The relativistic theory of gravitation and its consequences

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Usp. Fiz. Nauk **155**, 369–396 (July 1988)

A brief critical analysis of the general theory of relativity shows that its adoption leads to the abandonment of a number of fundamental principles underlying physics. A relativistic theory of gravitation is constructed. In it the gravitational field possesses all the attributes of physical fields, and the theory agrees completely with the fundamental physical principles and also the available experimental and observational facts. The consequences of the relativistic theory of gravitation, including, in particular, the development of collapse and the evolution of the universe, are considered.

CONTENTS

INTRODUCTION

From time to time papers have been published in this journal presenting different views about the general theory of relativity (GR).[5,45] A detailed critical analysis of these papers has been given in the specialist journals on theoretical physics (Refs. 11, 12, 16–18, 36, and 46). But since the reader of Uspekhi has not been precisely informed about our views, and much of what has been published in Uspekhi on this question is based either on an insufficiently deep penetration into the essence of the problem of gravitation or on the desire, come what may, to fit everything into the old channel of the general theory of relativity, we have felt the obligation to share our ideas about the development of the theory of gravitation with the readers of Uspekhi, preceding this with a critical analysis of the difficulties of GR. Our position is precisely formulated in the framework of the relativistic theory of gravitation, which agrees with the known experimental data and is in complete agreement with general physical principles; it can be changed only under the pressure of new facts if they are found to be in disagreement with it.

1. ANALYSIS OF THE BASIC PROPOSITIONS OF THE GENERAL THEORY OF RELATIVITY

We shall see below that the adoption of the fundamental conceptions of GR amounts to the abandoning of a number of fundamental principles that underlie physics. In the first place there is the abandonment of the conservation laws for energy, momentum, and angular momentum and (in complete agreement with the equivalence principle) the abandonment of the idea of the gravitational field as a physical field of Faraday-Maxwell type. However, we shall first consider briefly the history of the question.

The discovery by Poincaré and Minkowski of the four-dimensional world gave in principle the possibility of showing that in the general case different frames of reference correspond to different space-time metrics $\gamma_{\mu\nu}(x)$ (dependent on the coordinates x^μ of the frame and not necessarily diagonal). For example, in an arbitrary nonertial frame of reference S' the metric coefficients $\gamma'_{\mu\nu}$ are functions of the coordinates x' of this frame, and this leads to the appearance of an acceleration of a free material particle with respect to S' and inertia forces which are expressed in terms of the derivatives of first order of the tensor $\gamma'_{\mu\nu}$ with respect to the corresponding coordinates. The kinematic nature of the inertia forces is reflected in the fact that the accelerations "generated" by them in free material bodies do not depend on the masses of these bodies. It is well known that gravitational forces possess the same property, since, as experiments show, the gravitational mass of a body is equal to its inertial mass. This circumstance was used by Einstein, who concluded (Ref. 1, p. 231) that the gravitational field must be described like the field of inertia forces, by a metric tensor $g_{\mu\nu}$, though in a Riemannian space-time. Later, Einstein wrote: "The entire theory arose on the basis of the conviction that in a gravitational field all physical processes take place in exactly the same way as without a gravitational field but in an appropriately accelerated (three-dimensional) coordinate system ("equivalence hypothesis")" (Ref. 1, p. 400). In this central point Einstein abandoned the concept of the gravitational field as physical reality, and this led subsequently to insuperable difficulties in GR.

One of them, which follows directly from what has been said above, is related to the nonlocalizability of the gravitational field. It is well known that in all physical theories one of the most important characteristics of a field has always been its energy-momentum tensor density, which, following Hilbert, one obtains by varying the density of the field Lagrangian with respect to the components of the space-time metric tensor. Such a characteristic reflects the fact of the existence of the field, namely, a nonvanishing energy-momentum tensor density in some space-time region is a necessary and sufficient condition for the presence in it of a physical field. In GR the gravitational field does not possess such a characteristic, and this is due to the fact that in Einstein's theory the quantities $g_{\mu\nu}$ have a double meaning—on the one hand they are field variables, on the other hand the components of the space-time metric tensor. Because of this physi-

 0038-5670/88/070581-16$01.80

cogeometrical dualism of $g_{\mu\nu}$ the expression for the density of the completely symmetric energy-momentum tensor must simultaneously represent the field equation. It is then obvious that the density of the total symmetric energy-momentum tensor of the system determined by the (generally covariant) Hilbert manner must be strictly zero in the whole of space-time, while outside the matter the density of the symmetric energy-momentum tensor of the gravitation field must be zero. Thus, in GR the gravitational field outside a source is devoid of a fundamental physical characteristic—the energy-momentum tensor. As a consequence of this, GR also lacks the conservation laws for energy, momentum, and angular momentum of the matter and the gravitational field taken together.

Understanding clearly the need for "energy-momentum" characteristics of the gravitational field and conservation laws, Einstein introduced in 1918 the concept of the energy-momentum pseudotensor τ_μ^ν of the gravitational field. However, in the same year Schrödinger[2] showed that by an appropriate choice of the coordinates of the three-dimensional space all components of τ_μ^ν outside a homogeneous sphere can be made to vanish. Answering Schrödinger, Einstein wrote: "With regard to Schrödinger's arguments, they have conviction by analogy with electrodynamics, in which the stresses and energy density of any field are nonzero. However, I can find no reason why this should be the same for gravitational fields. Gravitational fields can be specified without introducing stresses and an energy density" (Ref. 1, p. 627). This corresponded completely to his earlier assertion: "... For an infinitesimally small region the coordinates can always be chosen in such a way that there will be no gravitational field in the region" (Ref. 1, p. 423); later he confirmed this point of view, asserting (see Ref. 3, p. 124): "For any infinitesimally small neighorhood of a point in an arbitrary gravitational field one can find a local system of coordinates in a state of motion such that with respect to this local coordinate system a gravitational field does not exist (local inertial system)." *These assertions demonstrate that Einstein knowingly abandoned the concept of the gravitational field as physical reality.* At the same time, it is readily noted that inertial forces and gravitational forces are entirely different even in their mathematical structure; for the Riemann-Christoffel curvature tensor $R^{\,\alpha}_{\mu\nu\beta}$ for the former is identically zero but for the latter it is not. Later, in 1948, Einstein revised his view of the equivalence principle and no longer spoke of the equivalence of fields of inertial forces and gravitational forces and merely noted that fields of inertial forces are a special case of gravitational fields satisfying, the Riemann conditions $R^{\,\alpha}_{\mu\nu\beta} = 0$. It appears that this circumstance nevertheless escaped many.

Another fundamental difficulty in GR intimately related to the identification of the gravitational field with the metric tensor of a Riemannian space is the absence in it of not only local but also integral conservation laws for energy, momentum, and angular momentum. The first person who noted this characteristic feature of GR was Hilbert, who in 1917 wrote[4]: "I assert ... that for the general theory of relativity, i.e., in the case of general invariance of the Hamilton function, energy equations that ... correspond to the energy equations in orthogonally invariant theories do not exist at all. I could even note this circumstance as the characteristic feature of the general theory of relativity." However, neither Einstein nor other physicists reacted in any way to this remark of Hilbert. The fundamental fact that in GR conservation laws for energy, momentum, and angular momentum are in principle impossible because the Riemannian space introduced in GR does not possess the maximal group of motions of space-time escaped the notice of Einstein's contemporaries.

Some physicists still do not understand this.[1)] Other physicists, recognizing the absence of conservation laws in GR, regard this as a very important fundamental step in GR in the development of physical ideas. But neither in the macroscopic nor the microscopic world do we find a single experimental fact that directly or indirectly casts doubt on the validity of the conservation laws for matter. Indeed, Einstein himself recognized their fundamental importance very well. In Ref. 1 (p. 299) he wrote: "... One must certainly require that the matter and the gravitational field together should satisty the energy-momentum conservation laws" and later: "Experience forces us to seek a differential law equivalent to the *integral* conservation laws for momentum and energy" (Ref. 1 p. 651), and then: "I wish to show here that ... the concepts of energy and momentum can be established [in GR] just as clearly as in classical mechanics."

The investigation (Ref. 1, p. 650) made by Einstein (in the framework of GR) in 1918 and the subsequent mathematical confirmation in the same year by Klein[7] of the results that Einstein obtained created the impression that the energy-momentum problem in GR had been completely solved. The conclusion of these studies, with insignificant modifications, is still repeated in many textbooks and papers without recognition of the fact that the arguments of Einstein and Klein contained a simple but fundamental error. The point is that the entity J_σ, whose components Einstein identified with the energy and momentum, is found on closer examination to be a quantity that vanishes identically. A further unsatisfactory consequence of GR is the nonuniqueness of its predictions for gravitational effects. Such a conclusion can be drawn on the basis of the fact that for a chosen arithmetization of space the Hilbert-Einstein equations alone do not yet determine the metric of the Riemannian space-time (in the general case their solution can contain four arbitrary functions). The subsequent fixing of the metric by the imposition on the obtained solutions of coordinate conditions (which are always explicitly noncovariant) is a procedure that is far from unique, since no restrictions at all are imposed on the actual choice of the coordinate conditions in GR, whereas the functional structure of the metric coefficients depends strongly on the choice and is different for different choices. This is the one side of the matter. On the other side it follows from the theorem proved by Weyl,[8] Lorentz,[9] and Petrov,[10] according to which for given equations of all timelike and all isotropic geodesics in any coordinate system, i.e., for some chosen arithmetization of space, the metric tensor of space-time in this system is determined up to a constant factor,[2)] that different metric tensors in a given coordinate system must lead to different predictions for the motion of test bodies and light. And since in GR the functional structure of the metric tensor in a given coordinate system is different for different choices of the coordinate conditions, the predictions of the theory will depend on this choice, i.e., will not possess the property of uniqueness; this is a further fundamental shortcoming of GR.

We demonstrate the nonuniqueness of the predictions in GR in two examples—the calculation of the inertial mass and the example of the delay of a radio signal by a gravitational field.

Suppose a static spherically symmetric body of active mass M is the source of gravitational field. For what follows we adopt an arithmetization of space for which the center S of the source has the value $r = r_s = 0$ and any point of space-time is given a set of numbers x^μ (t,x^1,x^2,x^3). Then one of the general exterior (relative to the body M) solutions of the Hilbert-Einstein equations

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu}R = 8\pi G T^{\mu\nu}_M, \qquad (1.1)$$

where $R^{\mu\nu}$ is the Ricci tensor, $R = R^{\mu\nu}g_{\mu\nu}$, and $T^{\mu\nu}_M$ is the energy-momentum tensor of the matter (of all forms of matter except the gravitational field), will be the solution

$$g^{00} = \frac{1}{B}, \quad g^{kn} = \frac{1}{C}\gamma^{kn} + \left(\frac{1}{C} - \frac{1}{A}\right)\frac{x^k x^n}{r^2}, \quad g^{0k} = 0, \qquad (1.2)$$

or, in equivalent form,

$$g_{00} = B, \quad g_{kn} = C\gamma_{kn} + (C - A)\frac{x^k x^n}{r^2}, \quad g_{0k} = 0; \qquad (1.2')$$

here

$$B = 1 - \frac{2GM}{rC^{1/2}}, \quad A = C\left(1 + \frac{rC'}{2C}\right)^2\left(1 - \frac{2GM}{rC^{1/2}}\right)^{-1},$$
$$C' = \frac{dC(r)}{dr}; \qquad (1.3)$$

further $r^2 = -\gamma_{kn}x^k x^n$, $g = \det g_{\mu\nu} = -BAC^2$; with regard to the function $C(r)$, it is required merely to be smooth and such that

$$\lim_{r\to\infty} C(r) \to 1,$$

and otherwise it is fairly arbitrary.[3)]

Using the definition in GR for the inertial mass of the body (or its total energy),

$$P^0 = \lim_{r\to\infty} \oint ds_k h^{00k}, \qquad (1.4)$$

in which $ds_k = -(x_k/r)r^2 \sin\theta \, d\theta \, d\varphi$, and

$$h^{00k} = -\frac{1}{16\pi G}\frac{\partial}{\partial x^n}[g(g^{00}g^{kn} - g^{0k}g^{0n})], \qquad (1.5)$$

we find after simple calculations

$$P^0 = -\frac{1}{2G}\lim_{r\to\infty}\left[r^2 C\left(\frac{\partial C}{\partial r} + \frac{C-A}{r}\right)\right]. \qquad (1.6)$$

