Acceleration of cosmic rays by shock waves

E. G. Berezhko and G. F. Krymskiĭ

Institute of Space-Physics Research and Aeronomy, Yakutsk Branch, Siberian Division, Academy of Sciences of the USSR

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Theoretical work on various processes by which shock waves accelerate cosmic rays is reviewed. The most efficient of these processes, Fermi acceleration, is singled out for special attention. A linear theory for this process is presented. The results found on the basis of nonlinear models of Fermi acceleration, which incorporate the modification of the structure caused by the accelerated particles, are reported. There is a discussion of various possibilities for explaining the generation of high-energy particles observed in interplanetary and interstellar space on the basis of a Fermi acceleration mechanism. The acceleration by shock waves from supernova explosions is discussed as a possible source of galactic cosmic rays. The most important unresolved questions in the theory of acceleration of charged particles by shock waves are pointed out.

CONTENTS

1. INTRODUCTION

A characteristic feature of a collisionless space plasma is the occurrence of processes which result in the generation (or acceleration) of fast charged particles with an energy far above the thermal energy. These particles can be observed directly in interplanetary space.[1] The presence of fast accelerated particles in interplanetary space and in various astrophysical objects has been established by radio, x-ray, and γ-ray astronomy (see, for example, Ref. 2 and the bibliography there). Among the most obvious manifestations of the acceleration processes are the galactic cosmic rays.

There is particular interest in the acceleration processes which occur near the fronts of shock waves which are propagating through a space plasma, primarily because shock waves are a fairly common phenomenon in space. As examples we might cite the shock waves from chromospheric solar flares and from supernova explosions. Furthermore, a large amount of energy is usually released in the form of a directed motion of plasma in the processes which lead to the formation of shock waves. A significant fraction of this energy may go into the acceleration of a small fraction of the plasma particles, with the result that particles appear with energies many orders of magnitude above the thermal energy.

A theory for the acceleration processes must be derived in order to reach an understanding of the fundamental properties of the plasma and also to reconstruct the overall picture of such phenomena as chromospheric flares and supernova explosions. Furthermore, there are strong arguments for believing that shock waves from supernovae are among the primary sources of galactic cosmic rays.

The first experimental indications of an acceleration of charged particles by interplanetary shock waves[3-8] stimulated the development of theoretical ideas regarding possible mechanisms for the acceleration of particles near shock fronts.[9-16] Experiments which have been carried out on space vehicles directly in interplanetary space have conclusively proved that intense acceleration processes operate near the fronts of shock waves.[17,18] These acceleration processes are presently being studied widely and in detail.

Shock waves themselves are fairly complex physical phenomena.[19-24] In the narrow spatial region which is called the "shock front," magnetohydrodynamic (MHD) energy of the unperturbed medium is converted in part into thermal energy as the result of a variety of dissipative processes. In an ordinary gas the dissipation results from binary collisions.[20] For the conditions which prevail in a space plasma, "collisionless" shock waves are more typical. The dissipation at the front of such shock waves is of a collective nature and is a consequence of the onset of plasma instabilities.[21-24] The motion of the plasma particles in this case is determined not by binary collisions but by the interaction of the particles with a turbulence which is generated at the front; the length scale which determines the thickness of the front of an intense shock wave is the gyroradius of the thermal ions. The collective nature of the plasma processes which occur at the front

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and their definitely nonlinear nature pose significant difficulties in a theoretical description of collisionless shock waves. Except for cases of shock wave which are not very strong,[22,23] the theory here is very incomplete (see the review by McKee and Hollenbach[24] regarding the present state of this question).

The nature of the motion of fairly fast particles near the front of a collisionless shock wave is relatively independent of the particular structural details of the wave. The reason for this situation is that the motion of charged particles in a space plasma is determined primarily by the interaction of these particles with magnetic fields: the large-scale (or regular) field, on the one hand, and the random (or turbulent) field, on the other. The interaction with the random field component results in random changes in the directions in which the fast particles move, i.e., a scattering of these particles. The mean free path with respect to scattering of fairly high-energy particles is far greater than the thickness of the shock fronts, so the particular structural features of the fronts do not influence the fast particles. This assertion of course does not mean that there is no interest in studying the structure of a shock front as part of a larger study of the acceleration of charged particles. As we have already mentioned, the unperturbed plasma through which the shock wave propagates undergoes a heating at the front. After this heating, the fastest particles in the thermal distribution may be subject to acceleration by some mechanism or other. The processes which occur at the shock front and the structure of the front thus determine the rate at which particles are injected into the acceleration regime and, ultimately, such an important parameter as the number of accelerated particles.

The possibility that charged particles will be accelerated in a plasma stems from the electric fields in the plasma. These are primarily the induced fields which arise as a highly conducting plasma moves in a magnetic field. Acceleration of particles by such fields may occur near a shock front. Figure 1a illustrates the situation with a schematic diagram of the trajectory traced out by a fast charged particle in the rest frame of the front of a transverse plane shock wave which is propagating opposite the x axis. This particular diagram corresponds to the case in which there are no small-scale electromagnetic fields, or they play only a minor role (a laminar shock wave). For a fast particle whose velocity v is much greater than the plasma velocity $\mathbf{u}$, and for which the mean free path with respect to scattering (λ) and the gyroradius ρ_B are much greater than the front thickness l, the shock wave represents an MHD discontinuity in which the magnetic field B, the density ρ, and the plasma velocity u in the regions behind and ahead of the wavefront are related by[19]

$$\rho_2=\sigma\rho_1, \qquad u_2=\frac{u_1}{\sigma}, \qquad B_2=\sigma B_1, \tag{1.1}$$

where σ is the degree of compression of the matter at the shock front, Here and below, the subscripts 1 and 2 specify the regions ahead of and behind the front, respectively. As it intersects a shock front, a particle undergoes a gradient drift and is displaced along the electric field $\mathbf{E}= -[\mathbf{uB}]/c$ (c is the velocity of light), so its energy increases. The change in the energy of the particle is determined quantitatively by conservation of the adiabatic invariant[12,13,16] $\mu=p_\perp^2/B$, where $P_\perp$ is the component of the particle momentum perpendicular to the magnetic field. The conservation of μ in this case is not a trivial fact. Furthermore, it is not a consequence of the validity of the drift approximation, which is not applicable for the region of the shock front, where the magnetic field undergoes rapid changes. The conservation of μ for a particle intersecting a shock front has been established as the result of a detailed study of the trajectory of such a particle.[12,1] The results have been supported by numerical calculations.[25]

If the magnetic field is oriented obliquely with respect to the shock front, particles with sufficiently large pitch angles can be reflected by the front, since the magnetic field which is intensified behind the front plays the role of a magnetic mirror. The reflection of the particles is accompanied by an increase in their energy.[10,16] Even if the magnetic field is in the most favorable direction, however, the energy of the particles—except for a negligible fraction of these particles—increases by no more than an order of magnitude.[25,26] Under conditions with a laminar, quasitransverse shock wave, with the magnetic field making a small angle with respect to the surface of the front, a reflection of ions may also occur as the result of an electric field which arises because of a charge separation near the shock front.[22,23,27,28] These processes undoubtedly play a governing role in shaping the structure of a shock wave. However, the one-shot nature of these mechanisms limits their possibilities in the generation of high-energy particles.

The laminar shock wave we have been talking about is of course an idealization. For space plasmas, there would typically be a random magnetic field, which results from the development of a plasma turbulence, in addition to the regular magnetic field. The scattering of particles by irregulari-

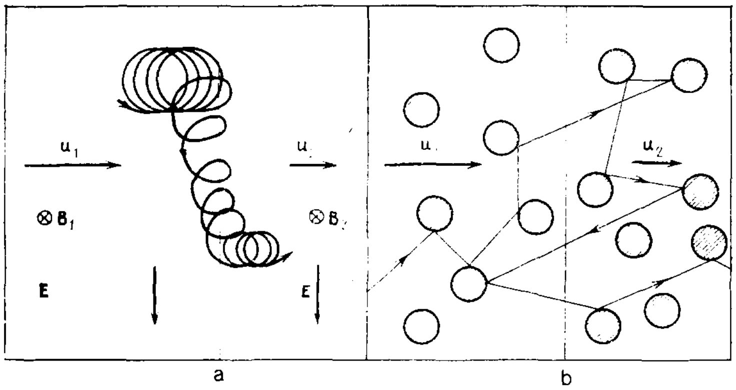


FIG. 1. Motion of a fast charged particle near the shock front of a laminar shock wave ($\mathbf{E}= -[\mathbf{uB}]/c$) (a) and a shock wave in a turbulent medium (b).

ties of the magnetic field tends to make the particles isotropic, and it also presents the possibility of a repeated intersection of a shock front (Fig. 1b). The particles may thus undergo an acceleration of a cyclic nature. Each cycle—a double intersection of the front—is accompanied by an increase in the energy of a particle, even in the absence of a regular magnetic field.[29-33] Since the typical velocities of turbulent fluctuations are of the order of the Alfvén velocity $c_a = B/(4\pi\rho)^{1/2}$, and since the relation $c_a \ll u_{1,2}$ holds in the case of strong shock waves, we can treat the irregularities of the magnetic field—the scattering centers—as being frozen in the plasma. Furthermore, the scattering of the particles will be an elastic scattering in the frame of reference which is moving with the scattering centers, if the effect of the fast particles on the medium is ignored.

