Gravitational-wave astronomy[1)]

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CONTENTS

1. INTRODUCTION. GRAVITATIONAL-WAVE ASTRONOMY IN ACTION

Gravitational-wave astronomy is frequently seen as a very attractive field of science that promises grandiose discoveries and achievements but ... in the very distant future.

This judgement is made because gravitational waves have never yet been directly observed. An experiment with the emission and detection of gravitational waves under laboratory conditions is as yet unrealizable. The attempts at the direct detection on earth of gravitational radiation from cosmic sources have also been unsuccessful as yet. Despite tremendous efforts on the part of experimentalists it is not yet clear when the required level of sensitivity of the ground-based detectors will be achieved.

Despite this, there is much indisputable, though admittedly indirect, evidence that gravitational radiation plays an important part in real astrophysical processes. Observations confirm theoretical predictions based on allowance for gravitational-wave effects not only qualitatively but also quantitatively. In this sense gravitational-wave astronomy already actually exists and is being successfully developed. The present status of gravitational-wave astronomy is characterized by reliable observations of indirect manifestations of gravitational waves (change in the orbit of the binary pulsar PSR 1913 + 16[3]), the recognition of the decisive importance of gravitational waves in specific observed astrophysical phenomena (evolution of binary systems leading to the formation of cataclysmic variables[4] and type I supernovas[5]), and the deduction of nontrivial bounds on the parameters of the early universe from gravitational-wave considerations (comparison of a variety of observational data and theoretical predictions about the spectrum and energy density of relic gravitons).[6,7] One can say that gravitational-wave astronomy is already an effective tool for studying nature.

The successes of gravitational-wave astronomy would be more significant if systematic and continuous gravitational-wave surveys of the sky were made at the best sensitivity level now achieved. The absence of such observations was recently acutely felt when a supernova in the Large Magellanic Cloud (LMC SN 1987A) was detected in electromagnetic and neutrino radiation. Given a favorable combination of circumstances and optimistic assumptions about the gravitational energy release in the supernova explosion this event could have been observed by the best gravitational detectors. At the least a nontrivial upper bound on the energy released in the form of a gravitational-wave pulse could have been established:

$$\Delta E < M_{\odot}c^2 = 2\cdot 10^{54}\ \text{erg}.$$

However, the most sensitive antennas were not operating at the time, and the suggestions of a signal found by the Italian group[8] with an insufficiently sensitive antenna could hardly have been due to gravitational waves even if they did have some connection with the event. Attempts have been made to organize observations with the existing laser interferometers at Caltech and elsewhere in the hope of observing gravitational radiation from a pulsar that may have formed as a remnant of the supernova SN 1987A. These observations will probably also be unsuccessful, since even under the most optimistic assumptions about the pulsar the sensitivities of the interferometers are still insufficient[9] (see also Sec. 4 below). This example emphasizes once more the urgent need to improve significantly the detector sensitivity and organize systematic monitoring of the sky.

In this paper we first consider astronomical manifestations of gravitational waves (Sec. 2). We then present briefly some new theoretical results important for gravitational-wave astronomy (Sec. 3). In Sec. 4 we discuss cosmic

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sources of gravitational waves; predictions about the form and amplitude of the signals radiated by them are compared with the existing experimental limits. Particular attention is devoted to stochastic gravitational-wave radiation; this is due to the fundamental significance of the possible detection of a background of relic gravitational waves (Sec. 5). In Sec. 6 we explain the very simple principles of detection of gravitational waves and point out the main reasons that limit the sensitivity of detectors. Finally, some new experimental ideas are briefly discussed in Sec. 7.

2. ASTRONOMICAL MANIFESTATIONS OF GRAVITATIONAL WAVES

2.1. The binary radio pulsar PSR 1913+16

A binary star system necessarily radiates gravitational waves. Radiative corrections appear in the equations of motion in the $(v/c)^5$ approximation, where v is the characteristic speed of the orbital motion. In this approximation a binary system radiates energy and angular momentum, while the total momentum of the system remains unchanged. A consequence of the emission of gravitational waves is a systematic change of the parameters of the Keplerian orbit. The eccentricity and size of the orbit, and also the period of revolution of the bodies decrease secularly.[10] The radiative corrections become particularly significant in compact systems containing massive bodies.

The pulsar PSR 1913 + 16, which is a component of a close binary system, has truly become a gravitational laboratory in space.[3,11] The relativistic nature of the system is illustrated by the fact that the orbital velocity of the pulsar exceeds 300 km/sec, i.e., $v/c \approx 10^{-3}$. The entire system, which consists of two neutron stars that each have mass $1.4M_\odot$, has a diameter about that of the diameter of our sun. In addition, the high stability of the periodic radiation of the pulsar makes it possible to observe the time of arrival of individual pulses with tremendous accuracy.

Observations of the pulsar have been continued more or less regularly since 1974. The main relativistic effect, which appears in the equations of motion already in the terms of order $(v/c)^2$ and give rise to a displacement of the periastron of the orbit, is in this system 4° per year, instead of the 42″ per century for Mercury. The displacement of the periastron can be readily detected during an observation time of order 10 days. Chronometry of the arrival time of the pulsar pulses makes it possible to obtain a set of data more than sufficient to determine all the parameters of the binary system, including the masses of the companions. The relativistic effects measured in the system include the secular decrease of the orbital period, $\dot{P}$, the displacement of the orbital periastron, the time delay in the propagation of signals in the gravitational field of the neighboring neutron star, the quadratic Doppler effect, and relativistic corrections to the radial velocity, which make it possible to determine the inclination of the orbit. The accuracy and diversity of the observational data are so great that besides determining the Newtonian and relativistic effects in the motion of the pulsar one can obtain very stringent restrictions on the possible deviations from the simplest and most natural model for the complete system, namely, the model in the form of a pair of isolated neutron stars without any third bodies and also without any significant matter or gas flows. The possible deviations, expressed in relative units, do not exceed a few percent.

According to the latest data, the mass of the pulsar PSR 1913 + 16 is[2)] (see Ref. 3) $(1.445 \pm 0.007)M_\odot$, and the mass of its companion is $(1.384 \pm 0.007)M_\odot$. The observed value of $\dot{P}$ agrees with the theoretical value based on the Einstein quadrupole expression, with all the currently achieved accuracy, to better than 4%. At the present stage of the development of astrophysics the binary pulsar PSR 1913 + 16 is used not so much to test the predictions of the general theory of relativity, including the emission of gravitational waves, but rather to determine the physical properties of the system at an increasingly sophisticated level.

2.2. Cataclysmic variables

Cataclysmic variable stars, and also some x-ray sources of moderate luminosity, are very close binary systems. One of the components of the system, called the primary component, is a compact star—a degenerate dwarf, a neutron star, or even a black hole. The secondary component is most often a low-mass main sequence star. The mass of the primary component is usually estimated at $(0.6–1.5)M_\odot$, and that of the secondary component at between $0.3M_\odot$ and $0.03M_\odot$. In these systems a transfer of mass from the low-mass star to the more massive star is observed, the rate of accretion being about $10^{-10}M_\odot$ per year. Another important observational fact is the abrupt cutoff in the distribution of cataclysmic variables with respect to their orbital periods. Only a few systems with periods shorter than 80 min are observed. These observations can be qualitatively and quantitatively explained when gravitational radiation is taken into account.[4,13]

Stable and prolonged mass transfer in such systems is possible only because the orbital angular momentum of the system is reduced by radiation in the form of gravitational waves. Under the influence of the gravitational-wave damping the stars approach each other, the low-mass star fills its Roche lobe, and a flow to the primary component commences. Mass transfer from the low-mass star to the more massive companion under the condition of conservation of the orbital angular momentum should be accompanied by an increase in the radius of the orbit. In other words, if there were no gravitational radiation the distance between the stars would increase and matter transfer would cease. However, the gravitational-wave effect is dominant, and as a result the accretion does not cease and the radius of the orbit continues to decrease, although somewhat slower than before the onset of mass transfer.

A gradual and slow approach of the stars takes place over about $5 \cdot 10^9$ yr. During this time of evolution the star losing mass continues to remain states for main sequence stars. For this reason the approach of the stars continues, and a stable rate of accretion at the level $(10^{-10}–10^{-11})M_\odot$/yr i.e., near the observed value, is ensured. However, when the mass of the secondary star has been reduced to about $0.08M_\odot$ the star comes out of thermal equilibrium and becomes degenerate. Decrease of the mass of the star is then accompanied by an increase of its radius. As a result, the tendency for the radius of the orbit to increase becomes predominant, and the system passes through a stage of a minimal orbital period. According to calculations made under the simplest assumptions the minimal orbital period must be 65–70 min, and this is in good agreement with the observations and explains the cutoff in the period distribution. It is remarkable that a few well-known binary systems with much shorter periods (~ 11 min) can also be explained in the framework of these ideas. In these systems the secondary component is a helium star, and not a main sequence hydrogen star. The diameters of helium stars are approximately four times less, and this makes it possible for them to approach the primary component more closely. As a result, the binary system has a shorter minimal period. Similar models in which the dynamical role of gravitational radiation is decisive also explain the evolution and properties of some compact x-ray objects.

2.3. Type I supernovas

In the context of gravitational-wave astronomy supernova explosions are usually mentioned as powerful sources of gravitational radiation. However, gravitational waves also have a bearing on supernovas in a different sense. Indeed, they can be expected to play a decisive role in the dynamical evolution of stars, leading to supernova explosions of a definite type.

