A simple method of preparing pure states of an optical field, of implementing the Einstein-Podolsky-Rosen experiment, and of demonstrating the complementarity principle[1)]

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A description is given of a device that periodically transforms a field from the vacuum state to a pure excited state that corresponds to the propagation of two correlated photons. It employs the phenomenon of parametric scattering, i.e., the emission of pairs of photons by a nonlinear crystal excited by a pulsed coherent pump under phase-matched conditions. In accordance with the well-known Einstein-Podolsky-Rosen gedanken experiment, the device can be used to observe the correlation of either the transverse momenta of the photons (when the two detectors are located in the far-field zone) or their transverse coordinates (when the detectors are in the near-field zone). The device may be of interest in photometry, and also from the methodological point of view as a clear demonstration of the EPR paradox and the complementarity of the transverse coordinate and momentum of a photon.

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"Every physicist thinks that he knows what is a photon. I have spent my entire life trying to figure out what is a photon, and I still don't know."
Albert Einstein

1. INTRODUCTION

Consider the following experiment (Fig. 1) that is readily carried out in the optical range. A plane pump wave of frequency ω_0 and wave vector $\mathbf{k}_0$ excites a transparent piezoelectric crystal which, because of its nonlinear polarizability, radiates correlated pairs of photons ("diphotons") in accordance with the scheme $\omega_0 \rightarrow \omega_1 + \omega_2$, $\mathbf{k}_0 \rightarrow \mathbf{k}_1 + \mathbf{k}_2$. The radiation usually has a wide spectrum and is reasonably directional, being confined to a cone of the order of 10°. This effect is referred to as *spontaneous parametric scattering* or, in other words, *spontaneous parametric frequency down-conversion*. The correlation between the photons is recorded by two photon-counting photomultipliers, located at points $\mathbf{r}_1$ and $\mathbf{r}_2$, and a coincidence circuit.

When the detectors are located at a large distance from the crystal, or in the focal plane of an objective, they record photons with particular angles of emission, and correlation between their readings is observed only for $\mathbf{p}_1 = -\mathbf{p}_2$, where $\mathbf{p} \equiv \mathbf{k}_\perp \equiv \{k_x, k_y\}$. This relationship may be looked upon as a consequence of the conservation of the transverse momentum of the photons, which is a consequence of the assumed homogeneity of the model in the transverse plane (the vector $\mathbf{k}_0$ lies along the z axis and is perpendicular to the plane-parallel crystal).

On the other hand, when the detectors lie in the immediate neighborhood of the plane-parallel crystal, or in the region of its image produced by a collecting lens, the correlation between the readings should be observed only at closely spaced points with almost equal transverse coordinates:

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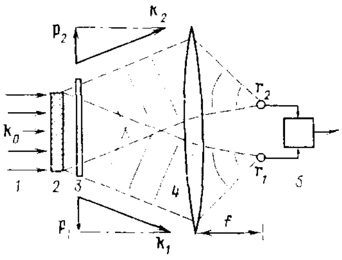

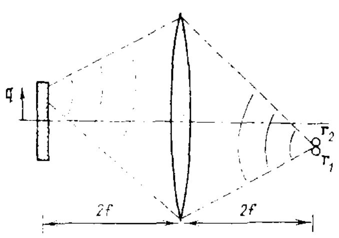

FIG. 1. Principle of an experiment demonstrating complementarity in measuring the transverse coordinates **q** and momenta **p** of two photons: a—the directions of emission of the photons, i.e., their transverse momenta $\mathbf{p}_1 = -\mathbf{p}_2$ are measured with the detectors located in the focal plane of the objective; *1*—incident wave (pump), *2*—nonlinear crystal, *3*—pump filter, *4*—objective of focal length *f*, *5*—coincidence circuit; b—the coordinate of the common point of creation of the photons, $\mathbf{q} = -\mathbf{q}_1 = -\mathbf{q}_2$, is measured with the detectors located in the region of the real image of the plane crystal.

$\mathbf{q}_1 = \mathbf{q}_2$, where $\mathbf{q} \equiv \mathbf{r}_\perp \equiv \{x, y\}$. This "transverse bunching" of photons is explained by the local character of the interaction, which results in the emission of the two twin photons from a common point in the crystal.

Thus, a correlation is observed either between the lateral momenta or lateral coordinates of photons, depending on the longitudinal coordinates z_n of the detectors. Strictly speaking, the second detector is superfluous because it does not provide new information: if we record a count in the "distant" detector 1 with coordinate $\mathbf{r}_1$, we may be sure that the other photon has a plane phase front near the crystal, with a definite transverse component of the wave vector, given by $\mathbf{p}_2 = -\mathbf{q}_1 k_1/r_1$ (we assume that the modulus of the wave vector is known). Similarly, when a count is recorded in the "near" detector 1′ with the coordinate $\mathbf{q}_1'$, we know that a second photon with a spherical phase front has been emitted from the same point.

By varying the longitudinal position of detector 1 (or by observing the successive counts in the two detectors 1 and 1′), we indirectly measure the transverse coordinate or the momentum of the second photon, without affecting it in any way. In other words, we are preparing photons with a predetermined initial wavefront curvature.

In order to dispose of "spurious" photons that appear because detector 1 has a finite aperture and imperfect efficiency, an optical shutter, operated at the appropriate time by the amplified output pulse of detector 1, can be inserted into the path of the second photon.

The above arrangement reproduces the Einstein-Podolsky-Rosen (EPR) gedanken experiment, proposed about 50 years ago[1] as a means of demonstrating the incomplete character of the quantum-mechanical description of physical reality (the Einstein-Podolsky-Rosen paradox is discussed, for example, in Refs. 2–9). However, the historical irony is that such experiments are now regarded as direct demonstrations of the *validity* of the quantum-mechanical description. They constitute a clear manifestation of Bohr's complementarity principle[2] or, in Fock's formulation,[10] the *principle of relativity with respect to the means of observation*. According to Bohr, prior to a count, the photons in the pair cannot be assigned any kind of attribute, such as, for example, the direction of propagation or position of the point of creation, and frequency or time of creation.

Many modified Einstein-Podolsky-Rosen experiments have now been carried out (see Refs. 9 and 11–13) in which the complementary observables are the orthogonal components of spin (of photons or protons). These experiments have attracted considerable attention because they have revealed a violation of Bell's inequality,[14] which means that the hidden variable theory (in its local version) cannot be valid and the quantum-mechanical rule for calculating probabilities is confirmed. Photon pairs are generated in these experiments as a result of cascade transitions in free atoms,[11] or as a result of the annihilation of positronium,[6,9] so that the polarization of the photons before measurement is undetermined, but there is strict correlation between the measured polarizations of the photons in a given pair (because of the law of conservation of angular momentum). At the same time, the angular correlation between the directions of emission of the photons is usually weak, but the points at which photons in a given pair are created are always correlated.

In contrast to this, in spontaneous parametric scattering the type of polarization is fixed, but there is relatively strict correlation either between the points of creation or the momenta of the photons (respectively due to the local character of the crystal nonlinearity and the macroscopic dimensions of the coherently radiating region, $a_0^2 l$, where a_0 is the radius of the pump beam and l is the crystal length), and also between the times at which the photons are created or between their frequencies. Spontaneous parametric scattering can thus be used to perform a new type of Einstein-Podolsky-Rosen experiment, i.e., an experiment with a continuous spectrum of mutually complementary observables q and p (in accordance with the original idea of the experiment) and, in principle, ω and t. An important feature of spontaneous parametric scattering is the high degree of directionality and the high intensity of the radiation, which can usually be observed visually for a mean pump power of less than 0.1 W.