Choosing, for example,

$$C(r) = \left(1 + \frac{GM}{r}\right)^2, \qquad (1.7)$$

we obtain from (1.6) exact equality of the inertial mass P^0 of the body to its gravitational mass M. But if we take

$$C(r) = \left[1 + \alpha^2\left(\frac{8GM}{r}\right)^{1/2}\right]^2, \qquad (1.8)$$

where α is a free real parameter, then (1.6) gives the value

$$P^0 = (1 + \alpha^4)M, \qquad (1.9)$$

and this directly indicates the nonuniqueness of the predictions of GR for the inertial mass of the considered system[4)] and does not agree with the experimental confirmation of the equality of the gravitational and inertial masses, which, incidentally, was taken by Einstein as the foundation of his theory. *This result shows that not only local but also integral energy-momentum conservation laws do not hold in GR.*

Turning to the illustration of the nonuniqueness of the predictions of GR for the gravitational radio signal delay effect, and retaining the arithmetization of space chosen above, we take the position of the source of the radio pulses (the earth) to be the point $e(r_e,\varphi_e,\theta_e = \pi/2)$, the position of the receiver or reflector (Mercury) (which reflects the signal back to the point e) to be the point $p(r_p,\varphi_p,\theta_p = \pi/2)$, and the points of the surface of the body M (assuming it to be spherical) to have the coordinate value $r = r_f$. For simplicity, we consider two special cases, taking for the adopted arithmetization of space $C(r) = 1$ in the first case and $C(r) = [1 + (GM/r)]^2$ in the second. Then in the first case (a)

$$ds_a^2 = \left(1 - \frac{2GM}{r}\right)dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1}dr^2$$
$$- r^2(d\theta^2 + \sin^2\theta \, d\varphi^2), \qquad (1.10)$$

and in the second (b)

$$ds_b^2 = \left(\frac{r - GM}{r + GM}\right)dt^2 - \left(\frac{r + GM}{r - GM}\right)dr^2$$
$$- r^2\left(1 + \frac{GM}{r}\right)^2(d\theta^2 + \sin^2\theta \, d\varphi^2). \qquad (1.11)$$

Since the substitution $r = \rho - GM$ reduces the expression (1.11) to the form (1.10),

$$ds_b^2 = \left(1 - \frac{2GM}{\rho}\right)dt^2 - \left(1 - \frac{2GM}{\rho}\right)^{-1}d\rho^2$$
$$- \rho^2(d\theta^2 + \sin^2\theta \, d\varphi^2), \qquad (1.11')$$

it might seem that the consequences of (1.10) and (1.11) would be identical. In reality, this is not the case, since (1.10) and (1.11′) differ significantly; for whereas in (1.10) the values $r = r_S = 0$, $r = r_f$, $r = r_{e,p}$ are by virtue of the adopted arithmetization associated with the positions of the center S of the body M and its surface and the positions of the source and reflector of the radio pulses, in (1.11′) these positions will will correspond to $\rho = \rho_S = GM$, $\rho = \rho_f = r_f + GM$, $\rho = \rho_{e,p} = r_{e,p} + GM$, and this will undoubtedly affect the results of the calculations and lead to different consequences of (1.10) and (1.11). This conclusion also follows directly from the Weyl-Lorentz-Petrov theorem, since the metric coefficients in (1.10) and in (1.11) differ and, therefore, the motions along the geodesics in the metrics (1.10) and (1.11) will be quite different. This can also be shown by direct calculation.

Using the standard methods and restricting ourselves in the calculations to the first order in G, we obtain for the time of propagation of a radio signal to one end (for $\varphi_p - \varphi_e > \pi/2$ and under the assumption that at the pericenter its trajectory touches the point r_f of the surface of the body M) the expressions[13,14]

$$t_a = (r_p^2 - r_f^2)^{1/2} + (r_e^2 - r_f^2)^{1/2}$$
$$+ GM\left[2\ln\frac{r_p + (r_p^2 - r_f^2)^{1/2}}{r_e - (r_e^2 - r_f^2)^{1/2}} + \left(\frac{r_p - r_f}{r_p + r_f}\right)^{1/2}\right.$$
$$\left. + \left(\frac{r_e - r_f}{r_e + r_f}\right)^{1/2}\right] \qquad (1.12)$$

in the case of the solution (1.10) and

$$t_b=(r_p^2-r_f^2)^{1/2}+(r_e^2-r_f^2)^{1/2}+2GM\left[\ln\frac{r_p+(r_p^2-r_f^2)^{1/2}}{r_e-(r_e^2-r_f^2)^{1/2}}+\left(\frac{r_p-r_f}{r_p+r_f}\right)^{1/2}+\left(\frac{r_e-r_f}{r_e+r_f}\right)^{1/2}\right] \tag{1.13}$$

in the case of the solution (1.11). For $r_f \ll r_{e,p}$ we then obtain (to the chosen accuracy) the following expressions, taking into account the deflection of the signal by the gravitational field,

$$t_a=R+2GM\ln\frac{r_e+r_p+R}{r_e+r_p-R}-2GM, \tag{1.12'}$$

$$t_b=R+2GM\ln\frac{r_e+r_p+R}{r_e+r_p-R}, \tag{1.13'}$$

in which

$$R=(r_e^2-r_\perp^2)^{1/2}+(r_p^2-r_\perp^2)^{1/2} \tag{1.14}$$

is the relative distance (along the straight line) between the points e and p, and $r_\perp$ is the coordinate of the point of intersection of the straight lines joining e and p on the one hand and S and the pericenter of the trajectory on the other.

Since by virtue of the adopted arithmetization the numbers which occur in (1,12), (1.13) or (1.12'), (1.13') are the same, we obtain the conclusion, confirming the one drawn above, that the predictions of GR for this effect are nonunique in the variables x^μ, the nonuniqueness arising already in the first order in G!

We now show that the transition in the obtained results from the arithmetization numbers to the observed physical quantities does not change the conclusion of nonuniqueness. For this, using, respectively, the metrics (1.10) and (1.11), we calculate in the first order in G the physical radial distances (measured experimentally) from the surface of the body M to the source e and the reflector p of the radio pulses:

$$l_{e,p}=\int_{r_f}^{r_{e,p}} dr\,(-g_r)^{1/2}\approx r_{e,p}-r_f+GM\ln\frac{r_{e,p}}{r_f}, \tag{1.15}$$

and the relative frequency shift (measured experimentally) in the field of the body M:

$$\left.\frac{\Delta\omega}{\omega}\right|_{e,p}^{f}\equiv\delta_{e,p}\approx GM\left(\frac{1}{r_f}-\frac{1}{r_{e,p}}\right). \tag{1.16}$$

It can be seen that both the distances l and the relative frequency shifts δ are the same for the two metrics in the first order in G. This means that the transition by means of (1.15) and (1.16) in $t_{a,b}$ from r to the observed physical quantities l and δ leaves the conclusion of nonuniquenss of the predictions of GR for this effect untouched.[5)]

There is a belief that if the time Δt of the gravitational delay is expressed in terms of the times of revolution T of the source e (the earth), the reflector p (Mercury), and some test body revolving around M in a circular orbit with $r=r_f$, then it will not depend on the choice of the metric, i.e., will be the same for the metrics (1.10) and (1.11). We demonstrate the error of this belief.

Suppose for simplicity that all the bodies revolve around M in circular orbits. Then for the arithmetization of space adopted above we obtain in the case of the metrics (1.10) and (1.11), respectively, in the first order in G the periods[14–16]

$$T^{(a)}_{f,e,p}=2\pi\frac{r^{3/2}_{f,e,p}}{(GM)^{1/2}}, \tag{1.17}$$

$$T^{(b)}_{f,e,p}=2\pi\frac{r^{3/2}_{f,e,p}}{(GM)^{1/2}}\left(1+\frac{3}{2}\frac{GM}{r_{f,e,p}}\right). \tag{1.18}$$

It can be seen that in accordance with GR the times T of revolution of the bodies in their orbits in the variables x^μ are, like the times t, different for the different metrics. If in (1.17) and (1.18) we go over from the numbers r to the observed physical quantities l and δ, then in these measurable variables too the nonuniqueness of the theoretical values of T remains. The difference between the times $T^{(a)}$ and $T^{(b)}$ of revolution corresponding to the metrics (1.10) and (1.11) is explained here by the fact that although the physical radial distances to the orbit in the different metrics are the same (in the first order in G), the speeds of the motion of the body in it are different in the different metrics.[16,17]

If now the propagation times t_a and t_b are expressed in terms of the times of revolution $T^{(a)}$ and $T^{(b)}$, respectively, and the notation $L\equiv[T(GM)^{1/2}/2\pi]^{3/2}$ is introduced to simplify the expressions, then for the two metrics the identical connection between t and T is obtained:

$$t=(L_p^2-L_f^2)^{1/2}+(L_e^2-L_f^2)^{1/2}+GM\left[2\ln\frac{L_p+(L_p^2-L_f^2)^{1/2}}{L_e-(L_e^2-L_f^2)^{1/2}}+\left(\frac{L_p-L_f}{L_p+L_f}\right)^{1/2}+\left(\frac{L_e-L_f}{L_e+L_f}\right)^{1/2}\right]. \tag{1.19}$$

To determine the actual gravitational delay Δt, which is the quantity of true physical interest, we must in (1.19) also separate the time t_0 that would be required by the signal to traverse the path from the emitter e to the reflector p in the absence of the gravitational influence on the signal of the central body M. This can be done by calculating the time t_0 for the chosen arithmetization of space in the flat metric $\gamma_{\mu\nu}(r)$:

$$t_0=(r_p^2-r_\perp^2)^{1/2}+(r_e^2-r_\perp^2)^{1/2}, \tag{1.20}$$

and, using the connections (1.17) and (1.18), expressing it in terms of the revolution times T. In the case of the metrics (1.10) and (1.11), this gives[18 6)]

$$t_0^{(a)}=(L_p^2-L_\perp^2)^{1/2}+(L_e^2-L_\perp^2)^{1/2}, \tag{1.21}$$

$$t_0^{(b)}=(L_p^2-L_\perp^2)^{1/2}+(L_e^2-L_\perp^2)^{1/2}-GM\left[\left(\frac{L_p-L_\perp}{L_p+L_\perp}\right)^{1/2}+\left(\frac{L_e-L_\perp}{L_e+L_\perp}\right)^{1/2}\right]. \tag{1.22}$$

Thus, the gravitational delay time Δt, determined by the difference between t and t_0, is different in the metrics (1.10) and (1.11). For $GM, L_\perp, L_f \ll L_{e,p}$,

$$\Delta t_a=2GM\ln\frac{L_e+L_p+L_0}{L_e+L_p-L_0}-2GM, \tag{1.23}$$

$$\Delta t_b=2GM\ln\frac{L_e+L_p+L_0}{L_e+L_p-L_0}, \tag{1.24}$$

where $L_0\equiv ct_0^{(a)}$, and this proves the error of the belief that Δt is determined uniquely in GR when it is expressed in terms of the revolution times.

If the time t is calculated[18] by going over right at the start from the coordinates $x^\mu=(t,r,\theta,\varphi)$ the variables $\xi^\alpha=(t,\rho,\theta,\varphi)$, where $\rho\equiv r(C(r))^{1/2}$ and in which ds^2 will be determined by the expression (1.11'), then in the first order in G

$$t=(\rho_p^2-\rho_f^2)^{1/2}+(\rho_e^2-\rho_f^2)^{1/2}+GM\left[2\ln\frac{\rho_p+(\rho_p^2-\rho_f^2)^{1/2}}{\rho_e-(\rho_e^2-\rho_f^2)^{1/2}}+\left(\frac{\rho_p-\rho_f}{\rho_p+\rho_f}\right)^{1/2}+\left(\frac{\rho_e-\rho_f}{\rho_e+\rho_f}\right)^{1/2}\right], \tag{1.25}$$

$$T_{f,e,p}=2\pi\rho_{f,e,p}^{3/2}(GM)^{-1/2}. \tag{1.26}$$

Despite the resulting unique connection (given by the expression (1.19)) between t and the experimentally observed values of T, the delay time Δt will again be different for the different metrics $g_{\mu\nu}(r)$. Indeed, calculating t_0, the meaning of which was discussed above, in the original arithmetization of space with the introduction of the flat metric $\gamma_{\mu\nu}(r)$, and going over in the final result from r to ρ, using the connection $r(C(r))^{1/2}=\rho$, we again, taking into account (1.26), arrive in the cases $C(r)=1$ and $C(r)=[1+(GM/r)]^2$ at the expressions (1.21) and (1.22), i.e., for Δt we obtain (1.23) and (1.24), respectively. But if the time t_0 is calculated by introducing the flat metric $\gamma_{\mu\nu}(\rho)$, taking ρ formally as radial coordinate, a nonuniqueness in Δt arises from the nonuniqueness in the choice of $\rho=\rho_S$. For example, for $\rho_S=0$ the value of t_0 will be determined by the expression (1.21) and Δt by the expression (1.23) (which does not agree, incidentally, with the experimental data[19]), while for $\rho_S=GM$ we obtain the results (1.22) and (1.24). In addition, it must be borne in mind that the solution $g_{\mu\nu}(\rho)$ of the form (1.11′) in the variables ξ^α is by no means the unique solution of the Hilbert-Einstein equations. Indeed, for any particular form of $C(r)$ the function $\rho=r(C(r))^{1/2}$ can always be taken as one of the variables in terms of which the Hilbert-Einstein equations (1.1) are expressed. But then it will also be solved by solutions of the form (1.2) and (1.3) except that now the part of r in them will be played by ρ and the part of $C(r)$ by the function $\widetilde{C}(\rho)$, i.e., the nonuniqueness in t and Δt will arise because of the arbitrariness of $\widetilde{C}(\rho)$, as occurred earlier in the original arithmetization x^μ.

