when talking to Prof. V. I. Voyachek who listened suspiciously to my questions.

In the same year 1897 Aleksandr Fridman entered the Second St. Petersburg High School located not far from the Isaac's Square about a 20-minute walk from his home at 35 Moĭka Embankment, where at the time he was living with his father. This house is state-protected as an architectural monument of the previous century and is located in one of the most beautiful parts of the city conserved in its unchanged form during the past century.

The State Historical Archives of Leningrad have extensive materials on the Second High School, which was the oldest in the city. Examination of the material in this collection for the years when Aleksandr Fridman was at the School provides a full picture about Fridman's teachers and pupils in the same class. Among the teachers one should mention particularly the headmaster of the School A. I. Davidenkov[4)] and teachers of mathematics (Ya. V. Iodynskiĭ and P. N. Genzel') and physics (I. V. Glinka). Minutes of the teachers' meetings reveal the progressive views of A. I. Davidenkov and of his skillful leadership of the School in the turbulent years of the 1905 Revolution, the years of strikes, clandestine meetings of students and school pupils, and riots. Davidenkov was always protecting the interests of his pupils. Iodynskiĭ provided a brilliant characteristic of Fridman attached to his school-leaving certificate and transmitted to the university. I. V. Glinka, whose personal life is evidence of his active sympathies with progressive forces, came out against the autocracy in 1905; he was an outstanding Russian teacher, author of textbooks on physics published in the first decade of the century. P. N. Genzel' also

wrote a book on algebra for high schools.

Fridman's class was very strong in ability: five members of this class received gold medals. Naturally, Fridman was among the medallists and so was his closest friend, Yakov Tamarkin, later coauthor with Fridman of a paper on Bernoulli numbers written while still at school and published by D. Hilbert while the authors were still pupils (Ref. F1).[5] Records of the progress (or lack there of) of this class, reported annually to the teachers' meeting can be used to follow the dynamics of Fridman's studies. He ended the first year with overall just "satisfactory" results (this also included arithmetic, but with "excellent" marks in religion), but beginning from the third year he and Tamarkin were the best pupils invariably with "excellent" markings. The documents of the Second School kept in the collection include also the annual conduct books. Quotations from these conduct books were frequently (though not always) used to supplement the school-leaving certificate and were presented by the candidates to a higher education establishment which they wanted to join. However, I looked in vain to find the name of Fridman in the yearly records of those pupils who committed offences which were sometimes dramatic and now seem just funny. Young Fridman was clearly not a naughty pupil in the early years at the School, and when he became older he became totally preoccupied by mathematics and physics on the one hand, and by his political activity (which was far from naughtiness) on the other. Only once does his name appear in these conduct books and it was presented by one of the teachers to the meeting describing the behavior of the pupils during the turbulent years of October 1905. The record says: 'on October 13th, during a meeting, father of Fridman, pupil of the eighth year, called out his son and holding his hands begged him to return home saying "Your mother is ill, let's go!".[6] The pupil tore away from his father with the words "My colleagues are dearer to me" and returned to the meeting.' We must stress here Aleksandr Fridman's answer to his father showing solidarity with his colleagues, which is very characteristic of him. Following Fridman junior, we can doubt the arguments of Fridman senior about the state of health of his stepmother: a claim of illness of a relative is the usual and not a very original way of making young ones see reason!

The revolutionary activities of A. A. Fridman were recalled by his colleague (later Academician) V. I. Smirnov. He wrote that in the home of his grandfather Fridman on the Dvorets Embankment, occupied by persons who were in one way or another in service of the imperial court, the grandson kept revolutionary leaflets which he produced with his colleagues. S. A. Dianin describing in his book the revolutionary youth of St. Petersburg/Petrograd[18] reported that Fridman and Tamarkin participated actively in the revolutionary movement of schoolboys and were members of the Central Committee of the Northern Social-Democratic organization of St. Petersburg high schools. Both of them were following the traditions or perhaps customs of these years and had their own party names: Fridman was called "Lilovyĭ" ("purple"; it is now impossible to decipher the origin of this name!). The Central Committee met every week, on Sunday evening, at the home of one of its members. We shall just note that the Central Committee was linked directly to the St. Petersburg Committee of the Russian Social-Democratic Workers party and that the schoolboys had three duplicators which they used to produce leaflets in hundreds of copies.

After graduating from high school, Fridman became much less politically active and he concentrated his work on the lectures which were attracting him more and more. However, this first inoculation which he received during the years of the 1905 Revolution was not lost and it was manifested subsequently by his direct participation in the organization of the Russian air force during the First World War, his activity at Perm̀ University, as well as titanic efforts to organize Soviet science in the last five years of his life.

3. In 1906 A. A. Fridman was admitted to the Mathematics Department of the Physicomathematical Faculty of St. Petersburg University, as expected. A happy circumstance was that it is in this year that V. A. Steklov became a professor at this university and this was the man who probably had the greatest influence on Fridman's progress.

Our information on Fridman during his university years comes primarily from his extensive documented work retained in the University Collection and transferred to the Leningrad Archives mentioned earlier. I would like to quote some extracts from this documentation in the form of a list of books which Fridman and Tamarkin studied when still at school and which they brought with them as first-year students. This list was provided much later by Fridman in one of his annual postgraduate reports. It consisted of the following sections: theory of numbers (work of R. Dedekind, P. Dirichlet, J. Lagrange, A. Legendre, L. Euler, and among the Russian scientists E. I. Zolotarev, A. A. Markov, Yu. V. Sokhotskiĭ, and P. L. Chebyshev); analysis and special functions (J. Bertrand, R. Baire, A. Cauchy, C. E. Picart, J. Serret, and others); mechanics and mathematical physics (P. E. Appell, W. Weber, R. Clausius, H. Poincaré, G. Riemann, etc.); geometry (L. Bianchi, G. Lamé, K. A. Andreev, and B. Ya. Bukreev). Fridman included the following comment about this section: "unfortunately, in our knowledge of synthetic and non-Euclidean geometry (by "our" the author means together with Ya. D. Tamarkin—V. F.) there are serious gaps, but we hope to fill this in later studies." The next comment is "at the end of our time at high school and in the beginning of our university course we were interested in the foundations of our science. We studied the following works . . ." and this is followed by an impressive list of books by E. Borel, P. du Dubois-Reymond, R. Dedekind, G. Cantor, and A. Schoenflies. The list ends with books on the theory of probability and finite differences (V. Ya. Bunyakovskiĭ, A. A. Markov) studied by Fridman and Tamarkin.

We deliberately cited the authors of the books above (dropping the names of their works), to give an idea of the rate and volume (increasing from year to year) of Fridman's studies. In the reports on his postgraduate work (1911–1913) Fridman always gave equally "frightening" lists of books he studied (sometimes with his colleagues).

There is no direct information on what exactly Fridman read during his student years, but this information can be obtained from his record book (called "Notes on the attended courses"). We find from his record book that he attended the full course of mathematics read by professors Yu. V. Sokhotskiĭ, D. F. Selivanov, I. I. Ivanov, and V. A. Steklov (integration of equations with partial derivatives), N. M. Gyunter, etc. Physics was taught to Fridman and his colleagues by O. D. Khvol'son and N. A. Bulyakov and me-

chanics by D. I. Bobylev. In all these subjects Fridman received the top grade of "excellent." Against some of the courses in mechanics and mathematics (all the names of the courses and of the professors are in print-like script) and there is a note "excused." One would expect the progress of student Fridman in the study of such subjects to be so evident to professors that they did not require any formal tests.

We shall now return to Vladimir Andreevich Steklov (1864–1926). He not only was a brilliant mathematician and science organizer, but also had musical and literary talents. He wrote outstanding books on Lomonosov and Galileo, as well as lively and interesting descriptions of his travels beyond the ocean. For our purposes it is more important that for over two decades Steklov wrote daily notes in his diary. His diaries are stored carefullly at the Leningrad Division of the Archives of the USSR Academy of Sciences. Studies of these diaries make it possible to view a whole epoch through the eyes of a careful observer and to judge the St. Petersburg/Petrograd science, the life at the Academy of Sciences and at the University through the eyes of a prolific and energetic scientist. These fluent notes require a careful editor and commentator, and can provide Fridman's biographers with an invaluable source of information on this young scientist. The first mention of A. A. Fridman is in a note of Steklov dated January 13, 1908. He writes on this day: "at 4 pm I met Tamarkin and Fridman (students) who brought further lectures on integral calculus which they recorded. They took away the corrected (i.e., quickly reviewed) versions. There is no possibility for a thorough revision! They said that they will come to a lecture on the 16th. They asked whether it is possible to have the Mathematics Circle approved without a leader. I said submit a proposal. We shall see!"

We can see that there is an independent circle of mathematics students where they don't want to admit teachers. Such circles existed in those years at all the faculties of the University. Much information on circles of this kind can be found in the printed minutes of the meetings of the Council of St. Petersburg University for 1907. They include circles on physics, aeronautics, political economy, studies of the Roman law, hunting and nature: over twenty different circles with a variety of programs. They combined to form an Intercircle Organization chaired by V. L. Komarov, the future president of the USSR Academy of Sciences and at the time associate professor of the university. A. F. Gavrilov writes that Fridman was an active member of some of these circles, obviously of the physics and mathematics ones: "I recall his lecture at one of the circles on the "Channels on Mars," because the discovery of channels was published not so long ago. Aleksandr Aleksandrovich ended his lecture with the following words which were greeted with loud applause: "The channels appeared almost abruptly, at least they were constructed very fast. Does not this mean that socialism is already established on Mars?" (Ref. 19, p. 418). This is a good illustration of the frame of mind of the students and of Fridman himself!

Authorization of the Mathematics Circle without a leader was not obtained: the official leader was Prof. D.F. Selivanov, but he did not appear at any of the meetings so that students attained the desired goal if not de jure then de facto.

Fridman belonged to a physics circle already in his high school. The school circle met once a week in the physics laboratory and reports were presented at these meetings; the circle also published its own journal. Fridman attended the meetings of the circle at his school when he was already a university student and in particular he presented a report on the nature of ball lightning.

Fridman, Tamarkin, and other students of Steklov visited him at home approximately once a month and this went on for many years.

The note of Steklov dated January 13, 1908 mentions lectures written down by Fridman and Tamarkin. In the Physicomathematical Faculty there was for some time a Publications Commission whose task was to print the course of lectures of university professors. Fridman was one of the active members of the office of this Commission. The office prepared a selection of lectures, negotiated with professors, assigned able students to be lecture compilers, and dealt with the printers who published the lectures. A major role in this activity was the financial side: work on preparation of the courses for publication was relatively well paid and this was a great help to needy students. The courses prepared and published by the Commission have been preserved: by law copies of the courses had to be deposited at the Public Library and at the University Library. Unfortunately, the title pages of these courses do not give the names of the compilers, but it is known that Fridman prepared for publication D.F. Selivanov's "Differential Calculus" (there were four editions of this book) and also his "Integration of Functions"; he also dealt with "Higher Algebra" of Yu. V. Sokhotskiĭ and "Theory of Numbers" of I. I. Ivanov (in the last case the compilers were Fridman and Tamarkin). The diary notes of V. A. Steklov also mention the publication of "Equations with Partial Derivatives."

In the case of Fridman the educational activity in the Publications Commission was also largely stimulated by the financial difficulties after the death of his father[7] and retirement of his grandfather. In the search for paid work he wrote reviews for the journal "Russkaya Shkola" (Russian school) published in St. Petersburg (Refs. F2–F4), was a proofreader at the editorial office of "Zhurnal Russkogo Fiziko-Khimicheskogo Obshchestva" (ZhRFKhO), which is mentioned on the title page of one of the volumes, and wrote reviews for this journal (Refs. F7–F9, F11, F12, and F22). Among these reviews of Fridman one should mention his response to a book on the Norwegian mathematician Niels Henrik Abel (Ref. F10). As it frequently happens, Fridman addresses those parts of Abel's life described by the author of the book (L. de Pesloyuan) which found resonance in his own thoughts and circumstances: "In the life of a scientist, apart from his activity as a thinker, one should always distinguish two very important aspects: the material situation and the personal life" writes the reviewer. Speaking of the material circumstances, Fridman points out that "this great scientist had to be satisfied with 40 kopeks per day, which was a very unsatisfactory sum." This review of the biography of Abel was written by a mature man (Fridman at that time was 23), who was a thoughtful and discriminating reader.

In autumn 1910 Fridman experienced the greatest financial difficulties. At that time he was commissioned to write a paper in one of the collections which were being prepared by V. K. Lebedinskiĭ. The collections were on the physics of electromagnetic oscillations and included both translations of classical papers on the subject (Hertz, Kel-

vin, etc.) and reviews written by Russian physicists. In the second collection (there were six in this series) it included a paper by Fridman "Integration of second-order nonlinear equations"—Ref. F6) based on an equation describing processes occurring in an oscillatory circuit. The paper is methodological in nature and is attractive by its clarity and perceptiveness of the presentation. He discusses thoroughly some of the statements accepted intuitively and without protest, without even bothering about the need of their rigorous justification. For example, Fridman shows that the usual form of the solution of such an equation represented by a linear combination of two particular solutions is not only sufficient, but also *necessary*: it follows rigorously from the structure of the initial equation. The paper is interesting also because it can be regarded as a prologue to the original investigation of the characteristics of the discharge of a capacitor tackled by A. A. Fridman and V. I. Smirnov somewhat later (Ref. F15).

Some idea about the difficult conditions under which Fridman was living as a student and postgraduate can be gained from the official papers directed by V. A. Steklov to the Rector of St. Petersburg University. It includes a petition to retain Fridman and two of his colleagues in the same course, Ya. D. Tamarkin and V. V. Bulygin, as postgraduates in preparation for professorial positions. This is what V. A. Steklov wrote at the end of 1910: "I have the honor to present to your Excellency, in accordance with Article 47, Sec. IV, Subsec. 4 of the University Statutes of 1884 a request to retain for two years in the Department of Mathematics of St. Petersburg University Aleksandr Aleksandrovich Fridman selected by the Physicomathematical Faculty for preparation to professorial grade: 1) from September 1, 1910 to January 1, 1911 and to give him a stipend of 400 roubles for this period. I am attaching eight documents, diploma of the first degree . . . " and so on. The request was accompanied also by a separate testimonial in which Fridman and Tamarkin are praised very highly: "In their abilities and industry both of them are equal and already impress one as being young scientists rather than students who have just completed their university course."