We shall suppose that the crystal is pumped by individual identical and coherent (according to Glauber[15]) pulses of length $2\tau_0 \gg 1/\omega_0$, so that, after each pump pulse traverses the crystal, the scattered field undergoes a transition from the vacuum state $|0\rangle$ to a particular and practically pure "diphoton" state $|2\rangle$. This provides us with a relatively unusual possibility of preparing an essentially quantum-mechanical object, i.e., the optical field, in a pure state with macroscopic coherence scales. In the experiment employing the optical shutter, we actually prepare the single-photon state $|1\rangle$, which is not only of methodological interest, but also provides us with new possibilities in quantum photometry,[16–19] data transfer, and so on.[20–22]

In what follows, we give a simple theoretical description of the experiment and its various interpretations. Photon correlation in spontaneous parametric scattering has been analyzed for the far zone by a number of workers.[16,17,23–25] The corresponding experiments are described in Refs. 26–28. Some of the properties of the near-zone field in spontaneous parametric scattering are discussed in Refs. 18 and 29. In this paper, the theory of spontaneous parametric scattering is presented in a clear space-time form, illustrating the Einstein-Podolsky-Rosen paradox and the complementarity principle. The model is distinguished by both experimental and theoretical simplicity, and appears to be the optimum model for discussing "perpetual" problems and for refining concepts associated with the Einstein-Podolsky-Rosen paradox, i.e., concepts such as determinism, causality, locality, action at a distance, elements of physical reality, and so on.[8]

2. SCATTERING OPERATOR

In a nonlinear transparent crystal without a center of symmetry, the macrofield energy density contains the term $EP^{(2)}$, where $P^{(2)} = \chi E^2$ is the quadratic polarization, so that we can start with the following phenomenological interaction Hamiltonian for the field modes:[17]

$$V(t) = -\frac{1}{2}\int d^3r\chi E_0^{(+)}(\mathbf{r},\ t)\,[E^{(-)}(\mathbf{r},\ t)]^2 + \text{h.c.},\quad (2.1)$$

where χ is the contraction of the quadratic polarizability tensor of the crystal and the field polarization unit vectors (we neglect the dependence of χ on the frequency and direction of the field), E_0 and E are operators representing the pump field ($\omega \sim \omega_0$) and the scattered field ($\omega < \omega_0$) in the interaction representation, and the signs $\pm$ refer to the positive and negative frequency parts.[15] The rapidly oscillating terms are not taken into account in (2.1). We assume that the phase-matching condition $\mathbf{k}_0 \simeq \mathbf{k} + \mathbf{k}'$ is satisfied only for the extraordinary pump waves and the ordinary scattered waves. We note that the effective Hamiltonian (2.1) with excluded matter operators yields the same results in the crystal transparency region as the usual perturbation theory for three-photon processes[17] or other forms of perturbation theory.[24]

Spontaneous parametric scattering is described by the *scattering operator*, taken in the linear approximation (see, for example, Ref. 17):

$$S(t,\ t_0) = 1 + (i\hbar)^{-1}\int_{t_0}^{t} dt'\,V(t').\quad (2.2)$$

This operator determines the perturbed field wave function $|t\rangle$ in the interaction representation in terms of the initial wave function $|t_0\rangle$, and relates the operators in the Heisenberg and interaction representations:

$$|\,t\rangle = S\,|\,t_0\rangle,\quad E_\Gamma = S^+ES.\quad (2.3)$$

Suppose that the initial state of the field corresponds to the coherent state[15] of the pump modes[2)] with the eigenvalue $\mathscr{E}_0(\mathbf{r},t)$, so that we can replace the operators $E_0^{(\pm)}$ with classical quantities $\mathscr{E}_0^{(\pm)}$ [in moments normal with respect to $E_0^{(\pm)}$]. We shall suppose that the pump is pulsed, i.e., nonzero in the crystal interior only for times between $-\tau_0$ and τ_0, so that, for $t > \tau_0$, the wave function is $|t\rangle \equiv |\rangle$ and the scattering operator ceases to be time-dependent:

$$S = 1 + \frac{i}{2\hbar}\left(\hat{\chi}_1^{(+)}E_1^{(-)2} + \hat{\chi}_1^{(-)}E_1^{(+)2}\right) \equiv 1 + S^{(1)};\quad (2.4)$$

where $E_1 \equiv E(x_1)$, $x_1 \equiv \{\mathbf{r}_1, t_1\}$; the integral operators $\hat{\chi}_1^{(\pm)}$ are defined by the rule

$$\hat{\chi}_1^{(\pm)} f(x_1) \equiv \chi\int d^4x_1\mathscr{E}_0^{(\pm)}(x_1)\,f(x_1)\quad (2.5)$$

(where the integral is evaluated over the excited volume of the crystal, $a_0^2 l$, and over the time interval between $-\tau_0$ and τ_0). Equations (2.3)–(2.4) determine all the characteristics of the scattered field that is produced as a result of the passage of the pump pulse through the crystal. At the same time, the operators $E^{(\pm)2}$ ensure that the initial state of the field $|t_0\rangle$ acquires a small time-independent admixture $S^{(1)}|t_0\rangle$ (for $t > \tau_0$ in the interaction representation).

3. MOMENTUM REPRESENTATION

To transform to the momentum representation, we shall express the operators $E^{(\pm)}$ in terms of the operators representing the creation $[a_\mathbf{k}^{(-)} \equiv a_\mathbf{k}^{(+)}]$ and annihilation $[a_\mathbf{k}^{(+)} \equiv a_\mathbf{k}]$ of photons in mode $\mathbf{k}$:

$$E^{(\pm)}(x) = \sum_\mathbf{k} u_\mathbf{k}^{(\pm)}(x)\,a_\mathbf{k}^{(\pm)}(x),\quad u_\mathbf{k}^{(\pm)}(x) \equiv \pm\frac{i}{2\pi}(\hbar\omega_\mathbf{k})^{1/2}e^{\pm ikx};\quad (3.1)$$

where $kx \equiv \mathbf{k}\cdot\mathbf{r} - \omega_\mathbf{k} t$, $\omega_\mathbf{k} \equiv c|\mathbf{k}'|/n_\mathbf{k}$, $n_\mathbf{k}$ is the refractive index of the crystal, and the quantization length is chosen to be 2π (in which case, $\sum_\mathbf{k} \ldots = \int d^3k \ldots$). For simplicity, the difference between $n_\mathbf{k}$ and 1 will be taken into account only in the difference $\mathbf{k}_0 - \mathbf{k} - \mathbf{k}'$. The summation over $\mathbf{k}$ in (3.1) and in the subsequent formulas is carried out only over modes with frequencies smaller than $\omega_0 - 1/\tau_0$.

Suppose that the initial state of the field (ignoring the pump modes) is the vacuum state $|t_0\rangle = |0\rangle$, so that the pump pulse and the operator $E^{(-)2}$ [see (2.4)] force the field into the "diphoton"[3)] state $|2\rangle$. The probability amplitude for the appearance of one photon in each mode $\mathbf{k}$ and $\mathbf{k}'$ is then equal to the matrix element of the scattering operator S, relating the initial state $|0\rangle$ to the final two-photon state:

$$|\mathbf{k}\cdot\mathbf{k}'\rangle = |\mathbf{k}'\cdot\mathbf{k}\rangle \equiv a_\mathbf{k}^+ a_{\mathbf{k}'}^+|0\rangle = \ldots|1\rangle_\mathbf{k}\ldots|1\rangle_{\mathbf{k}'}\ldots|0\rangle_{\mathbf{k}''}\ldots\quad (3.2)$$

(the case $\mathbf{k} = \mathbf{k}'$ need not be considered because integration over the modes is implied). According to (2.4) and (3.1),

$$\langle\mathbf{k}\cdot\mathbf{k}'|S|0\rangle = \langle\mathbf{k}\cdot\mathbf{k}'|2\rangle = \frac{(\omega_\mathbf{k}\omega_{\mathbf{k}'})^{1/2}}{4\pi^2 i}\,\hat{\chi}_1^{(+)}\exp[-i(k+k')x_1].\quad (3.3)$$

The set of vectors $|\mathbf{k}\cdot\mathbf{k}'\rangle$ forms an orthonormal basis of the "two-photon subspace" (common Hilbert space of the states of the field) to which the vector $|2\rangle - |0\rangle$ belongs:

$$|2\rangle = |0\rangle + \sum_{\mathbf{k}\mathbf{k}'}{}' |\mathbf{k}\cdot\mathbf{k}'\rangle\langle\mathbf{k}\cdot\mathbf{k}'|2\rangle.\quad (3.4)$$

The prime on $\sum$ indicates that terms obtained by interchanging $\mathbf{k}$ and $\mathbf{k}'$ must be taken into account only once because the two photons in a pair are indistinguishable. Thus, the field transforms from the vacuum state to the diphoton state $|2\rangle$, which can be represented as a superposition of the vacuum $|0\rangle$ and the two-photon states $|\mathbf{k}\cdot\mathbf{k}'\rangle$.