Bearing in mind that Eqs. (1.1) in the original arithmetization of space are satisfied by the class of solutions[16]

$$C(r)=\left[1+\frac{(\lambda+1)GM}{r}\right]^2$$

with arbitrary real parameter λ, we can, choosing λ, make the gravitational delay time Δt in GR arbitrary, in particular, equal to zero; in the case of the Earth and Mercury $\Delta t\approx 0$ for $\lambda\approx-11.2$.

We end by demonstrating the nonuniqueness of the predictions of GR for the gravitational delay effect in a thought experiment. Imagine two test bodies, in each of which there are a source, a reflector, and a detector of radio signals, separated by a certain distance and fixed at the points e and p, and that at the point S, which is fairly close to the line ep but approximately equally distant from e and p, there is fixed a "needle" capable of reflecting signals sent to it and on which one can when necessary place a (small) massive spherically symmetric body M. Removing the body M from the "needle" to a great distance (to "infinity"), we use the times of propagation of the radio signals between the individual pairs of points to establish (taking $r_S=0$) the physical distances r_e from S to e, r_p from S to p, and L_0 from e to p. In principle, this method can be used to realize a physical arithmetization of the entire region of space occupied by our "facility." The numbers r obtained in this manner can then be taken as the values of the variable r in the Hilbert-Einstein equations. Then the physical distances between arbitrary points of space in the presence of the body M (when its center is fixed on the "needle") will be expressed in terms of the numbers r, which are the radial distances from S to the chosen point in the absence of M. Using for the adopted arithmetization of space (1.10) or (1.11) as solutions of the Hilbert-Einstein equations, we obtain for the time t of propagation of the signal from e to p in the field of M the expressions (1.12′) or (1.13′), in which R will be equal to L_0. Thus, the gravitational delay time $\Delta t=t-t_0$ predicted by GR is nonunique.

This analysis shows that nonuniqueness of the predictions for gravitational effects is an organic feature of GR.

Thus, the absence in GR of conservation laws for energy, momentum, and angular momentum, the abandonment of the notion of the gravitational field as a physical field, and also the nonuniqueness of the predictions for gravitational effects render GR a physically unsatisfactory theory and require a fundamental review of ideas about gravitation.

2. THE RELATIVISTIC THEORY OF GRAVITATION[7]

Since the time of Newton it has been known that the geometry of space is an inseparable part of physical theory. According to the apt remark of Gauss, "... geometry should not be considered with arithmetic, which exists purely *a priori*, but rather with mechanics." Therefore the study of mechanical phenomena at low velocities (compared with the velocity of light) is a test of not only the law of mechanics but also of the Euclidean nature of the geometry that occurs organically in Newton's theory.

The study of electromagnetic phenomena and the motion of particles with velocities near the speed of light made it necessary to give up the concepts of absolute space and time separately and led to the conception of a single four-dimensional space-time, in which the scales of length and time cease to be absolute but depend on the velocities relative to the motion of the coordinate system. This naturally required the transition from Euclidean geometry of space to pseudo-Euclidean geometry of space-time.

Thus, as long as we considered nonrelativistic physical processes the experiments confirmed the Euclidean structure of the geometry of space and the concept of time as an independent parameter. But as soon as relativistic physical processes were considered, experiments indicated a different, pseudo-Euclidean structure of space-time.

The discovery of the pseudo-Euclidean geometry of space-time enriched physics as a whole and was reflected, first, in a generalization of Newtonian mechanics to the relativistic mechanics of Poincaré, and then in all physical theories of both the macroscopic and the microscopic world except gravitation. Moreover, in all theories fundamental physical concepts such as energy, momentum, angular momentum, and their conservation laws were retained.

In 1921 Einstein, analyzing the properties of space-time (Ref. 3, p. 85), correctly noted that "... the question of whether this continuum has a Euclidean, Riemannian, or other structure is a physical question that must be answered by experiment and not a question of convention of the choice on the basis of simple expediency." In our opinion, the solution to this problem must be based, not on special observational data on the motion of light and test bodies, but on deeper fundamental properties of matter irrespective of its particular forms. Indeed, if the geometry of space-time were determined by studying the motion of test bodies and light, then in principle one could establish for it a Riemannian structure. But this would automatically lead to the abandonment of fundamental laws of nature—the conservation laws for energy, momentum, and angular momentum—since Riemannian space does not in the general case have the group of motions required for their fulfillment.

It is our conviction that in establishing the structure of the space-time geometry we should not use particular (and different for different sources) facts about the motion of light and test bodies but rather the most general dynamical properties of matter—its conservation laws, which are not only of fundamental importance but are also confirmed experimentally. *It is obvious that the existence of the ten conservation laws (for energy, momentum, and angular momentum) objectively reflects the property of our material world that is manifested in the homogeneity and isotropy of space-time.*

There are three known types of space that admit the introduction of ten integrals of motion. They are the space of constant negative curvature (Lobachevskiĭ space), the space of zero curvature (Euclidean space), and the space of constant positive curvature (Riemann space). The first two spaces are infinite, while the third is a closed space, although it does not have boundaries. If of any theory, including the theory of the gravitational field, we require that in it all ten conservation laws should hold, then it is obviously necessary to give up a Riemannian geometry of general form and choose as basis one of the geometries listed above.[8] Since all currently known experimental data on the electromagnetic, weak, and strong interactions unambiguously indicate a space-time with pseudo-Euclidean geometry (which underlies the theory of the corresponding fields[9]) and there are no facts that cast doubt on this, it is natural to assume that this space is common to all physical theories, so that no exception is made for the gravitational field. Then fulfillment of the conservation laws for energy and momentum and, separately, angular momentum will be guaranteed.

On the basis of this, we formulate the basic propositions of the relativistic theory of gravitation (RTG).

PROPOSITION I. As the fundamental, base, space in the RTG we take the Minkowski space x^μ with the metric $\gamma^{\mu\nu}(x)$.

This proposition reflects the property inherent in all matter, irrespective of its nature, of the universality of the conservation laws for energy and momentum and, separately, angular momentum.[10]

PROPOSITION II. In the RTG the gravitational field is regarded as a true real (with zero rest mass) physical field in Minkowski space possessing all the attributes inherent in other physical fields; it is associated with a symmetric field tensor $\Phi^{\mu\nu}$ of second rank with representations corresponding to the spin states 2 and 0.

The elimination from the states of the field $\Phi^{\mu\nu}$ of its representations corresponding to spin values 1 and 0 is achieved (see Refs. 24–27) by making $\Phi^{\mu\nu}$ satisfy the field equation

$$D_\mu \Phi^{\mu\nu} = 0, \tag{2.1}$$

where D_μ is the covariant derivative with respect to the metric $\gamma^{\mu\nu}$ of the Minkowski space. This equation not only eliminates from consideration the unphysical spin states of the gravitational field $\Phi^{\mu\nu}$ but also makes it impossible to eliminate the metric $\gamma^{\mu\nu}$ of Minkowski space from the theory,[11] simultaneously making it possible to separate noninertial effects from manifestations of the gravitational field.

PROPOSITION III (geometrization principle). *The Lagrangian density of all forms of matter except for the gravitational field is, by virtue of the universality of gravitational interactions and the tensor nature of the gravitational field, constructed in the RTG on the basis of contractions with an effective tensor $g^{\mu\nu}$ determined by "adding" the gravitational field $\Phi^{\mu\nu}$ to the metric tensor $\gamma^{\mu\nu}$ in accordance with the rule*

$$\tilde{g}^{\mu\nu} \equiv \tilde{\gamma}^{\mu\nu} + \tilde{\Phi}^{\mu\nu}, \tag{2.2}$$

where

$$\tilde{g}^{\mu\nu} \equiv (-g)^{1/2} g^{\mu\nu}, \quad \tilde{\gamma}^{\mu\nu} \equiv (-\gamma)^{1/2} \gamma^{\mu\nu}, \quad \tilde{\Phi}^{\mu\nu} \equiv (-\gamma)^{1/2} \Phi^{\mu\nu}, \tag{2.3}$$

$g \equiv \det g_{\mu\nu}$, $\gamma \equiv \det \gamma_{\mu\nu}$; *the derivatives of the nongravitational physical fields Φ_a that occur in the Lagrangian $\mathscr{L}_M(g^{\mu\nu}, \Phi_a)$ are assumed to be derivatives ∇_μ covariant with respect to the effective metric $g^{\mu\nu}$.*

The geometrization principle introduces into the theory, as a consequence of the universality of the gravitational interactions and the tensor nature of the gravitational field, the secondary concept of an effective Riemannian space with metric $g^{\mu\nu}$, which is defined (and this is very important) in a single chart (sheet?). This space has a purely field origin; the primary concepts in the theory are still the Minkowski space with metric $\gamma^{\mu\nu}$ and the gravitational field $\Phi^{\mu\nu}$ in it. The geometrization principle of the RTG falls short of the equivalence principle of GR, since in the RTG, as in other physical field theories, all physical quantities have a tensor (and not pseudotensor) nature and, therefore, in particular, the energy density of the gravitational field at a point cannot be made to vanish by any coordinate transformations, although the force effect of the gravitational field on a material point can be compensated.

PROPOSITION IV. The Lagrangian density of the free gravitational field is assumed in the RTG to be a quadratic function of the derivatives of first order $D_\lambda g_{\mu\nu}$ covariant with respect to the Minkowski space tensor $\gamma^{\mu\nu}$; it is also required that under transformations of the gravitational field $\Phi^{\mu\nu}$ of the form

$$\delta_\varepsilon \tilde{\Phi}^{\mu\nu} \equiv \delta_\varepsilon \tilde{g}^{\mu\nu} = \tilde{g}^{\mu\lambda} D_\lambda \varepsilon^\nu(x) + \tilde{g}^{\nu\lambda} D_\lambda \varepsilon^\mu(x) - D_\lambda(\varepsilon^\lambda \tilde{g}^{\mu\nu}), \tag{2.4}$$

where $\varepsilon^\nu(x)$ is an infinitesimal 4-vector, $\mathscr{L}_g(\tilde{\gamma}^{\mu\nu}, \tilde{g}^{\mu\nu}, D_\lambda g_{\mu\nu})$ should change solely by a divergence (gauge principle):

$$\mathscr{L}_g \to \mathscr{L}_g + D_\nu Q^\nu(x). \tag{2.5}$$

The gauge principle makes the local noncommutative gauge Lie algebra of supercoordinate transformations (2.4) of the gravitational field the basis of the construction of $\mathscr{L}_g$. One can readily show that the operators δ_ε form a Lie algebra by calculating, using (2.4), the Lie commutator:

$$(\delta_{\varepsilon_1}\delta_{\varepsilon_2}-\delta_{\varepsilon_2}\delta_{\varepsilon_1})\,\tilde{g}^{\mu\nu}(x)=\delta_{\varepsilon_3}\tilde{g}^{\mu\nu}(x), \tag{2.6}$$

where

$$\varepsilon_3^{\mu}=\varepsilon_1^{\nu}D_{\nu}\varepsilon_2^{\mu}-\varepsilon_2^{\nu}D_{\nu}\varepsilon_1^{\mu}. \tag{2.7}$$

It should be noted that by virtue of the universality of the field equation (2.1) required by proposition II the gauge transformations (2.4) will hold only on a manifold $\varepsilon^{\nu}(x)$ that satisfies the equation

$$g^{\alpha\beta}D_{\alpha}D_{\beta}\varepsilon^{\nu}(x)=0. \tag{2.8}$$

We note also that although the expression (2.4) for $\delta_{\varepsilon}\,\tilde{g}^{\mu\nu}(x)$ is formally, in its form, identical to the expression for the infinitesimal increment of $\tilde{g}^{\mu\nu}$ under the coordinate transformation

$$x^{\mu}\rightarrow x^{\mu}+\xi^{\mu}(x), \tag{2.9}$$

for the field $\tilde{\Phi}^{\mu\nu}$ it is completely different from the infinitesimal increment

$$\delta_{\xi}\tilde{\Phi}^{\mu\nu}=\tilde{\Phi}^{\mu\lambda}D_{\lambda}\xi^{\nu}(x)+\tilde{\Phi}^{\nu\lambda}D_{\lambda}\xi^{\mu}(x)-D_{\lambda}(\xi^{\lambda}\tilde{\Phi}^{\mu\nu}), \tag{2.10}$$

which arises under the transformation (2.9). Thus, the gauge transformations of the fields that we have introduced have an entirely different content from the coordinate transformations, and therefore fixing of the gauge cannot bear any relation to fixing of the choice of the coordinate system.[12)]

The relativistic theory of gravitation can be constructed uniquely on the basis of propositions I–IV.