This request was accepted and Fridman received a fairly high stipend (100 roubles per month; the monthly salary of a university professor was about 300 roubles in the early decades of this century) and Fridman remained in the Physicomathematical Faculty. He had to write an annual report on his work and his supervisor, V. A. Steklov, had to judge his progress. All these materials have been preserved. Steklov usually wrote simultaneously concerning the aforementioned three mathematicians and we therefore frequently find that the beginning of the same (usually handwritten) testimonial is in the personal file of Bulygin, continuation in Fridman's file, and the conclusion in Tamarkin's file. Sometimes Steklov added to these three the names of V. I. Smirnov, M. F. Petelin (also former pupils of the Second St. Petersburg High School, who were a year ahead of Fridman), and Ya. A. Shokhat (who was a year younger), known in the West as J. A. Shohat. We shall now quote the most eloquent excerpt from one of such testimonials of Steklov (dated 1912): "It should be noted that the class of 1910 is rather exceptional. In the class of 1911 and among the students of the fourth year of the previous class there is none equal in learning and ability to Messrs. Tamarkin, Fridman, Bulygin, Petelin, Smirnov, Shokhat, and others. In my 15 years of teaching at Khar'kov University I never met such a case. This favorable circumstance should be utilized for the benefit of the University." The mathematicians listed by Steklov should be supplemented also by the names of A. S. Bezikovich (Besicovitch) and A. F. Gavrilov.[8] The former was younger and the latter was approximately coeval with each of the "magnificent six." For completeness, we must mention a later pupil of V. A. Steklov, who although was not a friend of the just-listed young scientists, but worked with Steklov in 1917–1918. This was a colleague of Fridman from the high school political activity, S. A. Dianin (in the thirties he worked in a theoretical division of one of the institutes of the Consortium of Physicotechnical Institutes headed by the institute now known as the A.F. Ioffe Physicotechnical Institute in Leningrad).

The Fridman's postgraduate reports are striking, as already mentioned, because of the lists of the mathematical literature mastered by him. They also include information on his own independent work on mathematical physics and mechanics, partly published on various occasions in Russia and abroad. For example, during the first years of his postgraduate post he carried out the already mentioned work on an oscillatory discharge of a capacitor (together with V. I. Smirnov). In cooperation with M. F. Petelin, Fridman solved the problem, posed sometime ago by W. Bjerknes, of determination of the law governing the changes in the volumes of two spheres immersed in a liquid and pulsating in it so that the interaction between them obeys the law of the inverse square of the distance between them (Ref. F14). Some of this work dealt with the theory of elasticity, specifically with isodynamic surfaces in a solid (Ref. F13) and elastic equilibrium of bodies. We shall quote here from what is an interesting comment from our point of view, made by Fridman in the report for 1911/12 (in the second year of his postgraduate studies): 'In derivation of the equations of hydrodynamics and of the theory of elasticity we concluded that the equations of motion of a viscous liquid represent a transition stage between the equations of a liquid without internal friction and the equations of the theory of elasticity. We were struck (apparently this refers to Fridman and Tamarkin—V. F.) by the circumstance that the theory of elasticity does not require an equation of continuity, which is of major importance in hydrodynamics, because without it the problems in hydrodynamics would have been indeterminate. On the other hand, in the theory of elasticity this equation is quite superfluous and may not be satisfied at all, since the frequently used equations of the theory of elasticity have a unique solution. It is quite clear that the "equation of continuity" simply represents conservation of the mass of a substance and it should apply to any deformable substance whether it is a liquid or an elastic solid.'

Fridman studied the work of Jacobi and Kirchhoff and used them as the source material for further independent research. He studied also "Lectures on Hydrodynamics" of N. E. Zhukovskiĭ (Joukowski) and he simplified the proof given by this outstanding Moscow professor that any flow of an incompressible liquid characterized by a potential velocity is steady-state. He also considered the problems of planar flow and jet formation. Apparently, he reported his research to Zhukovskiĭ because we shall see later that the latter knew the young Petrograd scientist already in 1916–1917 and had

a high opinion of him.

In cooperation with Ya. A. Shokhat, Fridman worked on problems in technical mechanics (calculation of the sag of a rectangular plate clamped at the edges and subjected to constant and uniformly distributed surface load). In this case the young scientist cooperated with professor of the Naval Academy I. G. Bubnov. Tamarkin and Fridman studied the complex case of bending of a beam under a variable load. He was interested in the problems of approximate calculations and in this case he cooperated with mathematician N. M. Krylov (a future academician).

Developing the problem outlined by V. A. Steklov in his doctoral dissertation, Fridman investigated special cases of the solution of the Laplace equation and dealt successfully with the task set by his supervisor (Ref. F11). Steklov, commenting on the work of his pupil, mentioned particularly the soundness and the elegance of the execution of this task.

We can see that the spectrum of interest and research of Fridman was very wide. One should recall that this research was carried out against the background of intensive work in preparation for examinations for the master's degree because this degree opened up the opportunity to become a professor.

The annual reports of Fridman as well as those of Bulygin, Petelin, Tamarkin, and Shokhat contain information that, during their years as postgraduate students, they founded a Mathematics Circle, which met every week for collective discussion of difficult mathematical problems and for working together on current mathematical literature, mainly in the form of books. The books particularly valued by the members of the Circle were the course of mathematical analysis by E. Goursat, mechanics of P. E. Appell, and the work of N. E. Zhukovskiĭ (Joukowski). Basically this St. Petersburg circle of seminars was similar to the Olympia Academy of Einstein and his colleagues, which met regularly in Bern some years earlier. University professors did not participate and this was also true in Bern. The St. Petersburg scientists also tackled philosophical problems, particularly those related to the foundations of mathematics. There was another similarity between the Olympia Academy and the members of the Mathematics Circle and this was their love of music. V. A. Steklov frequently mentions in 1911–1916 the musical evenings at his home, where he invited his young pupils including naturally Fridman. Smirnov and Tamarkin played the piano excellently. The host (this is clear from his memoirs) had a good voice (some thought that he could have become an opera singer) and he sang to the accompaniment of the piano. Fridman was not a pianist, but he understood and liked music. Prof. N. N. Mirolyubov (in a letter to the author of the present paper) recalled that in the early twenties he frequently saw Fridman and Tamarkin at the concerts of the Petrograd Philarmonia and that they usually both followed the symphony orchestra with music scores in their hands.

Naturally, Fridman and his colleagues did not suspect the existence of the Olympia Academy. However, they knew while still students that there was in St. Petersburg a circle devoted to new theoretical physics organized in 1908 by P. S. Ehrenfest who came from Germany. Fridman, who was eight years younger than Ehrenfest, regarded him as one of his teachers. He and Tamarkin (and another mathematician, S.N. Bernshteĭn) regularly attended the meetings of the Ehrenfest circle, which included physicists A. F. Ioffe, D. S. Rozhdestvenskiĭ, K. K. Baumgart, V. R. Bursian, Yu. A. Krutkov, L. D. Isakov, T. A. Afanas'eva-Ehrenfest, V. M. Chulanovskiĭ, and some others. The earliest of the three preserved letters of Fridman are addressed to Ehrenfest and are evidence of the sympathy with which Ehrenfest treated his young colleague and helped him by "word and deed." Apparently it was Ehrenfest who arranged for proofreading work for Fridman (for "Zhurnal Russkogo Fiziko-Khimicheskogo Obshchestva") and recommended books for review. Fridman and Tamarkin visited the Ehrenfests frequently at home and the eldest daughter of the Ehrenfests gave them affectionate nicknames which stuck later. This girl called Tamarkin "little elephant" because of his tendency to stoutness and Fridman was called by her "little spider" because of the thin "musical" fingers and his lean body.

Fridman started teaching during his postgraduate years: first at the Mining Institute and then at the Institute of Transport Engineers. Some documentation about his work as a teacher was preserved in the form of a book of problems on higher mathematics, the first edition of which appeared already in 1912 and a later edition in 1931 (Refs. F45 and F69).

As already mentioned, immediately after graduating from St. Petersburg University, Fridman began to prepare for exams for the master's degree (in pure and applied mathematics). Materials on the examinations for this degree have been preserved and we can easily establish the days when he took various tests. Using these dates, if we turn back to the diary of Steklov, we can see that the latter regarded the examinations of his pupil as important events and reported the successes of Fridman and his colleagues by mentioning the excellent results in these exams.

Finally, in 1913 Fridman began to work at the Main Geophysical Observatory (which was then headed by Academician B.B. Golitsyn) or, more exactly, at one of the divisions of the Observatory located in Pavlovsk which was a suburb of St. Petersburg. It was during this period that Fridman's interest in physics of the atmosphere, in hydrodynamics and aerodynamics became set. This lively interest (concerning the origins of which we can only speculate) is surprising in a man who beginning from his school days and right through his time as an undergraduate and a postgraduate was interested in the theory of numbers, analysis, and mathematical physics and would seem to be preparing for a career of an indoor scientist. However, his interest in the fifth ocean has remained a distinguishing mark of the rest of the life of Fridman.

In Pavlovsk, Aleksandr Aleksandrovich was not only preoccupied by theoretical meteorology (Refs. F17–F21). He undertook flights on airships, participated in sending up of pilot balloons and kites[9)] carrying instruments for the determination of the direction and velocity of wind and for the study of other processes taking place in the atmosphere. His work was supported by B.B. Golitsyn, on whose recommendation Fridman was sent to Leipzig, where W. Bjerknes (Director of the Meteorological Institute of Leipzig University) and H. T. Hesselberg were working at the time (Ref. F19). Fridman returned to St. Petersburg on the eve of the First World War.

4. Although A. A. Fridman's transition from pure mathematics to applied meteorology may cause some sur-

prise, even more unexpected is his metamorphosis during the First World War and the subsequent years. Aleksandr Aleksandrovich demonstrated high organizational abilities and undoubtedly enjoyed activities of this kind.

V. A. Steklov writes the following in his diary on August 6, 1914: "Fridman appeared unexpectedly. He is going to war in the air force and is seconded there from the Main Geophysical Observatory." In his autobiography Fridman wrote in 1925 that "I aimed to introduce aerological observations into air force practice and in this way to help aviation to the best of my ability, and at the same time to increase the number of aerological stations. I volunteered for the air force with the permission and approval of the Director of the Observatory B. B. Golitsyn. I worked initially on the northern front near the towns of Osowiec (Osovets) and Lyko, and then also on the other fronts organizing aerological observations and generally the aerological service" (Ref. F62; Ref. F73, p. 387).

Thus began the war odyssey of A. A. Fridman which lasted more than three years. Its stages can be judged by his published letters to V. A. Steklov and B. B. Golitsyn. These letters showed that Fridman was a very brave man, unusually devoted to science, who found time and strength for science in the lulls between the battles in which he participated from the first weeks of the war. Fridman was frequently sent to Petrograd and every time he visited his teacher V. A. Steklov, as noted by the latter in his diary (during the war months of 1914 we find such notes on September 2, November 2, and November 23; there are many such mentions also in the later years). For example, on November 2, 1914 Steklov writes: "Walking to the University I met Fridman! He was sent by the Grand Duke Aleksandr Mikhaĭlovich to organize the air reconnaisance in various armies. He is one of the candidates for the Georgiĭ Cross (the Cross of St. George, the highest military honor in pre-revolutionary Russia—translator) fourth class, for successful reconnaisance. He told me much. We had dinner at our house. He says that the morale of the soldiers is good, but naturally they would like to finish the war successfully as soon as possible. He plans to spend probably two weeks here and is then traveling to L'vov . . . . On the whole he is satisfied with the situation and his work."

If we compare the original of Fridman's autobiography mentioned earlier, stored at the Archives of the Academy of Sciences of the USSR in Leningrad, with the printed text of 1927 and 1966, we can see that some parts of this autobiography have been omitted. They include also the phrase from which it follows that he was awarded the Georgiĭ Cross. Other sources indicate that he received also a second Georgiĭ Cross and was also awarded the hononary Georgiĭ sidearms (Ref. 19, p. 421; Ref. F63, p. 343). The graduate of St. Petersburg University with a master's degree turned out to be a brave airman!

At the front Aleksandr Aleksandrovich was also preoccupied with the problems of the theory of accurate bomb aiming and compiled suitable tables the use of which increased considerably the probability that the bomb hit the target. He discusses the calculation method in his letters to Steklov. It happened that at the fortress of Peremyshl' (Przemyśl) in Western Galicia held by Austrians in February 1915 one of the soldiers was a future professor and meteorologist G. Ficker (who met Fridman later in the peaceful twenties). Ficker, answering Steklov sending news of Fridman's death, wrote in 1925 that the only successful targeting of a bomb which he witnessed in Peremyshl' was from an aeroplane piloted personally by Fridman.[10] In the articles published about Aleksandr Aleksandrovich it is mentioned that when Peremyshl' was being bombarded and targets were hit, the German soldiers said "Fridman is flying today"[11] (Ref. 12, p. 433).

We shall give at least two excerpts from the frontline letters of Fridman to Steklov (Ref. F73). On February 5, 1915, he wrote:

"My life is fairly quiet apart from such happenings as a shrapnel exploding at a distance of twenty steps, explosion of a detonator of an Austrian bomb at a distance of half a step, when I got off nearly scot-free, and a fall on my face and head resulting in nicking my upper lip and suffering some headaches."

On February 28, 1915 he wrote: "I was able to check personally my ideas (on accurate bomb targeting using the tables he compiled—V. F.) during one of the flights above Peremyshl'; we found that the bombs fell as predicted by the theory. To confirm the theory finally, I'll soon fly again" (Ref. F73, p. 341). It should be mentioned that the equation for the motion of a bomb derived by Fridman included a certain coefficient related to the resistance of air to the bomb and the magnitude of this coefficient had to be found empirically: it turned out to be different for bombs of weights amounting to 5, 25, and 40 Russian pounds, which were "hurled" by Fridman from the aeroplane.

The reputation of Fridman as airman was made: he was sent as a teacher to the school for airmen in Kiev.

To be objective, we must point out that although in all the previous notes of Steklov we find Fridman in the best light, beginning from the second half of 1915 there appear notes of some annoyance: the teacher finds that his pupil has some negative qualities. Vladimir Andreevich shared his feelings and fears with Ya. D. Tamarkin who was the closest friend of Fridman and asked that their conversations should not be kept secret from Aleksandr Aleksandrovich but rather should be passed on to him. The next time that Fridman came to Petrograd, he visited Steklov again. This is what Vladimir Andreevich wrote on February 14/27, 1916 in his diary: 'Fridman came Tamarkin at 3 pm. Tamarkin talked to Fridman about his not always satisfactory behavior; Fridman came to "explain himself." I pointed out my unhappiness about his growing bragging and boasting, beginnings of careerism . . . . I said that he should watch himself and if possible control himself and suppress these low instincts which are in his nature . . . . It is useful to remonstrate[12] with him. Maybe this will have some effect.'

