Nonconservation of baryon number under extremal conditions

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In gauge theories with left-right asymmetric fermion composition (e.g., in the standard model of electroweak interactions), the fermion number F is not conserved because of the anomaly $\partial_\mu j_\mu^F \sim F_{\mu\nu} \tilde{F}_{\mu\nu}$. In models with small coupling constants, the amplitudes for processes with anomalous nonconservation of the fermion number are exponentially suppressed under normal conditions. It is shown that these are fast processes under extremal conditions, i.e., in the field of a magnetic monopole and at high densities and temperatures. An F-nonconservation mechanism associated with a level crossing phenomenon in external gauge fields is described. The theory and the experimental consequences of the monopole catalysis of proton decay are discussed. It is shown that both Abelian and non-Abelian gauge theories have a critical density above which cold fermion matter becomes absolutely unstable. The absence of the suppression of the anomalous electroweak nonconservation of fermion number at high temperatures is demonstrated, and the cosmological consequences of this phenomenon are discussed. The strong nonconservation of the fermion number in the decay of heavy fermions and technibaryons is also considered.

CONTENTS

1. INTRODUCTION

Modern theories of elementary particles are based on the principles of gauge symmetry. The gauge theory of colored quarks and gluons, i.e., quantum chromodynamics, is now a generally accepted theory of strong interactions. Weak and electromagnetic interactions are successfully described by the standard electroweak theory. Definite advances have been made toward the unification of interactions: several realistic grand unification models have been constructed, in which strong, weak, and electromagnetic interactions are the low-energy manifestations of a unified gauge interaction. Attempts based on supergravity and superstring theories have also been made to include gravitation in the unification scheme.

In addition to attempts to broaden gauge theories, i.e., to unify the different interactions, efforts have also been made to extend the theories "in depth," i.e., to investigate effects associated with the complex dynamic properties of gauge theories. It is now clear that the dynamic content of gauge theories is not exhausted by perturbation theory even in models with weak coupling. Among nonperturbative aspects of gauge theories, we may emphasize the complex structure of vacuum and the consequent nonconservation of fermion quantum numbers (such as the baryon and lepton numbers), and the existence of solitons, i.e., particle-like solutions of the classical field equations, which correspond to extended particles at the quantum level.

One of the most interesting predictions of unified gauge theories is the nonconservation of the baryon number. At least two possible mechanisms are being discussed at present for the nonconservation of the number of baryons. The first arises naturally in grand unification theories within the framework of perturbation theory[1,2] (see the reviews in

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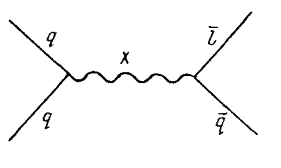

FIG. 1. Two quarks transform into an antiquark and a lepton as a result of the exchange of a hyperheavy boson.

Refs. 3 and 4). In its simplest version, it is due to the exchange of superheavy vector and scalar bosons, as shown in Fig. 1. Possible manifestations of this mechanism include the spontaneous decay of the proton[1,2] and neutron-antineutron oscillations.[5]

Another mechanism arises already in the standard electroweak theory[6] and is due to the complex structure of vacuum[6–8] and the triangular anomaly.[9,10] Under normal conditions, the amplitudes with nonconserved baryon number that are due to this mechanism are suppressed by the factors $\exp(-8\pi^2/g_w^2) \sim 10^{-77}$, where $g_w^2 = e^2/\sin^2\theta_w$ is the coupling constant of the electroweak gauge group $SU(2)_L$ [of the subgroup $SU(2)_L \times U(1)$]. Direct observations of processes with electroweak B-violation are impossible at low energies. However, there is a number of situations in which the rate of anomalous processes with nonconservation of the baryon number may not be small. Reactions with electroweak nonconservation of the baryon number can occur with high probability under extremal conditions, e.g., at high fermion densities[11–14] and at high temperatures.[15] The fast nonconservation of the baryon number is also possible in the decay of particles with large enough mass ($M \gtrsim 10$ TeV).[16–20]

Effects associated with the complex structure of vacuum in gauge theories are also found to play a significant part in interactions between fermions and magnetic monopoles. These monopoles appear in grand unification theories as static solutions of classical field equations, i.e., solitons[21] (see the reviews in Ref. 22). In most grand unification models, the interaction of quarks and leptons with monopoles leads to the decay of the proton with a high cross section (of the order of the cross section typical for strong interactions).[23–25] This phenomenon, now called monopole catalysis of the proton decay, is also an example of the strong nonconservation of the baryon number under extremal conditions (in the strong magnetic field of the monopole). Similar properties should also be exhibited by drops of anomalous matter that appear in Abelian V-A theories[11,14]: when they come into contact with ordinary matter, such drops should absorb nucleons, and this should be accompanied by the release of an amount of energy approaching the rest energy of the proton.

It is therefore expected that a considerable acceleration (by tens of orders of magnitude!) of processes with the nonconservation of the baryon number will occur in many extremal situations. Apart from its purely theoretical interest, this possibility has also attracted attention in connection with experimental, cosmological, and astrophysical applications. Monopole catalysis of proton decay, which should occur with a large cross section, comparable with the nuclear cross section, and should be accompanied by the release of an amount of energy approaching the rest energy of the proton, is one of the processes that can be exploited in the search for monopoles.

In many cases, the characteristic energy scales (temperatures, chemical potentials, and masses of decaying particles) are of the order of 10^2–10^4 GeV, which is much less than the energies typical for grand unification theories (10^{15} GeV). There is therefore some hope of a direct verification of the theoretical predictions. The search for magnetic monopoles and for the monopole catalysis of proton decay is being carried out at most underground installations. At present, the best upper limit for the flux of ultraheavy magnetic monopoles has been obtained using the underground scintillation-counter telescope at the Baksan Neutrino Observatory of the Institute of Nuclear Research of the Academy of Sciences of the USSR[26] and the Baĭkal underwater detector.[27] These installations can also be used to search for drops of anomalous matter. Searches for heavy-particle decays accompanied by the nonconservation of baryon number can also be made on accelerators (although, it must be admitted, only in the relatively distant future).

So far, the only observational argument in favor of the nonconservation of baryon number in nature is the baryon asymmetry of the Universe[28,29] (see the review in Ref. 30). The electroweak nonconservation of the baryon number, which should proceed sufficiently rapidly at temperatures of the order of a few hundred GeV or more, has a direct relation to the problem of the generation of baryon asymmetry. It may well be that the observed baryon asymmetry of the Universe arose precisely as a result of electroweak processes[15,31] at relatively low temperatures (of the order of a few hundred GeV). This possibility is closely related to the question of the nature of the electroweak phase transition and the mass of the Higgs boson. The fast nonconservation of baryon number at high fermion densities is significant for nonstandard cosmological models with an intermediate cold stage (such models have recently been proposed, for example, in connection with the supersymmetric unification theory[32]). Effects connected with the nonconservation of baryon number can lead to the formation of inhomogeneous anisotropic phases in such models.[33] The observable manifestation of the presence of such phases in the early Universe is the relatively intensive gravity-wave relic noise.[34]

In this review, we shall examine theoretical ideas and descriptions that lead to the conclusion that fast nonconservation of the baryon number may be possible under extremal conditions. The key property of gauge theories in this context is the complex structure of vacuum and the associated nonconservation of fermion quantum numbers. These questions are discussed in Section 2. This is followed by a presentation of the results relating to concrete physical situations in which the nonconservation of baryon number should occur with high probability, e.g., the monopole catalysis of proton decay (Section 3), V-A theories at high fermion densities (Section 4), the electroweak theory at high temperatures (Section 5), and electroweak decays of heavy particles (Section 6). Section 7 contains the concluding remarks.

2. GAUGE VACUUM AND THE NONCONSERVATION OF FERMION QUANTUM NUMBERS

2.1. The structure of vacuum in gauge theories

The complex structure of vacuum is a common property of four-dimensional non-Abelian gauge theories[6,7] and of a number of two-dimensional gauge models of field theory[8,35] (see also the reviews in Refs. 36 and 37). To be specific, let us consider a four-dimensional model with the SU(2) gauge group and Higgs field φ, which transforms in accordance with some nontrivial representation T of SU(2) (this representation will not be specified for the moment). We shall use the matrix field

$$A_\mu = -ig\frac{\tau^a}{2}A_\mu^a,$$

where g is the gauge coupling constant, A_μ^a ($a = 1,2,3$) are real vector fields, and τ^a are the Pauli matrices. Under gauge transformations $\omega(x)$ from SU(2), the fields transform as follows:

$$A_\mu \to A_\mu^\omega = \omega A_\mu \omega^{-1} + \omega\ \partial_\mu \omega^{-1},$$

$$\varphi \to \varphi^\omega = T(\omega)\,\varphi.$$

It is convenient to use the gauge $A_0 = 0$. In this gauge, there is a residual gauge freedom with respect to transformations with gauge functions $\omega(\mathbf{x})$ that are independent of time. We shall confine our attention to gauge functions that tend to a direction-independent constant at infinity[1)]:

$$\lim_{|\mathbf{x}|\to\infty} \omega(\mathbf{x}) = \omega_0. \quad (2.1)$$

Under this condition, the gauge functions are characterized by an integer (the topological number of the gauge transformation), namely,

$$n[\omega] = \frac{1}{24\pi^2}\int d^3x\,\varepsilon^{ijk}\,\mathrm{Tr}\,(\partial_i\omega\cdot\omega^{-1}\partial_j\omega\cdot\omega^{-1}\partial_k\omega\cdot\omega^{-1}). \quad (2.2)$$

This number is the degree of mapping of the three-dimensional space with an identified [by virtue of (2.1)] infinity into the gauge group SU(2). The gauge transformations $\omega(\mathbf{x})$ and $\omega'(\mathbf{x})$ can be obtained from one another by the successive application of infinitesimal gauge transformations when, and only when, they belong to the same class, i.e., $n[\omega] = n[\omega']$. An example of a gauge function with a topological number is provided by the function

$$\omega_n(\mathbf{x}) = \exp\left(i\,\frac{\boldsymbol{\tau}\mathbf{n}}{2}\,\Omega(r)\right), \quad (2.3)$$

where $\mathbf{n} = \mathbf{x}/r$ and $\Omega(r)$ is an arbitrary real function satisfying the conditions

$$\Omega_n(r = 0) = 0, \quad \Omega_n(r = \infty) = 2\pi n. \quad (2.4)$$

Classical vacuums, i.e., the field configurations with minimum energy, are pure gauge configurations, $\mathbf{A} = \omega \mathbf{d}\omega^{-1}$, $\varphi = T(\omega)\varphi_0$, where φ_0 is the value of the Higgs field for $\mathbf{A} = 0$. If we do not distinguish between configurations that differ from one another by infinitesimal gauge transformations, the different vacuums correspond to gauge functions with different topological numbers. We thus have a discrete set of classical vacuums labeled by the integer n and differing from each other topologically, i.e., by nontrivial gauge transformations (Fig. 2).

An important feature of gauge theories is the height of the energy barrier between neighboring vacuums. This height is equal to the static energy of the saddle configuration $\mathbf{A}^s,\varphi^s$) (Fig. 2 shows only one direction in the functional space ($\mathbf{A},\varphi$), so that ($\mathbf{A}^s,\varphi^s$) looks like a maximum of the static energy functional; in reality, ($\mathbf{A}^s,\varphi^s$) is a saddle, since the static energy increases in other directions, not shown in Fig. 2). The saddle point is an unstable solution of

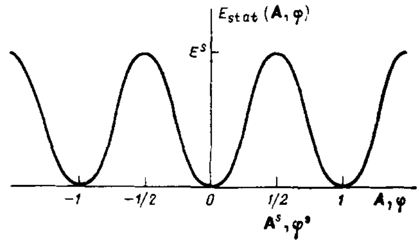

FIG. 2. Schematic representation of the dependence of static energy on boson fields. The minima correspond to classical vacuums. The point ($\mathbf{A}^s,\varphi^s$) determines the height of the barrier between different vacuums. It constitutes the saddle point of the static energy functional (the sphaleron).

the static field equations. This solution was found (for a model with a doublet Higgs field) in Ref. 39, and was rediscovered in Ref. 40; the fact that the solution was unstable was discovered in Ref. 41; the role of the saddle solution as a configuration determining the minimum height of the barrier between neighboring vacuums was elucidated in Ref. 42, where this solution was called a sphaleron[2)] (from the Greek σφαλεροϛ = ready to fall, unstable). The sphaleron configuration has the form

$$A_i^s = i\,\frac{\varepsilon_{ijk}x_j\tau_k}{r^2}\,f(\xi),$$
$$\varphi^s = \frac{\varphi_0}{\sqrt{2}}\,h(\xi)\,\frac{i\tau^a x^a}{r}\begin{pmatrix}0\\1\end{pmatrix}, \quad (2.5)$$

where $\xi = g\varphi_0 r$ and the functions $f(\xi)$ and $h(\xi)$ have the following asymptotic behavior:

$$f(0) = h(0) = 0, \quad f(\infty) = h(\infty) = 1.$$

The energy of the sphaleron (the height of the barrier between the vacuums) is given by the expression

$$E^S = \frac{2M_W}{\alpha_W}\,B\left(\frac{\lambda}{g^2}\right), \quad (2.6)$$

where M_W is the mass of the vector boson which appears as a result of the Higgs mechanism, λ is the self-interaction constant of the scalar field, and $\alpha_W = g^2/4\pi$. In the model with the doublet Higgs field, the function $B(\lambda/g^2)$ varies from 1.56 to 2.72 as λ/g^2 varies from zero to infinity.[42] We note that, in the standard electroweak theory (in which $M_W \approx 80$ GeV, $\alpha_W = \alpha/\sin^2\theta_W \approx 1/30$), the height of the barrier amounts to $E^S \approx 10$ TeV. We shall see later that this quantity determines the energy scale for processes with fast nonconservation of baryon number.

Since, in theories with a small coupling constant, the height of the barrier is much greater than the energy scale M_W typical for perturbation theory, each classical vacuum in the quantum theory corresponds to a state $|n\rangle$ whose wave function is concentrated near the nth minimum. In particular, the trivial classical vacuum ($\mathbf{A} = 0$, $\varphi = \varphi_0$) corresponds to the perturbation theory vacuum $|0\rangle$. The state $|n\rangle$ is obtained from $|0\rangle$ by the application of the unitary operator $U[\omega_n]$, which performs the gauge transformation with topological number n. Because of the gauge invariance of the Hamiltonian, the states $|n\rangle$ are degenerate among themselves.

Gauge theories must satisfy the requirement that all physical states must be invariant (to within the phase) under the gauge transformation. Invariance under topologically trivial transformations, produced by the successive application of infinitesimal transformations, is assured by the imposition of the additional condition (Gauss's law) on physical states:

$$(D_i E_i + J_0)\,\text{phys.}\rangle = 0, \tag{2.7}$$

where $E_i = F_{0i}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$, and J_μ is the gauge current of matter (in the present case, of the Higgs field). However, condition (2.7) does not ensure invariance under topologically-nontrivial gauge transformations. For example, each of the $|n\rangle$ vacuums can be chosen so that Gauss's law is satisfied for it, but the state $|n\rangle$ becomes $|n+n'\rangle$ under the gauge transformation $U[\omega'_n]$. The ground state (θ-vacuum), which is invariant (to within the phase) under all gauge transformations, is a linear superposition of the form

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta}\,|n\rangle \tag{2.8}$$

for which $U[n]|\theta\rangle = e^{in\theta}|\theta\rangle$.

Different θ-sectors of the theory split off because the matrix elements of any gauge-invariant operator $\hat{O}$ between vacuums with different values of θ are zero:

$$\langle\theta' \mid \hat{O} \mid \theta\rangle = 0 \quad \text{for} \quad \theta' \neq \theta. \tag{2.9}$$

Actually, the $|n\rangle$ vacuums can be taken in the form $|n\rangle = U_1^n|0\rangle$, where $U_1 \equiv U[\omega_1]$ performs the transformation with unit topological number (any other choice differs from this by a topologically-trivial gauge transformation that is insignificant by virtue of the Gauss condition). For the gauge-invariant operator $\hat{O}$, we have $\hat{O}$

$$\langle\theta'|\hat{O}|\theta\rangle = \sum_{n,\,m} \exp(-im\theta' - in\theta)\,\langle 0|\hat{O}U_1^{n-m}|0\rangle,$$

and hence (2.9) follows.

