Louis de Broglie (1892–1987)

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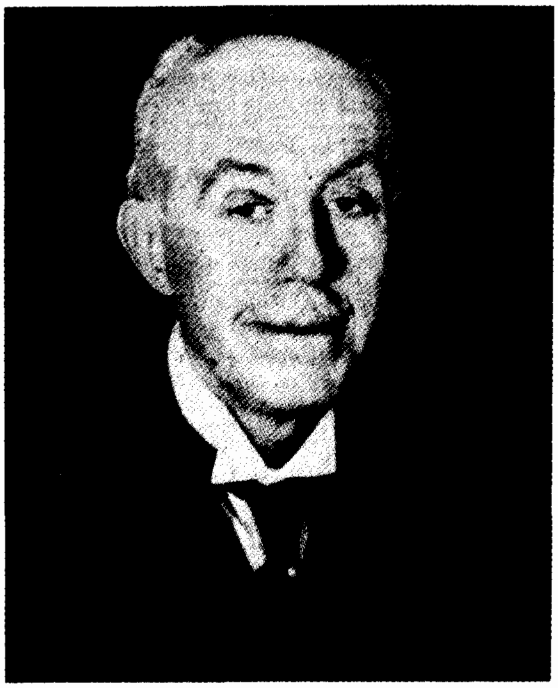
Louis de Broglie, Nobel Laureate, Member of the Academie Française, Foreign Member of the Academy of Sciences of the USSR, and the recipient of many other honors, failed to reach his ninety-fifth birthday by less than six months. His death marked the departure of the last of the legendary galaxy of creators of quantum mechanics. However, even among these extraordinary people, Louis de Broglie was unusual.

His long life was outwardly exceptionally uneventful: he continued to live in the same city (Paris), studied and worked in the same institution (the university), and was occupied with the same topic (theoretical physics), but, at the same time, his life was full of paradoxes and profound inner drama. Although he was a physicist by vocation, he was brilliantly educated in the humanities and, as he admitted to a pupil in his declining years, he had read more books on history than on physics.[1] A democrat by conviction, he was the only prince among his collegues. He devoted his life to science, published forty-three books (the last in 1982) and more than two hundred papers (many in his fifties and sixties), but his fame began when, in 1923, as a novice theoretical physicist, he wrote three papers and, in 1924, he generalized them in his thesis entitled "Studies in the Theory of Quanta."[5] Although he was one of the most implacable opponents of the probabilistic interpretation of quantum mechanics, he abandoned his initial ideas and, for almost twenty years, taught this interpretation. However, in his declining years, he returned to the ideals of his youth, and did so in the face of almost total disbelief from the physics community. His last book[6] was a record of his lectures in which he gave an account of the Copenhagen interpretation, but the book carried on its pages some questions which the author tried to answer outside the framework of his interpretation.



LOUIS DE BROGLIE
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In his papers published in 1923–1924, Louis de Broglie was the first to suggest that particles had wave properties and, being logical, he formulated the idea of universal particle-wave duality. The essence of this was that corpuscular and wave properties of matter were not mutually exclusive but, on the contrary, were part of some mysterious but totally real unity.

In 1924, he defended his thesis at the Sorbonne before a commission consisting of the leading scientists of France, namely, P. Langevin, J. Perrin, E. Cartan, and C. Maugin. The thesis bore the essential hallmark of greatness: none of those present could fully understand it. However, Langevin intuitively appreciated its significance and wrote in his report that "the candidate tried with surprising mastery to overcome the difficulties facing physics."[7] Toward the end of 1924, it was he who sent a copy of the thesis to Albert Einstein.[1)] He found the right addressee. The creator of the theory of light quanta was the first to appreciate the importance of these new ideas. Einstein immediately noted that "de Broglie recognized something more than a simple analogy."[9]

Einstein made this remark in a paper,[9] published in 1925, on the wave properties of a monatomic gas, which he wrote under the influence of a paper by S. M. Bose which Einstein received in the summer of 1924.[10] De Broglie's ideas have also played a part in the writing of this paper by Einstein. Indeed, Einstein was one of the first to consider the statistical properties of a gas of photons, and came very close to what is now known as Bose-Einstein statistics.[2)]

It was Einstein who drew Schroedinger's attention to de Broglie's thesis. Schroedinger's report at Debye's seminar on de Broglie's thesis or, more precisely, on how he, Schroedinger, understood this thesis, already contained the foundations of what subsequently became known as the wave equation, or the Schroedinger equation (Ref. 8, p. 254, and

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Ref. 12). It is thus clear that, on the genealogical tree of quantum mechanics, de Broglie's thesis was a direct precursor of both Bose-Einstein statistics and the Schroedinger equation.