The amplitude $\langle\mathbf{k}\cdot\mathbf{k}'|2\rangle$ of the state $|\mathbf{k}\cdot\mathbf{k}'\rangle$ is appreciable, according to (3.3), only for mode pairs satisfying the

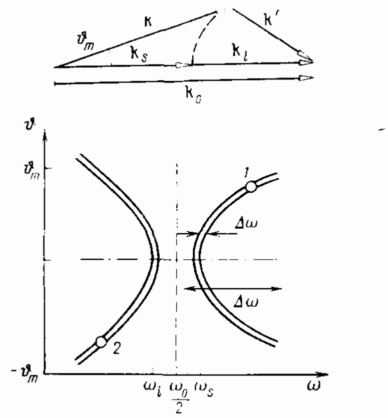

FIG. 2. The frequency-angle spectrum $\omega(\vartheta)$ in parametric scattering is determined by the phase-matching conditions (conservation of energy and momentum in the three-photon interaction), refractive index dispersion, and birefringence of the crystal. For a given orientation of the crystal, collinear phase matching occurs for frequencies ω_s and ω_i close to half the pump frequency $\omega_0/2$ ($\vartheta_m \sim 5$–$10°$ is the maximum scattering angle, $\Delta\omega$ is the common frequency band, $\Delta\omega_i$ are the band frequencies radiated in a narrow angular range; broken and dot-dash curves—boundaries between signal and idler photon spectra for frequency and angular filtration, respectively; points *1* and *2* are connected by the phase-matching conditions.

matching condition

$$|\omega_{\mathbf{k}}+\omega_{\mathbf{k}'}-\omega_0|\tau_0 \leqslant 1,\ \ |\mathbf{p}+\mathbf{p}'|a_0 \leqslant 1,\ \ |k_z+k_z'-k_0|l \leqslant 1 \tag{3.5}$$

[where $\mathbf{p}\equiv\{k_x, k_y\}$]. This condition gives a definite relation $\omega(\vartheta)$ between the frequency and the scattering angle $\vartheta\equiv\arcsin(p/k)$. We are interested in the case where the matching condition is satisfied for $\omega\sim\omega_0/2$ in a certain range of scattering angles, between zero and ϑ_m (Fig. 2). Usually, $\vartheta_m \sim 5$–$10°$) so that the quasioptical approximation is valid.

The square of the amplitude $\langle \mathbf{k}\cdot\mathbf{k}'|2\rangle$ can be interpreted as the joint probability that one photon will appear in each of the modes $\mathbf{k}$ and $\mathbf{k}'$. Multiplying it by the mode density, we obtain the probability density for the detection of photons (per unit frequency and solid angle intervals):

$$\frac{dP(\mathbf{k}\cdot\mathbf{k}')}{d\omega\, d\omega'\, d\Omega\, d\Omega'} = |\langle \mathbf{k}\cdot\mathbf{k}'|2\rangle\, kk'c^{-1}|^2. \tag{3.6}$$

We note that the momentum representation describes directly only those experiments in which the detectors are located in the far region in the direction of $\mathbf{k}$ and $\mathbf{k}'$. For the usual types of spontaneous scattering by individual particles, we can confine our attention to this case because the concept of the near region is meaningful only in the case of macroscopic dimensions of the coherently excited volume. In this respect, spontaneous parametric scattering, using a laser pump, is a unique extended source of a countable number of optical photons with plane and spherical wavefronts of arbitrary initial curvature and orientation (see below). We note that spontaneous parametric scattering (SPS) is a reversible process, i.e., by directing the diphotons onto a second nonlinear crystal, we can again obtain a coherent field ω_0, $\mathbf{k}_0$ (Refs. 29 and 30). The unusual statistical properties of the SPS field can be examined not only by recording coincidences between photons, but also by measuring the high efficiency with which they produce two-photon transitions.[18,29,30]

4. SPACE-TIME DESCRIPTION

This description was first given in Ref. 31, where the SPS intensity was determined in the far zone. Here, we shall use a general formalism that exhibits certain properties of SPS that are important for the purposes of interpretation.

Let us express the Heisenberg ("perturbed") operators E_{H} in terms of the operators E in the interaction representation. In the linear approximation, (2.3) and (2.4) yield

$$E_{\mathrm{H}1}^{(+)} = E_1^{(+)} + [E_1^{(+)},\ S^{(1)}] = E_1^{(+)} + \hat{\chi}_2^{(+)} D_{12} E_2^{(-)}, \tag{4.1}$$

where

$$D_{12} = D(x_1,\ x_2) \equiv \frac{i}{\hbar}\,[E_1^{(+)},\ E_2^{(-)}] \tag{4.2}$$

is the Green function for the wave equation. When $t_1 > t_2$, it is equal to the retarded positive-frequency field $\mathscr{E}_1^{(+)}$ at the point (event) x_1, produced by a dipole-moment δ-pulse from the point x_2, or the advanced negative-frequency field $\mathscr{E}_2^{(-)}$ at the point x_2, produced from the point x_1. The same function determines the "vacuum fluctuations":

$$\langle 0|E_1^{(+)}E_2^{(-)}|0\rangle = -i\hbar D_{12}. \tag{4.3}$$

In the case of a homogeneous space (the effect of the lens is discussed in Ref. 18),

$$D_{12} = \frac{i}{4\pi^2}\sum_{\mathbf{k}} \omega_{\mathbf{k}} \exp[ik(x_1-x_2)]. \tag{4.2'}$$

Next, we must find the normally-ordered correlation functions[15] (normal moments) of the scattered field. The first moment is nonzero if, apart from the pump, the crystal intercepts a coherent field $\mathscr{E}(\mathbf{r},t)$ containing frequencies less than ω_0. Suppose that, in the zero-order approximation, all the fields are in the coherent state, i.e., the initial wave functions satisfy the equation

$$E^{(+)}(\mathbf{r},\ t)\ |\ t_0\rangle = \mathscr{E}^{(+)}(\mathbf{r},\ t)\ |\ t_0\rangle,$$

so that (2.3) and (4.1) yield the mean perturbed fields at x_1 for $t_1 > \tau_0$:

$$\langle E_1^{(+)}\rangle \equiv \langle t_0|E_{\mathrm{H}1}^{(+)}|t_0\rangle = \langle |E_1^{(+)}|\rangle = \mathscr{E}_1^{(+)} + \hat{\chi}_2^{(+)} D_{12}\mathscr{E}_2^{(-)}. \tag{4.4}$$

This formula provides the quantum-mechanical space-time description of the well-known optical effect of difference-frequency generation.

The second and fourth moments will now be determined for the vacuum initial state of the scattered field, i.e., for $|t_0\rangle = |0\rangle$ and $|\rangle = |2\rangle$. From (4.1) and (4.3), we find that, in the first nonvanishing approximations,

$$G_1 \equiv \langle E_1^{(-)}E_1^{(+)}\rangle = -i\hbar\hat{\chi}_2^{(-)}\hat{\chi}_3^{(+)} D_{12}^{*} D_{13} D_{23}, \tag{4.5}$$

$$F_{12} \equiv \langle E_1^{(+)}E_2^{(+)}\rangle = -i\hbar\hat{\chi}_3^{(+)} D_{13} D_{23}, \tag{4.6}$$

$$G_{12} \equiv \langle E_1^{(-)}E_2^{(-)}E_2^{(+)}E_1^{(+)}\rangle = |F_{12}|^2, \tag{4.7}$$

where G_1 can also be expressed in terms of F (Refs. 18 and 24). We note that the operators $E_{\mathrm{H}1}^{(+)}$ and $E_{\mathrm{H}2}^{(+)}$ do not commute for $t_{1,2} < \tau_0$ and $t_1 \neq t_2$, i.e., $F_{12} \neq F_{21}$. This is one of the reasons why it is convenient to assume a pulsed pump in the theory. This condition is probably not essential in the actual experiments.

The "diffraction" integrals in (4.5)–(4.7) were estimated in Ref. 18 for a number of typical cases (using the approximate quasioptical Green's functions for the near and far zones and for the space behind the collecting lens).[4] Here, we shall confine our attention to a qualitative analysis.

5. CONNECTION BETWEEN THEORY AND EXPERIMENT

According to photon counting theory (see, for example, Ref. 15), the normal moments G are the link between theory and optical experiments because it is precisely these moments that determine the observable quantities, i.e., the probabilities of photon counts at the photomultiplier output and their coincidences. The essential point is that that probabilities are expressed in terms of the field wave function alone, i.e., without taking into account the reaction of the detector, which can be looked upon as a classical object. We shall determine the probabilities per pump pulse, i.e., the ratio of the number of successful trials m_1 (when the pump pulse is accompanied by a pulse at the detector output) to the total number m of trials for a sufficiently large value of the latter. The situation thus contains all the significant components of any quantum-mechanical model, namely, classical procedures for repeated preparation and observation of the object, and the quantum-mechanical object itself, i.e., the electromagnetic field.

When the pump energy is low enough, each detector produces zero counts in most trials and only rarely a single count. Events in which one detector absorbs the entire diphoton and produces two counts per pump pulse are also described by first-order perturbation theory [see (4.7) with $\mathbf{r}_1 = \mathbf{r}_2$], but they can be excluded by frequency or spatial filters that separate the field into two parts (i.e., distinguish between the photons in the pair).[5] For example, only part of the scattered field with frequencies $\omega > \omega_0/2$ may be directed into detector 1 (this part will be referred to as the "signal component"), whereas detector 2 intercepts the "idler" field with $\omega < \omega_0/2$. It is also possible to distinguish between the photons according to the sign of the component k_x of the wave vector along one of the transverse axes (see Fig. 2). Selection according to k_x is preferable because of the symmetry and simplicity of implementation.

