FROM THE CURRENT LITERATURE

Phase transition in quark-gluon plasma and hydrodynamic theory

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The phase transition from quark-gluon plasma to hadronic matter is extensively examined in relation to the hadronization problem in the hydrodynamic theory of multiple production. Existing models of hadrons and thermalization mechanisms in ultrarelativistic collisions are briefly discussed. When surface interaction effects are taken into account, the phase transition is nontrivial in the bag model: metastable states of matter are possible. Possible hadronization scenarios are discussed, and a kinetic analysis is given of cooling and hadronization processes. It is shown that, when the initial plasma energy density is close to the critical value ($\simeq 4$ GeV/fm^3), the more realistic scenarios are those based on the nonequilibrium hadronization of supercooled plasma, which involves an appreciable increase in the mean transverse momentum of secondary particles. Traditional (equilibrium) mechanisms should predominate in hotter plasmas. Theoretical estimates are compared with JACEE cosmic-ray data.

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1. INTRODUCTION

The hydrodynamic theory of multiple production of particles[1-3] is different in character from other models of interaction between hadrons. Its principal feature is that hadronic matter is looked upon as a continuous heated medium, so that classical equations and concepts from continuum mechanics can be employed, including the temperature T, the energy density ε, the entropy s, and so on. For a long time, this was practically the only theory of this kind.

The situation has changed in recent years. The concept of a continuous medium has become entrenched in quantum chromodymamics (QCD).[4] Physical (observable) vacuum is now interpreted as nontrivial, nonperturbative matter that ensures confinement, i.e., the absence of colored objects (solitary quarks and gluons) in the spectrum of physical excitations. Whatever the specific nature of this vacuum matter (instanton fluid,[5,6] quasi-Abelian magnetic field,[7] or Higgs field;[8] see Refs. 4, 9, and 10 for the relevant reviews), this state is found to be energetically more favorable than vacuum in its previous interpretation (absolute emptiness). It has negative energy density[1)] $\varepsilon_V \approx -0.5-1$ GeV/fm^3 (Refs. 11 and 12) and excess pressure $p_V = -\varepsilon_V > 0$ (Refs. 10 and 11).

The so-called perturbative vacuum ($p_{PV} = \varepsilon_{PV} = 0$) is found to be unstable and can exist only in the interior of hadrons, i.e., within regions of ~ 1 fm^3 from which physical vacuum is excluded[2)] (Ref. 13). In applications, it is convenient to assign positive energy density $\varepsilon_{PV} = |\varepsilon_V|$ and negative pressure $p_{PV} = -|\varepsilon_V|$ to the perturbative vacuum; the observable vacuum then corresponds to $\varepsilon_V = p_V = 0$.

Finally, we consider the idea of the quark-gluon plasma, i.e., the state of deconfinement of superdense or superhot hadronic matter. We shall restrict our attention to hot matter with chemical potential $\mu = 0$. The plasma is then a hot medium in thermodynamic equilibrium, in which massless free quarks (q) and gluons (g) move against the background of the perturbative vacuum. The equation of state of this matter[14-16] as

$$\begin{aligned} &\varepsilon = \varkappa T^4 + |\varepsilon_V|,\\ &p_0 = \frac{1}{3}(\varepsilon - 4|\varepsilon_V|),\\ &\varkappa \equiv \frac{\pi^2}{3^{\prime}}\left(G_g + G_q N_f \frac{7}{8}\right), \end{aligned} \tag{1.1}$$

where G_q, G_g are the numbers of degrees of freedom and N_f is the number of quark flavors. For the color group SU(3) with two light quarks (u and d), we have $\varkappa = 12.2$.

From the point of view of vacuum structure, transition to this state is interpreted as the breaking up of the vacuum condensates responsible for the quark masses (i.e., the breaking of chiral symmetry) and confinement.[10] Cooling down to a certain temperature T_C results in the reconstitu-

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tion of the condensates, the quarks and gluons combine into colorless formations, i.e., hadrons, and we have a phase transition to the confinement state.

Hydrodynamic theory is the only theory in which cooling of hot matter is considered. This means that hydrodynamic theory provides a natural framework for the description of the plasma phase and of the phase transition.

It is useful to note one further circumstance that has stimulated the development of the hydrodynamic approach to phase transitions in plasma. It involves the experiments on collisions between heavy nuclei, planned for the near future. These experiments are expected to provide extensive data on the plasma phase and on transient processes. Although the validity of the hydrodynamic theory in relation to hadronic collisions is still being discussed[3)] (Ref. 4), the description of such complicated systems as colliding nuclei is naturally carried out within the framework of the continuum formalism.[18] The hydrodynamic process consists of three stages.

1. *Formation of the initial state.* It is assumed[1-3] that this is a very hot state in thermodynamic equilibrium. Its lateral dimensions are of the order of the dimensions of the hadron ($R \sim 1$ fm), and it is highly compressed in the longitudinal direction $\Delta \ll R$.

2. *Isentropic expansion, described by the equations of an ideal relativistic fluid.* This occurs preferentially in the longitudinal direction[2] (one-dimensional flow), because the initial state is compressed. The expansion regime depends on the equation of state. Hadronic matter is usually described by the equation of state of the ultrarelativistic hadronic (pionic) gas

$$p = \frac{1}{3}\varepsilon = \frac{1}{3}\varkappa_\pi T^4, \quad \varkappa_\pi = 1. \tag{1.2}$$

A phase transition is introduced into the plasma model when the equation of state has a discontinuity.

3. *Termination of interaction and free motion apart of hadrons as the system cools to the temperature* $T \approx m_\pi = 140$ MeV.

Several questions are being discussed. First, there is the mechanism responsible for the formation of the initial state as a result of collisions. It is clear that theoretical information on the fraction of energy remaining in the thermalized blob, the distribution of velocity in it, and its relative compression can be obtained only by considering a specific model.

Second, the mechanism responsible for the thermalization of the initial state is still unclear. It was assumed in setting up the hydrodynamic theory that shock waves thermalize the system instantaneously,[2] but this hypothesis requires confirmation. Since the advent of the theory of dynamic chaos[19] it has become clear that stochastization is a consequence of the development of the global instability of systems. This instability arises under certain conditions and in a finite time, both of which are also model-dependent.

Finally, it is still not entirely clear what is the effect of a phase transition during hydrodynamic expansion, although substantial advances have been made in this direction.[20-27] This problem has recently attracted considerable attention among theoretical groups in connection with searches for plasma signals, including phase-transition signals. Many experimental characteristics are sensitive to the transition stage, but information relating to high-temperature plasma may be "erased" during the phase transition. The distribution of secondary particles over transverse momenta and multiplicity[25,27] are the most informative in this respect.

The aim of this review is to discuss existing theoretical data on the nature of the phase transition (Section 3) and to examine possible scenarios for its realization during hydrodynamic expansion (Section 4). We shall confine our attention to a qualitative description of the hydrodynamic picture in different scenarios (detailed quantitative descriptions would be too laborious).

Before we proceed to the phase transition themselves, it will be useful to examine the most widely used models of the hadron and of the hydrodynamic state that arises as a result of collisions. This is done in Section 2.

2. MODELS OF HADRONIC COLLISIONS

2.1 Perturbative QCD model[28]

A moving hadron constitutes a current of valence quarks (most energetic, or hard) and a sea of virtual quarks and gluons (constituting the so-called soft component). Hard and soft components exhibit different interactions during collisions between hadrons. Hard components pass through without intensive interaction and form the fragmentation region. Soft components (mostly gluons) interact strongly and multiply; this creates slow (in the center of mass system) and thermalized particles that form the central region. During this process, hard components retain their connection with the central region by interacting with their soft "tails". In the end, the creation of excited and thermalized matter extends over the region occupied by gluon clouds and the trail of the hard component. This picture corresponds to the so-called scaling of the initial distribution in which all thermodynamic variables depend on only the proper (for a given element) time $\tau = (t^2 - z^2)^{1/2}$, where z is the position coordinate along the collisions axis and the velocity of the elements is $v = z/t$. The scaling regime for isotropic expansion is then found to have the following very simple form (independent of the equation of state):

$$S = S_{\text{in}}\tau_{\text{in}}\tau^{-1}, \tag{2.1}$$

where in refers to the initial state and τ_{in} is in general an undetermined parameter that depends on the thermalization mechanism and thermalization time. It is assumed in this model that thermalization occurs as a result of the multiple interaction of soft gluons (although the quantum-field mechanism of the process is not entirely clear).[4)] In the language of the theory of dynamic chaos, this can be formulated as follows. Evaluation of Feynman diagrams (including diagrams with complex topology) leads to results that are unstable under small variations of parameters. The system then ceases to be pure in the quantum-mechanical sense, and becomes stochastic.[29] While this is happening, entropy increases to its equilibrium value. The time taken to reach equilibrium is determined by an increase in instability. This problem has not been correctly solved, but the scenario seems realistic. In the absence of data on the relaxation time, it has usually been assumed[28] that $\tau_{\text{in}} \simeq 1$ fm. However, it is more natural to suppose that τ_{in} is of the order of the mean free path λ, which in turn is of the order of the reciprocal of temperatue:[25] $\lambda \sim T_{\text{in}}^{-1}$. Finally, it is assumed that the fraction of energy transferred to the thermalized subsystem is

$\sim 1/2$. It is difficult to provide a more accurate estimate in this model.[30]

2.2. Phenomenological models (bags[31,32] and strings[33])

In the MIT bag model,[31] the hadron is a bag filled with perturbative vacuum (with quarks moving against this background) and located in the physical vacuum. The mass of a bag is given by

$$m = (\varepsilon_T + \varepsilon_U + B)\, v, \tag{2.2}$$

where B is the bag constant, v is its volume, and ε_T, ε_U are the thermal and potential energy densities. The question of the medium that carries the positive vacuum energy is usually not discussed. It is important to note that a satisfactory description of the spectra of nucleons and resonances is achieved by assuming that $B \approx 0.13\,|\varepsilon_V|$, which is lower by almost an order of magnitude than the energy density of the observable (condensed) vacuum. This suggests that the vacuum inside the bag cannot be regarded as empty: it must have the properties of a continuous medium.