The vacuum angle θ is thus seen to be a further parameter of the theory (in addition to the coupling constants and the vacuum expectation values of the Higgs field). The appearance of this parameter is nonperturbative in character. In some theories, for example in quantum chromodynamics, it leads to the violation of CP symmetry. The possibility of a solution of the strong CP problem by the introduction of additional symmetry[43] and the axion[43,44] was discussed in the review given in Ref. 36b.

The structure of the vacuum (2.8) can be approached somewhat differently by considering tunneling between n-vacuums. The quasiclassical amplitude for the tunneling transition between a trivial vacuum ($n = 0$) and a vacuum with topological number n is determined by the minimum of the Euclidean action on boson fields with the asymptotic behavior

$$\begin{aligned} &\mathbf{A} \to 0, \quad \varphi \to \varphi_0, \quad t \to -\infty, \\ &\mathbf{A} \to \omega_n \vec{\partial}\omega_n^{-1}, \quad \varphi \to T(\omega_n)\,\varphi_0, \quad t \to \infty. \end{aligned} \tag{2.10}$$

For these configurations, we have

$$Q\,[\mathbf{A}\,(\mathbf{x},\,t)] = n\,[\omega_n] \equiv n,$$

where

$$Q\,[A] = \frac{1}{32\pi^2}\int d^3x\,dt\,\mathrm{Tr}\,F_{\mu\nu}\tilde{F}_{\mu\nu} \tag{2.11}$$

is the topological number of the gauge field.[45] Minimum action is achieved on (many-) instanton configurations[45] converted to the gauge $A_0 = 0$. We then have $S_{\min} = 8\pi^2|n|/g^2$ (inclusion of the Higgs field leads to small corrections to the instanton value[6]), so that the tunneling amplitude is suppressed by the factor $\exp(-8\pi^2|n|/g^2)$.

Since tunneling between n-vacuums is possible, none of them is an eigenstate of the Hamiltonian. The Hamiltonian of the theory is gauge-invariant and can be diagonalized simultaneously with the operator U_1. The θ-vacuums (2.8) are, in fact, the corresponding eigenstates.

The exponential suppression of the tunneling amplitude means that, under normal conditions, the effects of the complex structure of vacuum are exceedingly small. However, we shall see later that they become significant under extremal conditions. Before we proceed to the examination of this question, let us pause to consider the mechanism responsible for the anomalous nonconservation of fermion quantum numbers.

2.2. Level crossing and the nonconservation of fermion quantum numbers

The nonconservation of fermion quantum numbers will now be discussed by considering the example of the gauge model with the group SU(2) and left-handed massless doublets of fermions $\psi_L^{(i)}$ ($i = 1,\ldots,n_F$) that do not interact with the Higgs field (the necessary condition for the absence of global anomaly[46] is that the number n_F of doublets is even). The gauge-invariant current of each doublet $J_\mu^{(i)} = \bar{\psi}_L^{(i)}\gamma^\mu\psi_L^{(i)}$ is conserved at the classical level, but is anomalous at the quantum level[9]:

$$\partial_\mu J_\mu^{(i)} = \frac{1}{32\pi^2}\,\mathrm{Tr}\,F_{\mu\nu}\tilde{F}_{\mu\nu}. \tag{2.12}$$

The right-hand side of this equation is the total divergence

$$\frac{1}{32\pi^2}\,\mathrm{Tr}\,F_{\mu\nu}\tilde{F}_{\mu\nu} = \partial_\mu K_\mu,$$

where

$$K_\mu = \frac{1}{16\pi^2}\,\varepsilon_{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(F_{\nu\rho}A_\sigma - \frac{2}{3}A_\nu A_\rho A_\sigma\right). \tag{2.13}$$

We can therefore determine the conserved current which, however, will not be gauge-invariant:

$$j_\mu^{(i)} = J_\mu^{(i)} - K_\mu, \quad \partial_\mu j_\mu^{(i)} = 0. \tag{2.14}$$

Equation (2.12) indicates that the number of fermions may not be conserved. Integrating (2.12) over the four-dimensional space between the three-dimensional hyperplanes $t = t_1$ and $t = t_2$, we obtain

$$N^{(i)}(t_2) - N^{(i)}(t_1) = N_{CS}(t_2) - N_{CS}(t_1), \tag{2.15}$$

where $N^{(i)} = \int J_0^{(i)} d^3x$ is the fermion number and

$$N_{CS} = \int K_0\,d^3x = \frac{1}{16\pi^2}\int d^3x\,\varepsilon^{ijk}\,\mathrm{Tr}\left(F_{ij}A_k - \frac{2}{3}A_iA_jA_k\right)$$

is the Chern-Simons number for the gauge field A. Relation (2.15) was obtained on the assumption of a sufficiently rapid decrease in the field A_μ at infinity (for a theory operating in a finite volume, it is sufficient to assume periodic boundary conditions for A_μ). Equation (2.15) implies that, if the gauge field evolves so that its Chern-Simons number varies, there is also a variation in the number of fermions in the

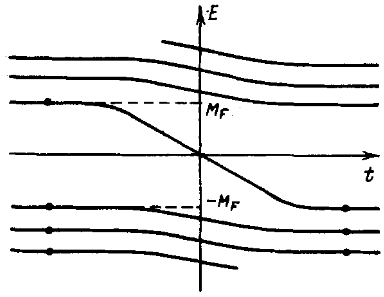

FIG. 3. Fermion level crossing.

system. This nonconservation of fermion number arises as a result of the phenomenon of level crossing (see Refs. 47–49 and the references therein). Let us now consider a system in an external gauge field $\mathbf{A}(\mathbf{x},t)$ which changes adiabatically from $\mathbf{A}(\mathbf{x},t_1)$ to $\mathbf{A}(\mathbf{x},t_2)$. At each intermediate time t, we can calculate the fermion spectrum (the set of eigenvalues of the Dirac Hamiltonian in the external field for fixed t). The spectrum varies in the course of time, and some of the levels cross zero from above and some from below. The difference $N_+ - N_-$ between the number of levels crossing zero from above (N_+) and the number of levels crossing zero from below (N_-) is, in general, nonzero.

For each value of $\mathbf{A}$, the ground state of the fermion system is a state in which all negative-energy states are filled, whereas positive energy states are unoccupied. A real fermion corresponds to a filled positive level, and an antifermion corresponds to a free negative level. The net effect of the level crossing phenomenon is that the number of real fermions will change (Fig. 3) so that, if, initially, the system contains N fermions, the number of real fermions at the end is given by

$$N(t_2) = N - (N_+ - N_-).$$

The difference $(N_+ - N_-)$ is due to the difference between the Chern-Simons numbers of the gauge field, $N_{\rm CS}(t_2) - N_{\rm CS}(t_1)$. We shall now consider two types of the external field $\mathbf{A}(\mathbf{x},t)$ that are important for the ensuing analysis. In the first case, the gauge field of the group SU(2) changes to a pure gauge:

$$\begin{aligned} \mathbf{A}(\mathbf{x},\ t_1) &= \mathbf{A}(\mathbf{x}), \\ \mathbf{A}(\mathbf{x},\ t_2) &= \omega \mathbf{A}(\mathbf{x})\omega^{-1} + \omega\vec{\partial}\omega^{-1}. \end{aligned} \tag{2.16}$$

Direct evaluation will verify that, for this field,

$$N_{\rm CS}(t_2) - N_{\rm CS}(t_1) = n[\omega],$$

where $n[\omega]$ is the topological number of the gauge transformation. The consequence of the Atyah-Patodi-Singer theorem[50] is the equation

$$N_+ - N_- = -n[\omega],$$

which ensures that (2.15) is satisfied.

For the sake of simplicity, we have so far confined our attention to adiabatically varying external fields $\mathbf{A}(\mathbf{x},t)$. However, the results can be extended to the case of rapidly varying fields. They are also valid when the fields $\mathbf{A}(\mathbf{x},t)$ arise spontaneously, e.g., during a tunneling transition between classical vacuums with different topological numbers.

A special case of a field with boundary values (2.16) is the field with the asymptotic behavior defined by (2.10), which describes a transition between topologically different classical vacuums of boson fields. The sphaleron configuration (2.4) plays an important role here. If the trajectory in the space of the field passes through it at some time t_0, we have

$$Q = \frac{1}{32\pi^2}\int_{t_1}^{t_0} d^3x\, dt\, {\rm Tr}\, F_{\mu\nu}\tilde{F}_{\mu\nu} = \frac{1}{2}.$$

The crossing of fermion levels will then occur precisely at time t_0. The number of fermions is not conserved and

$$\Delta N^{(i)} = n. \tag{2.17}$$

The alternative derivation of this relationship, which relies on the properties of zero fermion modes in an instanton-type field, is given in the well-known paper by 't Hooft[6] and in subsequent publications.[51]

We now introduce a few remarks in relation to this important result.

(1) Equation (2.17) appears in the theory with left-handed fermion doublets. In the theory with right-handed doublets, we have, instead, $\Delta N^{(i)} = -n$. In general,

$$\Delta N_{\rm L}^{(i)} = -\Delta N_{\rm R}^{(i)} = n, \tag{2.18}$$

where $\Delta N_{\rm L}^{(i)}$ ($\Delta N_{\rm R}^{(i)}$) is the fermion number of the ith left-handed (ith right-handed) doublet. It follows that the fermion number is conserved in vector-like theories (quantum chromodynamics and quantum electrodynamics), but there is a violation of chirality $Q^5 = N_{\rm L} - N_{\rm R}$. The fermion number is not conserved in V-A theories.

(2) In theories with massive fermions, the result depends on how the fermions acquire mass. If the mass is introduced explicitly into the Lagrangian, the right-hand side of (2.12) acquires an additioinal term that is proportional to the fermion mass. For fields with space-time dimensions greater than the fermion quantum wavelength, these contributions cancel the anomalous term $F\tilde{F}$, in which case there is no level crossing and the fermion quantum numbers are conserved. In standard electroweak theory, fermions acquire mass as a result of the interaction with the Higgs fields. When the mass is introduced in this way, the level crossing picture for fields with the asymptotic behavior defined by (2.10) remains unaltered.[17,52] In particular, (2.17) remains valid for each doublet. This equation determines the selection rules for electroweak processes with nonconserved fermion (baryon, letpon) number:

$$\begin{aligned} \Delta N_{\rm e} &= \Delta N_\mu = \Delta N_\tau = \ldots = n, \\ \Delta N_{\rm q} &= N_{\rm c} N_{\rm g} n, \end{aligned} \tag{2.19}$$

where $N_{\rm e}$ (N_μ,...) is the electron (muon, ...) lepton number, $N_{\rm q}$ is the number of quarks, $N_{\rm c} = 3$ is the number of colors, and $N_{\rm g}$ is the number of generations. Since n is always an integer, the baryon number must change by at least 3. The proton is stable in the electroweak theory, but the deuteron can decay, at least in principle (although its lifetime is exponentially large). We also note that the electroweak theory conserves $B - L$, where $L = N_{\rm e} + N_\mu + N_\tau + \cdots$ is the total lepton number.

(3) The presence of massless fermions in the model

completely suppresses the tunneling between vacuums with different topological numbers (the same applies to the standard electroweak theory). Actually, when the system is in the classical boson vacuum with $n = 0$ and, at the same time, is in the fermion vacuum (no fermions or antifermions), tunneling into the classical boson vacuum with $n > 0$ must be accompanied by the creation of fermions in the system. This process cannot occur spontaneously because of energy conservation. Let us elucidate all this in a somewhat different way. Consider the conserved, but gauge-noninvariant, fermion number

$$N_0^{(i)} = \int j_0^{(i)}\, d^3x, \qquad (2.20)$$

where j_μ is given by (2.14). Under the gauge transformation $\omega(x)$, we have

$$N_0^{(i)} \to U[\omega]\, N_0^{(i)} U[\omega^{-1}] = N_0^{(i)} - n[\omega]. \qquad (2.21)$$

The gauge transformation operator U_n with topological number n carries nonzero $N_0^{(i)}$, where

$$[N_0^{(i)},\ U_n] = nU_n. \qquad (2.22)$$

The vacuum $|n\rangle \equiv U_n|0\rangle$ has $N_0^{(i)} = n$, so that transitions between different $|n\rangle$ vacuums are impossible. In the exactly soluble two-dimensionless massless quantum electrodynamics, the operators U_n have been constructed explicitly[8] and relations such as (2.21) and (2.22) have been obtained as a consequence of this construction.

Let us now consider a different case that is of importance in the present context, namely, the Abelian gauge field of the form[10,13,14]

$$\mathbf{A}(\mathbf{x}) = a\,(\mathbf{e}_1 \cos \mathbf{k}\cdot\mathbf{x} - \mathbf{e}_2 \sin \mathbf{k}\cdot\mathbf{x}), \qquad (2.23)$$

where $\mathbf{k}$ is an arbitrary vector and $\mathbf{e}_{1,2}$ are two real polarization vectors perpendicular to the one another and to $\mathbf{k}$. The fermion Lagrangian will be taken in the form

$$\mathcal{L}_F = \sum_{i=1}^{f} \psi_L^{(i)} i\gamma^\mu \left(\partial_\mu - i\frac{\tau^3}{2} A_\mu\right) \psi_L^{(i)},$$

where

$$\psi_L^{(i)} = \begin{pmatrix} \psi_L^{(i,+)} \\ \psi_L^{(i,-)} \end{pmatrix}$$

is the doublet of left-handed fermions ($\psi_L^{(+)}$ and $\psi_L^{(-)}$ have charges $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively). The analog of (2.15) is then

$$\Delta N^{(i)} = \Delta N_{CS}, \qquad (2.24)$$

where

$$N_{CS} = -\frac{1}{32\pi^2}\int d^3x\, \varepsilon^{ijk} F_{ij} A_k = -\frac{1}{16\pi^2}\int d^3x\, \mathbf{H}\cdot\mathbf{A} \qquad (2.25)$$

is the Chern-Simons number of the Abelian field. For the configuration defined by (2.23), we have $\mathbf{H} = k\,\mathbf{A}$, so that

$$N_{CS} = -\frac{L^3}{16\pi^2} ka^2, \qquad (2.26)$$

where L is the linear size of the box in which the system is located.

Let us now explicitly follow the level crossing, assuming that

$$H \gg k^2 \qquad (2.27)$$

or, equivalently, $a \gg k$. Let us take the momentum $\mathbf{k}$ along the first axis and $\mathbf{e}_1, \mathbf{e}_2$ along the third and second axes, respectively. Condition (2.27) then signifies that near, say, $\mathbf{x} = 0$, the vector potential can be written in the form

$$\mathbf{A} = a\mathbf{e}_1 - kax^1\mathbf{e}_2, \qquad (2.28)$$

i.e., the magnetic field can be regarded as uniform $H_1 = H_2 = 0$, $H_3 = H$. The spectrum of left-handed fermions in the external field (2.28) is characterized by two continuous variables p_3 and p_2 [momentum along the third axis and position of the orbit on the (x^1, x^2) plane; Ref. 53] and one discrete variable $n = 0,1,2,\ldots$ (number of orbits). The energies are given by

$$\begin{aligned} &n = 0, \quad E = p_3 - \frac{a}{2}, \\ &n > 0, \quad E = \pm\left[\left(p_3 - \frac{a}{2}\right) + nH\right]^{1/2}. \end{aligned} \qquad (2.29)$$

For a given n, the number of levels in the interval is given by[53]

$$2\,\frac{L^3}{(2\pi)^2}\,\frac{H}{2}\,dp_3 \qquad (2.30)$$

(the factor 2 represents the presence of two types of fermion in the doublet). Equation (2.29) shows that, when the amplitude is changed from a to $a + da$, fermion levels with $n = 0$, $a/2 < p_3 < (a + da)/2$ cross zero from above and there are no crossings of zero from below. In view of (2.30), we find that the number of levels that appear in the Dirac sea as the amplitude is varied from zero to a is

$$N_+ = 2\,\frac{L^3}{(2\pi)^2}\int_0^a \frac{1}{2} H \cdot \frac{1}{2}\, da = \frac{L^3}{16\pi^2} ka^2,$$

which is in complete agreement with (2.24) and (2.26).

We now note an important difference between Abelian four-dimensional theories and nonAbelian theories. In Abelian theories, a nonzero Chern-Simons number requires the presence of a magnetic field in the system [see (2.25)]. The number of fermions will change only when a magnetic field appears in the system. This field "remembers" the initial number of fermions: when the magnetic field is turned off, the fermion levels again pass from the negative part of the spectrum to the positive part, and the number of fermions is restored. In non-Abelian theories, the Chern-Simons number is nonzero even for pure gauge fields for which all the gauge-invariant quantities are zero. It follows that, in processes such as transitions between topologically-different classical vacuums, a fermion field vanishes without trace.