This fact is physically obvious for the case in which a particle interacts with a magnetic-field irregularity which is frozen in a plasma, since the magnetic field performs no work. A detailed study of the interaction of a charged particle with MHD waves[1,2,34] leads to the same result. In this case the change in the magnitude of the momentum of a particle as a result of its scattering is

$$\Delta p = (\mathbf{p}_f - \mathbf{p}_i)\frac{\mathbf{u}}{v}, \tag{1.2}$$

where $\mathbf{u}$ is the velocity of the plasma, and $\mathbf{p}_i$ and $\mathbf{p}_f$ are the momenta of the particle respectively before and after the scatterings. Since Δp is small (by virtue of the relation $u/v \ll 1$), we can assume $p_f = p_i = p$. The change in the momentum of a particle after a double crossing of a front is $\Delta p = (\mathbf{p}_k - \mathbf{p}_i)\mathbf{u}_1 v^{-1} + (\mathbf{p}_f - \mathbf{p}_k)\mathbf{u}_2 v^{-1}$. Averaging this expression over the particle flux, which we assume to be isotropic, for angles from $\pi/2$ to π between the vectors $\mathbf{p}_i$, $\mathbf{p}_f$ and $\mathbf{u}$ and from 0 to $\pi/2$ between $\mathbf{p}_k$ and $\mathbf{u}$, we find the average change in the momentum of the particle over the cycle:

$$\langle \Delta p \rangle = \frac{4}{3}(u_1 - u_2)\frac{p}{v}. \tag{1.3}$$

After a given cycle, the particle has a definite probability for not returning to the front. The number of particles therefore decreases with increasing index of the cycle. The integral spectrum of accelerated particles $N(p)$, i.e., the number of particles with a momentum greater than p in a unit volume, can be found from the balance equation

$$\frac{dN}{dp} = \frac{P_c - 1}{\langle \Delta p \rangle} N, \tag{1.4}$$

where P_c is the probability for the completion of the given cycle. This probability follows from the obvious relation $N(p + \Delta p) = P_c N(p)$, which shows that the number of particles capable of undergoing the $(i + 1)$st front-crossing cycle is equal to the product of the number of particles which have undergone i cycles and the probability for the completion of the next cycle. The probability P_c satisfies the relation $P_c = P_1 P_2$, where $P_{1,2}$ are the probabilities for a particle which has entered the regions ahead of and behind the front to return to the front. The probability P_1 is equal to unity, since all the particles from region 1 are carried by convection to the front. The probability P_2 can be expressed in terms of J_{12}, which is the flux of particles coming out of region 1 and going into region 2, and the directed flux of particles into region 2: $P_2 = (J_{12} - J_2)/J_{12}$. The assumption that the particle distribution behind the front is approximately isotropic and homogeneous yields $J_{12} = nv/4$ and $J_2 = nu_2$, where $n = dN/dp$ is the differential particle density. We thus find an expression for the probability for the completion of the next cycle:

$$P_c = 1 - \frac{4u_2}{v}. \tag{1.5}$$

Using (1.3)–(1.5), we find an equation for the density n:

$$\frac{d\,pn}{dp} + 3\frac{u_2}{u_1 - u_2} n = 0. \tag{1.6}$$

A solution of this equation is a power-law function $n \sim p$ with an exponent $\gamma = (\sigma + 2)/(\sigma - 1)$.

The above analysis, which can also be found, aside from inconsequential differences, in Refs. 31, 32, and 35–39, illustrates the physical content of the type I Fermi acceleration mechanism[29-33] (we will say simply "Fermi mechanism"; this process is also called the "diffusive acceleration of particles by a shock wave" in the non-Soviet literature and the "regular mechanism" in the Soviet literature).

Even this simple analysis shows an important advantage of the Fermi mechanism: The spectrum of the accelerated particles is independent of the properties of the medium through which the shock wave is propagating. When we further note that the degree of compression is in the interval $\sigma = 3$–4 for strong shock waves, we see that the exponent on the accelerated-particle spectrum is $\gamma = 2$–3, precisely what is observed in the galactic cosmic rays and in the relativistic electrons in the remnants of supernovae.[40] This circumstance makes the Fermi mechanism extremely attractive for explaining several astrophysical phenomena, and this mechanism has accordingly attracted much research interest in recent years. Despite the fact that the theory of Fermi acceleration has been covered in several reviews[39,41-44] (see also Refs. 1 and 2), it is worthwhile to review the recent results because of the intense activity in this field. This is our purpose in the present paper.

We should also point out some other acceleration processes which could operate in a space plasma. These are the statistical acceleration mechanisms. They operate if the scattering centers undergo random motions, regardless of whether these motions constitute a displacement of masses of matter with a frozen-in magnetic field (magnetized clouds)[45] or wave motions of turbulent fluctuations[13-15,34] (see also Ref. 1). All versions of the statistical mechanisms have a common physical content. The fast particles and the scattering centers in a sense constitute two distinct gases. Since the scattering centers are macroscopic plasma volumes, they correspond to an infinitely high temperature. Their thermal contact with the fast particles through scattering events results in a transfer of energy from the scattering centers to the particles, i.e., an acceleration of the particles. In other words, the acceleration process here is an analog of the ordinary heating which occurs in a collisional plasma. Interestingly, this analogy is quite general: The heating in a plasma with collisions corresponds to an acceleration of fast particles in a plasma without collisions. As confirmation of this statement we could also cite the Fermi and friction[46,47] acceleration mechanisms. The efficiency of statistical acceleration processes in specific space-physics objects is a rather complicated question, since the efficiency is determined primarily by the level and nature of the plasma turbulence, about which we would usually have extremely limited information. Furthermore, most of the energy in interstellar space is in the form of motion of large volumes of matter, in particular, large-scale shock waves.[42] For this reason, the Fermi acceleration mechanism, through which energy of the directed motion of the medium is transferred directly to fast particles, is preferable in many regards.

To avoid any misunderstanding, we should state that in either case energy of a macroscopic volume of matter is transferred to individual charged particles through an interaction of the particles with scattering centers. Consequently, the role played by the turbulent field and the problem of finding a theoretical description of it are of equal importance to the two acceleration mechanisms. The physical reason why the Fermi acceleration mechanism has a high efficiency in many cases is that the same particle-scattering events draw energy from the directed motion of the matter, and the amount of this energy available in the case of a strong shock wave is far greater than the energy of the random motions. In this regard, the role played by the statistical mechanisms may be a governing one in the formation of the population of superthermal particles, which are then injected into the Fermi acceleration process. This interpretation is supported, in particular, by an analysis of experimental results obtained in interplanetary space.

In taking up the theory of Fermi acceleration we note that the term "cosmic rays" in the title of this paper is being used in its broad sense: It is to be understood as synonymous with "fast" or "accelerated particles".

2. LINEAR THEORY OF FERMI ACCELERATION

The transport of fast charged particles in a space plasma can be described quite comprehensively and systematically on the basis of a diffusion transport equation, because of the random magnetic field.

The diffusion description method is applicable if the angular distribution of the particles is rendered approximately isotropic by frequent events in which fast particles are scattered by irregularities of the magnetic field. It then becomes possible to restrict the analysis to the first two angular moments in the expansion of the distribution function:

$$f(\mathbf{r},\ \mathbf{p},\ t)=f(\mathbf{r},\ p,\ t)+f_\alpha(\mathbf{r},\ p,\ t)\,p_\alpha p^{-1}.$$

The isotropic part of the distribution function, $f(\mathbf{r}, p, t)$, satisfies the equation[48-50]

$$\frac{\partial f}{\partial t}=\nabla_i\varkappa_{ij}\nabla_j f-\mathbf{u}\nabla f+\frac{\nabla\mathbf{u}}{3}\,p\,\frac{\partial f}{\partial p}+Q, \tag{2.1}$$

and the first moment is given by[41]

$$f_\alpha=-3\,\frac{\varkappa_{\alpha\beta}}{v}\,\nabla_\beta f-\frac{u_\alpha}{v}\,p\,\frac{\partial f}{\partial p}\,. \tag{2.2}$$

Here $\varkappa_{ij}$ is the particle diffusion tensor, $\mathbf{u}$ is the hydrodynamic velocity of the plasma, and Q is the strength of the particle source, which describes the creation (injection) and annihilation (escape from the system) of particles. The creation and annihilation may also result from any possible processes which would change the energy of the particles, other than an adiabatic process. An adiabatic change in the energy of the particles is described by the third term on the right side of the equation. That term shows that the energy of an individual particle, ε, varies in accordance with the equations[48-54]

$$\left\langle\frac{dp}{dt}\right\rangle=-\frac{\nabla\mathbf{u}}{3}\,p,\quad \left\langle\frac{d\varepsilon}{dt}\right\rangle=-\frac{\nabla\mathbf{u}}{3}\,pv, \tag{2.3}$$

where the angle brackets mean an average over a time interval much longer than the time scale between scattering events.

It is specifically the use of Eq. (2.1) which has made it possible to study and reach a correct understanding of a long list of phenomena and processes involving cosmic rays in interplanetary space.[1,51,52,55]

A necessary condition for the applicability of transport equation (2.1) is that the mean free path with respect to scattering be small in comparison with the length scale ($l=|u/\nabla u|$) of the variations in the plasma velocity u. In the case of discontinuous flows—in particular, a shock wave represents a discontinuous flow for fast particles—this condition breaks down. A detailed analysis shows that Eq. (2.1) must be supplemented with a boundary condition at the shock front which relates the solutions of this equation on the two sides of the front. For a quasiparallel shock wave, with the regular magnetic field making a small angle with the normal to the shock front, $\mathbf{h}$, the boundary conditions become particularly simple[1,56,57]:

$$f_1=f_2,\quad f_{1\alpha}h_\alpha=f_{2\alpha}h_\alpha. \tag{2.4}$$

These conditions reflect the circumstance that the crossing of the shock front by a particle does not change the momentum of the particle, so the particle density $n=4\pi p^2 f$ and the normal component of the density of the directed flux, $j_\alpha=(4\pi/3)p^2 v f_\alpha$, must be continuous.

For an arbitrary orientation of the magnetic field, the density of the particles has a jump at the front because of a reflection of these particles by the front.[57] Although there are no fundamental difficulties in treating the general case, we will restrict the analysis here to the mathematically simpler case of quasiparallel shock waves.

