Numerical solution of the Fokker-Planck equation

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J. Killeen, G. D. Kerbel, M. G. McCoy, and A. A. Mirin. *Computational Methods for Kinetic Models of Magnetically Confined Plasma.* Springer-Verlag, New York; Berlin; Heidelberg; Tokyo (1986) pp. 196, (Springer Series in Computational Physics).

Kinetic effects play a determining role in the physics of straight mirror traps. In the case of toroidal systems for a long time it was thought that effects associated with a deviation from the Maxwellian distribution are important only in low-density regimes, when an appreciable number of runaway electrons appears. However, the development of methods of heating plasma with the aid of high frequency waves and the injection of a beam of hot neutrals led to the necessity of studying the kinetics of plasma also in systems with toroidal geometry. The complexity of the problem required the development both of analytic and of numerical methods. During the last decade an extensive literature on this subject has grown up, and the numerical solution of kinetic equations has developed into an independent scientific field.

The book under review is devoted to a consistent description of the mathematical models of a plasma with a Coulomb interaction between particles and of the methods of numerical solution of the posed problems. The kinetic equation with such an interaction is commonly referred to as the Fokker-Planck (FP) equation. At least one half of the book is devoted to a presentation of the authors' own results. They are leading specialists in the field of numerical solution of the FP equation.

A brief Introduction contains the history of the problem and reviews the existing models and numerical codes.

The second chapter discusses the kinetics of a plasma in a uniform magnetic field. In this case the kinetic equation turns out to be two-dimensional or one-dimensional. The authors provide complete expressions for the collision operator in the spherical system of coordinates. For a compact way of writing this operator one utilizes the "Rosenbluth potentials" which are functionals of the functions being sought. The construction of numerical methods is discussed on the example of the one-dimensional equation. Two-dimensional equations are solved by two methods. The first consists of the expansion of the solution in terms of the eigenfunctions of the angular operator (the Legendre functions) with a subsequent difference solution of a system of one-dimensional equations with respect to the expansion coefficients. The second method consists of the solution of the FP equation on a two-dimensional network. As examples problems are examined concerning losses of particles in a mirror trap, concerning the deviation of the distribution of the ions from a Maxwellian one in the TFTR (tokamak fusion test reactor) and a number of other interesting problems.

The third chapter is the central one. Here are discussed the kinetic models of a plasma in a nonuniform magnetic field of toroidal systems. In such traps the trajectories of particles can deviate far from "their" magnetic surfaces and as a result of the motion in the velocity and configurational space turn out to be bound. This leads to the necessity of a multidimensional description of the process which until now is unacceptable even for the most highly developed computers. Operations of averaging are utilized to simplify the problem. At first averaging over the rapid Larmor rotation of the particles is carried out. As a result one obtains the generalized drift kinetic equation. Then in this equation averaging is carried out over the periodic or quasiperiodic motion in the longitudinal direction. Here the characteristic frequency is the frequency of oscillations of confined particles between the points of reflection (the bounce-frequency). After the second averaging the kinetic equation again, as in the case of a uniform magnetic field, becomes two-dimensional. The same averaging procedures are employed in analyzing the interaction of electromagnetic waves with plasma. The developed method is illustrated on a number of problems. Neoclassical corrections to the conductivity are evaluated, modeling of an experiment on determining the ion temperature on the basis of the spectrum of charge-exchange neutrals is carried out.

In the last fourth chapter a hybrid model is discussed which joins the FP equations to the transport equations and which contains a description of the injection of hot neutrals into the plasma. The distribution function of hot ions in this case depends on four variables: two velocity coordinates, the radial coordinate and time. In order to describe the hot ions the nonlinear FP operator is used. Results of calculations using the hybrid model are compared with experiments in the PLT (Princeton large torus), DITE (direct injection tokamak experiment), and TFTR installations.

Among the deficiencies of the book one should include the unutilized possibility of describing the neoclassical transport of particles in carrying out the operation of the second averaging in the third chapter. There are practically no references to the work of Soviet authors. In particular, the theory of convective transport in the corrugations of the longitudinal magnetic field developed by A. V. Gurevich and Ya. S. Dimant has not been reflected.