Supernova explosions are usually subdivided into types I and II. In their turn, the type I explosions have recently been distinguished into two subtypes.[14]

On the basis of all observational manifestations, type II supernovas correspond to the final stage of evolution of isolated massive stars, $M \gtrsim 9M_{\odot}$, that have not yet completely exhausted their hydrogen. In contrast, type supernovas represent explosions of not too massive but strongly evolved stars. The spectra of such explosions reveal no hydrogen or helium, but heavy elements are present. Type II supernovas are mainly observed in spiral arms of galaxies, where there are many young stars. In contrast, type I supernovas are distributed over the volume of galaxies more or less uniformly. Only type I supernovas are observed in elliptical galaxies, where, as a rule, the process of star formation has long ceased. The light curves of these supernovas, their spectra, and the expansion rates of the shells are rather similar. All this indicates a universal mechanism leading to these explosions. Type I supernovas are most probably the result of evolution of not too massive stars, $M \lesssim 8M_{\odot}$, in binary systems in which mass transfer processes take place actively.[5] (Indeed, one can see no reason for the explosion of a quiescent, long-lived star situated somewhere on the edge of an elliptical galaxy if it is not due to interaction with a close neighbor.) Evolution calculations in the framework of this model convincingly terminate with explosions having the characteristics of type I supernova explosions. These evolution tracks necessarily contain a stage in which gravitational waves play an important dynamical part. In this stage two degenerate white dwarfs are present in the system, and they approach each other solely under the influence of gravitational-wave damping. In the course of the subsequent evolution matter accretion from the less massive to the more massive star begins. The supernova explosion itself occurs when the accreting white dwarf has acquired a mass that exceeds the Chandrasekhar limit, $M \approx 1.4M_{\odot}$. The calcuated characteristics of the explosions, including the expected number of events in a typical galaxy, agree well with the actual observations.

3. THEORY AND SOME NEW RESULTS

3.1. Mathematical description of gravitational waves

The theoretical basis of gravitational-wave astronomy is the general theory of relativity. Gravitational waves are an unavoidable consequence of this theory. Einstein was the first who investigated them.[15]

It is well known that in the framework of general relativity the same quantities play the role of gravitational field variables and components of the space-time metric. In the usual, "geometrical" formulation of the theory the metric tensor $g_{\mu\nu}(x)$ of a curved space-time plays this dual role. The mathematical description of gravitational waves is usually constructed by a specification of the quantities $g_{\mu\nu}(x)$.

In the weak-field approximation the components $g_{\mu\nu}$ are represented in the form

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}, \tag{1}$$

where $\eta_{\mu\nu}$ can be regarded as the components of the metric of a flat Minkowski world. The quantities $\eta_{\mu\nu}$ have the values $\eta_{00} = -\eta_{11} = -\eta_{22} = -\eta_{33} = 1$, and the remaining $\eta_{\mu\nu}$ are zero. The oscillating corrections $h_{\mu\nu}(x,t)$ represent weak gravitational waves in the linear approximation. In this case one speaks of weak gravitational waves propagating on a flat background. The description of nonlinear effects would require allowance to be made for the subsequent terms in the expansion (1).

On the earth we are always concerned with weak gravitational waves. The weak-field approximation is also valid near the majority of astronomical sources of gravitational waves. It can break down only near sources that include supercompact massive bodies.

In cosmology one takes into account a background gravitational field of the homogeneous isotropic (Friedmann) universe, and therefore the expansion (1) is replaced by

$$g_{\mu\nu} \approx \gamma_{\mu\nu} + h_{\mu\nu}, \tag{2}$$

where $\gamma_{\mu\nu}$ is the metric of the Friedmann cosmology. We require this description only in Sec. 5, where we discuss cosmological gravitational waves.

It should be said that the notion of a dynamical gravitational field $h^{\mu\nu}$ defined on the background of some auxiliary space-time with metric tensor $\gamma_{\mu\nu}$ can be made exact and rigorous and is not simply approximate and valid only in some cases. In this way one can construct an entirely equivalent "field" formulation of general relativity. A convenient connection between $\gamma^{\mu\nu}$, $h^{\mu\nu}$, and $g^{\mu\nu}$ is given by

$$(-g)^{1/2} g^{\mu\nu} \equiv (-\gamma)^{1/2} (\gamma^{\mu\nu} + h^{\mu\nu}),$$

where in the case of a flat background space-time the quantities $\gamma_{\mu\nu}$ are the components of the metric of a Minkowski world expressed in arbitrary curvilinear coordinates. The resulting theory possesses all the necessary attributes of a "field" theory, namely, it contains a Lagrangian and action; the variational principle leads to field equations for dynamical variables that are completely equivalent to Einstein's equations; the standard rules can be used to obtain an energy-momentum tensor of the gravitational field (a tensor and not a pseudotensor!) and conservation laws that reflect the symmetry of the background space-time; and the theory admits a coordinate group and gauge group of transformations. In connection with the theory of gravitational radiation such an approach is helpful because it makes the analogy between gravitational and electromagnetic waves particularly clear.[29]

Using the gauge freedom, one can achieve fulfillment of the convenient (but not obligatory) subsidiary conditions $h^{\mu\nu}_{;\nu} = 0$ just as the gauge freedom in the theory of the electromagnetic field makes it possible to achieve fulfillment of the Lorentz gauge condition. After this, the (exact!) Einstein equations can be reduced to equations of a manifestly wave type:

$$\Box h_{\mu\nu} = \frac{16\pi G}{c^4}(t_{\mu\nu} + \tau_{\mu\nu}),$$

where $\Box$ is the d'Alembertian wave operator, $t_{\mu\nu}$ is the energy-momentum tensor of the gravitational field, and $\tau_{\mu\nu}$ is the energy-momentum tensor of the material sources.

Being identical to the "geometrical" formulation of general relativity in all experimentally verifiable conclusions, the "field" formulation of general relativity has the form of an ordinary field theory and is convenient in theoretical investigations. This approach is interesting, in particular, in that it gives an additional justification of the standard results derived earlier by means of the so-called energy–momentum pseudotensor. (For more details about the field formulation of general relativity, its properties, and the comparison of it with the ordinary, "geometrical" formulation, see Ref. 16.)

3.2. Relativistic celestial mechanics

In recent years there has been a lively discussion of the equations of motion of gravitating bodies with allowance for the periodic and secular relativistic corrections. This problem has not only fundamental but also practical interest, since its solution makes it possible to take into account rigorously and systematically all relativistic corrections up to the radiative approximation, inclusively. An example of an astronomical system for which this is necessary is the already mentioned binary pulsar PSR 1913 + 16. The most detailed results have been obtained for binary systems consisting of spherically symmetric nonrotating bodies. It is assumed that the relative velocity of the bodies is small, i.e., the slow-motion approximation, $v/c \ll 1$, is used. This problem has now been solved with a degree of rigor and completeness that is standard in ordinary celestial mechanics. The equations of motion for the bodies, each considered as a whole, are derived with allowance for all relativistic corrections to terms of order $(v/c)^5$ inclusively. All the periodic and secular relativistic corrections to the Keplerian orbit have been found. An explicit expression has been obtained for the gravitational radiation reaction force, and expressions have been found for dE/dt, dP^i/dt, and dL^i/dt, where E is the total energy of the binary system, P^i is its momentum, and L^i is its angular momentum. An explicit and complete expression has been found for the Lagrangian of the system, valid in the approximation up to $(v/c)^4$ inclusively, in which the system is still conservative. It has been shown that the theory can be applied to both extended and compact objects (such as neutron stars or black holes) irrespective of their internal structure. This is due to the fact that the equations of motion contain a single characteristic of the bodies, namely, their relativistic (Tolman) mass. It must be specially emphasized that the conclusions relating to the secular variation of the orbital parameters agree exactly with the conclusions obtained in the usual manner using Einstein's quadrupole expression. The exhaustive analysis of this problem was the outcome of many independent studies made by different methods but leading to partly overlapping or identical results (see, for example, Ref. 17).

4. SOURCES OF GRAVITATIONAL WAVES AND MODERN EXPERIMENTAL LIMITS

Sources of gravitational waves are usually classified as pulsed, periodic, and stochastic. A pulsed signal represents several oscillations of the field $h^{\mu\nu}$, the duration of the variable part of the signal being, at the least, short compared with the observation time.[3)] A periodic signal is modeled by a monochromatic wave, i.e., it is assumed that the amplitude and frequency of the signal vary little during the time of observation. A stochastic signal consists of steady-state noise with a fairly broad spectrum. We shall consider all forms of radiation successively, analyzing however only the astrophysical sources that seem to be the most interesting and certainly exist.

4.1. Pulsed sources

The greatest attention is usually devoted to pulsed gravitational radiation accompanying cosmic catastrophes. It is to this type of radiation that modern experimental programs are largely directed. Pulsed sources are represented in Fig. 1, in which we also indicate the achieved experimental limits and the expected sensitivity of some detectors.

