Proton-(anti)proton cross sections and scattering amplitudes at high energies

I. M. Dremin

P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow
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This note was motivated by some new experimental results which were first reported[1] at the International Conference on High-Energy Physics (Uppsala, Sweden, 25 June–1 July 1987), organized by the European Physical Society. This was the first measurement[2] of the ratio of the real part to the imaginary part of the amplitude for the forward elastic scattering of protons by antiprotons at the Sp$\bar{\rm p}$S collider at CERN at an energy $s^{1/2} = 546$ GeV. The ratio turned out to be unusually large[1)]:

$$\rho_{p\bar{p}} = 0{,}24 \pm 0{,}04. \tag{1}$$

This figure is roughly twice the average estimates which had been found previously from the dispersion relations under the standard assumptions regarding the energy behavior of the total cross sections for the interaction of protons with protons and with antiprotons (Ref. 3–6, for example).

If such a large value of ρ is confirmed by future, more-accurate measurements, there will accordingly be two consequences: First, the present understanding of this situation will have to be extensively revised. Second, there will be some change (although not very large) in the present estimates of the total cross sections. This change will have effects on theoretical extrapolations to ultrahigh energies.

There is a point to be stressed here, however: So far, this is the result of only a single experiment; the measurement error is rather large; and the very value of ρ depends slightly on the procedure by which it is extracted from the experimental data (more on this below).

We first need to recall the general situation. In measurements of the differential elastic cross section for the scattering of hadrons with a small momentum transfer one obtains information on four basic characteristics of the hadron interaction: the total cross section $\sigma_{\rm tot}$, the ratio (ρ) of the real part of the elastic scattering amplitude to its imaginary part, the slope (B) of the diffraction cone, and the total elastic cross section $\sigma_{\rm el}$.

The total cross section is related to the imaginary part of the amplitude for forward elastic scattering, $A_{\rm n}$, by the unitarity relation

$$\mathrm{Im}\, A_{\rm n}\,(s,\ t=0) = \sigma_{\rm tot}(s), \tag{2}$$

where, as usual, s is the square of the energy in the center-of-mass frame, and t is the square of the momentum transfer. The differential cross section for the elastic scattering of hadrons is written in the form

$$\frac{d\sigma_{\rm el}}{dt} = \frac{1}{16\pi}\,|\,A_{\rm n} + A_{\rm C}\,|^2, \tag{3}$$

where $A_{\rm n}$ and $A_{\rm C}$ are respectively the nuclear and Coulomb parts of the scattering amplitude. The nuclear scattering cross section is parametrized in the following way:

$$\frac{d\sigma_{\rm n}}{dt} = \frac{1}{16\pi}\,|\,(\rho + i)\,\sigma_{\rm tot}e^{Bt/2}\,|^2 \equiv \frac{1}{16\pi}\,|\,A_{\rm n}\,(s,\ t)\,|^2. \tag{4}$$

The elastic cross section is thus found through an integration[2)] over t:

$$\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} \approx \frac{\sigma_{\rm tot}\,(1+\rho^2)}{16\pi B}. \tag{5}$$

At a very small momentum transfer, the purely Coulomb scattering (as a result of the exchange of a photon) is dominant. The cross section for that process is given by[3)]

$$\frac{d\sigma_{\rm n}}{dt} = \frac{1}{16\pi}\left|\pm G^2(t)\,\frac{8\pi\alpha}{t}\,e^{\pm i\varphi}\right|^2 \equiv \frac{1}{16\pi}\,|\,A\ \ (s,\ t)\,|^2, \tag{6}$$

where $G(t)$ is the electromagnetic form factor of the proton, the phase[4)] is $\varphi = \ln(0.08|t|) - 0.577$, and $\alpha = 1/137$.

The Coulomb amplitude falls off rapidly with increasing momentum transfer, and even at a comparatively small momentum transfer the nuclear scattering with a characteristic diffraction cone described by expression (4) is dominant. In the intermediate region, where these components are comparable in magnitude, and where they interfere, according to (3), the relative phase of these amplitudes is important. The phase φ can be found theoretically.[4)] The quantity ρ can be found from the interference of the nuclear and Coulomb scattering. The interference term in cross section (3) reaches a maximum at small values of t:

$$|t|_{\rm int} \approx \frac{8\pi\alpha}{\sigma_{\rm tot}}, \tag{7}$$

corresponding to angles of about 0.12 mrad at the Sp$\bar{\rm p}$S collider energy $s^{1/2} = 546$ GeV or 2 μrad at the SSC collider at $s^{1/2} = 40$ TeV (if the cross section is assumed to be about 120 mb at this energy). We thus see what a complicated matter it becomes to find ρ at such high energies, where the measurements must be taken at angles less than 0.01° even at the Sp$\bar{\rm p}$S collider. Nevertheless, it is possible to cope with this problem, as we will see below. Measurements have been carried out[2] to angles of 0.165 mrad (i.e., slightly greater than 0.12 mrad).