3. MONOPOLE CATALYSIS OF PROTON DECAY

We begin our discussion of processes with fast nonconservation of baryon number with the decay of the proton, induced by a magnetic monopole in grand unification theories. The basic idea will first be illustrated by considering the example of the SU(2) model (Sections 3.1–3.4). We will then consider the interactions of fermions with the simple (fundamental) monopole of SU(5) theory (Secton 3.5). In Section 3.6, we briefly touch upon the question of the model dependence of the catalysis effect.

3.1. Magnetic monopole in the SU(2) model

We first recall the basic properties of the monopole solution of t' Hooft and Polyakov[21] in the model with the SU(2) gauge group and a triplet of Higgs fields (see the reviews in Ref. 22). We shall use the matrix notation for the Higgs field, i.e., $\varphi = \tau^a \varphi^a$ ($a = 1,2,3$), so that the vacuum expectation value in the unitary gauge will be $\langle \varphi \rangle = v\tau^3$. In this gauge, the vector bosons $V_\mu^\pm = (1/\sqrt{2})\ (A_\mu^1 \pm iA_\mu^2)$ acquire mass $M_V = gv$ and the boson A_μ^3 remains massless ("photon"). Under the gauge transformations ω, the field φ transforms as follows: $\varphi \to \omega\varphi\omega^{-1}$. In the unitary gauge, the unbroken subgroup $U(1)_{EM}$ has the generator τ^3 that commutes with $\langle \varphi \rangle$. This model is unrealistic, but it is useful to discuss it before the results are generalized to realistic theories.

In the gauge in which all th e fields are regular, the classical monopole configuration is

$$\begin{aligned} &A_0^{cl} = 0, \\ &A_i^{cl} = \frac{\varepsilon_{aij}\tau^a n_j}{2ir}(1 - K(r)), \\ &\varphi^{cl} = \tau^a n^a v\,(1 - H(r)), \\ &\mathbf{n} = \frac{\mathbf{x}}{r}. \end{aligned} \tag{3.1}$$

The functions K and H satisfy equations that follow from the classical field equation and have the asymptotic behavior

$$\begin{aligned} &K(0) = H(0) = 1, \\ &K(\infty) = H(\infty) = 0, \end{aligned} \tag{3.2}$$

and K and H tend to zero exponentially. The characteristic size of the region (monopole core) in which K and H are nonzero is of the order $r_M \sim M_V^{-1}$ (if the mass of the Higgs boson is $M_H \sim M_V$). The mass of the monopole [classical energy of the configuration (3.1)] is $M_M \sim M_V/\alpha$, where $\alpha = g^2/4\pi$. It will be important for the subsequent discussion that the configuration (3.1) is spherically symmetric if rotations in space are simultaneously augmented by global gauge transformations.

In the regular gauge (3.1), the unbroken subgroup $U(1)_{EM}$ has the generator $(\boldsymbol{\tau}\mathbf{n})$ that commutes with φ^{cl}. When $r \gg r_M$, the field tensor for (3.1) is

$$\begin{aligned} &F_{0i}^{cl} = 0, \quad F_{ij}^{cl} = \varepsilon_{ijk}\mathcal{H}_k^{cl}, \\ &\vec{\mathcal{H}}^{cl} = \mathbf{n}(\boldsymbol{\tau}\mathbf{n}) \cdot \frac{1}{2ir^2}. \end{aligned} \tag{3.3}$$

This field does actually describe a monopole with magnetic charge $4\pi g^{-1}$: the magnetic field $\vec{\mathcal{H}}^{cl}$ is directed along the radius vector $\mathbf{n}$ in ordinary space and is proportional to $(\boldsymbol{\tau}\mathbf{n})$, i.e., it has only the electromagnetic component.

3.2. Absence of suppression of nonconservation of fermion number

In contrast to the vacuum sector, processes with nonconservation of the fermion number are not suppressed when the magnetic monopole is present. We now present some simple arguments in favor of this assertion.[23,24] We shall include n_f left-handed massless fermion doublets in the SU(2) model. As discussed in Section 2.2, the nonconservation of the fermion number is due to transitions between states that differ by topologically nontrivial gauge transformations. In complete analogy with the vacuum sector, let us consider monopole states $|M,n\rangle = U[\omega_n]|M,0\rangle$, where $|M,0\rangle$ is the monopole state in perturbation theory. Transitions between the states $|M,0\rangle$ and $|M,n\rangle$ are described by configurations of boson fields with the following asymptotic behavior[3)] (as before, we use the gauge $A_0 = 0$):

$$\begin{aligned} &\mathbf{A}(\mathbf{x},t) \to \mathbf{A}^{cl}(\mathbf{x}),\ \varphi(\mathbf{x},t) \to \varphi^{cl}(\mathbf{x}) \\ &(t \to -\infty), \\ &\mathbf{A}(\mathbf{x},t) \to \omega_n \mathbf{A}^{cl}(\mathbf{x})\,\omega_n^{-1} + \omega_n \vec{\partial}\omega_n^{-1}, \\ &\varphi(\mathbf{x},t) \to \omega_n \varphi^{cl}(\mathbf{x})\,\omega_n^{-1}, \quad n[\omega_n] = n \\ &(t \to +\infty). \end{aligned} \tag{3.4}$$

We shall show that the Euclidean action for Euclidean configurations with this asymptotic behavior can be as small as convenient. This will mean that the transition process will not be a tunneling transition and will not be suppressed exponentially. Since the fermion number is not conserved in this process [we note that the fields (3.4) are a special case of (2.16)], the nonconservation of the fermion number is also not suppressed.

As an example, consider the configuration

$$\begin{aligned} &A_0 = 0, \\ &\mathbf{A} = h\mathbf{A}^{cl}h^{-1} + h\vec{\partial}h^{-1}, \\ &\varphi = h\varphi^{cl}h^{-1} = \varphi^{cl}, \end{aligned} \tag{3.5}$$

where

$$h(\mathbf{x},t) = \exp\left(i\frac{\boldsymbol{\tau}\cdot\mathbf{n}}{2}S(r,t)\right). \tag{3.6}$$

To ensure that (3.5) has the asymptotic behavior defined by (3.4), we must ensure that S has the following asymptotic behavior:

$$\begin{aligned} &S(r,t) \to 0 \ \text{при}\ t \to -\infty, \\ &S(r,t) \to \Omega_n(r) \ \text{при}\ t \to +\infty, \end{aligned}$$

where $\Omega_n(r)$ is an arbitrary function satisfying (2.4). When the action is evaluated, we must take into account the mass of the monopole, i.e., we must consider $S - S_M$, where $S_M = M_M T$, and T is the normalizing time. We thus obtain

$$S - S_M = \frac{2\pi}{g^2}\int_{-\infty}^{\infty} dt \int_0^{\infty} dr\,[r^2(\partial_r\partial_t S)^2 + K^2(\partial_t S)^2]. \tag{3.7}$$

Since $K(\infty) = 0$, this integral is finite (this is not the case in the vacuum sector in which $K = 1$: since $\partial_t S(r = \infty) \neq 0$, the analogous integral diverges). The action (3.7) can be made as small as convenient by a scaling transformation of time $t \to \lambda t$, for which $(S - S_M) \to \lambda^{-1}(S - S_M)$.

Thus, action for configurations such as (3.5) can be as small as convenient, and there is no exponential suppression of processes with nonconservation of fermion number. In other words, configurations such as (3.5) provide a large contribution to the functional integral for the Green functions that produce the nonconservation of the fermion number. Let us now consider the configurations (3.5) in somewhat greater detail.

If we select $S(r,t)$ so that this function is nonzero only outside the monopole core, the field tensor for the configuration (3.5) assumes the form

$$F_{ij} = F_{ij}^{cl}, \tag{3.8a}$$

$$F_{0i} = \frac{1}{2i} (\tau^a n^a) \xi_i, \quad (3.8b)$$

$$\xi = \mathbf{n}\partial_r \partial_t S. \quad (3.8c)$$

It follows from (3.8c) that the "electric" field is radial and points along φ^{cl} in group space. In other words, this field is of purely electromagnetic origin. The possibility that purely electromagnetic radial fluctuations in the "electric" field will produce nonconservation of fermion number in SU(2) is a specific property of the monopole sector. Actually, it is precisely because of the presence of the radial magnetic field that the topological number of such fluctuations is nonzero:

$$Q \propto \int \vec{\mathscr{H}}^{cl} \xi d^3x\, dt \propto \int \xi\, dr\, dt.$$

The monopole sector is special because, in contrast to the vacuum sector, the gauge functions (2.3) now commute with φ^{cl}, i.e., they lie wholly in $U(1)_{EM}$.

The absence of suppression of transitions with nonconservation of the fermion number signifies that neither perturbation theory nor the standard quasiclassical method can be used to investigate the monopole-fermion interaction. We shall discuss later some of the approaches to the investigation of this interaction. For the moment, we shall examine these interactions on the basis of selection rules.

3.3. Asymptotic states of fermions in the presence of a monopole

In order to consider the scattering of fermions by a monopole with allowance for nonconservation of the fermion number, we must first find the asymptotic states of fermions in the presence of the monopole. Let us therefore consider the Dirac equation in the external field of the monopole, well away from the monopole core. In a regular gauge, this equation is

$$i\sigma^\mu (\partial_\mu + A_\mu^{cl}) \psi_L = 0, \quad (3.9)$$

where $\sigma^\mu = (1, \boldsymbol{\sigma})$ and ψ_L is a two-component Weyl spinor. As noted in Section 3.1, the field of the monopole is invariant under rotations in space, augmented by global SU(2) transformations. We therefore have conservation of angular momentum:

$$\mathbf{J} = \mathbf{L} + \mathbf{S} + \frac{1}{2}\boldsymbol{\tau}, \quad (3.10)$$

where L and S are the usual orbital angular momentum and the spin angular momentum, respectively, and $\boldsymbol{\tau}$ operates on the SU(2) indices. Because of the presence of the additional term $\boldsymbol{\tau}/2$ ("spin from isospin" [54]), the angular momentum (3.10) must assume integer values. This agrees with the well-known result reported by Tamm[55] to the effect that the angular momentum in the field of the Dirac monopole must be an integer.

In the unitary gauge, the fermion doublet contains the left-handed fields a_+ and b_- with electric charges $+g/2$ and $-g/2$, respectively. The charge operator in this gauge is identical with τ^3 (to within the factor $g/2$). In the regular gauge (3.1), the charge operator is $(\boldsymbol{\tau}\mathbf{n})$. It may be verified that, well away from the monopole center, this operator commutes with the Dirac operator $\sigma^\mu(\partial_\mu + A_\mu^{cl})$, i.e., the fermion charge does not change as it moves away from the monopole center. Near the monopole center, the fermion charge is not conserved at the level of quantum mechanics. This is due to the presence of the charged fields $V^\pm$ in the monopole core, and these fields appear in (3.9) as external classical fields. We shall return to this question later.

Let us now consider the solutions of the Dirac equation with the lowest angular momentum (s-waves, $J = 0$). The most general form of the spherically symmetric wave function contains two radial functions $\chi_{1,2}(r,t)$ (Ref. 54)

$$\psi_{\alpha i} = \frac{1}{\sqrt{8\pi}\, r} (\varepsilon_{\alpha i}\chi_1 - i\tau^a_{\alpha\beta}\varepsilon_{\beta i} n_a \chi_2), \quad (3.11)$$

where $\alpha = 1,2$ and $i = 1,2$ are the Lorentz and SU(2) indices. We now introduce the column

$$\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix},$$

and write the radial equation outside the monopole core in the form

$$(i\tilde{\gamma}^0 \partial_t + i\tilde{\gamma}^1 \partial_r)\chi = 0, \quad (3.12)$$

where $\tilde{\gamma}_0 = \tau^1$ and $\tilde{\gamma} = -i\tau^3$ are two-dimensional γ-matrices. In terms of this notation, the electric charge operator is equal to the two-dimensional γ^5-matrix $\tilde{\gamma}^5 = \tau^2$. Its conservation is obvious from (3.12).

An important property of s-wave fermions that distinguishes them from fermions with higher angular momenta is the absence of the centrifugal barrier. Outside the monopole core, the independent solution of (3.12) has the form

$$\chi_+ = \frac{1}{\sqrt{2}} \exp[-iE(t+r)] \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad (3.13a)$$

$$\chi_- = \frac{1}{\sqrt{2}} \exp[-iE(t-r)] \begin{pmatrix} 1 \\ -i \end{pmatrix}. \quad (3.13b)$$

When $E > 0$, the solution given by (3.13a) describes a positively charged fermion a_+, whereas (3.13b) describes the negatively charged fermion b_+. It follows from (3.13) that χ_+ contains only the incident wave and χ_- only the reflected wave. The left-handed positively charged s-wave fermions can only fall on the monopole, whereas the negatively charged can only issue from the center.[56,57] For right-handed s-wave fermions (in our model, these are antifermions), we have the reverse situation: positively charged fermions are emitted by the center and negatively charged fermions are incident on the center. The possible asymptotic states of the s-wave fermions are listed in Table I. These properties are also valid for massive Dirac fermions, in which fermion helicity plays the role of chirality.[56,57]

The absence of the centrifugal barrier and the properties of asymptotic states of s-wave fermions listed in the table lead to two important consequences. First, the s-wave fermions, even those with low energies, can readily "explore"

TABLE I.

Fermion charge	Chirality (helicity for $mN_F \neq 0$)	Direction of motion
+	Left-handed (+)	Toward the center
−	Left-handed (+)	Away from the center
+	Right-handed (−)	Toward the center
−	Right-handed (−)	Away from the center

the structure of the monopole core. Actually, the solutions given by (3.13) are valid everywhere outside the monopole core, and it is only the interactions at short distances (of the order of the size of the monopole **M**) that lead to the transformation of the incident into the reflected wave. Secondly, the scattering of waves by the monopole cannot be investigated within the framework of fermion quantum mechanics in an external monopole field. At the level of the Dirac equation (3.9), the left-handed fermion cannot be transformed into a right-handed antifermion. This means that, at the quantum mechanical level, the scattering of the incident s-wave fermion a_+ looks as follows (we recall that there are no right-handed fermions in our model; the reflected s-wave can only be associated with the fermion b_-):

$$a_+ + M \to b_- + M. \tag{3.14}$$

This process is in conflict with the law of conservation of electric charge. This conclusion is confirmed by the explicit solution[58] of the Dirac equation (3.9): at large distances from the core, the s-wave is a superposition of solutions (3.13a) and (3.13b) with equal amplitudes. The scattering of a fermion by the monopole can be desribed only within the framework of quantum theory if the interactions between the fermions and the boson-field fluctuations are taken into account. Processes involving nonconservation of the fermion number, which are due to the ground-state anomaly and complex structure, then play a fundamental part.

We note that these properties are characteristic only for s-wave fermions. Fermions with higher angular momenta have both an incident and a reflected wave associated with them, whatever the sign of the charge. They are reflected from the monopole because of the centrifugal barrier well before they reach the core.

3.4. Scattering of s-wave fermions: selection rules

Since the process defined by (3.14), which arises at the level of the Dirac equation in the external field of the monopole, is in conflict with the conservation of electric charge, we must consider alternative final states for the scattering of the s-wave a_+ by the monopole, taking into account quantum field theory effects.

(1) a_+ transforms into b_- and the surplus charge is shifted to the monopole (the monopole becomes a dion, i.e., an electrically charged monopole). However, the dion is heavier than the monopole, the mass difference being of the order of g^2/r_M. Consequently, this process is in conflict with the conservation of energy for incident-fermion energies that are small in comparison with g^2/r_M (in grand unification theories, $g^2/r_M \sim 10^{13}$ GeV).

(2) The s-wave a_+ can transform into a_+ with a higher angular momentum as a result of the emission of a photon. This process was examined in Ref. 59, where it was shown that its amplitude was small (proportional to g^2 and finite). Processes involving a change in the angular momentum are thus seen to play an insignificant role.

(3) The only remaining possibility is that the scattering process occurs with nonconservation of the fermion number, due to the complex structure of the ground state and the anomaly. This conclusion is in total agreement with the arguments presented in Section 3.2, which show that this nonconservation process proceeds intensively in the absence of the monopole.