The hot bag ($\varepsilon_T \gg \varepsilon_U$) is filled[34] with almost-free equilibrium quarks and gluons, i.e., it actually contains the plasma state. The volume of a stable bag and its mass are related by $v = m/4B$, which corresponds to zero pressure in the bag (the negative vacuum pressure is compensated by the thermal pressure) and $\varepsilon_T = 3B$. Thus, in the approximation that is sufficient for the hydrodynamic description, hadrons may be looked upon as hard spheres of diameter R, filled with matter whose equation of state is $\varepsilon = \text{const} = 4B, p = 0$.

Central collisions between such objects have been examined in the Landau model (see Ref. 1). Let us recall it.

When the hadron walls collide in the center of mass system, two shock waves are produced and propagate in opposite directions with velocity 1/3 along the collision axis. They are accompanied by the formation of thermalized matter at rest, which is interpreted as the initial state. The longitudinal size of the system is $\Delta_0 = R\gamma^{-1}$, where $\gamma = E_{\rm CM}/2m_{\rm h}$ is the Lorentz factor of the colliding hadrons and the time taken by the process is $T_0 = 3\Delta_0$[5]. It is assumed that the entire energy of the system is released in the form of heat. The initial temperature is then given by $T_{\rm in} \sim m_\pi \gamma^{1/2}$, and the size of the system turns out to be less than the mean free path[6].

The law of expansion in the central region is close to the scaling law. The difference between them is that all the thermodynamics variables depend on the rapidity $y = \frac{1}{2} \ln[(t+z)/(t-z)]$ ("bell jar" rather than a "plateau"). This picture contains a single hypothetical element, namely, instantaneous thermalization in the shock wave, and it is precisely this point that has been criticized. Thus, the width of the shock wave front (just like the thermalization time $t_{\rm st}$) is of the order of the reciprocal temperature. For times $t < t_{\rm st}$, the system evolves dynamically like elastically compressed matter with the equation of state[36] $\varepsilon = p$. Thermalization of the compressed state can occur as a result of the formation and interaction of phonon-type elementary excitations. We are essentially dealing here with thermalization due to the global Kolmogorov-Sinaĭ instability.[19,35] The time scale for the establishment of the equilibrium state is $t_{\rm st} \sim T_{\rm in}^{-1} \propto \gamma^{-1/3}\,{\rm fm} > t_0$.

In the string model,[33] which is conceptually not very different from the bag model, the hadron is looked upon as a set of quarks held together by gluon tubes in physical vacuum. Collisions between such objects present a somewhat different picture than in the bag model: the strings become tangled up and stretched, and break off. Quarks detached in this way form the fragmentation region, and the remaining stretched string can have one of two possible fates:

(a) A single stretched string is likely to be formed in collisions between hadrons with a large impact parameter. A few growing inhomogeneities are formed on the string as a result of its relaxation (in classical physics, an example of this evolution is provided by the formation of dissipative structures in nonequilibrium systems[37]). They then evolve and decay independently of one another.

(b) In the case of collisions between nuclei, the stretched strings merge into a single (thick) stretched rod. The rod is stable against the formation of small-scale inhomogeneities. Its principal mode of relaxation is contraction as a whole. The process subsequently develops as in the bag model.

2.3 "Classical models"[8,38]

By classical models we mean the interpretation of the hadron as a self-localized stationary solution of the classical nonlinear field equations. Such models are significantly different from phenomenological models although they lead to a similar picture, namely, the spherical solution corresponds to a bag and the cylindrically symmetric solution corresponds to a string. Self-localized solutions have finite energy that is naturally associated with the mass $m_{\rm h}$ of the particle. Physical fields are concentrated in the region of size R, and fall off exponentially outside this region, i.e., confinement follows from the very existence of the solution. The parameters of this particle-like solution ($m_{\rm h}$, R, and the diffuseness of the boundary) depend only on the parameters of the Lagrangian and on the requirement that the solution exists (there is a discrete set of such parameters).

An example of a spherical solution of this type for the QCD Lagrangian with the Higgs field is provided by the 't Hooft-Polyakov monopole.[39] We note that its existence and stability are due to the existence of topological charge and the nontrivial structure of vacuum outside the monopole.

The hadron does not have such a ("monopole") charge, but it can be assigned a topological number that is associated with the behavior of the homoclinic trajectory describing the hadron in phase space. When this trajectory has a node, the corresponding solution is stable and does not spread out in space. This situation is encountered in the description of nerve impulses.[37] The approach has not as yet been implemented in QCD gauge theory. However, solutions of this type do exist in chiral models ("skyrmions"[38]) and in gauge models operating with effective (model) Lagrangians. An example of a solution of this type is provided by the "field-theoretic" bag.[8] We note (and this will be useful below) that the solution then has a relatively sharp boundary which enables us to introduce a "surface tension" energy for the bag. The collision between such objects is a purely dynamic process, and the state that appears as a result of the collision must also follow from the solution of the corresponding (nonlinear) equations. However, the stochastization of this state due to its instability under small perturbations is possible, and likely. Fractal structures[35] can also be formed. It is important to note that the characteristic stochastization time is then determined by the parameters of the equations, and need not be of the order of the mean free path,[7)] i.e., $t_{st} > T_{in}^{-1}$.

It follows from the foregoing analysis that, for a wide class of models,[8)] collisions between hadrons lead to the formation of homogeneous compressed thermalized matter whose lateral dimensions and evolution times are related by

$$\Delta \sim t_{st} \backsim \tau_{in} \backsim K T_{in}^{-1}, \tag{2.3}$$

where K is a numerical factor of the order of unity.

The initial distribution of velocity, entropy, and so on, is model-dependent, so that the experimental consequences of the two models are in general different.

We note that the phase transition process is not very sensitive to the initial state. We shall show later that analysis of transition kinetics in different scenarios requires only information on the rate of cooling, which is already present in (2.3). Since (2.3) is approximately valid for all models, the phase transition picture is model-independent. It follows that, to obtain a qualitative description of the phase transition, it is sufficient to approximate the cooling process by the simplest scaling law (2.1).

3. PHASE TRANSITION IN THE QUARK-GLUON PLASMA

Theoretical analyses of the plasma $\leftrightarrow$ hadrons phase transition process has evolved mostly along three directions.

First, there is the so-called QCD lattice formulation.[40] This has resulted in substantial advances in the study of QCD effects at finite (high) temperatures. A data base has accumulated for pure gauge SU(N) theories (without quarks). Let us briefly summarize the main results.[41] At the temperature $T_C = 200 \pm 50$ MeV, matter undergoes a phase transition with respect to confinement, where in SU(3) this is a first-order phase transition (metastable states have also been observed[42]). When $T > T_C$, the thermodynamic plasma parameters assume asymptotic values (to within perturbative corrections[16]) that are also satisfactorily reproduced by computer experiments (Fig. 1).

The energy density falls sharply for $T < T_C$. Analysis of vacuum effects shows that the quantity $(1/4)(\varepsilon - 3p) = |\varepsilon_V| \to \mathrm{const}\,(T)$ is equal to 0.5–1 GeV/fm^3, which is in agreement with theoretical ideas.

Recently, data for SU(N) theories with light (dynamic) quarks have also become available, but they are only preliminary and mutually contradictory. The character (type) of the phase transition remains unclear (a more detailed analysis of the situation can be found, for example, in Ref. 41). For the moment, the results presented above must serve as reference data for phase-transition models.

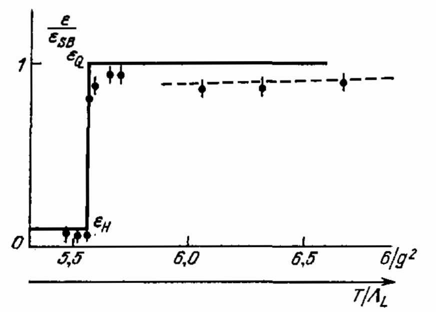

FIG. 1. Dependence of the energy density ε in SU(3) [in units of the asymptotic value $\varepsilon_{SB}(T^4)$] on $\beta^{-1} = 6g^2 \leftrightarrow T/\Lambda_L$ (taken from Ref. 42). Broken line represents the perturbative evaluation of $\varepsilon(T, g^2)$; solid line corresponds to the "ideal" ε/T^4. ε_{QH}, and ε_H are the critical values of the energy density in plasma and the hadronic phase, respectively.

The second direction involves studies of nonperturbative QCD effects in vacuum polarization and screening.[43,44] We cannot pause to examine these problems in detail here (see, for example, Ref. 45).

Finally, the third direction involves phenomenological models of the phase transition. We shall now examine in detail the bag model because it is closer than other models to the hydrodynamic treatment of the phase transition.

3.1. Elements of the general theory of phase transitions

The character of any process involving a change of phase is determined by two factors,[46] namely, (a) the equilibrium parameters of the phases, i.e., the choice of the stationary state and (b) the way the new equilibrium is approached.

The first problem is purely thermodynamic and involves a choice of the most probable state (under the given conditions). The second problem involves more detailed properties of the system (in general, microscopic properties) and is treated in kinetics. Despite the different scales of the two problems, they must be solved (or at least posed) simultaneously, since otherwise much of the information invested in the formalism of thermodynamics is lost.

Let us examine this in greater detail by considering a system occupying a given volume V; the temperture T is controlled from outside (extensive parameters)[9)]. The thermodynamic description of the system is then based on the partition function

$$Z(T, V) = \int dr e^{-\beta \mathscr{H}(r)}, \quad \beta \equiv T^{-1}, \tag{3.1}$$

where $\mathscr{H}(r)$ is the Hamiltonian and the integral is evaluated over the entire phase space. We then have

$$\begin{aligned} &F(T, V) = -T \ln Z(T, V) - \text{free energy},\\ &\varepsilon = -\lim_{V\to\infty} \frac{1}{V} \frac{\partial}{\partial \beta} \ln Z(T, V) - \text{energy density},\\ &p = \lim_{V\to\infty} \frac{T}{V} \ln Z(T, V) - \text{pressure}. \end{aligned} \tag{3.2}$$

On the other hand, the theory of phase transitions frequently employs the partition function density $Z(T, V; \chi)$ which involves the order parameter χ such that

$$\begin{aligned} Z(T, V) &\equiv \int d\chi \tilde{Z}(T, V; \chi) \equiv \int e^{-\beta\mathscr{H}} d\chi\, dr\, \delta(r - r(\chi))\\ &= \exp(-\beta F_{\min}(\tilde{\chi})) \int d\chi \exp[-\beta(F(\chi) - F_{\min}(\tilde{\chi}))]. \end{aligned} \tag{3.3}$$

Since F is additive, the argument of the exponential under the integral sign is proportional to $V(\chi - \tilde{\chi})^2$:

$$F(T, V) = -(\lim_{V\to\infty} T) \ln Z(T, V) = F(\tilde{\chi})(1 + O(V^{-1} \ln V)). \tag{3.4}$$

The equilibrium value of $\tilde{\chi}$ corresponds to a minimum of free energy, and is the only one to survive as $V \to \infty$.