We shall now try to defend Fridman. Basically a bookish man, who has just started practical work at the Pavlovsk Observatory, finds himself in August 1914 under very different circumstances, sees death alongside him, is exposed to mortal danger because aeroplane flights flights at that time even under peaceful conditions were risky undertakings and much more so during the war. The situation is that the enemy tries to bring down an aeroplane and to kill you if possible. Fridman left his colleagues working in the way he had been accustomed to on manuscripts and books. And he at the front is "working" in the sky above Peremyshl'. In the corridors of the University he meets his colleagues and his

professors and it is as if there is no war. But, in the sky of Galicia he meets the enemy aeroplanes face to face.

At the same time he is continuing his research and is doing this successfully. His thoughts and work give rise to a number of papers (Refs. F23, F24, F26, and F27), and in no way is he falling behind his colleagues. In addition to this he gets the Georgiĭ Cross for bravery and a very concrete visible and tangible success in the form of establishment of aviation workshops, construction of new instruments, and calculation of tables needed for accurate bomb aiming at the front (Ref. F28). From a destitute undergraduate and a postgraduate student making both ends meet with difficulty he becomes in 1916 a man with a salary comparable with or sometimes exceeding that of a professor (it is probably during his Petrograd trips that Fridman assembled his excellent library of German, English, French, and Russian classics on mathematics and mechanics, which has been preserved to this day). We think that, instead of boasting, Aleksandr Aleksandrovich simply told Vladimir Andreevich Steklov with inner satisfaction and justified pride what he had been and what he has become assuming that the teacher would share with him this satisfaction and pride. He turned to his friends with frequent requests for help (and for this he was also told off by Steklov) because firstly that is what friends are for and should help one another and, secondly, we can be sure that he was as ready to accept and fulfill their requests and errands. We can say that with assurance because later at Perm' University it was he who didn't spare time or effort to help Gavrilov and Tamarkin to come to Perm' and then later in Petrograd it was he who, being himself the first one to enter the Naval Academy, arranged for Gavrilov, Smirnov, and Shokhat to join him there.

And Steklov scolded Fridman because he liked him: after all it is said that only people who are close can have real quarrels. If Vladimir Andreevich had been indifferent to his pupil and found Fridman unusually smug and a limited man he would have given him up as a bad job. Instead, in a fatherly manner he warned him of the danger of finding himself rejected for his boasting after he had survived the dangers of the front. Aleksandr Fridman had to admit that whether the misgivings of his teacher were right or not, they were dictated by worry about his fate and by care for him. Therefore, he did not break off his relationship with Steklov and as before continued to come to him for advice and encouragement.

Just a small piece of evidence remained indicating that Aleksandr Aleksandrovich was at least slightly angry with Steklov ("Aleksandr, you are angry, this means . . . "). The evidence is this: up to February 1916, the letters to his teacher began with "Much-esteemed and dear Vladimir Andreevich," but after their conversation in February, the word "dear" disappeared and reappeared only at the end of 1918.

During his time in Kiev (1916–1917) Fridman arranged in the Military School for airmen observers courses on air navigation, navigational instruments, and services for weather forecasting at the front, and repair of aviation instruments in the army at the front.

The work of this School in Kiev and of the Central Air Navigation Station, commanded by Ensign A. A. Fridman, was inspected in May 1916 by Air Lieutenant General A. N. Krylov, who was appointed to command the Chief War Meteorological Administration of the Russian army. The file of Alekseĭ Nikolaevich Krylov in the Archives of the USSR Academy of Sciences includes "Summary of Lectures on Air Navigation" prepared by Fridman (Ref. F25) (which is a booklet of 43 pages), but not a single copy of this booklet has been found in the main libraries of Leningrad or Kiev! In the same file can be found some other printed instructions prepared for the staff of the Air Navigation Station and signed by Fridman.

In addition to this intensive war work, Fridman gave a number of lectures at Kiev University intending to become an assistant professor at this university. In accordance with the regulations then in force, he started with what were known as trial lectures. One of them was entitled "Motion of a liquid at a variable temperature" and another "Curvilinear coordinates." Aleksandr Aleksandrovich was appointed an assistant professor and this was the position which he accepted at Perm' University in 1918 with the support of V. A. Steklov. Another area of activity of Fridman in Kiev was a local physicomathematical society (the members at the time were B. N. Delone, A. P. Kotel'nikov, V. P. Linnik, and O. Yu. Shmidt). Here again Fridman frequently presented lectures and became a full member of the society.

In 1917 he was transferred to Moscow.

5. Back in 1915, workshops for repair of air navigation instruments were organized in Moscow. Among the small staff assembled in these workshops was N.N. Andreev[13] recommended by Fridman whom he knew as a younger colleague at the University and who became the author of unusually lively memoirs of Aleksandr Aleksandrovich.[20] Soon after it was decided to replace the workshops with a factory and in summer of 1917 the whole staff, including naturally A. A. Fridman, of the Central Air Navigation Station was transferred from Kiev. Andreev recalled that as soon as he began to work in the workshops, it was sufficient to mention the name of Fridman, to N.E. Zhukovskiĭ (Joukowski) to ensure the latter's support of a number of projects undertaken at the workshops, particularly on the design and equipment of a small wind tunnel for the calibration of instruments.

At this "Aviapribor" factory Fridman was head of one of the divisions and later Director. In the course of over six months spent in Moscow, Fridman strengthened the links between the factory and N. E. Zhukovskiĭ, organized the mathematical office where improved tables for bomb targeting were compiled, and had done much other work. However, when soon after the October Revolution, some hope appeared for the ending of the war and for a period of peaceful growth, the factory's activity was reduced.

Fridman decided to go from Moscow to Perm', where by this time a University was already organized, first as a branch of Petrograd University and then as an independent higher educational establishment, which was the first in the Urals. V. A. Steklov wrote an excellent recommendation for A. A. Fridman and it was this recommendation that undoubtedly played its role in the appointment of Aleksandr Aleksandrovich as a professor of Perm' University. In letters that Fridman wrote to Steklov from Perm' he frequently mentioned that the scientific work he was conducting there was the "payment" for the flattering opinion that Steklov gave in his reference.

Immediately on arrival in Perm' (April 1918) Fridman with his usual energy entered into University affairs. Sup-

ported by the Perm' City Soviet and with the help of industrial establishments in the city, he organized a well-fitted-out mechanical laboratory. The first meeting of the Perm' Physicomathematical Society took place in June 1918. Fridman accepted the very burdensome duties of Secretary. Fulfilling these duties, he organized publication of a journal of the Society and managed to prepare two volumes of this journal for publication (1919 and 1921), and included his own papers (Refs. F29 and F30) as well as papers by his colleagues. A strong team of mathematicians, graduates of St. Petersburg University, assembled in Perm': A. S. Bezikovich (Besicovitch), I. M. Vinogradov, N. M. Gyunter, A. F. Gavrilov, R. O. Kuz'min, and Ya. D. Tamarkin. Over 60 people participated in the work of the Physicomathematical Society, including an astronomer (future academician) G. A. Shaĭn, physicists who were members of the Ehrenfest circle in St. Petersburg, G. G. Veĭkhardt, V. V. Doĭnikova, and others. In one of his letters to Steklov, Fridman mentioned that "... some ability in managing practical affairs frequently serves me badly because my colleagues tend to give me any responsible practical work" (letter dated June 9, 1918; Ref. F73, p.352). In fact, the load on Aleksandr Aleksandrovich was unusually high: during the last part of his stay in Perm' he organized anew the faculty, creating there three divisions and two institutes (mechanical and geophysical), undertook to give a series of lectures on a number of subjects ranging from theoretical mechanics to a general course of physics, including also exercises (on meteorology, descriptive geometry), and laboratory work on mechanics. In addition to that he had many problems which he had to solve as Assistant Rector for management of the university plant. (!). Finally, the last but not the least was his research on mechanics, mathematics, and hydrodynamics (including preparation of his dissertation for the master's degree).

This work was complicated also by the circumstance that during his transfer from Moscow Fridman lost a trunk with books and manuscripts, which he in vain expected to recover. He had to do much writing and many calculations all over again.

Fridman remained in Perm' when it was occupied by Kolchak's army. In the Archives of the USSR Academy of Sciences there is a version of his autobiography (largely the same as that published). We shall give an excerpt from this autobiography not included in the published version: "in the spring of 1919 I was directed by the council of Perm' State University to travel for scientific work during the summer vacation to the Ekaterinburg Magnetic and Meteorological Observatory. This coincided with evacuation of the personnel of Perm' University. In Ekaterinburg because of the difficult position of the Observatory during the transition time, I accepted duties of the head of the Aerological Division of the Observatory and a member of its Council and I performed these duties to the middle of August 1919 when I returned to Perm' University."

Documents found in the Perm' archives indicate that he was frequently attacked by the reactionary professors at the University for his links to the Soviet government, but he did not deny his views or sympathies during the months of occupation. For example, minutes of one of the meetings of the University Council record that in March 1919 both Fridman and Veikhardt voted against a pro-Kolchak anti-Soviet communication from Perm' University to foreign higher educational establishments.

Although it is clear from the letters of Fridman to Steklov written from Perm' that he was enthusiastic about his work there, it appeared that he tended more and more to recognize the need to return to Petrograd. He did so much for Perm' University, that he regarded it as his moral right to carry out his plan to return to Petrograd. As before, the diary entries of Vladimir Andreevich Steklov allow us to establish the time of return of Aleksandr Aleksandrovich to his home city: it was the end of the second decade of May 1820. This was the time of fierce battles with white-guard Poles, the civil war had not yet ended, but the life in the city began to recover slowly. Steklov wrote on May 20, 1920: "Tamarkin and Fridman appeared unexpectedly at 11:30 am; they arrived after 12 days in a special heated freight car." They were bringing books which the library of Perm' University was returning to Petrograd University. They also brought some produce which was nearly confiscated from the two mathematician friends: this was the time of the war against profiteers and bag-men. However, all turned out well.

Fridman asked Steklov to support his application for the position of a junior member of the staff in the Physicomathematical Faculty of Petrograd University. Steklov grumbled about this in his diary, but he did help and he talked to Prof. A.A. Ivanov (astronomer) and already on July 12, 1920 Fridman was appointed lecturer in the Department of Mathematics and Mechanics. Earlier on July 1, he began to give a course on applied mathematics and lead exercises on higher mathematics at the Faculty.

The higher educational establishments of Petrograd opened their doors to those who could not go to university before the Revolution, such as workers and peasants. There was a very serious shortage of lecturers in the city. It is therefore not surprising that Aleksandr Aleksandrovich received invitations from other higher educational establishments. At the Institute of Transport Engineers, with which Fridman had close links before the war, he began to give a series of lectures at the just opened Faculty of Air Transport as a professor of Applied Aerodynamics. In addition to these lectures on aerodynamics and air navigation, he also gave a course on approximate calculations and in 1925 coauthored with Ya. S. Bezikovich (Besicovitch), a book which was republished in 1930 (Refs. F55 and F67). N. M. Gyunter recalls that this was an excellent course.

On the recommendation of A.N. Krylov, Aleksandr Aleksandrovich began to work also at the Physicomechanical Faculty of Petrograd Polytechnic Institute. This faculty was opened in 1919 due to the efforts of a number of scientists in the city, particularly A.F. Ioffe, A.N. Krylov, F. Yu. Levinson-Lessing and others. On August 2, 1920, at the meeting of the Council of the Faculty (the secretary of which at that time was P. L. Kapitza) Fridman was elected (by nine votes against one) Professor of Theoretical Mechanics. In the short available time Aleksandr Aleksandrovich established at this Institute, as recalled by his pupils L. G. Loĭtsyanskiĭ and A. I. Lur'e, a "school of mechanics, educated a large number of pupils, who developed further the ideas and teaching methods of their mentor. The existing (this was written in 1949—V. F.) specialties of hydroaerodynamics and the strength of machines go back in their roots to the school of Fridman" (Ref. 22, p. 84). All the courses on mechanics given by Fridman were saturated with vector

analysis, which at that time was a great rarity. The lectures were compiled by L. G. Loĭtsyanskiĭ and then were used to write and publish a book on "Theoretical Mechanics" (Ref. F51) published by the Naval Academy. Fridman lectured to students of the second and third year of the Physicomechanical Faculty and, according to one of his students, N. D. Devyatkova,[23] in addition to "academic" problems of mechanics he introduced much that was new into the course: elements of nonlinear mechanics, a series of lectures on approximate calculations, and methods for compiling tables. Particularly interesting is the fact that he told his students at the Polytechnic Institute about his original work on the theory of relativity and cosmology.

N. D. Devyatkova recalled also a somewhat unexpected event, which illustrates the manner in which Fridman gave his lectures. He never used any notes, but on one occasion coming to a lecture he said that he had left at home the summary and cancelled the lecture!

However, the teaching activity of Fridman was not limited to just three institutes! A. N. Krylov, who in 1919 was Head of the Naval Academy (from which he graduated about 30 years earlier) invited Fridman to join the work of the Academy: Fridman became his assistant in the Department of Mechanics. As already mentioned, he invited his colleagues—A. F. Gavrilov, Ya. D. Tamarkin, and Ya. A. Shokhat—to join the Academy. A. F. Gavrilov describes Fridman's lectures at the Academy as "most interesting."

One had to know Aleksandr Aleksandrovich to understand that he fulfilled all these duties in a responsible manner. How did he find the time for all this? Were there sufficient hours in a day? After all there was still work in the Atomic Commission of the State Optics Institute (calculations of various models of many-electron atoms, studies of adiabatic invariants) and particularly his research work: on meteorology at the Main Geophysical Observatory and on cosmology!

6. In his autobiography of 1925, A. A. Fridman mentions that in 1922 he was sent abroad to Germany and Norway (Ref. F73, p. 389). This is a slip of the pen (which has reappeared in the subsequent few papers about him): in fact, Aleksandr Aleksandrovich traveled abroad in 1923. It became possible to establish this on the basis of the materials that reached recently the Leningrad Division of the Archives of the USSR Academy of Sciences: they were letters dating back to 1923–1925 written by A. A. Fridman to N. E. Malinina. Before we quote some important excerpts from them, we must go back a little to the past: we shall return to 1911. In July of 1911 A. A. Fridman married Ekaterina Petrovna Dorofeeva, who was a home tutor (this follows from a preserved document in the form of a letter from Aleksandr Aleksandrovich to the Rector of the University with a "very respectful request to allow him to get married"). Such a title of home tutor was obtained by E. P. Dorofeeva at the Higher (Bestuzhev) Course for Women, which was completed by her. Ya. D. Tamarkin wrote to Steklov on July 13, 1911: "The marriage of Aleksandr Aleksandrovich was as unexpected for me as for you. His wife is quite a likeable person, although slightly older than him. At this stage I can say that the marriage had only a beneficial effect on Aleksandr Aleksandrovich; it reduced his usual nervousness, made him more calm, and in any case did not at all disturb our work, which continues almost without interruptions five days per week."