The scattering of s-wave fermions by the monopole can be found in many models by starting with the selection rules and the properties of asymptotic states.[60–62] For example, consider the SU(2) model with two left-handed fermion doublets (the number of such doublets must be even in order to cancel the global anomaly[46])

$$\psi_L^{(1,2)} = \begin{pmatrix} a_+^{(1,2)} \\ b_-^{(1,2)} \end{pmatrix}_L .$$

It follows from the properties of s-wave fermions listed in the table and from the conservation of electric charge that incidence of an a_+ on the monopole leads to the appearance of a $\bar{b}$ in the final state (the antifermion $\bar{b}$ is right-handed and positively charged). This process occurs with nonconservation of fermion number and arises because of the complex structure of the ground state and the anomaly. Next, it follows from the selection rule (2.17) that the change in the number of fermions of the first and second type should be the same. The only possible process involving the s-wave $a^{(1)}$ in the initial state is therefore the process

$$a^{(1)} + \mathbf{M} \to \bar{b}^{(2)} + \mathbf{M}\ (+\bar{b}b\text{-pairs}).$$

The cross section for this is determined by the probability of finding the s-wave fermion $a^{(-1)}$ in the incident plane wave. It is given by[57]

$$\sigma_{a\to b} = \frac{\pi}{2k^2}, \tag{3.15}$$

where **k** is the momentum of the incident fermion. It follows that the cross section for processes with nonconservation of fermion number is actually quite high: it does not contain suppressing factors due to the small magnitude of the coupling constant or the size of the monopole core.

A somewhat more complicated situation arises in the model with four left-handed fermion doublets. Here, the conservation of electric charge and the selection rule (2.17) do not allow a single final state with a definite (integral) number of fermions for the incident s-wave $a^{(1)}$. This paradox is resolved in the theory with strictly massless fermions by the fact that, when the monopole is present, the theory allows states with fractional fermion numbers.[61,62] In the strictly massless theory, the allowed process occurs with nonconservation of fermion quantum numbers and has the form

$$a^{(1)} + \mathbf{M} \to \frac{1}{2} b^{(1)} + \frac{1}{2}\bar{b}^{(2)} + \frac{1}{2}\bar{b}^{(3)} + \frac{1}{2}\bar{b}^{(4)} + \mathbf{M}. \tag{3.16}$$

When the fermion masses are finite, but small, there are no asymptotic scattering states with fractional fermion numbers. However, part of the selection rules (2.17) is then violated by the mass terms. Processes such as (3.16) arise as intermediate processes, at distances from the center that are smaller than m_F^{-1} and, in the final state, the number of fermions is always an integer. However, in theories with massive fermions, anomalous nonconservation of fermion quantum numbers is also very considerable. This is clear from the fact that, at distances from the center much smaller than m_F^{-1}, the fermion masses can be neglected.

As already noted, systematic studies of interactions between fermions and monopoles are possible only within the framework of quantum field theory. Standard perturbation theory and quasiclassical methods are invalid for this problem. Nevertheless, a relatively detailed analysis can still be carried out. It is based on the fact that the properties of the fermion-monopole interaction are related to spherically-symmetric fluctuations in boson fields (see Section 3.2) and the s-wave fermion degrees of freedom. We can therefore confine our attention to the s-wave sector of the theory and include fields with higher angular momenta. There is a number of results[24,59,63] showing that the influence of fields with high angular momenta on s-wave dynamics is actually small.

In the s-wave approximation, the system becomes effectively two-dimensional, i.e., the angular dependence of all the fields is known explicitly, and the only significant variables are the distance to the center of the monopole and the time. The methods developed for two-dimensional quantum-field models have been used to investigate the situation. An exact solution is possible in the theory with massless fermions.[23-25] In particular, it is possible to calculate the Green's functions with nonconservation of fermion number. In the model with four left-handed doublets, the simplest operator bearing the fermion number and satisfying the selection rules (2.17) is the four-fermion operator $a^{(1)}a^{(2)}b^{(3)}b^{(4)}$, where all the fields are taken at a single space-time point. The expectation value of this operator over the state of the monopole (fermion condensate) is nonzero and is equal to

$$\langle (a^{(1)}a^{(2)}b^{(3)}b^{(4)})\,(\mathbf{x})\rangle = \frac{\text{const}}{r^6}, \tag{3.17}$$

where the numerical constant is of the order of unity. The functional integral for the condensate (3.17) contains contributions of Euclidean configurations of boson fields with $Q = 1$ (Refs. 23 and 24). The fact that the size of the condensate is not suppressed is a further argument in favor of fast nonconservation of fermion number.

Condensates that conserve the fermion number, but are associated with a change in the fermion flavor, are also nonzero in the field of the monopole. For example,[64]

$$\langle a^{(1)}\bar{b}^{(1)}\bar{a}^{(2)}b^{(2)}\rangle = \frac{\text{const}}{r^6}. \tag{3.18}$$

The great importance of these expectation values is also due to the properties of the interaction between fermions and the monopole: analogous Green's functions arise in the vacuum sector because of the exchange of heavy vector bosons $V_\mu^\pm$, and are suppressed by the inverse powers of M_V. In the monopole sector, the fermions interact intensively with the monopole core, which contains heavy bosons, and this leads to the absence of suppression.

The s-wave dynamics in the theory with massive fermions is not amenable to exact solution. However, a number of qualitative results can be obtained in this case by using the bosonization method.[25] We shall not pause to consider these results, and refer the reader to the original papers of Refs. 25, 61, 65.

3.5. Monopole catalysis of proton decay in SU(5) theory

The model with the SU(5) gauge group[2] is one of the simplest grand unification models. The colored SU(3) and weak SU(2) groups are embedded in SU(5) as follows:

$$SU(3)_c = \begin{pmatrix} SU(3) & 0 \\ 0 & 1 \end{pmatrix},$$

$$SU(2) = \begin{pmatrix} 1 & 0 \\ 0 & SU(2) \end{pmatrix},$$

and the electromagnetic charge matrix is

$$Q_{EM} = \text{diag}\left(-\frac{1}{3},\ -\frac{1}{3},\ -\frac{1}{3},\ 1,\ 0\right).$$

The left-handed fermions and antifermions in each generation form $\bar{5}$- and 10-plets. For example, for the first generation, we have

$$\bar{5} = (\bar{d}_1,\ \bar{d}_2,\ \bar{d}_3,\ e^-,\ \nu_e)_L,$$

$$10 = \begin{pmatrix} 0 & \bar{u}_3 & -\bar{u}_2 & u^1 & d^1 \\ & 0 & \bar{u}_1 & u^2 & d^2 \\ & & 0 & u^3 & d^3 \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}_L,$$

where the bar indicates an antiparticle and the indices 1, 2, 3 refer to the color.

SU(5) breaks down to $SU(3)_c \times U(1)_{EM}$ in two stages. In the first stage, the breaking down continues to $SU(3)_c \times SU(2) \times U(1)$ as a result of the formation of the Higgs 24-plet condensate (associated representation):

$$\langle \varphi_{24}\rangle = V\ \text{diag}\ (2,\ 2,\ 2,\ -3,\ -3),$$
$$V \sim 10^{15}\ \text{GeV}.$$

In the second stage, $SU(3)_c \times SU(2) \times U(1)$ breaks down to $SU(3)_c \times U(1)_{EM}$. The simplest method is to introduce the Higgs 5-plet with the vacuum expectation value

$$\langle \varphi_5\rangle = v\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},\quad v \sim 250\ \text{GeV}.$$

Although the minimal SU(5) is probably excluded by searches for proton decays,[66] we shall examine it for the sake of simplicity. The discussion can be extended to other variants of the model.

The Higgs 24-plet does not directly interact with the fermions. The fermions acquire mass as a result of the interaction with the 5-plets.

The simplest (fundamental) monopole in the SU(5) model[67] is essentially the same as the monopole discussed in Section 3.1, which belongs to the group $SU(2)_M$ embedded in SU(5) as follows:

$$SU(2)_M = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ & & SU(2)_M & \\ 0 & & & 1 \end{pmatrix}.$$

The only unbroken $SU(2)_M$ generator in the unitary gauge (analog of τ^3 in Section 3.1) is the generator

$$t_M^3 = \text{diag}\,(0,\ 0,\ 1,\ -1,\ 0) = Q_{EM} + \frac{1}{3}Y_c, \tag{3.19}$$

where Y_c is the color hypercharge. In accordance with (3.19), the fundamental monopole has both ordinary and colored magnetic charges. The massive vector bosons X of the $SU(2)_M$ groups (analogs of $V_\mu^\pm$ of Section 3.1) are superheavy ($M_X \sim 10^{14}$ GeV), so that the mass of the monopole and the size of its core are determined by the grand unification scale:

$$M_M \sim \frac{M_X}{\alpha} \sim 10^{16}\ \text{GeV}, \quad r_M \sim M_X^{-1} \sim 10^{-28}\ \text{cm}.$$

The first-generation fermions form four left-handed doublets with respect to $SU(2)_M$:

$$\begin{pmatrix} -\bar{u}_2 \\ u^1 \end{pmatrix}_L, \begin{pmatrix} \bar{u}_1 \\ u^2 \end{pmatrix}_L, \begin{pmatrix} d^3 \\ e^+ \end{pmatrix}_L, \begin{pmatrix} e^- \\ \bar{d}^3 \end{pmatrix}_L.$$

Other first-generation fermions are singlets with respect to $SU(2)_M$ and do not interact with the monopole at distances greater than the size of its core. They will be of no interest to us here. If we neglect fermions belonging to subsequent generations, and interactions not included in $SU(2)_M$, the model will be exactly the same as the SU(2) model with four left-handed doublets, which was discussed in Sec. 3.4. The condensates (3.18) include those that violate the baryon number, for example,

$$\langle(\bar{u}_{1L}\bar{u}_{2L}\bar{d}_{3L}e_L^+)(x)\rangle = \frac{\text{const}}{r^6}. \quad (3.20)$$

Nonconservation of baryon number is also found to occur in condensates such as (3.18). An example of this is

$$\langle(\bar{u}_{1L}\bar{u}_{2R}\bar{d}_{3R}e_L^+)(x)\rangle = \frac{\text{const}}{r^6}. \quad (3.21)$$

The quantum numbers of these condensates correspond to the following processes (monopole catalysis of nucleon decay):

$$p + M \to e^+ + M \ (+\ \text{pions}), \quad (3.22)$$

$$n + M \to e^+ + \pi^- + M \ (+\ \text{pions}). \quad (3.23)$$

Large values of these condensates signify that these processes occur with high cross sections. Since the only significant energy scale is the characteristic scale of strong interactions, it is expected that the cross sections for processes (3.22) and (3.23) will be of the order of the cross sections typical for strong interactions.[23-25]

The fast nonconservation of baryon number can also be seen at the level of selection rules. Reactions such as (3.16) include processes with the violation of baryon number, e.g.,

$$d_L^3 + M \to \frac{1}{2} e_L^+ + \frac{1}{2}(\bar{u}_{1R} + \bar{u}_{2R}) + \frac{1}{2} d_{3R} + M. \quad (3.24)$$

The quantum numbers of this process correspond to the reaction

$$p + M \to \frac{1}{2} e^+ + \frac{1}{2} p + M. \quad (3.25)$$

The final-state wave function is interpreted as a linear superposition of a proton and a positron: the process defined by (3.25) effectively describes the following two reactions:

$$p + M \to e^+ + M,$$

$$p + M \to p + M,$$

which have roughly equal cross sections. It is particularly clear in the scattering picture that processes with nonconservation of baryon number occur with high cross sections. We recall that (3.24) is the only possible reaction for the initial d_L^3 (in the limit of massless fermions).

The inclusion of fermions belonging to the subsequent generations leads to the appearance of other channels of monopole catalysis[23,60,68] [in addition to (3.22)]:

$$p + M \to e^+ + \mu^+\mu^- + M, \quad (3.26a)$$

$$p + M \to \mu^+ + K^0 + M, \quad (3.26b)$$

$$p + M \to \mu^+ + K^+ + \pi^- + M. \quad (3.26c)$$

It is expected that the cross sections for these processes have the same order of magnitude as the cross section for (3.22) [in particular, (3.26a) is not the radiative correction to (3.22)].

Effects due to interactions not included in $SU(2)_M$ were taken into account in Refs. 25, 69, and 70. It was shown there that the main conclusion, i.e., that the catalysis cross section was high, remained in force although some of the details of the behavior of the fermions near the monopole were altered.

The catalysis cross section is quite difficult to estimate and only partial results are available at present. The problem is that we have to match large distances ($r \gg 1$ fm), at which nucleons interact with the monopole because they have a magnetic moment, to intermediate distance ($r \sim 1$ fm), at which the quark structure of the nucleon comes into play, and short distances ($r \ll 1$ fm), which are responsible for the nonconservation of the baryon number. Electromagnetic interactions lead to a specific dependence of the cross section on the relative velocity of the monopole and the nucleon at low velocities. This dependence is different for the proton and the neutron[60,71]:

$$\sigma_p = \frac{\sigma_0}{\beta^2}, \quad \sigma_n = \frac{\sigma_0}{\beta}, \quad \beta = \frac{v}{c}. \quad (3.27)$$

The β^{-2} dependence for the proton is analogous to the enhancement of the inelastic cross section by the attractive Coulomb potential [cf., (3.15)]. We note that, for some nuclei, the dependence on β is weaker for the neutron.[71] The range of validity of formulas such as (3.27) is examined in Ref. 71.

Another effect of interactions at large distances is that the monopole can capture a proton or nucleus into a quasi-stationary orbit.[72] The cross section for this process is of the order of 1–0.1 mb for $\beta \sim 10^{-3}$. This effect can lead to a higher catalysis cross section.

There is only one published[73] calculation of the constant σ_0 in (3.27), based on the nonrelativistic quark model. According to the results reported in Ref. 73, $\sigma_0 \sim 0.1$ mb. However, much more work has to be done to obtain a more reliable estimate.

3.6. Model dependence of monopole catalysis

We have seen that, in the SU(5) model, proton decay stimulated by the fundamental monopole should occur with a high cross section (of the order of the cross sections typical for strong interactions). The existence of magnetic monopoles is a common property of grand unification models based on simple or semi-simple gauge groups.[74] The question therefore arises as to which unified theories and which types of monopole typically involve the monopole catalysis of proton decay.

A large number of publications has been devoted to this question. It has been shown that, in the SU(5) model, proton decay is stimulated not only by the fundamental monopole but also many other types of monopole.[60,75] Actually, this model, does not provide a single example of a monopole that does not lead to catalysis. Supersymmetrization of the model does not alter the properties of the catalysis process[76]: the cross section has the typical hadronic scale, as before, and (3.22), (3.23), and (3.26) remain as the principal modes. We note that the principal proton decay modes in the supersymmetric SU(5) theory are $p\to K^+\bar{\nu}_\mu$, $p\to K^0\mu^+$. Monopole catalysis will also occur in the SO(10) model[75] and in the SU(4)×SU(2)×SU(2) Pati-Salam model.[77] Catalysis will probably occur in all models that do not contain exotic heavy fermions.[75,78] However, catalysis may be absent from a model with exotic heavy fermions.[69,75] An example of this type of model is provided by the SU(5) theory with twice the number of generations, suggested in another context in Ref. 79. It has been shown[69] that monopole catalysis will not occur in this model, at least not for low relative velocities of the proton and monopole. Monopole catalysis of proton decay is thus seen to be a model-dependent effect.

Another aspect of the question of model dependence is that of catalysis in Kaluza-Klein type models, in which monopole solutions are also known.[80] The solutions of the Dirac equation in the field of a monopole in the five-dimensional Kaluza-Klein model were examined in Refs. 81 and 82, and it was shown that the nonconservation of charge, which occurs at the level of the Dirac equation in the field of the t'Hooft-Polyakov model, did not occur in the Kaluza-Klein model. Instead, there was nonconservation of chirality. This has been regarded[81,82] as an indication for the absence of the catalysis process. However, the question cannot be regarded as finally settled because the above five-dimensional model has a number of features that make it unrealistic (the presence of a massless scalar field interacting with fermions, vector coupling of the gauge field to the fermions instead of the V-A coupling, and so on). These undesirable features of the model are also very significant for interactions between fermions and monopoles. It may well be that monopole catalysis of proton decay will occur in more realistic Kaluza-Klein type models.

4. NONCONSERVATION OF FERMION NUMBER IN A COLD DENSE FERMION MEDIUM

In many ways, effects associated with nonconservation of the fermion number and the triangular anomaly determine the properties of a cold dense fermion matter in V-A theories.[11-14] Here, we shall examine both the case of fermion matter that is neutral in all the gauge charges (Sections 4.1–4.3) and the case of a fermion medium with nonzero density of Z^0-charge in the standard electroweak theory (Section 4.4).