The effect caused by the real part of the amplitude can be seen in Fig. 1, which shows the deviation from unity of the ratio of the measured cross sections to its expected value at $\rho = 0$, i.e., the quantity $R(t) = (d\sigma/dt)_{\rm exptl}/(d\sigma/dt)_{\rho=0} - 1$. It should be noted that the customary assumption is that ρ and B in (4) do not depend on the momentum transfer t at such small angles, and $\sigma_{\rm tot}$ is assumed to be known. The interference contribution to the cross section turns out to be proportional to $\rho\sigma_{\rm tot}$.

At this point we should perhaps warn the reader that result (1), written above, which is the result of a single experiment, has a fairly large error; furthermore, as is empha-

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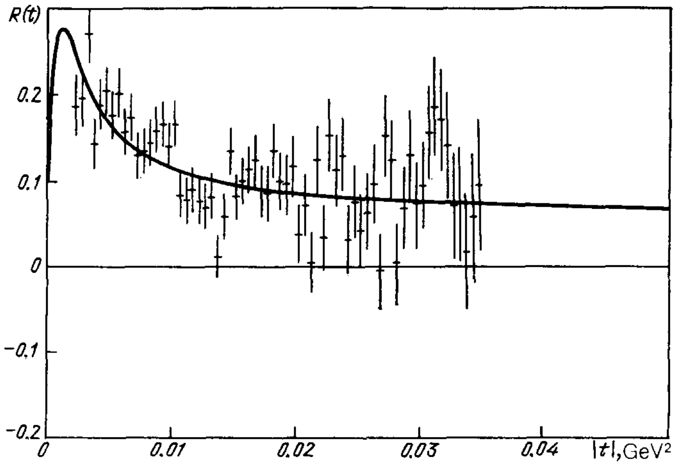

FIG. 1. Difference between the measured differential cross section for elastic scattering and its theoretical value for $\rho = 0$.

$R(t) = (d\sigma/dt)_{\text{exptl}} (d\sigma/dt)_{\rho=0}^{-1} - 1.$

The solid line is the best fit of estimate (1) to the experimental values.

sized by Bernard *et al.*[2] themselves, that result depends on the assumption that B and ρ remain constant. If, for example, the slope parameter of the diffraction cone, B, changes, say from 15.3 GeV^{-2} at $|t| = 0.06$ GeV^2 to 17.2 GeV^{-2} at $t = 0$, then according to their estimate one should reduce the value of ρ by[5] 0.02. If, on the other hand, the slope parameter for the real part turns out to be twice that for imaginary part, then one should reduce ρ by 0.03. Unfortunately, we are not presently in a position to resolve the uncertainties which stem from these assumptions.

With these comments in mind, and without making any further stipulations, we will thus attempt to discuss the consequences which such a large value of ρ might have if it were to be confirmed and if the assumptions which have been made were to be justified.

We begin with the total cross sections. Measuring total cross sections accurately is not a trivial problem, since the quantity which emerges directly from an experiment is a count rate, rather than the cross section. In an elastic-scattering experiment, for example, one determines the number of counts per unit time, $N(t)$, in a given momentum-transfer interval near t. This count is naturally proportional to the differential scattering cross section:

$$N(t) = L\frac{d\sigma}{dt}, \tag{8}$$

where the normalization coefficient L is called the "luminosity." From (8) and (4) we find

$$\sigma_{\text{tot}}(1+\rho^2)^{1/2} = 4\left(\frac{\pi N(0)}{L}\right)^{1/2}. \tag{9}$$

The quantities $N(0)$ and B are found through a linear extrapolation [in accordance with (4)] of the logarithm of the measured count rate $\ln N(t)$ from the nuclear-scattering region to the point $t = 0$ (without a measurement of the luminosity).

There are various ways to determine L. For example, there is the method which Van der Meer has proposed for measuring the luminosity directly on the basis of the transmission of colliding beams through each other. A more accurate method makes use of the Coulomb region, where the normalized cross section $d\sigma_C/dt$ in (6) is known, so the luminosity can be determined by dividing the count rate measured at very small angles in the Coulomb region by this cross section. In any case, once we have measured the luminosity in one way or another we find information on the quantity $\sigma_{\text{tot}}(1+\rho^2)^{1/2}$, in accordance with (9). Knowing the luminosity, we can find the total cross section directly if we measure the count rate for any interaction:

$$N_{\text{tot}} = L\sigma_{\text{tot}}. \tag{10}$$

Frequently, however, either the luminosity is not known at all, or the error in the measurement is large. In such a case one resorts to a method which does not require knowledge of that quantity. It can be seen from (8) and (10) that through simultaneous measurements of the total count rate and the differential count rate (in the nuclear-scattering region) one can eliminate L, finding

$$\sigma_{\text{tot}}(1+\rho^2) = \frac{16\pi N(0)}{N_{\text{tot}}}. \tag{11}$$

Working by this method, which does not require a measurement of the luminosity, one finds the product $\sigma_{\text{tot}}(1+\rho^2)$. It can be seen from (5) that the ratio of the elastic cross section to the total cross section can also be determined approximately without a measurement of the luminosity. This method of course wins more popularity at high energies, since it requires neither direct measurements of the luminosity nor working at very small angles in the deep Coulomb region.