The quantity $\tilde{Z}(T, V; \chi)$ plays a fundamental part when $F(\chi)$ has several extrema of which one corresponds to true

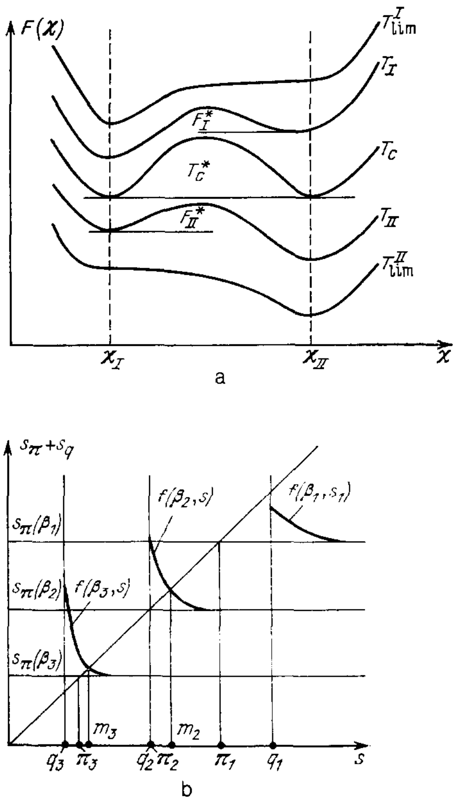

FIG. 2. a—Dependence of the free energy density $F(T,\chi)$ on the order parameter χ for different temperatures; $T^{\rm II}_{\rm lim}$—thermodynamic limit of existence of phase II, $T_{\rm C}$—equilibrium transition temperature, $T^{\rm I}_{\rm lim}$—thermodynamic limit of existence of phase I. b—Singularities of the partition function $\hat{Z}(T,s)$ for different temperatures: $T_1 > T_{\rm C}$, $T_2 = T'_{\rm C}$, $T_3 < T_{\rm C}$; $\beta \equiv T^{-1}$

equilibrium and the others to local equilibrium or to metastable states. It is precisely this situation that corresponds to the phase transition. Figure 2a shows $F(T, V; \chi)$ plotted as a function of χ for different temperatures, typical for first-order transitions. The temperature $T_{\rm C}$ corresponds to the phase transition, i.e., to the coexistence of phases. For temperatures $T_{\rm C} < T < T^{\rm I}_{\rm lim}$, state I is stable, but state II may be metastable. For temperatures $T > T^{\rm II}_{\rm lim}$ state II is absent. The condition $F_{\rm II} \geqslant F_{\rm I}$ is therefore necessary, but not sufficient, for the I→II transition, and further factors, sufficient to overcome the energy barrier are required. On the other hand, when there is no barrier, or when $\chi_{\rm I}$ and $\chi_{\rm II}$ approach one another and merge, the sufficient condition for the change of phase is also a necessary condition. This situation corresponds to a phase transition of order 2 or higher. Kinetics has no independent significance in this case, in the sense that it determines only the characteristics of the phase transition, and not the fact that it actually occurs.

The following remark will be useful at this point. The quantity χ can be taken to be any state variable (or set of variables), but it is desirable for relaxation in χ to be slower than in all other (integrated) variables. The choice of χ is thus dictated by kinetic considerations. We are essentially dealing with the problem of selecting the main variable or, equivalently, the process responsible for the particular effect. The correct procedure is known in mathematics as the Tikhonov theorem.[47]

The barrier in F determines the lifetime of the metastable state: $\tau \infty T^{-1} \exp(\beta F^*)$. When χ is chosen so that the spatial homogeneity of the system is unaffected by the transition, then $F^* \infty V$ is very large and a simultaneous first-order transition throughout the system us unlikely. The physical picture of the transition involves the evolution of a focus (bubble) of a new phase inside the old phase. The quantity F^* is then the work done in producing this bubble, and remains finite in the thermodynamic limit as $V \to \infty$.

We have examined the question of the order parameter in some detail because most papers devoted to the plasma ↔ hadrons phase transitions do not analyze the partition function density in relation to this parameter.

3.2. Phase transition in a gas of bags

Among semiphenomenological models, the bag gas has the distinctive feature that the plasma phase (a gas of free massless quarks and gluons) is introduced into the model *a priori*. Because of this, there is no chirality phase transition, and the transition to the deconfinement phase constitutes a growth of the individual bags, their coalescence, and the eventual emergence of a single bag occupying the entire accessible volume. This is most clearly implemented in Refs. 48–51 (see also Refs. 52 and 53).

The partition function in this case is taken to be

$$Z(T, V) = \sum_{N=1}^{\infty} \frac{1}{N!} \int_0^{\infty} \prod_{j=1}^{N} \left\{ \frac{d^3 p_j}{(2\pi)^3}\, dm_j\, dv_j\, \rho(m_j, v_j) \times \exp\left[\beta (p^2 + m^2)^{1/2}\right] \right\} \times \left(V - \sum_{j=1}^{N} v_j\right) \Theta\left(V - \sum_{j=1}^{N} v_j\right), \qquad (3.5)$$

where N is the number of "objects" in the system, m_j are their masses, v_j their volumes, and $\mathbf{p}_j$ their momenta. The quantity ρ represents the spectrum or the partition function of one "object". It is taken in the form

$$\rho(m, v) = \frac{\pi^2}{3} \delta(m - m_\pi) \delta(v - v_\pi) + \rho_{\rm B}(m, v). \qquad (3.6)$$

where the first term corresponds to pions; the factor $\pi^{2/3}$ represents the pion charge state. At high temperatures, for which $T > m_\pi$, we can put $m_\pi = 0$, $v_\pi = 0$.

The second term, $\rho_{\rm B}$, represents the bags. The spectrum of a bag,[34] modified to represent unstable (inflating) bags, is given by

$$\rho_{\rm B}(m, v) = A\pi^2 v^{-3-\alpha} \left(\frac{m}{v} - B\right)^{-7/4} \Phi(v) \times \exp\left\{\frac{4}{3} [\varkappa (m - Bv)^3 v]^{1/4}\right\} \qquad (3.7)$$

where A is a numerical factor close to unity, $m - Bv = \varepsilon_{\rm T} v$ is the internal (thermal) energy of a bag [see (2.2)], and α is a model parameter ($\alpha = 0$ in Ref. 34). The factor $\Phi(v)$ decreases with decreasing volume, and can be approximately represented by $\Phi(v) = \Theta(v - V_0)$ where V_0 is the minimum possible volume of the bag (but still sufficiently large).

The expression given by (3.5) becomes very much simpler if we neglect the excluded volume (i.e., assume that $\sum_j v_j \ll V$). The partition function can then be factored and takes the form

$$Z(T, V) = \exp(V\varphi(\beta)), \tag{3.8}$$

where

$$\varphi(\beta) = \int \frac{d^3p}{(2\pi)^3} dm\, dv \rho(m, v) \exp[-\beta(\mathbf{p}^2 + m^2)^{1/2}]$$

$$\equiv \varphi_\pi(\beta) + \varphi_B(\beta).$$

$\varphi_\pi(\beta) = \frac{1}{3}\beta^{-3}$ — the contribution of the π-gas to pressure,

$$\varphi_\pi(\beta) \equiv s_\pi, \tag{3.9}$$

$$\varphi_B(\beta) = \frac{1}{3}\int_{V_0}^{\infty} dv v^{-3-\alpha}\tilde{\varphi}(\beta)\exp(s_q(\beta)v),$$

$$\tilde{\varphi}(\beta) \equiv \beta^3\left(\frac{\varkappa}{\beta^4} + B\right)^{3/2},$$

and

$$s_q \equiv \frac{\varkappa}{3\beta^3} - B\beta = \frac{1}{T}\left(\frac{1}{3}\varkappa T^4 - B\right), \tag{3.10}$$

where the last expression represents the pressure in the bag, written in terms of temperature, and $\varepsilon = (\varkappa\beta^{-4} + B)$ is the energy density in the bag. The functions $\varepsilon(T)$, $p(T)$ reproduce the plasma equation of the state: $p = (\varepsilon - 4B)/3$. At low temperatures $T < T_0 \equiv (3B/\varkappa)^{1/4}$ (such that $s_q < 0$) $\varphi_B(\beta, V_0)$ is exponentially small), the equation of state has a form typical for a pion gas: $p = 1/3$, $\varepsilon = (1/3)T^4$. When $T > T_0$, the integral in (3.9) is found to diverge, and this is the basis for Hagedorn's idea[52] of a "limiting" temperature. However, the divergence is removed when the factor $(V - \sum_j v_j)$ is introduced into (3.5).

An elegant technique that allows us to take into account the excluded volume (the so-called isobaric ensemble formalism) is developed in Refs. 50 and 51. The function $Z(T, V)$ is written as an integral over the complex variable s:

$$Z(T, V) = \frac{1}{2\pi i}\int_{s-i\infty}^{s+i\infty} ds\, e^{sV} Z(T, s), \tag{3.11}$$

where the contour of integration lies to the right of the singularities $\hat{Z}(T, s)$, and

$$\hat{Z}(T, s) \equiv \int_0^{\infty} dV e^{-sV} Z(T, V) \tag{3.12}$$

is the Laplace transform of the partition function.

The positions of the singularities of $\hat{Z}(T, s)$ depend on temperature (by analogy with moving poles in the Regge formalism[3]). As $V \to \infty$, only the right hand (leading) singularity contributes to (3.11). Phase transition is interpreted as the collision of singularities (pinch effect) at the temperature $T = T_C$, which is looked upon as the transition temperature.