The gravitational-wave field produced by a source can be expressed on the earth in terms of a characteristic dimensionless amplitude h as a function of the frequency ν in Hz. The great uncertainty in the structure of the source makes it possible to use simplified expressions to describe its radiation. If it is assumed that the two polarizations of a gravitational wave are on an equal footing and an averaging over the time is performed, then the energy flux density in a plane gravitational wave is given by

$$I = \frac{\pi c^3}{4G} h^2 \nu^2.$$

A source that is at distance r from the earth and during the time $T \approx 1/\nu$ radiates an energy $\Delta E = 4\pi^2 I/\nu$ gives rise to a characteristic amplitude

$$h \approx \frac{1}{r}\left(\frac{G}{\pi^2 c^3}\frac{\Delta E}{\nu}\right)^{1/2}. \qquad (3)$$

A typical source of pulsed gravitational radiation is a supernova explosion. A sufficiently massive isolated star with mass $M \gtrsim 8M_\odot$ can end its evolution in the form of a type II supernova explosion leading to the formation of a neutron star. Calculations of the gravitational-wave energy release are rather complicated and ambiguous (see Ref. 18 and the recent review of Ref. 2), but the results are grouped near certain mean values that follow from the most general arguments. Indeed, the characteristic time scale for the final collapse stages and oscillations of the stellar core is $3\cdot 10^{-3}$–$3\cdot 10^{-4}$ sec, i.e., $\nu \approx 3\cdot 10^2$–$3\cdot 10^3$ Hz. At the same time the nonsphericity of the core due to rotation, a magnetic field, or

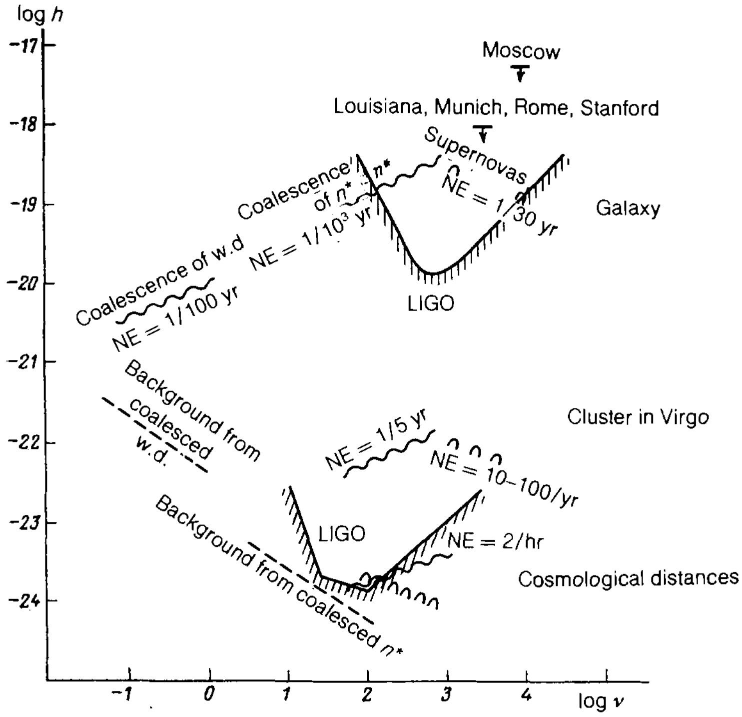

FIG. 1. Pulsed sources (w.d. stands for white dwarfs, NE for the number of events).

possible instabilities is hardly less than 1%, and this leads to the estimate $\Delta E = 10^{-4} M_\odot c^2$ of the gravitational-wave energy release. Such an estimate is not excessively great; it is about 0.1% of the expected neutrino energy release. We use this estimate of ΔE and the expression (3) to find the values of h:

$$h \approx 2 \cdot 10^{-19} \cdot \frac{10\,\mathrm{kpc}}{r} \left(\frac{\Delta E}{10^{-4} M_\odot c^2}\right)^{1/2} \left(\frac{10^3\,\mathrm{Hz}}{\nu}\right)^{1/2}.$$

In principle, similar values of h can also be characteristic for type I supernova explosions and "quiet" collapses that are not manifested in the optical range.

The amplitude from an individual event decreases with increasing distance as $1/r$. The expected number of events (NE) increases in proportion to the considered volume of space. In our Galaxy the events in which we are interested may occur about every 30 years. Assuming, very roughly, that in each volume 3 Mpc^3 there is one galaxy and that events occur in it with the same rate, we can estimate h and the number of events for different distances. In Fig. 1 these quantities are given for events occurring, respectively, in the center of the Galaxy ($r = 10$ kpc), in a volume that includes the cluster of galaxies in the constellation Virgo ($r = 20$ Mpc, three thousand galaxies), and in distant galaxies at cosmological distances ($r = (1\text{–}6) \cdot 10^3$ Mpc). Unfortunately, there are large uncertainties, both in the signal amplitude and in the number of expected events.

Better known are the amplitude and form of the gravitational field produced in the process of approach to each other through the radiation of gravitational waves and the subsequent coalescence of a pair of compact objects—neutron stars, white dwarfs, or even black holes. In the initial stage of this process a quasiperiodic signal is radiated, and the final stage has the nature of a burst. The importance of this process is emphasized by Thorne.[2]

Two bodies of comparable mass m in a circular orbit produce at distance r from them the characteristic amplitude[19]

$$h \approx 4 \frac{G^{5/3}}{c^4} \frac{1}{r} m^{5/3} \nu^{2/3}, \tag{4}$$

and this can be rewritten in the form

$$h \approx 4 \frac{r_g}{r} \left(\frac{v}{c}\right)^2,$$

where $r_g = 2Gm/c^2$ is the gravitational radius of a body of mass m, and v is the orbital velocity. The coalescence of neutron stars certainly ends when they touch, and this corresponds to a maximal radiated frequency of $\nu \approx 10^3$ Hz and $(v/c) \approx 10^{-2}$. Under these conditions $h = 5 \cdot 10^{-19}$ from a source at distance $r = 10$ kpc.

The coalescence of neutron stars is a powerful but very rare process. In a galaxy like ours one event may occur during several thousand years. The number of events becomes acceptable from the point of view of observations if one includes in a survey galaxies at cosmological distances (see the estimate of the number of events in Fig. 1). At frequencies $10^2 \lesssim \nu \lesssim 10^3$ Hz the radiation takes the form of individual events, i.e., not more than one radiating system is in the frequency range $\Delta\nu \approx \nu$. The signal from an individual event is shown in Fig. 1 by the wavy line with legend "Coalescence of n^*." Events occurring at different distances correspond to the different wavy lines. At frequencies $\nu \lesssim 10^2$ Hz there are many radiating systems, their signals overlap, and they form a stochastic background. The characteristic values of $h(\nu)$ at these frequencies are $N^{1/2}$ times greater than for an individual system (N is the number of systems in the range of frequencies $\Delta\nu \approx \nu$). The spectrum of the rms amplitude of all such sources at all distances up to the Hubble radius is shown in Fig. 1 by the broken line "Background from coalesced n^*".[20]

The quasiperiodic nature of the radiation from an individual coalesced binary system can be used to enhance the sensitivity of a detector in a specially arranged experiment. The point is that one can repeat measurements during several tens of periods n before the signal frequency changes appreciably. In other words, one can reduce the band $\Delta\nu_n$ of frequencies that contribute to the detector noise, $\Delta\nu_n = \nu/n$. As a result, the sensitivity to amplitude h is increased by $n^{1/2}$ times, where $n = \nu^2/\dot{\nu}$, in which ν is the variable frequency of the radiated wave.[2] In the final stage of coalescence n20–30.

The well-developed theory of the coalescence process makes it possible in principle to extract important additional information from the observations. For example, simultaneous measurement of the dependences $h(t)$ and $\nu(t)$ could enable one to determine the absolute distance to the source, since one can eliminate the unknown masses of the stars that form the binary system.[21]

The coalescence of white dwarfs is a more frequent evolutionary process than the coalescence of neutron stars. However, because of the greater radius of white dwarfs the coalescence ends at a lower frequency and smaller amplitude of the radiated gravitational wave. An individual event at the center of the Galaxy gives the signal shown in Fig. 1 by the wavy line with the legend "Coalescence of w.d. (white dwarfs)." The totality of such events in the volume of space with Hubble radius forms a stochastic background. With allowance for the number of systems radiating in the given range $\Delta\nu \approx \nu$, we obtain the spectrum shown in Fig. 1 by the line "Background from coalesced w.d."[20]

The experimental limits achieved by different groups are indicated in Fig. 1 by the arrows. Also indicated is the level of the expected accuracy of the Laser-Interferometric Gravitational Observatory (LIGO) created by the collaboration of Caltech and MIT in the United States.[44] It can be seen from Fig. 1 that the second stage of the experiment (lower curve) must permit reliable detection of numerous astrophysical sources of gravitational waves. It should be emphasized that analogous projects are being realized in several countries. (The experimental possibilities are discussed in more detail in Sec. 6.)

4.2. Periodic sources

We now turn to periodic sources of gravitational waves (Fig. 2). A typical source is a close binary star, for example, ι Bootae. It follows from Eq. (4) that the characteristic amplitude of the gravitational waves on the earth is

$$h \approx 10^{-21} \cdot \frac{100\,\mathrm{pc}}{r}\left(\frac{m}{M_\odot}\right)^{5/3}\left(\frac{\nu}{10^{-4}\,\mathrm{Hz}}\right)^{2/3}.$$

For simplicity we consider a pair of stars of the same mass in a circular orbit. For compact systems radiating relatively strongly the orbits always become circular. A list of specific binary systems in the Galaxy is given in Ref. 22.

The complete set of binary stars in the Galaxy produces a stochastic background. Its characteristics depend on the distribution of the stars with respect to the masses, the orbital parameters, and the evolutionary scenarios. The results of the new calculations of the spectrun of the rms amplitude are given in Fig. 2 (from Ref. 20, see also Ref. 23). The maximum of the spectrum is produced by a large number of systems: $N \approx 3 \cdot 10^7$. The spectrum ceases to be continuous in the region of frequencies $\nu \approx 10^{-2}$ Hz. At higher frequencies there is more than one system in a range of frequencies $\Delta\nu \approx \nu$.