In 1913 Fridman settled with his wife in Pavlovsk close to his work at the Observatory. The Steklovs were their frequent guests. It is clear from the diary of Vladimir Andreevich Steklov that in the subsequent years the evenings held at the Steklov house for students were attended by Aleksandr Aleksandrovich together with Ekaterina Petrovna. In the autumn of 1925 Steklov is specially concerned about obtaining a pension for Fridman's widow, E. P. Dorofeeva. The relevant document (which is in the Steklov collection) is surprising for a number of reasons. First of all, the initial version of the well-known obituary (published in 1927 and 1966)[24] basically formed an extensive preamble to the petition for a personal pension to E. P. Dorofeeva, which was directed to the National Commissariat for Education, dated September 29, 1925, whereas the obituary is dated October 1925. In this petition, Steklov wrote: "He left a widow, Ekaterina Petrovna Fridman, (who was) his direct helper in the continuous scientific and organizational activity of the deceased, carrying out a variety of tasks, proofreading, etc."

We must mention here that two years before this document was composed, A. A. Fridman officially divorced E. P. Dorofeeva and was married for the second time (and this was not only a civil marriage but also a church one, so as to "strengthen it," as Fridman once said) to Nataliya Evgen'evna Malinina, member of the staff of the Main Geophysical Observatory.[14)] Letters to her (1923–1925) are unusually emotional and contain important biographical material. They show that contrary to the widely held views (based on the meager data of recorded memoirs of his contemporaries), Fridman was a man of major passions. Having fallen deeply in love with N. E. Malinina he was unable to be perfectly happy in this second marriage, but suffered with the thought of the grief he had caused to E. P. Dorofeeva and vacillated between the two women who in this difficult situation behaved very nobly. In one of his letters to Malinina, after marrying her, Fridman wrote: "In my path, as a symbol of extreme points of my vacillations there are you and Ekaterina Petrovna." Another letter contains the following phrase penetrating in its sincerity: "I cannot kill myself, I lack the spiritual force to do it." Not long before his death, Fridman learned that Malinina was expecting a child. On July 1, 1925 he wrote to her: "Now all have gone from the Observatory, I remain alone among the statues and portraits of my great and small predecessors, my soul is relaxing after the daily bustle, and I am glad to think that a few thousand miles away there beats a loved heart, a tender soul lives, and a new life is growing, . . . , a life the future of which is a mystery and it has no past."

N. E. Malinina bore a boy who was named Aleksandr in the tradition of the Fridman family. He was born after the death of his father (Fridman's son died in 1983 in Leningrad).

A major part of the exchange of letters with Malinina occurred in 1923 and consists of letters from abroad. We shall now cite some parts of these letters and make the necessary comments.

July 29, 1923. "Berlin is astounding in its contrasts: on the one hand the luxurious wealth, faced with which our piglet profiteers are little boys, and on the other hand the hardship of the middle classes and the poverty of the workers . . . . Queues for butter, potatoes, bread, and sugar are grow-

ing every day. Berlin is undoubtedly on the eve of major events, like the whole of Germany, because it is time to shake off the bourgeois rags and all the clever prejudices. The currency in Berlin has gone mad; recently (only three days ago) a dollar was worth 1 million marks, today it is 4 million and a further rise is expected, so that in three days the mark has fallen 400%; this does not worry those that have dollars, but for Germans living on salaries this is a catastrophe and malnutrition, which is unlike our malnutrition because the shops are overflowing with goods and there are people eating their fill and living in comfort ."

August 6, 1923, Berlin. "I was yesterday at the Stacken airfield and talked to Kurt Wegener: he is a very interesting man and told me much of interest, but to have a clear idea of the German air navigation, one would have to live in Germany for at least a month; their progress in air navigation is hardly reflected in the literature and it is difficult to print anything in Germany . . . . My itinerary is now as follows: up to August 8th I'll be in Berlin, on the 8th I'll travel to Hamburg where I'll be up to the 11th, on the 11th I'll return to Berlin, while in Berlin, I'll go to the Stacken airfield and then once again to Potsdam, on the 13th I'll travel to Lindenberg where I'll be to the 18th, on the 18th I'll return to Berlin and on the 19th I'll travel to Christiania (now called Oslo—V.F.) and Bergen. After approximately a week, on August 26th I'll return to Berlin, on the 27th I'll go to Göttingen, where I'll be for three days up to the 30th or return to Berlin or travel to Fulda in Thuringia, where there is an airfield for gliders. Gliders will soon play a major role in the investigation of the atmosphere and it would be a good idea to learn about the organization of the relevant work."

In the next undated letter but apparently written on August 8, Fridman reports that he saw Ficker, Director of the Prussian Meteorological Institute and discussed with him the possibility of the latter working in Russia. Ficker very much wanted this. After the death of Aleksandr Aleksandrovich, V. A. Steklov officially invited Ficker to become Director of the Main Geophysical Laboratory, but Ficker after long hesitation (reflected in an exchange of letters to Steklov) declined. In August 1923 Fridman arranged to publish the papers of his colleagues in a German meteorological journal. Ensuring such publication in various meteorological, mechano-mathematical, and astronomical journals was one of the tasks which Aleksandr Aleksandrovich set himself. He described this as follows: "If I do the same for mathematical and mechanical papers, then three-quarters of the target of my trip will be fulfilled."

However, the remaining one quarter was very large! First of all, scientific contacts and discussion of work with his colleagues, then, buying of instruments and books, ...

August 11, 1923. "(Alfred) Wegener told me much of interest about his work on the genesis of continents . . . , his hypotheses are well supported by experimental data, continents float in a liquid magma, all the continents were initially together and they gradually separated; such a movement of continents must have had a major influence on the magnetic phenomena and possibly on secular variations, but this is my and not Wegener's view. In Seewarte[15] they showed me operation of an excellent machine for the prediction of tides, which is very ingenious.

Wegener told me that the meteorological division which he is heading has no program of research; when I told him about a program of work at the Theoretical Meteorology Division, he was very surprised and doubted whether such a program could be carried out. I answered that we have been doing it for three years."

On August 12 Fridman returned from Hamburg to Berlin and on the same date he wrote to Malinina:

"The trip was very dreary. It was particularly dreary to look at German forests: tree next to another tree, all carefully aligned, all swept out, there is no twig on the ground, and the twigs are arranged in piles in accordance with size. Confound it! This is not a forest but an archive. This is what infidels converted God's forests into . . . . Candidly, our life with all its coarseness and its shortcomings suits me better than the bourgeois life in Germany. This is undoubtedly because with us capitalism is a necessary evil, whereas in the West it is the Moloch, it is all, an omnipresent god and this is the origin of the numerous prejudices of the West ."

The day of August 18 Fridman spent at Lindenberg Observatory and lectured there on the work of the Theoretical Meteorology Division which he headed at the Observatory in Petrograd. Professor G. Hergessel who met him, introduced him as follows (we are citing here Fridman's letter) ". . . there is nowhere a division such as the Theoretical Meteorology Division of the Main Geophysical Observatory." Fridman added that he arranged for publication of the work of his colleagues in a German journal.

This was followed by Norway, valuable discussions with Prof. Hans Hesselberg, whom Fridman knew from his first trip to Leipzig in 1914. They formulated clearly the plans for future cooperation, in particular, these colleagues (and friends!) agreed to write a book on dynamic meteorology, but fate decided otherwise.

In Göttingen Fridman went to the laboratory of the famous L. Prandtl, Director of the Institute of Hydrodynamics. A characteristic written piece of evidence remained concerning one of their interactions: in 1925 a book was published by a German engineer, J. Ackeret, a colleague of Prandtl; the book was called "Rotor Ship, A New Way of Utilizing the Wind Force" (two rotating cylinders were intended to replace a sail on the deck of such a ship; their interaction with air should provide a driving force). This book appeared in the Soviet Union under the editorship of A. A. Fridman.[25] It is worth noting this book also because the invention of a rotor ship (by A. Flettner) in its own time equally impressed A. Einstein, who devoted a special paper to the subject.[26] Is it not surprising that both Einstein and Fridman were equally attracted not only by the global problems in physics and technology, but also by much more modest topics? We shall now quote another example of similarities between the two men. In 1911, Fridman published an excellent popular scientific paper on "A theory of an aeroplane" (Ref. F12). Some years later, in 1916, Einstein wrote on a similar subject in the journal "Naturwissenschaften" and this paper is included in the fourth volume of his collected works published in the Soviet Union (see Ref. 27). Moreover, in the 1920s Einstein devoted much time to inventions, whereas in the middle 1910s he put forward the idea of a special shape of an aircraft wing, which increases its lift.[28] It is interesting that this idea was tested (without success) in practice. The two scientists could hardly have known of each other's corresponding interests.

During his trip to the West, Fridman bought many in-

struments for the Main Geophysical Observatory and books and journals for its library. The luggage was so large that Aleksandr Aleksandrovich had to return by sea. On September 12, 1923 Krutkov described to his sister an evening spent in Berlin in the company of Fridman and two other friends from Russia (A.B. Feringer, who was working at the Main Geophysical Laboratory, and Academician F. I. Shcherbatskiĭ, a well known specialist on India) said that on the fifteenth Fridman was sailing home from Szczecin.

After a year Aleksandr Aleksandrovich traveled for the third time abroad, this time to Holland. The First International Congress of Applied Mechanics took place in Delft on April 22–28, 1924. Apart from Fridman, the Soviet Union was represented by A. I. Ioffe, A. N. Krylov, N. M. Gyunter, and Ya. D. Tamarkin. In an interview with a correspondent of "Pravda" (on July 23, 1924) Fridman reported that 19 papers out of a total of 70 were presented by Soviet scientists.[16] His own paper (coauthored with L.V. Keller from the Main Geophysical Observatory) dealt with problems of turbulence in a compressible liquid (Ref. F54). In a letter to V. A. Steklov, written from Berlin on May 2, 1924 Fridman said that "the Congress went very well and the relationships with the Russians were excellent; in particular, I was included among the members on the committee organizing the next congress. The paper of A. F. Ioffe was particularly successful.[17] N. M. Gyunter's contribution attracted great interest of Lichtenstein, who is currently editing "Zeitschrift für Mathematik." The contributions of Tamarkin attracted the attention of Courant from Göttingen. My own work and that of my colleagues interested Blumenthal, von Karman, and Levi-Civita. If you have the time, I will tell you happily and in detail about the Congress" (Ref. F73, p. 363).

In addition to this paper, Fridman also presented a paper on behalf of N. E. Kochin and separately described the work of his own and his colleagues B. I. Izvekov, N. E. Kochin, O. A. Kostyreva, P. Ya. Polubarinova (Kochina) from the Main Geophysical Observatory, G. A. Grinberg (his former graduate student), and L. G. Loĭtsyanskiĭ from the Polytechnic Institute. Before returning to Leningrad on May 21, Aleksandr Aleksandrovich spent over two weeks in Germany.

Professor G. Shaw (from London), President of the International Commission on the Study of the Upper Layers of the Atmosphere, wrote to the Main Geophysical Observatory in the autumn of 1925, already after the death of Fridman:. "I recall a large number of papers which he brought to the International Congress of Applied Mechanics in Delft (Ref. 13, p. 77).

The rate of activity of Aleksandr Aleksandrovich in Holland and Germany can be judged by the fact that soon after his return and before the next academic year he asked officially the Dean of the Physicomathematical Faculty of the University to release him from lectures in 1924–1925 because of "completely upset health." His request was granted and the lectures previously given by Fridman were given by a well known mechanics expert, G. V. Kolosov.

However, Aleksandr Aleksandrovich continued to work at his usual health-destroying rate performing all his other multifarious research, teaching, and organizational activities.

7. Apart from the great importance of the work of A. A. Fridman on meteorology and hydrodynamics, his wide recognition— which will grow with years—rests on his research on relativistic cosmology. The main publication of Aleksandr Aleksandrovich on this subject, "On the curvature of space" (Ref. F36), was initially criticized by Einstein. The founder of the theory of relativity was wrong, not only this time,[18] but, according to his own admission, his rejection of considering the possibility of existence of a nonsteady-state universe and the incorrect sharp criticism of the work of Fridman was one of his major oversights. We shall therefore consider in detail the polemics between Einstein and Fridman, especially since we now have a number of new documents on this topic.

There is no doubt that Fridman with his interest in problems of mechanics back in the university years (and possibly earlier) was familiar with the "lesser principle of relativity," which was the name he used later for the special theory of relativity. The relevant problems have been the object of lively discussions in the St. Petersburg circle of Ehrenfest, in which Aleksandr Aleksandrovich participated. The first direct proof of his interest in this range of problems can in fact be found in a letter from Fridman to Ehrenfest soon after his return from Perm' to Petrograd. In the letter in the form of a report (dated August 6, 1920), addressed to a man regarded by Fridman—together with B. B. Golitsyn and particularly V. A. Steklov—as one of his teachers, Aleksandr Aleksandrovich described his work in the Atomic Commission, as well as teaching and research carried out at the Main Geophysical Observatory, and at that time his most fruitful technical work at the "Aviaprom" factory. Fridman then tackled the problem of interest to us. He wrote: "I worked on the axiomatics of the lesser principle of relativity. Starting from two assumptions that: 1) uniform motion remains uniform for a uniformly moving world and 2) the velocity of light is constant (and the same in moving and nonmoving worlds), I obtained uniform—in the sense of world space—formulas more general than the Lorentz transformation with an extra parameter. In two or more spatial dimensional world the formulas for the addition of velocities (representing the properties of the transformation group) can be used to derive the Lorentz formulas. This difference between one-dimensional and other worlds seemed to me ridiculous.[19] In recent years I began to think what arithmetic would have been invented by people if their velocities were close to the velocity of light. I would like very much to study the great principle of relativity (i.e., the general theory of relativity—V. F.), but I do not have the time."[30]

The beginning of his interest in axiomatics goes back probably to his time in Perm'. On return to Petrograd, Fridman established very close contacts with colleagues who were up to date on the subject. This applies particularly to Prof. V. K. Frederiks. During the First World War Frederiks was a "civil prisoner of war" in Germany, but because of the help from David Hilbert, he was able to work as his personal assistant at the Göttingen University. Therefore, he was aware of the latest research on the theory of relativity and its "great principle," he knew about the discussions between Hilbert and Einstein. On arrival in Petrograd, he began a course of lectures on the theory of relativity at the University and in the Polytechnic Institute. At the Institute the corresponding problems were considered also in the lectures of Ya. I. Frenkel' beginning from 1921. V. A. Fok

(Fock), whom Fridman brought to work in his division at the Main Geophysical Observatory, was a witness to lively discussions of the problems in the theory of relativity between Frederiks and Fridman, which took place at a seminar of the Physics Institute at the University. Vladimir Aleksandrovich Fok writes as follows about this seminar: "The main papers on the theory of relativity were presented by V. K. Frederiks and A. A. Fridman, but there were also some contributions from Yu. A. Krutkov, V. R. Bursian, and others. I recall vividly the papers of Frederiks and Fridman. The style of these papers was different: Frederiks understood deeply the physical aspects of the theory and did not like mathematical treatments. Fridman stressed not the physics, but mathematics. He aimed at mathematical rigor and attached much importance to complete and rigorous formulation of the initial assumptions. The discussions between Fredericks and Fridman were very interesting" (Ref. 6, p.399).