The function $\hat{Z}(T, s)$ is reduced to the form

$$\hat{Z}(T, s) = (s - f(\beta, s))^{-1}, \tag{3.13}$$

where

$$f(\beta, s) = \int \frac{d^3p}{(2\pi)^3} dm\, dv \rho(m, v) \exp[-\beta(\mathbf{p}^2 + m^2)^{1/2} - sv]$$

$$\equiv f_\pi(\beta, s) + f_B(\beta, s);$$

$$f_B(\beta, s) = \tilde{A}\varphi(\beta)\int_{V_0}^{\infty} dv v^{-1-\alpha}\exp[-v(s - s_q)], \tag{3.14}$$

$$f_\pi(\beta, s) = \frac{1}{3}\beta^{-3}\int_0^{\infty} dv\, \delta(v - v_\pi) e^{-sv} \approx \frac{1}{3}\beta^{-3} \tag{3.15}$$

[practically identical with s_π; see (3.9)].

The expression given by (3.13) contains two singularities, namely, a pole and a singularity of the form $f_B(\beta, s) \propto (s - s_q)^\alpha \Gamma(V_0(s - s_q), \alpha)$ at $s = s_q$. The pole (we shall denote it by s_m) is a root of the equation $s_m - f(\beta, s_m) = 0$.

When $\alpha \leqslant 0$, the pole always lies to the right of the singularity and, strictly speaking, there is no phase transition. When $\alpha > 0$, the function $f_B(\beta, s)$ is finite at $s = s_q$, and phase transition is possible. Figure 2b shows the graphical solution of the equation $s_m - f(\beta, s_m) = 0$ for the three temperatures $T_1 > T_C$, $T_2 = T'_C$ and $T_3 < T_C$.

For $T = T_1$, the leading singularity occurs at $s = s_q$ and corresponds to plasma. For $T = T_C$, the pole and the branch point coincide (not shown in the figure). The precise value of T_C is set by the equation

$$s_q(T_C) - s_\pi(T_C) = f_B(T_C, s_q(T_C)) = \int_{V_0}^{\infty} \frac{dv}{v^{1+\alpha}}. \tag{3.16}$$

For large V_0 and $\alpha > 0$, we have $f_B(T_C, s_q(T_C)) \to 0$, i.e., $T'_C \equiv [3B/(\varkappa - 1)]^{1/4}$ is a good approximation to T_C. When $T_2 = T'_C$, the pole at $s = s_m(\beta)$ lies to the right of $s_q(\beta)$ and $s_\pi(\beta)$. When $T_3 < T_C$, the pole s_m approaches $s_\pi(\beta)$, but always remains greater than the latter.

We can now use (3.5) to analyze the partition function density for its dependence on the order parameter χ, which is conveniently taken to be the ratio of the bag volume v to the total volume V $(\chi = v/V)$. The partition function density can be written in the form

$$\begin{aligned}\tilde{Z}(T, V; \chi) &= \sum_{N=1}^{\infty}\frac{1}{N!}\sum_{k=1}^{N}\frac{1}{k}\int_{V_0}^{\infty}\prod_{j\neq k}\hat{\rho}(v_j)\left(V - \sum_{j=1}^{N} v_j\right)\\ &\times\Theta\left(V - \sum_{j=1}^{N} v_j\right) dv_j \hat{\rho}(v_k)\left(V - \sum_{j\neq k}^{N} v_j - v_k\right)\\ &\approx \sum_{N}^{\infty}\frac{1}{N!}\sum_{k=1}^{N}\frac{V}{N}\int dv_j\prod_{j\neq k}\hat{\rho}(v_j)\left(V - \sum v_j\right)\\ &\times\ \hat{\rho}(V\chi)\left(1 - \sum_{j=1}^{N-1}\chi\right),\end{aligned} \tag{3.17}$$

where

$$\hat{\rho}(v) \equiv s_\pi\delta(v - v_\pi) + \tilde{\varphi}(\beta) v^{-1-\alpha}\Theta(v - V_0) e^{s_q v}. \tag{3.18}$$

For small χ, a large number of the objects contribute significantly; the density $Z(\chi)$ decreases with increasing χ (Ref. 51), and $\sum_{j=1}^{N-1}\chi_j \to 0$. For large χ, the principal contribution to the partition function is due to the $N = 1$ term, and $\sum_{j=1}^{N-1}\chi_j = 0$. Thus, approximate information about the nature of the function can be obtained by analyzing the factor

$$\tilde{\rho}(V\chi)(1-\chi)$$

$$= [s_\pi\delta(\chi) + \tilde{\varphi}(\beta)\chi^{-1-\alpha}\theta(\chi - \chi_0) e^{s_q\chi V}](1-\chi). \tag{3.19}$$

which corresponds to the contribution of one bag of arbitrary volume. When $T \gtrsim = T_C$, there are two maxima, namely, one for $\chi \to 1$, which corresponds to pure plasma, and the other for $\omega \simeq \chi_\pi$, which corresponds to the pionic fluid.

The minimum of (3.19) as a function of χ occurs at $\chi = \chi_{\min} \approx (1+\alpha)\ (s_q\ V)^{-1}$. At the temperature T_0 (for which $s_q = 0$), we have $\chi_{\min} \to 1$, and the two extrema coalesce and vanish. All that remains is the extremum corresponding to the hadronic fluid. The temperature T_0 is the analog of $T^{\mathrm{II}}_{\lim}$ of Fig. 2a. For $T \approx T_C\{1 + [(1+\alpha)/(\varkappa \chi_0^3]\}^{1/4} \equiv T^{\mathrm{I}}_{\lim}$ we have $\chi_{\min} = \chi_\pi$, i.e., the maximum that corresponds to the hadronic fluid vanishes in the range $\chi_\pi < \chi < 1$. This temperature is the analog of $T^{\mathrm{I}}_{\lim}$ of Fig. 2a. The maxima of the partition function density in v correspond to minima of free energy, and the barrier between them corresponds to the minimum of $\widetilde{Z}\ (\chi)$. The presence of these extrema at $T \simeq T_C$ indicates that we are dealing with a first-order transition. To conclude this Section, we note the following points.

(a) The excluded volume and the unstable bags ($u \neq m/4B$) are fundamental to the model.[51] A phase transition cannot be described in a self-consistent manner without them.

(b) For temperatures $T \lesssim T_C$, for which the leading singularity is still the pole $s_m\ (\beta)$, the system is actually in a mixed phase in which bags and pions exist in dynamic equilibrium. The bags "breathe" (some contract, others expand); this ensures that the elasticity and, correspondingly, the velocity of sound, are small in this state ($c_s \to 0$). The equation of state of matter is

$$p = \beta^{-1} s_m (\beta) \approx p_C, \quad c_S^2 \equiv \frac{\partial p}{\partial \varepsilon} \to 0,$$
$$\varepsilon_\pi < \varepsilon = -\frac{\partial}{\partial \beta} s_m (\beta) < \varepsilon_q, \tag{3.20}$$

which is qualitatively different from the case of "pure" phases.

(c) The above approach is correct in the thermodynamic limit as $V \to \infty$. Important effects such as, for example, the existence of metastable and "pure" states are then no longer relevant. Actually, in an infinitely large system, the probability that a focus of extraneous phase will not appear anywhere is exceedingly small. Pure states must therefore provide a small contribution to the total partition function. In reality, the volume of the system is finite, or even not very large, during the hydrodynamic expansion stage. The probability that a pure state will arise in this case is then not small. Moreover, it depends on the kinetics of the process, and may be significantly greater than one would expect from the analysis of the equilibrium partition function. The question whether these effects are taken into account within the framework of the above method requires separate examination. We note, in particular, that the partition function (3.5) contains only simply connected bags, i.e., it does not contain bags with cavities filled with hadronic matter. This "mirror asymmetry" affects the description of the transition in supercooled plasma whose physics involves precisely such objects.

The above procedure can be used to describe the kinetics of metastable states by including in the partition functions not only the leading term, but also terms that vanish in the limit as $V \to \infty$:

$$Z(T, V) = \sum_j c_j (T, V)\, e^{s_j(T) V} \approx c_\pi e^{s_\pi V} + c_q e^{s_q V} + c_m e^{s_m V}, \tag{3.20'}$$

where $c_j\ (T, V)$ are coefficients whose values depend on prehistory and tend to zero as $V \to \infty$.

(d) The MIT bag model does not take into account the bag surface energy (which is not significant for large bags). On the other hand, effects due to the nucleation of a bubble of new phase, which determine the kinetics of metastable states, are due to surface energy. Surface energy has a clear physical significance in "field-theoretic" bag models[8] (interaction of internal fields with nontrivial ambient vacuum) and is given by $\sigma \equiv \omega B^{3/4}$, where ω is of the order of unity (see also Ref. 61).

The expression for the bag mass now assumes the form

$$m = \varepsilon_T v + Bv + \sigma v^{2/3}. \tag{3.21}$$

Changes in the partition function thus reduce to the replacement of $s_q\ v$ with $(s_q\ v - \sigma v^{2/3})$, which does not affect the position of the singularities, but becomes important in two ways. First, the function $f\ (\beta, s)$, remains finite at $s = s_q$, independently of α. The first-order phase transition will therefore take place for any value of α. Second, surface tension affects the position and size of the minimum of the density $\widetilde{Z}\ (\chi)$, and this is accompanied by an appreciable expansion of the temperature range within which metastable states exist.

3.3. Kinetics of phase transitions

We must now examine the realization of any particular state. The result will depend on initial conditions and on kinetics. First, we recall the fundamentals of first-order phase transitions. The transition rate is related to the probability that a focus (or bubble) of a new phase will appear. The free energy, or work for this to happen, is given by

$$A = (p_{\mathrm{I}} - p_{\mathrm{II}})\, \frac{4}{3} \pi R^3 + 4\pi\sigma R^2. \tag{3.22}$$

where R is the radius of the bubble, p_{I} and p_{II} are the pressures corresponding to phases I and II, respectively, σ is the surface energy density, given by $\sigma = \omega B^{3/4}$, and ω will be looked upon as a phenomenological parameter.

When $p_{\mathrm{I}} - p_{\mathrm{II}} \equiv \Delta p > 0$, both terms in (3.21) are positive, and $A \to \infty$ as R increases. This means that phase II is thermodynamically unfavorable, and the bubble collapses.