If the quantity ρ^2 is not known accurately, but if it is small, then the uncertainties in the determination of the total cross section by both these methods will also be small. Here we see the importance of result of (1) for measurements of the total cross sections at the Sp$\bar{\text{p}}$S collider. It has previously been assumed on the basis of the predictions of the dispersion relations that the quantity ρ^2 makes a contribution of about 2% to expression (11), so a corresponding correction has been introduced during the extraction of the total cross section. The measured value $\sigma_{\text{tot}}(1+\rho^2) = 63.3 \pm 1.5$ mb has thus yielded[8] the value $\sigma_{\text{tot}} = 61.9 \pm 1.5$ mb. According to (1), however, this correction is 6%; the effect is to reduce significantly the total cross section, which can now be estimated as follows for an energy $s^{1/2} = 546$ GeV;

$$\sigma_{\text{tot}} = 60 \pm 2 \text{ mb}. \tag{12}$$

On the other hand, the new data indicate that the real part of the amplitude for the elastic scattering of protons by

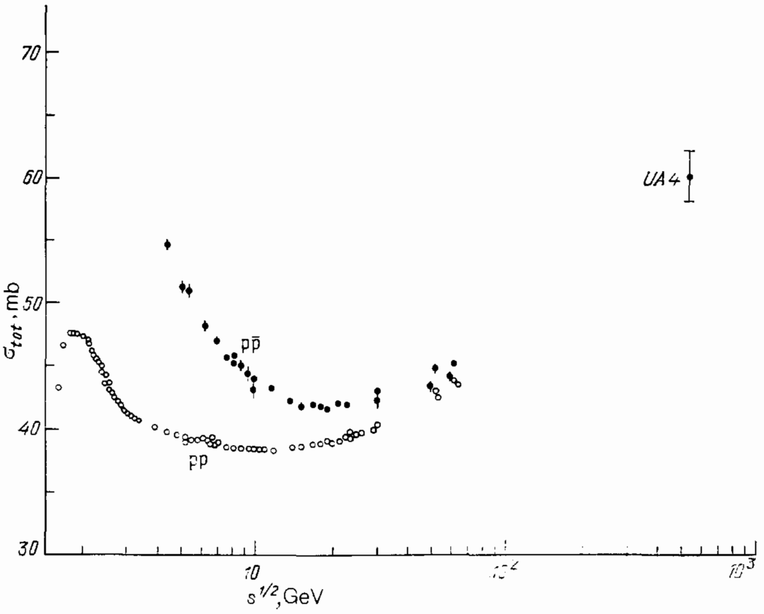

FIG. 2. Behavior of the total proton-proton and proton-antiproton interaction cross sections as functions of the energy.

antiprotons increases rapidly. If the forward scattering amplitude is expressed in millibarns and is normalized in accordance with (2) as $A \equiv r + i\sigma_{tot}$, then its real part r increases[7] by a factor of more than three or by 10 mb as we go from the ISR at $s^{1/2} = 50$ GeV (where the value is 4.4 ± 0.8 mb) to the Spp̄S energies $s^{1/2} = 546$ GeV (where its value reaches 14.4 ± 1.4 mb). The imaginary part (or the cross section σ_{tot}) increases by about 40% or by 17 mb here. Figures 2 and 3 show experimental data on the total cross sections and on the ratios of the real and imaginary parts of the amplitudes in the energy interval $5 \leqslant s^{1/2} \leqslant 546$ GeV.

We will supplement those data by noting that the ratio of the elastic cross section to the total cross section increases from 0.175 ± 0.001 to 0.215 ± 0.005 as we go from the ISR to the Spp̄S, and the slope of the diffraction cone at $|t| < 0.1$ GeV^2 in pp̄ also increases[6)] from 14 to 15.3 GeV^{-2}.

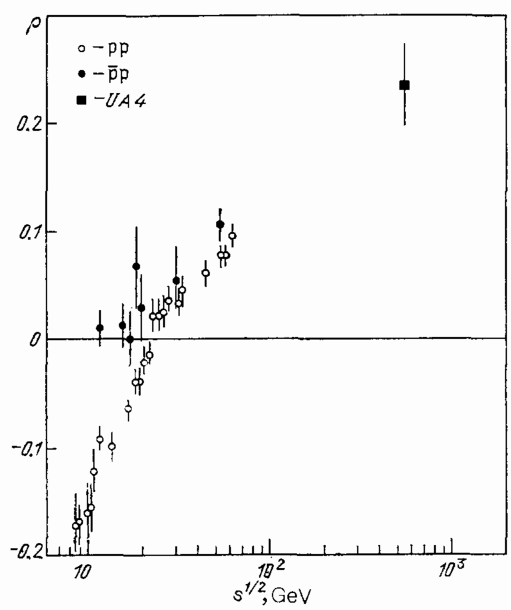

FIG. 3. Behavior of the ratios of the real part of the amplitude for forward elastic scattering to its imaginary part in collisions of protons with protons and with antiprotons as functions of the energy.