We recall that, during the first half of this century, when Louis de Broglie put forward his ideas, the great majority of physicists considered that quanta were no more than a convenient way of describing the properties of radiation, and had little connection with anything real. Max Planck himself, who nominated Einstein for the Prussian Academy of Sciences in 1913, seemed to be apologizing for the fact that "in his speculative constructions, he sometimes may well go too far, as, for example, in the hypothesis of light quanta . . . " (Ref. 8, p. 54). This view was shared by other leading German scientists who joind Planck in signing the nomination, and there was very little significant change during the next decade insofar as this point of view was concerned (it will suffice to recall the attitude of Niels Bohr). Actually, it may well have happened that the formula for the momentum of a photon, put forward by Stark[13] in 1909, could have been immediately generalized to the then only known subatomic particle, i.e., the electron. However, this did not happen because practically no one looked upon the quantum of light as a particle. Virtually all physicists continued to believe that light was motion of the ether and that the properties of the photon-wave and the electron-particle were separated by a chasm. On the other hand, in his thesis, Louis de Broglie did indeed look upon the photon of light as a particle, and a set of quanta as a gas, so that both the electron and quantum of light were regarded by him as particles of energy which, in some strange way, filled all space: "The electron seems to us to be a portion of energy which we suggest, perhaps erroneously, to be the best known; in accordance with the ideas that we have developed, the energy of the electron must extend over all space, but with high concentration within a region of very small size . . . " (Ref. 5, p. 21). The transition to the consideration of the wave properties of the corpuscle, made by de Broglie in the face of existing ideas and, it appeared, existing experimental evidence (the corresponding experiments were not adequately interpreted until much later), was possible because of his initial point of departure: de Broglie had no doubt about the reality of quanta and was therefore constrained to look for an actual process to which the frequency in Planck's formula could be attributed. The answer to this question could not be found within the framework of classical ideas. Louis de Broglie started with the proposition that the properties of the quantum could not be fundamentally different from those of ordinary particles. However, he knew that all the then known particles were similar to one another and had nonzero rest mass. He therefore introduced a further hypothesis which was illogical and, as we now know, unnecessary, namely, that the quantum also had a rest mass, in which case it did indeed become similar to other particles. This hypothesis was especially necessary for de Broglie because no one at the time could conceive of the quantum as a particle if it had zero rest mass; the question of the mechanics of the quantum was ignored.

Louis de Broglie knew, of course, that his hypothesis was "unsafe" and, in order to avoid contradiction with existing facts, he restricted the mass of the quantum by the condition $m_0 < 10^{-50}$ g, thus removing it from the region accessible to experiment. However, if the quantum has a rest mass, there must be a coordinate frame in which it is at rest and, if we transform to this frame, we can more readily examine it. Louis de Broglie then wrote as follows: "Being guided by the general relationship between the concepts of frequency and energy, we shall allow in this paper the existence of a periodic phenomenon whose nature is still to be determined and which is associated with each isolated portion of energy and depends on the proper mass in accordance with the Planck-Einstein equation" (Ref. 5, p. 9). If we denote the rest mass by m_0, then, in the frame in which the quantum is at rest, the proposed, "periodic phenomenon" will have the frequency $\nu = \epsilon/h$ and, consequently, $\nu_0 = m_0c^2/h$. For an observer relative to whom the quantum, i.e., the conveyor of the oscillations, moves with velocity $v = \beta c$, the frequency is lower. In this model, the quantum is a clock and clocks run slow in a moving frame. This means that the frequency should also be *lower*. Hence, the frequency of the "oscillations of the quantum," measured by the observer relative to whom the quantum moves with velocity $v = \beta c$, should be $\nu_0(1-\beta^2)^{1/2}$.

However, the above argument is not the only one possible. If we look upon the quantum not as a particle but as a wave, we must describe it by some function such as $\sin[\nu_1 t - (x/\lambda)]$, where ν_1 is the frequency of the wave and λ its wavelength. According to the Lorentz transformation, the frequency in the frame in which the quantum is at rest (we are dealing with the quantum as a particle) and the frequency ν_1 in the frame of the observer are related by $\nu_1 = \nu_0/(1-\beta^2)^{1/2}$.

However, this frequency is quite different from that obtained earlier. The two differ by the factor $(1-\beta^2)^{-1}$. It follows that our discussion has reached an impasse and, if we are to follow the usual logic of proof, we must reject the entire argument. However, as in a game of chess, Louis de Broglie found a good move and thus saved an apparently hopeless situation.