When $\Delta p < 0$, the quantity A has a maximum at $R = R_{\mathrm{cr}} = 2\sigma/|\Delta p|$. It is readily verified that the state of a bubble of size $R = R_{\mathrm{cr}}$ is unstable: a small increase in the radius leads to a further expansion, whereas a small reduction leads to collapse. The quantity $F^* = A\ (R_{\mathrm{cr}}) = (16\pi/3)\sigma^3/|\Delta\rho|^2$ is the energy of a fluctuation that is necessary and sufficient for a transition from the metastable to the stable state. The probability (per unit time) that this fluctuation will appear is

$$W_A \backsim \tau_T^{-1} e^{-F^*/T}, \tag{3.23}$$

where $\tau_T \sim T^{-1}$ is the thermal fluctuation time. The lifetime of the metastable state (or, equivalently, the time taken to form a transition focus) is

$$\tau_f = W_A^{-1} \backsim T^{-1} e^{F^*/T}. \tag{3.24}$$

In equilibrium, i.e., for $T = T_C$, we have $\Delta p = 0$, and the transition continues for an infinite time if $\omega \neq 0$. When

$\omega = 0$, there is no barrier, and the nucleus forms in a time $\tau_C \sim T_C^{-1}$ even near equilibrium. When the barrier in F^* is not zero, but is small, so that $F^* \sim T$, the lifetime of the metastable formation is of the order of the fluctuation time $\tau_f \sim T_C^{-1}$. This means that the question

$$\frac{16\pi}{3} \frac{\sigma^3}{T\,|\Delta p|^2} = 1 \qquad (3.25)$$

can be looked upon as the condition for a spontaneous phase transition from the metastable to the stable state. The transition itself is explosive in character under these conditions.

This approach is fruitful for the hadrons $\leftrightarrow$ plasma phase transition, despite its schematic nature:[54] important qualitative conclusions can be drawn by introducing a single phenomenological parameter into the theory. The properties of supercooled metastable plasma must then be analyzed in terms of the "mirror" picture (singly-connected bubbles of hadronic phase in plasma), which was discussed above.

Analysis of (3.23)–(3.25) leads to the following picture of the behavior of the limiting temperatures with the parameter $\omega = \sigma/B^{3/4}$ (Fig. 3b).

(a) For $\omega \sim 10^{-1}$ ($\sigma \ll B^{3/4} \to 0$) the spontaneous hadronization temperature is $T^{\mathrm{q}}_{\mathrm{lim}} \approx T_C\ [1 - (\pi/3\omega^2)^{1/2}]$, and the spontaneous ionization temperature is $T^{\mathrm{h}}_{\mathrm{lim}} \approx T_C\ (1+\omega^3)$, i.e., the temperature interval within which supercooled plasma can exist (for a given ω) is wider than the ΔT for superheated hadronic matter.

(b) Negative plasma pressure for $T < 0$ does not in itself ensure spontaneous hadronization when ω is not too low ($\omega \gtrsim 2\times 10^{-1}$).

(c) The value $\omega \equiv \omega^* \sim 0.3$ is critical: there is no spontaneous hadronization temperature $\omega > \omega^*$; supercooled plasma can exist up to $T_q = 0$. This is so because W_A has a maximum as a function of temperature at $T = T^* = 0.57\,T_C$ (see Fig. 3a), and the position of this maximum is independent of ω; the hadronization probability falls for $T_q < T^*$. In the absence of reliable data on ω, we can take $T^* \equiv T^{\mathrm{h}}_{\mathrm{lim}}\ (\omega^*) = 0.57\ T_C$ as a reasonable estimate for the spontaneous hadronization temperature of supercooled plasma.

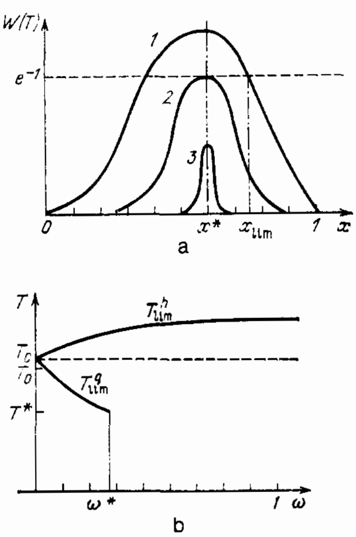

FIG. 3. a—Probability of formation of a nucleus of hadronic phase as a function of the dimensionless temperature $\omega = T_q/T_C$ of supercooled plasma; curves *1–3* correspond to different values of the parameter ω: $\omega_1 < \omega^* < \omega_3$. b—Dependence of the limiting temperature of plasma and of the hadronic phase, $T^{\mathrm{q}}_{\mathrm{lim}}$ and $T^{\mathrm{h}}_{\mathrm{lim}}$, on the parameter ω.

Our discussion thus far leads to the following conclusion: the transition plasma $\leftrightarrow$ hadronic phase is a first-order transition in the bag model.[51] This is indicated by the presence of the different singularities that correspond to different phases, and by the presence of extrema on the partition function as a function of the order parameter. This means that metastable states and the associated hysteresis phenomena can arise in the course of transitions (both forward and reverse).

We now turn to the realization of these phenomena during the hydrodynamic evolution of plasma.

4. HADRONIZATION IN THE HYDRODYNAMIC INTERACTION PICTURE

The possibility of incorporation of phase transitions into the hydrodynamic picture of interactions between ultrarelativistic nuclei (hadrons) is inherent in the formalism of hydrodynamics. Actually, the evolution of hot matter formed as a result of collisions, must satisfy the following conditions[3]:

(1) conservation of the energy-momentum tensor

$$\partial_\mu T^\nu_\mu = 0, \quad T^\nu_\mu = (\varepsilon + p)\, u^\mu u^\nu - p g^{\mu\nu} \quad (\mu,\ \nu = 0,\ 1,\ 2,\ 3), \qquad (4.1)$$

where ε and p are, respectively, the energy density and pressure in the rest frame of the element of matter, and u^μ is the hydrodynamic velocity of the element $\{\gamma;\ \gamma\,\mathbf{v}\}$;

(2) conservation (or nondecrease) of entropy $\hat{S}$

$$\partial_\mu (s u^\mu) \geqslant 0, \quad s \equiv \frac{\varepsilon + p}{T}, \qquad (4.2)$$

where s and T are, respectively, the entropy density and temperature in the rest frame.

To the above set of equations we must add the equation of state of matter, which, in this context, appears as an additional condition or a free parameter. Two-phase matter can be described by introducing a discontinuous equation of state. We know in advance that this leads to the development of discontinuous processes against the background of the state formed during an early stage of hydrodynamic expansion (i.e., well away from the phase transition region). It is convenient to take for this stage the scaling regime of expansion discussed in Section 2. We recall that

$$\frac{s}{s_{\mathrm{in}}} = \left(\frac{T}{T_{\mathrm{in}}}\right)^3 = \frac{\tau_{\mathrm{in}}}{\tau}, \quad \tau_{\mathrm{in}} \approx \frac{1}{T_{\mathrm{in}}}. \qquad (4.3)$$

Before we examine the effect of the two-phase state of matter on this regime, let us consider steady-state transition processes.

4.1. Steady-state theory of combustion of a continuous medium

Steady-state regimes of hydrodynamic evolution of two-phase matter are well-known in the theory of combustion of continuous media.[23,55] Let us illustrate the character of these regimes by a simple one-dimensional model. Consider an element of volume traversed by a phase-transition front that takes the medium from the initial phase i to the final phase f (Fig. 4).

We shall assume that (a) both media are homogeneous,

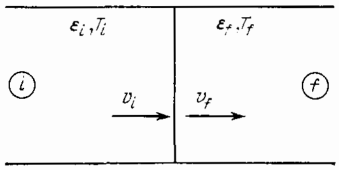

FIG. 4. The front of a model discontinuous process.

(b) the transition front has an infinitesimal thickness, and
(c) all processes are steady-state ones.

The hydrodynamic equations for this process yield the conservation laws for the separation boundary. In the frame in which the boundary is at rest,

$$(\varepsilon+p)_i\,\gamma_i^2 v_i=(\varepsilon+p)_f\,\gamma_f^2 v_f,\qquad (\varepsilon+p)_i\,\gamma_i^2 v_i^2+p_i=(\varepsilon+p)_f\,\gamma_f^2 v_f^2+p_f, \tag{4.4}$$

which is readily solved for the relative velocities of the two flows:

$$v_i^2=\frac{p_i-p_f}{\varepsilon_i-\varepsilon_f}\,\frac{\varepsilon_f+p_i}{\varepsilon_i+p_f},\qquad v_f^2=\frac{p_i-p_f}{\varepsilon_i-\varepsilon_f}\,\frac{\varepsilon_i+p_f}{\varepsilon_f+p_i}. \tag{4.5}$$

The formulas given by (4.5) are valid for any discontinuous process, so long as we have not specified the equation of state. There are, therefore, two distinct cases, namely, $v_i<v_f$ and $v_i>v_f$. The physical difference between them is particularly clear in the rest frame of phase *i*. In the first case, the final phase flows away from the transition front (it is evaporated from the surface), and the process is referred to as *deflagration*. In the second case, the final phase moves in the same direction as the front, and effectively exerts a pressure upon it. This is referred to as *detonation or explosion*.

Turning now to the specific question of hadronization, we consider the equations of state of the ideal plasma (parametrized in accordance with the bag model) and of the ideal gas of the lower hadronic states (pions; see Fig. 4). For phase *i* ($i=q$) we have

$$\varepsilon_q=\varkappa_q T^4+B,\quad p_q=\frac{1}{3}(\varepsilon_q-4B);\quad \varkappa_q\approx 12. \tag{4.6}$$

For phase *h* ($f=h$),

$$\varepsilon_h=T^4\varkappa_\pi,\quad p_h=\frac{1}{3}\varepsilon_h,$$

$$c_S=\frac{1}{\sqrt{3}}\ \text{— velocity of sound},\quad \varkappa_\pi=1.$$

The curves cross when $p_h(T)=p_q(T)$, and the point intersection is interpreted as the equilibrium phase transition temperature

$$T_C=\left(\frac{3}{\varkappa-1}B\right)^{1/4}.$$

Agreement with lattice QCD[41] can be achieved by assuming that $T_C\simeq 200$ MeV and $B\simeq 1$ GeV/fm^3. The expression is given by (4.5) and the condition for increasing entropy now take the form

$$v_q^2=\frac{1}{3}\,\frac{\varepsilon_q-\varepsilon_h-4B}{\varepsilon_q-\varepsilon_h}\,\frac{3\varepsilon_h+\varepsilon_q-4B}{3\varepsilon_q+\varepsilon_h},$$

$$v_h^2=\frac{1}{3}\,\frac{\varepsilon_q-\varepsilon_h-4B}{\varepsilon_q-\varepsilon_h}\,\frac{3\varepsilon_q+\varepsilon_h}{3\varepsilon_h+\varepsilon_q-4B}, \tag{4.7}$$

$$\varepsilon_q-1\leqslant\frac{\varepsilon_h}{\varkappa}\left(\frac{3\varepsilon_q+\varepsilon_h}{3\varepsilon_h+\varepsilon_q-4B}\right)^2.$$

These conditions determine the range of values of ε_q, ε_h for which the process is allowed both thermodynamically ($\Delta\tilde{S}>0$) and kinematically ($0\leqslant v_{q,h}^2\leqslant 1$). The allowed regions are shown shaded in Fig. 5. In one of them, $v_q<v_h$, i.e., deflagration is possible, whereas in the other, $v_q>v_h$, which corresponds to the explosive process. The two regions are separated, i.e., there is a range of ε_q and, correspondingly, of T_q, in which both processes are forbidden. Let us examine them in further detail.