The Fermi acceleration of particles by a shock wave consists of a transfer of energy of the directed motion of the plasma to fast particles, as we have shown. The fast particles may in turn influence both the internal properties—the turbulence level—and the structure of the shock front—the distribution of the hydrodynamic velocity $\mathbf{u}(\mathbf{r})$. The problem of finding the energy and spatial distributions of the accelerated particles must therefore be solved simultaneously with a determination of the self-consistent turbulence spectrum and the structure of the shock front. Only when the energy density of the accelerated particles is insignificant in comparison with the energy of the directed motion of the plasma can the effect of the fast particles be ignored. This situation may be realized, for example, if the number of accelerated particles is small, because of a low rate of injection of thermal particles into the acceleration process. The corresponding version of the theory—the linear theory or the test-particle approximation—is presented in this section of the review.

2.1. Acceleration by a plane shock wave

In the one-dimensional case with a plane shock wave, which is propagating opposite the x axis in a homogeneous medium, the particle distribution function depends on only the coordinate x. The component $\varkappa_{xx}$ is thus the sole component of the diffusion tensor which appears in Eq. (2.1); we will accordingly adopt the simpler notation $\varkappa$.

Transport equation (2.1) takes the following form in the rest frame of the shock front, where it occupies the position $x = 0$:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\varkappa\frac{\partial f}{\partial x} - u\frac{\partial f}{\partial x} - \frac{\Delta u}{3}\delta(x)\,p\frac{\partial f}{\partial p} + Q, \qquad (2.5)$$

where $\Delta u = u_1 - u_2$. Written in generalized form [$\nabla \mathbf{u} = -\Delta u \delta(x)$], this equation already contains the boundary conditions (2.4). However, since the solution of this equation is sought separately in the region ahead of and behind the shock front in real situations, we will write the boundary conditions explicitly, also taking account of the possible presence of a lumped source $Q = Q_0\delta(x)$ at the shock front. These conditions can be found by integrating Eq. (2.5) over x term by term from $-\delta$ to δ and then letting δ go to zero:

$$f_1 = f_2, \quad \varkappa_1\frac{\partial f_1}{\partial x} + \frac{u_1}{3}p\frac{\partial f_1}{\partial p} = \varkappa_2\frac{\partial f_2}{\partial x} + \frac{u_2}{3}p\frac{\partial f_2}{\partial p} + Q_0. \qquad (2.6)$$

As before, the subscripts 1 and 2 specify the quantities ahead of and behind the front, and the values of the functions f_1 and f_2 and their derivatives are taken at the points $x = -0$ and $x = +0$, respectively.

In real situations we would expect that the injection of the particles would occur in one of two ways: Either the injection occurs at the shock front, where the plasma is heated, and then the fastest particles become involved in the acceleration process; or the fastest particles already present in the unperturbed medium undergo acceleration. The first of these possibilities is described by the lumped source $Q = Q_0\delta(x)$, and the second by the specification of the boundary condition $f_1(-\infty, p) = f_\infty(p)$, where $f_\infty(p)$ is the spectrum of fast particles in the unperturbed medium. If we consider only these two possibilities, we can assume that there are no sources of particles away from the shock front. The steady-state solution of Eq. (2.5) will then have the form

$$f_i = A_i + B_i \exp\int\limits_0^x \frac{u_i}{\varkappa_i}\,\mathrm{d}x. \qquad (2.7)$$

The requirement that the function be bounded is satisfied with $B_2 = 0$, and the boundary conditions at $x = -\infty$ and $x = 0$ are satisfied with $A_1 = f_\infty(p)$, $B_1 = A_2 - A_1$. The function $f_2(p) = A_2(p)$ and the function $f_1(x,p)$ along with it can be found by using the second of conditions (2.6). For a monoenergetic source $Q_0 = (N_0/4\pi p^2)u_1\delta(p - p_0)$ and for a spectrum $f_\infty = (N_\infty/4\pi p^2)\delta(p - p_0)$ in the unperturbed medium, this condition becomes the equation

$$u_1 f_2 + \frac{\Delta u}{3}p\frac{\partial f_2}{\partial p} = \frac{N_0 + N_\infty}{4\pi p^2}u_1\delta(p - p_0).$$

Solving this equation and (2.7), we find

$$f_1(x, p) = \frac{N_\infty}{4\pi p^2}\delta(p - p_0)\left[1 - \exp\left(-\int\limits_x^0 \frac{u_1}{\varkappa_1}\,\mathrm{d}x\right)\right] + f_2(p)\exp\left(-\int\limits_x^0 \frac{u_1}{\varkappa_1}\,\mathrm{d}x\right),$$

$$f_2(p) = \frac{N_0 + N_\infty}{4\pi p_0^3}q\left(\frac{p}{p_0}\right)^{-q}\theta(p - p_0), \qquad (2.8)$$

where $q = 3u_1/\Delta u$, and $\theta(x)$ is the unit step function (Heaviside function). We thus see that the two injection mechanisms operate additively. For arbitrary spectra of the injected particles $f_0 = Q_0(p)/u_1$ and $f_\infty(p)$, the distribution function of the accelerated particles behind the shock front takes the form

$$f_2(p) = \int\limits_0^\infty (f_0(p') + f_\infty(p'))\,G(p, p')\,\mathrm{d}p',$$

where

$$G(p, p') = \left(\frac{p}{p'}\right)^{-q}\frac{q}{p'}\theta(p - p')$$

is the Green's function of the problem of the acceleration of particles by a plane shock wave.

Expression (2.8) shows that an important aspect of Fermi acceleration is the universal shape of the spectrum of accelerated particles: the exponent $\gamma = q + 2$ in the density expression $n \sim p^{-\gamma}$,

$$\gamma = (\sigma + 2)(\sigma - 1)^{-1} \qquad (2.9)$$

is determined entirely by the extent of compression of the matter at the shock front. In the case at hand, of quasiparallel shock waves, in which the magnetic field is dynamically inconsequential,[19,40] the extent of compression,

$$\sigma = (\gamma_g + 1)\left(\gamma_g - 1 + \frac{2}{\mathrm{Ma}_1^2}\right)^{-1},$$

is determined by the adiabatic index of the medium, γ_g, and the Mach number $\mathbf{Ma}_1 = u_1/c_s$, where $c_s = \left(\gamma_g \frac{P_{g1}}{\rho_1}\right)^{1/2}$ is the sound velocity, and P_{g1} is the thermal pressure in the medium ahead of the front. For strong waves ($\mathbf{Ma}_1 \gg 1$) which are propagating through a fully ionized plasma ($\gamma_g = 5/3$) we would have $\sigma = 4$ and thus $\gamma = 2$.

The universal—exclusive—shape of the spectrum of accelerated particles over the entire momentum range from p_0 to ∞ is of course a consequence of our idealized formulation of the problem. In real situations, the finite dimensions of the shock wave and the finite thickness of the shock front would impose certain restrictions on the acceleration process.

The restrictions which stem from the finite dimensions of the shock wave, R, can be described at a qualitative level in the following way: If a particle, in the course of its acceleration, goes a distance greater than R away from the front, then it will have a small probability for returning to the front and for continuing to be accelerated. This effect can be dealt with approximately in the one-dimensional problem. For this purpose, we add to the situation discussed above an absorbing surface, at a distance R ahead of the front. The boundary condition $f_1(-\infty, p) = 0$ ($f_\infty = 0$) is replaced by $f_1(-R, p) = 0$, and the steady-solution of Eq. (2.5) becomes, instead of (2.8)

$$f_1(x, p) = f_2(p)\frac{\exp\left(-\int\limits_x^0 \frac{u_1}{\varkappa_1}\,\mathrm{d}x\right) - \exp\left(-\int\limits_{-R}^0 \frac{u_1}{\varkappa_1}\,\mathrm{d}x\right)}{1 - \exp\left(-\int\limits_{-R}^0 \frac{u_1}{\varkappa_1}\,\mathrm{d}x\right)},$$

$$f_2(p) = \frac{N_0}{4\pi p_0^3}q\left(\frac{p}{p_0}\right)^{-q}\theta(p - p_0)$$

with a spectral exponent

$$q=\frac{3\sigma}{\sigma-1}a, \quad a=\left[1-\exp\left(-\int_{-R}^{0}\frac{u_1}{\varkappa_1}dx\right)\right]^{-1}. \quad (2.10)$$

We see that the presence of an absorbing boundary steepens the spectrum since we have $a>1$. If the diffusion coefficient is a growing function of the momentum, as it essentially always is for fast particles, the steepening of the spectrum is important in practice for particles with momenta $p \gtrsim p_m$, where p_m is found from the relation $g_1 \equiv Ru_1/\varkappa_1(p_m)=1$. Since the spectrum falls off rapidly at $p>p_m$, in this case, the value of p_m represents a maximum momentum of the accelerated particles. The effect of a finite thickness of the shock front can be studied by examining the acceleration by a shock wave with a smooth velocity profile

$$u(x)=\frac{u_1+u_2}{2}-\frac{u_1-u_2}{2}\,\mathrm{th}\,\frac{x}{l}, \quad (2.11)$$

which varies from the value $u_1=u(-\infty)$ to the value $u_2=u(\infty)$ over a length scale l.

In the steady state, with a constant diffusion coefficient, Eq. (2.1) can be put in the following form through a change of spatial variable[58]:

$$\varkappa\frac{\partial}{\partial u}\left(\frac{du}{dx}\frac{\partial f}{\partial u}\right)-u\frac{\partial f}{\partial u}+\frac{p}{3}\frac{\partial f}{\partial p}=0, \quad (2.12)$$

where $du/dx=-2(u_1-u)(u-u_2)/l(u_1-u_2)$. The particle source Q has been set equal to zero, and the acceleration problem has been reduced to one of finding a solution of Eq. (2.1) under the boundary condition $f(u=u_1, p)=f_\infty(p)$. As before, we can restrict the discussion to the case of a monoenergetic spectrum of injected particles, $f_\infty(p)=(N_\infty/4\pi p_0^2)\delta(p-p_0)$, without any loss of generality.