The most direct way of constructing a scalar density $\mathscr{L}_g$ for the free gravitational field in Minkowski space satisfying proposition IV (with allowance for propositions I–III) would be to represent it as a general superposition of all possible "contractions of forms quadratic in the derivatives of first order $D_{\lambda}g_{\mu\nu}$ with the tensors $\tilde{g}_{\alpha\beta}$ and $\tilde{\gamma}_{\alpha\beta}$. Adopting the gauge principle, one can, after a lengthy investigation, establish, as is shown, for example, in Ref. 28, the uniquely determined form of $\mathscr{L}_g$. Here, we shall give a somewhat different, simplified way to construct $\mathscr{L}_g$.

It is easy to show that under the transformations (2.4) the quantities $(-g)^{1/2}$ and $\tilde{R}\equiv(-g)^{1/2}R$, where R is the scalar curvature of the effective Riemannian space, change in accordance with the law

$$\begin{aligned}&(-g)^{1/2}\rightarrow(-g)^{1/2}-D_{\nu}[\varepsilon^{\nu}(-g)^{1/2}],\\&\tilde{R}\rightarrow\tilde{R}-D_{\nu}(\varepsilon^{\nu}\tilde{R})\end{aligned} \tag{2.11}$$

and, therefore, satisfy the gauge principle. Representing $\tilde{R}$ in the form

$$\tilde{R}=-\tilde{g}^{\mu\nu}(G^{\lambda}_{\mu\nu}G^{\sigma}_{\lambda\sigma}-G^{\lambda}_{\mu\sigma}G^{\sigma}_{\nu\lambda})-D_{\nu}(\tilde{g}^{\mu\nu}G^{\sigma}_{\mu\sigma}-\tilde{g}^{\mu\sigma}G^{\nu}_{\mu\sigma}) \tag{2.12}$$

or

$$\tilde{R}=-\tilde{g}^{\mu\nu}(\Gamma^{\lambda}_{\mu\nu}\Gamma^{\sigma}_{\lambda\sigma}-\Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\lambda})-\partial_{\nu}(\tilde{g}^{\mu\nu}\Gamma^{\sigma}_{\mu\sigma}-\tilde{g}^{\mu\sigma}\Gamma^{\nu}_{\mu\sigma}), \tag{2.12$'$}$$

where we have the third-rank tensor

$$G^{\lambda}_{\mu\nu}=\frac{1}{2}g^{\lambda\sigma}(D_{\mu}g_{\sigma\nu}+D_{\nu}g_{\sigma\mu}-D_{\sigma}g_{\mu\nu}), \tag{2.13}$$

and the Christoffel symbols

$$\Gamma^{\lambda}_{\mu\nu}=\frac{1}{2}g^{\lambda\sigma}(\partial_{\mu}g_{\sigma\nu}+\partial_{\nu}g_{\sigma\mu}-\partial_{\sigma}g_{\mu\nu}), \tag{2.13$'$}$$

we note that in (2.12) each group of terms behaves separately under general coordinate transformations as a scalar density. At the same time it should be noted that whereas in the complete expression $\tilde{R}$ the dependence on the metric $\gamma^{\mu\nu}$ of the Minkowski space is identically eliminated, in the individually considered first and second group of terms in (2.12) it cannot be eliminated. Noting further that by virtue of (2.1) the gauge principle is also satisfied by a scalar density $\sim\gamma_{\mu\nu}\tilde{g}^{\mu\nu}$, we represent the Lagrangian density of the free gravitational field in the form

$$\begin{aligned}\mathscr{L}_g=\lambda_1(\tilde{R}+D_{\nu}Q^{\nu}(x))+\lambda_2(-g)^{1/2}+\lambda_3\gamma_{\mu\nu}\tilde{g}^{\mu\nu}\\+\lambda_4(-\gamma)^{1/2}.\end{aligned} \tag{2.14}$$

Here, the divergence term with vector density $Q^{\nu}(x)$ constructed from $\tilde{g}^{\mu\nu}$ and $D_{\lambda}\tilde{g}_{\mu\nu}$ has been added in order to eliminate (in accordance with proposition IV) from $\mathscr{L}_g$ the terms with derivatives higher than the first order. This is done by the choice

$$Q^{\nu}(x)=\tilde{g}^{\mu\nu}G^{\sigma}_{\mu\sigma}-\tilde{g}^{\mu\sigma}G^{\nu}_{\mu\sigma}.$$

As a result, we obtain a density that is a scalar with respect to all coordinate transformations:

$$\begin{aligned}\mathscr{L}_g=-\lambda_1\tilde{g}^{\mu\nu}(G^{\alpha}_{\mu\nu}G^{\beta}_{\alpha\beta}-G^{\alpha}_{\mu\beta}G^{\beta}_{\nu\alpha})+\lambda_2(-g)^{1/2}\\+\lambda_3\gamma_{\mu\nu}\tilde{g}^{\mu\nu}+\lambda_4(-\gamma)^{1/2}.\end{aligned} \tag{2.15}$$

The values of the factors λ will be established below.

In accordance with the principle of least action, we then obtain the equation[13)]

$$\frac{\delta\mathscr{L}_g}{\delta\tilde{g}^{\mu\nu}}=\lambda_1R_{\mu\nu}+\frac{1}{2}\lambda_2g_{\mu\nu}+\lambda_3\gamma_{\mu\nu}=0, \tag{2.16}$$

with Ricci tensor

$$R_{\mu\nu}=D_{\lambda}G^{\lambda}_{\mu\nu}-D_{\mu}G^{\lambda}_{\nu\lambda}+G_{\mu\nu}G^{\lambda}_{\sigma\lambda}-G^{\sigma}_{\mu\lambda}G^{\lambda}_{\nu\sigma}. \tag{2.17}$$

Determining now, using (2.15), the energy-momentum tensor of the gravitational field in the Minkowski space:

$$\begin{aligned}t_g^{\mu\nu}&=-2\frac{\delta\mathscr{L}_g}{\delta\gamma_{\mu\nu}}\\&=2(-\gamma)^{1/2}\left(\gamma^{\mu\alpha}\gamma^{\nu\beta}-\frac{1}{2}\gamma^{\mu\nu}\gamma^{\alpha\beta}\right)\frac{\delta\mathscr{L}_g}{\delta\tilde{g}^{\alpha\beta}}\\&\quad+\lambda_1J^{\mu\nu}-2\lambda_3g^{\mu\nu}-\lambda_4\tilde{\gamma}^{\mu\nu},\end{aligned} \tag{2.18}$$

where

$$J^{\mu\nu}=D_{\alpha}D_{\beta}(\gamma^{\alpha\mu}\tilde{g}^{\beta\nu}+\gamma^{\alpha\nu}\tilde{g}^{\beta\mu}-\gamma^{\alpha\beta}\tilde{g}^{\mu\nu}-\gamma^{\mu\nu}\tilde{g}^{\alpha\beta}), \tag{2.19}$$

we arrive, taking into account (2.16), at a different form of the dynamical equations of the free gravitational field that is equivalent to (2.16):

$$\lambda_1J^{\mu\nu}-2\lambda_3\tilde{g}^{\mu\nu}-\lambda_4\tilde{\gamma}^{\mu\nu}=t_g^{\mu\nu}. \tag{2.20}$$

To ensure that Eqs. (2.16) and (2.20) are satisfied identically in the absence of a gravitational field, we must set

$$\lambda_2=-2\lambda_3,\quad \lambda_4=-2\lambda_3.$$

The values of λ_1 and λ_3 can be readily identified by transforming (2.20) by means of (2.1) and (2.2) to the form

$$\gamma^{\alpha\beta}D_\alpha D_\beta\tilde{\Phi}^{\mu\nu}+2\,\frac{\lambda_3}{\lambda_1}\,\tilde{\Phi}^{\mu\nu}=-\frac{1}{\lambda_1}\,t_g^{\mu\nu}, \tag{2.21}$$

and, even more transparently, in Galilean coordinates:

$$\square\Phi^{\mu\nu}+2\,\frac{\lambda_3}{\lambda_1}\,\Phi^{\mu\nu}=-\frac{1}{\lambda_1}\,t_g^{\mu\nu}. \tag{2.21$'$}$$

It is clear that the factor $2\lambda_3/\lambda_1\equiv m^2$ is naturally associated with the square of the graviton rest mass, while $-1/\lambda_1$ must in accordance with the correspondence principle be taken [14] equal to 16π, i.e.,

$$\lambda_1=-\frac{1}{16\pi}, \quad \lambda_2=\lambda_4=-2\lambda_3=\frac{m^2}{16\pi}.$$

Thus, the Lagrangian of the free gravitational field in the Minkowski space constructed on the basis of the gauge principle has in the general case the form

$$\mathcal{L}_g=\frac{1}{16\pi}\,\tilde{g}^{\mu\nu}\,(G^\lambda_{\mu\nu}G^\sigma_{\lambda\sigma}-G^\lambda_{\mu\sigma}G^\sigma_{\nu\lambda})\\-\frac{m^2}{16\pi}\left[\frac{1}{2}\,\gamma_{\mu\nu}\tilde{g}^{\mu\nu}-(-g)^{1/2}-(-\gamma)^{1/2}\right]. \tag{2.22}$$

The dynamical equations of the gravitational field corresponding to it, which augment Eqs. (2.1) to the complete system of the RTG, can be represented in the two equivalent forms

$$R^{\mu\nu}-\frac{m^2}{2}\,(g^{\mu\nu}-g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta})=0 \tag{2.23}$$

or

$$D_\alpha D_\beta\,(\gamma^{\alpha\beta}\tilde{g}^{\mu\nu}+\gamma^{\mu\nu}\tilde{g}^{\alpha\beta}-\gamma^{\alpha\mu}\tilde{g}^{\beta\nu}-\gamma^{\alpha\nu}\tilde{g}^{\beta\mu})\\+m^2\,(\tilde{g}^{\mu\nu}-\tilde{\gamma}^{\mu\nu})=16\pi t_g^{\mu\nu}. \tag{2.24}$$

It follows from (2.24) that a necessary (and sufficient) condition for the existence of energy-momentum conservation laws,

$$D_\mu t_g^{\mu\nu}=0\,, \tag{2.25}$$

for a gravitational field with nonzero rest mass is fulfillment of the field equations (2.1). We particularly emphasize that Eqs. (2.23) and (2.24) are not invariant with respect to the gauge transformations (2.4). This means that the introduction into the Lagrangian of the mass term takes away from the geometry of the effective Riemannian space-time, and also the energy-momentum tensor of the gravitational field, the gauge arbitrariness,[15] making them uniquely determined in the indicated sense. Because the mass term eliminates the degeneracy, its introduction can also be regarded as a technical device used in calculations, with the rest mass subsequently set equal to zero.