These results are very important for reaching an understanding of the physics of hadronic processes and for the possibility of generating predictions about their behavior at even higher energies. In my opinion, and the most important and most likely consequence of a growth of ρ is the assertion that the cross sections for proton-(anti)proton interactions should increase dramatically as we move to the Tevatron and the UNK and SSC colliders without any apparent saturation in this growth. The proton-proton cross sections may become larger than the proton-antiproton cross sections (in contrast with the behavior at energies up to the ISR; Fig. 2).

Just what arguments are there in favor of such a strong assertion? We would first like to take a careful look at the predictions which were generated before the appearance of data from the Spp̄S collider. It was usually assumed that the cross sections for proton-proton and proton-antiproton interactions would continue to converge in accordance with an $s^{1/2}$ law as a result of the fading of the contribution from secondary Regge trajectories with a negative signature (of negative P and C parity). It was assumed that their growth would be either a Froissart growth, in proportion to $\ln^2 s$, or even a power law s^Δ in some limited energy interval. This confidence was based on the circumstance that at the ISR energies the pp and pp̄ cross sections were converging (although the measurements of the cross-section difference $\Delta\sigma$ were not very accurate), while the cross sections themselves were growing, and this growth continued even at the Spp̄S energies. Working from this behavior and the dispersion relations, one could calculate how the ratio of the real part of the amplitude to the imaginary part would behave at higher energies.[5] Beginning at the energies of the Spp̄S collider, there should have been essentially no difference between pp and pp̄. The results are shown in Fig. 4. The power-law growth of the cross sections due to the supercritical pomeron leads to the largest values of ρ extrapolation 1, taken from Ref. 9, which leads to σ_{tot} (546 GeV) ≈ 66 mb; extrapolation 2, taken from Ref. 10, which gives σ_{tot} (546 GeV) ≈ 62 mb],

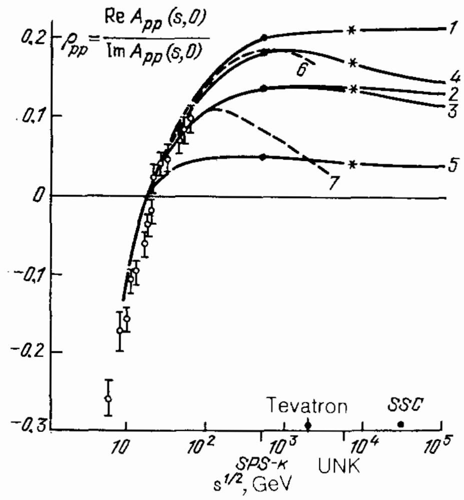

FIG. 4. Theoretical predictions of the behavior of the ratio of the real part of the amplitude for forward elastic scattering of protons by protons, ρ_{pp} as a function of the energy obtained on the basis of the dispersion relations and through various extrapolations of the total cross sections to higher energies. Each curve is labeled with the index of the extrapolation, which is explained in the test proper.[3]

although the difference from a Froissart regime is small. Froissart extrapolations 3, 4, and 6, taken from Refs. 4, 11, and 12, differ in the growth rate and nature of the preasymptotic corrections. A more rapid growth of the cross sections led to an increase in ρ. A logarithmic growth of the cross sections was ruled out [extrapolation 5; Ref. 13) since it did not describe the experimental data which were available at the time and led to values of ρ which were too small. The ratio ρ also fell off sharply in cases in which it was required that the amplitude growth reach saturation, e.g., in the case in which extrapolation 6 was replaced by extrapolation 7, which differed in that the Froissart growth of the cross sections terminated and the cross section assumed a constant value of about 85 mb. Figure 4 clearly shows the decrease in ρ as a result of this change.

A general conclusion which follows from an analysis of the dispersion-relation results in Fig. 4 and from the new data from the Sp$\bar{\text{p}}$S collider, shown in Fig. 3, is that the conventional assumption that the amplitudes for the interaction of protons with protons and with antiprotons at high energies would rapidly converge is hardly correct for any plausible and reasonably smooth extrapolations of the total cross sections. Under this assumption it is not possible to find the large value in (1) of the ratio ρ as measured at the Sp$\bar{\text{p}}$S collider.