The method of separation of variables with $f(u,p)=\sum_{n=0}^{\infty}F_n(u)p^{-q_n}$ is used to reduce Eq. (2.1) to a Sturm-Liouville problem of seeking the eigenfunctions F_n and the eigenvalues q_n. The function F_0, which corresponds to the smallest eigenvalue, q_0, determines the behavior of the solution $f(u,p)$ at large momenta. This function has no nodes in the interval (u_1, u_2), so it can be sought in the form $F_0=(u_1-u)^\alpha$. Substituting $f=F_0p^{-q_0}$ into Eq. (2.12), we find

$$\alpha=\frac{lu_1}{2\varkappa}, \quad q_0=\frac{3\sigma}{\sigma-1}\left(1+\frac{lu_2}{2\varkappa}\right).$$

We see that at large momenta the spectrum is a power spectrum, with an exponent which increases in magnitude with increasing front thickness. The steepening of the spectrum is important under the condition $l \gtrsim \varkappa/u_2$. The complete solution of the problem, found by Laplace transforms, is given in Ref. 58.

The limitations which are imposed by the finite dimensions of the shock wave, R, and the finite thickness of the shock front, l, can thus be summarized as follows: The particles which are effectively accelerated are those for which the diffusion length $L(p)=\varkappa_1(p)/u_1$ lies in the interval

$$l \ll L(p) \ll R. \quad (2.13)$$

2.2. Rate of Fermi acceleration

The role played by one acceleration mechanism or another under specific physical conditions depends on the efficiency of the mechanism, which in turn depends on the rate at which the accelerated particles acquire energy. As the acceleration rate increases, a progressively smaller role is played by competing processes, primarily the various types of energy loss.

The rate of Fermi acceleration can be found by studying the time evolution of the spectrum of accelerated particles. In Eq. (2.5) and in boundary conditions (2.6) it is convenient to take Laplace transforms,[43,59-64] $\bar{f}(x,p,s)=\int_0^\infty f(x,p,t)e^{-st}dt$:

$$\frac{\partial}{\partial x}\varkappa\frac{\partial\bar{f}}{\partial x}-u\frac{\partial\bar{f}}{\partial x}-s\bar{f}+\frac{1}{s}\frac{N_0}{4\pi p_0^2}\delta(p-p_0)=0, \quad (2.14)$$

$$\bar{f}_1=\bar{f}_2, \quad \varkappa_1\frac{\partial\bar{f}_1}{\partial x}+\frac{u_1}{3}p\frac{\partial\bar{f}_1}{\partial p}=\varkappa_2\frac{\partial\bar{f}_2}{\partial x}+\frac{u_2}{3}p\frac{\partial\bar{f}_2}{\partial p}, \quad (2.15)$$

where we have assumed that the particle source is "turned on" at the time $t=0-Q=(N_0/4\pi p_0^2)\delta(p-p_0)\theta(t)$. If the diffusion coefficients $\varkappa_{1,2}$ do not depend on the coordinate x, a solution of Eqs. (2.14), (2.15) is

$$\bar{f}_i(x,p,s)=\frac{f_2(p)}{s}\exp\left\{-\frac{3}{2}\int_{p_0}^{p}\frac{u_1A_1+u_2A_2}{\Delta u}\frac{dp}{p}+x\frac{u_i}{2\varkappa_i}\left[1-(-1)^i\left(1+\frac{4s\varkappa_i}{u_i^2}\right)^{1/2}\right]\right\}, \quad (2.16)$$

where $A_i=[1+(4s\varkappa_i/u_i^2)^{1/2}-1$, and $f_2(p)$ is the steady-state spectrum (2.8) at the shock front. The inverse transformation can not be made in the general case. However, at the long times $t \gg \varkappa_i/u_i^2$, of practical importance, it is a straightforward matter to find the behavior of the solution. In this case we can use the approximate expression $2s\varkappa_i/u_i^2$, for the quantities A_i, so that the inverse Laplace transformation can be made[43,63]:

$$f_1(x,p,t)=f_2(p)\exp\left(\frac{xu_1}{\varkappa_1}\right)\theta(t-t_a),$$
$$f_2(x,p,t)=f_2(p)\theta\left(t-t_a-\frac{x}{u_2}\right), \quad (2.17)$$

where

$$t_a=\int_{p_0}^{p}\frac{3}{u_1-u_2}\left(\frac{\varkappa_1}{u_1}+\frac{\varkappa_2}{u_2}\right)\frac{dp}{p} \quad (2.18)$$

is the time required to accelerate a particle from a momentum p_0 to p. We thus see that at each time t a steady-state universal spectrum $f \sim p^{-q}$ is established in the momentum region $p_0 < p \lesssim p_m(t)$ at the shock front. The maximum momentum, determined by the relation $t=t_a(p_m)$, increases in time in accordance with the equation $dp_m/dt=p_m/\tau_a$, so the quantity

$$\tau_a=\frac{3}{u_1-u_2}\left(\frac{\varkappa_1}{u_1}+\frac{\varkappa_2}{u_2}\right) \quad (2.19)$$

has the meaning of a characteristic acceleration time.

Figure 2 illustrates the relation between approximate results (2.17) and the exact result, found by taking the inverse Laplace transformation of expression (2.16) numeri-

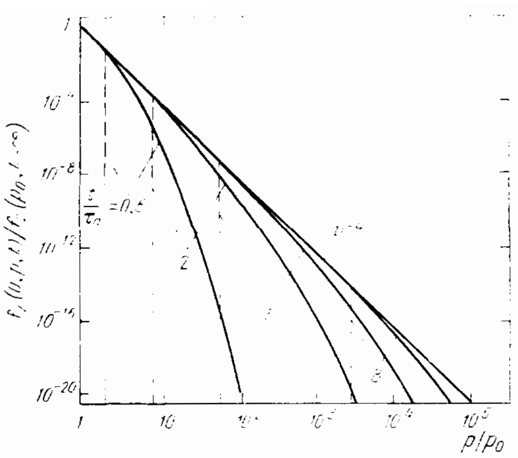


FIG. 2. Spectrum of particles accelerated by a shock wave with a degree of compression $\sigma = 4$ at various times t after the time of injection.

cally. Also shown here is the dynamics of the relaxation to a steady-state spectrum. This figure shows the distribution function of the accelerated particles at the front, $f_2(0, p, t)$ for the case of a strong shock wave ($\sigma = 4$) and with constant diffusion coefficients $\varkappa_1 = \varkappa_2$. The dashed lines are the positions of $p_m(t)$. We see that the momentum region in which the spectrum of the accelerated particles is approximately the steady-state spectrum $f_2(p)$ increases over time, in accordance with expression (2.17). At each instant there is a fairly smooth and extended region in the spectrum which corresponds to $p > p_m(t)$.

The time evolution of the Fermi acceleration process can also be studied in a random-walk model. All the basic results which have been obtained by that method[60,65–67] agree with the results reported above.

Competing processes such as statistical acceleration[62,68–70] and various types of energy loss[71–73] can be taken into account systematically on the basis of Eq. (2.1), if that equation is supplemented with terms to describe these processes. However, without resorting to the procedure of solving the transport equations, which is a fairly laborious task in these cases, it is possible to determine the momentum region for which Fermi acceleration is the dominant mechanism and in which we should not expect to find any changes in the spectrum of accelerated particles. To do this it is sufficient to compare the acceleration time τ_a with the scale time $\tau_i = |p/\langle dp/dt \rangle_i|$, which determines the rate of change of the momentum, $\langle dp/dt \rangle_i$, for the competing process.[74] In the region of particle momenta in which the relation $\tau_a \ll \tau_i$ holds, the Fermi-acceleration process proceeds without any substantial changes. For statistical mechanisms, the characteristic acceleration time[1,2] $\tau_s \sim \varkappa \tilde{u}^{-2}$ is determined by the random component of the velocity of the scattering centers, $\tilde{u}$. Under the conditions prevailing in a space plasma, the role of scattering centers is played primarily by the MHD turbulence, for which we have $\tilde{u} \sim c_a$. We thus find $\tau_a/\tau_s \sim \mathbf{Ma}_1^{-2}$, which tells us that Fermi acceleration outweighs the statistical acceleration in the vicinity of strong shock waves, for which the Mach number is large.

For relativistic electrons the basic types of loss in a space plasma are the synchrotron and Compton losses.[75] Since the rate of these losses increases rapidly with the energy, we would expect to find a sharp cutoff in the spectrum of accelerated electrons at a certain energy $p_m c$, at which the rates of loss and Fermi acceleration would become comparable.[71–74] Estimates carried out for the shock waves from supernovae yield[2,74] $p_m c = 10^{13}$–10^{15} eV.

Both the electrons, on the one hand, and the protons and nuclei of heavier elements, on the other, are subject to energy losses through Coulomb collisions in a plasma. For relativistic particles the Coulomb loss is slight in the space medium and does not play an important role.[75] The losses may prove important for particles with an energy of the order of the thermal energy of plasma. If the particles are injected into the acceleration process directly from the thermal distribution behind the shock front, the velocity of the injected particles will be $\gtrsim u_1$, which is larger by a factor of $\mathbf{Ma}_1$ then the thermal velocity of the ions in the region ahead of the shock front. Consequently, in the case of strong shock wave ($\mathbf{Ma}_1 \gg 1$) the time scale of the energy loss, due primarily to collisions with electrons, can be found from[76]

$$\tau_q = \frac{3T_1^{3/2}}{8\sqrt{2\pi}} \frac{m_i}{m_e^{1/2} Z_i^2 e^4 N_1 L},$$

where e and m_e are the charge and mass of the electron, T_1 and N_1 are the temperature and density of the plasma, L is the Coulomb logarithm, and m_i is the mass of the particle. Using the values $T_1 = 5 \cdot 10^{-11}$ erg ($3 \cdot 10^5$ K) and $N_1 = 3 \cdot 10^{-3}$ cm^{-3} for the interstellar plasma—this so-called corona phase fills 70% of the volume of the galactic disk[77]—we find $\varkappa_1 < u_1^2 \cdot 10^{12}$ cm^2/s for protons from the condition $\tau_a < \tau_q$. Using $u_1 = \mathbf{Ma}_1 c_s$, and taking the sound velocity to be $c_s \approx 200$ km/s, we see that this relation holds for essentially any Mach number, since in the galactic disk[2] we would have $\varkappa \sim 10^{27}$ cm^2/s even at an energy ~ 1 GeV. It is not difficult to show that incorporating the Coulomb loss in the region behind the front of a strong shock would not introduce any further limitations, especially since the perturbed nature of the medium would apparently lead to a relation $\varkappa_2 \ll \varkappa_1$.