4.1. Instability of normal symmetric fermion matter at high-densities[11]

The instability of normal neutral matter at sufficiently high fermion density arises both in Abelian and non-Abelian V-A theories. For simplicity, let us consider the Abelian V-A model with left-handed fermions, and assume that the vector fields acquire mass as a result of the Higgs mechanism. The Lagrangian for the model is

$$\mathscr{L} = -\frac{1}{4g^2}F_{\mu\nu}^2 + |(\partial_\mu - iA_\mu)\varphi|^2 - \lambda(|\varphi|^2 - c^2)^2 + \sum_{i=1}^{f}\bar{\psi}_L^{(i)} i\gamma^\mu\left(\partial_\mu - iA_\mu\frac{\tau^3}{2}\right)\psi_L^{(i)},$$

where

$$\psi_L^{(i)} = \begin{pmatrix}\psi_L^{(i,+)}\\ \psi_L^{(i,-)}\end{pmatrix},$$

and the fermions $\psi_L^{(i,\pm)}$ have charges $\pm g/2$. The condition for a symmetric fermion medium is

$$N_F^{(i,+)} = N_F^{(i,-)} \equiv \frac{N_F}{2f},$$

where $N_F^{(i,\pm)}$ are the numbers of positively and negatively charged fermions of type i for $\mathbf{A} = 0$. We shall assume that the system is placed in a large box of sixe L, so that N_F is large but finite.

As noted in Section 2.2, the appearance of the classical gauge fields (condensates) leads to the level crossing phenomenon, so that the number of real fermions may change. It is therefore necessary to distinguish between the initial number N_F of fermions and the number N_R of real fermions. The basic parameter of the system under investigation is N_F, which, by definition, is conserved. The physical meaning of N_F is quite clear. Suppose that the system was initially in a very large volume $(L')^3$, $L'\gg L$, so that the fermion density was low and matter was in the normal state with boson condensates $\varphi = c$, $\mathbf{A} = 0$. In this state, N_F is the number of real fermions in the system. As the size of this box is reduced from L' down to L, a condensate of the gauge field can appear within the box, and the number of real fermions can change, but the properties of the system are determined by N_F, as before.

In non-Abelian theories, the normal states will, in general, have boson fields of the form

$$\varphi = T(\omega)\varphi_0,\quad \mathbf{A} = \omega\vec{\partial}\omega^{-1},$$

where ω is a gauge function. The number of real fermions is then given by

$$N_F = \hat{N}_0 + n[\omega],$$

where $n[\omega]$ is the topological number (2.2) and N_0 is determined by (2.20). It follows that N_F is a gauge-invariant quantity [see (2.21)]. States with different ω are physically indistinguishable, so that we may suppose, without loss of generality, that $\omega = 1$. Hence,

$$N_F = N_0.$$

Outwardly, this relation defines N_F as a quantity invariant under topologically nontrivial gauge transformations, but this invariance is not, in fact, required because we have put $\omega = 1$.

The instability of normal matter against the formation of the condensate of gauge fields arises at high enough densities for the following reasons. The appearance of the condensate with negative N_{CS} leads to a reduction in the number of real fermions, according to (2.24). This, in turn, leads to a reduction in the energy of the fermions by the amount

$$-\Delta E_F = f\mu_0 N_{CS}, \tag{4.1}$$

where μ_0 is the chemical potential (Fermi energy) in the normal state. At the same time, the energy of the boson field increases, but this increase does not depend on the fermion density.[4)] It is clear that the gain in energy (4.1) is much greater than the increase in the boson energy for sufficiently large μ_0, so that the precipitation of the condensate is energetically favored at high densities.

In accordance with the foregoing, the effective Hamiltonian for static boson fields in the one-loop approximation is[5)]

$$E_B^{eff} = \int d^3x \left[\frac{1}{4g^2} F_{ij}^2 + (\partial_i \varphi)^2 + A^2\varphi^2 + \lambda(\varphi^2 - c^2)^2 - \mu_0 f \varepsilon_{ijk} \frac{F_{ij}A_k}{32\pi^2} \right], \quad (4.2)$$

where we have used the unitary gauge and assumed that the field **A** is small and that the characteristic momentum of the gauge field is $k \ll \mu_0$. It then follows from (4.2) that, when $\mu > \mu_{crit}$, where

$$\mu_{crit} = \frac{16\pi^2 M_V}{g^2 f},$$

the normal state becomes unstable: a negative mode of the form

$$\mathbf{A} = a(\mathbf{e}_1 \cos \mathbf{k}\cdot\mathbf{x} - \mathbf{e}_2 \sin \mathbf{k}\cdot\mathbf{x}) \quad (4.3)$$

with $k = M_V$ is found to appear (a is a small amplitude and $\mathbf{e}_{1,2}$ and $\mathbf{k}$ form an orthogonal basis). We shall see in Sections 4.2 and 4.3 that this instability leads to significantly different states in Abelian and nonAbelian theories.

4.2. Drops of anomalous matter in Abelian V-A theories[11,14]

It was noted in Section 2.2 that, in Abelian V-A theories, the crossing of the fermion levels and the vanishing of real fermions require the appearance of a magnetic field in the system. The final state to which the development of instabilities leads is therefore characterized in the Abelian case by nonzero classical magnetic fields. The ground state is inhomogeneous[14]: drops of "anomalous matter" are found in this state and are surrounded by normal vacuum, whereas a gauge field condensate with the structure given by (4.3) is formed in the interior of the drops. This condensate ensures that practically all the fermions pass into the Dirac sea, so that there are practically no real fermions either inside or outside the drops.

The properties of the drops can be determined as follows. Suppose that a gauge field condensate with the structure indicated by (4.3) has appeared in a region of size R, and that the amplitude a and momentum k of the condensate have not as yet been determined. If the appearance of the condensate leads to the transfer of N_F fermions to the Dirac sea, its Chern-Simons number should be N_F/f:

$$N_{CS} \sim \frac{R^3}{16\pi^2} ka^2 = \frac{N_F}{f}. \quad (4.4)$$

The energy of the drop is determined by the energy of the boson field (the contribution of the Dirac sea to the energy can be neglected for small coupling constants). When $\lambda \ll g^2$, the vanishing of the scalar condensate is energetically favored. It is then found that $\varphi = 0$ inside the drop (when $\lambda \gg g^2$, the boson field condensates have a more complicated structure, analogous to the mixed state of a superconductor of the second kind in a magnetic field; the final formulas for $\lambda \gg g^2$ are not very different from those reproduced below). Consequently, the energy of the boson fields contains two terms associated with the energy of the magnetic field and the energy of the Higgs field for $\varphi = 0$:

$$E = R^3 \frac{k^2a^2}{g^2} + \lambda c^4 R^3, \quad (4.5)$$

where we have discarded constants of the order of unity and have neglected surface terms (this approximation is valid for $N_F \gg 1$).

Minimization of the energy with respect to k for fixed R and subject to the additional condition (4.4) shows that k assumes the smallest possible value

$$k \sim \frac{1}{R}. \quad (4.6)$$

It then follows from (4.4) that

$$a \sim \frac{1}{R}\left(\frac{N_F}{f}\right)^{1/2}. \quad (4.7)$$

For these values of the momentum and amplitude, we find from (4.5) that

$$E = \lambda c^4 R^3 + \frac{N_F}{fg^2}\frac{1}{R}.$$

Minimization of this expression with respect to R determines the size and energy of the drop:

$$R \sim \frac{1}{c}\left(\frac{1}{f\lambda g^2}\right)^{1/4} N_F^{1/4}, \quad E \sim c\left(\frac{\lambda}{f^3g^6}\right)^{1/4} N_F^{3/4}. \quad (4.8)$$

The magnetic field inside the drop does not depend on N_F:

$$H \sim \sqrt{\lambda} g c^2. \quad (4.9)$$

The energy of the drop of anomalous matter is thus seen to increase with N_F more slowly than linearly. This means that the coalescence of drops is energetically favored, and the state with the lowest energy is that corresponding to a single drop. Next, the chemical potential of the fermions inside the drop is small:

$$\frac{\partial E}{\partial N_F} \sim N_F^{-1/4}, \quad (4.10)$$

so that there are actually practically no real fermions either inside or outside the drop. In theories in which fermions acquire mass as a result of interaction with the Higgs field, the relation given by (4.10) signifies that, for sufficiently high N_F, the drops are stable even when there are no fermions in their exterior. The stability condition is then

$$\frac{\partial E}{\partial N_F} < m_F. \quad (4.11)$$

where N_F is the fermion mass. This condition is satisfied when

$$N_F \gg \frac{\lambda}{f^3g^6}\left(\frac{c}{m_F}\right)^4.$$

We note that, for parameters (coupling constants and vacuum expectation values) of the order of the parameters of the standard electroweak theory, the number of fermions, the drop size, and the drop mass on the stability boundary are of the order of $N_F \sim 10^{16}$, $M \sim 10^{15}$ GeV, and $R \sim 10^{-12}$ cm, respectively.

Stable extended objects in the form of drops of anomalous matter are thus seen to arise in Abelian V-A theories. It follows from (4.11) that the energy of a fermion inside the drop is lower than the rest energy of the free fermion. Consequently, such drops should absorb fermions when they come into contact with ordinary matter. This process should be accompanied by an energy release of the order of the fermion rest mass.

The above results are valid not only for V-A theories, but also for other Abelian models that do not have a vector structure. Among known interactions, these properties are exhibited by the interaction associated with the U(1) subgroup of the standard gauge group SU(2) × U(1). However, it has the same selection rules as the SU(2) interaction, but is characterized by a smaller coupling constant. It is therefore problematic as to whether the drops of anomalous matter associated with the SU(1) subgroup can actually be formed. Abelian interactions that do not have the vector structure, and differ from U(1), are predicted by a number of grand unification theories, including models based on superstrings.[83]

4.3. Fate of the anomalous state in non-Abelian theories[14]

In non-Abelian gauge theories, nonconservation of the fermion number due to level crossing can occur as a result of a transition of the system to the topologically nontrivial gauge vacuum. The ground state of the symmetric medium with N_F fermions is therefore actually the state without real fermions above the vacuum with topological number N_F/f [in the SU(2) model with left-handed doublets, f is the number of doublets]. At low densities, the normal state (degenerate Fermi gas) is metastable, but the number of fermions in the system decreases as a result of instanton-type tunneling transitions, and the lifetime of the normal state is exponentially long.

The question then arises as to whether the anomalous Abelian matter discussed in Section 4.2 is metastable or whether it can undergo a fast (classical) transition to a state with a small fermion number above the topologically nontrivial value. This question is discussed in Ref. 14, where it is shown that the second possibility occurs in non-Abelian theory. We note that a wholly analogous result occurs in two-dimensional gauge theories with complex structure of vacuum and nonconservation of fermion number,[12,13] namely, they also exhibit a classical transition to the normal state with a small number of real fermions above the topologically nontrivial vacuum.

In four-dimensional nonAbelian theories, absolute instability of Abelian anomalous matter arises in the following way. Consider the SU(2) model, in which the anomalous Abelian state is characterized by a gauge condensate with $\mathbf{A}^1 = \mathbf{A}^2 = 0$, $\mathbf{A}^3 = a(\mathbf{e}_1 \cos \mathbf{kx} - \mathbf{e}_2 \sin \mathbf{kx})$, and the amplitude and momentum are given by (4.6) and (4.7), respectively. It then follows from (4.8) and (4.9) that the magnetic field is almost uniform and $H^{1/2} \gg k$. In a uniform magnetic field $\mathbf{H}^3$, perturbations of the fields $\mathbf{A}^{1,2}$ have negative modes.[84] These modes have the property that their excitation does not alter the Chern-Simons number of the gauge field.[11,14] Consequently, the excitation of negative modes reduces the energy of the boson fields and does not alter the number of real fermions (which is practically zero) or their energy, which means that the anomalous state is unstable.

It can be shown (see Ref. 14 for further details) that the development of this instability leads to a transition of the system to a state with low (much lower than the critical) number of fermions above the topologically nontrivial vacuum. It follows that, in the non-Abelian four-dimensional theory, the density of cold symmetric fermion matter cannot exceed the critical value

$$n^{\text{crit}} = \frac{2f}{3\pi^2} (\mu_{\text{crit}})^3.$$

The attainment of critical density is followed by intensive nonconservation of fermion number with the result that the system undergoes a transition to the normal state with a low density of fermions.

The foregoing results are directly applicable to the standard electroweak theory without right-handed neutrinos. The neutrality condition in this theory has the form

$$n_u^{(\alpha)} = n_d^{(\alpha)} = n_e = n_{\nu_e} = \dots, \qquad (4.12)$$

where $n_u^{(\alpha)}$ is the density of u-quarks of color α ($\alpha = 1,2,3$), and so on. In the standard model, with three generations of quarks and leptons ($f = 12$), the critical density

$$n_{\text{crit}} = 3 \cdot 10^{12} \ \text{fm}^{-3},$$

which is greater by roughly 12 orders of magnitude than the nuclear density.

4.4. Nonsymmetric matter: anisotropic condensate of W-bosons

In the asymmetric case in which fermion matter has nonzero density of Z^0-charge, there is another mechanism that leads to the instability of the normal state and to the Bose condensation of W-bosons.[85] For example, consider the SU(2) × U(1) model with a single doublet of leptons (without quarks). Only the neutrinos are present in the electrically neutral normal (without the W-boson condensate) fermion medium. When the chemical potential (Fermi energy) of the neutrinos exceeds $M_W + m_e$, the following reaction becomes energetically convenient:

$$\nu_e \to W^+ + e^-.$$

As a result of this reaction, electrons appear in the medium and the W-bosons produced in this way precipitate into the condensate. This is a second-order phase transition.[85] The critical value of the chemical potential is $M_W + m_e$ and is much smaller than μ_{crit} of the symmetric neutral medium.

An analogous situation is also found to arise in standard electroweak theory (in which there are both leptons and quarks) in the case of an asymmetric medium for which (4.12) is satisfied. The W-boson condensate that forms if we ignore the nonconservation of the fermion number is homogeneous and isotropic.[85] Inclusion of this nonconservation leads to a qualitatively new effect, namely, the W-boson condensate becomes anisotropic[33] and has the structure

$$\mathbf{A} = a \ (\mathbf{e}_1 \cos \mathbf{k} \cdot \mathbf{x} - \mathbf{e}_2 \sin \mathbf{k} \cdot \mathbf{x}), \qquad (4.13)$$

where the amplitude a varies slowly in comparison with the homogeneous case, and

$$k \sim \alpha_W \mu.$$

The appearance of the anisotropic condensate can be explained as follows (see Ref. 33 for further details). If we neglect effects associated with the nonconservation of the fermion number, the effective bosonic Hamiltonian can be schematically written in the form (using the unitary gauge)

$$E_B^{\text{eff}(0)} = \int d^3x \left[\frac{1}{4g^2} (F_{ij}^a)^2 + \frac{1}{2} (d\varphi)^2 + \mathcal{U}(A^a, \varphi) \right], \quad (4.14)$$

where $\mathbf{A}$ and F_{ij}^a is the vector potential and the strength of the W-boson field, φ is the Higgs field, and $\mathcal{U}$ is a relatively complicated function that includes both the classical energy of the bosonic fields and contributions that arise due to the interaction between fermions and the condensate. The presence of the W-boson condensate is due to the fact that $\mathcal{U}$ has a minimum for

$$|\mathbf{A}| = a \neq 0.$$

The inclusion of effects associated with nonconservation of the fermion number leads to the appearance in the effective Hamiltonian of a term that is linear in the derivative of the gauge field:

$$E_B^{\text{eff}} = E_B^{\text{eff}(0)} + E_B^{\text{eff}(1)},$$

where

$$E_B^{\text{eff}(1)} = \frac{\bar{\mu}}{32\pi^2} \int \varepsilon_{ijk} F_{ij}^a A_k^a \, d^3x, \quad (4.15)$$

where $\bar{\mu} \sim \mu$ (the explicit expression for $\bar{\mu}$ is quite complicated and will not be reproduced here; see Ref. 33). The origin of (4.15) is the same as that of (4.1): when the gauge field with nonzero Chern-Simons number appears, the energy of the fermions in the medium is reduced because of level crossing and the reduction in the number of real fermions. The expression given by (4.15) can be obtained formally by evaluating the leading contribution to the one-loop polarization operator in the fermion medium.