Let us begin by considering a perfect detector, i.e., a detector with zero inertia and negligible size. This means that $T \ll t_{\mathrm{coh}}$ and $A \ll a_{\mathrm{coh}}^2$, where T and A are, respectively, the time constant and cross-sectional area of the detector, and t_{coh} and a_{coh} are the coherence time and radius of the field, determined from (4.5) and (4.7). It can be shown[18] that, in the near zone (in the immediate vicinity of the crystal or in the region of its image produced by the lens), $t_{\mathrm{coh}} \sim 1/\Delta\omega$, $a_{\mathrm{coh}} \sim \lambda/\vartheta_m$ ($\Delta\omega$ and ϑ_m are the widths of the frequency and angular spectra in spontaneous parametric scattering; see Fig. 2), whereas, in the far zone, $t_{\mathrm{coh}} \sim 1/\Delta\omega_l = |\tau_s - \tau_i|/2\pi \sim 1$ ps, where $\tau_{si} = l/u_{si}$ are the times taken by the signal and idler photons to cross the crystal, and $u_{s,i}$ are the group velocities at frequencies corresponding to collinear matching of ω_s, ω_i.

Suppose that the pump pulse length is much greater than t_{coh}, so that we can define the "time of emission" of the diphoton as lying in the range $-\tau_0 \ldots \tau_0$. the probability that a count will be recorded at time $t_1 \pm T/2$ at the output of detector 1, located at $\mathbf{r}_1$, is proportional to the second normal moment of the field at $x_1 \equiv \{\mathbf{r}_1, t_1\}$:

$$P(x_1) \equiv \lim_{m\to\infty} \frac{m_1}{m} = C_1 G_1, \quad C_1 \equiv \frac{\eta_1 V_1}{2\pi\hbar\bar{\omega}}; \tag{5.1}$$

where η_1 is the quantum yield, $V_1 \equiv cT_1A_1$ is the detection volume, and $\bar{\omega} \sim \omega_0/2$ is the mean frequency. The probability $P(x_2)$ of a count in detector 2 has a similar form. It is assumed in all this that the detectors do not shade one another (it is convenient to use a lightguide to collect the light flux).

The probability $P(x_1)$ for $V_1 \ll V_{\mathrm{coh}}$ is[17] of the order of $\eta_1(\Gamma l)^2 V_1/V_{\mathrm{coh}}$, where $\Gamma \equiv \pi k_0 \chi \mathscr{E}_{00}/2$ is the parametric gain exponent and $\mathscr{E}_{00}$ is the slowly-varying amplitude of the pump in the region corresponding to x_1. The necessary condition for the validity of perturbation theory is that $P(x_1) \ll 1$. The quantity $(\Gamma l)^2$ can be interpreted as the mean number of photons per mode (or per coherence volume). It reaches unity only for pump intensities of the order of 10 MW/cm^2.

The relative number of doubly successful trials, m_{12}, when both detectors produce one count in a given trial within time intervals $t_1 \pm (T_1/2)$ and $t_2 \pm (T_2/2)$, is proportional to the fourth normal moment of the field (4.7):

$$P(x_1, x_2) = \lim_{m\to\infty} \frac{m_{12}}{m} = C_1 C_2 G_{12}. \tag{5.2}$$

"Random coincidences," occurring with probability P_1P_2, and other effects that are quadratic in the pump are not taken into account because they can be excluded by reducing $V_1V_2|\mathscr{E}_0|^2$.

The ratio m_{12}/m_1 can be interpreted as the conditional probability

$$P(x_2|x_1) = \frac{P(x_1, x_2)}{P(x_1)}. \tag{5.3}$$

It is natural to define the *effective field* of photon 2 that describes the space-time distribution of the probability amplitude for its detection for fixed x_1 by the condition $C_2|\mathscr{E}_2^{(+)}|^2 \equiv P(x_2|x_1)$, from which it follows (apart from a constant phase factor) that

$$\mathscr{E}_2^{(+)} \equiv \langle E_1^{(+)} E_2^{(+)} \rangle \langle E_1^{(-)} E_1^{(+)} \rangle^{-1/2} = \frac{F_{12}}{G_1^{1/2}}. \tag{5.4}$$

In the system incorporating the optical shutter that is open at the appropriate time by the amplified output pulse of detector 1, the probability of a count in detector 2 is also given by (5.2) (this requires, of course, that, in any frame of reference, $t_2 > t_1$). Here, the shutter replaces the coincidence circuit, and the ratio of counts in the two detectors, m_2/m_1, is equal to the quantity given by (5.3).

According to the postulates of quantum theory, the number of a successful trial and the position of a count within the time interval $r_1/c \pm \tau_0$ are unpredictable. However, once a count has appeared in detector 1 at some time t_1, the count in detector 2 can appear in the same trial only within a certain interval (Fig. 3), determined by (4.7) to within $\pm t_{\mathrm{coh}}$ (see Ref. 18). Of course, this precision is unattainable in practice because, $T \gtrsim 1\,\mathrm{ns} \gg t_{\mathrm{coh}} \sim 1\,\mathrm{ps}$ for currently available photomultipliers. To allow for the last inequality,

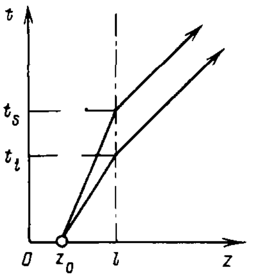

FIG. 3. The frequency spectrum of the SPS field consists of two parts ($\omega \sim \omega_s$ and $\omega \sim \omega_i$), propagating inside the crystal with different group velocities, so that the difference between their emission times in the crystal, $t_s - t_i$, is determined by the position z_0 of the point at which the diphoton is created inside the crystal.

we must arrange the moments G over the corresponding interval (see Ref. 18). In practice, it is much easier to measure the correlation between the spatial components of the points at which the photons are detected.

Similarly, for $A_n > A_{\rm coh}$, we must average G over the detector cross sections.[18] If detector 2 can then easily "see" all the modes coupled by the matching conditions to the modes of detector 1 (in other words, A_2 covers the entire beam of the effective field $\mathscr{E}_2$), then (5.3) gives η_2. This means that, after a count is recorded in Ref. 1, the second photon must definitely enter detector 2, which enables us to perform an absolute measurement of its efficiency, using the formula $\eta_2 = m_{12}/m_1$ (Refs. 16–19 and 27).

6. DIFFERENT INTERPRETATIONS

So far, the formulas given above have been confirmed experimentally only in the case where the pump is continuous and the detectors are located in the far zone,[26–28] but there are no reasons for doubting their validity in the more general case. Any interpretation of such experiments must not, therefore, be in conflict with the joint distribution of the points of detection of the two photons that follows from (4.7) and (5.2):

$$P(x_1,\ x_2) \sim |F_{12}|^2 = |\hbar\chi \int \mathrm{d}^4 x D(x_1,\ x)\, D(x_2,\ x)\, \mathscr{E}_0^{(+)}(x)|^2. \tag{6.1}$$

This formula, taken together with (4.5) and (5.1), describes completely all the observed spontaneous effects, i.e., effects of the first order in the energy of the pump pulse, and the role of quantum theory is exhausted once the formula has been derived. The theory claims to predict only the average results of the entire experiment that involves a large number of trials. The direct result of the experiment with fixed x_1, x_2 takes the form of a certain set of pairs of numbers $\{M_1, M_2\}$ that assume values 0 or 1. Questions such as "What is the structure of the field $\mathscr{E}(x)$ produced under the influence of a given pump pulse?" or "What happens to the field after the count has been recorded in detector 1?" are regarded as incorrect in quantum theory, and remain unanswered.

However, this "quantum agnosticism" is incompatible with the familiar classical ideas about the existence of a field with certain objective parameters that are independent of the detectors. This conflict is also reflected in the terminology used, i.e., instead of the more correct "a state of the field has arisen"[6)] we say "a photon has been emitted" or "a pair of photons has been emitted", and imagine a wave chain (wave packet) propagating from the source. It is difficult to avoid the temptation to describe the field as in some way "objectively existing" in an individual trial within the time interval t_1, t_2 between the times at which the pulses appear in the detectors. However, we cannot then avoid acknowledging that detector 1 then has a mysterious "influence" on this field.

The violation of Bell's inequality,[14] discovered in the 1970s and confirmed very reliably in a number of EPR polarization experiments,[11–13] has demonstrated that hidden-variable theories with a local interaction (i.e., without the above "influence") cannot be valid. The remaining possibilities are: either (1) locality ("separability"[7)]) must be abandoned, and we have to accept determinism with hidden variables and classical statistical theory, or (2) we must reconcile ourselves to the quantum-mechanical rules for calculating the probabilities of observable events. The Copenhagen interpretation of these rules is essentially local (see, for example, Ref. 32 and the discussion given below), but a nonlocal treatment of these rules is also possible and presupposes a certain instantaneous "nonforce"[33,34] and "noninformative"[35] interaction and even the possibility of telekinesis[36] and telepathy (see Ref. 7, p. 218). Einstein and Schroedinger rejected the possibility of influence, referring to it as "telepathy"[37] and "magic" (see Ref. 38) (this was before the experiments on the verification of Bell's inequalities).