In Fig. 2 we also give the expected values of h from individual pulsars under possible assumptions about their deformed state. The upper limit for h is obtained under the assumption that the entire observed spin-down of the pulsar is due to gravitational-wave radiative damping. More realis-

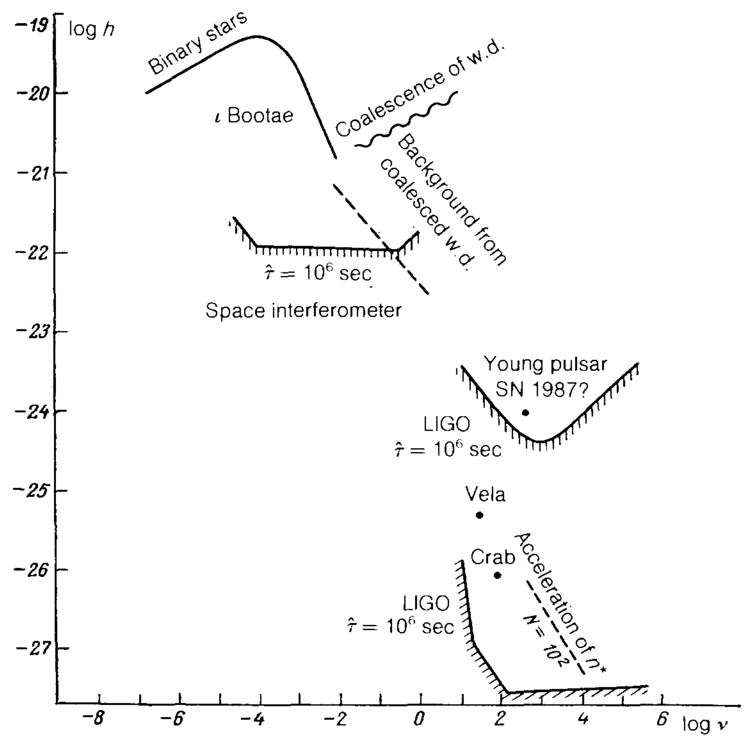

FIG. 2. Periodic sources.

tic estimates of the deviations from axial symmetry for some known pulsars lead to the values of h given in Fig. 2.[24]

The greater the asymmetry of a rotating star, the more strongly (under otherwise equal conditions) it radiates and the sooner the rotation energy is carried away by gravitational waves. The rotational energy is $J\Omega^2/2$, where J is the moment of inertia, $\Omega = 2\pi\nu/2$ is the circular frequency of the radiation, and ν is the frequency of the radiated gravitational wave. We denote the spin-down time due to the radiation of gravitational waves by $\tau/2$. Then, equating $(J\Omega^2/\tau)/4\pi r^2$ to the energy flux density I, we find the characteristic h:

$$h \approx \frac{1}{r}\left(\frac{GJ}{c^3\tau}\right)^{1/2}.$$

Instead of uncertainty in the asymmetry of the star we here have an uncertainty in τ. The typical value of J for neutron stars is $J \approx 10^{45}$ g·cm^2. We make some estimates of h.

It is widely believed that pulsars are born slowly rotating and do not have appreciable deformation. But the opposite point of view, according to which the birth of neutron stars does not take place "smoothly," is not ruled out. To be definite, let us take $\tau \approx 10^8$ sec = 3 yr. Then, as follows from the expression for h, a young neutron star born at distance $r = 50$ kpc produces $h = 10^{-24}$ on the earth, and this is shown in Fig. 2. The "de-excitation" time $\tau = 10^8$ sec is not chosen at random. On the one hand, it corresponds to a not too large relative deformation of the star, $\varepsilon \approx 10^{-2}$, and gives a comparatively large h. On the other hand, during this time it would be possible, if an optical identification of the source were made, to organize gravitational-wave observations, using fully, moreover, the accumulation of the quasiperiodic signal. It is assumed, of course, that during this time the asymmetry does not relax because of any internal reasons. These considerations may be especially topical in connection with the recent supernova explosion in the Large Magellanic Cloud ($r \approx 55$ kpc) and the expected creation (during 1990?) of laser-interferometric observatories of the LIGO type. It can be seen from Fig. 2 that such detectors could readily observe gravitational waves from this object if a source with parameters close to the expeted values was formed as a result of the supernova SN 1987A.

In Fig. 2 we also give the expected level of the radiation from an accreting neutron star whose angular velocity exceeded the critical value, causing the star to be strongly deformed and become a strong source of gravitational waves.[25] The broken line drawn in Fig. 2 takes into account the factor $N^{1/2}$, where $N \approx 100$ is the expected number of such systems in the Galaxy.

Periodicity of a signal means that the experimentalist has the possibility of prolonged separation of the signal from the noise. Figure 2 gives the expected sensitivity of ground-based laser-interferometric antennas (a project of LIGO type) for accumulation time $\hat{\tau} \sim 10^6$ sec.

Particularly promising is the project of a laser interferometer in space.[26] The lengths of the arms of a space interferometer could be 10^6–10^8 km. Such a detector is most sensitive in the range of frequencies in which binary stars radiate. A detector could be used to observe the radiation of an individual system and of the stochastic background. The sensitivity of a space laser interferometer to periodic signals for accumulation time $\hat{\tau} \approx 10^6$ sec is shown in Fig. 2.

We note that the technique of Doppler tracking of spacecraft can also work in the regime of an interferometer. In this case the requirements on the stability of the frequency oscillator, the basis of the Doppler tracking technique, are not so stringent, and as a result the sensitivity of this technique to gravitational-wave signals may be increased.

5. STOCHASTIC BACKGROUND OF GRAVITATIONAL WAVES AND THE EARLY UNIVERSE

As we have seen, a stochastic background of gravitational waves can be formed by a collection of periodic or pulsed sources existing at the present epoch. However, most interesting of all is the primordial, or cosmological, stochastic background of gravitational waves whose origin is associated with quantum processes in the very early universe. The detection of such a background of relic gravitons promises to give most important information about the physical conditions that obtained in the Planck epoch. It might even make it possible to establish experimentally whether or not the creation of the universe was associated with a process of quantum-mechanical tunneling.[27]

Besides relic gravitational radiation, there could exist other stochastic components produced at later epochs: in the period of nucleosynthesis, in phase transitions, in the stage of formation of galaxies or primordial stars, etc. However, we shall not consider them in detail here.

5.1. Quantum production of gravitons

We recall the main features of the mechanism of quantum production of gravitons leading to the formation of a relic stochastic background.[28,29,6] It is particularly appropriate to recall the details of this mechanism because it is precisely the same principles that are today widely used in modern inflationary cosmology to explain the origin of the primordial fluctuations of the scalar fields capable of leading to density perturbations and, ultimately, to the formation of the observed galactic structure (see, for example, Refs. 30 and 31).

A gravitational-wave perturbation

$$h = \frac{1}{a(\eta)}\,\mu(\eta)\,e^{inx},$$

interacting with the background gravitational field of the isotropic universe,

$$ds^2 = a^2(\eta)\,(d\eta^2 - dx^2 - dy^2 - dz^2),$$

satisfies the equation

$$\mu'' + \mu\left(n^2 - \frac{a''}{a}\right) = 0, \tag{5}$$

where $a(\eta)$ is the scale factor of the homogeneous isotropic cosmological model, and n is the dimensionless wave number, expressed in fractions of the scale factor $a(\eta)$. The wavelength of a gravitational wave with wave number n is $\lambda_g = 2\pi a/n$. The spatial dependence of the perturbation always maintains the form e^{inx}.

The physical interpretation of Eq. (5) is obvious—it is the equation for an oscillator parametrically excited by interaction with the external gravitational field by a change of the oscillator frequency. This same equation can be regarded as a Schrödinger equation with potential $U(\eta) = a''/a$ for a particle with energy n^2. In Eq. (5) the variable η plays in this

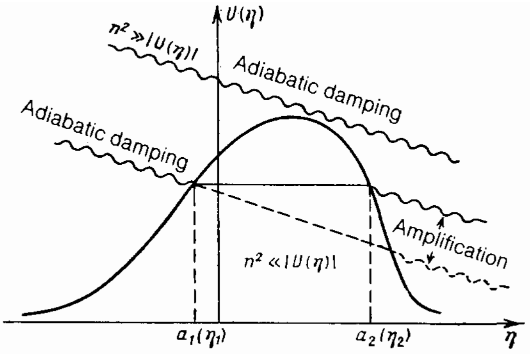

FIG. 3. Amplification of waves.

case the role of a spatial coordinate. An arbitrarily varying scale factor $a(\eta)$ creates a potential $U(\eta) \neq 0$ if $a''/a \neq 0$.

Suppose there is a continuous expansion of the universe, i.e., an increase of $a(\eta)$ with increasing η. A typical potential is shown in Fig. 3. In the regions in which the wave with a given n satisfies the condition $n^2 \gg |U(\eta)|$ Eq. (5) takes the form $\mu'' + n^2\mu = 0$, and the wave number changes in accordance with the law

$$h = \frac{A}{a} \sin(n\eta + \varphi) e^{inx}, \quad A = \text{const}, \quad \varphi = \text{const}. \quad (6)$$

On the background of a slowly increasing function $a(\eta)$ the numerical value of the wave amplitude decreases adiabatically in proportion to a^{-1}, and the energy density in proportion to a^{-4}. For simplicity we assume in this discussion that the conditions $n^2 \gg |U(\eta)|$ and $n^2 \gg (a'/a)^2$ are approximately the same, although in the general case this is not so.