It follows from (4.14) and (4.15) that configurations such as (4.13) do, in fact, minimize the effective bosonic Hamiltonian, where $k = g^2/16\pi^2\bar{\mu}$ and the amplitude a is found by minimizing the function

$$\mathcal{U}(a, \varphi) - \frac{1}{2} \left(\frac{\bar{\mu} g}{16\pi^2} \right)^2 a^2.$$

It can be shown that the second term in this expression is a small correction to the first, so that inclusion of (4.15) does not alter the amplitude of the condensate in the leading order in g.

The state with the anisotropic boson condensate is stable with respect to small perturbations near it, but unstable with respect to transitions to a state with a smaller number of fermions above the topologically-nontrivial vacuum.[33] In contrast to the case of a symmetric medium, such transitions can occur only by tunneling. The problem of evaluating the probabilities of these transitions is very complicated and it may well be that the rate of these tunneling processes will turn out to be much higher than the rate of instanton transitions in vacuum.

The anisotropy of the energy-momentum tensor in the state with the W-boson condensate (4.13) is of the order of 10^{-4} in the standard model. In standard cosmological models with an intermediate cold stage, this anisotropy is sufficient to generate the relic gravitational noise with amplitude of 10^{-18}–10^{-20} in a period of 3h–10 d.[34] This is one of the most powerful possible sources of gravity waves in this range.

The detectors of gravity waves that are being discussed at present have sensitivities that will be sufficient for the detection of noise with these parameters (see Ref. 34 and the references therein).

5. NONCONSERVATION OF BARYON NUMBER IN THE STANDARD ELECTROWEAK THEORY AT HIGH TEMPERATURES AND BARYON ASYMMETRY OF THE UNIVERSE

5.1. Rate of processes with nonconservation of fermion number at high temperatures

The electroweak nonconservation of the baryon number due to the complicated structure of vacuum should be very intensive at high enough temperatures.[15] Whereas, at zero temperature, transitions between topologically different vacuums (for example, with $n = 0$ and $n = 1$; see Fig. 2) are tunneling processes, such transitions can occur at nonzero temperatures as a result of thermodynamic fluctuations occurring above the barrier.

Let us examine one of the equilibrium configurations of gauge and scalar fields at a temperature T that is small in comparison with the height E^S of a barrier located "near" a vacuum with $n = 0$. The system can "jump over" the barrier as a result of a thermodynamic fluctuation, and may find itself near the $n = 1$ vacuum. This process is accompanied by a change in the fermion (baryon, lepton) number.

Let us estimate the probability of this hop in the high-temperature approximation in which $T \gg M_W$. We shall consider a *quasi-equilibrium* thermodynamic ensemble constructed above one of the gauge-equivalent vacuums, say, the $n = 0$ vacuum. (We shall suppose that the only nonequilibrium processes are those with B-nonconservation.) The probability of thermodynamic fluctuations with $\Delta n = \pm 1$ is then equal to the rate of decay of this metastable state.[15] The latter is determined by the probability that the system will be found in the neighborhood of the saddle point $\mathbf{A}^s, \varphi^s$, i.e., the probability of forming the sphaleron configuration. Roughly speaking, this probability is determined by the Boltzmann factor $\exp(-E^s/T)$, where $E^s = (2M_W/\alpha_W)B$ is the energy of the sphaleron. This estimate can be improved by taking into account the fact that the expectation value of the scalar field (and, hence, of M_W) depends on temperature,[86,87] so that the exponential suppression factor is[15]

$$\exp\left(-\frac{2M_W(T)}{\alpha_W T} B\left(\frac{\lambda}{g^2}\right) \right), \quad (5.1)$$

where we have neglected the slow (logarithmic) dependence of the coupling constant on temperature. The evaluation of the pre-exponential factor is very laborious.

The general formula for the probability of decay of the metastable state per unit volume per unit time in the one-loop approximation[6)] (which, in our case, is the same as the probability of transition between topologically different vacuums) is[88–91]

$$\Gamma = Z_0 \frac{\omega_-^S}{2\pi} \operatorname{Im} \prod_k \frac{\sinh(\omega_k^0/2T)}{\sinh(\omega_k^S/2T)} \exp\left(-\frac{E^S}{T}\right), \quad (5.2)$$

where ω_k^S are the eigenfrequencies of the Bose excitations in the background field of the sphaleron, ω_k^0 are the corresponding eigenfrequencies in the absence of the sphaleron, and ω^s is the only negative mode (the sphaleron is unstable in the "direction" of the change in the topological number). The factor Z_0 represents the normalization of the zero modes. The basic effect due to ω_k^s is the renormalization of the zero-temperature parameters of the sphaleron, which leads to the replacement of M_W with the temperature-dependent effective mass [see (5.1)],[15] i.e., (see also Refs. 92 and 91),

$$\operatorname{Im} \prod_k \frac{\sinh(\omega_k^0/2T)}{\sinh(\omega_k^s/2T)} \exp\left(-\frac{E^S}{T}\right) \rightarrow (2M_W(T))^3 \varkappa \times \exp\left(-\frac{2M_W(T)}{\alpha_W T} B\right), \qquad (5.3)$$

where the factor $(2M_W(T))^3$ is the effective reciprocal volume of the sphaleron[42,91] and has been introduced to ensure the correct dimensions of $\Gamma, \varkappa \sim 1$, i.e., it is a slowly-varying function of the ratio (λ/α_W). The factor Z_0 was found in Ref. 91:

$$Z_0 = N_{tr} N_{rot} \left(\frac{M_W(T)}{2\pi\alpha_W T}\right)^3, \quad N_{tr} = 26, \quad N_{rot} = 5.3 \cdot 10^3. \qquad (5.4)$$

The numerical values of N_{tr} and N_{rot} are given for the special case $\lambda = g^2$ and the result is, in fact, a slowly-varying function of the ratio λ/g^2. Altogether, the sphaleron has six zero modes, three translational modes, and three modes associated with the noninvariance of the configuration (2.5) under a combination of space and isotopic SU(2) rotation.

The final expression for the probability of thermodynamic fluctuations is

$$\Gamma = \frac{T^4 \omega_-^S}{M_W(T)} \left(\frac{\alpha_W}{4\pi}\right)^4 N_{tr} N_{rot} \left(\frac{2M_W(T)}{\alpha_W(T)}\right)^7 \times \exp\left(-\frac{E^S(T)}{T}\right) \varkappa. \qquad (5.5)$$

This form of the pre-exponential factor in Γ was obtained in Ref. 91, and a renormalization scheme in which the determinant in (5.2) leads to the parametrization of (5.3) was noted in Ref. 92. An evaluation of the analog of the parameter $\varkappa$ for the two-dimensional model theory is given in Ref. 93.

Formula (5.5) is valid in a relatively narrow temperature interval:

$$M_W(T) \ll T \ll E^S(T).$$

At low temperatures, the main contribution to processes with nonconservation of the fermion number is provided by tunneling transitions. On the other hand, when $\Gamma \sim (\alpha_W T)^4$, we can no longer use the analysis of the decay of metastable states with allowance for only the saddle point of the energy functional.[88]

It follows from (5.1) and (5.5) that the exponential suppression factor decreases with increasing temperature both because of the presence of the factor $1/T$ in the exponential and because of the reduction in $M_W(T)$. When $T > T_c$ [T_c is the phase transition point with the breaking of the SU(2)×U(1) group], the W-bosons become massless and the argument of the exponential becomes formally zero. This leads to the conclusion that the rate of processes with nonconservation of B is not exponentially suppressed for $T \gg T_c$ although it cannot be calculated in the quasiclassical approximation. The fact that the "electric" components of the gauge field have a mass of the order of gT for $T \gtrsim T_c$ (Debye screening) is not significant for estimates of the argument of the exponential: the sphaleron contains only the "magnetic" field components. The natural mass scale for static magnetic components is the quantity $\alpha_W T$ (Refs. 94 and 95), which is none other but the unique dimensional coupling constant of the three-dimensional gauge theory that is the high temperature limit of the four-dimensional theory. In approximate estimates, we can replace M_W in (5.5) with $\alpha_W T$, i.e., $\Gamma \sim (\alpha_W T)^4$ for $T \gtrsim T_c$. Of course, this type of discussion cannot be regarded as a derivation of the formula for the probability of fluctuations above the phase transition point. The exact evaluation of Γ for $T > T_c$ remains an open question.

5.2. Fate of the baryon asymmetry that arises in GUT[15,96]

Fast electroweak processes with nonconservation of the baryon number at temperatures of the order of a few hundred GeV or more have a significant effect on the baryon asymmetry of the Universe (BAU). Two possibilities are of particular interest in this connection.[15] First, the baryon asymmetry that is produced at GUT temperatures (of the order of 10^{15} GeV) may be masked by electroweak processes. Second, BAU can arise directly in electroweak theory[31] or a modification of it (see Section 5.3).

The kinetic equation describing the washing out of the baryon and lepton charges has the following form for massless fermions:

$$\frac{d\,\Delta B}{dt} = -V_B \Delta B, \quad \frac{d\,\Delta L}{dt} = -V_B \Delta L, \qquad (5.6)$$
$$\Delta B = B - B_0, \quad \Delta L = L - L_0,$$
$$B_0 = \frac{4}{13}\zeta (B-L)_{in}, \quad L_0 = B_0 - (B-L)_{in},$$
$$V_B = \frac{13 N_f \Gamma}{2T^3} \xi,$$

where B and L are the densities of the baryon and lepton numbers, respectively, and $(B-L)_{in}$ is the initial value of the $(B-L)$ asymmetry. The parameters ξ and ζ assume the following values for $T \gtrsim M_W, M_H$:

$$\xi = \frac{66N_f + 39}{65N_f + 39}, \quad \zeta = \frac{104N_f + 52}{88N_f + 52}. \qquad (5.7)$$

A very nontrivial dependence of the right-hand side of (5.7) on the number N_f of generations arises when we take into account the fact that, in $B-L$ asymmetric plasma, its electrical neutrality is attained for nonzero chemical potential of the scalars.[71]

It follows from (5.7) that the characteristic time for processes with nonconservation of B is

$$\tau_A = \frac{2T^3}{13 N_f \Gamma \xi}. \qquad (5.8)$$

The fact that $\tau_A \gg 1/T$ for $T \gtrsim M_W$ explains the impossibility of calculating the effect just discussed within the framework of the Matsubara formalism for the temperature Green's functions. In this formalism, fields are periodic (antiperiodic in the case of fermions) functions of the Euclidean time t_E with period $\beta = 1/T$. The analytic continuation of the *approximate* answer for the Matsubara Green's function to *large* real times is an exceedingly complicated problem that has not as yet been solved. In the formalism used here for the decay of the metastable state, the problem is formulated right at the beginning in real time (Refs. 88 and 90), and there are no difficulties with the analytic continuation.

The solution of (5.7) for the case of an expanding Universe with initial conditions

$$\left(\frac{B}{n_\gamma}\right)_{\text{in}}=b_{\text{in}},\quad \left(\frac{L}{n_\gamma}\right)_{\text{in}}=l_{\text{in}}$$

gives

$$\Delta=\frac{B_0}{n_\gamma}+\frac{(\Delta B)_{\text{in}}}{n_\gamma}\exp(-A),\qquad A=\int_{t_0}^{t}V_B(t)\,dt, \tag{5.9}$$

where the time is related to temperature by $T_U=M_0/T^2$, $M_0=M_{Pl}/1.66N_{\text{eff}}^{1/2}$, and N_{eff} is the number of massless degrees of freedom. Since $A\gg 1$, only the B–L-asymmetric part of BAU is found to survive.

This conclusion has to be modified to some extent for plasmas that are asymmetric in the quark and lepton flavors.[96] The BAU that arises in GUT is usually proportional to the Yukawa coupling constants between fermions and Higgs fields (and, thereby, the quark and lepton masses).[97] Thus, at the time of the grand unification, BAU is concentrated mostly in the third generation of fermions. If the GUT interactions do not lead to an effective breaking of the conservation laws (2.18), the surviving baryon asymmetry will also be nonzero in GUT with B–L conservation $(b-l)_{\text{in}}=0$ (Ref. 96):

$$b_{\text{out}}\approx-\frac{6}{13\pi^2}\sum_{i=1}^{N_f}\frac{\overline{m}^2_{l_i}(T_*)}{T_*^2}\Delta_i^{\text{GUT}}, \tag{5.10}$$

where Δ_i^{GUT} are the asymmetries relating to the quantum numbers L_i-B/N_f that are conserved in the electroweak theory, L_i is the electron, muon, etc., lepton number, $\overline{m}^2_{l_i}$ is the mean square of the mass of the ith generation lepton, and T_* is the temperature of quenching of anomalous electroweak processes with nonconservation of B, determined from the condition $\tau_A=t_U$. We shall see in Section 5.3 that $T_*=T_c$ for $M_H\lesssim M_{\text{crit}}\approx 45$ GeV (Refs. 31 and 92) and $T_*<T_c$ for $M_H\gtrsim M_{\text{crit}}$ (M_H is the mass of the Higgs boson). When $T_*>T_c$, the fermions acquire mass due to one-loop corrections. The increment that is asymmetric in the flavors and contributes to (5.10) has the form ($M_H\lesssim M_{\text{crit}}$):

$$\frac{\overline{m}^2_{l_i}(T_*)}{T_*^2}=\frac{\pi\alpha_W}{3}\frac{m^2_{l_i}(0)}{M_W^2(0)}.$$

If, on the other hand, $M_H\gtrsim M_{\text{crit}}$, then

$$\frac{\overline{m}^2_{l_i}(T_*)}{T_*^2}=\left[\frac{\pi\alpha_W}{3}+\frac{2}{3}\left(\frac{45\alpha_W}{2B}\right)^2\right]\frac{m^2_{l_i}(0)}{M_W^2(0)}.$$

In the case of three fermion generations, we have[8)]

$$\begin{aligned}\Delta&=8\cdot 10^{-7}\Delta_3^{\text{GUT}}, && M_H\ll M_{\text{crit}},\\ &=(2-4.5)\cdot 10^{-6}\Delta_3^{\text{GUT}}, && M_H\gg M_{\text{crit}}.\end{aligned} \tag{5.11}$$

To obtain the observed value $\Delta\sim 10^{-9}$, we must demand that the microscopic asymmetry in leptoquark decays

$$\delta_X=\frac{\sum[B(i)(\Gamma(X\to i)-\Gamma(\bar X\to\bar i))]}{\Gamma_{\text{tot}}(X)} \tag{5.12}$$

should be of the order of unity, which seems unnatural. (We recall that BAU in leptoquark decays is $\Delta\approx 10^{-3}\delta_X$; Refs. 28–30.) The situation becomes significantly simpler in models with a heavy fourth generation of fermions, where, say, $m_{\tau'}\approx 100$ GeV. Here, it is sufficient to have $\delta_X\approx 10^{-3}$.

The observed BAU in models with three generations can therefore be due to grand unification, but only if B–L is not conserved in the grand unification theory (moreover, this nonconservation must be sufficiently intensive to ensure that the B–L asymmetry of the Universe is large enough). This is a further stringent criterion for choosing between grand unification theories. For example, the SU(5) model does not satisfy it. In grand unification theories that do not conserve B–L, it is natural to expect the suppression of B–L-violating processes such as neutron-antineutron oscillations, the decay of the nucleon along "nonstandard" channels such as $p\to\nu\pi$, νK, $n\to e\pi^+$, e^-K^+, double neutrino-free β-decay, neutrino oscillations, and so on.

We also note the closely related BAU scenario using fast electroweak nonconservation of the baryon number.[98] This mechanism is based on the assumption that there is a heavy lepton (with mass greater than a few tens of TeV) in whose decays the lepton number is not conserved. This leads to the generation of a leptonic asymmetry at higher temperatures, which is then transformed into the baryon asymmetry as a result of electroweak processes. The model predicts the existence of processes with the nonconservation of the lepton number at low energies (double neutrino-free β-decay, $\mu\to e\gamma$ decay) with rates approaching the experimental limits. Another possibility of generating the baryon number by electroweak processes in the modified theory is discussed in Ref. 15. We shall not pause to consider the possibility of constructing a grand unification theory with the required properties, and proceed directly to discuss the possibility of BAU arising directly within the framework of the standard electroweak theory.