It is clear at a qualitative level that this magnitude can be approached either by specifying a difference between the cross sections for the interaction of protons with protons and with antiprotons which does not fall off with the energy or by assuming a very rapid growth of the cross sections. This assertion might also be demonstrated at a quantitative level,[14] by examining the results of calculations with the help of the dispersion relations under these assumptions. Under these assumptions, the results found by this method are extremely reliable (Fig. 5). However, the integral dispersion relations suffer from the disadvantage that they are, comparatively speaking, not very graphic. A test of any new hypothesis requires new numerical calculations. The assertion regarding the role played by a rapid growth of the cross sections and by the difference between the pp and p$\bar{\text{p}}$ cross sections can be demonstrated graphically, and some simple estimates can be made, by making use of differential dispersion relations[7]) (Ref. 4, for example), which are conveniently written in the form[7]

$$\frac{dr_{p\bar{p}}}{d\eta} = \sigma_- + \frac{d^2}{d\eta^2}\left(\sigma_+ - \frac{1}{3}\sigma_-\right) + O\left(\frac{d^n\sigma}{d\eta^n}\right) \quad (n > 2), \quad (13)$$

where

$$\eta = \frac{2}{\pi}\ln\frac{s}{s_0}, \quad \sigma_\pm = \frac{\sigma_{pp} \pm \sigma_{p\bar{p}}}{2}, \quad s_0 = \text{const.}$$

Let us ignore the difference between the cross sections σ_- at high energies and assume that the growth of σ_+ stems from a term of the type $\beta \ln^2(s/s_0)$. For a linear dependence of the real part of the amplitude on $\ln(s/s_0)$, the experimental data on its growth which we presented above determine the left side of equality (13), so we have

$$3.4 \ \text{mb} = \frac{\pi^2}{2}\beta, \quad (14)$$

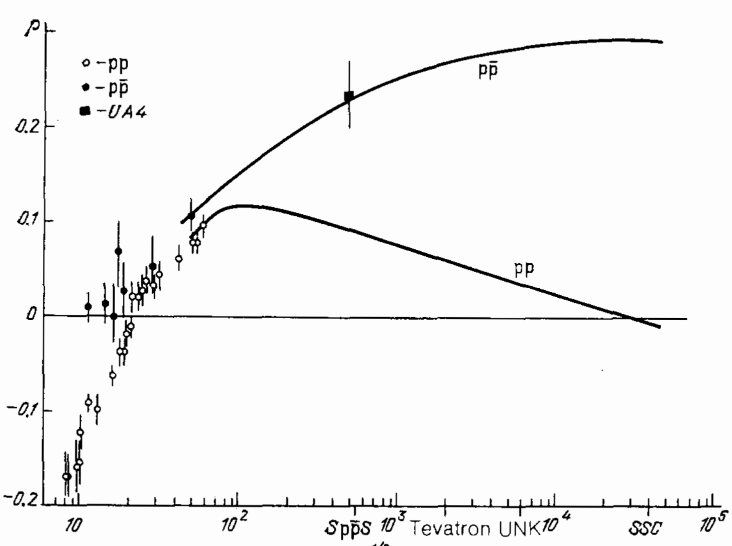

FIG. 5. Difference between ρ_{pp} and $\rho_{p\bar{p}}$ under the assumption $\Delta\sigma \equiv \sigma_{pp} - \sigma_{p\bar{p}} = 2\pi\varepsilon \ln(s/s_0)$, $\varepsilon = 0.1$ mb, and $s_0 = 350$ GeV^2

and thus $\beta \approx 0.7$ mb, and the growth of the cross sections σ_+ turns out to be nearly twice as pronounced as is observed experimentally.

If the data on the growth of the real part of the amplitude are correct, there are two ways out of the resulting contradiction. The first is to suggest an extremely irregular behavior of the total cross sections as functions of the energy, with terms with high derivatives in (13) beginning to play a significant role. The physical reason for such an event might be threshold processes, but they seen extremely unlikely at such high energies. Specific estimates are difficult to generate from (13). One can, however, suggest various models[15,16] with a very rapid growth of cross section which will lead to the necessary value of ρ. In a model with a threshold growth,[15] the total cross sections at the Tevatron energies increase sharply (by more than 20 mb) in comparison with the customary interpolations, which predict 70–80 mb. In a model with a very supercritical pomeron[16] (in which the cross section increases as $\varkappa s^{0.3} + c$), the sharp growth of the cross sections also leads to the prediction of a huge value of the cross section at the SSC collider: about 250 mb. By way of comparison, the customary predictions of this figure are in the range 100–160 mb. Measurements of the total cross sections at the Tevatron will soon bring some clarity into the problem of describing the energy dependence of ρ. If a sharp increase in the cross sections is not observed, we will be left with a second way out: to assume that the energy dependence of the cross sections is smooth enough that the higher derivatives in (13) can be ignored and to incorporate a positive contribution of the term σ_- to the right side of Eq. (13). It is clear from the estimates above that this contribution should be small, $\sim$ 1–2 mb; i.e., at the Sp$\bar{\text{p}}$S collider energies the cross section for proton-proton interactions should become 2–4 mb larger than the cross section for proton-antiproton interactions. This fact, however is of fundamental importance—at the ISR energies, the situation was just the opposite: The antiproton cross sections were larger than the proton cross sections.