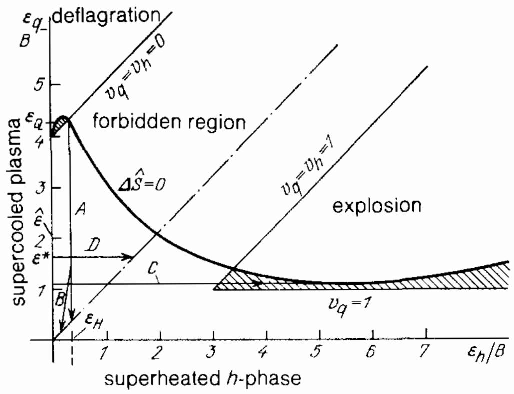

FIG. 5. Allowed phase-transition states. Arrow shows A–D type transitions.

Deflagration is a quasiequilibrium isothermal transition. The range of temperatures T_q (and values of ε_q) in which it is kinematically allowed is very narrow $T_{max}\geqslant T_q\geqslant T_0$, $T_{max}\approx T_C$, $T_0\approx 0.98\,T_C$ (T_0 corresponds to the point $\varepsilon_q=4B$, $\varepsilon_h=0$, and $p_q=0$). The process proceeds very slowly: the transition-front velocity lies in the range $0<v_f<0.04c_S$ [zero velocity corresponds to the dynamic phase equilibrium $p_h(T)=p_q(T)$]. Moreover, deflagration is accompanied by a change of volume: the volume of the hadronic phase exceeds the volume of the plasma by an order of magnitude (by a factor of $\simeq 12$). This means that the deflagration process is possible if the initial plasma element has vacuum adjacent to it (deflagration on the surface of the system). At any rate, supercooling of the plasma down to $T_q<T_0$ prevents deflagration.

Explosion is a nonequilibrium process whereby supercooled ($T_q\leqslant T_q^d=0.53\,T_C$) metastable plasma undergoes a transition to the superheated hadronic phase with $T_h\geqslant T_h^d=1.8\,T_C$. The transition front moves very rapidly: $v_f\simeq 1$. In the system in which the plasma is at rest, the hadronic phase moves in the same direction as the phase separation boundary. As a result, the hadronic phase presses against the boundary, and its volume is significantly smaller than the initial volume:

$$\frac{V_h}{V_q}=\frac{4(\varepsilon_q-B)}{3\varepsilon_h+\varepsilon_q-4B}\leqslant\frac{2}{\varkappa}.$$

The range of values of ε_q, ε_h that are allowed in the case of explosion is much wider than for deflagration. The point $\varepsilon_q\simeq 1.2B$, $\varepsilon_h\simeq 6.5B$ is a special one and corresponds to the Jouguet condition $v_h=c_S$. When the Jouguet condition is satisfied, the process is an "evolution" according to Landau.[55]

It is clear from the above analysis that both stationary regimes are possible not only in bounded intervals of ε_h and ε_q, but also under strictly defined kinematic conditions. In particular, they cannot develop in the longitudinal direction of plasma expansion, since the velocity of the cooling front ($T = \text{const}$ surfaces) relative to the incident flow is not equal to the velocity of the discontinuity under scaling conditions, either in deflagration or in the explosive state. The cooling front moves faster than light because cooling of the individual elements is causally independent. In principle, the evolution of stationary explosive processes in the lateral direction is allowed, but it must be considered against the background of longitudinal expansion, i.e., under inhomogeneous and nonstationary conditions. It is not clear *a priori* whether deflagration, which is essentially a slow surface phenomenon, can play a part in the hadronization of the system when

(A) the system is small and volume effects are comparable with surface effects or

(B) the cooling process is much slower than the hadronization process, $\tau_h \ll \tau_{exp}$, i.e., it may be considered that the phase transition occurs under equilibrium (isothermal) conditions.

On the other hand, the explosive process is possible if cooling occurs much more rapidly than the evolution of the explosive focus. There are also two further possibilities:

(C) the time for the explosive focus to evolve is relatively long for any $T_q < T_C$ (down to $T_q = 0$); the number of individual foci should then be small and effects associated with their growth should not overlap, and

(D) the probability that a focus will form depends on T_q in such a way that it has a sharp maximum for a certain $T_q = T^*$; a stationary collective process is then unlikely; hadronic flows arising from different foci mix and produce effective boiling of the entire plasma volume.

Thus, analysis of the above simple model problem enables us to identify four possible phase transition scenarios based on hypotheses (A)–(D). The realization of these hypotheses will be discussed later. For the moment, we note that the replacement of the above (ideal) equation of state with more realistic equations, i.e., equations that take into account interactions in the plasma and the spectrum of higher hadronic states,[16] will affect the structure of the diagram, but only qualitatively; the conclusion that there are two different regimes, i.e., "slow" and explosive, is valid for any matter.

It is also important to note that the abundance of possibilities (A)–(D) is due to the assumed possibility of plasma supercooling, i.e., the idea of the barrier-type phase transition.

4.2. Hadronization scenarios

A. Hadronization by deflagration was proposed in Ref. 27 and, later, by van Hove within the framework of the string model.[22] Since this is a surface phenomenon, it can occur under special initial conditions (when surface effects predominate). This situation arises if, as a result of collision, the strings connecting quarks in hadrons become tangled, and a loose ball of string is produced. In the course of expansion, the current tubes are compressed and stretch, and the vacuum islands between them expand, i.e., the medium takes the form of a net of strings. In the critical region $T \sim T_C$, string tension prevents further expansion and ruptures the individual strings. These two factors ensure that the plasma splits into a number of fine ($R \sim 1 - n$ fm) drops that have different rapidities ($\delta y \sim 1$) and cool very slowly. These drops subsequently evaporate independently of one another, producing strong fluctuations in the rapidity distribution of secondary particles. When a drop supercools down to $T \lesssim T_C$, this may result in a state involving two discontinuous processes that follow one another on the surface of the drop.[23] One of them is a shock wave that heats the supercooled plasma to $T \lesssim T_C$ and the other is the usual deflagration front. The rate of the process need not be close to zero, but v_f is nevertheless less than c_S. The principal manifestations, or "signals", of this scenario are: (1) fluctuations (peaks) in the distribution dn/dy and (2) an "effective temperature" that is responsible for the formation of the mean transverse momentum of the particles, which is somewhat lower than T_C but greater than m_π: $T_{eff} = T_C \times 2^{-1/4}$ which corresponds to $\langle p_\perp \rangle \lesssim 0.4$ GeV/c.

B. The scenario involving passage through a mixed phase[24–26] is realized for a smoothed phase transition (without an energy barrier) and/or infinitesimally slow cooling. The hadronic gas and plasma islands (drops) can then coexist in equilibrium. Supercooling does not occur, and the temperature is maintained at a constant value close to T_C (due to the release of the latent heat of transition and recombination). We may consider this to be a realistic regime because cooling occurs relatively slowly (according to the scaling law) at temperatures near to T_C. Thus, the fall in energy density for ε_Q to ε_H at the expense of longitudinal expansion alone requires a time τ_H, that is greater by an order of magnitude than that for cooling from ε_{in} to ε_Q: $\tau_H/\tau_Q \approx \varkappa^{3/4} \approx 10$. In other words, if the characteristic time taken to form a hadron is $\tau_f \sim 1$ fm $< \tau_Q$, the system spends much of its time in a mixed state.[10)] Moreover, dissipative effects are large in this region, due to the transient velocity coefficients (in particular, it is shown in Ref. 56 that volume viscosity is negligible both in the plasma and in the hadronic gas, but becomes significant in the mixed phase). Dissipative effects ensure that the expansion process may be accompanied by an increase in entropy.

This regime is thermodynamically special because $c_S^2 = \partial p/\partial \varepsilon \to 0$. The result is an instability to the development of shock fronts ("evolving" front[55] when $v_f > c_S > 0$). The instability can manifest itself in a discontinuous transition to a hadronic state of lower pressure (rarefaction shockwave)[11)] accompanied by collective lateral motion. All this ensures that the mean lateral momentum of the particles increases slowly as a function of ε_{in} in the range $\varepsilon_H < \varepsilon_{in} < \varepsilon_Q$, remaining close to $\langle p_\perp (T_C) \rangle$ (Ref. 25). This differs from the behavior of $\langle p_\perp \rangle$ for hadronic matter, and is therefore the phase transition signal.

We note that the concept of the phase transition front between the mixed phase and the hadronic gas is largely abstract because the mixed phase itself contains a multitude of phase separation boundaries (fractal structure). It would seem that the approximation of an infinitesimally thin front is not valid in this situation.

C. The "explosive bubble" scenario[23] can be realized when the energy barrier separating the phases is so high that the development of a fluctuation that takes the system to the new (stable) phase has a low probability at all plasma temperatures: $T_0 > T_q \to 0$. The dynamic regime corresponding to a growing bubble of this kind is stable[55,57] if the Jouguet condition is satisfied on the transition front: the hadronic flow velocity relative to the phase separation boundary is equal to the velocity of sound c_S. Hadronic matter behind the phase-transition front is then heated to $T_H \simeq 2.5 T_C$ and presses against the front; a rarefaction wave propagates away from the front and toward the center of the explosive focus, so that a normal hadronic state at rest ($T_H \lesssim T_C$) is established at the center. The boundary between the resting and excited hadronic gases constitutes a weak discontinuity that propagates with velocity c_S behind the explosion front.[12] These processes ensure that almost the entire energy stored in the deep metastable state is transformed into the energy of collective transverse motion of the hadronic gas, which leads to a substantial increase in the mean transverse momentum of the hadrons (and a certain increase in entropy). The "signal" that this scenario has been realized is therefore anomalously large[54] $\langle p_\perp \rangle \gtrsim 1$ GeV/c and is correlated with strong fluctuations in the rapidity distribution of secondary particles. The effect should have a threshold, i.e., it should appear discontinuously when energy densities sufficient for the plasma formation are reached: $\varepsilon_{in} \geqslant \varepsilon_Q$.