The Coulomb energy loss is even smaller in the interplanetary plasma, where we would have[1] $T = 10^5$ K, $N = 7$ cm^{-3}, and $c_s = 50$ km/s; the diffusion coefficient at energies $\lesssim 1$ MeV would be $\varkappa \lesssim 10^{21}$ cm^2/s.

We thus see that in a collisionless space plasma there is the possibility of an injectionless regime of Fermi acceleration: The absence of any significant energy loss would make it possible to accelerate particles directly from thermal energies, without the need for a preliminary acceleration of these particles.

2.3. Acceleration of particles by spherical shock waves

The results found in a study of Fermi acceleration by a plane shock wave have a limited range of applicability. Under actual conditions, the shock waves are not plane waves; they instead have complex spatial and temporal characteristics. It thus becomes necessary to study how the acceleration process is influenced by the nonzero dimensions, the curvature of the shock front, and the actual law of motion of the front. There is yet another important circumstance, which distinguishes the real wave from a plane wave in a fundamental way: Shock waves in space are formed primarily either as the result of processes of an explosive nature at stars (supernova explosions, chromospheric flares, etc.) or as the result of the slowing of supersonic streams of plasma issuing from the surfaces of stars (stellar winds). Consequently, either the region ahead of the shock front or behind the front will consist of diverging plasma streams. The expansion of the plasma ($\nabla u > 0$) leads to an adiabatic slowing of the particles in accordance with the law (2.3), and this effect may also have a strong influence on the spatial distribution and spectrum of the accelerated particles.

When we move up to the three-dimensional case, the computational difficulties increase inordinately. For this reason, all the basic results which have been established to date refer to the spherically symmetric case, in which the transport equation (2.1) can be written in the form

$$\frac{\partial f}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\varkappa r^2 \frac{\partial f}{\partial r}\right) - u\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u)\frac{p}{3}\frac{\partial f}{\partial p} + Q, \tag{2.20}$$

under the assumption that f, $\varkappa$, u, Q depend on only the single spatial coordinate r. In this case the boundary conditions at the shock front (2.4), take the form

$$f_1 = f_2, \quad \varkappa_1 \frac{\partial f_1}{\partial r} + \frac{u_1}{3} p \frac{\partial f_1}{\partial p}$$
$$= \varkappa_2 \frac{\partial f_2}{\partial r} + \frac{u_2}{3} p \frac{\partial f_2}{\partial p} + \mathrm{sign}\,[u(R-0) - u(R+0)]\,Q_0, \tag{2.21}$$

where $Q_0(r-R)$ is the part of the source Q which is lumped in the region $r = R$.

2.3.1. Standing shock wave. The acceleration of charged particles by a standing spherical shock wave is of research interest because stellar winds may undergo a shock transition as a result of an interaction with the interstellar medium.[52,78] This question is particularly important because the sun is a star which has a wind. The plasma velocity u can be written as a function of the distance to the star, r, in the following way[52,78]:

$$u(r < R) = u_1, \quad u(r > R) = u_2\left(\frac{R}{r}\right)^2, \tag{2.22}$$

where $u_2 = u_1/\sigma$. An analytic solution of the steady-state transport equation can be derived only in certain individual cases, with a special choice of the diffusion coefficient[79–81] $\varkappa(r, p)$. The basic features of this problem can be analyzed by specifying a cosmic-ray diffusion coefficient

$$\varkappa(r < R) = \varkappa_1\left(\frac{r}{R}\right), \quad \varkappa(r > R) = \varkappa_2\left(\frac{r}{R}\right). \tag{2.23}$$

This approach makes it possible to solve Eqs. (2.20) and (2.21) and, without any particular difficulty, to find the spectrum of accelerated particles with momenta $p > p_0$, i.e., with momenta above that (p_0) of the particles injected at the shock front:

$$f_2(r, p) = f_1(R, p)\left[1 - \exp\left(-\frac{g_2}{2}\frac{R^2}{r^2}\right)\right](1-\beta)^{-1}, \tag{2.24}$$

$$f_1(r, p) = A\left(\frac{r}{R}\right)^{-s}\left(\frac{p}{p_0}\right)^{-q}, \tag{2.25}$$

where

$$q = \frac{3\sigma}{\sigma-1}\left[1 + \frac{2}{(\sigma-1)g_1} + \frac{\sigma}{(\sigma-1)^2}\frac{\beta}{1-\beta}\right.$$
$$+ \left[\frac{1}{2} + \frac{1}{(\sigma-1)g_1}\right]$$
$$\left.\times\left\{\left[1 + \frac{8\beta\sigma^2}{(1-\beta)(\sigma-1)[2+(\sigma-1)g_1]^2}\right]^{1/2} - 1\right\}\right],$$

$$s = \frac{2}{\sigma-1} + g_1 + \frac{2+(\sigma-1)g_1}{2(\sigma-1)}$$
$$\times\left[\left\{1 + \frac{8\beta\sigma^2}{(1-\beta)(\sigma-1)[2+(\sigma-1)g_1]^2}\right\}^{1/2} - 1\right],$$

$\beta = \exp(-g_2/2)$, $g_{1,2} = u_{1,2}R/\varkappa_{1,2}$, and the amplitude ($A$) of the spectrum of accelerated particles is determined by the rate of their injection.

The primary feature of the solution which is found—a feature which is common to problems involving the acceleration of cosmic rays by shock waves with nonzero dimensions—is the fact that both the spatial distribution of the accelerated particles and the shape of their spectrum are determined by the values of the dimensionless parameters $g_{1,2}$. The physical meaning of these quantities follows from the circumstance that they primarily determine the extent at which the moving scattering medium, with length scale R, influences the spatial distribution of the cosmic rays. Specifically, if $g \ll 1$, the influence is slight, but if $g \gg 1$ it is governing. For this reason, the quantity g is called the "modulation parameter."

The circumstance that the dimensions of the shock wave are nonzero, which results in a further escape of particles from the vicinity of the shock front, and also the adiabatic slowing in the region $r < R$ reduce the acceleration efficiency. This reduction is reflected in a steepening of the spectrum of the accelerated particles. The role played by these two factors is determined by the magnitudes of the modulation parameters g_1 and g_2, respectively. Regardless of the values of these parameters, the spectrum of accelerated particles is steeper than in the plane-wave case. As the parameter g_1 increases, the role played by the slowing decreases. The reason is that at $g_1 \gg 1$ the length scale of the diffusive penetration of particles into the region $r < R$ is small: $L = \varkappa_1/u_1 \ll R$. The rate of adiabatic slowing here is $\tau_{ad} = |\nabla \mathbf{u}/3|^{-1} \approx 3R/2u_1$, according to (2.3). In order to compare this rate with the acceleration rate we need to allow for the circumstance that the particles spend only a part $\tau_{a1} = 3\varkappa_1/\Delta u u_1$ of the acceleration time τ_a [see (2.19)] in region 1. Incorporating the factor τ_{a1}/τ_a leads to $\tau_{ad}^{-1} \approx [2\varkappa_1/(\sigma-1)Ru_1]\tau_a^{-1}$, which is considerably lower than the acceleration rate τ_a^{-1} if $\varkappa_1 \ll Ru_1$.

At $g_1 \gg 1$, the difference between the exponent in the accelerated-particle spectrum,

$$q = \frac{3\sigma}{\sigma-1}\left[1 + \frac{\sigma\beta}{(\sigma-1)^2(1-\beta)}\right],$$

and that in the plane-wave case results from exclusively the nonzero dimensions of the shock wave. It can be seen that this effect can be described correctly at the qualitative level in the one-dimensional problem with an absorbing boundary [see (2.10)]. If both of the parameters g_1 and g_2 have large values, the exponent of the accelerated-particle spectrum, q, becomes approximately the same as that in the plane-wave case, $3\sigma/(\sigma-1)$.

The way in which the shape of the particle spectrum and the spatial distribution of the particles depend on the modulation parameters is illustrated by Fig. 3, which shows the distribution function $f(r,p)$ as a function of the particle momentum p/p_0 for several values of the distance r/R and for values of 0.1 and 10 of the modulation parameters $g_1 = g_2$. This figure clearly demonstrates the decrease in the relative number of slowed particles ($p < p_0$) with increasing $g_{1,2}$ and the tendency of the shape of the accelerated-particle spectrum ($P > p_0$) toward a plane-wave limit, $f \sim p^{-3\sigma/(\sigma-1)}$.

Although the applied value of solution (2.24), (2.25) is limited because of the particular choice of diffusion coefficients in (2.23), it still gives a clear picture of the basic aspects of the process of Fermi acceleration by a shock wave of nonzero dimensions.