We should mention here that lectures on the theory of relativity were presented by Fridman also at meetings of the Petrograd Physicomathematical Society (of which he was the Secretary). N. M. Gyunter recalls the words of Ya. D. Tamarkin on some occasion: "We shall soon sort out these problems: Fridman started studying Weyl."

The results of his original research, discussed in detail in the publications mentioned at the beginning of the present biography,[2–7] were published by Fridman in German and Russian. In a small note referring to this work Einstein, as reported by V. A. Fok, "with somewhat superior airs said that the results of Fridman seemed to him suspect and that he found an error, the correction of which reduced Fridman's solution to a steady-state one." (Ref. 6, p.401), i.e., to the same solution from which Einstein himself derived in 1917 a cylindrical world with a curvature constant in time by introducing a cosmological λ term in the equation for gravitational potentials.

We shall begin by considering directly the subject of the discussion, i.e., the actual work of Fridman. The tasks which he set himself are formulated by him in Sec. 1 of his paper (Ref. F36). Fridman writes: "The purpose of this note is to derive cylindrical and spherical worlds[20] as special types which follow from certain general assumptions and then to show the feasibility of deriving a special world the curvature of space which is constant relative to the three coordinates adopted as spatial, but varies with time, i.e., it depends on the fourth coordinate which is time; this new type of universe resembles in all other respects the cylindrical world of Einstein" (Ref. F73, p.229).

Fridman's assumptions on the three-dimensional space are the same as those made by Einstein and de Sitter, namely that the three-dimensional space is homogeneous and isotropic. Moreover, it has a positive curvature.[21] As far as the time behavior of the world, Fridman differed from Einstein and de Sitter by making no *a priori* assumptions whether the world can (or cannot) vary with time: this should follow from the equations.

It is appropriate to mention here an opinion on the work of Fridman given by Ya. B. Zel'dovich in a paper published in "Uspekhi Fizicheskikh Nauk" a quarter of a century ago in connection with the 75th anniversary of Fridman's birth (and reprinted in Zel'dovich's works[7]). Commenting on the initial assumptions of Fridman, Zel'dovich wrote: this minimal number of assumptions was used to obtain a grandiose theoretical conclusion: the galaxies cannot be at rest relative to one another. The relative velocities of motion of the objects increase proportionally to the distance between them . . . . The theory of Fridman predicted a grandiose phenomenon, the scale of which was a billion times greater than the scale of the phenomena in the solar system. Therefore, without exaggeration we can acknowledge a major scientific step made by Fridman: his work became the foundation of all modern cosmology" (Ref. 7, pp. 403–404). Later Zel'dovich wrote that the importance of the work of Fridman is particularly great if we bear in mind the cosmological investigations of his contemporaries, including Einstein. "Einstein began from a specific point of view that the universe should be a steady-state one, i.e., it should be on the average constant in time. When it was found that these equations do not give a solution, he began to alter arbitrarily the equations of the general theory of relativity (roughly speaking, he introduced something of a negative density and a negative pressure in vacuum[22] simply in order to save the steady-state universe" (Ref. 7, p.404).

It is interesting to consider the psychological details associated with Fridman's evaluation of his own conclusions. On the one hand, and this is discussed in several following pages, he energetically defended the correctness of his results in a discussion with Einstein. On the other hand, one cannot but note a certain sceptical veneer in some of his phraseology, relating to the conclusions of his work. For example, in the excerpt cited above it is apparently not fortuitous that he deliberately called his long paper a "note." More important is the commentary on this note in a book called "World As Space and Time" which appeared in 1923 (Ref. F40), a year later than the publication of the paper of Ref. F36. A non-steady-state universe, which he obtained, as the saying goes, "at the tip of his pen,"[23] reminded Fridman of a "tale from Indian mythology on life periods." It is also possible to speak of creation of the universe "from nothing," but one nevertheless should regard all these as curious facts which cannot be confirmed solidly by the insufficient astronomical experimental data. In the absence of reliable data of this kind it is pointless to give any numbers characterizing the age of the variable universe; however, if nevertheless we calculate, for fun the time from the moment when the universe was born at a point up to now, i.e., if we begin consequently to find the time from the creation of the world, we obtain numbers of tens of billions of ordinary years" (Ref. F73, p. 317).

Six years passed and the work of Hubble became that "astronomical experimental evidence" which confirmed the conclusions of Fridman on a non-steady-state expanding universe. The problem of whether such expansion continues without limits or changes to compression and coming together of galaxies is determined, as is known, by the average density of matter in the universe for which there are still no reliable estimates.

Another remarkable and well-known fact is that some of the results of Fridman's theory and of a non-steady-state homogeneous universe, and even the law determining the cosmological expansion of the universe (Hubble's law) can be derived on the basis of classical Newtonian concepts of theory of gravitation without recourse to the theory of relativity, as demonstrated first in 1934 in Ref. 30. I well remember the contribution of Ya. B. Zel'dovich in the summer of

1963 (or 1964) at a theoretical seminar at the A. F. Ioffe Physicotechnical Institute in Leningrad dealing with an expanding universe. Having obtained the main results by this "classical" method (in the paper cited above he points out that this "Newtonian approach is rigorous and exact," Ref. 7, p.408), Yakov Borisovich looked away from the blackboard on which he just wrote down the simple derivations, stood silent for a moment, and then said: "What a remarkable science classical mechanics is! I worked in different branches of physics, but I like it best!"

8. We shall now turn directly to the substance of the dispute between Fridman and Einstein. The time table is as follows.

1. Fridman's paper "On the curvature of space" (Ref. F36) reached the journal "Zeitschrift für Physik" on June 29, 1922.

2. The first response of Einstein to Fridman's paper was received by the editorial office of this journal on September 18, 1922.

3. A second note representing the response of Einstein (Ref. F73, p. 398) to the same paper of Fridman was received by the editorial office on May 21, 1923 (Ref. F73, p. 398).

Between September 18, 1922 and May 21, 1923 there were some "behind-the-scenes" events contributing to the drama of the ideas of these two scientists.

The journal "Zeitschrift für Physik" was the most popular physics journal in the twenties. Undoubtedly, Einstein looked at it immediately on receipt. Unfortunately, a detailed chronology of Einstein's activities has not yet been compiled. However, it is known that in the spring of 1922 he spent some time in France and in October of this year he was in Switzerland. Probably at some time (between July and September) he had a vacation and did not respond immediately to Fridman's paper about which he could hardly have heard before. However, having read it, he decided he had to answer it. His reaction, in the objective account of V. A. Fok is mentioned above.

A negative response from the lips of the man who had an exceptional authority must have affected Aleksandr Aleksandrovich. The issue of the journal with Einstein's note reached Russia in October; both Fridman and his colleagues could have learnt about it then, but in fact this happened earlier. On December 6, 1922 Fridman sent Einstein a detailed factual letter to Berlin. We know about this letter from Fok's account and from a paper by M. A. Loris-Melikov (Ref. 14, p. 61), but no copies of the this letter has been found in the Soviet archives. In the early seventies I turned to Dr. O. Nathan, Einstein's executor and keeper of his archives, with the request to search whether his archives include the letter from Fridman. The letter was found and a photocopy was kindly sent to me.[24] It began as follows:

"Much-esteemed Professor! From a letter of one of my friends, who is now abroad, I had the honor to find that you sent to press in Vol. 11 of "Zeitschrift für Physik" a short note where you point out that if all the assumptions made by me in my paper "On the curvature of space" are adopted, then from the world equations derived by you it would follow that the curvature of the universe is a quantity independent of time."

We interrupt this quotation. There is no doubt that "one of Fridman's friends" is Yu. A. Krutkov who went to Germany in September 1922 and spent there over a year.

In the rest of his letter, Fridman demonstrated by direct calculations that vanishing of the divergence of the energy-momentum tensor, pointed out by Einstein, does not imply at all that the radius of curvature of the world should be constant in time. "Bearing in mind the definite interest in the possibility of existence of a non-steady-state universe, writes Fridman, I venture to submit to you my calculations—for checking and critical review." At the end of his presentation, Fridman recalls that he recently investigated "the case of a world with a constant and a time-dependent negative curvature . . . . The results of these calculations showed that there may be a world with a constant (but a negative) curvature and one with a curvature that varies (with time). The possibility of deriving from your world equations a world of a constant negative curvature is for me of exceptional interest and, therefore, I earnestly ask you to answer my letter, although I know that you are very busy." Fridman concludes: "If you find the calculations presented in my letter correct, I would ask you not to refuse my request to inform the editorial office of "Zeitschrift für Physik" about it; it may be that in this case you would publish a correction to your opinion or provide an opportunity for reprinting parts of my letter" (Ref. 31, p. 11).

There is no doubt that Fridman discussed with his colleagues, particularly with V. K. Frederiks, his reasoning and was generally convinced of his correctness. However, it should be pointed out particularly that he did not send the letter directly to the journal and thus demonstrated an exceptional sensitivity to the feelings of his opponent.

Fridman correctly assumed that Einstein is "very busy," but he hoped to have his answer soon. However, weeks passed and there was no response to his letter. The reason for this is very simple: Einstein was out of Berlin in December 1922. At the end of September, when he sent his first note on Fridman's paper to the journal, Einstein and his wife went first to Switzerland and then to France and beginning from October 11 they were aboard a ship going to Japan. Einstein is known to have learned with some delay that he was awarded the Nobel Prize. He was unable to be present on December 11 in Stockholm at the ceremony of presentation of the Prize. He returned to Berlin only in March 1923. One would expect that it would take him some time to deal with mail that had accumulated for nearly six months . . . .

April passed and in May Einstein was invited to Leiden (he was at the time an honorary professor of Leiden University) to be present at the farewell lecture of Lorentz, who was retiring. Yu. A. Krutkov was at the time in Leiden. Thus there was every reason for Fridman to turn to Yuriĭ Aleksandrovich with a request to discuss with Einstein the dispute between them and to acquaint him with the arguments in his letter of December 6 (and he might have sent Krutkov a copy of the letter). In Leiden, Einstein stayed in the home of Ehrenfest whom Krutkov saw every day and with whom he was working. In 1923 Krutkov could be regarded as one of the best educated theoreticians in our country. His knowledge of the theory of relativity was excellent. We know of the mediation mission of Krutkov from the second note of Einstein on Fridman's paper, from the paper of V. A. Fok,[6] and from other sources.[32] This is supported also by direct diary records of Krutkov and by his letters to his sister Tat'yana Aleksandrovna, which were found among other personal

documents of Yu. A. Krutkov stored in the A. N. Krylov collection at the Archives of the Academy of Sciences in Leningrad. Krutkov's notebooks of 1923 are all covered with formulas from Fridman's paper and calculations relating to this paper. On April 29 Krutkov writes to his sister from Leiden: "Einstein should come any day. I am very interested in him." May 4: "I cannot write, because I must hurry to a paper presented by Einstein: he is a very pleasant man." A note in his exercise book: "On Monday May 7, 1923 I read with Einstein Fridman's paper in "Zeitschrift für Physik." Again in a letter to his sister: "May 13 . . . . I do not know why the arrival of my passport from Berlin was delayed because otherwise I would have left on the 15th with Einstein . . . .[25] Einstein is very nice." May 18: "At 5 pm Einstein described to Ehrenfest, Drost, and one Belgian his latest work . . . . I vanquished Einstein in an argument about Fridman. The honor of Petrograd is saved" (Ref. 32, p. 674).

We mentioned earlier that the second note of Einstein on the cosmological paper of Fridman was received by the journal on May 21, 1923. There is no doubt that the timing is not accidental and that this note was the result of a discussion between Einstein and Krutkov in Leiden.

We shall now give the text of Einstein's second note. It was published in Russian in the second volume of "Collection of Scientific Works" of the founder of the theory of relativity and is included as an Appendix to the volume containing "Selected Works" of Fridman. The note reads as follows:

"In my previous note I criticized the above mentioned work (Einstein used the title of Fridman's paper in the heading of his note—V. F.). However, my criticism, as I found from Fridman's letter delivered to me by Mr. Krutkov, was based on an error in calculations. I regard the results of Mr. Fridman as accurate and shedding new light. It appears that the field equations admit, apart from static, also dynamic (i.e., time-dependent) centrally symmetric solutions on the structure of space" ("Selected Works," p. 398).

The phrase "Fridman's letter delivered to me by Mr. Krutkov" can only have an obvious meaning: after a long journey, Einstein did not read Fridman's letter because of pressure of time; only after meeting Krutkov he learned of its contents, possibly because, as I mentioned earlier, Fridman acquainted Krutkov with the contents. The important point is that on May 7, Einstein *read* with Krutkov the paper of Fridman because Krutkov did not even mention a *letter* from Petrograd to Berlin.