However, in a region in which $n^2 \ll |U(\eta)|$ Eq. (5) takes the form $\mu'' - \mu(a''/a) = 0$, and its solution has the form[54]

$$h_1 = C_1 e^{inx}, \; h_2 = C_2 \int a^{-2}\, d\eta e^{inx};$$

$$C_1 = \text{const}, \quad C_2 = \text{const}.$$

After passing through the barrier and entering the region in which again $n^2 \gg |U(\eta)|$ the "typical" wave will have a greater amplitude than it would in accordance with the adiabatic law of variation. By the amplitude of a "typical" wave we understand the rms amplitude obtained by squaring, averaging with respect to the arbitrary initial phase φ, and taking the square root. One can say that below the barrier the wave amplitude remains practically constant in accordance with the dominant solution h_1, despite the increase of the scale factor $a(\eta)$. (If the wave were to behave in accordance with the adiabatic law in the region $a_1 < a < a_2$ occupied by the potential $U(\eta)$, then with increasing $a(\eta)$ its amplitude would decrease, as indicated by the broken line in Fig. 3.) Thus, whereas to the left of the barrier the wave had the form (6), after passing through the barrier the wave with given number n behaves approximately in accordance with the law

$$h \approx \frac{A}{a} \frac{a_2}{a_1} e^{in\eta} e^{inx}, \quad \frac{a_2}{a_1} \gg 1.$$

The amplitude of such a wave, after it has made one complete oscillation subsequent to its emergence from below the barrier (i.e., an oscillation with length comparable to the current Hubble radius), decreases adiabatically with increasing $a(\eta)$. The amplification coefficient, i.e., the ratio of the actual amplitude to what the wave would have, had it varied throughout the time in accordance with the adiabatic law, is approximately equal to $a_2/a_1 \gg 1$.

It is interesting that also in the case of contraction a "typical" wave is amplified, with, moreover the same coefficient a_2/a_1. This happens because in the case of contraction the below-barrier solution h_2 is dominant, and in an approximate description of the potential in the form $U(\eta) = \text{const}$ this solution is given by a quantity proportional to $(a_2/a_1)^2$. One can say that instead of adiabatic growth by a_2/a_1 times the wave amplitude actually grows below the barrier by $(a_2/a_1)^2$ times.

These conclusions are the essence of the phenomenon of superadiabatic amplification of classical waves and zero-point quantum fluctuations.

It can be seen from Fig. 3 that waves with different n are amplified to different degrees, and this leads to a transformation of the initial fluctuation spectrum (for example, the spectrum of zero-point quantum fluctuations) into the final spectrum. Different assumptions about the behavior of the scale factor in the early universe lead to different predictions about the amplitude $\langle h^2 \rangle$, the energy density ε_g, and the spectrum of the stochastic background at the present epoch.

5.2. Observational bounds on the intensity of the stochastic background and physics of the early universe

To facilitate the transition to the quantities with which the experimentalist operates, it is convenient to use the notation

$$h(x,\ t) \sim \int h_k e^{ikx} d^3k.$$

The statistical independence of waves with different wave vectors k and k' means that

$$\langle h_k h_{k'} \rangle = h_\nu^2 \delta(k - k'),$$

whence

$$\langle h^2 \rangle \sim \int h_\nu^2 \nu^2\, d\nu = \int h^2(\nu) \frac{d\nu}{\nu}, \quad h(\nu) \equiv (h_\nu \nu^3)^{1/2},$$

and the energy density is

$$\varepsilon_g = \int \varepsilon_\nu\, d\nu = \int \varepsilon(\nu) \frac{d\nu}{\nu} \sim \int h^2(\nu) \nu^2 \frac{d\nu}{\nu}, \quad \varepsilon(\nu) \equiv \varepsilon_\nu \nu.$$

The spectrum of the gravitational-wave background can be characterized by the dimensionless quantity $h(\nu)$ or by $\varepsilon(\nu)$, which has the dimensions erg/cm^3. Frequently, one also uses the parameter

$$\Omega_g = \frac{\varepsilon(\nu)}{\varepsilon_{cr}},$$

where ε_{cr} is the critical cosmological energy density.

The dependence $a(\eta)$ and, therefore, the form of the potential $U(\eta)$ can be expressed by virtue of Einstein's equations in terms of an effective equation of state $p(\varepsilon)$. If in the hadronic epoch, i.e., for 10^{-43} sec $< t < 10^{-6}$ sec, there were deviations from $p = \varepsilon/3$ in the direction of harder equations of state, i.e., $p > \varepsilon/3$, they must have led to "violet" spectra of the relic gravitons, in which the total energy density ε_g is determined by the high-frequency end of the spectrum. A negative effect pressure, $p < 0$, usually leads to "red" spectra.

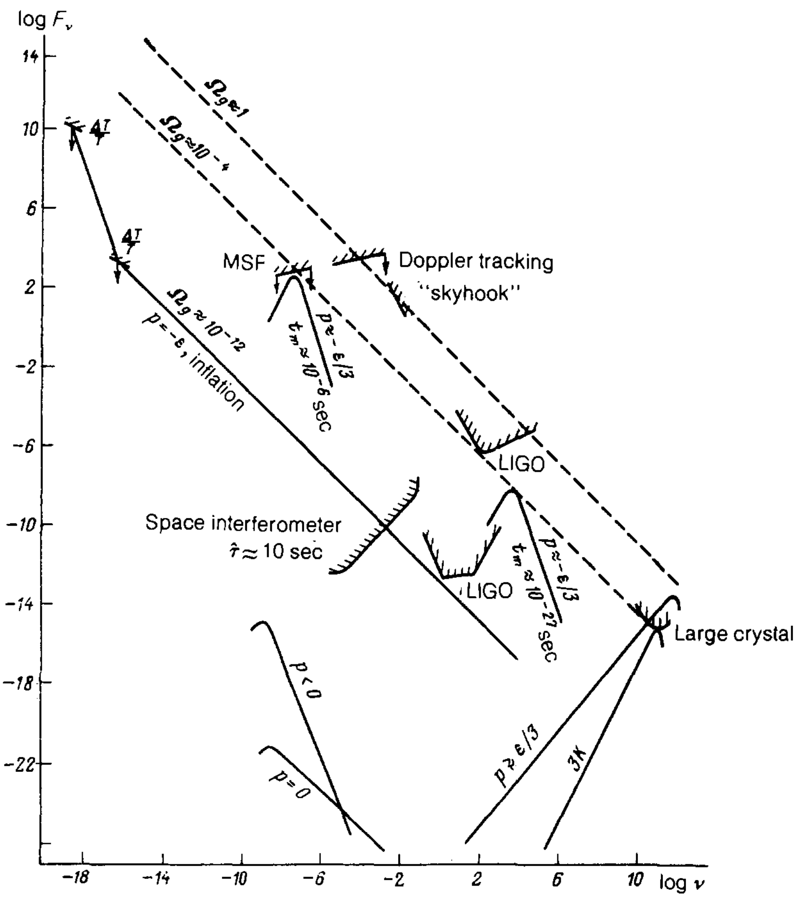

FIG. 4. The stochastic background.

Some possible spectra for different $p(\varepsilon)$ are shown in Fig. 4 by the thin continuous lines (see also Ref. 32). The spectra are expressed in terms of the spectral flux density F_ν ($\mathrm{erg \cdot sec^{-1} \cdot cm^{-2} \cdot Hz^{-1} \cdot sr^{-1}}$) as a function of the frequency, and

$$\varepsilon_g = \frac{4\pi}{c} \int F_\nu \, d\nu.$$

For comparison we show the spectrum of the 3-degree microwave background. We denote its energy density by ε_γ.

A too hard an effective equation of state, say $p \approx \varepsilon$, could not have occurred in the hadronic epoch, since it predicts an inadmissibly high density of relic gravitons, $\varepsilon_g/\varepsilon_\gamma \gg 1$, in the high-frequency range, $\nu \approx 10^{12}$ Hz. The parameters of the inflationary model of the early universe (effective equation of state $p = -\varepsilon$) are, in contrast, restricted by the amplitude of the gravitational waves in the region of the lowest frequencies, $\nu \approx 10^{-18}$–10^{-16} Hz. The spectrum of $\langle h^2 \rangle$ immediately after the end of the inflationary stage must be "flat," as readily follows from the elementary considerations presented above. Indeed, the spectrum of the zero-point fluctuations is determined by the spectral component

$$h_n = \frac{1}{(2n)^{1/2}} e^{in\eta}.$$

The scale factor has the form $a(\eta) = -1/\eta$ (the parameter η is negative and increases from $-\infty$). A wave with a given n begins to be amplified, i.e., its amplitude becomes constant in place of the adiabatic decrease, from a certain time η_i, which is determined by the condition $n^2 \eta_i^2 \approx 1$. Long waves, i.e., waves with smaller n, are below the barrier for a longer time. The amplification coefficient is equal to the ratio of the scale factors at the beginning and end of the amplification. In the given case, this reduces to multiplication of h_n by n^{-1}. As a result, at the end of the inflationary stage we obtain the spectrum

$$h_n \approx \frac{1}{(2n)^{1/2}} \frac{1}{n} e^{in\eta} \approx n^{-3/2} e^{in\eta}.$$