Such a situation does not contradict any basic principles of the theory, although untll recently it was regarded as unlikely. Such a situation arises if reggeon exchanges with a negative signature—odderons—begin to play a role.[17-19] The contribution of trajectories of this sort in the ordinary reggeon picture is exhausted by poles of the type ρ and ω, which have essentially faded away at the energies of the Sp$\bar{\text{p}}$S collider, but this circumstance does not rule out the appearance of odderons which do not fade away but in fact increase with the energy (analogs of a froissarton or pomeron), although at this point we cannot unambiguously specify particles of any sort which lie on this trajectory. In the limiting Froissart regime there may be limiting odderon components[18,19] in both the real part of the amplitude, which increases as the square of the logarithm of the energy, and the imaginary part of the amplitude, which increases in proportion to the logarithm of the energy. An important point is that these components appear with opposite signs in the pp and p$\bar{\text{p}}$ amplitudes.

Before we take up the reasons for this effect and other consequences of this behavior of the cross sections, we would like to discuss a simple analytic model[4,14] with growing cross sections in which it is possible to generate some specific numerical estimates and predictions for higher energies.

We choose the amplitude for forward scattering with a negative signature, A_-, in the form

$$A_- = \varepsilon \left(\ln \frac{s}{s_0} - \frac{i\pi}{2}\right)^2, \tag{15}$$

and we choose the amplitude with a positive signature, A_+, in the form

$$A_+ = i\left[C + \beta\left(\ln \frac{s}{s_0} - \frac{i\pi}{2}\right)^2\right], \tag{16}$$

by specifying the constants[8] $C = 41.7$ mb, $\beta = 0.47$ mb, and $\varepsilon = 0.1$ mb. We will use expressions (15) and (16) beginning at the Sp$\bar{\text{p}}$S energies, where the contribution of secondary Regge trajectories can be ignored. At the ISR energies, this component is still extremely noticeably; adding it to (15) and (16), we can generate a good description of the ISR experimental data. The cross sections and the real parts of the amplitudes are calculated from expressions (15) and (16) in the form

$$\sigma_{\bar{p}p/pp} = C + \beta\left(\ln^2 \frac{s}{s_0} - \frac{\pi^2}{4}\right) \mp \pi\varepsilon \ln \frac{s}{s_0}, \tag{17}$$

$$r_{\bar{p}p/pp} = \pi\beta \ln \frac{s}{s_0} \pm \varepsilon\left(\ln^2 \frac{s}{s_0} - \frac{\pi^2}{4}\right). \tag{18}$$

Their numerical values are given in Table I for the energies of the Sp$\bar{\text{p}}$S ($s^{1/2} = 546$ GeV), the Tevatron ($s^{1/2} = 2$ TeV), the UNK ($s^{1/2} = 6$ TeV), and the SSC ($s^{1/2} = 40$ TeV).

When the amplitudes are chosen in this way, the differential dispersion relations hold identically. Only the first two terms on the right side of Eq. (13) contribute; the rest vanish. Since the real part of the amplitude contains a term which increases as the square of the logarithm, the specific numerical estimates found with the help of (17) and (18) differ slightly from (14), but the qualitative conclusions reached earlier remain in force.

Looking at the numbers in Table I, we clearly see a growth of the cross sections with the energy, that the proton cross sections are larger than the antiproton cross sections, that the difference between the two increases with the energy, and that there is a noticeable difference in the behavior of the real parts of the amplitudes and their ratios to the imagi-

TABLE I.

	Sp$\bar{\text{p}}$S	Tevatron	UNK	SSC		Sp$\bar{\text{p}}$S	Tevatron	UNK	SSC
σ_{pp}, mb	64	84.7	107	156	$r_{p\bar{p}}$, mb	14.4	22	30	46.8
$\sigma_{p\bar{p}}$, mb	60	78.7	100	146	ρ_{pp}	0.09	0.06	0.04	0
$\Delta\sigma$, mb	4	6	7	10	$\rho_{p\bar{p}}$	0.24	0.28	0.30	0.32
r_{pp}, mb	5.8	5.1	4	0					

nary parts (the cross sections). The factor which is most critical with respect to the conclusions of the model is the difference between protons and antiprotons in collisions with protons.

Unfortunately, we have access to only two numbers (the second and last) in the first column [and these numbers were already used in the specification of the parameters in (15) and (14)]. From the four independent numbers in each of these columns we can hope to learn only two in the future, since present plans call for collisions of protons only with antiprotons or only with protons at some particular energy. Although the energy growth of the total cross sections is not by itself a critical test of a model, when combined with the large (and ever-increasing) value of $\rho_{p\bar{p}}$ or with the small value of ρ it clearly points to the odderon model.