2.3.2. Traveling shock wave. The primary reason for the interest in the acceleration of particles at the front of a traveling shock wave is that the shock waves generated in supernova explosions are regarded as a probable source of galactic cosmic rays.[2,42,43]

The important time variation makes this problem so complicated that it is not possible to derive an exact solution of transport equation (2.20). However, the basic features of the Fermi acceleration of particles by a traveling shock wave can be studied by working from approximate solutions found for wave expansion laws $R \sim t^{2/5}$ (Refs. 82 and 83) and $R \sim t^{1/2}$ (Ref. 63). We will treat this problem here by the method of Refs. 84 and 85; although that method yields only an approximate solution, it can be used for any expansion law $R(t)$. This method can be summarized by saying that the solution of transport equation (2.20) in the region ahead of the shock front, $f_1(r,p,t)$, and that behind the shock front, $f_2(r,p,t)$, are expressed in terms of the particle distribution function at the shock front, $f_R(p,t) \equiv f_{1,2}(R,p,t)$. This distribution function can then be found from boundary condition (2.21). It becomes a particularly simple matter to derive a solution in the region $r < R$ behind the shock front, if we make the customary assumption that the particle diffusion coefficient $\varkappa_2$ is small. In real situations, this coefficient may be small because the medium is highly perturbed in this region. If we assume $\varkappa_2 \ll R\dot{R}$, where $\dot{R} \equiv dR/dt$ is the velocity of the shock front, we can ignore the diffusion term in transport equation (2.21):

$$\frac{\partial f_2}{\partial t} = \frac{\nabla \mathbf{u}}{3} p \frac{\partial f_2}{\partial p} - u \frac{\partial f_2}{\partial r}.$$

It is not difficult to see that a solution of this equation can be written in the form

$$f_2(r, p, t) = f_R(t_R, ap), \tag{2.26}$$

where the factor

$$a = \exp \int_{t_R}^{t} \frac{\nabla \mathbf{u}}{3} \, dt \tag{2.27}$$

reflects the adiabatic change in the energy of the particles, and $s(t)$ is the solution of the equation $ds/dt = u(s,t)$ with the boundary conditions $s(t_R) = R(t_R)$, $s(t) = r$.

In the external region, $r > R$, where we have $u = 0$, the solution of the transport equation

$$\frac{\partial f_1}{\partial t} = \frac{\varkappa_1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f_1}{\partial r} \right)$$

with the boundary condition $f_1(R, p, t) = f_R(p, t)$ can be written in the form

$$f_1(r, p, t) = \frac{1}{4\sqrt{\pi}\, r} \int_0^t \frac{r - R(t')}{[\varkappa_1 (t - t')]^{3/2}} \times \exp \left\{ - \frac{[r - R(t')]^2}{4\varkappa_1 (t - t')} \right\} \mu(t') \, dt', \tag{2.28}$$

where the function $\mu(t)$ is the solution of the integral equation[86]

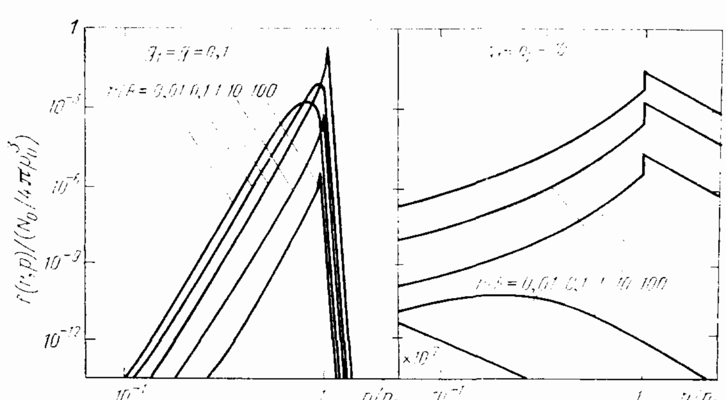
$$\frac{\mu(t)}{2\varkappa_1} + \frac{1}{4\sqrt{\pi}} \int_0^t \frac{R(t) - R(t')}{[\varkappa_1 (t - t')]^{3/2}} \times \exp \left\{ - \frac{[R(t) - R(t')]^2}{4\varkappa_1 (t - t')} \right\} \mu(t') \, dt' = \dot{R}(t) f_R(t). \tag{2.29}$$

Using the method of steepest descent, we can easily solve this equation for large values of the modulation parameter $g_1 = R\dot{R}/\varkappa_1$:

$$\mu(t) = \varkappa_1 \left(1 + \frac{b+1}{g_1} \right) \dot{R}(t) f_R(t) + O\left(\frac{1}{g_1^2} \right), \tag{2.30}$$

where $b = d\ln f_R / d\ln R$. We can then use the same method to carry out the integration in expression (2.28). As a result, we find, in the case $g_1 \gg 1$,

$$f_1(r, p, t) = f_R(p, t) \frac{R}{r} \times \exp \left\{ - g_1 \frac{r - R}{R} \left[1 + \frac{b + 1 - (\nu - 1)\nu^{-1}}{g_1} \right] \right\}, \tag{2.31}$$



a b

FIG. 3. Spectra of particles accelerated by a standing spherical shock wave ($\sigma = 4$) at various distances r in the cases of (a) weak and (b) strong modulation.

where $\nu = \mathrm{d}\ln R/\mathrm{d}\ln t$. Substitution of expressions (2.26) and (2.31) into boundary condition (2.21) yields an equation for $f_R(p, t)$:

$$p\frac{\partial f_R}{\partial p}\left(\frac{\Delta u}{3}-\frac{\nabla \mathbf{u}}{3g_2}R\right)$$

$$-u_1\left[1+\frac{b+2-(\nu-1)\nu^{-1}}{g_1}+\frac{b\nu}{g_2}\right]f_R=Q_0,$$

where $g_2 = Ru_2/\varkappa_2$, $u_2 = u_1/\sigma$, $u_1 = \dot{R}$ and $\Delta u = u_1 - u_2$. As before, we restrict the analysis to the case of a monoenergetic function $Q_0 = u_1N_0\delta(p-p_0)/4\pi p^2$. We then find the following solution of this equation, which holds within terms $\sim 1/g$:

$$f_R(p,\ t)=\frac{N_0(t)\,q}{4\pi p_0^3}\left(\frac{p}{p_0}\right)^{-q}\theta(p-p_0),$$
$$q=\frac{3\sigma}{\sigma-1}\left[1+\frac{d+2-(\nu-1)\nu^{-1}}{g_1}+\frac{d\nu+\nabla\mathbf{u}(R)(R/\Delta u)}{g_2}\right], \tag{2.32}$$

where the parameter $d = \mathrm{d}\ln N_0/\mathrm{d}\ln R$ determines the time dependence of the strength of the particle source.

Expressions (2.26), (2.31) and (2.32) thus show that the distribution near the shock front of the accelerated particles for which the modulation parameter is large ($g_{1,2} \gg 1$) is the same as that for a plane wave. If the particle diffusion coefficient depends on the momentum $\varkappa(p) = \varkappa_0(p/p_0)^\alpha$, $a>0$, $g_{1,2}(p_0) \gg 1$, the condition $g_{1,2}(p) \gtrsim 1$ corresponds to the region $p_0 \leqslant p \lesssim p_m(t)$, where the quantity $p_m(t)$ is determined from the relation $g_1(p_m) = 1$ if we assume $g_2(p) > g_1(p)$ ($\varkappa_2 < \varkappa_1$). It can be seen from expression (2.32) that in the case of a constant injection rate ($N_0 = \mathrm{const}$, $d = 0$) the exponent q increases significantly with increasing momentum p near the point $p = p_m$. In other words, the spectrum steepens. The quantity p_m thus represents a maximum momentum of the particles which have been accelerated at the given time t. If, on the other hand, the injection rate falls off quite rapidly with the time $N_0 \sim R^d$, $d < -2 + (\nu-1)\nu^{-1}$, then near the momentum p_m we will observe not a steepening but, on the contrary, a flattening of the spectrum, as can be seen from (2.32). The explanation for this effect runs as follows: Those particles which have reached a momentum $p>p_0$ by the time t are injected at the time $t' = t - \Delta t$; this time is earlier than t by an amount Δt which is of the order of the acceleration time t_a [see (2.18)]. Since the number of particles which are injected over the time t_a is $N = 4\pi R^2(t)Q_0(t)t_a$, and since the acceleration time t_a is an increasing function of the momentum, the accelerated-particle spectrum will flatten out if $N(t)$ is a decreasing function of the time. The sign of the derivative $\mathrm{d}N/\mathrm{d}t$ is the same as the sign of the quantity $d + 2 - (\nu-1)\nu^{-1}$. It then follows that at $d > -2 + (\nu-1)\nu^{-1}$ the accelerated-particle spectrum will be steeper, and at $d < -2 + (\nu-1)\nu^{-1}$ flatter, than in the plane-wave case. At the same time, this result means that if the injection rate near the front falls off sufficiently rapidly we should expect a significant number of particles with momenta $p > p_m(t)$, which will have been accelerated at earlier times $t' < t$ under the condition $p_m(t') > p_m(t)$.

Let us examine in more detail the important case of an expansion

$$R(t)=R_0\left(\frac{t}{t_0}\right)^{2/5}, \tag{2.33}$$

which corresponds to the adiabatic stage of the evolution of the shock waves from supernova explosions.[32] The velocity of the medium behind the shock front ($r<R$) can be taken to be[40]

$$u(r,\ t)=\frac{\sigma-1}{\sigma}\frac{r}{R}\dot{R} \tag{2.34}$$

in this case. When the time dependence of the degree of compression is taken into account $\sigma = 4/(1+3/\mathbf{Ma}_1^2)$, $\mathbf{Ma}_1 = \dot{R}/c_s$, the expression which can be found for the particle distribution function behind the shock front from Eqs. (2.26), (2.27) and (2.32) is quite complicated. In the simpler case of large Mach numbers, $\mathbf{Ma}_1 \gg 1$, we find

$$f_2(r,\ p,\ t)=\frac{N_0q}{4\pi p_0^3}\left(\frac{p}{p_0}\right)^{-q}\left(\frac{r}{R}\right)^{\sigma(3+d)}\theta\left[p-\left(\frac{r}{R}\right)^{1-\sigma}p_0\right]$$
$$\times\,\theta\left\{p_m\left[t\left(\frac{r}{R}\right)^{5/2\sigma}\right]-p\right\}, \tag{2.35}$$

where

$$p_m(t)=p_0\left(\frac{2}{5}\frac{R_0^2}{\varkappa_{10}}\right)^{1/\alpha}\left(\frac{t}{t_0}\right)^{-1/5\alpha}, \tag{2.36}$$

if we assume that the diffusion coefficient has a momentum dependence $\varkappa_1(p) = \varkappa_{10}(p/p_0)^\alpha$. At a constant injection rate ($d=0$), as can be seen from expression (2.35), the density of the accelerated particles falls off rapidly, in proportion to $(r/R)^{3\sigma}$, because of the adiabatic slowing. A situation of this sort may be realized if the injected particles are galactic cosmic rays with a uniform density N_0 in the region ahead of the shock front.[82,63]