In EPR experiments with photons, the events x_1 and x_2 are separated by a space-like interval (Fig. 4) and the influence–both classical and quantum-mechanical–can propagate along two paths: either with velocity of light through the common past according to the scheme $x_1 \to x \to x_2$ by means of an advancing wave[36,39–41] or, with the velocity greater than that of light, along the "straight line" between x_1 and x_2. The latter variant means that we abandon the basic postulate of relativity theory and return to action at a distance,[8] whereas the former variant involves a departure from the principle of causality.

The unusual nature of EPR experiments can be clarified with the aid of the following coarse analogy. Suppose that two identical telegrams are sent out simultaneously from Moscow to Kiev and to Vladivostok. As soon as he receives his own telegram, the Kiev recipient will instantly recognize

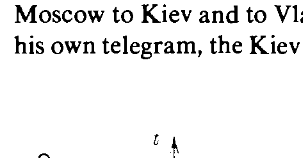

FIG. 4. When two photons are simultaneously emitted in different directions $\mathbf{r}_1$ and $\mathbf{r}_2$ from a single point x_0, the coordinates and times of their detection, x_1 and x_2, are separated by a space-like interval (broken line), so that the events x_1 and x_2 cannot be causally related. The broken line represents the effect of a mirror delay line that must be used to transmit information by modulating the state of the light field.

that the same telegram will be received in Vladivostok after a particular interval of time (from the standpoint of an observer in suitable motion, the telegram will reach Vladivostok before it reaches Kiev). Let us now suppose that this procedure is repeated periodically, but the text of the telegrams is altered randomly each time. If we admit that the Kiev addressee can decide in advance the text of the successive pair of telegrams, we shall have some idea of the significance of the violation of Bell's inequalities.[14] The case where the decision is made after the telegram has been sent corresponds to "retarded sampling" experiments.[13]

An elementary description of EPR polarization experiments and of Bell's inequalities is given in Refs. 42 and 43.

7. CHOICE OF SUBASSEMBLIES AND HIDDEN VARIABLES

When certain types of experiment are described, the concept of "influence" (of the position of one instrument on the readings of another) can be avoided if we use the usual theory of probability and assume that the crystal radiates photon pairs with random (but *a priori* determined) parameters, and that the detectors simply select certain subsets of particles. These parameters can be regarded as predictable, at least in principle (in the theory of hidden variables; see, for example, Refs. 5 and 9), or unpredictable ("statistical" interpretation of quantum theory[48]).

For example, consider the case where a correlation is observed between the readings of two detectors in the far zone of the radiating volume (see Fig. 1a), and both detectors produce a count in a given trial (pump pulse) only when they point in directions related by the condition $\mathbf{p}_1 + \mathbf{p}_2 = 0$. It is natural to suppose that a pair of photons is radiated in each such trial in the two directions linked by this condition, and that these directions fluctuate randomly from pulse to pulse. In the arrangement involving the optical shutter, we simply wait for the successive successful trial, when we are lucky and the photons are radiated in the required direction, i.e., detector 1 performs a selection of a certain classical subensemble. The distribution density

$$P(\mathbf{p}_1, \mathbf{p}_2) = P(\mathbf{p}_1)\delta^{(2)}(\mathbf{p}_1 + \mathbf{p}_2)$$

measured by performing a large number of trials, can be calculated from (3.6) and, in principle, a more complicated model with hidden variables can be constructed.

Similarly, when the detectors are located in the near zone, we may suppose that successive points with transverse coordinate $\mathbf{q}$ in the plane-parallel crystal radiate spherical waves that are focused by a lens on the detector in each successful trial (Fig. 1b). This procedure presupposes that there exists a distribution density for the points at which the photons are created, namely,

$$P(\mathbf{q}_1, \mathbf{q}_2) = P(\mathbf{q}_1)\,\delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2).$$

However, if we wish to describe both these types of experiment simultaneously within the framework of a single theory, we encounter difficulties: we must introduce the joint distribution $p(\mathbf{q}_1, \mathbf{p}_1)$, i.e., assume that the crystal radiates sometimes spherical and at other times plane waves. The question is then: how can we describe all the continuous sets of intermediate cases covered by (6.1)? Clearly, we must assume that the crystal radiates randomly a succession of pairs of all the possible converging and diverging spherical waves with centers covering all space. This quite artificial model must be further augmented by matching conditions relating pairs of centers that are mirror-symmetric relative to the plane of the crystal (see below). Such schemes clearly lose out in comparison with the simplicity and elegance of quantum theory.

A similar difficulty arises also when we attempt to use a model for choosing subensembles of two types of experiment in which the times $t_{1,2}$ of recorded counts or the frequencies $\omega_{1,2}$ are determined. In terms of classical concepts we have to assume that the crystal radiates sometimes short wave trains (at random times in the range $|t_n| < \tau_0$) and at other times long wave trains (with frequencies ω_1 and $\omega_2 = \omega_0 - \omega_1$ that fluctuate from pulse to pulse in the range $0 - \omega_0$). Wave trains of intermediate length must, of course, also arise.

These qualitative considerations show that the result of quantum-mechanical calculations based on (6.1) can hardly be described by a classical probability scheme within the framework of "local realism" or "latent determinism", without resorting to the idea of probability amplitudes and interference between them.

8. INFLUENCE THROUGH A COMMON PAST

We shall confine our attention to a single model of influence that is distinguished by clarity and predictive power. It follows naturally from the structure of (6.1), which gives the probability that each of the two detectors will record a count in one trial.

Suppose that the position vector $\mathbf{r}_1$ of detector 1 and the time t_1 at which the count appears are fixed, and that we wish to find the conditional probability distribution $P(x_2|x_1)$ for a count in detector 2 as a function of its position $\mathbf{r}_2$ and time t_2 of the count (the detector may be located behind the shutter). In the system without the shutter, the two detectors are equivalent, and the "influencing" detector 1 may be located further away from the crystal than detector 2.

Comparison of (4.4) and (4.6) shows that the effective field (5.4) for fixed x_1 is identical with the classical field $\mathscr{E}^{(+)}(x_2)$ produced during the parametric transformation in the same crystal of a fictitious field with negative-frequency part $\mathscr{E}^{(-)}(x) \equiv -i\hbar G_1^{-1/2} D(x_1, x)$. It follows from (4.2) that this field is a pulse with a spherical wave front that converges onto the point $\mathbf{r}_1$ at time t_1 (Fig. 5).

However, we are equally entitled to assume that this pulse is "radiated into the past" by means of an advanced Green's function from a point source at $\mathbf{r}_1$. The fictitious

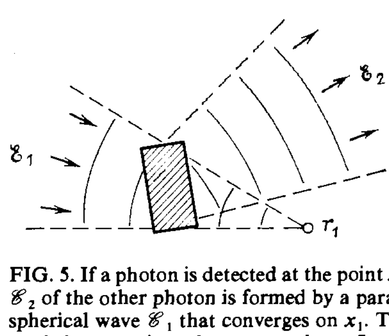

FIG. 5. If a photon is detected at the point $x_1 = \{\mathbf{r}_1, t_1\}$, the effective field $\mathscr{E}_2$ of the other photon is formed by a parametric transformation of the spherical wave $\mathscr{E}_1$ that converges on x_1. This transformation can be regarded, approximately, as specular reflection by the crystal of an advanced diverging wave emitted at x_1 into the past.

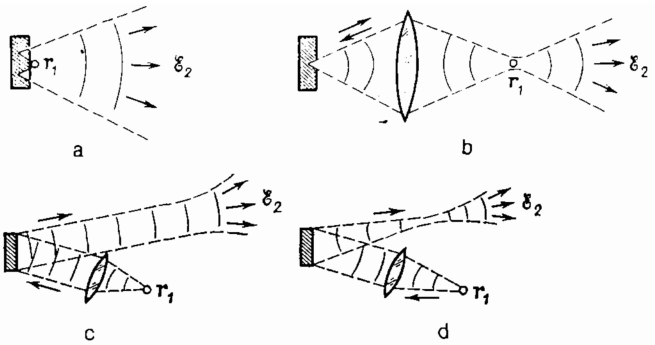

FIG. 6. "Influence" of the point of detection $\mathbf{r}_1$ of one photon on the effective field $\mathscr{E}_2$ of the other photon. When $\mathbf{r}_1$ lies near the plane crystal (a) or in the region of its real image produced by the lens (b), $\mathscr{E}_2$ is a spherical wave radiated from $\mathbf{r}_1$. If, on the other hand, $\mathbf{r}_1$ lies in the far-field zone, or in the focal plane of the lens (c), $\mathscr{E}_2$ is a plane wave with a diffractive divergence. A field with arbitrary initial wavefront curvature and beam axis orientation can be formed by moving the lens in front of the detector (d).

field $\mathscr{E}^{(-)}(x)$ radiated from the point of detection of the single photon, $x_1 = \{\mathbf{r}_1 \cdot t_1\}$, reaches the point $\mathbf{r}$ inside the crystal at a past instant of time $t < t_1$ and "beats" with the pump field $\mathscr{E}_0^{(+)}(x)$, creating the transformed field $\mathscr{E}^{(+)}(x_2)$, in accordance with (4.4). The latter determines the probability of finding the second photon at x_2.[8])

The parametric transformation of the fictitious advanced field $\mathscr{E}_1$ of the detected photon into the retarded field $\mathscr{E}_2$ of the other photon can be looked upon approximately as a reflection from the crystal (in the case of the near zone, this occurs at a definite plane inside the crystal; see Fig. 3). Actually, if we neglect the difference in frequency from $\omega_0/2$, then, according to (3.5), the "reverse" angle of incidence is equal to the angle of reflection: $\vartheta_1 \simeq \vartheta_2$. Thus, the point at which one photon is detected is, after reflection from the crystal, the effective source of the field of the other photon (Fig. 5).