Ignoring for the time being the fact that $n\eta \ll 1$, and assuming that the waves "already oscillate," we obtain

$$h^2(n) \approx h_n^2 n^3 \sim n^0, \quad \langle h^2 \rangle \sim \int \frac{dn}{n}.$$

In accordance with the standard terminology, this spectrum is called the "flat" Harrison-Zel'dovich spectrum.[33] (It is the analog of the $1/f$ spectrum well known in radio physics.) Since $h^2(n)$ does not depend on n, this spectrum can be characterized by the assertion that if in the post-inflationary epoch the waves were neither weakened nor amplified, then they would begin their adiabatic evolution (when the wavelength becomes less than the Hubble radius) with the same amplitude irrespective of the wavelength. In reality $U(\eta) = 0$ in the radiation-dominated stage, for which $p = \varepsilon/3$, but then, in the matter-dominated stage, when $p = 0$, the potential $U(\eta)$ is again nonzero. In this stage the waves with the longest wavelength—the waves whose present frequencies are less than $\nu \approx 10^{-16}$ Hz-are additionally amplified.[7] The remaining waves undergo an ordinary transition from the region in which their wavelength is greater than the Hubble radius to the regime in which the wavelength becomes less than the Hubble radius. In addition, the waves with shorter wavelengths are damped adiabatically for a longer time, and as a result the spectrum at the present epoch in the range of frequencies $10^9 > \nu > 10^{-16}$ Hz has the form $h(\nu) \sim \nu^{-1}$, $\varepsilon(\nu) \sim \nu^0$. (For more details about gravitons in the inflationary model, see Refs. 7, 31, and 34.)

The possible inflationary spectrum, restricted at the lowest frequencies by the existing observational limits on the angular anisotropy of the electromagnetic microwave background, $\Delta T/T$, is shown in Fig. 4. For waves with frequencies $\nu \gtrsim 10^{-16}$ Hz the corresponding Ω_g must be $\Omega_g \lesssim 10^{-12}$.

For comparison we have plotted in Fig. 4 two spectra of relic gravitons that could be formed in models having at a definite stage of evolution from $t = t_{\rm Pl}$ to a certain $t = t_{\rm m}$ the effective equation of state $p = -\varepsilon/3$. The effective equation of state $p = -\varepsilon/3$ corresponds to a scale factor $a(\eta) \sim e^{\eta}$. This equation of state is interesting not only because it gives the simplest potential $U(\eta)$ in the form of a Π-shaped step of finite width but also because $p = -\varepsilon/3$ arises naturally in some models of the early universe, for example, in a class of solutions for a massive scalar field. (The same equation of state characterizes a collection of cosmic strings.[36]) The actual spectra shown in Fig. 4 differ in the assumption made about the time $t_{\rm m}$ at which the effective equation of state $p = -\varepsilon/3$ is replaced by the ordinary $p = \varepsilon/3$. In the first case it was assumed that $t_{\rm m} = 10^{-6}$ sec, in the second $t_{\rm m} = 10^{-27}$ sec. Accordingly, in the first case the spectrum is concentrated in the region of the frequency $\nu \approx 3 \cdot 10^{-7}$ Hz, in the second of $\nu \approx 10^4$ Hz. Essentially, the situation reduces to an "injection" of gravitons with an energy density equal to the energy density of the background matter at the time $t_{\rm m}$ and a subsequent adiabatic cooling of the gravitons, beginning at $t_{\rm m}$, until our epoch.

The broken lines in Fig. 4 show the positions of the maxima of broad spectra, with frequency band $\Delta\nu \approx \nu$, that give when integrated $\varepsilon \approx \varepsilon_{\rm cr}$ ($\Omega_g = 1$) or $\varepsilon_g \approx \varepsilon_\gamma$ ($\Omega_g = 10^{-4}$). In the second case the characteristic $h(\nu)$ is expressed by the equation $h(\nu) \approx 10^{-20}/\nu$.

We now turn to the discussion of the existing experimental bounds on the intensity of the stochastic background. In Fig. 4 they are indicated by the lines with arrows. In the lowest frequency range the bounds follow from measurements of the angular variations of the microwave background, $\Delta T/T$. These measurements establish a bound on the amplitudes of waves with not only wavelength of the order of the Hubble radius but also with longer wavelengths.[37]

Waves with wavelength of the order of the present Hubble radius ($\nu_{\rm H} \approx 2 \cdot 10^{-18}$ Hz) make the largest contribution to the quadrupole component of $\Delta T/T$. Since $\Delta T/T < 10^{-4}$, we have approximately $h < 10^{-4}$, and Ω_g for these waves is $\Omega_g \approx h^2 < 10^{-8}$. Waves whose wavelength was comparable with the current Hubble radius at the "recombination" epoch, i.e., when the relic photons became free, make the largest contribution to $\Delta T/T$ in the range of angles of a few degrees. The present frequency of these waves is $\nu \approx 10^{-16}$ Hz. It again follows from the experimental data on $\Delta T/T$ that in this region of wavelengths $h < 10^{-4}$ at the recombination epoch and, therefore, $h < 10^{-8}$ today. Since the frequency is two orders of magnitude greater than $\nu_{\rm H}$, the corresponding $\Omega_g \approx h^2(\nu/\nu_{\rm H})^2 < 10^{-12}$. The bounds on Ω_g in the region of still shorter wavelengths are not so effective, since the $\Delta T/T$ produced by these waves is "washed out" by the noninstantaneous nature of the transition from the radiation-dominated to the matter-dominated stage.[38]

Monitoring the arrival time of the pulses of the millisecond pulsar PSR 1937 + 21 (MSP) gives information about waves with a period of a few years. This technique already leads to nontrivial bounds in terms of Ω_g. In accordance with the recent data (Ref. 3) $\Omega_g \leqslant 10^{-6}$ in the range of frequencies $\nu \approx 10^{-8}$ Hz. A further increase in the duration of the pulsar observations will significantly strengthen the bounds on Ω_g provided the experimentalist has a good frequency standard at his disposal. Such a standard is needed to hold throughout the entire time of observations the phase of the oscillations, which is then compared with the successively arriving pulsar pulses. The strengthening of the bounds on Ω_g occurs not so much because of the statistical reduction of the errors of the observations as through the fact that an increase in the duration of the observations amounts to a displacement of the maximal sensitivity to the region of longer and longer wavelengths. To the same bound on h there correspond ever more stringent bounds on Ω_g with increasing wavelength, as can be clearly seen in Fig. 4. In principle, this technique is very sensitive in terms of Ω_g to long waves, $\lambda_g \gg cT$, where T is the total time of observation. But long waves are manifested in very slow secular variations of the pulse arrival time. However, the same changes occur for a more prosaic reason—secular variation of the frequency of the pulsar itself. As we are unable to separate them, we are forced to attribute all low-frequency components to the pulsar and to discard them by means of "polynomial fitting" of the arrival time. As a result, this technique is most sensitive to waves with $\lambda_g \approx cT$.[39]

For waves with frequencies greater than 10^{-7} Hz indirect bounds at the level $\Omega_g \lesssim 10^{-4}$ are determined by the conditions of nucleosynthesis in the early universe.[40] The reason is that too high a density of gravitons, neutrinos, or other massless particles would change the expansion rate of the universe at the nucleosynthesis epoch, as a result of which the amount of synthesized helium would be too great and come into contradiction with observations.

Finally, certain experimental restrictions on the stochastic background at the level $\Omega_g \approx \Omega_{\rm cr}$ follow from Doppler space observations in the range of frequencies 10^{-2}–10^{-4} Hz.[41]

The prospects for stronger restrictions on Ω_g in different ranges of the spectrum appear promising. Figure 4 shows that the project of a laser interferometer in space is particularly impressive. However, it must be said that besides the overcoming of the technical problems there is also the problem of the stochastic noise produced by binary stars in the Galaxy. It will evidently be necessary to make a careful search for a suitable range of frequencies and, possibly, have a special orientation of the directional diagram of the gravitational antenna outside the plane of the Galaxy.

Figure 4 also shows the expected sensitivity of the other possible methods of observation: an improved variant of Doppler tracking of spacecraft,[42] the "skyhook,"[43] and ground-based laser interferometer of the LIGO type.[2,44]

The possibility of using special crystal antennas to detect the background in the high-frequency region of the spectrum[45] warrants particular attention. The expected sensitivity level is identified in Fig. 4 by the line "Large crystal." (This proposal is discussed in more detail below, in Sec. 7.2.) It is important to emphasize that in this range of frequencies ($\nu \approx 10^{10}$–10^{11} Hz) there may be concentrated not only the high-frequency end of the nonthermal spectra but also the maximum of the thermal background of relic gravitons with a temperature of order 1°K.

It should be mentioned that a stochastic background in various ranges can also be produced by other processes in the early universe,[46] but they appear more problematic.

6. DETECTION OF GRAVITATIONAL WAVES

6.1. Brief description of detectors

Gravitational-wave antennas can be nominally divided into two large groups: oscillators and free bodies. The first group includes the ordinary solid (resonant bar) antennas (Weber cylinders) and some natural detectors such as blocks in the earth's crust,[47] normal vibrations of the earth and the planets, binary systems,[48] etc.