Yu. B. Tatarinov in discussing recently[33] the Fridman-Einstein dispute in the light of the Krutkov materials quoted here and published in 1970 and 1974 (Refs. 31 and 32) decided to recheck the accuracy of the translation of Einstein's note given in the "Works" mentioned above. The phrase of interest to us translated more accurately reads as follows: "My objection was based on a calculation error, as I convinced myself with the encouragement of Mr. Krutkov and guided by the letter of Mr. Fridman." Hence, Tatarinov concludes, Einstein did read Fridman's letter. This was probably true. Krutkov discussed Fridman's paper with Einstein in Leiden, and supplemented the analysis of Fridman's arguments. Returning close to 20th of May to Berlin, Einstein found Fridman's letter (he could have also found it earlier and taken it with him to Leiden or could have become familiar with the letter on the basis of a copy held by Krutkov). Einstein met Krutkov on return from Leiden to Berlin. On June 8, 1923 Krutkov wrote from Berlin to his sister: "How are things with Fridman? I wrote to him about his dispute with Einstein and wonder what is he thinking of ? In any case he should have written to me." Alas, the letters exchanged between Krutkov and Fridman have not been found but from other letters (those of Krutkov to his sister and of Fridman to N. E. Malinina) we can deduce the necessary information on Fridman's stay in Berlin in August-September of the same year (1923) and his attempts to meet with Einstein. On August 9, Krutkov wrote: "Fridman is here today and in a few hours he is traveling to Hamburg. Einstein's note in which he is rehabilitated, at my instigation, has already appeared." Fridman wrote on August 19: "My trip is not working out: Einstein, for example, has gone on vacation and I shall not be able to see him." September 2: "I still have the following things to do: 1) go to Göttingen; 2) see von Pahlen (an astronomer, my former assistant); 3) see Mises (editor of "Zeitschrift für angewandte Mathematik") and Einstein; 4) make the necessary purchases." September 13: "I visited the astronomer von Pahlen, my old friend, today and there I met the astronomer Freundlich, a very interesting man with whom I spoke about the structure of the universe . . . . All were impressed with my dispute with Einstein and my subsequent victory, and I find it satisfying because my papers will be accepted more readily for publication."

A short comment about the last letter. The persons mentioned in that letter are well known. E. von Pahlen (1882–1952) and E. Freundlich (1885–1964) were German astronomers. Before the Revolution E. von Pahlen lived in St. Petersburg and was a member of the Russian Physicochemical Society. During the first World War he served in the air force (in L'vov and Kiev) under A. A. Fridman. After the Revolution he emigrated to Germany. Craters on the surface of the Moon are named after von Pahlen and Freundlich.[26] They are authors of books on cosmology and on the theory of relativity. The book of von Pahlen "Infinite World and the Theory of Relativity" published in Potsdam in 1924 (Ref. 34) was in Fridman's library. Freundlich was the author of one of the first popular science books on the theory of relativity, which was published in 1916 (it appeared in Russian translation in 1924) and was distinguished by a very benevolent preface from Einstein. R. Mises (1883–1953) was an Austrian mathematician and mechanics expert who studied aerodynamics. Fridman probably was also attracted to him because he was an airman.

Unfortunately, Fridman did not succeed in meeting Einstein in 1923 nor during his next trip to Holland and Germany in 1924 (which we described earlier), so the two men never did meet.

9. The years 1922–1925 represented a saturtion period of Fridman's publishing activity: books appeared which summarized his research and his teaching experience. The first that should be mentioned is his work on "Experiments on Hydromechanics of a Compressible Liquid" (Ref. F35), which he submitted as his master's thesis to Petrograd University. This book summarizes and develops Fridman's work on hydrodynamics and aerodynamics carried out over a number of years (Refs. F23, F24, F26, F27, F30–34, and F37–39) and published both before and (partly) somewhat later (Ref. F35). This work was reviewed in Refs. 8–10.

Another important result of the work of A. A. Fridman is the book on "Fundamentals of the Theory of Relativity" (Ref. F50) intended by him and Frederiks as a multi-volume series, of which only the first volume appeared and it contained the fundamentals of tensor analysis. The book was published by Academia, one of the most popular and prestigious publishers in the twenties and thirties.[27] We should mention here the books, referred to earlier, on approximate calculations (Ref. F55), theoretical mechanics (Ref. F51), and a course on "Solution of Numerical Equations" (F56).

One should add here Fridman's participation in the preparation of the first edition of the Great Soviet Encyclopedia. Aleksandr Aleksandrovich was the editor of contributions on geophysics and meteorology. The first volume of the Encyclopedia appeared already after Fridman's death and opened up with the sad annnouncement that three of the leading collaborators did not live to see the first volume: they were B. Ya. Bryusov (one of the founders of the Encyclopedia), M. V. Frunze (editor of the military studies section), and A. A. Fridman. However, in the subsequent volumes readers found three additional papers of Fridman which reached them like the light from a star that had expired: "Arithmetic" (Ref. F60, with a specially striking excellently written historical review), "Atmosphere" (Ref. F61, coauthored with B. I. Izvekov and E. I. Tikhomirov), and "Air Navigation" (Ref. F62), the last being the subject in which Aleksandr Aleksandrovich was the best specialist in the country.

10. The last year in the life of A. A. Fridman was marked by further strengthening of his ties with the Main Geophysical Observatory. On February 5, 1925 a resolution of the Chief Science Board of the National Comissariat for Education appointed a new team to run the Observatory. Fridman was appointed Acting Director of the Observatory (he was confirmed as Director on June 6).

At what is now the A. I. Voeĭkov Main Geophysical Observatory there is a diary of orders for this period. The diary is a good-quality account book. Various orders and instructions are written in longhand in the diary. Since, beginning from February 1925, they were signed by Fridman, we can easily establish that Aleksandr Aleksandrovich was practically every day at the Observatory. There was one important exception: he frequently traveled on Observatory business to Moscow. The frequency of such trips increased on approach of summer. For example, in the period from March to May he was in Moscow at least once every ten days. This was due to the intensive preparations and then the actual running, in May 1925, of the First All-Union Geophysical Congress, to which Fridman attached much importance. Parallel to this Congress or, more exactly, during it, there was a meeting on the initiative of Aleksandr Aleksandrovich of directors of the central geophysical establishments of the Russian Federation and of the Soviet republics. This meeting worked out the principles of organization of the weather service in the country, and considered coordination of the activities of the various establishments (annual conferences, publications, etc.). One of the last papers published by Fridman (Ref. F59) is probably a summary of his speech at this meeting.

One of the resolutions adopted by the Congress especially stressed the "correctness of the line taken by the Main Geophysical Observatory on the tasks of this Observatory and of the general meteorological service, and it was accepted that the planned performance by the Observatory of five groups of tasks, namely: 1) methodology of observations and their analysis, 2) acquisition of data for weather analysis, 3) publishing of such data, 4) investigation of the laws governing atmospheric phenomena, and 5) actual weather forecasting, with the associated—in all five groups—publishing activity, training of scientific personnel, and popularization of meteorology among the wide masses of workers, will make it possible for the Main Geophysical Observatory to remain in the forefront of meteorological establishments not only in the Soviet Union, but in the world" (Ref. 13, p. 23).

Naturally, there were many (over 200!) papers at this Congress, including those presented by the staff of the Main Geophysical Observatory.

In parallel to the preparations for the Congress, A. A. Fridman and his colleagues began planning a balloon flight. In addition to preparing a scientific program for the flight and the means to carry out this program, much organizational effort was required. The book of orders at the Main Geophysical Observatory includes the 685th order for July 16, 1925: "in view of the service mission of a balloon flight for scientific purposes planned for July 16, I transfer temporarily the prerogatives of Director of the Main Geophysical Observatory to L. V. Dashkevich, Assistant Director for Administrative and Economic Affairs." The order is signed both by Director of the Observatory and its Scientific Secretary P. Ya. Polubarinova (here and in similar texts we can sense a military man in Aleksandr Aleksandrovich—after all he spent over three years in the army!). The next day he wrote to N. E. Malinina: "I am very tired; I am still hoping to ride in a balloon, but I do not know whether this will be possible. I am so fatigued by the Observatory work that I wait impatiently for a rest and an opportunity to do nothing for several days." When this letter was written, Fridman naturally knew that the flight would not take place the same day or the next. The main purpose of the flight was to investigate atmospheric whirlwinds, but the crew, professor A. A. Fridman and the pilot P. F. Fedoseenko, planned to carry out also some biological investigations (determination of changes in the amounts of microorganisms in air with altitude) and make also some medical self-observations. The flight proved dangerous and the course of events was described by Aleksandr Aleksandrovich soon after in an excellent paper published in a journal " 'Khochu Vse Znat' " ("I Want to Know All," Ref. F58), which was reprinted in 1925 (Ref. 13, pp. 8–11) and in 1966 (Ref. F73). The courage shown by A. A. Fridman during the flight was later reported by P. F. Fedoseenko.[36] The crew reached record altitude, for the USSR, amounting to 7400 m (a previous record of 6400 m was established by a colleague of A. A. Fridman—N. A. Rynin from the Institute of Transport Engineers back in 1910). Aleksandr Aleksandrovich was unable to find the time needed to prepare a scientific report on the results of this flight. The popular paper mentioned above is very much a reflection of its author. It is written in a calm manner without any false pathos. We shall now quote two short excerpts from this paper: "The feelings and experience in the clouds are interesting. Complete quiet, complete rest, cannot see anything, and you do not know the locality above which you are flying. Nobody can see you and you cannot see anybody.

Complete isolation. True, initially you can hear the sounds of 'life' from the ground: you can hear the whistles of steam engines, bells, the cocks crowing, the dogs barking, etc. If you can still hear these sounds you feel comfortable, but soon they disappear. All is dead silence." "While we were fussing with breathing of oxygen, an accident occurred. A deafening explosion was heard in complete silence, we looked up and saw that the balloon was all covered by smoke. A quick thought occurred: we are on fire, so that chances for saving ourselves are very small. Then the smoke dispersed and we found that our 'oxygen trunk' fractured. This is what happened: at high altitudes when the pressure is low, the oxygen trunk expanded and broke up and moist gas escaped and cooled in the process, so that the moisture condensed in the form of a cloud which we took for smoke. Having established the cause of the explosion, I sighed with relief, although the reserve of oxygen decreased considerably and it was difficult to maintain our altitude for long" (Ref. F73, p. 383).

The balloon approached ground near a village of Okoroki in the Nizhegorod province (at that time Gubernya). The peasants working in the fields rushed toward the balloonists leaving the gondola. A. A. Fridman wrote that he and P. F. Fedoseenko gave an improvised lecture on the flight and its purposes. A. F. Vangengeĭm added (Ref. 13, p. 7) that Komsomol members from the village and Aleksandr Aleksandrovich began corresponding. Unfortunately, these letters have not been preserved. The balloonists reached Leningrad on July 21.

Soon after this flight Fridman had at long last a chance to rest: he could fulfill his dream of at least a short vacation in the Crimea. N. E. Malinina was working there at one of the outstations of the Main Geophysical Observatory in Feodosiya. On July 27, having just returned from the last trip, Fridman traveled to Moscow and then on July 30 he reached Simferopol and Feodosiya. L. G. Loĭtsyanskiĭ recalls that Aleksandr Aleksandrovich was very satisfied with his rest in Crimea which lasted less than three weeks. On August 17, he returned to Leningrad refreshed and apparently full of new strength. There was nothing to indicate a quick end.

One of the reasons why Fridman was hastening to Leningrad was that there were plans for celebrations of the bicentenary of the founding of the Academy of Sciences. V. A. Steklov, Vice-President of the Academy, invited A. A. Fridman to join the organizers of the celebrations and to prepare a scientific program, including a visit to the Main Geophysical Observatory by Soviet and foreign scientists (this visit did indeed take place; in particular, M. Planck, C. Raman, and G. Ficker came to the Main Geophysical Observatory in Leningrad and to the Academy station in Pavlovsk).[28)]

In the extensive documentation of preparations for the jubilee session (stored in the Leningrad Archives of the Academy of Sciences of the USSR) there is evidence of how carefully the celebrations were prepared. For example, among other documents, there was a plan for the participants of the session to see Glinka's opera "Ruslan and Ludmila" in the Opera and Ballet Theater and the program mentions that in one of the rows in the dress circle there should be seats for Ficker, Fridman, and some other Soviet and foreign scientists.

The diary of orders at the Main Geophysical Observatory shows that Fridman was at the Observatory on August 19, 20, 22, and 23. Later orders were signed by the Scientific Secretary of the Observatory, because apparently Aleksandr Aleksandrovich was occupied at the Academy or may have been unwell. On September 2, he understood that he was seriously ill: he then signed (the handwriting is somewhat different, because he was possibly already in bed) the order that during his illness the duties of Director of the Observatory were to be performed by Prof. E. I. Tikhomirov. The doctors diagnosed typhoid fever. By the beginning of September two weeks passed since the return of Fridman from Crimea which is exactly the incubation period of typhoid. Aleksandr Aleksandrovich recalled that on his way to Leningrad he bought appetizing pears at one of the stations and thoughtlessly failed to wash them. An absolute carelessness had a fatal result. A. A. Fridman died on September 16 in the hospital.

The announcements of his death appeared in "Pravda" and "Izvestiya" as well as in Leningrad newspapers. The evening edition of the Leningrad "Krasnaya Gazeta," published on September 18, 1925, had an interview with Dr. Ya. A. Bukhshtab, who treated A. A. Fridman. He said that early in the morning of September 16 an internal stomach bleeding started. His temperature was very high and Aleksandr Aleksandrovich was in a delirium. Surprisingly a description of his state was given in this newspaper: "The delirium of the deceased was very characteristic: he spoke of students, lectures, recalled his balloon flight, and tried to carry out some calculations. Occasionally it seemed he was giving a lecture."

The journal "Klimat i Pogoda" (Climate and Weather) in its chronicle section (Ref. 13, p. 76) described the funeral of Aleksandr Aleksandrovich. Participants at the funeral included President of the Academy of Sciences, A. P. Karpinskiĭ and its Permanent Secretary Academician S. F. Ol'denburg, demonstrating the scale of this tragedy to Soviet science. All the way from the hospital on Kamennyĭ Island to the Smolensk Cemetery the coffin with the body of Aleksandr Aleksandrovich was carried by his students and colleagues. Among those who spoke at the cemetery at the open grave were A. F. Vangengeĭm, author of one of the first obituaries, Prof. I. V. Meshcherskiĭ (who worked at a laboratory at the Polytechnic Institute where Fridman was a postgraduate student), Prof. N. A. Rynin in whose faculty Aleksandr Aleksandrovich worked at the Institute of Transport Engineers, his companion in the balloon flight P. F. Fedoseenko, and his closest colleague and pupil at the Polytechnic Institute L.G. Loĭtsyanskiĭ (Ref. 13, pp. 76–78).

At the end of October at the State Geographical Society, where A. A. Fridman spoke many times there was a mourning session dedicated to his memory and a number of papers were presented. Other sessions took place at the Physics Institute of Leningrad University and at the Leningrad Physicomathematical Society where Fridman was the Secretary.

When still alive A. A. Fridman was proposed for the prestigious Prize of the Chief Science Board of the National Commissariat for Education of the USSR for his work on "Experiments on Hydromechanics of Compressible Liquids" (Ref. F35). The decision to award the Prize was signed on October 29, 1925 when Aleksandr Aleksandrovich was no longer living.[37] The Lenin Prize was given to him posthumously in 1931 for "outstanding scientific

work." He was well known in the prewar years, but became even more widely known after his death. Without exaggeration we can say that in the sixties A. A. Fridman achieved world fame as one of the major scientists of our century.