In the presence of other forms of matter, the total Lagrangian density can be represented by virtue of Propositions III and IV in the form

$$\mathcal{L}=\mathcal{L}_M\,(\tilde{g}^{\mu\nu},\ \Phi_a)+\mathcal{L}_g\,(\tilde{\gamma}^{\mu\nu},\ \tilde{g}^{\mu\nu},\ D_\lambda g_{\mu\nu}), \tag{2.26}$$

where Φ_a are the matter fields (excluding the gravitational field), and $\mathcal{L}_g$ is given by (2.22). This leads to the equations

$$R^{\mu\nu}-\frac{m^2}{2}\,(g^{\mu\nu}-g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta})=\frac{8\pi}{(-g)^{1/2}}\left(T^{\mu\nu}-\frac{1}{2}\,g^{\mu\nu}T\right), \tag{2.27}$$

where $T^{\mu\nu}=-2(\delta\mathcal{L}_M/\delta g_{\mu\nu})$ is the energy-momentum tensor density of the nongravitational forms of matter in the effective Riemannian space, and $T=T^{\mu\nu}g_{\mu\nu}$. But if, using (2.26), we calculate the energy-momentum tensor density of the matter and the gravitational field in the Minkowski space,

$$t^{\mu\nu}=t_g^{\mu\nu}+t_M^{\mu\nu},\\t_g^{\mu\nu}=-2\,\frac{\delta\mathcal{L}_g}{\delta\gamma_{\mu\nu}}, \quad t_M^{\mu\nu}=-2\,\frac{\delta\mathcal{L}_M}{\delta\gamma_{\mu\nu}}, \tag{2.28}$$

then instead of (2.27) we can obtain a different form of the dynamical equations:

$$D_\alpha D_\beta\,(\gamma^{\alpha\beta}\tilde{g}^{\mu\nu}+\gamma^{\mu\nu}\tilde{g}^{\alpha\beta}-\gamma^{\alpha\mu}\tilde{g}^{\beta\nu}-\gamma^{\alpha\nu}\tilde{g}^{\beta\mu})\\+m^2\,(\tilde{g}^{\mu\nu}-\tilde{\gamma}^{\mu\nu})=16\pi t^{\mu\nu}, \tag{2.29}$$

which in its content is identical[16] to (2.27). Taking into account the field equations (2.1), we finally arrive at the following system of equally valid basic generally covariant dynamical equations of the RTG:

$$\gamma^{\alpha\beta}D_\alpha D_\beta\tilde{\Phi}^{\mu\nu}+m^2\tilde{\Phi}^{\mu\nu}=16\pi t^{\mu\nu}, \tag{2.30}$$

$$D_\mu\tilde{\Phi}^{\mu\nu}=0, \tag{2.31}$$

or, in equivalent form,

$$R^{\mu\nu}-\frac{m^2}{2}\,(g^{\mu\nu}-g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta})=\frac{8\pi}{(-g)^{1/2}}\left(T^{\mu\nu}-\frac{1}{2}\,g^{\mu\nu}T\right), \tag{2.32}$$

$$D_\mu\tilde{g}^{\mu\nu}=0. \tag{2.33}$$

We especially emphasize that the physical fields which occur in Eqs. (2.30), (2.31) or (2.32), (2.33) depend on the coordinates of the Minkowski space, and that the metric tensor of this space is contained in the equations organically, reflecting the fact that the physical phenomena take place in the Minkowski space.

Here, as before, a necessary and sufficient condition for fulfillment of the conservation laws

$$D_\mu t^{\mu\nu}=0 \quad \text{or} \quad \nabla_\mu T^{\mu\nu}=0 \tag{2.34}$$

is fulfillment of the field equations (2.31) or, equivalently, (2.33). The validity of the first of the equations in (2.34) is readily established by means of (2.30) and (2.31). To establish the validity of the second, it is sufficient to take into account in addition to (2.33) the identities

$$\nabla_\lambda\gamma_{\mu\nu}\equiv-G^\sigma_{\lambda\mu}\gamma_{\sigma\nu}-G^\sigma_{\lambda\nu}\gamma_{\mu\sigma},\\(-g)^{1/2}\,(D_\mu g^{\mu\nu}+G^\lambda_{\mu\lambda}g^{\mu\nu})\equiv D_\mu\tilde{g}^{\mu\nu}.$$

In the case of a massless gravitational field ($m=0$), its dynamical equations in the absence of matter will have the form

$$D_\alpha D_\beta\,(\gamma^{\alpha\beta}\tilde{g}^{\mu\nu}+\gamma^{\mu\nu}\tilde{g}^{\alpha\beta}-\gamma^{\alpha\mu}\tilde{g}^{\beta\nu}-\gamma^{\alpha\nu}\tilde{g}^{\beta\mu})=16\pi t_g^{\mu\nu}. \tag{2.35}$$

Here, the tensor $\gamma^{\mu\nu}$ of the Minkowski space is identically eliminated, i.e., these equations do not contain[17] the metric $\gamma^{\mu\nu}$; however, if in them we take into account Eqs. (2.31), then the tensor $\gamma^{\mu\nu}$ will occur in the system of equations without the capability of being eliminated.

Equations (2.35) are invariant with respect to the allowed gauge transformations, although the tensor $t_g^{\mu\nu}$ of the gravitational field is gauge noninvariant, like the gauge noninvariant interval of the effective Riemannian space and its curvature tensor:

$$\delta_\varepsilon\, ds^2 = (\delta_\varepsilon g_{\mu\nu})\, dx^\mu dx^\nu,$$
$$\delta_\varepsilon R_{\mu\nu\alpha\beta} = -R_{\sigma\nu\alpha\beta} D_\mu \varepsilon^\sigma - R_{\mu\sigma\alpha\beta} D_\nu \varepsilon^\sigma - R_{\mu\nu\sigma\beta} D_\alpha \varepsilon^\sigma - R_{\mu\nu\alpha\sigma} D_\beta \varepsilon^\sigma - \varepsilon^\sigma D_\sigma R_{\mu\nu\alpha\beta},$$

this indicating that the effective geometry of space-time and the field tensor $t_g^{\mu\nu}$ are undetermined in the absence of matter.[18] At the same time, because the change $\delta_\varepsilon t_g^{\mu\nu}$ under the gauge transformation (2.4) is, as is readily seen using (2.35), transformed into the divergence of an antisymmetric tensor of third rank:

$$\delta_\varepsilon t_g^{\mu\nu} = -\frac{1}{4\pi} D_\lambda D_\sigma (\delta_\varepsilon \Pi^{[\mu\sigma][\nu\lambda]}),$$
$$\Pi^{[\mu\sigma][\nu\lambda]} = \frac{1}{4}(\gamma^{\lambda\mu}\tilde{g}^{\sigma\nu} + \gamma^{\sigma\nu}\tilde{g}^{\lambda\mu} - \gamma^{\lambda\sigma}\tilde{g}^{\mu\nu} - \gamma^{\mu\nu}\tilde{g}^{\lambda\sigma}),$$

the gauge arbitrariness of $t_g^{\mu\nu}$ will not affect the integral physical characteristics of the gravitational field being defined.

In the presence of other forms of matter, the dynamical equations take the form[19]

$$D_\alpha D_\beta (\gamma^{\alpha\beta}\tilde{g}^{\mu\nu} + \gamma^{\mu\nu}\tilde{g}^{\alpha\beta} - \gamma^{\alpha\mu}\tilde{g}^{\beta\nu} - \gamma^{\alpha\nu}\tilde{g}^{\beta\mu}) = 16\pi t^{\mu\nu}, \quad (2.36)$$

and with allowance for (2.1) the complete generally covariant system of equations of the RTG can be represented either in the form

$$\gamma^{\alpha\beta} D_\alpha D_\beta \tilde{\Phi}^{\mu\nu} = 16\pi t^{\mu\nu}, \quad (2.37)$$
$$D_\mu \tilde{\Phi}^{\mu\nu} = 0, \quad (2.38)$$

or in the equivalent form

$$-g)^{1/2} R^{\mu\nu} = 8\pi \left(T^{\mu\nu} - \frac{1}{2} g^{\mu\nu} T\right), \quad (2.39)$$
$$D_\mu \tilde{g}^{\mu\nu} = 0. \quad (2.40)$$

We note in passing that the form of Eqs. (2.37) and (2.38) strongly resembles the form of Maxwell's equations in electrodynamics in the absence of a gravitational field:

$$\gamma^{\alpha\beta} D_\alpha D_\beta A^\mu = 4\pi j^\mu, \quad (2.41)$$
$$D_\mu A^\mu = 0, \quad (2.42)$$

except that in electrodynamics the electromagnetic field A^μ is a tensor of first rank and its source is the conserved density $j^\mu(x)$ of the electromagnetic current, whereas in the RTG the gravitational field $\Phi^{\mu\nu}$ is a second-rank tensor and its source is the conserved energy-momentum tensor density of all the matter in the Minkowski space. Because of this, i.e., because of the fact that $t^{\mu\nu}$ also contains the energy-momentum tensor density $t_g^{\mu\nu}$ of the gravitational field, the equations of even the free gravitational field will be nonlinear.

The dynamical equations (2.36), (2.37), or (2.39) of the RTG, like (2.29), (2.30), or (2.32), are gauge noninvariant, and this means that in the presence of matter the effective Riemannian geometry of space-time and the energy-momentum tensor of the gravitational field are determined uniquely (have no gauge arbitrariness). *Although the form of Eqs. (2.39) is the same as that of the Hilbert-Einstein equations, they differ fundamentally from these equations, since in all the equations of the RTG, including, of course, Eqs. (2.39) and (2.40), the field variables are functions of Minkowski space, and, moreover, the metric tensor $\gamma^{\mu\nu}$ of the Minkowski space occurs in an inseparable manner in any of the various forms of the complete system of RTG equations given above. It is this fundamental circumstance that permits the RTG to consider all physical fields, including the gravitational field, in a single Minkowski space[20] and specify the coefficients $g^{\mu\nu}$ of the effective Riemannian space in a single chart* (sheet?). In particular, one can choose global Cartesian (Galilean) coordinates. This is all reflected not only in the fundamental conservation laws but also in the uniqueness (in contrast to GR) of the description of all gravitational phenomena. For given boundary and initial conditions, the basic system of the RTG will possess the property of uniqueness, as a result of which the physical quantities and predictions obtained by means of it will also be unambiguous. With allowance for the equation of state of the matter, the system (2.37)–(2.38) or (2.39)–(2.40) becomes a closed system of equations determining the dynamics of both the field and the matter. It follows from what has been said that in the RTG the Minkowski space is not a fictitious space, since it is manifested both in the fundamental conservation laws and in the description of all gravitational phenomena; its characteristics can always be determined by an appropriate analysis of experimental data on the motion of light and test bodies in the effective Riemannian space. Already Fock said (Ref. 35, p. 296 of the Russian original) that "the decisive thing in definitions is not direct observability but the correspondence with nature, even if this correspondence is established by indirect ratiocination." Thus, observability must not be understood in a primitive but in a more general and deeper sense as correspondence with nature.

Because of the importance in the RTG of the field equation (2.1), we shall add to what was said above a few more words. As we have already noted, this equation has no connection with the coordinate conditions of GR, and the freedom in the choice of the coordinates in the RTG is maintained. It not only selects, as is required by proposition II, the physical states of the field corresponding to the representations with spins 2 and 0, but also, by virtue of the way in which the Minkowski metric occurs organically, separates everything due to the manifestation of the gravitational field from everything that has no connection with the manifestation of the field. For example, if in the case of a static centrally symmetric massive source M we establish in advance an arithmetization of space in which the center S of the source is associated with the point $r_S = 0$ (as is natural and generally accepted), the unique simultaneous solution of Eqs. (2.37)–(2.38) or (2.39)–(2.40) will be the solution (1.2) with the function $C(r) = [1 + (GM/r)]^2$, i.e., the metric (1.11). The coefficients $g_{\mu\nu}$. determining the metric (1.10) will not satisfy (2.40) and therefore cannot be regarded as solutions of the RTG. It follows from this that all predictions of the RTG for gravitational effects in the given field will be uniquely determined. In particular, for the time t of propagation of a radio signal from the point e to the point p in the field of M (see Sec. 1) the RTG gives[13] the results (1.13) and (1.13′), and not (1.12) and (1.12′). The introduction into the theory of the field equation (2.1), which makes it impossible to eliminate the metric of the Minkowski space from the theory, finds its reflection in the description of all the physical phenomena and leads to physical consequences qualitatively different from those of GR. We shall demonstrate this for the examples of the development of collapse and the evolution of a Friedmann universe.