He noted that not only the frequencies of the two processes, but also their velocities, should be different.[3] At this point, the logical chain of the argument broke off, and further advance required of the author a bold step that demonstrated his greatness. De Broglie formulated a requirement that was neither justified nor fully understood: "If the internal process in the moving field is initially with the wave, the phase harmony should persist indefinitely" (Ref. 5, p. 18).

Let us consider the phase of the two oscillations at time t or the point $x = vt$ (at which the quantum is located). The phase of the "internal" oscillation at this point is

$$\nu t = \nu_0 (1-\beta^2)^{1/2} \frac{x}{v}. \tag{1}$$

The phase of the "wave," on the other hand, is given by

$$\nu_1 t - \frac{1}{\lambda} x = \nu_0 \left(\frac{x}{v} - \frac{x}{u}\right) \frac{1}{(1-\beta^2)^{1/2}}, \tag{2}$$

if we assume that the phase of the wave propagates with velocity $\nu_1\lambda = u$ (now usually referred to as the "phase velocity").

The phases given by (1) and (2) are identical if the phase velocity is such that the following equation is satisfied:

$$(1-\beta^2)\frac{1}{v} = \frac{1}{v} - \frac{1}{u}.$$

Hence, recalling that $\beta^2 = v^2/c^2$, we find that $u = c^2/v$.

The phase velocity u is found to be greater than the velocity of light. However, this is not a new paradox because, as de Broglie noted, nothing material is propagating with this velocity. The mass (and energy) propagate with velocity v, so that the de Broglie hypotheses do not lead to contradiction with the principle of relativity.

De Broglie then makes one further remark which concludes his construction: the phase velocity of the waves is not constant, but depends on the wavelength λ. If this is so, then, as was shown much earlier by Rayleigh (to whom de Broglie refers), the energy is transported in the wave with the "group velocity," given by

$$v = \frac{d\nu}{d\lambda^{-1}}.$$

By substituting

$$\nu = \frac{u}{\lambda} = \frac{c^2}{v}\frac{1}{\lambda},$$

we obtain the group velocity which is precisely equal to the velocity v of the quantum! Everything is thus in place. It is surprising how much inventiveness was required to obtain a result which now seems to us to be trivial.

The correct result was, in fact, deduced from two arbitrary and unproved hypotheses, namely, the hypothesis of the mass of the photon and the hypothesis of the harmony of phases. The former divided the wave motion into two, and the latter restored their unity. The result was a group velocity, and wave motion was found to be inherent not only to quanta, but also to any other particles. Intuition turned out to be stronger than logic.

With striking perspicacity, Louis de Broglie fully appreciated the significance of this discovery. He published his work in 1924 in English in the Philosophical Magazine,[14] where he pointed out many of the consequences of his hypothesis, and formulated a program for its further development. For example, in response to Perrin's question about possible experimental verification of the wave hypothesis, de Broglie put forward the idea of electron diffraction. This phenomenon was actually discovered three years later, in 1927, in special experiments performed by C. J. Davisson and L. Germer[15] and, independently, by G. P. Thomson.[4)16]

As far back as 1924, de Broglie related the stability of the atom to the fact that an even number of wavelengths had to fit into a stationary orbit of an electron. The authorship of this suggestion seems to have been lost in history, but it provided de Broglie's waves with the first aspect of reality.

The picture facing de Broglie was strikingly different from that seen by his contemporaries. It was only due to de Broglie that the duality of wave and quantum properties did not seem to be a paradox, but a manifestation of a certain symmetry, i.e., a permanent, organic property of nature and evidence of its harmony.

A striking independence of mind was necessary to proceed alone along a totally unexpolored path. The imagination required for the 1923 papers and the fecundity of their conclusions bring to mind another comparable visionary who lived 400 years earlier, namely Johann Kepler. His belief in the perfection and harmony of the Universe led Kepler from the hypothesis of inherent world symmetry to profound truths about the structure of the solar system, whose mathematical significance remained undiscovered until Newton. It is precisely the symmetry between the wave and corpuscular pictures, first seen by de Broglie, that led him to profound truths about the structure of the microworld. The role of Newton in this case fell to Erwin Schroedinger. One of the most obvious manifestations of the harmony of the Universe was, for de Broglie, the identity between the principle of least action and the Fermat principle. "Guided by the idea of a profound identity between the principle of least action and the Fermat principle"—wrote de Broglie in his thesis—"I accepted right from the outset of my researches into this question that, for a given total energy of a moving body and consequently given frequency of its wave and phase, the dynamically possible trajectories of the body coincided with the possible wave rays" (Ref. 5, p. 33). Noting that Hamilton had already drawn attention to the similarity between the equations of wave optics and the kinematic description of particles, de Broglien saw in the Hamilton-Jacobi theory something he referred to as the embryonic theory of union between wave and particles.