This recipe for finding $P(x_2, x_1)$ for a given x_1 can also be used when lenses (Fig. 6) or other transparent optical elements are present. As a result, coincident counts should occur with maximum probability when the lens is used to image (with allowance for "reflection" by the crystal) the detector apertures one on another (Fig. 7). The collecting lens in front of detector 1 focuses photon 2: its effective field is concentrated in an Airy circle in a plane $z_2(z_1)$, determined by the lens formula (subject, of course, to the condition that A_1, l, and the chromatic aberrations on "reflection" are sufficiently small). This phenomenon can be referred to as "mutual focusing of photons."

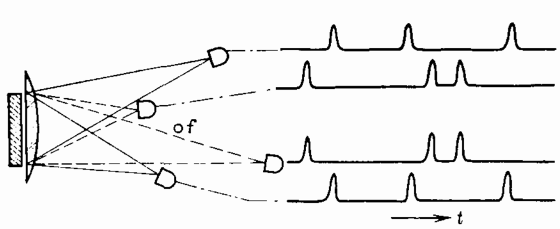

FIG. 7. Mutual focusing of photons in stimulated parametric scattering. If the apertures of the $2N$ perfect ($\eta = 1$) photon counters are imaged by the lens, one unto another in pairs (taking into account the "reflection" by the crystal), the output pulses in conjugate counters must appear only simultaneously in a given trial (provided the detector apertures and separations are much greater than the corresponding coherence lengths).

In the scheme involving a shutter, we can place different lenses and frequency and space filters in front of the controlling detector to prepare photons with arbitrary initial curvature (positive or negative) and wavefront orientation, and an arbitrary frequency spectrum in the range $0 - \omega_0$. It is clear that, by varying the filter parameters in time, it is possible to produce partial modulation of the radiated light and transfer of information (see below).

9. EINSTEIN-PODOLSKY-ROSEN PARADOX

We now return to Fig. 1. Suppose that we have two detectors 1 and 1′ with longitudinal coordinates f and $2f$. When detector 1 records a count, we indirectly recognize that the momentum of the photon is $2p_2$ (we assume that $\hbar = 1$). The above rule for constructing the effective field $\mathscr{E}_2$ (Fig. 6c) can now be used to estimate the uncertainty in this measurement: it is clear that this uncertainty is limited by the diffractive divergence of the plane wave, $\Delta p \sim 1/a_0$, where a_0 is the cross section (we omit the subscript 2). If, on the other hand, detector 1′ records a count, we determine the transverse coordinate of the point at which photon 2 is created with an uncertainty limited by the width of the angular spectrum: $\Delta q' \sim 1/\Delta p' \sim \lambda/\Delta\vartheta$; in the limit, $\Delta\vartheta \sim 1$ and $\Delta q' \sim \lambda$. Hence, we find that

$$\Delta p \cdot \Delta q' \sim \frac{\lambda}{a_0} \ll 1, \tag{9.1}$$

which would seem to be in conflict with the uncertainty relation. By increasing the pump-beam diameter and the transverse dimensions of the crystal, we can determine p to any desired precision for a constant precision in the measurement of q. The uncertainty relation is thus invalid for the standard deviations Δq and $\Delta p'$, measured for the two series of experiments that differ by the position of the detectors (Fig. 1). In each series, one of the detectors is fixed and the other is displaced in the transverse plane.

Similar considerations have led Einstein, Podolsky and Rosen to the conclusion that the wave function does not provide a complete description of physical reality (since both p and q can be measured). This aspect of the EPR paradox can readily be explained within the framework of quantum mechanics: photon 2 does not have "its own" wave function prior to the count in detector 1 and, on its own, it can be described only by the density matrix $\rho_2 = \mathrm{Sp}_1(|2\rangle\langle 2|)$ (this was immediately noted by Fock in the introductory paper[44] to the EPR paper translated into Russian;[1] see also Refs. 4, 45, and 46). It is only after the count has been recorded by the detector at x_1 that it can be assigned an individual characterization, namely, the function $\mathscr{E}_2 \sim F_{12}$ [see (5.4)]. Naturally, the parameters Δq, Δp of the effective field $\mathscr{E}_2$ for different x_1 are not connected by an uncertainty relation and are functions of z_1 (Fig. 6).

When the two detectors 1 and 1′ are present in the near and far zones, we cannot predict which of them will "fire" in successive trials and, by the definition given by Einstein, Podolsky, and Rosen themselves, this means that we cannot associate with q and p the elements of physical reality. It is only after a count has been recorded in one of the detectors that either q or p assumes reality, but this occurs in different trials.

Thus, the EPR experiment does indeed show that the description based on the wave function of a particle that has interacted with another quantum object is incomplete. We note, at this point, that the same conclusion is valid in the case of a closed system. The point is that $|t\rangle$ is a single-time characteristic of the system, so that, to find the multiple-time correlation functions $G(t_1,t_2,\ldots)$, we must also know the time dependence of the operators (in the Heisenberg or interaction representation). Thus, knowledge of $|2\rangle$ is not sufficient to enable us to calculate $P(x_1,x_2)$ for $t_1 \neq t_2$: the operators $E(t_1)$ and $E(t_2)$ must be determined. In other words, $G(t_1,t_2,\ldots)$ is not covered by the Schroedinger representation (except for $t_1 = t_2 = \ldots$). On the other hand, it may be supposed that, by specifying $|t_1\rangle$ and $|t_2\rangle$, we implicitly determine the evolution operator $U(t_1,t_2)$ that relates $E(t_1)$ to $E(t_2)$.

10. COPENHAGEN INTERPRETATION

The essence of this point of view, shared at present with certain variations[9] by the great majority of physicists, can be described in relation to the experiments considered here as follows. In general, it is meaningless to speak of the parameters of photons in a given pair (momenta, frequencies, positions, times of emission, and types of polarization) prior to measurement. These concepts classical in origin characterize not the photons themselves, but the type of macroscopic instrument with which the photons interact. Thus, when we record a photon transmitted by a polarizer with its axis lying along the x direction, we cannot maintain that the photon was x-polarized prior to entering the polarizer.[10] The results of the experiment depend on the experimental procedure, by analogy with the length of an object in the special theory of relativity.[2,10]

The unique and exhaustive characterization of the scattered field after the passage of a given pump field is its state, i.e., its membership in some particular statistical ensemble. When the pump and the crystal are stable, we may suppose that a particular pure state $|2\rangle$ is prepared during each pump pulse (fluctuations in the pump or crystal parameters lead to a mixed state which can be detected by measuring the variance of the number of counts in several series of trials).

The state vector $|2\rangle = S|0\rangle$ is thus the objective and maximally complete characterization of the field that is invariant under the measurement procedure. It does not depend on the coordinates of the detectors or lens parameters, and is determined entirely by the "preparing" part of the system, i.e., the pump and the crystal. As a result, the field does not have any space-time structure. The structure emerges as the "diphoton field" F_{12} in the eight-dimensional space $x_1 \cdot x_2$ only after the lens parameters have been specified, and as the effective field $\mathscr{E}_2$ in ordinary space-time only after the coordinates of one of the detectors have been fixed. We emphasize that F_{12} is a collective characteristic of both photons, which otherwise have no structure. We may suppose that, at $t_1 = t_2$, F_{12} plays the part of the diphoton wave function in the coordinate representation.[24] We note that the expression given by (4.6), looked upon as a function of the single argument x_1 (or x_2), satisfies the wave equation, and its modulus changes little within the coherence intervals.

It is clear that the information encoded in $|2\rangle$ and F_{12} is objective in character because it is determined by the macroscopic definition of the experiment and is therefore the same for all observers and is independent of their presence or absence.