If, on the other hand, the particles are injected into the acceleration process from a thermal distribution behind the shock front, then we would expect that the rate of their injection would fall off with the time ($d<0$), since the plasma temperature behind the front, $T_2 \sim \mathbf{Ma}_1^2$, is a decreasing function of the time. With $d = -3$, for example, the density of accelerated particles in the perturbed region does not depend on the distance r, as can be seen from expression (2.35). This case corresponds to the scaling solution of the transport equation which has been taken up previously.[83]

In the region ahead of the front, the distribution of particles with momenta $p_0 \leqslant p \lesssim p_m(t)$ is described by expressions (2.31) and (2.32). Since the magnitude of the momentum p_m falls off over time, according to (2.36), there will be a significant number of particles with momenta $p > p_m(t)$ ahead of the front, as we have already mentioned. Their distribution can be found approximately by using expressions (2.28)–(2.30). Here it is convenient to break up the integral over t' in expression (2.28) into two parts, from t_0 to t_p and from t_p to t, where the time t_p is determined by the relation $\varkappa_1(p) = R(t_p)\dot{R}(t_p)$. We will then solve the equation for $f_1(r, p, t)$ by an iterative method, finding[84]

$$f_1(r,\ p,\ t)\approx\frac{A}{4(\pi\varkappa_1)^{1/2}t^{3/2}}\exp\left(-\frac{r^2}{4\varkappa_1 t}\right), \tag{2.37}$$

where

$$A=\int\limits_{t_0}^{t_p} f_R(p,\ t')\,\mathrm{d}t',$$

and f_R is given by (2.32). We thus see that particles with momenta $p > p_m(t)$ fill a region with a length scale R_p

$\approx (\varkappa_1 t)^{1/2}$, which increases more rapidly then the size of the shock wave: $R(t) \sim t^{2/5}$.

The presence of runaway accelerated particles—which lead the shock front—is an important feature, which distinguishes the process of acceleration by a traveling wave from the plane case. These particles may carry off a significant fraction of the energy, so they may be an important factor influencing the structure and dynamics of the shock wave.

2.4. Acceleration of particles by an ensemble of shock waves

The generation of particles of the highest energies is of particular interest in connection with the problem of the origin of the galactic cosmic rays. As was shown above, the particles which are accelerated by an individual shock wave of nonzero dimensions begin to escape from the vicinity of the shock front at a rapid pace once a certain maximum momentum p_m has been attained. If there is a certain number of other waves in the system at this time, these particles may, as they interact with these other waves, continue to acquire energy. Such a process of repeated acceleration is possible if the diffusion coefficient is small ($g_2 \gg 1$) behind the front of each shock wave, as before.[83,87,88]

Working from very general considerations we can show that the spectrum of the particles accelerated in this manner will be the same as at the front of an individual shock wave. As was shown in §1, the universal shape of the spectrum of accelerated particles is a consequence of two conditions: 1) When the front is intersected twice, the increment in the momentum of particle is $\Delta p = (4/3)(\Delta u/v)p$. 2) The probability that a particle will return to the front from the region ahead of the front is $P_1 = 1$, while the probability that it will return from the region behind the front is $P_2 = 1 - (4u_2/v)$. It is totally irrelevant whether the particle interacts with the same front each time or with different fronts. Both these conditions will hold if the relation $g_2 \gg 1$ holds and also if the characteristic time (τ_e) which the particles spend in the volume V containing the shock waves is much longer than the acceleration time τ_a. To see which factors determine the characteristic acceleration time in this case, we take a more detailed look at this process.

For definiteness we assume that the shock waves expand in accordance with the scaling law (2.33). If we further assume that when a certain maximum dimension R_m is reached the shock waves dissipate (more precisely, they cease to have any effect on the cosmic rays), and new shock waves form in their place, we can assume that the number of shock waves in the system, N_S, remains constant. It is a simple matter to construct a steady-state equation for the density of accelerated particles, n, averaged over space. For this purpose we multiply transport equation (2.1) term by term by $4\pi p^2$ and integrate over the entire volume of the system (V). In the process we allow for the circumstance that the distribution of cosmic rays between shock waves is approximately uniform by virtue of the conditions assumed here ($g_1 \ll 1, g_2 \gg 1$) and that this distribution is described in the perturbed region by expression (2.35) with $d = 0$:

$$\frac{1}{\tau_a}\frac{dpn}{dp} + \frac{n}{\tau} + \frac{n}{\tau_e} = 0. \tag{2.38}$$

Here τ_e is the average time spent by the particles in the system. This time is limited by the escape of these particles by diffusion across the boundary of volume V. Here also,

$$\tau_a = V\left[\sum_{k=1}^{N_s} 4\pi R_k^2 \dot{R}_k \frac{\sigma_k - 1}{3(\sigma_k + 1)}\right]^{-1} \tag{2.39}$$

is the characteristic acceleration time due to the collective effect of all N_S shock waves; $\tau = V\left[\sum_{k=1}^{N_s} 4\pi R_k^2 \dot{R}_k/(\sigma_k + 1)\right]^{-1}$; and for simplicity we have ignored possible effects of an intersection of shock waves.

A solution of Eq. (2.39) is the power spectrum $n \sim p^{-\gamma}$. Here we would have an exponent $\gamma = (\tau_a/\tau) + (\tau_a/\tau_e)$ if all the shock waves were identically strong (σ_k σ), and we would have

$$\gamma = (\sigma + 2)(\sigma - 1)^{-1} + \tau_a \tau_e^{-1} \tag{2.40}$$

for the momentum region where we have $\tau_a < \tau_e$—the same as for the spectrum generated by an individual shock wave.

As the modulation parameter g_2 is reduced, the distribution of particles in the interior regions of the shock waves becomes progressively more nearly uniform, in contrast with (2.35). It is not difficult to see that this tendency leads to a relative increase in the role of adiabatic slowing and thus a decrease in the average acceleration rate. In the limiting case $g_2 = 0$, for example, we have $\langle \nabla u f \rangle = 0$, and there will be no acceleration at all. Since there is also a slight modulation of the distribution of particles in the interior region of a shock wave in the case $g_2 \ll 1$, there will also be some acceleration effect in this case. The effect will be essentially the same as that studied in Refs. 89–91, where the acceleration of particles by a supersonic turbulence was examined. For that acceleration process the acceleration rate $1/\tau_a$ is a small quantity of second order in the parameter $\dot{R}/v$ and significantly lower in the case under discussion here—specifically because of the weak modulation.

The maximum momentum of the particles accelerated by the collective mechanism,

$$p_m = \min\{p_{m1}, p_{m2}\}, \quad g_2(p_{m1}) = 1, \quad \tau_e(p_{m2}) = \tau_a, \tag{2.41}$$

is set by either an increase in the rate at which particles escape from the system (a decrease in the time spent in the system, τ_e, with increasing momentum) or a weakening of the modulation by the shock waves of particles with large momenta. A collective acceleration of particles will be important in those cases in which this value of p_m turns out to be greater than the maximum momentum of the particles generated by an individual shock wave.

3. NONLINEAR MODELS OF FERMI ACCELERATION

The hard spectrum of the particles accelerated by a strong shock wave is the reason why the energy calculated in the linear approximation,

$$E = 4\pi \int_{p_0}^{p_m} \varepsilon p^2 f \, dp$$

(ε is the kinetic energy of the particle), which is an increasing function of the cutoff momentum p_m, may formally exceed the total internal energy of the plasma. This circumstance corresponds to the fact that the accelerated particles, which have an anisotropic distribution in the region ahead of the shock front may excite MHD waves. The effect would be to increase the scattering properties of the medium and ultimately to lead to an increase in the maximum momentum p_m. All these arguments indicate that a systematic theory for Fermi acceleration should be definitely nonlinear. It should include the solution of the problem of a self-consistent plasma turbulence, and it should incorporate the modification of the structure of the shock wave by the pressure of the accelerated particles. This modification will in turn influence the acceleration process itself. No less important is the question of the rate at which plasma particles are injected into the acceleration process.

At present we are far from having a complete systematic solution of these problems, because of their complexity. The progress which has been made in this direction has resulted primarily from the development of models which incorporate one aspect or another of this nonlinear problem at a semiphenomenological level. Nevertheless, these models have been useful in obtaining several important results of fundamental value.

3.1. MHD structure of a shock wave in a gas with cosmic rays

The MHD description of the structure of a shock wave incorporating the effect of accelerated particles is based on the system of ordinary MHD equations which include, along with the quantities characterizing the state of the thermal plasma (the gas), macroscopic characteristics of the accelerated particles (the cosmic rays), specifically, the pressure

$$P_c = \frac{4\pi}{3}\int_0^\infty p^3 v f \, dp,$$

the energy flux

$$F_{c\alpha} = \frac{4\pi}{3}\int_0^\infty \varepsilon v f_\alpha p^2 \, dp,$$

and the pressure (P_w) and the energy flux (F_w) of the plasma turbulence which may be generated by cosmic-ray particles near the shock front. In the one-dimensional case of a plane shock wave which is propagating along the field lines of a regular magnetic field B, opposite the x axis, the MHD equations in the rest frame of the shock front are[30,92-96]

$$\begin{aligned} &\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0,\\ &\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x}(\rho u^2 + P_g + P_c + P_w) = 0,\\ &\frac{\partial}{\partial t}\left(\frac{\rho u^2}{2} + E_g + E_c + E_w\right)\\ &\quad + \frac{\partial}{\partial x}\left[\rho u\left(\frac{u^2}{2} + \frac{\gamma_g}{\gamma_g - 1}\frac{P_g}{\rho}\right) + F_c + F_w\right] = 0, \end{aligned} \tag{3.1}$$

where ρ, u, p_g, and γ_g are the density, velocity, pressure, and adiabatic index of the gas; and E_g, E_c, and E_w are the internal-energy density of the gas, that of the cosmic rays, and that of the turbulence.