Although de Broglie was a loner in his creativity, his sources were, nevertheless, in the external world. This takes us, following the trend of history, to the First International Solvay Congress held in Brussels between 30 October and 3 November 1911 under the chairmanship of H. A. Lorentz. The overall theme of the congress was "The theory of radiation and quanta." The congress was attended by leading physicists of the time, including H. W. Nernst, M. Planck, A. Einstein, M. Curie, P. Langevin, A. Sommerfeld, and M. Brillouin. However, even this congress failed to achieve significant advances in the understanding of the role and place of the quantum in electrodynamics and physical statistics.

The scientific secretary (and, later, one of the editors of the proceedings of the congress) was Maurice de Broglie, elder brother of Louis. Louis himself not only did not attend the congress, but was not even occupied with physics at the time, his attention being attracted by problems in the history of the Middle Ages and philosophy. However, it was no accident that it was precisely during 1911, i.e., in the year of the congress, that Louis de Broglie moved to the Faculty of Science at the Sorbonne.[5)] It is clear that the influence of his elder brother, who became familiar with the papers read to the congress, played a decisive role in the development of the younger man. Maurice de Broglie was seventeen years older than Louis. In 1904, he gave up the career of a naval officer and became head of the laboratory which he established in Paris and in which, from 1906 onward, he devoted all his time to the study of x-ray spectra and the photoelectric effect. The standard of the experiments performed in Maurice's laboratory was very high and the director himself was a pupil of P. Langevin. His election as the Scientific Secretary of the First Solvay Congress was clear evidence for the international standing of the elder of the two de Broglie brothers.

It was with Maurice that the younger de Broglie first discussed his scientific ideas, and it was under his influence that, in 1922, he came to the conclusion presented in his thesis in which, at the end of a historical introduction, he wrote "it would seem that the moment has arrived for an attempt to unify the corpuscular and wave points of view, and to deepen our understanding of quanta. This we have recently done, and the principal aim of this thesis is to present as completely as possible the new ideas that we have put forward, as well as the advances to which they lead and the numerous problems that they raise" (Ref. 5, p. 9).

When we speak of de Broglie's creativity, and note the unusual nature of his arrival in physics and his entire approach to the study of the world surrounding him, we cannot ignore his unusual genealogy. The scientific biographies of great physicists usually make no mention of genealogy but, in this case, this is appropriate: family traditions must have had a profound influence on Louis de Broglie's creative work.

The only prince in the many centuries of the history of physics,[6)] he was a member of the de Broglia family.[7)] Mention of his ancestors can be found in the Italian chronicles of the twelfth century. In the middle of the seventeenth century, the Piédmont Count François Maria Broglia entered the service of the French king and thus began three centuries of military and governmental activity of the de Broglies in France. Louis de Broglie's ancestors included delegates to the Assembly and ministers, marshals, and members of the French Academy. We note that the father of modern wave optics, A. J. Fresnel, is related (although distantly) to the de Broglie family: he was born on the de Broglie estate, of which his father was the manager.

Independence of mind was the most characteristic feature of the de Broglie's creativity. This sometimes meant that his profound ideas were not understood by the physics community, but we need not be too critical of them because Louis de Broglie himself was not always, by far, consistent in his announcements. For example, in 1923, he actually suggested an analog of what might be considered the probabilistic interpretation of stimulated emission by an atom when he continued the line originally pursued by Einstein.[20] De Broglie wrote: "When the phase wave traverses an excited atom, the latter has a certain probability of emitting a quantum of light, which is determined at each time by the intensity of the wave" (Ref. 14, p. 450). However, de Broglie himself did not wish to go too far along this line. Thus, when, in 1926, Max Born put forward the probabilistic interpretation of the solution of the Schroedinger equation for the scattering problem,[21] Louis de Broglie became one of the most convinced opponents of this interpretation. Throughout his life, except, it is true, for some temporary forced retreats in which he adopted the probabilistic treatment, probably for pedagogical reasons, Louis de Broglie tried to find another interpretation, much closer to the ideas of classical physics. His dream was to find a "clear picture of wave-particle duality that would be consistent with classical conceptions."[22] His attempts to construct the theory of the double solution by introducing an additional nonlinear equation, i.e., departure from the superposition principle—one of the cornerstones of quantum mechanics—is actually a departure from classical conceptions no less radical than ordinary quantum mechanics. A similar comment could be made about attempts to look upon a particle as a singularity of a wave, and to give the wave the role of directing the motion of the singularity. He invested this picture of a "pilot wave" with hopes that have been irresistible to his pupils to this day.