5.3. Baryon asymmetry of the Universe in the standard electroweak theory

The observed baryon asymmetry of the Universe may turn out to be wholly due to electroweak processes with nonconservation of the baryon number. A grand unification theory does not then need to be brought in to explain it. In principle, all three conditions for the generation of the baryon asymmetry are satisfied in the Universe at temperatures of the order of a few GeV, namely, (1) the baryon number is not conserved because of electroweak processes, (2) CP violation is also present in the standard electroweak theory (Kobayashi-Maskawa mechanism), and (3) thermodynamic nonequilibrium is present because of the expansion of the Universe. The question then is whether the baryon asymmetry is generated at the observed level.[9)] A complete answer to this question has not as yet been given and only partial results are available. It is shown in Ref. 15 that the standard model does not ensure the generation of the baryon asymmetry of necessary size if the phase transition in this model (accompanied by the formation of the Higgs conden-

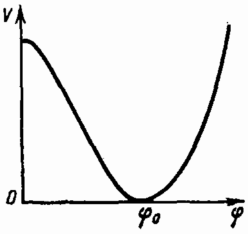

FIG. 4. Effective potential for the scalar field near the critical temperature.

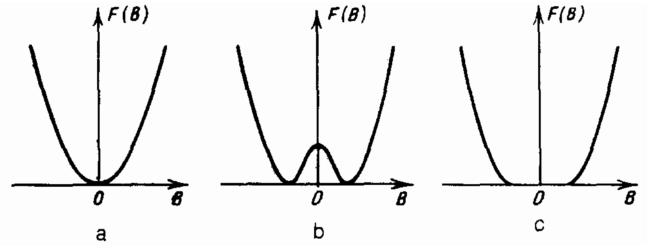

FIG. 5. Possible shapes of the effective potential for the density of the Chern-Simons number.

sate) is a second-order phase transition. The reason is that the nonequilibrium is too weak in the case of a second-order phase transition, and this leads to a relatively small baryon asymmetry. The appearance of the baryon asymmetry in the standard model is not, therefore, excluded, but only in the case of the second-order phase transition which will occur in models with a sufficiently light Higgs boson, i.e., $M_H \sim 10$ GeV (Ref. 15).

A scenario for the generation of the observed asymmetry within the framework of the standard model was proposed in Ref. 31. It was based on the assumption of a high-temperature degeneracy in the Chern-Simons number of the ground state of gauge theories. Let us examine this in greater detail.

We begin with the structure of the ground state for $T > T_c$ and $T < T_c$ in the case of the second-order phase transition (throughout our discussion, T_c will be the temperature at which the barrier between the phases with $\langle\varphi\rangle = 0$ and $\langle\varphi\rangle = \varphi_0$ disappears). During the expansion of the Universe, the system is in the state with $\langle\varphi\rangle = 0$ up to $T = T_c$ (Refs. 87 and 99), and thereafter this state becomes absolutely unstable and $\langle\varphi\rangle$ increases to φ_0, which corresponds to a minimum of the effective potential (Fig. 4). The question is: what happens to the gauge field during this transition? When $T < T_c$, symmetry is spontaneously broken, and W- and Z-bosons are massive. This means that the ground state of the system is dominated by configurations in the form of small deviations from pure gauges. When $T > T_c$, the vector bosons have zero bare mass. This leads to strong (power type) infrared divergences in the sector of static SU(2) magnetic fields and, apparently, to the existence of a nontrivial structure of the ground state. Thus, the authors of Refs. 94 and 100 have put forward arguments showing that the non-Abelian plasma can contain a nonzero SU(2) magnetic field $H \sim g^3 T^2$ at finite temperatures, and that the corresponding characteristic dimension of inhomogeneities is $(g^2 T)^{-1}$. This structure of the ground state must vanish during the first-order phase transition, i.e., the quantity

$$B = \frac{1}{32\pi^2}\frac{1}{V}\int_{t_0}^{\infty} d^4x \, \mathrm{Tr}\, F_{\mu\nu}\tilde{F}_{\mu\nu} \qquad (5.13)$$

must, in general, be nonzero for some characteristic equilibrium initial configuration. By virtue of (2.12), B is none other than the baryon number (apart from the factor N_f), created as a result of the SU(2) phase transition from some specific initial configuration. As usual, the integral in (5.13) is the difference between the Chern-Simons numbers of the initial and final states. The quantity $N_{CS}(t_0)$ can be estimated from

$$N_{CS}(t_0) \sim \alpha_W VHA \sim V\alpha_W^3 T^3,$$

and the baryon number density is[31]

$$\frac{n_B}{n_\gamma} \sim \alpha_W^3. \qquad (5.14)$$

Of course, for $T > T_c$, the electroweak plasma has configurations that decay both into baryons and antibaryons, and the total BAU is obtained after averaging over all the admissible states. Analysis of these states is conveniently based on the concept of the effection potential with B as the variable (the effective Chern-Simons number of the initial state). The potential $V(B)$ can be introduced[31,101] since B depends only on the initial state at time t_0 (and, in general, on the shape of the scalar potential[100]). Figure 5 shows three possible shapes of $V(B)$. We note that the symmetry of $V(B)$ under the replacement $B \to -B$ is dictated by CPT-invariance.

We still do not know which of the possible shapes of the potential is realized in nature. Preliminary results[102] obtained within the framework of the lattice formulation of gauge theories show that the high-temperature electroweak plasma is populated by quasiparticles, i.e., collective excitations of gauge and scalar fields, whose decay during the phase transition gives

$$\frac{1}{32\pi^2}\int \mathrm{Tr}\, F_{\mu\nu}\tilde{F}_{\mu\nu}\, d^4x = \pm 1.$$

In other words, the decay of a quasiparticle leads to the creation of twelve fermions (or antifermions) because of the triangular anomaly. Monte Carlo calculations[102] provide a very rough numerical estimate for the concentration of such quasiparticles:

$$n_{qp} \approx 0.1 - 0.2\,(\pi\alpha_W T)^3. \qquad (5.15)$$

When the Bose-Einstein condensate of quasiparticles is present, the potential for B is of the form indicated in Figs. 5b and c, which correspond to the spontaneous breaking of global symmetry responsible for the existence of the quasiparticles and antiquasiparticles. The possibility of condensation of quasiparticles can be settled by calculations using large lattices (say, 50^3), which can be done, at least in principle, using modern computers.

Let us now consider the cosmological properties of all three types of potential $V(b)$ (Refs. 31 and 101). In the first case (Fig. 5a), the state of the Universe before the phase transition is CP-symmetric. Consequently, the BAU that arises should be proportional to the measure of CP-violation in processes involving the nonconservation of baryon number. The breaking of C- and CP-symmetry in electroweak theory at high temperatures arises because of the Yukawa interaction between quarks and the Higgs fields. This part of the Lagrangian is

$$\mathscr{L}_Y = \frac{g_W}{\sqrt{2} M_W} (\bar{Q}_L K M_d D_R \varphi + \bar{Q}_L M_U U_R \tilde{\varphi} + \text{h.c.}), \quad (5.16)$$

where Q_D^α are the left-handed quark doublets, α is the generation index, $U_R^{(\alpha)}$ and $D_R^{(\alpha)}$ are the right-handed quarks with electric charges of 2/3 and $-1/3$, respectively, K is the Kobayashi-Maskawa matrix, and M_u and M_d are the diagonal mass matrices of the fermions. It is well known[28-30] that the amplitude for the CP-violation is proportional to the product of the imaginary part of the diagram (without the coupling constants) and the imaginary part of the product of all the coupling constants on the same diagram. It is readily seen that, in our case, in which we have to sum over the quark flavors, the imaginary part of the product of the Yukawa coupling constants arises only in twelfth-order perturbation theory[31] (Fig. 6). However, the sum of all the diagrams in this order does not manifest the effects of CP-violation because of the mutual cancellation of graphs with the replacement $u_R \leftrightarrow d_R$, $\varphi \leftrightarrow \tilde{\varphi}$, $\tilde{\varphi}_i = \varepsilon_{ij} \varphi_j^*$ (Ref. 101). Thus, CP becomes significant beginning with the fourteenth order in the Yukawa coupling constants h_k (or twelfth order in h_k and second order in the hypercharge interaction). Numerically,

$$\begin{aligned} D &= \operatorname{Im} \operatorname{Tr} \mathcal{M}_u^3 \mathcal{M}_d^2 \mathcal{M}_u \mathcal{M}_d \\ &= \left(\frac{g_W^2}{2M_W^2} \right)^7 s_1^2 s_2 s_3 \sin \delta m_t^6 m_b^4 m_c^2 m_s^2 \approx 10^{-22}, \\ \mathcal{M}_d &= K M_d^2 K^+, \quad \mathcal{M}_u = M_u^2; \end{aligned} \quad (5.17)$$

where $s_i = \sin \theta_i$, θ_1 is the Cabibbo angle and δ is the CP-violation phase. We have used the upper limit[103] for the product $s_1^2 s_2 s_3 \lesssim 3 \times 10^{-4}$, $\sin \delta \approx 1$. Despite the large number of diagrams contributing to B-nonconservation (the combinatorial factor is $\sim 10^4$), the number 10^{-22} seems too small to explain BAU in the case of the B-trivial structure of the high-temperature ground state[11]).

The potentials of Fig. 5b actually correspond to spontaneous CP-violation at high temperatures. In the case of Fig. 5b, domains with different B will be formed throughout the Universe. Baryon asymmetry will arise in regions with $B > 0$, and antibaryon asymmetry in regions with $B < 0$. Generation of the observed baryon excess is possible, in principle, but a mechanism has to be found for stretching the domains to dimensions of the order of the visible part of the Universe. This question is discussed in detail in Ref. 101. We note, at this point, that this stretching is not possible during standard inflation[104] because an exponential reduction in temperature

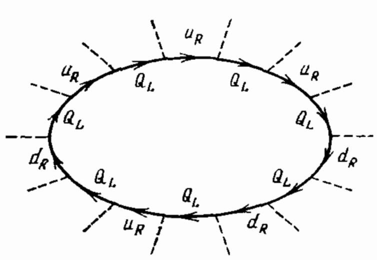

FIG. 6. Fermion loop of the lowest nonvanishing order that arises when CP-violation effects are described for processes with nonconservation of baryon number. Broken lines correspond to scalars.

leads to the vanishing of the gauge fields. New domains with dimensions $O(\alpha_W T)^{-1}$ form during the subsequent heating. The vanishing of the domain structure and the formation of the "pure" state [in one of the minima of the potential $V(B)$ throughout the Universe] is possible in the presence of a strong CP-nonconservation (~ 1) at ultrahigh temperatures ($T \sim 10^{15}$ GeV).

If the ground state is infinitely degenerate in B at high T (Fig. 5c), BAU can arise in the standard model even for a very small [such as in (5.17)] CP-violation amplitude.[31] The point is that, when $T > T_c$, the nonequilibrium expansion of the Universe and the CP-violation in interactions with nonconservation of baryon number lift the degeneracy in B. The resulting increment in $V(B)$ is

$$\Delta V \approx \delta_{ms} \frac{T^2}{M_0} B, \quad (5.18)$$

where δ_{ms} is the microscopic asymmetry in processes with nonconservation of the baryon number, which is proportional to D in (5.17). It follows that, depending on the sign of δ_{ms} (related to the sign of the CP-violation in kaon decays), the state with minimum (maximum) value of B becomes the most favored one energetically. It may be shown that, when[31]

$$\rho = 4 \left(4\pi \frac{M_0}{T_c} \alpha_W^2 \delta_{ms} \right)^{1/2} \gg 1 \quad (5.19)$$

the system will be in the state with the maximum (in absolute value) value of B, independently of the position on the plateau from which it started. At the same time, the magnitude of the BAU will not depend on the CP-violation amplitude (it is determined exclusively by the infrared properties of the gauge theories), whereas the sign of BAU is related to the sign of the CP-violation in neutral-kaon decays.

The final answer for the BAU is

$$\Delta = \frac{45}{4\pi^2 N_{eff}} N_f B(T_c) S(M_H), \quad (5.20)$$

where $S(M_H)$ is the factor representing the macroscopic suppression of asymmetry, taking into account the heating of the Universe after the first-order phase transition and the washing out of the baryon excess by processes with nonconservation of baryon number.

The evaluation of the magnitude and sign of δ_{ms} (and, thereby, of the parameter ρ) is difficult because of the high perturbation-theory order. Rough estimates show that $\delta_{ms} \sim 10^{-16}$–$10^{-22}$,[12]) which leads to $\rho \sim 10$–10^{-2}. If we use the numerical result of Monte Carlo calculations to estimate the condensate of Chern-Simons density, we find that

$$\Delta \approx (10^{-6} - 10^{-11}) S(M_H), \quad (5.21)$$

which is not too far from the observed result $\Delta_{obs} \approx 10^{-8}$–$10^{-10}$ (Ref. 30). The dependence of the suppression factor on M_H is very specific[31] (Fig. 7). When $M_H \gtrsim M_{crit} \approx 45$ GeV, the temperature after the phase transition is found to be higher than the quenching temperature for anomalous electroweak processes, and practically the entire BAU disappears. On the other hand, when $M_H \approx M_{CW}$ (M_{CW} is the mass of the Higgs boson[13]) in the Coleman–Weinberg theory[105]), the phase transition occurs with the production of entropy,[106,107] which again reduces the ratio n_B/n_γ. BAU generation within the framework of the electroweak theory

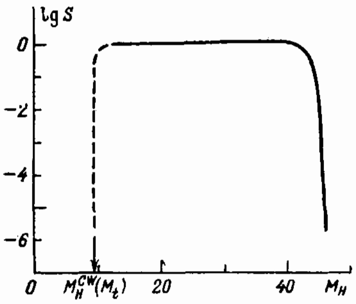

FIG. 7. Macroscopic asymmetry-suppressing factor $S(M_H)$ as a function of the mass of the Higgs boson for $m_t \lesssim M_W$.

is therefore possible only for[31]

$$M_{CW} \leqslant M_H \leqslant M_{crit} \approx 45 \text{ GeV}. \qquad (5.22)$$

If the entire observed BAU is due to the anomalous electroweak nonconservation of baryon number, the inequality (5.22) must be looked upon as a cosmological restriction on the mass of the Higgs boson. A discussion of the analogous restrictions, taking into account the mass of the t-quark, is given in Ref. 92, where it is found that the value $M_{crit} \approx 45$ GeV is practically independent of m_t and is not sensitive to the emission of new fermion generations (the discussion is based on the standard theory with one Higgs doublet). The case where BAU without the macroscopic factor $S(M_H)$ is greater than the observed value Δ_{obs} is also discussed in Refs. 31, 92, and 101. Here, cosmology gives a practically unambiguous prediction for the mass of the Higgs boson ($M_H \approx M_{CW}$ or $M_H \approx M_{crit}$). We also reproduce the relation that arises in a Coleman–Weinberg-type theory between the theoretical BAU without taking into account the entropy release Δ_{max}, the temperature of the phase transition in SU(2)×U(1), which, in this case, is equal to the temperature T_{ch} for chirality violation in quantum chromodynamics with six massless quarks,[107] and the mass of the Higgs boson

$$M_H \approx \frac{\pi T_{ch}^2}{\sigma}\left(\frac{8N_{eff}}{15}\right)^{1/2}\left(\frac{\Delta_{max}}{\Delta_{obs}}\right)^{2/3}, \qquad (5.23)$$
$$\sigma \approx 250 \text{ GeV}.$$

This is discussed in Ref. 101.

We note, in conclusion of this Section, that a confirmation of the above scenario of the emergence of BAU in electroweak theory would require Monte Carlo calculations analogous to those performed in Ref. 102, but using larger lattices.

6. ANOMALOUS ELECTROWEAK DECAYS OF HEAVY PARTICLES

The fast nonconservation of fermion quantum numbers, due to transitions between topologically different vacuums, is possible not only at high densities and temperatures, but also in the decay of heavy particles.[16,17] The energy necessary to overcome the barrier shown in Fig. 2 is then provided by the mass of the decaying particle. To prevent the suppression of the decay process by the tunneling exponential, the necessary mass must exceed the height E^S of the barrier (in electroweak theory, this is of the order of 10 TeV).

Let us illustrate this possibility by considering the decay of a technibaryon in the technicolor model.[16–20] Technicolor[108] (Refs. 4 and 109) is an alternative to the Higgs mechanism of spontaneous symmetry breaking. Instead of the Higgs field, this model introduces massless techniquarks (in the simplest case, the U, D that transform under the action of the electroweak group SU(2)×U(1) in the same way as ordinary quarks u, d). It is assumed that the techniquarks interact with one another by new strong (technicolor) interactions that are completely analogous to the ordinary color forces except that the scale of the technicolor interactions, Λ_{TC}, is of the order of a few hundred GeV (in contrast to chromodynamics, in which $\Lambda_{KXD} \approx 100$ MeV). It is considered that all this leads to vacuum expectation values $\langle \bar{U}U \rangle = \langle \bar{D}D \rangle \sim 500$ GeV, i.e., there is spontaneous breaking of chiral symmetry. Since these expectation values break down the electroweak group SU(2)×U(1) as well, the W- and Z-bosons acquire mass. The longitudinal components of these bosons are the technipions (which would be massless goldstone bosons in the theory without the SU(2)×U(1) gauge interactions).