It is reported that Aleksandr Aleksandrovich sometimes spoke somewhat ironically about the signficance of his results. It seems to us that this was not pride masking as disparagement: it was simply a manifestation of modesty so characteristic of him. However, people close to Fridman (particularly Corresponding Member of the USSR Academy of Sciences G. A. Grinberg)[29] said that actually Fridman understood well the scale of his achievement including that on relativistic cosmology. Nevertheless, Fridman probably would have been surprised and pleased if he could have learned of the world acknowledgment of his work which we are witnessing today. The achievements of A. A. Fridman are permanently part of the golden treasure of science and at the same time are part of its living tissue.

In conclusion, I would like to thank the staff at the Leningrad Archives where I worked on preparing this paper (State Historical Archives of Leningrad, Leningrad Division of the Archives of the Academy of Sciences of the USSR, archives at the M. I. Kalinin Leningrad Polytechnic Institute and at the A. I. Voeĭkov Main Geophysical Observatory) for their help. The text of the paper was read by A. D. Chernin and K. S. Shifrin and I am grateful for their comments.

Throughout the work on this paper I was continuously supported by Yakov Borisovich Zel'dovich with whom I was able to discuss a detailed plan of this paper. He, who has done so much for the later development of astrophysics and cosmology, was deeply interested in the monumental personality of A. A. Fridman. I acknowledge with gratitude the kind help of Yakov Borisovich.

[1]J. A. Wheeler, who knew Einstein well, writes that Fridman's famous prediction of a pulsating universe "Einstein for a time regarded as too horrific to accept" (Ref. 1, p. 19).

[2]The numbers identify the citations at the end of the article.

[3]A. A. Fridman's birth certificate is among other documents in the collection of the St. Petersburg University stored at the State Historical Archives of Leningrad. We shall not give detailed references to the archival documents and simply identify the collection where they are stored.

[4]He was the uncle of well-known Soviet scientists Academicians N. N. Davidenkov (physicist) and S. N. Davidenkov (neuropathologist).

[5]The letter F in front of a reference number will be used to denote items in the full bibliography (73 citations) of the work of A. A. Fridman, compiled by the present author and added as an appendix to the present paper.

[6]Fridman senior married for the second time in 1897.

[7]The exact date of the death of his father, A. A. Fridman senior, could not be established. All that known is that he died earlier than A. I. Fridman, grandfather of the scientist.

[8]The subsequent fate of members of the University Mathematics Circle, friends of A. A. Fridman, varied. Vasiliĭ Vasil'evich Bulygin (1888–1919) did not live long but managed to publish several papers on mathematics and was one of the coauthors of a book on problems in higher mathematics (Ref. F69). Mikhail Fedorovich Petelin (1886–1921) participated in the work on military meteorology and was a man of great promise. He also carried out a number of investigations, one of them as coauthor of Fridman (Ref. F14), and died young, like Bulygin. Yakov Davidovich Tamarkin (1889–1945), an outstanding mathematician and coauthor with V. I. Smirnov of early editions of the first volume of an excellent "Smirnov course of higher mathematics," was an author of a number of papers and books. In 1925 he emigrated to the USA and becane one of the leading mathematicians, member of the National Academy of Sciences and Arts. Yakov Aleksandrovich Shokhat (1886–1944) born in Warsaw, returned in the twenties to Poland and from there went to the USA where he was known as J. A. Shohat. He also became well known for his work (one of his books was written together with Tamarkin) and had books published in the USA, France, and Germany. Abram Samoĭlovich Bezikovich (known as Besicovitch in the West) (1891–1970) cooperated closely with H. Bohr, emigrated to England, became a Fellow of the Royal Society; his work on almost-periodic functions, theory of probability, and geometry are well known. Aleksandr Feliksovich Gavrilov (1887–?) became professor of mathematics and taught at the Universities of Perm' and Nizhniĭ Novgorod, and from 1920 to the end of his life at various higher educational establishments in Leningrad. He served during the First World War together with Fridman and the two of them compiled exact tables for bomb release; Gavrilov wrote a number of textbooks on higher mathematics. One who became best known was a remarkable scientist and man Vladimir Ivanovich Smirnov (1887–1974) who like Ya. D. Tamarkin was one of the closest friends of Fridman. His name is well known in the Soviet Union: he was Academician, Hero of Socialist Labor, a major mathematician at the Leningrad University, and author of a five-volume course on higher mathematics published repeatedly in the Soviet Union and abroad.

[9]In one of the notes of A. A. Fridman there is a mention of a paper published in some journal on "Equilibrium shape of a string holding a kite." It proved impossible to identify this paper.

[10]It is natural to ask the question: how did G. Ficker know this? The answer to that is given by an excerpt from an exceptionally sympathetic interview given by G. Ficker already after A. A. Fridman's death to a correspondent of the Leningrad "Krasnaya Gazeta" (evening edition of September 18, 1925): "I met A. A. Fridman in Berlin three times. Once talking to him I found that just before the fall of Peremyshl' (Przemyśl), Prof. A. A. Fridman, then a member of the Russian air force dropped a large bomb above my home in Peremyshl'. At this time I was in the German air force and reporting to my superior for the next task. I well remember that the bomb dropped by Fridman was the only one which the Russians managed to drop on Peremyshl'. During the first meeting with Fridman in Berlin I established accurately the time and place of this unusual and unpleasant meeting on the battlefield."

[11]It is not clear how they could know who was flying. However, even if this is a legend, it is typical!

[12]According to the "Dictionary of the Russian Language" the word "proborka" (used by Steklov in this connection) is a colloquial expression for a severe, sharp talking-to or reprimand.

[13]He should not be confused with an acoustician N. N. Andreev.

[14]N. E. Malinina lived to an old age and died in 1981. She was doctor of physicomathematical sciences and for many years headed the Leningrad Division of the Institute of Terrestrial Magnetism, Ionosphere, and Propagation of Radiowaves of the USSR Academy of Sciences. She was a very well educated and versatile woman who in her youth knew V. V. Mayakovskiĭ and K. I. Chukovskiĭ. I am obliged to A. D. Chernin for information about her.

[15]Naval Observatory (German).

[16]The Proceedings of the Congress[29] consisted of 51 papers presented by the participants, including four contributions from the Soviet scientists.

[17]The paper by A. F. Ioffe dealt with the problems of plasticity and strength of crystals (Ref. 29, pp. 64–66).

[18]Among the documents in the four-volume selective collection of works of Einstein (which was published in the Soviet Union in the sixties) there are six notes about errors or inaccuracies in the papers of Einstein written before 1922.

[19]It is not clear what Fridman meant when he wrote about an extra parameter in the Lorentz transformation formulas for the one-dimensional case. Knowing the scale of Fridman's talents, it is difficult to imagine that this would have been some elementary error. On the other hand, Fridman did not publish any comments on this subject neither in his papers on the theory of relativity nor in his book (Ref. F40).

[20]The usual terminology is the Einstein universe and the de Sitter universe.

[21]The case of a world with a negative curvature, not considered by Einstein and de Sitter, is discussed in Ref. F43.

[22]This is the celebrated λ (cosmological) term in the system of equations for gravitational potentials.

[23]An analysis of the cubic polynomial, which occurs in the expression governing the age of the universe, was used by Fridman to produce several variants of its evolution [see Eq. (10) in Ref. F73 on p. 234 and the following pages].

[24]Complete translation of this letter can be found in Ref. 31.

[25]In fact, Einstein left Leiden somewhat later.

[26]There is also a crater on the Moon named after A. A. Fridman.

[27]One should mention Fridman's book on the "World as Space and Time" which appeared in 1923 and was also published by Academia (Ref. F40). It gave a very physical presentation of the problems in the theory of relativity and cosmology ("world pictures") derived using a mini-

mum of mathematics: only this extraneous aspect of the work of Aleksandr Aleksandrovich led him to regard his book as popular science. Understanding of the range of topics selected by the author and the approaches adopted by him in the presentation requires considerable effort from a reader. The book was republished twice (Refs. F72 and F73). The 1923 edition is in the personal library of V. I. Lenin in the Kremlin.[35]

[28]In the Leningrad Division of the Archives of the USSR Academy of Sciences there is a draft (in Russian) of a letter by V. A. Steklov written to G. Ficker after the jubilee session, probably in answer to the letter by Ficker with condolences on the death of Fridman. Steklov says: "The death of Fridman did indeed darken the end of our jubilee, which he prepared with such an enthusiasm. For our Observatory his death is a great shock."

[29]G. A. Grinberg was supervised by A. A. Fridman in his diploma work on the theory of elasticity and hydrodynamics in the special theory of relativity, which Fridman described at the 1924 Congress in Delft.

[1]J. A. Wheeler, *Einstein's Vision*, Springer Verlag, Berlin, 1968 [Russ. transl., Mir, M., 1970].

[2]Ya. B. Zel'dovich and I. D. Novikov, *Relativistic Astrophysics, Vol. 2: The Structure and Evolution of the Universe*, University of Chicago Press, 1983 [Reprint of 1971 ed.] [Russ. original, Nauka, M., 1975].

[3]S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, Wiley, N. Y., 1972 [Russ. transl., Mir, M., 1975].

[4]L. E. Gurevich and A. D. Chernin, *Introduction to Cosmology* [in Russian], Nauka, M., 1978.

[5]I. D. Novikov, *Evolution of the Universe*, Cambridge Univ. Press, 1983. [Russ. original, Nauka, M., 1979].

[6]V. A. Fok (Fock), "Fridman's work on Einstein's theory of gravitation," in: *A. A. Fridman, Selected Works* [in Russian], Nauka, M., 1966, pp. 398–402. (This book will be cited later as *Selected Works.*)

[7]Ya. B. Zel'dovich, "Theory of the expanding universe founded by A. A. Fridman," *ibid.*, p. 402.

[8]A. S. Monin and A. M. Yaglom, *Statistical Fluid Mechanics* MIT Press, Cambridge, MA, 1975 (*Mechanics of Turbulence*) [Russian original, Part 1, Nauka, M., 1965].

[9]N. E. Kochin, *Collection of Works* [in Russian], Vol. 1, Izd. AN SSSR, M.,-L., 1949.

[10]I. A. Kibel', "Hydrodynamic (numerical) short-term weather forecasting," in: *Mechanics in the Soviet Union in the Last 50 Years* [in Russian], Vol. 2, Nauka, M., 1970, p. 561.

[11]V. N. Trostnikov, *A. A. Fridman, an Outstanding Soviet Scientist* [in Russian], Znanie, Moscow (1963).

[12]L. S. Polak, "Life and scientific work of A. A. Fridman," *Selected Works*, p. 427.

[13]Klim. Pogoda No. 2–3 (1925).

[14]Geofizich. Sb. **5**, No. 1 (1927).

[15]P. Ya. Kochina, *Memoirs* [in Russian], Nauka, M., 1974.

[16]P. Ya. Kochina, *Nikolaĭ Evgrafovich Kochin* [in Russian], Nauka, M., 1979.

[17]I. B. Cohen, "Meeting with Einstein," in: *Einstein Collection 1967* [Russ. transl.], Nauka, M., 1967, p. 45.

[18]S. A. Dianin, *Revolutionary Youth in St. Petersburg 1897–1917* [in Russian], Priboĭ, L., 1926.

[19]A. F. Gavrilov, "Recollections of Fridman," in: *Selected Works*, p. 417; "To the memory of A. A. Fridman (obituary)," Usp. Fiz. Nauk **6**, 73 (1926).

[20]N. N. Andreev, "My meetings with A. A. Fridman," (In Russian) Priroda No. 6, 88 (1967).

[21]K. N. Loskutov, "Activities of A. A. Fridman at Perm' University," Uch. Zap. Perm. Univ. No. 163 (Fizika), 149 (1968).

[22]L. G. Loĭtsyanskiĭ and A. I. Lur'e, "Aleksandr Aleksandrovich Fridman (1888–1925)," Tr. Leningr. Politekh. Inst. No. 1, 83 (1949).

[23]E. D. Devyatkova, *Recollections of A. F. Ioffe* [in Russian], Nauka, L., 1973, p. 70.

[24]V. A. Steklov, "A. A. Fridman (obituary)," *Selected Works*, p. 394; see also Ref. 14, p. 7.

[25]J. Ackeret, *Rotor Ship: A New Way of Utilizing the Wind Force* (ed. by A. A. Fridman) [Russian transl.], L., 1925.

[26]A. Einstein, "Flettner's ship," [Russian translation], Izobret. Ratsionalizat. No. 6, 17 (1965).

[27]A. Einstein, "Elementary theory of flight and of water waves," in: A. Einstein, *Collected Scientific Works* [Russ. transl.], **4**, Nauka, M., 1967, p. 22.

[28]V. Ya. Frenkel' and B. E. Yavelov, *Einstein as an Inventor* [in Russian], Nauka, M., 1981.

[29]*Proc. First Intern. Congress of Applied Mechanics*, Delft, 1924 (ed. by C. Beizano and J. M. Bürgers), Delft, 1925.

[30]W. H. McCrea and E. A. Milne, "Newtonian universe and the curvature of space," Q. J. Math. **5**, 17 (1934).

[31]V. Ya. Frenkel', "New documents on a dispute between Einstein and Fridman," in: *Einstein Collection 1973* [in Russian], Nauka, M., 1974, p. 5.

[32]V. Ya. Frenkel', "Yuriĭ Aleksandrovich Krutkov," Usp. Fiz. Nauk **102**, 639 (1970) [Sov. Phys. Usp. **13**, 816 (1971)].

[33]Yu. B. Tatarinov, "Fifty years of the theory of an expanding universe," Vopr. Istorii Estestvozn. Tekh. No. 3, 88 (1982).

[34]E. von Pahlen, *Der unendliche Weltraum und die Relativitättheorie*, Potsdam (1924).

[35]*Catalog of the Library of V. I. Lenin in the Kremlin* [in Russian], Politizdat, M., 1961.

[36]P. Fedoseenko, "At an altitude of 7400 m," Vestn. Vozd. Flota **8**, No. 9, 23 (1925).

[37]Chronicle, Sov. Nauka, No. 3, 174 (1925).

BIBLIOGRAPHY OF THE WORKS OF A. A. FRIDMAN* (Friedmann or Friedman in the Western references)

1906

F1"Sur les congruences degré et les nombres de Bernoulli," Math. Ann. **62**, pp. 409–412 [coauthor Ya. D. Tamarkin].

1908

F2*Review of a book: S. Santerre, *Psychologie du nombre et des operations elementaires de l'arithmetique*, Paris, 1906, Russkaya Shkola Otd. iII, No. 1, pp. 27–28.