The experiment illustrated in Fig. 1 clearly demonstrates the complementarity principle and the idea of indirect quantum-mechanical measurement. In each trial in the interval $\tau_0 \ldots t_1$ (i.e., so long as the diphoton is "on the way"), the concept of transverse coordinates $\mathbf{q}$ and momenta $\mathbf{p}$ of the photons has no operational meaning. It is only the detection of a photon in detector 1′ that enables us to assign its twin a definite coordinate $\mathbf{q}_2 = \mathbf{q}_1$ of the point of creation (of course, only to the extent allowed by diffraction). In the same trials in which detector 1 "fires," the second photon "acquires" a definite transverse momentum $\mathbf{p}_2 = -\mathbf{p}_1$. By displacing detector 1 from the near zone to the far zone, we can form photons of an intermediate type, with arbitrary wavefront curvature.

In a system incorporating a shutter, the latter can be controlled by detector 1 or detector 1′, as desired, and the shutter can be operated in the inverval $\tau_0 \ldots t_1$ ("retarded selection" experiment[13]). From the point of view of "local realism," the photons acquire a particular wavefront shape after creation.

Complementarity manifests itself here in that the emitted photon cannot have simultaneously a plane and a spherical wavefront. The question "what happens to the field at the time t_1 of the count?" is usually either refused by the followers of the Copenhagen school,[11] or they say that there is a change not in the field, but only in our information about it. The potential possibility (in our case, the possibility that the photons can have a spherical wavefront with any curvature that can be described by a function of two arguments F_{12}) becomes a reality[12] in the form of one of the alternatives (the function F_{12} for fixed x_1). This change in information is conveniently described as the *reduction* of the wavefunction.

11. REDUCTION OF WAVE FUNCTION

In a photon-counting experiment, a reduction can be visualized as follows. Suppose we measure the correlation function

$$K_{12} \equiv \langle 2|E_1^{(-)} f(t_2)\, E_1^{(+)}|2\rangle. \tag{11.1}$$

If $f(t_2) = E_2^{(-)} E_2^{(+)}$, then K_{12} is equal to G_{12} and describes the correlation between the counts produced by the two photon counters (in principle, f can also be a multiple-time operator). The average in (11.1) is evaluated over the time-independent state $|2\rangle$ of the free field, prepared at $t < t_{1,2}$, and the operators are taken in the interaction representation.

The definition (11.1) can be rewritten in the form

$$K_{12} = \langle 1|f(t_2)|1\rangle, \quad |1\rangle \equiv E_1^{(+)}|2\rangle. \tag{11.2}$$

This form of the correlation function (it is also suitable for $t_2 < t_1$) enables us to use the following treatment: at the time at which the count appears in detector 1, the initial state of the field $|2\rangle$ is transformed by the operator $E_1^{(+)}$ into a new single-photon state $|1\rangle$, and the observable f is then determined by this new *reduced* state.[13]

We must now stipulate that, usually, the word "reduction" is used in quantum mechanics in a somewhat narrower sense, i.e., it is taken to mean the result of the reaction of the measuring instrument on the wave function when the instrument shows a particular value of some observable (the re-

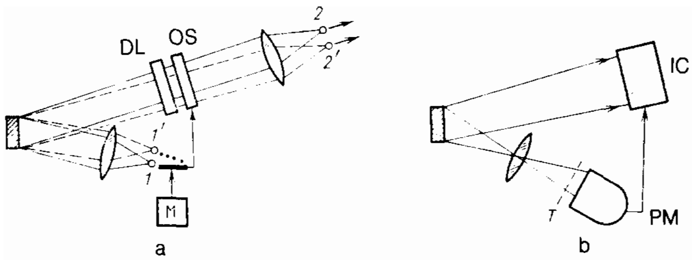

FIG. 8. Transmission of information by modulation of the photon state vector: a—transmission using a binary code ["1" and "0" are transmitted when photomultipliers *1* and *1′* are connected to the optical shutter (OS), respectively; M—modulator used to switch the photomultipliers, DL is the delay line]; b—transmission of a two-dimensional image in which the transparency T is imaged (with allowance for reflection by the crystal) on the photocathode of an image converter (IC), opened only when a count is produced at the photomultiplier output.

duction is then formally accomplished by the operator $|\mathscr{E}\rangle\langle\mathscr{E}|$). Here, the detector no longer measures the field strength, so that $|1\rangle$ is not the eigenstate of the operator $\mathscr{E}$ (i.e., it is not coherent).

According to (11.2), it may be considered that, when detector 1 produces a count in the system incorporating the shutter, the system prepares the field in the pure single-photon state $|1\rangle$, i.e., it radiates a photon with a known space-time structure (Fig. 8). In momentum representation, (3.1) and (3.4) show that this state is

$$|1\rangle = \sum_{\mathbf{k}} |\mathbf{k}\rangle \langle \mathbf{k}|1\rangle, \tag{11.3}$$

where

$$\langle \mathbf{k}|1\rangle \equiv \sum_{\mathbf{k}'} u_{\mathbf{k}'}^{(+)}(x_1)\langle \mathbf{k}\cdot\mathbf{k}'|2\rangle = \frac{(\hbar\omega_{\mathbf{k}})^{1/2}}{2\pi i}\,\hat{\chi}_3^{(+)} D_{13} e^{-ikx_1}$$

and $|\mathbf{k}\rangle \equiv a_{\mathbf{k}}^{+}|0\rangle = |1\rangle_{\mathbf{k}}|0\rangle_{\mathbf{k}}\dots$. In contrast to $|2\rangle$, the state $|1\rangle$ is not normalized and does not have the vacuum component $|0\rangle$. The latter means that the photon is definitely emitted [but, according to (11.3), its momentum is undetermined]. If we apply the operator $E_2^{(+)}$ to (11.3), and use (3.1) and (4.6), we obtain $|0\rangle F_{12}$, which is in accordance with the previous results.

When $V_1 \gtrsim V_{\text{coh}}$, we must integrate (11.1) over all points x_1 in the detector with a certain weight $\eta(x_1)$. Replacing the integral with the sum, we obtain

$$\overline{K}_{12} \equiv \sum_{x_1} \eta(x_1) K_{12} = \mathrm{Sp}\,[\rho(t_1) f(t_2)], \tag{11.4}$$

where

$$\rho(t_1) \equiv \sum_{x_1} |1\rangle\, \eta(x_1)\, \langle 1| \tag{11.5}$$

is the diagonal matrix with elements $\eta(x_1)$, which plays the part of the density matrix for the output field. We may therefore suppose that, in the general case, the scheme incorporating the shutter prepares the field in the mixed state that is determined by the density matrix (the weight of the different pure states $|1\rangle$).

It is natural to consider the reduction $|2\rangle \to |1\rangle$ and the transformation from the joint probability $P(x_1,x_2)$ to the conditional probability $P(x_2|x_1)$ as a change in the way we see the role of detector 1: the detector and shutter (if present) are then looked upon not as the measuring part of the system, but as the preparing part. This approach immediately exposes the weak link in the EPR paradox, i.e., when we measure q_2 and p_2, we are dealing with different states $|1\rangle_{x_1}$ and $|1\rangle_{x_{1'}}$ of photon 2, whereas the uncertainty relation connects the parameters of a single state.

When the shutter is absent, detectors 1 and 2 are equivalent and it may turn out that the function $|1\rangle_{x_1}$ (like $\mathscr{E}_2$) is subjective because it reflects only the information possessed by observer 1; from the point of view of observer 2, on the other hand, we have a different reduction, namely, reduction to the function $E_2^{(+)}|2\rangle \equiv |1\rangle_{x_2}$. This terminology does not, however, take into account the operational significance of the symbols, namely, that the actually observed quantity, i.e., the number of coincidences m_{12} given by (5.2), requires the counting of the readings of both detectors. The introduction of the functional $|1\rangle_{x_2}$ signifies simply a different sequence of the evaluation of the moment G_{12}, which is symmetric in the indices 1, 2. The functions $\mathscr{E}_1$ and $\mathscr{E}_2$ correspond to different normalizations of the number of coincidences, i.e., m_{12}/m_2 and m_{12}/m_1, that can be interpreted as the conditional probabilities $P(x_1|x_2)$ and $P(x_2|x_1)$. The apparent subjectivism, due to the asymmetry in the indices 1, 2, is removed by the statistical nature of the predictions of quantum mechanics.

The description in terms of reduction (the $|2\rangle \to |1\rangle$ replacement) and the influence through the past (fixed argument x_1 in the function F_{12}) are, of course, mathematically equivalent. However, since the reduction occurs "instantaneously" at the time at which the count is recorded, and the wave function describes the field throughout space, it would appear that we are again dealing with nonlocality, this time action at a distance[8] (instead of the "telegraph into the past," we now have the "superluminous telegraph"). However, if we look upon the wave function simply as information, a change in this function is not a process in real space-time, so that the reduction is not a manifestation of nonlocality (a change in the information that we have when we receive, say, a telegram can, of course, cover space-like intervals, as well). This point of view is supported by the fact that information cannot be transferred at superluminous velocities in EPR-type experiments.