It would be desirable to find the characteristics of the scattering of protons by antiprotons, which would provide additional evidence about the presence of effects due to odderons. The growth of the elastic cross section and the increase in the slope of the diffraction cone do not by themselves lead to any obvious unambiguous conclusions.[4] More interesting is an effect which unambiguously shows the need to introduce odderons and which stems from the behavior of the first minimum just beyond the elastic-scattering diffraction cone.[9)] The experimental data at the ISR energies provide evidence that such a minimum exists in the pp scattering at $|t|\approx 1.3\ \mathrm{GeV}^2$, while there is no such minimum in this region in the pp cross section; there are instead a "shoulder" and, possibly, a very slight minimum at $|t|\approx 1.5\ \mathrm{GeV}^2$. The only way to explain this difference is in terms of exchanges with $C=-1$. At the Sp$\bar{p}$S energies, the cross section increases by more than an order of magnitude at the shoulder, and the shoulder itself begins at $|t|\approx 0.9\ \mathrm{GeV}^2$. The difference between the pp and p$\bar{p}$ results at the ISR energies and also the growth of the shoulder at the Sp$\bar{p}$S energies led to an active discretion of the contributions of negative-signature amplitudes: odderons[14-16] (in particular, the "maximal" odderon).

In the odderon-exchange models it is usually found that at a small momentum transfer the situation is dominated by an amplitude with a positive signature, which describes the basic diffraction cone. With increasing momentum transfer (at $|t|\sim 1.2\ \mathrm{GeV}^2$), the amplitude with a negative signature becomes the primary one; furthermore, the real part of this amplitude turns out to be considerably larger than the imaginary part. Near the minimum or dip, where both of the components are important, they interfere, giving rise to oscillations of a sort in the depth of the dip as the energy varies. This prediction is extremely interesting, but a test of it will require data over a fairly wide energy range.

Unfortunately, the use of the interactions of mesons with photons or protons would be of no help in clarifying the odderon question, since exchanges with a negative signature do not contribute to such interactions.

The possibility of studying proton-proton interactions at the Tevatron colliding-beam energies or at higher energies would be unusually valuable for a final resolution of this problem, although even here the situation is not very simple. It can be seen from Table I that the difference in the cross sections may also be rather small (about 6 mb), and the product $\sigma_{\mathrm{tot}}(1+\rho^2)$, which is found experimentally, turns out to be essentially the same for pp and p$\bar{p}$. In order to extract a value for ρ, on the other hand, we need to work in the region of the interference of the Coulomb and nuclear amplitudes, which is at very small angles in this energy range.

According to general theorems about the asymptotic properties of hadronic interactions, the total cross sections cannot grow more rapidly than $\ln^2 s$, and the difference between cross sections cannot grow more rapidly than $\ln s$. This limiting case for the contribution of an odderon (maximal odderon) has been analyzed in the example of Eqs. (15) and (16). The estimates in (14) were based on an analysis of the amplitude of an odderon with a logarithmically growing real part and a constant difference between cross sections. In general, one can study this amplitude in the more general form[10)]

$$A\sim\left(\ln\frac{s}{s_0}-i\,\frac{\pi}{2}\right)^{\gamma}, \tag{19}$$

where $\gamma\leqslant 2$, but for our illustrative purposes here it is sufficient to consider one more interesting case, $\gamma=0$, in addition to the two examples discussed above. Such an odderon does not lead to a difference in cross sections; it simply changes the real part of the amplitude.

If there is thus no difficulty in classifying the odderons, we still lack an understanding of the their physical nature. The physics of odderons might be deciphered either by taking a phenomenological path seeking those inelastic processes which are responsible for the difference which arises between the cross sections,[11)] or by taking a more thorough approach on the basis of quantum chromodynamics, determining those diagrams which lead to exchanges with a negative signature and C parity.

In quantum chromodynamics, the simplest candidate for an odderon is three-gluon exchange (Refs. 21 and 22, for example). In specific calculations, however, one obtains figures which are completely at odds with the experimental data. Three-gluon exchange leads to a purely real amplitude [$\gamma=0$ in (19)] with a comparatively small and negative real part,[22] $A_-\approx -0.8$ mb. Experimentally, in contrast, this value is large and positive, and it increases with the energy. The cross sections for the interaction of protons and antiprotons with protons tend toward the same limit in this case (i.e., $\gamma=0$). Correspondingly, the values of ρ turn out to be small. The same comments apply to the quasipotential approach with eikonal rescattering, which is reviewed in Ref. 23.

Theoretical arguments have been advanced[24] in favor of the appearance of odderons with a trajectory near 1 at $t=0$ on the basis of effective Langrangians in gauge theories of Yang-Mills fields.

In some of the models which have been developed, the cross sections for the interaction of antiprotons with protons have turned out to be larger than the proton-proton cross sections at arbitrarily high energies.[25] A model of this sort has even been interpreted[26] by means of a picture of inelastic processes based on a dual topological approach in which the difference between cross sections is explained on the basis of a contribution of annihilation processes which does not fade away with increasing energy and which results from diagrams with three "ladder" annihilations of quarks with antiquarks—which are not present in proton-proton interactions. The values found as a result, $\rho\sim 0.1$, however, rule out any hope that it will be possible to describe the experimental data at the energies of the Sp$\bar{p}$S collider.