D. The "boiling" scenario is conceptually close to the explosive scenario, but exhibits a number of substantial differences: the process is not a steady state nor a collective one; the concept of a macroscopic front cannot be introduced; and the closest picture that we can imagine is the instantaneous boiling-up of the plasma volume.[54,60]

This scenario is based on the assumption that the segregation of the hadronic phase is energetically forbidden up to a certain temperature T_q^* at which the barrier separating the phases can be overcome by energy fluctuations in the system ($E_f \sim T$). When this happens, a large number N of foci of hadronic phase is likely to form and grow rapidly in a nonstationary manner. Kinematic constraints do not apply to them. A single collective motion cannot then be established: the independently expanding hadronic bubbles produce a cellular structure in the hadronic phase, which is then destroyed by collisions between individual microfronts (bubble walls) and by mixing of hadronic flows. All the processes become turbulent at this stage, which is typical for the end of a metastable state in any physical process. The result is an averaging of ε_h over the volume of the element, and an instantaneous ($\tau_h \sim N^{-1}$) transition to a new (hadronic) state of equilibrium. The energy density in the given volume is conserved, and the temperature of hadronic matter substantially exceeds T_C. The entire process is accompanied by an increase in entropy: when $T^* \approx 0.6\, T_C$ (see Section 3), we have

$$T_h = T^* (\varkappa + B (T^*)^{-4})^{1/4} \approx 1.5 T_C, \tag{4.8}$$

$$\frac{\hat{S}_h}{\hat{S}_q} = \frac{T_h^3}{\varkappa T_q^3} = \frac{1}{\varkappa^{1/4}} \left[1 + \frac{1}{3} \left(\frac{T_C}{T^*}\right)^3\right] \approx 1\ 5. \tag{4.9}$$

It follows from (4.9) that this process requires a particular degree of supercooling, i.e., $T_q < \hat{T} \approx 0.75\, T_C$ ($\hat{\varepsilon}_q \approx 2B$) (see Fig. 5). The physics of the initial boiling-up stage is related to the striction of the volume ($V_h\ V_q$), which is possible only for $\varepsilon_q \leqslant \varepsilon_h$; the process is forbidden $T_q > \hat{T}$ for ($\Delta \hat{S}(\hat{T}) = 0$).

We note that, in this scenario, the phase transition process does not affect the "external" parameters of the hydrodynamic fluid element (volume V or velocity u^μ), but simply increases the temperature and entropy in the element. In other words, the evolution of hadronic matter after the phase transition simulates the absence of plasma in the initial state and the evolution of pure hadronic matter. The important point is that the initial temperature of this "simulated matter", $\tilde{T}_{in}$, is greater than both T_{in}^q ($\tilde{T}_{in} \approx 2.5\, \tilde{T}_{in}^q$) and the initial temperature $\tilde{T}_{in}^h$ of the actual hadronic phase corresponding to the given ε_{in}: $\tilde{T}_{in} \approx 1.33\, \tilde{T}_{in}^h$.

Thus, in this scenario, the phase transition "signal" is the rapid rise in $\hat{S}$ (and, hence, in multiplicity) and in the mean transverse momentum $\langle p_\perp \rangle$ as compared with the analogous characteristics for the expansion of pure hadronic (initially) matter. Moreover, the turbulent stage of the phase transition process should be accompanied by strong fluctuations, both in the distributions of secondary particles over $p_\perp$ and in the dependence of $\langle p_\perp \rangle$ or ε_{in}.

However, it is possible that the reverse phase transition will begin in the heated hadronic gas, i.e., fine drops of hot plasma ($T_q = 1.5 T_C$) will appear once again. This process is again accompanied by volume striction. If the system were to exist for an infinite period of time, an equilibrium (for given ε) would be established in the form of a mixed phase. In the present case, this state is reached as a result of damped periodic oscillations. The formation of a hot hadronic phase from highly supercooled plasma is the first stage of this periodic process. Detailed examination of this interesting and separate problem is outside the scope of the present review.

In practice, the more probable scenario seems to be that, as a result of intensive lateral motion, the system splits at the turbulization stage into noninteracting objects (hadrons and fine plasma drops)[58]. In other words, matter "hardens"[4] at the temperature of $1.5 T_C$, which corresponds to $\langle p_\perp \rangle \approx 0.75$ GeV/fm^3 (whatever the dependence on ε_{in}), i.e., this boiling evolution results in a plateau of $\langle p_\perp \rangle$ after $\varepsilon_{in} \simeq \varepsilon_Q$. We note that the relationship between ε_{in} and $dN/dy|_0$ is disturbed in this situation because the entire process is accompanied by a substantial increase in entropy.

To conclude our review of possible forms of phase transition, we note that the real hadronization process may proceed in the hydrodynamic system in a much more complicated manner than indicated by scenarios **A–D**: different hadronization mechanisms may superimpose. For example, the following situation is quite realistic. The central region ($y = 0$) cools rapidly and explodes. Some of the explosion byproducts (hot hadrons) penetrate lateral regions, heat them, and slow down the cooling process. The result is that their hadronization is due to the soft mechanism (**C** + **A**,**B**). Or: the boiling process results in three-dimensional expansion and the "hardening" of hadronic matter on the periphery of the system. The mixed phase appears at the center, expands, and transforms into cold hadrons (**D** + **B**).

In either case it is very difficult to predict the observable characteristics of secondary particles. All that can be done is to suggest that they (and in particular $\langle p_\perp \rangle$) must lie between values typical for the pure scenarios **A–D**, but closer to the most probable (competing) scenario under these conditions.

The questions as to which of the scenarios **A–D** is the most probable can only be determined on the basis of a kinetic analysis. Before turning to this question, we note that scenario **B**, i.e., equilibrium transition with the formation of the mixed phase, is the most popular among theoreticians.[24–26]

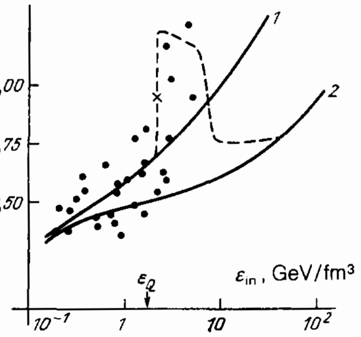

FIG. 6. Mean transverse momentum $\langle p_\perp \rangle$ as a function of the energy density ε_{in} in the initial state. Points represent the JACEE experimental data.[59] Curve *1* refers to purely hadronic matter, *2*—mixed state of plasma (theoretical calculations[25]). Broken curve—probable character of the function $\langle p_\perp \rangle \sim \varepsilon_{in}$ with allowance for nonequilibrium phase transitions[54] (see text).

There are several reasons for this. First, the deflagration mechanism comes into play much earlier ($T \sim T_C$) than the explosive mechanism ($T \lesssim 0.5\,T_C$). In other words, **C** and **D** are possible only when **B** has already occurred. Second, the size of the phase barrier was estimated on the basis of general considerations and was regarded as "not too high",[25] which meant that the turning on of **B** could be considered as not too slow. In the bag model with surface tension, however, the size of the barrier depends on the phenomenological parameter ω (see Section 3). We note that indirect estimates[61] show that this parameter can be quite high ($\omega \sim 0.5 - 1$), which corresponds to a very large barrier. Finally, the energy density ε_{in} attained experimentally (so far) is often estimated as $\varepsilon_{in} \sim 2$–4 GeV/fm^3. This may not be sufficient for the complete ionization of matter, but covers the mixed-state range $\varepsilon_H < \varepsilon_{in} < \varepsilon_Q$, so that the "signals" indicating the formation of the mixed state should be perceptible.

A detailed analysis (based on a computer calculation) of hydrodynamic processes in the mixed phase was reported in Ref. 25. Maximum attention was given to the relation $\langle p_\perp \rangle \propto dN/dy \propto \varepsilon_{in}$ and to the distribution of different particles over the transverse momentum because the characteristic behavior of $\langle p_\perp \rangle$ (slow increase in the range $\varepsilon_H < \varepsilon_{in} < \varepsilon_Q$) is the principal "signal" indicating the presence of the mixed state. Comparison of the results reported in Ref. 25 with experimental data[59] (Fig. 6) showed, however, that there was a large discrepancy between the proposed regime and the actual values. This discrepancy can be ascribed to the inadequacy of the expression

$$\varepsilon_{in} = \frac{3}{4\pi\tau_{in}} (p_\perp^2 + m^2)^{1/2} \frac{dN}{dy} A^{-2/3}, \tag{4.10}$$

which is valid only on the assumption of constant entropy at all stages of expansion and certain other effects.[13)] We note, however, that the experimental points are in better agreement with the qualitative effects of boiling or explosive growth of the hadronic bubble.

4.3. Kinetic analysis of phase transition scenarios[14)]

As already noted, the conclusion that a particular scenario is realistic must be based on kinetic analysis, i.e., on the comparison of the characteristic times for cooling, the "turning on" of the hadronization process, and collective hadronization. By the "turning-on" time of the transition process we mean the characteristic time τ_f necessary for the formation of the focus of the (h) phase. This is usually assumed to be 1 fm. However, the kinetic analysis given in Section 3 shows that τ_f depends significantly on the size of the barrier between the phases ($\sim\omega$) and on the temperature T_q. In particular,

$$\tau_f \sim W_A^{-1}(T_q) \sim T_C^{-1} \frac{1}{x} \exp\left[\frac{16\pi}{3}\left(\frac{\varkappa-1}{3}\right)^{1/4} \frac{\omega^3}{x(1-x^4)^2}\right], \tag{4.11}$$

where $x = T_q/T_C$ is the degree of supercooling of the plasma. The time τ_f has a minimum as a function of the temperature (x), which corresponds to the maximum of W_A (see Fig. 3a) at $x = 0.57 = x^*$. When $\omega \lesssim 10^{-1}$, the minimum is broad (half-width $\Delta \sim (30\,\omega)^{-3/2} \sim 1$), i.e., the turning on of the phase transition is equally probable for all temperatures below T_C. Under these conditions, $\tau_f \lesssim 1$ fm, which agrees with the generally adopted estimate. We note that the temperature T^q_{lim} for which the barrier in F^* is comparable with the energy fluctuation ($\sim T$) is then close to T_C (see Fig. 3b). However, this does not mean that hadronization is an avalanche process ("boiling") because this is forbidden in the region $T_q > \hat{T} \approx 0.75\,T_C$ by thermodynamic considerations ($\Delta\hat{S} < 0$). Only the equilibrium coexistence of phases is possible in this range of ω.