F3*Review of a book: S. Pellico, *Duties of Man* [Russ. transl.], M., 1907, *ibid.*, Otd. III, No. 7–8, pp. 2–3.

F4*Review of a book: S. Mel'gunov, *Student Organizations of 1880–1890 at Moscow University* [in Russian], M., 1908, *ibid.*, Otd. III, No. 7–8, p. 28.

1909

F5"Quelques formules concernant la théorie de la function et les nombres Bernoulli," J. Reine Angew. Math. (Crelle J.) **135**, pp. 146–156.

1910

F6*"Integration of second-order linear equations," in: *Electrical Oscillations* (ed. by V.K. Lebedinskiĭ) [in Russian], No. 2, St. Petersburg, pp. 109–122.

F7*Review of a paper: V. F. Kagan, "Mathematics," in: *History of Russia in the First Half of the Nineteenth Century*, Granat, St. Petersburg, 1909 [in Russian], Zh. Russ. Fiz. Khim. O-va Chast. Fiz. **42**, No. 2, p. 79.

F8*Review of a paper: "Work of student scientific circles of the Physico-mathematical Faculty of St. Petersburg University" [in Russian], *ibid.*, **42**, No. 2, p. 80.

F9*"Return of Halley's comet" [in Russian], *ibid.*, **42**, No. 3, pp. 113–116.

F10*Review of a book: L. de Pesloyuan, *Life and Work of Abel* [in Russian], *ibid.*, **42**, No. 6, pp. 244–246.

1911

F11"Sur la recherche des surfaces particulières de l'équation Laplace," Soobshch. Khark. Mat. Ob-va Ser. 2 **12**, No. 6, pp. 244–268.

F12"A theory of the aeroplane" [in Russian], Zh. Russ. Fiz. Khim. O-va Chast. Fiz. **43**, No. 9, pp. 362–376; No. 10, pp. 400–410.

1912

F13"Sur la recherche des surfaces isodynamiques," C. R. Acad. Sci. **154**, pp. 864–865 (April 1).

F14"Sur un probleme hydrodynamique de Bjerknes," Soobshch. Khark. Mat. Ob-va Ser. 2, 1912–1913, No. 6, pp. 253–262 [coauthor M. Petelin].

1913

F15"Problem of an oscillatory discharge of a capacitor" [in Russian], Zh. Russ. Fiz. Khim. O-va Chast. Fiz. **45**, No. 5, pp. 276–282 [coauthor V. I. Smirnov].

1914

F16*Abstract of paper: W. M. Schmidt, "What forms of reflection of sound are important in thunder" [in Russian], Ezhemes. Meteorol. Byull. **22**, No. 4, p. 9.

F17"Zur Theorie der Vertikaltemperaturverteilung," Meteorol. Z. **31**, pp. 154–156.

F18"Importance of the lines of air flow for aeronautics" [in Russian]*, Tekh. Vozdukhoplav. No. 6–8, pp. 246–258.

F19"Die Grössenordnung der meteorologische Elemente und ihrer räumlichen und zeitlichen Abteilungen," Veroeff. Geophys. Inst. Univ.

Leipzig Ser. 2 **1**, No. 1, pp. 14–40 [coauthor H. T. Hesselberg].

[F20] "Sur la distribution de la temperature aux diverses hauteurs," Geofizich. Sb. **1**, No. 1, pp. 35–55.

1915

[F21] "On the problem of the velocity of sound" [in Russian], Geofizich Sb. **2**, No. 1 pp. 69–72 [coauthor V. Ya. Al'tberg].

1916

[F22] "One case of adiabatic motion of a heavy gas" [in Russian] [††].

[F23] "Sur les tourbillons dans liquid a temperature variable," C. R. Acad. Sci. **163**, No. 9, pp. 219–222 (July 28).

[F24] "Vortices in a liquid with a variable temperature " [in Russian], Soobshch. Khark. Mat. Ob-va **15**, No. 4, pp. 173–176.

[F25] Summary of Lectures on Air Navigation [in Russian], Press of the Staff of the Kiev Military Region, Kiev, 43 pp. (published by the order of His Excellency, Commander of Aviation and Aeronautics in Active Army).

[F26] "Atmospheric whirlwinds" [in Russian], Geofiz. Sb. **3**, pp. 90–120.

1917

[F27] "Determination of the vertical flow of air by observations made using pilot balloons at one point" [in Russian], Geofiz. Sb. **3**, No. 2, pp. 75–83 [coauthor N. Putyata].

[F28*] "General principles of bomb aiming" [in Russian], Proc. Commission on Studies of Air Artillery at the Headquarters of the Air Force (formed by the order of the Field General Inspector of the Military Air Force at the Staff of the Supreme Commander), No. 3, Petrograd, pp. 3–8.

1918

[F29] "Problem of proving the parallelogram of forces " [in Russian], Zh. Perm. Fiz.-Mat. Ob-va Perm. Univ. No. 1, pp. 33–43.

1919

[F30] "Vertical flow in the atmosphere" [in Russian], Zh. Perm. Fiz.-Mat. Ob-va Perm. Univ. No. 2, pp. 67–104.

1920

[F31] "Distribution of the temperature with altitude in the presence of radiative heat exchange between the Earth and Sun" [in Russian], Izv. Gl. Geofiz. Obs. No. 2, pp. 42–44.

1921

[F32] "Atmospheric whirlwinds with vertical and horizontal axes" [in Russian], Izv. Gl. Geofiz. Obs. No. 3, pp. 3–4.

[F33] "Concept of a rotating fluid in atmospheric motion" [in Russian], Meteorol. Vestn. **31**, pp. 69–88.

[F34] "Die vertikalen Temperaturgradienten in der Atmosphäre," Beitr. Phys. Freien Atmos. **10**, No. 4, pp. 185–201.

1922

[F35] *Experiments on Hydromechanics of a Compressible Liquid (Dissertation)* [in Russian], Petrograd, 516 pp. (litho-graphic publication); review of the above*: V. A. Kostitsyn, Usp. Fiz. Nauk. **3**, No. 4, 495 (1923).

[F36] "Über die Krümmung des Raumes," Z. Phys. **10**, No. 6, pp. 377–387.

[F37] "Atmospheric whirlwinds and sudden variations of the wind" [in Russian], Tr. Aérol. Obs. Pavlovsk. 1920–1922 No. 1, pp. 8–12.

[F38] "Sur la cinématique des lignes des tourbillon," Bull. Acad. Sci. Cracovie Ser. A, pp. 25–44.

1923

[F39] "Über die vertikale Temperaturgradienten in der Atmosphäre," Ann. Hydrograph. Marit. Meteorol. **51**, No. 1, pp. 12–16.

[F40] *World as Space and Time* [in Russian], Academia, Petrograd, 130 pp.

[F41] "On the motion of a compressible liquid" [in Russian], Izv. Gidrol. Inst. No. 7, pp. 21–28.

1924

[F42] "Über eine Methode der Bestimmung der vertikalen Windgeschwindigkeit," Meteorol. Z. **41**, No. 3, pp. 90–91 (coauthor Ya. D. Tamarkin).

[F43] "Über die Möglichkeit einer Welt mit konstanter negativer Krummung des Raumes," Z. Phys. **21**, pp. 326–333 [translated in Usp. Fiz. Nauk **80**, No. 3, pp. 447–452 (1963)].

[F44] "Über Wirbelbewegung in einer kompressiblen Flüssigkeit," Z. Angew. Math. Mech. **4**, No. 2, pp. 102–107.

[F45] *Collection of Problems on Higher Mathematics* (ed. by Prof. N. M. Gyunter, Ya. D. Tamarkin, Ya. V. Uspenskiĭ, and A. A. Fridman) [in Russian], Gosizdat, L., 226 pp. [†††]

[F46] "Propagation of a discontinuity in a compressible liquid" [in Russian], Zh. Russ. Fiz. Khim. O-va Chast. Fiz. **56**, No. 1, pp. 40–58.

[F47] "Curvature of space" [in Russian], *ibid.*, **56**, No. 1, pp. 59–68 (this is a translation of Ref. F36); reprinted later in Usp. Fiz. Nauk **80**, No. 3, pp. 439–446 (1963) and again in Usp. Fiz. Nauk **93**, No. 2, pp. 280–287 (1967).

[F48] "Possible configurations of electrons in a Rutherford atom" [in Russian], Tr. Gos. Opt. Inst. **3**, No. 21, pp. 1–14 (coauthor Ya. D. Tamarkin).

[F49*] Review of a book: P. Molchanov, *Methods and Instruments for Modern Air Navigation Service* [in Russian], Vestn. Vozd. Flota Nos. 6-7, p. 81.

[F50] *Fundamentals of the Theory of Relativity* [in Russian], Academia, L., 166 pp. (coauthor V. K. Frederiks).

[F51] *Theoretical Mechanics (Kinematics)* [in Russian], Izd. Voennomorskoĭ Akademii, L., 277 pp. (lithographic printing, coauthor L. G. Loĭtsyanskiĭ).

[F52] "Über die Geometrie der halbsymmetrischen Übertragungen," Math. Z. **21**, No. 3-4, pp. 211–223 (coauthor J. A. Schouten).

[F53] "Über atmosphärische Wirbel und die Turbulenz des Windes," Beitr. Phys. Freien Atmos. **11**, No. 4, pp. 154–163.

1925

[F54] "Differentialgleichungen für die turbulente Bewegung einer kompressiblen Flüssigkeit," Proc. First Intern. Congress of Applied Mechanics, Delft, 1924 (ed. by C. Beizano and J. M. Bürgers), Delft, 1925, pp. 395–405 (coauthor L. V. Keller).

[F55] *Approximate Calculations* [in Russian], Leningrad, 132 pp. (coauthor Ya. S. Bezikovich).

Solution of Numerical Equations [in Russian], Leningrad (lithographic printing).

[F56] [45]

[F57] "Sur le mouvement d'un fluide partait compressible," Izv. RAN **19**, pp. 351–361.

[F58] "At an altitude of 7400 m" [in Russian], Khochu Vse Znat' No. 10, pp. 18–20; shortened version (included in Ref. F73) in Klim. Pogoda No. 2-3, pp. 8–11.

[F59] "Trends in the work on organization of a meteorological service" [in Russian], *ibid.*, No. 1, pp. 18–22.

1926

[F60*] Arithmetic, in: Bol'shaya Sovet-skaya Éntsiklopediya [in Russian], Ist ed., Vol. 3, Moscow, pp. 338–346.

[F61*] Atmosphere [in Russian], *ibid.*, Vol. 3, p. 779 (coauthors B. N. Izvekov and E. I. Tikhomirov).

[F62*] Air Navigation [in Russian], *ibid.*, Vol. 4, pp. 204–206.

1927

[F63] Curriculum vitae [in Russian], Geofiz. Sb. **5**, No. 1, pp. 11–13.

[F64] "Théorie du mouvement d'un fluide compressible et ses applications aux mouvements de l'atmosphère," *ibid.*, **5**, No. 1, pp. 16–56.

[F65] "Intermittent features of planar motion of an incompressible liquid" [in Russian], *ibid.*, **5**, No. 2, pp. 9–24 (coauthor P. Ya. Polubarinova-Kochina).

[F66] "Approximate conditions of the dynamic feasibility of motion of a compressible liquid" [in Russian], *ibid.*, **5**, No. 2, pp. 24–32 (coauthor N.E. Kochin).

1930

[F67*] *Approximate Calculations* [in Russian], 2nd rev. ed., Leningrad (coauthor Ya. S. Bezikovich).

1931

[F68*] "Can microcyclones be predicted?" [‡] [in Russian], Meteorol. Vestn. No. 9–12, pp. 112-113.

[F69*] *Collection of Problems on Higher Mathematics* [in Russian], GTTI, Moscow-Leningrad [‡‡] (coauthors A. A. Adamov, V. V. Bulygin, A. P. Vilizhanin, N. M. Gyunter, A. N. Zakharov, V. M. Melioranskii, V. I. Smirnov, V. F. Tochinskiĭ, and Ya. V. Uspenskiĭ).

1934

[F70] *Experiments on Hydromechanics of a Compressible Liquid* [in Russian], 2nd ed. (ed. comments by N. E. Kochin and with additional papers by B. I. Izvekov, I. A. Kibel', and N. E. Kochin), Moscow-Leningrad, 1934, 370 pp. Review of the above*: E. Tikhomirov, Meteorol. Vestn. No. 5-6, 49 (1935).

1959

[F71] "Letters to B. B. Golitsyn and V. A. Steklov" [in Russian], Tr. Inst. Ist. Estestvozn. Tekh. Akad. Nauk SSSR **22**, pp. 324–388.

1965

[F72] *World as Space and Time* [in Russian], 2nd ed., Nauka, Moscow, 112 pp. (Popular Science Series).

1966

[F73] *Selected Works* (ed. by L. S. Polak) [in Russian], Nauka, M., 462 pp. (Classics of Science Series), with supplements including papers of V. A. Steklov, E. P. Fridman, P. L. Kapitza, A. Einstein, V. A. Fok (Fock), Ya. B. Zel'dovich, P. Ya. Polubarinova-Kochina, A. F. Gavrilov, and L. S. Polak, and also author's own abstracts of his papers, his autobiography, literature about him, and a list of his works (51 citations).

Compiled by V. Ya. Frenkel'

*The works (including reviews and abstracts) found by the author of the present paper and missing from *Selected Works* of A. A. Fridman (Ref. F73) and from Ref. 14 are identified by an asterisk in the above list. Italic numbers are used for the items which are included in Ref. F73.

†The following comment is added to this paper by N. N. Andreev: "Since the author departed to the front, the paper was completed in his absence, which necessitated slight changes in the positions and numbers of graphs."

††This work is mentioned in Ref. 14, but without an indication where it was published. A printed copy is in the library of the A. I. Voeĭkov Main Geophysical Observatory (under the number R.5/11.a.45). There are grounds for assuming that this was published in 1916 by one of the publishers active at that time in Kiev.

†††Judging by the preface, this is the third edition of the collection. It was not possible to find the first two editions; see Ref. F69.

‡This represents a letter to E. I. Tikhomirov published by the recipient and furnished with a brief preface by him.

‡‡Details apply to the sixth (and probably the last) edition of this collection. The first edition, by a team of authors which was the same as above, with the exception of Bulygin, Smirnov, and Fridman, appeared in 1912 in St. Petersburg. In the preface to this edition it is pointed out that great help was received from Ya. D. Tamarkin and A. A. Fridman. In the second edition the authors included V. V. Bulygin (1915). Beginning from the fourth edition (1929) the authors included also A. A. Fridman. The fifth edition was published in 1930.

Translated by A. Tybulewicz