The present limiting sensitivity of solid detectors to gravitational wave bursts is at the level $h \sim 5 \cdot 10^{-18}$–10^{-18}. There is a gradual increase in the sensitivity—slow but steady.[49]

In the second group of detectors we have ground-based and space laser interferometers, Doppler tracking of spacecraft, chronometry of the arrival of pulsar pulses, and measurement of the angular anisotropy of the 3°K microwave electromagnetic background. These systems, and also "true" electromagnetic detectors in the form of electromagnetic resonators are distinguished by the essential use in some form or other of electromagnetic fields. These detectors are called "electromagnetically coupled" detectors. The description of the principles of operation of these detectors is based on the well-developed theory of the propagation of electromagnetic waves in a weak gravitational-wave field for given boundary conditions (see, for example, the reviews of Refs. 32 and 50).

Optimistic and imminent hopes are associated with the construction of ground-based laser interferometers. The construction of these detectors has now become a matter of international effort. Phototypes of these systems already successfully compete with the solid detectors. Similar projects for large systems are at different stages of development in the United States, Great Britain (Glasgow), the German Federal Republic (Munich), and France (Orsay).[4)] We shall briefly consider one of the projects: LIGO.[2,44]

The observatory will be situated in two places—on the western and eastern coasts of the United States. At each location there will be one, two, or several interferometers. The maximal length of an interferometer arm is 4 km. The entire system will be kept under vacuum conditions and seismically insulated. The interferometers will be equipped with mirrors with very low losses. Later it is intended to use new technical methods—such as "light circulation," this being equivalent to raising the laser power with the ultimate possibility of increasing the detector sensitivity. It is intended to commission the first stage of the observatory by 1990 or a little later. The detectors will probably work at the maximal sensitivity level by the year 2000. The two corresponding sensitivity levels are shown by the upper and lower lines LIGO in Figs. 1, 2, and 4.

6.2. Noise and sensitivity

As usual, the sensitivity of gravitational-wave detectors is determined by the ratio of the useful signal to the noise. In solid detectors the first type of noise with which the experimentalist must cope is the unavoidable thermal noise of the antenna, which has the consequence that the antenna executes randomly modulated vibrations. In laser interferometers shot noise, i.e., the fluctuations of the light intensity and the associated fluctuations of the phase, is unavoidable. Of course, the subsequent cascades of detectors also generate noise, and therefore the minimal detectable h, $h_{\min}$, calculated with allowance for only the thermal or shot noise, is often too optimistic. Such a calculation presupposes that the contribution of other noise can be ignored. Nevertheless, such estimates do give an idea of the potential sensitivity of the various detectors.

The detector sensitivity depends on the form in which the signal is realized—as periodic wave, short burst, or stochastic noise. It is clear that the possibility of prolonged accumulation of the signal in the case of detection of a periodic wave or stationary noise rasies the detector sensitivity. This is why the numerical value of $h_{\min}$ can differ strongly for the same detector depending on the signal that is defected.

Simple estimates of $h_{\min}$ for three types of gravitational-wave radiation and with allowance for the thermal noise of the solid antennas can be found, for example, in Ref. 29. Clarification of the possibilities of the large class of "electromagnetically coupled" detectors requires consideration of the electrodynamic equations in the field of gravitational waves.

In the approximation of geometrical optics the equations are very simple. The change in the frequency ω of an electromagnetic signal emitted at time t_e by one free body and received at time t_0 by another free body is

$$\frac{\Delta\omega}{\omega} \equiv z(t_0) = \int_{t_e}^{t_0} \frac{\partial h_{ik}}{\partial t} n^i n^k \, dt, \qquad (7)$$

where n^i is the unit vector in the direction of the source.

In a gravitational-wave field with components h_{ik} that are arbitrary functions of the argument $u = (\omega_g/c)(x_0 + k_i x^i)$, $k_i k^i = 1$, the expression (7) reduces to

$$\frac{\Delta\omega}{\omega}(t_0) = \frac{1}{2} \frac{n^i n^k}{1 - k_i n^i} (h_{ik}(0) - h_{ik}(\mathrm{e})), \qquad (8)$$

where $h_{ik}(\mathrm{e})$ and $h_{ik}(0)$ are, respectively, the values of h_{ik} at the point events of transmission of the electromagnetic signal from the one body and its reception at the other. If the signal is emitted from one body, reflected from the other, and received back at the first, as in the procedure of Doppler tracking of spacecraft, then in the expression (8) the event e is replaced by the signal reflection event and to the right-hand side of (8) there is added a further term containing the values of h_{ik} at the time of emission of the signal from the first body. In connection with the detection of short gravitational-wave bursts such an expression was first discussed in Ref. 51.

For a continuously radiating source of electromagnetic waves z is a continuous function of the current time of reception t_0. As follows directly from (7), the frequency of a continuous electromagnetic signal received at the point of observation acquires in the field of a monochromatic gravitational wave a periodic correction with period equal to the period $T_g = 1/\nu$ of the gravitational wave. The amplitude of the frequency variations depends on l/λ_g, where l is the distance between the source (reflector) of the signal and the receiver. If we neglect angle factors and numerical coefficients of order unity, the amplitude $\Delta\omega/\omega$ is expressed by

$$\frac{\Delta\omega}{\omega} \approx h \frac{l}{\lambda_g} \tag{9}$$

if $l \ll \lambda_g$, and

$$\frac{\Delta\omega}{\omega} \approx h \tag{10}$$

when l is comparable with λ_g or $l \gg \lambda_g$. Thus, depending on the phase of the gravitational wave at the time of observation, $\Delta\omega/\omega$ may be near zero or reach the positive and negative maximal values (9) or (10). The rms value of $\Delta\omega/\omega$ is $\sqrt{2}$ times smaller than the maximal value. If (as in an interferometer) there are mirrors separated by distance l and the initial value of $\Delta\omega/\omega$ is near (9), then for a not too large number of reflections B, $B \lesssim \lambda_g/2l$, the initial value of $\Delta\omega/\omega$ is increased by B times and may reach the value (10), i.e., the amplitude of $\Delta\omega/\omega$ obtained in the case of one passage of light, but from a large distance, is of order λ_g.

The phase of the electromagnetic oscillations also acquires in the field of a gravitational wave a periodic correction with period T_g. The variation of the phase accumulated from a certain $t = 0$ to the current t is given by

$$\Delta\varphi = \int_0^t \Delta\omega(t)\, dt.$$

During a time of order $T_g/2$ the variation of the phase reaches the maximal value (for $l \ll \lambda_g$)

$$\Delta\varphi \approx \omega h \frac{l}{\lambda_g} T_g \approx 2\pi h \frac{l}{\lambda_e}, \tag{11}$$

where λ_g is the wavelength of the electromagnetic wave.

During an observation time $T \ll T_g$ the variation of the phase is the fraction T/T_g of its maximal value (11), i.e.,

$$\Delta\varphi(T) \approx \omega h \frac{l}{\lambda_g} T. \tag{12}$$

We note that since the variation of the phase of the electromagnetic signal is expressed by a periodic function, there exist times of commencement of measurement of the phase for which the accumulation of the phase is proportional to t^2, and not t. (In the literature one can sometimes find the incorrect assertion that one of these laws is correct and the other incorrect. In reality both are correct but relate to different initial phases.)

The rms value of $\Delta\varphi$ is $\sqrt{2}$ times less than the maximal value. After B reflections between mirrors separated by the distance l, the initial value of $\Delta\varphi$, which is close to (11), is increased by B times and is the signal value of the phase:

$$\Delta\varphi \approx 2\pi B h \frac{l}{\lambda_e}. \tag{13}$$

(For actual calculations of the response of laser interferometers, see, for example, Ref. 52.)

In a laser interferometer the signal change of the phase (13), doubled on account of the two arms of the interferometer, must be compared with the fluctuation drift of the phase $\delta\varphi$, the "shot" noise. For laser light in a coherent state, $\delta\varphi \cdot \delta N \approx 1$, where $\delta N \approx N_\gamma$, N_γ being the number of photons emitted by the laser and absorbed by the photon detector during the considered time $T_g/2$. If the laser has power W and the detector efficiency η, then

$$\delta\varphi \approx \left(\frac{2\hbar\omega}{W\eta T_g}\right)^{1/2},$$

whence

$$h_{\min} \approx \frac{1}{lB}\left(\frac{\hbar c^2}{W\eta}\frac{\nu}{\omega}\right)^{1/2}.$$

It is this estimate that determines the sensitivity of laser interferometers to bursts in the frequency range $10^2 \lesssim \nu \lesssim 10^4$ Hz. At lower frequencies seismic noise is dominant.[2]

In the case of pulsar chronometry one can speak of a variation of the phase or a variation of the arrival time $R(t)$ of individual pulses:

$$R(t) = \int_0^t z(t')\, dt'.$$

In a field of stochastic gravitational radiation $\langle R(t)\rangle = 0$ but $\langle R^2(t)\rangle \neq 0$. With moderate accuracy observational limits on h and, therefore, on Ω_g can be readily obtained by using the expressions given above. As follows from (12), gravitational waves with wavelength λ_g less than the distance l between the pulsar and the earth give rise to a variation Δt between the calculated and actual arrival of the pulse of order

$$\Delta t \approx hT, \quad cT \ll l, \tag{14}$$

where T is the total time of observation. In principle, the relation (14) gives the same bounds on h for all waves in the range $cT \lesssim \Lambda_g \lesssim l$. However, as noted above, "polynomial fitting" effectively cuts out the low frequencies, so that under reasonable assumptions about the spectrum of the stochastic background the relation (14) actually applies to waves with wavelength $\lambda_g \approx cT$. Since the observed rms variation ΔT (after "polynomial fitting") does not exceed one microsecond, and T is several years, these considerations yield approximately limits on h at the level $h \lesssim 10^{-13}$–10^{-14} and $\Omega_g < 10^{-6}$ for waves with frequencies $\nu \approx 10^{-8}$ Hz.