The model predicts the existence of a large number of technihadrons, i.e., particles consisting of techniquarks by analogy with ordinary hadrons.[14)] In particular, there should be technibaryons containing N_{TC} techniquarks (N_{TC} is the number of technicolors). In the simplest variant, the lightest technibaryon is stable if we ignore the anomalous electroweak nonconservation of the technibaryon color. Its mass can be of the order of 10 TeV. Let us consider whether the decay of the technibaryon due to the triangular anomaly and the complex structure of vacuum in electroweak theory can occur without exponential suppression.

It will be convenient to describe the technibaryon within the framework of the Skyrme model[110] (see also Ref. 111). The main field in this model is the nonlinear sigma-field $V(x)$ with values in SU(2). In terms of the technipion fields Π^a ($a = 1,2,3$), it can be written in the form $V = \exp(i\tau^a \Pi^a / 2F_\pi)$, where F_Π is the technipion constant (the analog of $f_\Pi = 186$ MeV). In the absence of electroweak gauge fields, the technibaryon is a static topologically stable soliton (the skyrmion). The topological number of the field $V(x)$, which is identified with the technibaryon number, has the form [see (2.2)]

$$n[V] = \frac{1}{24\pi^2}\int d^3x\, \varepsilon^{ijk}\, \mathrm{Tr}\,(V\,\partial_i V^{-1} V\,\partial_j V^{-1} V\,\partial_k V^{-1}).$$

In the absence of gauge fields, the Hamiltonian for the Skyrme model in the case of static configurations of the field V is

$$E = \int d^3x \left\{ -\frac{1}{16F_\Pi^2}\mathrm{Tr}\,(\partial_i V V^{-1})^2 - \frac{1}{32e_{TC}^2}\mathrm{Tr}\,[\partial_i V V^{-1},\ \partial_j V V^{-1}]^2 \right\}, \qquad (6.1)$$

where e_{TC} is the dimensionless techniskyrmion constant [the last term in (6.1) is necessary for the stability of the soliton with respect to contraction to a point). We note that, in the limit of a large number of technicolors, the Hamiltonian for the sigma-model is proportional to N_{TC}, so that $F_\Pi \sim N_{TC}^{1/2}$, $e_{TC} \sim 1/N_{TC}^{1/2}$.

In the model with the Hamiltonian given by (6.1), the topological soliton (skyrmion) is a minimum of the energy functional in the sector with topological number $n[V] = 1$. The mass of the skyrmion can be estimated as follows. Substituting $y = F_{\Pi} e_{TC} x$, we have

$$E = \frac{F_{\Pi}}{16 e_{TC}} \int d^3x \left\{ -\mathrm{Tr}\,(\partial_i^{(y)} V V^{-1})^2 - \frac{1}{2} \mathrm{Tr}\,[\partial_i^{(y)} V V^{-1},\ \partial_j^{(y)} V V^{-1}]^2 \right\}. \quad (6.2)$$

The expression for the topological number is the same as before. The integral in (6.2) does not contain any parameters, so that the energy (mass) of the soliton is

$$M_{TB} = \frac{F_{\Pi}}{16 e_{TC}} C,$$

where C is of the order of unity. As already noted, the soliton is a model of the technibaryon. In the limit as $N_{TC} \to \infty$, its mass is proportional to N_{TC}, as predicted by technicolor dynamics.[112]

We must now include weak interactions. For the sake of simplicity, we shall ignore the subgroup U(1) of the group $SU(2) \times SU(1)$, i.e., we shall assume that $\sin^2 \theta_W$ is a small parameter, and will work in the lowest order in this parameter. The Lagrangian for the Skyrme model is invariant under global $SU(2)_L \times SU(2)_R$ transformations, where the field V transforms as follows: $V \to \omega_L V \omega_R^{-1}$.

When electroweak interactions are taken into account, $SU(2)_L$ becomes a gauge group and $\omega_L = \omega_L(x)$. The ordinary derivatives of the field V in the expression for the action are then replaced by the covariant derivative $D_\mu V = (\partial_\mu + A_\mu) V$ and the expression for action takes the form

$$S = \int d^4x \left\{ \frac{1}{2g^2} \mathrm{Tr}\, F_{\mu\nu}^2 - \frac{F_{\Pi}^2}{16} \mathrm{Tr}\,(D_\mu V V^{-1})^2 + \frac{1}{32 e_{TC}^2} \mathrm{Tr}\,[D_\mu V V^{-1},\ D_\nu V V^{-1}]^2 \right\}.$$

It will be convenient to take the unitary gauge with $V = 1$. In this gauge, we have

$$S = \int d^4x \left(\frac{1}{2g^2} \mathrm{Tr}\, F_{\mu\nu}^2 - \frac{F_{\Pi}^2}{16} \mathrm{Tr}\, A_\mu^2 + \frac{1}{32 e_{TC}^2} \mathrm{Tr}\,[A_\mu,\ A_\nu]^2 \right). \quad (6.3)$$

It is clear from this expression that the vector bosons acquire mass (in the limit as $\sin^2 \theta_W \to 0$, we have $M_Z = M_W$):

$$M_W = \frac{F_{\Pi}}{4} g,$$

and the technipions disappear from the spectrum and become the longitudinal components of the vector bosons.

The energy functional for static fields, which corresponds to the action given by (6.3), is

$$E = \int d^3x \left(-\frac{1}{2g^2} \mathrm{Tr}\, F_{ij}^2 - \frac{F_{\Pi}^2}{16} \mathrm{Tr}\, A_i^2 - \frac{1}{32 e_{TC}^2} \mathrm{Tr}\,[A_i,\ A_j]^2 \right), \quad (6.4)$$

where we have put $A_0 = 0$ (it may be shown[18] that this does not result in a loss of generality in our subsequent discussion). In the limit as $g \to 0$, the skyrmion is restored in this gauge, as follows.[16,17] The configuration energy of the field **A** is finite in this limit, provided **A** is a pure gauge field: $\mathbf{A} = V \vec{\partial} V^{-1}$. For such configurations, the first term in the integrand of (6.4) must vanish, and the second and third must be identical with the Hamiltonian (6.1), whose minimum in the sector with $n[V] = 1$ is, in fact, the skyrmion.

It is clear that, for small but finite g^2, the skyrmion minimum of the energy functional will exist as before. However, the skyrmion will be unstable with respect to instanton-type tunneling transitions.[113] Actually, when g is small, the skyrmion is a configuration of the form $\mathbf{A} = V \vec{\partial} V^{-1}$ with topologically nontrivial $V(x)$. The instanton transition transforms this configuration into the $\mathbf{A} = 0$ vacuum (Section 2.1). We shall show that, when the mass of the skyrmion is high enough, the energy minimum ceases to exist, i.e., the technibaryon becomes classically unstable.[16,17]

Substituting $y = F_{\Pi} e_{TC} x$, $B_i = A_i / F_{\Pi} e_{TC}$, we find that the Hamiltonian (6.4) can be written in the form

$$E = \frac{F_{\Pi}}{16 e_{TC}} \int \left(-\frac{8 e_{TC}^2}{g^2} \mathrm{Tr}\, B_{ij}^2 - \mathrm{Tr}\, B_i^2 - \frac{1}{2} \mathrm{Tr}\,[B_i,\ B_j]^2 \right) d^3y. \quad (6.5)$$

In the limit as $e_{TC}/g \to 0$, the first term in the integrand vanishes, and the remaining two decrease under the scaling transformatioin $B \to \lambda B$. The functional (6.5) does not have minima in this limit.

It is thus clear that the technibaryon becomes classically unstable for

$$\frac{e_{TC}^2}{\pi \alpha_W} < \xi_{crit}, \quad (6.6)$$

where $\alpha_W = g^2/4\pi$ and ξ_{crit} is a critical value. Numerical calculations[18] yield $\xi_{crit} = 10.35$. For fixed g and F_{Π} (the constant F_{Π} is fixed by the mass of the W-boson), the instability condition (6.6) reduces to the following condition for the skyrmion mass:

$$M_{TB} > C_{crit} \frac{M_W}{\alpha_W}.$$

The numerical result is $C_{crit} = 6$ (Ref. 18), which corresponds to the maximum mass of the metastable techniskyrmion, approximately equal to 14 TeV. The technibaryon is classically unstable when its mass is large. Rough estimates of the technibaryon width for masses of this order yield[17] $\Gamma_{TB} \sim M_W$.

It is natural to expect that, when the mass exceeds the critical value, the lifetime of the metastable technibaryon will be a rapidly-varying function of the mass, and will be suppressed by the factor $\exp(-4\pi/\alpha_W)$ for small masses. Numerical calculations,[19] performed within the framework of the Skyrme model, are in complete agreement with this picture: a reduction in M_{TB} by only 17% produces an increase in the lifetime by more than 20 orders of magnitude, and the lifetime exceeds 1 sec. When $M_{TB} \approx 10$ TeV, it is of the order of the lifetime of the Universe.

Of course, numerical estimates depend on the choice of the techniskyrmion model. However, calculations[20] based on another soliton model show that the critical mass of the technibaryon is not very dependent on the model parameters, and varies between 10 and 15 TeV.

The nonconservation of the technibaryon number that we have considered occurs because of effects associated with the structure of vacuum and the triangular anomaly in electroweak theory. Technibaryon decay is accompanied by the formation of an SU(2) gauge field configuration with topological number $Q(A)$ equal to -1. As discussed in Sec. 2, this process occurs with the creation of ordinary quarks and leptons, subject to the selection rules given by (2.19). In other words, the decay of the technibaryon is the many-particle process

$$\mathrm{TB} \to 3\bar{q}_1 + 3\bar{q}_2 + 3\bar{q}_3 + \bar{l}_1 + \bar{l}_2 + \bar{l}_3 + \ldots,$$

where $\bar{q}_i$ and $\bar{l}_i$ are antiquarks and antileptons in the ith generation and the ellipsis denotes W-bosons, photons, $\bar{q}q$, $\bar{l}l$ pairs, and so on. The perturbative production of a technibaryon-antitechnibaryon pair for high enough M_{TB} should lead to a relatively exotic event, namely, the production of a large number of leptons and quarks (jets) with high transverse momenta.

The possibility of the fast anomalous decay that we have examined in the technicolor model is probably not entirely realistic. Actually, inequality (6.6), under which this decay occurs, can be satisfied only for a very large number N_{TC} of technicolors. The necessary number of technicolors can be estimated as follows. As already noted, $e_{\mathrm{TC}} \propto N_{\mathrm{TC}}^{-1/2}$, so that

$$\frac{e_{\mathrm{TC}}^2}{e_{\mathrm{C}}^2} \sim \frac{N_{\mathrm{c}}}{N_{\mathrm{TC}}}, \tag{6.7}$$

where $N_{\mathrm{c}} = 3$ is the number of colors in chromodynamics and e_{C} is the Skyrme constant for strong interactions. Different estimates[115,116] show that $e_{\mathrm{c}} = 3.2$–4.5. Using (6.6) together with (6.7), we obtain

$$N_{\mathrm{TC,\,crit}} \sim N_{\mathrm{c}} \frac{e_{\mathrm{C}}^2}{\pi\alpha_{\mathrm{W}}\xi_{\mathrm{crit}}} = 25-50.$$

Another method of estimating this number is based on the fact that $F_{\Pi} \propto N_{\mathrm{TC}}^{1/2}$, $M_{\mathrm{TB}} \propto N_{\mathrm{TC}}$, so that

$$N_{\mathrm{TC}} \approx N_{\mathrm{c}} \left(\frac{M_{\mathrm{TB}}/F_{\Pi}}{M_{\mathrm{B}}/f_{\Pi}}\right)^2, \tag{6.8}$$

where M_{B} is the nucleon mass and f_{Π} is the pion decay constant. We then find that (6.8) leads to the following estimate for the critical number of technicolors: $N_{\mathrm{TC,crit}} \sim 90$. In any event, the necessary number of technicolors appears to be too high.

Whether realistic models with heavy particles are capable of undergoing fast anomalous decay with nonconservation of baryon and lepton number is still an open question. In particular, it is not clear whether elementary fermions interacting with the Higgs field with a high Yukawa constant can undergo this decay. The last question is answered (positively) at present only in the two-dimensional theory.[117]

7. CONCLUSION

We have seen that, in many models, extremal external conditions lead to intensive nonconservation of baryon number. In standard electroweak theory, the characteristic scale for the chemical potentials, i.e., the masses of the decaying particles, is of the order of 10 TeV. The nonconservation of the fermion number at temperatures ~ 100 GeV is a fast process. The question therefore arises as to whether it is possible to have fast nonconservation of the baryon number in collisions between ordinary particles at energies of the order of a few tens of TeV. This question has not as yet been answered, but there are some arguments[48] against this possibility. This question has been attracting increasing attention because electroweak processes with nonconserved baryon number are many-particle reactions for which the selection rules (2.19) are satisfied. These processes would lead to very exotic events involving tens of quarks and leptons with high transverse momenta in the final state.

It may well be that the nonperturbative mechanisms for nonconservation of the baryon number that we have discussed in this review do not exhaust *all* the possibilities. The gauge theory dynamics may turn out to be much richer than appears at present.

[1)]This restriction may turn out to be too strong in some models.[38] The role of transformations that do not satisfy this condition is still not clear.

[2)]An estimate for the height of the barrier between the vacuums was obtained independently in Ref. 17.

[3)]Transitions between the states $|M,0\rangle$ and $|M,n\rangle$ are literally forbidden by the selection rule (2.17) and by energy conservation (see the discussion in Section 2.2); here, we have in mind transitions between excitations above $|M,0\rangle$ and $|M,n\rangle$.

[4)]Strictly speaking, we are assuming that there is no screening of magnetic fields in the cold medium (Meissner effect) at the two-loop level, i.e., that there is no term of the form $g^2\mu_0^2\mathbf{A}^2$ in the effective Hamiltonian for the boson fields. Arguments suggesting that there is no Meissner effect in perturbation theory are given, for example, in Feynman's book.[114]

[5)]Formally, the last term in (4.2) is obtained by evaluating the one-loop contribution to the polarization operator for the field $\mathbf{A}$ in the leading order in momentum[11]; of course, it is exactly the same as the result given by (4.1).

[6)]A systematic scheme for evaluating higher-order corrections has been constructed by S. Yu. Khlebnikov and one of the present authors (M.E.Sh.).

[7)]We are indebted to S. Yu. Khlebrikov for drawing our attention to this fact. We note that the kinetic equations obtained in Refs. 15, 92, and 91, and the free energy obtained in Ref. 96, do not take into account the effect of asymmetry in the scalar sector. However, this has no effect on the numerical results.

[8)]The conclusion that, for $M_{\mathrm{H}} \lesssim M_{\mathrm{crit}}$, the baryon asymmetry in B–L-symmetric GUT will vanish was reported in Ref. 96 in the tree approximation for the masses of the fermions, and is violated by quantum-mechanical corrections; see (5.11).

[9)]There is also the hope that we will be able to relate the sign of the baryon asymmetry (excess of the number of baryons over the number of antibaryons, but not vice versa) and the sign of CP-violation in neutral-kaon physics.

[10)]It may be shown[101] that the dependence on the form of the scalar potential will be lost when the vacuum expectation value of the scalar field after the phase transition is high enough.

[11)]There is no summation over the flavors of the initial and final states when CP-violation is considered for neutral kaon decays. This explains the difference in the numerical values of the amplitudes of CP-violation.

[12)]Electroweak theory with four generations of fermions or two Higgs doublets[31] leads to much larger values of δ_{ms}.

[13)]We recall that, in the standard theory with one Higgs doublet and three generations of fermions, $M_{\mathrm{CW}}^2 = (3/8\pi^2\sigma^2)(2M_{\mathrm{W}}^4 + M_{\mathrm{Z}}^4 - 4m_t^4)$, where $\sigma = 250$ GeV. For the "light" t-quark $m_t \lesssim M_{\mathrm{W}}$) $M_{\mathrm{CW}} \approx 10$ GeV.

[14)]We shall not pause here to consider the serious difficulties of technicolor models. This is discussed in Refs. 4 and 100.

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