One should also remark on priorities. As is well known, the kinetic equation for particles with a Coulomb interaction

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was obtained for the first time by L. D. Landau in 1937 with the aid of an expansion of the Boltzmann equation. Twenty years later it was independently constructed by M. Rosenbluth *et al.* starting with statistical considerations. Unfortunately, in the Introduction this side of the history of the problem has received no mention. The authors cite Ref. 2 as the primary source, although Ref. 1 is known to them.

On the whole the book under review is of great interest for a wide circle of readers who deal with the kinetics of plasma and who use numerical methods. In the Soviet literature this subject so far has not received sufficient attention.

[1]L. D. Landau, Zh. Exp. Teor. Fiz. **7**, 203 (1937).
[2]M. N. Rosenbluth, W. M. Macdonald, and D. L. Judd, Phys. Rev. **107**, 1 (1957).

Excitons

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Excitons: Selected Chapters. Eds. E. I. Rashba and M. D. Sturge. North-Holland, Amsterdam; Oxford; New York; Tokyo (1987) pp. 485, (North-Holland Personal Library).

The book under review is dedicated to the 55th anniversary of exciton spectroscopy. The concept of an exciton was introduced in 1931 by Ya. I. Frenkel' for currentless electronic excitations associated with the correlation of the motion of electrons and holes. Excitons are divided into three fundamental types: Frenkel excitons, Wannier-Mott excitons and excitons associated with charge transport. At the present time excitons have been observed and investigated in all the basic types of nonmetallic crystals.

In the first chapter (the author is M. D. Sturge) basic concepts and definitions of exciton physics are introduced (Frenkel and Wannier-Mott excitons, polaritons, bound excitons, exciton complexes, exciton optical phenomena, exciton-phonon interaction), and also a brief historical review is given.

The following three chapters are devoted to the investigation of exciton polaritons. In the second chapter (J. L. Birman) electrodynamics of polaritons and non-local optical phenomena (response function, structure of a polariton mode, exciton-polariton spectroscopy) associated with them are examined. The third chapter (E. S. Koteles) is devoted to the experimental investigation of the dispersion of exciton-polaritons. The fourth chapter (E. L. Ivchenko) investigates the influence of spatial dispersion on the properties of exciton-polaritons (reflection and transmission spectra in massive and thin crystals, the influence of a magnetic field etc.).

The electrooptics of excitons (the Franz-Keldysh effect for simple and degenerate energy bands, exciton electroabsorption, etc.) is discussed in the fifth chapter (A. G. Aronov, A. S. Ioselevich).

Of great interest is the sixth chapter (V. B. Timofeev) in which free multiparticle exciton complexes in indirect gap semiconductors are investigated. Such complexes (exciton molecules and exciton trions) are analogous to ordinary molecules and molecular ions. The equilibrium of complexes with electron-hole liquid, and the effect on them of uniaxial pressure and magnetic field are examined.

Self-trapping of excitons is investgated in the seventh chapter (E. I. Rashba). The main models, classification and criteria of self-trapping are presented. Self-traping in low-dimensional systems, the self-trapping rate, the contribution of self-trapping to optical spectra and other problems are investigated.

In the two concluding chapters Frenkel excitons are examined. Properties of excitons in magnetic insulators (optical properties of paramagnetic ions, exciton-magnon transitions, etc.) are investigated in the eighth chapter (Y. Tanabe, K. Aoyagi). The ninth chapter (M. V. Belousov) is devoted to an examination of electron-vibrational (vibronic) intramolecular excitations serving as excitons.

The authors of the book under review have examined practically all the problems of exciton physics (one should only regretfully note the absence of a very interesting, in our opinion, and rapidly developing at the same time field of investigation—excitons in semiconductor heterostructures and superlattices). Specialists involved in studying excitions will find reviews useful to themselves not only in their own narrow specialty, but will also be enriched by interesting data from neighboring fields. The review papers were written by well-known specialists in these fields. The book is excellently illustrated and contains an extensive, well-selected bibliography.

The contents of the book are of essential interest for a broad circle of theoreticians and experimenters specializing in solid state physics.

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