However, Louis de Broglie was, of course, unique in his attempts to find a more logical (from the point of view of classical theory) explanation of quantum mechanics. In this sense, he was at one with Einstein and Schroedinger, who, like he, could not, to their last days, accept the world that they themselves created.

No one knows how much truth there is in the doubts of these great individualists. Vigorous discussions on the nature and meaning of physical reality, which have died down for some time, are now attracting the attention of the new generation, but a conclusion is still not within reach. We shall therefore illustrate the problems that troubled these great scientists so much, and continue to be topical to this day, by reproducing two fragments from the correspondence between de Broglie and Einstein in February and March of 1954.[23]

"Dear de Broglie,

Last evening I read the German translation of your paper, with which I was already familiar, on the problem of "quanta and determinism," and I found great pleasure in the clarity of the thoughts . . .

. . . Actually, I am convinced, just as you are, that we must look for a substructure whereas modern quantum mechanics artificially conceals this necessity by adopting a statistical form.

However, I have long been convinced that this substructure cannot be found in a *constructive* way, starting from the empirically established behavior of objects, because the efforts necessary for this would exceed human possibilities. I have arrived at this conclusion not as a consequence of the futility of many years of my own efforts, but because of my experience in the theory of graviation. The equations of the theory of graviation could be discovered *only* on the basis of a purely formal principle (of general covariance), i.e., on the basis of a conviction that the laws of nature are based on maximum possible logical simplicity . . . " (from Einstein's letter of 15 February 1954).

Here is an excerpt from de Broglie's answer, dated 8 March 1954:

Dear M. Einstein,

. . . I am again inclined to consider that the statistical interpretation adopted at the present time is "incomplete" and that we must seek exact space-time pictures of particle-wave duality that would enable us to justify the success of statistical laws in quantum mechanics.

The fact that, in your letter, you speak of your attitude to the problem of the quanta and your belief in "logical simplicity" has been in my mind ever since. Indeed it seems to me that these general logical connections that have led you to such magnificent results in the general theory of relativity and in unified field theories will in future enable us to achieve a better understanding of the significance of quanta and of wave-particle duality . . .

I thank you once again for the benefits gained by reading your valuable letter and for the enormous support that it affords me in my new researches."[23]

[1)]A. F. Ioffe has provided evidence that Einstein was told about de Broglie's work by Langevin as far back as the spring of 1924 during the Fourth Solvay Congress. He showed interest in this work at that time and asked for reprints of papers.[8]

[2)]It is clear that one of the first applications of a gas consisting of quanta is due to A. F. Ioffe. It was made in 1910.[11] Our attention was drawn to this paper by T. Aleksakhina.

[3)]We note that, without the hypothesis that the quantum has a mass, there would not be two frequencies. The paradox would then not arise, but there would not be a discovery either!

[4)]The question of the interpretation of the angular disribution of scattered electrons arose as far back as 1921 in the course of a study of secondary electron emission in the experimental work of Davisson and Kunsman.[17] Attempts to explain the resulting picture in terms of the Bohr quantum theory were clearly unsatisfactory. In 1925, a young physicist, W. Elsasser, interpreted these experimental results in terms of de Broglie's ideas which, until then, were not seriously considered by anyone.

[5)]A special paper by Max Planck was devoted at the congress to the contradiction between the hypothesis of the quanta of light due to Einstein and Maxwell's theory of the electromagnetic field. Planck expressed great hope that this contradiction would be overcome by using the principle of least action. Louis de Broglie recalled fifty years later that this was the question that interested him at the time more than any other: "At that early age I was overcome by these problems and decided to devote all my efforts to try to understand the true nature of the mysterious quanta which more than ten years earlier Max Planck introduced in theoretical physics, but whose profound significance was not understood" (Ref. 9, p. 459).

[6)]The elder brother, Maurice, was a duke; the younger brother was a prince. Louis de Broglie's attitude to his aristocratic origins is typified by the fact that he never resided as an adult in his inherited estate and tried, as he wrote himself, to avoid contact with the aristocratic world.

[7)]The name, de Broglie, is still sometimes pronounced as de Broglia.

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Translated by S. Chomet