12. QUANTUM CORRELATION AND THE TRANSMISSION OF INFORMATION

The fact that the system incorporating an optical shutter enables us to control indirectly the structure of each emitted photon by manipulation in the reference channel, can be looked upon as providing a new method of modulating light ("quantum modulation" or "modulation of state"). Figure 8a illustrates an application of this method to signal transmission in binary code. It is clear that two-dimensional images can also be transmitted and can be recorded with the help of an image converter in which a controlling electrode replaces the optical shutter (Fig. 8b) and performs partial frequency modulation. Modern electronics can readily cope with an average photon-counting rate of about $10^6\ \mathrm{s}^{-1}$ in the modulator, which produces a comparable flux of photons at exit from the shutter. If the beam cross section is of the order of 1 cm^2, this flux can readily be recorded on the screen of an image converter, or even directly by eye.

Experiments such as those illustrated in Fig. 8 can be treated as the selection by the detector of a certain subensemble of diphotons but, here again, it is natural, for the sake of uniformity, to adopt phrases such as "reduction" or "modulation through the common past."

We emphasize that, although the time at which each successive pulse appears is a random quantity, the transmission of information in the presence of a shutter is not, in principle, a random event: when $\eta_2 = 1$ and the aperture of the receiving detector is large enough, each modulator pulse in Fig. 8a is accompanied, with suitable delay, by a pulse at the output of detector 2 or 2′. It is only when $\eta_2 < 1$ that photon-counting statistics contribute a Poisson noise to the transmission. The possibility of transmission of information with superluminous velocity was considered earlier in connection with EPR experiments (Refs. 8 and 46). Actually, the formula given by (6.1) involves the field propagation functions $D(x_n,x)$, so that the events x_1, x_2 rely on the same light cone with apex at x inside the crystal, i.e., they are separated by a space-like interval (Fig. 4). The "influence" of x_1 on x_2 must therefore propagate with superluminous velocity.

However, even if we admit the instantaneous nature of the reduction process, or the existence of influence through the past, the "superluminous telegraph" is still unattainable. This is clear from Fig. 8a: in the absence of the shutter, detectors 2 and 2′ will "fire" independently of the position of the modulating switch, and there will be no transmission of information. We need the shutter to select at the output of the transmitter the sequences of units and zeros defined by switching detectors 1 and 1′. When the shutter and the optical delay line in channel 2 are both present (the latter is necessary to compensate for the delay in the electronics), the time taken to transmit information between the shutter and the receiver of information is determined by the velocity of light (Fig. 4).

If we wish to detect the correlation between counts produced by the two detectors in the absence of the shutter, we must connect them in the usual way to a coincidence circuit. The event "coincidence of pulses" is then separated from the point of transmission by a time-like or space-like interval.

13. CONCLUSION

We must emphasize once again that the choice between the interpretations examined above (except for the model of choosing the subensembles) is a matter of taste, since they are all operationally equivalent to (6.1) and there is no doubt that experiment does not enable us to identify which is to be preferred. The great wealth of experience accumulated by experimental quantum physics is evidence for the "victory of formalism against modelism."[39] In Feynman's words,[6] "for many people . . . this circumstance [i.e., the absence of a procedure for measuring the attributes of a photon—D.K.] is very worrying. . . . However, in all probability, nature does not notice this paradox."

There is, naturally, some doubt as to the usefulness of an examination of interpretations that do not admit of an experimental confirmation or refutation. On the other hand, we must admit the potential usefulness of some treatments (and of the related interpretational or semantic paradoxes) in education, in the development of physical intuition, in the refinement of terminology and of concepts, and in the unavoidable process of change in existing paradigms (a well known historical example is provided by Faraday's lines of force). For example, the treatment involving the advanced wave enables us to predict qualitatively the results of different experiments with diphotons, using conventional optical ideas (Fig. 6).[14)]

Essentially, the EPR paradox has two aspects, usually examined separately: the contradiction within the formalism of quantum mechanics, formulated in Ref. 1 (apparent violation of Heisenberg's inequalities) and the conflict with "common-sense" (violation of Bell's inequalities). The latter contradiction is a true paradox, demanding the abandonment of "local realism" and "hidden determinism" in favor of the Copenhagen interpretation (or, if you like, action at a distance). It has been estimated[45] that these two aspects have been discussed $\sim 10^6$ times.

Nevertheless, it seems that the model proposed here deserves attention since the case of the continuous spectrum leads to a clearer picture of the two contradictions. It may also be found useful in the teaching of quantum physics. Finally, we note the simplicity of the system: it does not require vacuum technology or tunable lasers.

Several experiments with two-photon radiation were carried out in the recent past.[51] They can also be described by formulas such as (6.1) and, accordingly, can be clearly interpreted in terms of advanced waves. We note the experiment of Rarity *et al.*,[52] which implemented the idea of the generation of a controllable number of photons, using spontaneous parametric scattering and an optical shutter.[16,23] We also note the bibliographic review of recent work on the EPR paradox and the interpretation of quantum mechanics, given in Ref. 53.

The connection between stimulated parametric scattering and the EPR paradox first emerged in the course of a discussion of the coherence properties of the SPS field with R. V. Khokhlov in 1976. Sadly, this was our last discussion . . . . However, ever since then, I have continued to think about this connection, and have come to the conclusion that macroscopic nonlinear optics, which owes so much to Khokhlov, offers us an unusual possibility of elucidating the basic concepts of quantum mechanics in optical terms familiar to all. At any rate, I now know exactly what a photon is: a photon is the entity designated above by the symbols $\mathscr{E}(x_2)$ and $|1\rangle$, which can be prepared in a pure form by stimulated parametric scattering.

In conclusion, I wish to thank V. B. Braginskiĭ and G. Ya. Myakishev for fruitful discussions.

[1)] Based on a paper read by the author to a seminar held in memory of R. V. Khokhlov at Moscow State University on 23 May 1986.

[2)] We can proceed to a more general case by averaging the final results with the aid of the *P-representation* of the density matrix.[15]

[3)] We use this designation to distinguish the real multimode state $|2\rangle$ from the stationary basis state with two excited modes, $|\mathbf{k}\cdot\mathbf{k}'\rangle$.

[4)] In the case of the far zone, (4.7) gives a result equivalent to (3.6). The effect of lenses, prisms, mirrors, etc., is taken into account by replacing plane waves in the expression given by (4.2′) for the Green's function with the eigenwaves of the corresponding boundary-value problem.

[5)] In some crystals with large birefringence, the two photons in the pair can have different polarizations (phase-matching of the form $\mathbf{k}_0^e = \mathbf{k}_1^0 + \mathbf{k}_2^e$).

[6)] This means that a field in a given trial belongs to a certain statistical ensemble of fields described by the wave function $|2\rangle$.

[7)] Separability implies the absence of superluminous influence.

[8)] We note the similarity between (6.1) and the Feynman perturbation theory formulas, in which it is assumed that positrons [in our case, negative-frequency field $\mathscr{E}^{(-)}$] are electrons propagating backward in time (see Refs. 36, 39, and 40).

[9)] There does not appear to be unanimity as to precisely what the Copenhagen interpretation is supposed to be. Sometimes, it is understood to mean a general conjunction of the views of Bohr and Heisenberg.[41,47,48] Cramer[41] has formulated the Copenhagen interpretation in the form of five basic concepts, including the interpretation of a wave function as an expression of an observer's information and reduction as a change in this information. He also gives a detailed analysis of the weak (including subjective) points of the Copenhagen interpretation, and proposes an alternative interpretation (see also Refs. 48–50). The most common is undoubtedly the objective variant of the Copenhagen interpretation, which, to use Einstein's expression, denies the necessity of an observer for the existence of the Moon.

[10)] In the case of single photons, all that we can say is that the photon had some particular polarization. On the other hand, if it was created together with another photon, then even this statement has no meaning.

[11)] It is, of course, possible to describe the interaction of the field with detector 1 in terms of formal quantum theory, but this provides us with a common wave function for the field and the detector and, if we do not postulate reduction (which enables us to speak of the wave function of a single field), the question posed above remains unanswered.

[12)] Similar ideas, developed by Heisenberg, can be traced back to Aristotle (see Refs. 7, 10, 48, and 50).

[13)] The effective wave function $|1\rangle$ describes the state of the second photon after the count. Prior to the count, the photon did not have "its own" wave function, i.e., it was in a mixed state.

[14)] An interesting treatment of different quantum-mechanical paradoxes, using an "advanced wave function" $\Psi^*(x)$ in the coordinate representation, propagating in real space, has recently been developed by Cramer.[41] We hope that the realistic experiment proposed here, with its transparent formal description provided by (6.1), will serve as a good "touch-stone" for this and the others treatments described in Ref. 41 (we note that it is incompatible with the "neoclassical" field theory; see Refs. 9 ad 24).

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