It is well known that in accordance with GR a star with total mass $M_0 > 3M_\odot$ that has exhausted its nuclear fuel must "collapse" during a finite proper time into a point object with infinite matter density.[21] Such objects, which have become known as "black holes", are completely cut off from an external observer in the sense that no physical signals from the region bounded by the Schwarzschild sphere (with radius $\tilde{r} = 2GM_0$) can pass beyond its boundary; so-called gravitational "self-closure" of the body occurs.[22] Essentially, this conclusion of GR is equivalent to the recognition of forms of existence of matter in which it is in principle uncognizable (since no physical information about the internal processes in a "black hole" can be obtained in any way).

The RTG radically changes ideas about the evolutionary nature of gravitational collapse,[36] and this occurs because of the field equation (2.1). We shall show when and how this happens. In accordance with the RTG, the required metric coefficients $g_{\mu\nu}$, which are related, on the one hand, to the required gravitational field $\Phi^{\mu\nu}$, and, on the other, determine the interval ds^2 of the effective Riemannian space, must be functions of the coordinates x^μ of the Minkowski world as primary variables. We choose $x^\mu = (t,r,\theta,\varphi)$ in such a way that the center of the collapsing star corresponds to $r = r_S = 0$. Then with allowance for the symmetry of the problem we can represent ds^2 in the form.

$$ds^2 = g_{00}(r, t)\, dt^2 + 2g_{01}(r, t)\, dt\, dr + g_{11}(r, t)\, dr^2 - W^2(r, t)(d\theta^2 + \sin^2\theta\, d\varphi^2). \tag{2.43}$$

Going over then from the primary variables x^μ to the variables $\xi^\nu \equiv (\tau,R,\theta,\varphi)$ of the comoving system, i.e., setting

$$\tau = \tau(r, t), \quad R = R(r, t), \tag{2.44}$$

we transform (2.43) to

$$ds^2 = d\tau^2 - e^{\omega(\tau, R)}\, dR^2 - W^2(d\theta^2 + \sin^2\theta\, d\varphi^2). \tag{2.45}$$

In making this transition, we must also (and we draw particular attention to this!) match the region of allowed values of τ and R (or τ and W) to the region of allowed values of the primary variables t and r. This is achieved by the solution of the field equation (2.40), which in the comoving coordinates ξ^ν takes the form

$$\frac{1}{(-g(\xi))^{1/2}} \frac{\partial}{\partial \xi^\mu}\left[(-g(\xi))^{1/2} g^{\mu\nu}(\xi) \frac{\partial x^\lambda}{\partial \xi^\nu}\right] + \gamma^\lambda_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} g^{\alpha\beta}(\xi) = 0, \tag{2.46}$$

with Christoffel symbols in the Minkowski space

$$\gamma^\lambda_{\mu\nu} = \frac{1}{2}\gamma^{\lambda\sigma}(\partial_\mu \gamma_{\sigma\nu} + \partial_\nu \gamma_{\sigma\mu} - \partial_\sigma \gamma_{\mu\nu}).$$

It is well known that the simplest nonstatic solution of Eqs. (2.39), or the Hilbert-Einstein equations (under the assumption of zero pressure and spatial homogeneity of the matter energy density: $\rho = \rho(\tau)$) is the Tolman solution[37]

$$W_{\mathrm{in}} = R\left(1 - \frac{\tau}{\tau_0}\right)^{2/3} \tag{2.47}$$

in the interior region $0 \leqslant R \leqslant R_0$ and

$$W_{\mathrm{ext}} = \left(R^{3/2} - R_0^{3/2}\frac{\tau}{\tau_0}\right)^{2/3} \tag{2.48}$$

in the exterior region $R \geqslant R_0$; at the same time

$$\exp\omega(\tau, R) = \left(\frac{\partial W}{\partial R}\right)^2; \tag{2.49}$$

here $R_0 \equiv [(9/2)GM_0\tau_0^2]^{1/3}$ determines the position of the material surface of the collapsing object at $\tau = 0$.

In GR there are no restrictions on the range of variation of the variable W, which has the meaning of a radial coordinate. It can therefore have all values from zero to infinity. Hence, using the equation

$$W^3(\tau)\,\rho(\tau) = W^3(0)\,\rho(0) \tag{2.50}$$

and (2.47), we arrive in GR at the following law of variation of the energy density with the time τ:

$$\rho(\tau) = \rho(0)\left(1 - \frac{\tau}{\tau_0}\right)^{-2}, \tag{2.51}$$

from which it follows that after a finite proper time $\tau = \tau_0$ all matter has collapsed into the point $W = 0$ and its energy density $\rho(\tau_0)$ has become infinite.

In the RTG the meaning of W is different, since W contains not only the metric characteristics of the Minkowski space but also the physical characteristics of the gravitational field $\Phi^{\mu\nu}$, which satisfies the field equations (2.46). In the case of the solutions (2.47)–(2.49), they are transformed to the form

$$\frac{\partial}{\partial \tau}(W^2 W' \dot{t}) = \frac{\partial}{\partial R}\left(\frac{W^2}{W'} t'\right),$$
$$\frac{\partial}{\partial \tau}(W^2 W' \dot{r}) = \frac{\partial}{\partial R}\left(\frac{W^2}{W'} r'\right) - 2rW', \tag{2.52}$$

where $f' \equiv \partial f/\partial R$, $\dot{f} \equiv \partial f/\partial \tau$.

We follow the dynamics of the material surface of the collapsing object setting in (2.48) $R = R_0$, i.e., $W_{\mathrm{ext}} \equiv W_f$. Then Eqs. (2.52) are solved by

$$r_f = W_f - GM_0, \tag{2.53}$$

$$t - t_0 = \tau - 2(2GM_0 W_f)^{1/2} + 2GM_0 \ln\left|\frac{W_f^{1/2} + (2GM_0)^{1/2}}{W_f^{1/2} - (2GM_0)^{1/2}}\right|, \tag{2.54}$$

where t_0 is introduced to synchronize the measurement of the times τ and t from zero. It can be seen from this that by virtue of the field equations of the RTG the region of allowed values of W_f is bounded below[23]:

$$W_f > 2GM_0, \text{ i.e., } r_f > GM_0 \equiv r_g. \tag{2.55}$$

With allowance for (2.48), it follows from this for $R = R_0$ that in accordance with the RTG any position of the surface of the collapsing object ($r_f > r_g$) will always correspond to a reading τ of the proper-time clocks strictly less than τ_0[24]:

$$\tau < \tau_c = \tau_0 - \frac{4}{3}GM_0. \tag{2.56}$$

Using the conservation law for the total energy in the RTG

$$r_f^3(\tau)\,\varepsilon(\tau) = r_f^3(0)\,\varepsilon(0) \tag{2.57}$$

and noting that by virtue of (2.48) for $R = R_0$

$$r_f(\tau) = R_0\left(1 - \frac{\tau}{\tau_0}\right)^{2/3} - r_g, \tag{2.58}$$

we obtain for the energy density $\varepsilon(\tau)$ averaged over the volume the expression

$$\varepsilon(\tau)=\frac{3M_0}{4\pi}\left[R_0\left(1-\frac{\tau}{\tau_0}\right)^{2/3}-GM_0\right]^{-3}. \qquad (2.59)$$

Therefore, the maximal mean energy density of the collapsing body allowed in the RTG is finite and equal to

$$\varepsilon_{\max}\equiv\varepsilon(\tau_c)=\frac{3M_0}{4\pi r_g^3}. \qquad (2.60)$$

The obtained results can be explained by following the dynamics of the surface layer of the collapsing body. For this it is sufficient to find its velocity dr_f/dt and acceleration d^2r/dt^2. We can do this readily by noting that for $R=R_0$ we have in accordance with (2.48) and (2.53)

$$\frac{dW_f}{d\tau}=\frac{dr_f}{d\tau}=-\left(\frac{2GM_0}{W_f}\right)^{1/2}, \qquad (2.61)$$

and by virtue of (2.54) and (2.61)

$$d\tau=\frac{r_f-GM_0}{r_f+GM_0}\,dt. \qquad (2.62)$$

Therefore,

$$\begin{aligned}\frac{dr_f}{dt}&=-\frac{r_f-GM_0}{r_f+GM_0}\cdot\left(\frac{2GM_0}{r_f+GM_0}\right)^{1/2},\\ \frac{d^2r_f}{dt^2}&=-\frac{GM_0(r_f-GM_0)(r_f-5GM_0)}{(r_f+GM_0)^4}.\end{aligned} \qquad (2.63)$$

It can be seen from this that as the body collapses the negative acceleration of its surface layer is replaced at $r_f=5r_g$ by a positive acceleration, and that as $\tau\to\tau_c$, i.e., $r_f\to r_g$, its velocity and acceleration tend to zero (see also Ref. 38).

Thus, according to the ideas of the RTG a collapsing body can contract only to a definite finite size, tending asymptotically to a state with finite radius (always larger than r_g) and finite density, i.e., no gravitational self-closure of the body occurs and the interior region of the body has a definite structure and remains in principle accessible to study. *Summarizing, we can say that the RTG denies the existence of both static and nonstatic spherically symmetric bodies with radii less than or equal to r_g.*

The picture of collapse that we have considered is a classical idealized picture. A more systematic treatment of it must take into account quantum processes, which begin to play an important part when $r_f\to r_g$ (see, for example, Ref. 39), and, of course, the real equations of state of the matter (nonzero internal pressure, etc.). All these factors will hinder the process of contraction and, therefore, the sizes of real collapsed bodies will exceed the ideal size. Therefore, according to the RTG the time evolution of a collapsing object is not terminated by the ending of its contraction (during a finite proper time and a finite time of the external observer) but goes over to a new stage with normal subsequent flow of both the proper time and the time of the external observer.

It is helpful to note that in the general case too, the physical space-time region Ω of variation of the variables ξ^ν, in which the solution of the Hilbert-Einstein equations is obtained, must be established by the connection between ξ^ν and the coordinates x^μ of the Minkowski space found by solving the field equations (2.40). Here it is necessary to seek only those solutions that establish a one to-one correspondence of ξ^ν and x^μ (with Jacobian of the transformation everywhere nonzero), since only such solutions enable one to regard the variables ξ^ν as one of the possible coordinate systems of the Minkowski space. The region Ω^* of the variation of ξ^ν determined solely on the basis of the solutions of the Hilbert-Einstein equations is not in the general case identical to the region Ω. In other words, whereas in the variables x^μ with metric tensor $\gamma_{\mu\nu}(x)$ Eq. (2.39) will have a solution $g_{\mu\nu}(x)$, the solution $g'_{\mu\nu}(\xi)$ in the variables ξ^ν with metric tensor

$$\gamma_{\mu\nu}(\xi)=\frac{\partial x^\alpha}{\partial\xi^\mu}\frac{\partial x^\beta}{\partial\xi^\nu}\gamma_{\alpha\beta}(x)$$

will hold only in the region $\Omega\subset\Omega^*$.

In the special case of the considered problem of gravitational collapse

$$\begin{aligned}\gamma_{00}(\xi)&=\left(1-\frac{2GM_0}{W}\right)^{-2}-\dot W^2,\\ \gamma_{01}(\xi)&=W'\dot W\left[\left(1-\frac{2GM_0}{W}\right)^{-2}-1\right],\\ \gamma_{11}(\xi)&=W'^2\left[\dot W^2\left(1-\frac{2GM_0}{W}\right)^{-2}-1\right],\\ \gamma_{33}(\xi)&=\gamma_{22}\sin^2\theta=-(W-GM_0)^2\sin^2\theta,\end{aligned}$$

and the elements of the tensor $\varkappa_{kn}$ that determined the interval dl^2 are equal to $\varkappa_{kn}=-\gamma_{kn}+(\gamma_{0k}\gamma_{0n}/\gamma_{00})$, for example,

$$\varkappa_{11}=W'^2(1-\dot W^2)\left[1-\dot W^2\left(1-\frac{2GM_0}{W}\right)^2\right]^{-1}.$$

It can be seen from this that for $W=2GM_0$ the quantities $\gamma_{\mu\nu}$ with $\mu,\nu=0,1$ become singular, and with allowance for (2.61) $\varkappa_{11}=0$. However, neither the one possibility nor the other is allowed either physically or mathematically. Thus, the variables ξ^ν are variables of the Minkowski space only if $W>2GM_0$.

We now consider the evolution of a Friedmann universe that follows from the RTG, retaining a nonvanishing graviton mass.