Igi and Kroll[31] have shown that at low energies the dispersion relations without subtractions for a C-odd amplitude hold quite accurately if we use the experimental data available today on the proton-proton and proton-antiproton amplitudes and if we assume that the difference between the pp and p$\bar{p}$ cross sections is parametrized at high energies by ordinary Regge poles without any contribution of an odderon with $J = 1$. This circumstance is an argument against the presence of such an odderon. Furthermore, Igi and Kroll[31] also point out that an odderon contribution to the dispersion relations with subtractions would be possible within the errors, but it could not be as large as in Ref. 27.

Let us summarize the results of this analysis. The new experimental result obtained at the Sp$\bar{p}$S collider at the energy $s^{1/2} = 546$ GeV—a large ratio of the real part of the amplitude for the forward scattering of protons by antiprotons to the imaginary part of this amplitude—was unexpected and is fraught with implications. This ratio is about twice as large as most of the predictions, although it has appeared that these predictions have been based on the general principles of the theory and the soundest extrapolations of the cross sections. The "refractive index" of the hadronic medium turns out to increase with the energy much more rapidly than its "absorption coefficient" does.

If the value found for ρ is verified by more-accurate measurements, and if the nuclear elastic cross section retains its exponential form at small values of t also, then the day may be saved either by assuming an extremely unusual and irregular behavior of the total cross sections as functions of the energy or (if their behavior is regular) by assuming that the proton-proton cross sections exceed the proton-antiproton cross sections at high energies. Neither of these assumptions would contradict the basic postulates of the theory. However, an irregular behavior of the cross sections (with, say, a sharp growth of a threshold nature or oscillations) would seen overly exotic. On the other hand, a constant positive difference between the cross sections σ_{pp} and $\sigma_{p\bar{p}}$ (or even a difference which increases with the energy) would seem to contradict the tendencies which we have seen experimentally at lower energies, up to the ISR energies, where this difference has been negative and has decreased with the energy. Nevertheless, the second of these possibilities seems preferable. It can be described theoretically by means of the exchanges of states with a negative signature and C parity: odderons.

Although the odderon concept does make it possible to generate reasonable values for the total cross section and the real part of the amplitude, the physical interpretation of an odderon remains an open problem. More generally, the physics of the growth of the real part of the amplitude, the ratio of the elastic cross section to the total cross section, and even the energy growth of the total cross sections remain open questions. The relationship between these phenomena and such new facts in inelastic processes as the violation of Feynman scaling, the violation of KNO scaling, the large fluctuations, and the strong correlations, is essentially not yet understood. The only point which is beyond doubt is that all these facts reflect a structure of hadrons which is considerably more complex than that which we had tacitly assumed previously and to which we had become accustomed.

[1)]The total error given here is a combination of the statistical error, of 0.024, which is the only error which was stated in Ref. 7, and a systematic error of 0.025 (Ref. 2).

[2)]We are not considering here the possible weak dependence of B on t.

[3)]The upper and lower signs refer to pp and p$\bar{p}$, respectively.

[4)]Phases of this magnitude are ordinarily adopted in the analysis of experimental data. It should be noted, however, that this magnitude may change, as a result of (for example) the use of the eikonal approximation.[28]

[5)]There exist elastic-scattering models[29] in which the diffraction peak differs from a purely exponential peak. Soffer[30] has asserted that a model of this sort is capable of describing the experimental data of Ref. 1 on the behavior of the differential elastic cross section at a small momentum transfer if the comparatively small value $\rho = 0.13$ is given. However, it appears to me that the curve given there[29] is not a very good fit of the experimental data in specifically the most critical region, i.e., the interference region. An increase in ρ could improve the fit.

[6)]Indicating that the proton becomes blacker and larger.

[7)]The criticism of the differential dispersion relations[15] which is sometimes encountered is not terribly convincing since the physical estimates ordinarily use either threshold amplitudes,[15] whose higher derivatives are generalized functions which can be determined only with the help of the integral relations, or analytic approximations of the cross sections, for which the series with higher derivatives is taken into account correctly in (13).

[8)]Since our purpose here is to demonstrate the general trends, we have not fitted these constants to minimize the mean square deviations.[4] After this paper had already been written, I learned of a preprint[27] with corresponding estimates with a fit of the data. The values $\beta \approx 0.38$ and $\varepsilon \approx 0.109$ were found there (with $\sigma_{tot} = 61.2$ mb and $\rho = 0.205$ at the Sp$\bar{p}$S). The qualitative conclusions remain the same.

[9)]This question is discussed in Ref. 32, for example.

[10)]It was shown in Ref. 27 that it is necessary to set $\gamma \approx 2$ in order to describe the experimental data discussed above.

[11)]Models of inelastic processes with an exchange of many "ladders" (DTU, strings, etc.) usually lead to well-known trajectories, and for the value of ρ at the Sp$\bar{p}$S they predict[20] values of about 0.12.

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