The generality of the principles of detection and the estimates given here demonstrate once more the great difficulties that must be overcome if gravitational radiation is to be successfully detected.

7. NEW IDEAS AND PROSPECTS

Although several gravitational-wave detectors have long existed and others are in the stage of technical implementation, the field of gravitational-wave astronomy is still far from having being transformed into a purely engineering matter. Here there is still great scope for new ideas and proposals even at the level of first principles.

7.1. Kinematic resonance and the memory effect

As an example we consider recently discussed effects in the motion of free test bodies—kinematic resonance and the memory effect.[53] Both effects relate to the final state of a pair of bodies that are at rest at the initial time and are subject to the action of a gravitational wave. And both effects follow from an analysis of the basic equation of mechanics, $m\ddot{x} = F(t)$, to which the equation of motion of a particle under the influence of a gravitational-wave force reduces in the simplest case.

The essence of kinematic resonance is that two particles that are in the field of a gravitational wave and are made free at a certain initial time will be subject not only to a relative oscillatory motion but also to a systematic approach to or separation from each other depending on the initial phase of the gravitational wave (a form of the drift effect[54]). After a time $\hat{\tau}$ the systematic change Δl of the distance is given by

$$\frac{\Delta l}{l} = \frac{1}{2} h\omega_g \hat{\tau} \cos\psi,$$

where l is the initial distance, and h, ω_g, and ψ are, respectively, the amplitude, frequency, and phase of the gravitational wave. Note that $\hat{\tau}$ can satisfy the condition $\omega_g \hat{\tau} \gg 1$, and, therefore, the resulting Δl can be much greater than hl.

The essence of the memory effect is that after the passage of a gravitational pulse having a definite temporal shape a pair of free particles that were at rest at a certain distance from each other are still at rest but with a different separation. In this a gravitational pulse with "memory" differs from an ordinary pulse (without "memory"), after the passage of which a pair of free particles is returned to the original relative position. In terms of the components h_{ik}, a pulse with "memory" is characterized by $h_{ik} = 0$ before the arrival of the pulse, followed by variable h_{ik} for a certain time and, finally, $h_{ik} = \text{const} \neq 0$ after the passage of the variable part of the pulse. The change in separation Δl after the action of a pulse with "memory" is determined by the simple formula

$$\Delta l = \frac{1}{m} \int \left(\int^{t} F(\tau)\, d\tau \right) dt$$

(see also Ref. 55). Pulses with "memory" must be radiated when gravitating bodies have an encounter that is not head-on and in fact in all processes in which the values of the second derivative of the quadrupole moment of the source differ asymptotically as $t \to \pm\infty$.

It is important that, as is shown in Ref. 53, both effects can in principle be used in ground-based and space interferometers, and also in observations with the POINTS program.[56] For example, the mirrors in an interferometer do not return to the original position immediately after the passage of a pulse but for a certain time ($\sim 10^{-1}$ sec) are removed from the equilibrium position. Thus, the experimentalist has the possibility of using a much longer averaging time ($\sim 10^{-1}$ sec) than the pulse duration ($\sim 10^{-3}$ sec). Differences also arise in the Doppler tracking technique. As follows from the very simple expression (8), the passage of a short pulse with "memory" is accompanied by $\Delta\omega/\omega \neq 0$ values not only because the variable part of the pulse passes through the source or receiver of the radiation, but $\Delta\omega/\omega = \text{const} \neq 0$ for a long time also because $h_{ik}(0) = h_{ik}(e) \neq 0$ due to the constant part of the pulse. Actual estimates of the possibility of observing pulses with "memory" in the Doppler tracking technique are considered in Ref. 57.

It should be emphasized that the pulses with "memory" considered here should more accurately be called pulses with "position memory," since it is the distance between the particles and not their relative velocity that changes. In principle, there can also exist pulses with "velocity memory," i.e., such that after they have passed, the relative velocity of the particles is changed. For this the asymptotic values of the third derivative of the quadrupole moment in the source of the gravitational radiation must change. It is difficult to estimate how often such pulses could arise in nature, but it is clear that their existence would open up entirely new possibilities in the formulation of the detection problem.

7.2. Possibilities of detection of high-frequency relic gravitons

Another direction of search is associated with estimating the possibility of detecting the high-frequency part of the spectrum of the relic gravitational-wave noise, $\nu \approx 10^{10}$–10^{11} Hz. We consider a new proposal that gives grounds for optimism.[45] This suggestion is based on the use of a composite crystal antenna ("large crystal") and is based essentially on calculation of the cross section for absorption of gravitational waves by such systems.

Suppose that a gravitational wave is incident along the normal to a thin rod (one-dimensional crystal). The wavelength λ_g is large compared with the length l of the rod. It is well known that if the wave excites the lowest acoustic mode $n = 1$, i.e., $\nu \approx v_s/2l$, then the absorption cross section σ_1, integrated over the frequencies of the resonance profile of the oscillator, is given by

$$\sigma_1 \approx \frac{G}{c^3} M v_s^2,$$

where M is the mass of the rod, and v_s is the speed of sound.[58,59] Now suppose that there is incident on the same rod a high-frequency gravitational wave that excites the nth mode of acoustic vibrations, $n \gg 1$ (but, as before, $l \ll \lambda_g$). If it is assumed that the ends of the rod are free, i.e., the tension at the ends of the rod is zero, then in this mode we obtain $\sigma_n = \sigma_1/n^2$ (Ref. 59). In other words, we obtain a cross section that is smaller, the larger is n.

However, for crystals and quite generally systems in which volume effects, and not effects on the boundaries, are important, periodic boundary conditions and not free-end conditions are more appropriate. Repeating the calculation of the absorption cross section with periodic boundary conditions, we obtain in this case $\sigma_n \approx \sigma_1$, i.e., the same cross sections for all n. In the case of incidence of a gravitational signal with a wide frequency band, $\Delta\nu \approx \nu$, it is necessary to take into account the number of modes excited in the rod, Δn, and calculate the total absorption cross section σ_{tot}. In the one-dimensional case $\Delta n \approx n$ and

$$\sigma_{tot} \approx \frac{G}{c^3} M v_s^2 \cdot n = \frac{G}{c^3} m v_s^2 \cdot n^2.$$

In the last equation we have introduced the mass m of a small element of the rod (elementary quadrupole) having dimension of the order of the acoustic wavelength λ_s; then $M = mn$. It can now be seen that σ_{tot} is proportional to the square of the number of such elementary quadrupoles.

In the case of a three-dimensional system with dimensions $l \ll \lambda_g$ one can also expect σ_{tot} to be determined by the expression

$$\sigma_{tot} \approx \frac{G}{c^3} m v_s^2 \cdot N^2,$$

where m is the mass of a cube having sides of order λ_s, N is the total number of such cubes, and the total mass of the crystal is $M = mn^3 = mN$.

The above expressions are valid as long as l is less than λ_g. To obtain a large composite antenna with total volume greater than λ_g^3, it is necessary to take many such crystals, operating independently. The total absorption cross section increases in proportion to the number of these crystals.

It should be said that a detailed calculation can make the above estimates both better and worse. The expression

$$\sigma_{tot} \approx \frac{G}{c^3} m v_s^2 \cdot N^2$$

for one crystal with dimensions $l \lesssim \lambda_g$ presupposes that in it all modes of acoustic vibrations in the frequency interval $\Delta\nu \approx \nu$ are excited. However, the tensor nature of gravitational waves may have the consequence that some of the modes are not excited, and then the dependence on N will be weaker than N^2 but not worse than $N^{4/3}$. On the other hand, in determining the absorption cross section of the composite antenna we made the most modest assumption, according to which this cross section is simply the sum of the cross sections of the individual crystals. However, it is not impossible that when allowance is made for the couplings between the crystals the cross section will be greater.

We apply without any changes the estimates made above of the absorption cross section of a composite antenna to the problem of detecting the stochastic background. We assume that in the considered range of frequencies an energy density $\varepsilon_g \approx \varepsilon_\gamma$ is present in the form of gravitational-wave noise. As detection criterion we adopt absorption by the complete system of the energy $\hbar\omega_g$, equal to the energy of one phonon. Then it can be shown that the total mass of the system must be $M \approx 10^4$ g and the accumulation time $\hat{\tau} \approx 10^5$ sec. These estimates are based solely on basic requirements and completely ignore possible technical problems. However, on the basis of the obtained numbers the problem of detecting the relic gravitational-wave background does not appear hopeless.

I hope that the contents of this paper demonstrate the attraction, importance, and fruitful development of gravitational-wave astronomy. Perhaps the most interesting thing is that we probably do not yet suspect the most remarkable phenomena in this field.

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[1)] The paper is a revised text of the plenary paper "Gravitational-wave astronomy"[1a] (see also Ref. 1b) given by the author at the Eleventh International Gravitational Conference GRG (Stockholm, July 1986). There was recently published the good review of Ref. 2, which considers in more detail some aspects of gravitational radiation and its detection.

[2)] In Ref. 12 the values $m_1 = 1.451 \pm 0.007$ and $m_2 = 1.378 \pm 0.007$, respectively, are given.

[3)] For a discussion of pulses with "memory," which contain a constant part of the field h^{m}, see Sec. 7.1.

[4)] Unfortunately, so far as the author knows, nothing similar is as yet planned in the Soviet Union.

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