In the general case the space-time interval ds^2 must with allowance for the assumed symmetries be taken in the form

$$ds^2=B(t)\,dt^2-A(t,r)(dx^2+dy^2+dz^2), \qquad (2.64)$$

where t,x,y,z are the coordinates of the pseudo-Euclidean space, i.e., $r^2=x^2+y^2+z^2$. Using the field equation (2.33), we can readily show that the function A will not depend on r, and

$$B(t)=A^3(t). \qquad (2.65)$$

By virtue of the latter $\tilde g^{00}=1$; and since also $\tilde\gamma^{00}=1$, we have by virtue of (2.2) $\tilde\Phi^{00}=0$. Substituting this in (2.30), we conclude that in accordance with the RTG *the total energy density of the matter and the gravitational field in a Friedmann universe must always be equal to zero, and that the universe itself is infinite and "flat."* This conclusion also remains valid for zero rest mass of the graviton. In contrast to the RTG, GR admits three models of a Friedmann universe: the model of a closed universe with finite volume and two models of an infinite universe. The choice of the particular model depends essentially on the mean matter density in the universe, which in GR cannot be predicted.

We consider the evolution of the universe under the assumption that $T^{\mu\nu}$ in (2.32) can be approximated by the energy-momentum tensor density of an ideal fluid,

$$T^{\mu\nu}=(-g)^{1/2}[(\rho+p)u^\mu u^\nu-g^{\mu\nu}p], \qquad (2.66)$$

where $\rho(t)$ and $p(t)$ are its density and isotropic pressure. Since $u^{\mu}u^{\nu}g_{\mu\nu} = 1$ and $u^k = 0$, it follows that $u^0 = A^{-3/2}(t)$. Going over in (2.32) to the proper time $d\tau \equiv A^{3/2}(t)\,dt$ and introducing for convenience the dimensionless scale factor $R(\tau) \equiv (A(\tau))^{1/2}$, we obtain the equations[40]

$$\left(\frac{1}{R}\frac{dR}{d\tau}\right)^2 = \frac{8\pi G}{3}\rho(\tau) - \frac{m^2}{6}\left(1-\frac{1}{R^2}\right)^2\left(1+\frac{1}{2R^2}\right), \tag{2.67}$$

$$\frac{1}{R}\frac{d^2R}{d\tau^2} = -\frac{4\pi G}{3}\rho(\tau) - 4\pi G p(\tau) - \frac{m^2}{6}\left(1-\frac{1}{R^6}\right), \tag{2.68}$$

which can also be reduced to the form[25]

$$\frac{1}{R}\frac{dR}{d\tau} = -\frac{1}{3(\rho+p)}\frac{d\rho}{d\tau}. \tag{2.69}$$

To close the system, we must also specify the equation of state of the matter. In the simplest case, we can take[42]

$$p(\tau) = \nu\rho(\tau), \tag{2.70}$$

regarding $\nu < 1$ in different time stages of the evolution of the universe as a constant (different for each stage) quantity. Under this assumption it follows from (2.69) that

$$\rho(\tau) = \frac{a(\nu)}{R^{3(1+\nu)}(\tau)}, \tag{2.71}$$

where $a(\nu)$ is a constant of integration corresponding to the particular stage.

Analysis of Eqs. (2.67) and (2.68) shows that for $m \neq 0$ the factor R varies with τ cyclically, increasing to $R_{\max} < \infty$ and then decreasing to $R_{\min} > 0$; the lower bound is due to the inequality $\nu < 1$, while the upper bound arises because of the monotonic decrease of the density $\rho(\tau)$ with increasing $R(\tau)$. We agree to measure the time τ from some state with $R = R_{\min}$ and divide the half-period of evolution of the universe from this state to the state with $R = R_{\max}$ into three stages: Stage I corresponds to values $1/3 < \nu < 1$ (if, of course, it is actually realized), stage II corresponds to $\nu = 1/3$, i.e., to the radiation-dominated stage in the evolution of the universe, and stage III corresponds to $\nu \approx 0$, i.e., the nonrelativistic stage. It is obvious that if stage I is realized, then only at a very early stage of the evolution and evidently (in accordance with the hypothesis of Markov[43]) at a limiting (Planck) value of the matter density ($\rho_{\rm Pl} = G^{-2} \approx 5\cdot 10^{93}$ g/cm^3).

Approximate integration of Eq. (2.67) with allowance for (2.71) in the stage I gives

$$R(\tau) \approx Z^{-1/3(1-\nu)} + \tau^2\frac{m^2}{16}(1-\nu)Z^{5/3(1-\nu)} \tag{2.72}$$

for $(R - R_{\min})/R_{\min} \ll 1$ and

$$R(\tau) \approx \tau^{2/3(1+\nu)}\left[\frac{3m^2Z}{16(1+\nu)^2}\right]^{1/3(1+\nu)} \tag{2.73}$$

for $R_{\min} \ll R \ll Z^{1/3(1+\nu)}$ and neglecting R and τ of the previous stage where

$$Z \equiv \frac{32\pi G a(\nu)}{m^2}. \tag{2.74}$$

In stage II for $R \gg Z^{-1/2}$, i.e., neglecting R and τ of the first stage,

$$R(\tau) \approx \tau^{1/2}\left(\frac{32\pi}{3}Ga\left(\frac{1}{3}\right)\right)^{1/4}, \tag{2.75}$$

while in stage III for $\tau \gg \tau_{\rm II}$, where $\tau_{\rm II}$ is the time of evolution of the universe from the beginning of the "expansion" to the onset of the nonrelativistic stage,

$$R(\tau) \approx \left(\frac{Z}{3}\right)^{1/3}\sin^{2/3}\frac{m\tau\sqrt{3}}{2\sqrt{2}}. \tag{2.76}$$

The last expression will be valid, as additional estimates show, for $\tau < \tau_0$, where

$$\tau_0 \equiv \sqrt{\frac{2}{3}}\frac{\pi}{m} \tag{2.77}$$

can be regarded as the half-period of evolution of the universe during which it goes over from the state with maximal density to the state with mimimal density, after which the opposite process of "contraction" commences; such cycles will be repeated continually.

It should be noted that the graviton mass m plays a role in the evolution of the universe only in the very beginning (see (2.72)) and the very end (see (2.76)) stages of its expansion (and contraction); in the remaining stages it has no actual effect. However, it is only through the nonvanishing mass that cyclical evolution is realized—for $m = 0$ the universe will evolve monotonically to a state with zero matter density (and at each time its density will be equal to the critical density).

From the condition that the age of the contemporary universe, $\tau_c = 2/3H_0$, where H_0 is the Hubble constant, cannot exceed the half-period τ_0, we obtain an upper bound for the graviton mass:

$$m < \sqrt{\frac{3}{2}}\pi H_0 \sim 10^{-65}\text{ g}. \tag{2.78}$$

And since in accordance with (2.67) the contemporary matter density $\rho(\tau) = \rho_c(\tau) + \rho_m$, where $\rho_c(\tau) \equiv (3/8\pi G)(\dot R/R)^2 \equiv (3/8\pi G)H^2(\tau) \approx 2\cdot 10^{-29}$ g/cm^3 and $\rho_m \approx m^2/16\pi G \approx 2.44\cdot 10^{-29}$ g/cm^3, this leads to a prediction for the maximal possible amount of hidden mass of matter in the universe, which exceeds by about 90 times the observed mass.

For the experimentally verifiable parameter $a \equiv -R\ddot R/\dot R^2$ of the deceleration of the expansion of the universe the RTG gives the value

$$a = \frac{1}{2}\left(1 + 3\frac{\rho_m}{\rho_c}\right) > \frac{1}{2}, \tag{2.79}$$

which also depends on the graviton mass.

An increase in the accuracy of the measurements of the Hubble constant and the deceleration parameter would enable us to draw more definite conclusions about the mass of the graviton, which plays, as can be seen from the above, an important part in the evolution of the universe and in other gravitational processes.

We also consider briefly the emission of gravitational waves. Investigating this question, Einstein wrote: "One could suppose that through an appropriate choice of the coordinate system one can always achieve the vanishing of all components of the gravitational field energy, and this would be extremely interesting. However, it is easy to show that in general this is not so" (Ref. 1, p. 631). It can be seen that Einstein expected the possibility of the vanishing of the energy of gravitational radiation on the basis of the equivalence principle that he advanced; he therefore regarded the expected result as "extremely" interesting. But he did not succeed in establishing it. I was nevertheless shown in Ref. 44 that gravitational radiation, as defined in GR by Einstein, can indeed be annihilated by an appropriate choice of an allowed coordinate system. However, if this is so, then, recognizing the validity of Einstein's formula for quadrupole radiation, one cannot regard it as a consequence of GR. It was Einstein's deep physical intuition that led him to the construction of this formula rather than the logic of his theory. General relativity cannot lead to the conclusion of the existence of gravitational waves, as is confirmed in Ref. 44.

Einstein's formula for quadrupole radiation follows naturally from the RTG. This is explained by the fact that in the RTG the gravitational field is a real physical field that, even locally, cannot be annihilated by a choice of the coordinate system. According to the RTG there can exist in nature gravitational waves that transport energy and momentum, and they can be detected experimentally.

At first glance it might seem that the introduction of Minkowski space does not agree with the effect of the "change" of the frequency of photons in a gravitational field, since in accordance with the conservation laws the total energy $\omega_1 \equiv E_{n'n}$ of a photon emitted by an atom at point 1 of the gravitational field must remain unchanged everywhere. The apparent paradox is very easily explained. Following, for example, Ref. 39 and writing down in Minkowski space the Dirac equation (for simplicity for a hydrogen atom in the approximation of a fixed nucleus) in the gravitational field of a static centrally symmetric source M with $r_S = 0$, we can readily see that it is not the energy E of the electron that plays the part of the quantized quantity but rather $P_0 \equiv E(f_+/f_-)^{1/2}$, where $f_\pm \equiv 1 \pm (GM/r)$, and the coordinate r can be regarded as the coordinate of the nucleus of the atom in the gravitational field of M (for the case of field inhomogeneities that do not have an effect over distances of the order of atomic distances). Therefore, the energy spectrum of the hydrogen atom takes (in the Schrödinger approximation) the form

$$E_n = -\frac{R}{n^2}\left(\frac{f_-(r)}{f_+(r)}\right)^{1/2}. \tag{2.80}$$

This result could have been foreseen by noting that in a gravitational field the role of the electron rest mass, which occurs in the Rydberg constant R, must be played by its rest energy $m(f_-/f_+)^{1/2}$ in the field. Thus, the energy (or frequency) of the photon emitted by the atom (in a definite transition) will be determined at the point r_1 by the expression

$$\omega_1 = \omega_0\left(\frac{f_-(r_1)}{f_+(r_1)}\right)^{1/2}, \tag{2.81}$$

where ω_0 is the corresponding frequency of the radiation "when the field is switched off." At the point r_2 the same electron transition will give a photon of frequency

$$\omega_2 = \omega_0\left(\frac{f_-(r_2)}{f_+(r_2)}\right)^{1/2}. \tag{2.82}$$

Since in an experiment one compares frequencies corresponding to identical transitions, an observer at the point r_2 will find for a photon that reaches him from the point r_1 a frequency ω_1 not equal to ω_2 with

$$\frac{\omega_2-\omega_1}{\omega_1} = \frac{\Delta\omega_{12}}{\omega_1} \approx GM\left(\frac{1}{r_1}-\frac{1}{r_2}\right), \tag{2.83}$$

and this is usually interpreted as a change of the frequency in the gravitational field.

Analyzing the question of the observability of Minkowski space on the basis of fundamental principles,[21] we can say that the unambiguous and deep connection between the experimentally verified conservation laws and the structure of space-time reflects precisely the objectivity and observability of the properties that are inherent in Minkowski space, i.e., the experimentally confirmed fact of the universality of the conservation laws reflects the objectivity of Minkowski space and its observability.