For larger ω, the probability distribution contracts rapidly ($\sim\omega^{-3/2}$), and W_A itself falls off as $e^{-\omega^3}$, i.e., the nucleation of the hadronic phase becomes probable only for $T_q \approx T^*$. The limiting temperature is also reduced: $T^q_{lim} = T^*$ at $\omega = \omega^* \simeq 0.3$, where $\tau_f = 1$ fm, as before, but outside the region $T \sim T^*$ we have

$$\tau_f(x) = \tau_f^* \exp\frac{(x-x^*)^2}{2\Delta^2} \tag{4.11*}$$

so that τ_f exceeds 1 fm by orders of magnitude.

As ω increases further, T^q_{lim} vanishes (the end of the plasma phase does not occur until $T_q = 0$), i.e., the barrier exceeds the energy noise level, and the probability that the barrier will be overcome is always low.

Analysis of the function $\tau_f(\omega, x)$ is thus seen to enable us to assign an approximate range of values of ω to each of the scenarios **A–D**:

$0 < \omega < 0.2$—mixed phase.

$\omega = 0.3$—boiling at $T \simeq T^*$.

$\omega > 0.5$—growth of single explosive bubbles [$\tau_f(x^*) \gg 10$ fm].

More specific choice between the scenarios must be based on a comparison between the rate of cooling and the rate of hadronization of the system as a whole in each of the four scenarios.

The rate of cooling can be obtained from (4.3):

$$\Delta\tau \sim \frac{\Delta T}{T} \tau_{in} \left(\frac{T_{in}}{T}\right)^3 \sim 1 \text{ фм} \cdot \frac{\Delta T}{T_C} \left(\frac{T_{in}}{T_C}\right)^2 x^{-3}. \tag{4.12}$$

where $\Delta\tau$ is the time spent by the system in the temperature interval ΔT. It follows from (4.12) that the cooling of ($T_{in} \gtrsim T_C$) systems which are initially not hot occurs relatively rapidly. Thus, the cooling from T_C to T^* occupies $\Delta_\tau^* \sim 5$ fm, which is less than $\tau_f(T_C)$ when $\omega \gtrsim 0.25$. In this case, the working transition mechanisms are those associated with supercooling turbulent "boiling" or the dynamic growth of combined bubbles. The choice between them is dictated by the value of ω. At this point, we may encounter a paradoxical effect. If, initially, $\varepsilon_{in} \simeq \varepsilon_q$ (plasma close to the mixed state), its hadronization under supercooling conditions will occur rapidly and in a nonequilibrium manner. The phase transition process "erases" all the information on the initial "soft" state ($p_{in} \ll \varepsilon_{in}$),[27] the quantity $\langle p_\perp \rangle$ evolves during the explosive transition stage and turns out to be greater than the corresponding values for hadronic matter. In other words, the graph of $\langle p_\perp \rangle \sim \varepsilon_{in}$ that is typical for nonequilibrium phase-transition mechanisms (broken curve in Fig. 6) shows a sharp (threshold type) increase at energy densities ε_{in} sufficient for the formation of the plasma ($\varepsilon_{in} \gtrsim \varepsilon_Q \sim 4$ GeV/fm^3), instead of the "soft" behavior typical of the mixed phase (curve 2 in Fig. 6).

We note that this is valid for $\omega \lesssim 10^{-1}$, i.e., when barrier effects are negligible. We then have $\tau_f(T_C) \simeq 1$ fm $\ll \Delta_\tau^*$; the transition is necessarily of the equilibrium type and the relation $\langle p_\perp \rangle \propto dN/dy\, \varepsilon_{in}$ should reproduce curve *2* of Fig. 6.

"Soft" phase transition mechanisms may come into play during the cooling of hotter plasma, including the coexistence of phases and deflagration. Predictions are difficult to make because of the competition and superposition of different phase-transition mechanisms that "mix" effects typical of each of them.

Moreover, even within the framework of the nonequilibrium scenario alone, the number of nucleating bubbles is relatively small and is subject to considerable fluctuation. Consequently, in the region above the threshold ($\varepsilon_{in} \gtrsim \varepsilon_Q \sim 4$ GeV/fm^3), there are unavoidable strong fluctuations in the observable quantities (dN/dy, $dN/dy\,dp_\perp$, $\langle p_\perp \rangle$), both in individual events and in the ensemble of events. In other words, the "signal" from the plasma above the threshold is the chaotic behavior of observable quantities or intermittance.[35] We note in this connection that there is considerable interest in detailed information on individual events, including in particular the data reported by JACEE group[59] (where, because of relatively poor statistics, the values of $\langle p_\perp \rangle$ were analyzed without averaging over the ensemble of events).

As ε_{in} increases further, the soft mechanism is more likely to come into play, i.e., scenario **B** becomes more likely. One then expects a reduction in $\langle p_\perp \rangle$ and an asymptotic approach to the equilibrium dependence on ε_{in} (broken curve in Fig. 6).

Finally, the hadronization of very hot systems is dominated by deflagration. The range in which this is significant is limited by two factors, namely, $(\Delta\tau)_{def}$ is small, i.e., $\sim 5 \times 10^{-2}\,(T_{in}/T_C)$ fm, and the collective hadronization time is large: ($v_f \lesssim 0.04\, c_S \Rightarrow \tau_h \gtrsim 40$) fm.

Comparison of $(\Delta\tau)_{def}$ with τ_h shows that the deflagration scenario is realistic at ultrahigh energies ($T_{in} \gtrsim 30\, T_C$) or for very specific (and rare) initial conditions.[20]

Analysis of the dependence of the character of hadronization on the size of the barrier (within the phenomenological model involving the single parameter ω) and on the initial conditions is thus seen to yield interesting information on the behavior of the system.

5. CONCLUSION

The presence of plasma in the initial state does not have an unambiguous effect on the hydrodynamic process: qualitatively different scenarios of the phase transition to hadronic matter are possible.

We emphasize that this ambiguity arises if the phase transition is hard (first order), i.e., it involves the overcoming of an energy barrier between the phases. It is only in this case that hysteresis phenomena (metastable states of the system) are possible and generate nonequilibrium (explosive) transitions to hadronic matter.

A phase transition is of the barrier type in the bag model when bag instability[51] and surface effects,[54] are taken into account.

The surface tension parameter $\omega = \sigma/B^{3/4}$ is currently regarded as phenomenological. A more accurate theoretical estimate is possible in the field-theoretic bag model,[8] i.e., in the self-localized solution of the nonlinear field equations. Further studies along this direction would seem to be very desirable because the reality of any particular scenario depends significantly on the value of ω.

Scenarios based on the explosive mechanism of "boiling" and explosive bubble growth, have common features: the transition is accompanied by an increase in entropy (and, consequently, in the secondary-particle multiplicity), and generates a surplus of secondary hadrons with high transverse momenta. Experimental predictions for scenarios based on the equilibrium transition (deflagration, transition through a mixed phase) are qualitatively different: here "softness" of the plasma state near the transition point[25] manifests itself in a reduction in $\langle p_\perp \rangle$ as compared with the hydrodynamic expansion of pure hadronic matter.

Which particular scenario is actually realized will depend (for reasonable $\omega \gtrsim 0.25$) on the initial state of the system (T_{in}, τ_{in}, and degree of homogeneity and ideality). We emphasize that, for plasmas above the threshold ($T_{in} \gtrsim T_C$, $\varepsilon_{in} \gtrsim \varepsilon_Q \sim 4$ GeV/fm^3), the soft and hard phase-transition mechanisms are found to compete. The onset of a particular scenario is a probabilistic process, which leads to strong fluctuations in observable quantities (dN/dy, $\langle p_\perp \rangle$). Phenomena such as intermittance[35] may serve as a "signal" from plasma above threshold. Equilibrium behavior (Fig. 6, curve 2) may be expected for initially hot plasmas ($\varepsilon_{in} \gg \varepsilon_Q$). However, detailed predictions require more precise specification of initial conditions, and these depend on the particular model of colliding hadrons (nuclei) that is employed.

Detailed information on the evolution of plasma and its experimental manifestations is thus seen to involve a wide range of factors, including the model of the hadron, the theoretical analysis of the nature of the phase transition, and the hydrodynamic evolution scenario. Hydrodynamic theory may be regarded as the link between hadron models that are close to it in spirit, and experiment.

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[1)]Here and henceforth we assume that $\hbar = c = k = 1$.
[2)]We note, however, that there is evidence[11,12,17] that nonperturbative vacuum fluctuations are not completely suppressed inside the hadron, i.e., "point" valence quarks are dissolved in the continuum. This will be discussed in greater detail below.

[3)]However, we shall apply the ensuing analysis of the hydrodynamic process to hadron collisions as well (see Refs. 18 and 35).

[4)]See, however, Ref. 35.

[5)]Note that $\tau_0 = (t_0^2 - \Delta_0^2)^{1/2} = \sqrt{2}\,\Delta_0 \sim R\gamma^{-1}$.

[6)]It is precisely this point, that the time, gave rise to criticism of the Landau model.

[7)]The very concept is no longer valid.

[8)]The only exception is the string (peripheral) model.

[9)]The conditions are chosen so that it is precisely this problem that is considered in lattice calculations.

[10)]We note, however, that τ_r is a nontrivial function of the temperature T and may exceed 1 fm by orders of magnitude in the region of $T \sim T_C$ (see Sec. 3).

[11)]The stability of rarefaction waves and the corresponding plasma hadronization mechanism are examined in greater detail in Ref. 57.

[12)]The above picture reproduces the solution of the classical problem of gas combustion (detonation) in a tube closed at one end.[55]

[13)]It is usually assumed in applications that $\tau_{in} \sim 1$ fm, which can also lead to an incorrect estimate of ε_{in}.

[14)]This section presents the results of Ref. 54.

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