SUPPLEMENT

The authors of the recent preprint Ref. 47 show that in GR transformations of the radial variable do not change the results of the predictions for gravitational effects. By itself this fact is trivial and does not need proving. However, it in no way leads to the conclusion drawn by the authors of uniqueness of the predictions of GR for gravitational effects. In their arguments, the authors omitted the most important thing—they did not note the fundamental difference between coordinate transformations of solutions and the multiplicity of equally valid solutions of the Hilbert-Einstein equations *in given coordinates*, an example of which is provided by the solutions (1.10) and (1.11). Their conclusions are therefore incorrect. In a given arithmetization the times t_a and t_b of propagation of a radio signal from e to p, represented in integral form

$$t_a = \int_{r_0}^{r_e} dr\, f_a(r) + \int_{r_0}^{r_p} dr\, f_a(r), \tag{1.12''}$$

$$t_b = \int_{\tilde r_0}^{r_e} d\tilde r\, f_b(\tilde r) + \int_{\tilde r_0}^{r_p} d\tilde r\, f_b(\tilde r), \tag{1.13''}$$

where

$$f_a = \left(1-\frac{2GM}{r}\right)^{-1}\left[1-\frac{r_0^3\,(r-2GM)}{r^3\,(r_0-2GM)}\right]^{-1/2},$$

$$f_b = \left(\frac{\tilde r+GM}{\tilde r-GM}\right)\left[1-\frac{(\tilde r_0+GM)^3\,(\tilde r-GM)}{(\tilde r+GM)^3\,(\tilde r_0-GM)}\right]^{-1/2},$$

and r_0 and $\tilde r_0$ correspond to the pericenters of the signal trajectory, contain as upper limits *the same* arithmetization numbers r_e and r_p. The arithmetization numbers do not bear any relation to the concepts of length; they simply label the points of the manifold. In Ref. 47 the multiplicity of solutions of the Hilbert-Einstein equations in given coordinates is essentially ignored, and therefore the criticism of the authors in no way refutes our proof of the nonuniqueness of the predictions of GR. We consider this question once more.

Making in (1.13″) the substitution $\tilde r = r - GM$, we obtain

$$t_b = \int_{\tilde r_0+GM}^{r_e+GM} dr\, f_b(r) + \int_{\tilde r_0+GM}^{r_p+GM} dr\, f_b(r),$$

and, subtracting from this (1.12″), we find in the first order in G and for $r_0 \ll r_e, r_p$

$$t_b - t_a \approx \int_{r_e}^{r_e+GM} dr\, f_a(r) + \int_{r_p}^{r_p+GM} dr\, f_a(r) \approx 2GM.$$

This is the essence of our conclusion of nonuniqueness of the predictions of GR. Since in the first order in G the behavior of clocks in the case of the metrics (1.10) and (1.11) is the same, this result must also be interpreted as nonuniqueness of the predictions of GR for the time of propagation of a radio signal from e to p in experimentally observed quantities.

But we shall now give all the arguments on the basis of the principles adopted by those who hold views similar to those of the authors of Ref. 47 and show that our conclusion of nonuniqueness of the predictions of GR does not depend on this. Thus, suppose two investigators (let us call them Chuk and Gek) have obtained instructions from the same experimentalist to calculate the time of propagation of a radio signal from the earth (e) to Mercury (p), expressing it in terms of physical quantities *at the disposal* of the experimentalist, for example, the radial distances $l^{(\exp)}_{e,p,0}$ from the earth, mercury, and pericenter to the surface of the sun and the gravitational frequency shifts $\delta^{(\exp)}_{e,p,0}$. Suppose also that, guided by personal motives, Chuk decided to do the calculations on the basis of the metric (1.10), while Gek took the metric (1.11), replacing *all* r by $\tilde{r}$. Then in accordance with their plans Chuk and Gek would obtain the results

$$t^{(a)}_{ep} = \int_{r_0}^{r_e} dr\, f_a(r) + \int_{r_0}^{r_p} dr\, f_a(r),$$

$$r^{(b)}_{ep} = \int_{\tilde{r}_0}^{\tilde{r}_e} d\tilde{r}\, f_b(\tilde{r}) + \int_{\tilde{r}_0}^{\tilde{r}_p} d\tilde{r}\, f_b(\tilde{r})$$

with the functions $f_a(r)$ and $f_b(\tilde{r})$ determined above. Making calculations of the integrals in the first order in G, Chuk obtains the result (1.12) with r_f in it replaced by r_0, while Gek obtains the result (1.13) with $r_{e,p}$ in it replaced by $\tilde{r}_{e,p}$ and r_f by $\tilde{r}_0$. Expressing then the arithmetization numbers $r_{e,p,0}$ and $\tilde{r}_{e,p,0}$ in terms of the physical quantities $l^{(\exp)}_{e,p,0}$ and $\delta^{(\exp)}_{e,p,0}$ proposed by the experimentalist, and using (each in his own metric and in the first order in G) the connection

$$l^{(\exp)}_k = \rho_k - \rho_f + GM \ln \frac{\rho_k}{\rho_f},$$

$$\delta^{(\exp)}_k = GM\left(\frac{1}{\rho_f} - \frac{1}{\rho_k}\right),$$

where $k = e,p,0$ and $\rho = r$ in Chuk's case and $\rho = \tilde{r}$ in Gek's case, they would obtain accordingly for the arithmetization numbers the same values. Meeting when they come to the experimentalist, Chuk and Gek would suddenly find that the functional connections they obtained between t_{ep} and $l^{(\exp)}$ and $\delta^{(\exp)}$ were different. Thus, the nonuniqueness of the theoretical predictions would be manifest. The data of the measurements t_{ep}, $l^{(\exp)}_{e,p,0}$, and $\delta^{(\exp)}_{e,p,0}$ supplied to the experimentalist make it possible by combined efforts to establish that the experimental data agree with the formula obtained by Gek and not by Chuk (if on their left- and right-hand sides the results of the observations are substituted in place of t_{ep} and $l^{(\exp)}_k$ and $\delta^{(\exp)}_k$). Thus, Chuk's formula would be found to be unacceptable for its adoption as basis. In our opinion, this is due to the fact that the choice of the solution of the Hilbert-Einstein equations is physically not identical to choice of a particular element in the equivalence class of diffeomorphic metrics. It is because of this difference that the ambiguity in the predictions of GR for gravitational effects arises.

We note in conclusion that in their section "Note Added" the authors actually confuse arithmetization numbers with distances. The numbers of the space arithmetization are nothing more than the names of objects; in the case $M = 0$ and $M \neq 0$ they are the same, and therefore $A = A'$ and $B = B'$. What is important is something different—the distance between the points A and B for $M \neq 0$ is not, of course, equal to the distance between these points when $M = 0$ because of the change of the metric coefficients, i.e., the change of the geometry. Thus, the thought experiment that we proposed at the end of Sec. 1 remains valid. As regards formulas (2.8) and (3.7) of Ref. 47, they were given in our paper cited by the authors of Ref. 47 in Ref. 9 of the second paper. (Their formulas follow from Eqs. (31a) and (32a) of our paper for $\lambda = -1$). In this connection we should also note that their criticism of Weinberg's method, which we also follow in the calculation, is without basis.

In 1986, Ya. B. Zel'dovich and L. P. Grishchuk published in Uspekhi a number of critical remarks about the RTG.[5] The editorial board of the Uspekhi did not acquaint the authors of the RTG with this paper before its publication and did not suggest that they should comment on it in the same number. In the middle of 1987 we sent a paper to Uspekhi. Recently, in April 1988, we became acquainted with a new paper of Zel'dovich and Grishchuk, which is an answer to our paper and is published in this present issue. We shall give a detailed critical analysis of their paper specially. Here we merely note that it contains numerous errors, and therefore its conclusions concerning the RTG and GR are completely incorrect.

[1)] In particular, the authors of Refs. 5 and 6 are confused when they assert (Ref. 5, p. 695 of the Russian original and p. 780 of the translated article) that GR has "... all attributes— ... energy-momentum tensor, and conservation laws."

[2)] From the physical point of view this means that by studying the motion of test bodies and light one can experimentally establish the structure of the space-time geometry.

[3)] It is precisely this arbitrariness (for given arithmetization of space) that leads in accordance with the Weyl-Lorentz-Petrov theorem to the ambiguity in the predictions of GR.

[4)] For more details about the indeterminacy of the inertial mass in GR and its dependence on the choice of the three-dimensional coordinates, see Refs. 11 and 12.

[5)] Therefore the assertion of the authors of Ref. 5 (on p. 706 of the Russian original and on p. 786 of the translated article) that in the given effect "... there is no ambiguity in the predictions of GR and no contradictions with it" must be regarded as incorrect.

[6)] Here, L_1 is introduced formally, although if the pericenter of the signal trajectory is sufficiently far from the surface of the body M it can be perfectly well realized by means of an auxiliary test body revolving around M in a circular orbit with $r = r_1$.

[7)] For many of the questions touched upon in this section, we follow mainly Refs. 20 and 21; they can also be found in Refs. 16 and 22–29.

[8)] Some aspects of the theory of gravitation in Minkowski space were considered earlier; see, for example, Refs. 30–34. However, they were all incomplete. Even the authors who originally set out on an original path in the construction of a theory did not carry on to the end and turned to a different path that did not give finished conclusions.

[9)] The physical equations describing the properties of matter always also contain organically the structure of space-time determined by the metric tensor; the meaning of the assertion that some particular phenomenon takes place, for example, in Minkowski space finds its precise reflection in the way in which the Minkowski metric is contained organically in the corresponding equations.

[10)] It also follows directly from this proposition that the special theory of relativity is valid in both inertial and noninertial frames of reference.[20] The widely held opinion, which goes back to Einstein, that special relativity holds only in inertial frames of reference is a confusion.

11) By virtue of what has been said, Eq. (2.1) cannot have any relation to coordinate conditions.

12) The gauge principle does not require invariance of the theory with respect to the gauge transformations (2.4); this is an important difference between gauge transformations in the RTG and gauge transformations in electrodynamics.

13) As is shown, for example, in Ref. 27, allowance for the "constraint" (2.1) does not change the result.

14) We have chosen a system of units with $c = \hbar = G = 1$.

15) Without the mass term, Eq. (2.24) is gauge invariant (see below).

16) One can show that (2.29) can be converted identically into (2.27) and vice versa.

17) This follows from the fact that for $m = 0$ the Lagrangian (2.22) of the free gravitational field is transformed with allowance for (2.12) into a sum of two terms, one of which does not contain the metric coefficients $\gamma^{\mu\nu}$, while the other, which depends on $\gamma^{\mu\nu}$, can be expressed as the divergence of a vector and therefore does not influence the equations.

18) In this the gauge transformations (2.4) differ significantly from the gauge transformations in electrodynamics, which do not affect observable physical quantities.

19) Equations (2.36) can be transformed identically into Eqs. (2.39).

20) Attempts to introduce into GR, without going beyond its ambit, the Minkowski metric by, as is proposed, for example, in Refs. 5 and 6, the simplest separation of the metric tensor $\tilde{g}^{\mu\nu}$ of the Riemannian space into the two parts of (2.2) without the introduction of any new physical hypotheses do not stand up at all to criticism, since the dynamical equations expressed using the separation (2.2)—they take the form of (2.36)—will actually contain the Minkowski space metric $\gamma_{\mu\nu}$ fictitiously, since they can be identically transformed to the Hilbert-Einstein equations (see (2.36) and the footnote relating to it). Therefore, in such an approach it is in principle impossible for there to be any talk of the Poincaré group of motions or conservation laws. All this is rather obvious if one bears in mind the incompatibility of the concepts of a Riemannian space and a global Minkowski space, and also the fact that a theory that does not contain the given metric organically cannot describe phenomena in the space-time determined by this metric. In the RTG the Riemannian space is an effective space that owes its origin to the physical gravitational field, and the Minkowski space metric $\gamma^{\mu\nu}$ occurs organically in the theory by virtue of the field equation (2.1). This is the fundamental difference between the RTG and GR.

21) Wheeler regarded gravitational collapse and the resulting singularity as "one of the greatest crisis of all times of fundamental physics."

22) Black holes do not have a material surface and bodies that fall into them encounter nothing but "empty" space when they cross the Schwarzschild sphere. Not even light can escape from the interior region of a black hole through its Schwarzschild sphere.

23) Each internal layer will tend to its own limiting position.

24) Thus, in accordance with the RTG the solutions (2.47)–(2.49) are physically invalid when $\tau \geqslant \tau_c$. All this shows that the assertions of the authors of Ref. 5 (on p. 704 of the Russian original and on p. 785 of the translated article) to the effect that every solution of the Hilbert-Einstein equations satisfies the field equations (2.40) are groundless.

25) Similar (but for $p = 0$) equations were obtained in Ref. 41. However, because the authors of Ref. 41 did not give the gravitational field the meaning of a real physical field in the fundamental Minkowski space, their theory of gravitation with nonzero graviton rest mass was, as the authors themselves concluded, not generally covariant (in contrast to our theory, which is always generally covariant). In addition, their approach to the construction of the theory is essentially heuristic and not based on rigorous physical and mathematical principles that lead unambiguously to a definite structure of the Lagrangian and the equations.

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Translated by J. B. Barbour