Equations relating the pressure and the energy flux of the cosmic rays are found through a term-by-term integration of transport equation (2.1) over $4\pi\varepsilon p^2 dp$. Here we use expression (2.2) for the first moment of the distribution function:

$$\begin{aligned} &\frac{\partial}{\partial t}\left(\frac{P_c}{\gamma_c - 1}\right) + \frac{\partial F_c}{\partial x} = w\frac{\partial P_c}{\partial x} - \bar{Q},\\ &F_c = \frac{\gamma_c}{\gamma_c - 1} w P_c - \frac{\bar{\varkappa}}{\gamma_c - 1}\frac{\partial}{\partial x}P_c, \end{aligned} \tag{3.2}$$

where

$$\bar{Q} = 4\pi\int_0^\infty \varepsilon Q p^2 \, dp$$

is the density of the energy source of the cosmic rays; the effective diffusion coefficient of the cosmic rays is given by

$$\bar{\varkappa} = \int_0^\infty dp\, p^3 v \varkappa \frac{\partial f}{\partial x}\left(\int_0^\infty dp\, p^3 v \frac{\partial f}{\partial x}\right)^{-1}$$

and is assumed in this theory to be a given positive-definite constant; and w is the velocity of the scattering centers, which has previously been assumed equal to the plasma velocity u in all cases. If the primary type of plasma turbulence is one consisting of Alfvén waves, which are excited as the result of an anisotropic distribution of cosmic rays ahead of the shock front,[31,32] the velocity of the scattering centers will instead be[97] $u - c_a$. Equations for the pressure $P_w = \delta B^2/8\pi$ and the energy flux of the Alfvén waves have been found in the quasilinear theory[94,95,98,99]:

$$\begin{aligned} &2\frac{\partial P_w}{\partial t} - u\frac{\partial P_w}{\partial x} + \frac{\partial F_w}{\partial x} = u\frac{\partial P_c}{\partial x} - \bar{L},\\ &F_w = P_w(3u - 2c_a), \end{aligned} \tag{3.3}$$

where δB is the amplitude of the Alfvén waves, and $\bar{L}$ is the density of the energy sink which describes the damping of the Alfvén waves.

The comparatively simple form of MHD equations (3.1)–(3.3) is a consequence of the assumption that the particles of the gas and the cosmic rays have quite distinct energies. Specifically because the cosmic-ray particles have energies much higher than the thermal particles we can ignore their contribution to the density ρ. For the same reason, even if the injection of the cosmic rays is from the thermal plasma, the energy source $\bar{Q}$ is usually ignored. In the steady state, Eqs. (3.1) reflect the conservation of the fluxes of mass, momentum, and energy, respectively, and are actually a generalization of the Rankine-Hugoniot relations:

$$\begin{aligned} &\rho u = \text{const},\\ &\rho u^2 + P_g + P_c + P_w = \text{const},\\ &\rho u\left(\frac{u^2}{2} + \frac{\gamma_g}{\gamma_g - 1}\frac{P_g}{\rho}\right) + F_c + F_w = \text{const}. \end{aligned} \tag{3.4}$$

The limiting case in which the plasma ahead of the front is cold $[P_{g1} \equiv P_g(-\infty) \ll P_{c1}]$ is a particularly graphic case for tracing the nature of the modification of a shock wave by cosmic rays. If we ignore the dynamics of the Alfvén turbulence, it turns out that we can derive a steady-state solution of Eqs. (3.1) and (3.2) in analytic form. It can be shown that these equations reduce to an equation for the velocity:

$$\bar{\varkappa}\frac{\partial u}{\partial x} = \frac{\gamma_c + 1}{2}(u - u_1)(u - u_2). \tag{3.5}$$

The solution of this equation,[93]

$$u = \frac{u_1 + u_2}{2} - \frac{u_1 - u_2}{2}\,\text{th}\,\frac{(\gamma_c + 1)(u_1 - u_2)x}{4\bar{\varkappa}} \tag{3.6}$$

describes a smooth transition from the value $u(-\infty) = u_1$ to $u(\infty) = u_2 = u_1/\sigma$ over a length scale $L \sim \tilde{\varkappa}/u_1$, where

$$\sigma = \left[\frac{\gamma_c - 1}{\gamma_c + 1} + \frac{2}{\mathrm{Ma}_1^2(\gamma_c + 1)}\right]^{-1}$$

is the degree of compression of the matter at the shock front, and $\mathbf{Ma}_1 = \rho_1 u_1^2/\gamma_c P_c$ is the Mach number. All the internal energy of the plasma behind the shock front is that of the cosmic rays; $P_{c2} = P_2 = P_{c1}[1 + \mathbf{Ma}_1^2 \gamma_c (\sigma - 1)/\sigma]$.

In the general case of arbitrary values of the parameters of the medium ahead of the front (P_{g1}, P_{c1}, and $\mathbf{Ma}_1$),[92,93] and when we take the dynamics of the turbulent field δB into account,[94,95] we find that the structure of the shock wave is more complex. In addition to the region in which all the parameters vary smoothly over a length scale $L \sim \tilde{\varkappa}/u_1$ (the prefront), there is a thermal front. As in the ordinary hydrodynamic theory which ignores viscosity and thermal conductivity,[19,20] the thermal front is a discontinuity in the behavior of the gas parameters[1)] ρ, u, and P_g as functions of x. Their values on the two sides of the discontinuity are related by Rankine-Hugoniot relations, (3.4), in which the parameters of the cosmic rays are everywhere continuous. In contrast with (3.5), the equation for u to which system (3.1)–(3.3) reduces in the steady state is given in the general case by[94]

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{(u_1 - u)\,\Phi(u)}{\mathrm{d}P_c/\mathrm{d}u}, \qquad (3.7)$$

where the function Φ depends on the parameters of the medium ahead of the shock front: P_{g1}, P_{c1}, P_{w1}, and $\mathbf{Ma}_1$. Whether the structure of the shock wave has a discontinuity—a thermal front—depends on the relation between $u_\infty < u_1$—the root of the equation $\Phi(u) = 0$—and u_*, the value at which the cosmic-ray pressure reaches a maximum and at which the derivative $\mathrm{d}P_c/\mathrm{d}u$ vanishes. The meaning of the quantity u_* is also determined by the circumstance that the cosmic-ray pressure $P_c(u)$ satisfies the equation[94]

$$\frac{\mathrm{d}P_c}{\mathrm{d}u} = \frac{\rho_1 u_1}{u^2}(c_s^2 - u^2),$$

where

$$c_s = \left[\frac{\mathrm{d}}{\mathrm{d}\rho}(P_g + P_w)\right]^{1/2}$$

plays the role of the sound velocity when there is an Alfvén turbulence in the medium.[100] At the point u_*, where the pressure P_c reaches a maximum, the plasma velocity u is comparable to the local sound velocity. If $u_* < u_\infty$, there is no thermal front, and the shock wave has a smooth structure: a smooth transition from the value u_1 to $u_2 = u_\infty$ over a distance $L \sim \tilde{\varkappa}/u_1$. The gas flow behind the shock front remains supersonic: $u_2 > c_s$. In the case $u_* > u_\infty$, a thermal front necessarily appears in the structure of the shock wave; it is a jump in the function $u(x)$ at some point x_0 from the value $u_s > u_*$ to $u_2 < u_\infty$. Figure 4 shows schematic diagrams of the cases of smooth and mixed structures of the shock wave.

The relations between u_s and u_∞ and between u_s and u_2 depend on the parameters $K = P_{c1}(P_{c1} + P_{g1})^{-1}$ and $\mathbf{Ma}_1$ and on the nature of the function $\bar{L}$, which determines the damping of the Alfvén waves. An important point is that the function $\bar{L}$ actually determines the degree of validity of Eqs. (3.3): If the damping is slight, the amplitude of the Alfvén waves in the prefront region may reach large values $\delta B/B \gtrsim 1$. If so, the quasilinear theory used in deriving Eqs. (3.3) would become inapplicable.

Since the nonlinear theory of Alfvén waves is far from completion, it is interesting to note the studies of the MHD structure of a shock wave which have been carried out for the case of strong damping, in which the amplitude of the Alfvén waves is limited to a level $\delta B/B < 1$ and in which a balance between the generation rate and the damping rate, $\bar{L} = c_a \partial P_c/\partial x$, is struck at each point (in a case with shock waves which are not too strong, with Mach numbers $\mathbf{Ma}_1 \lesssim 10$, nonlinear Landau damping limits the growth of the turbulence at the level $\delta B/B \lesssim 1$; (Ref. 90). The work performed by the pressure gradient of the cosmic rays is expended in this case on heating the gas. Consequently, the gas may be heated substantially not only as it crosses the thermal front but also in the prefront region, where its state is described by the equation[94]

$$\frac{u\rho^{\gamma_g - 1}}{\gamma_g - 1}\frac{\mathrm{d}}{\mathrm{d}x}\frac{P_g}{\rho^{\gamma_g}} = \bar{L}.$$

The additional heating of the gas lowers the acceleration efficiency, i.e., lowers the fraction of the energy which is transferred to the cosmic-ray particles. Nevertheless, the acceleration efficiency remains quite high, as is shown by numerical solutions of Eqs. (3.1)–(3.3). This high efficiency can be seen in the example in Fig. 5, which shows results of calculations[95] of σ, $P_{g2}(P_{c1} + P_{g1} + \rho_1 u_1^2)^{-1}$, and

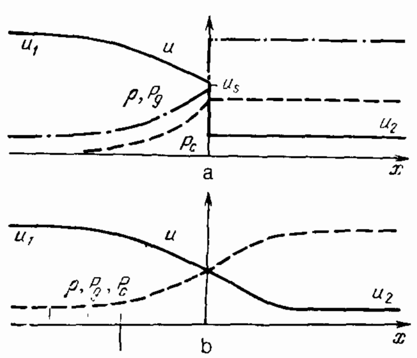


FIG. 4. Schematic diagrams of the cases of (a) a smooth and (b) a mixed structure of a shock wave modified by cosmic-ray pressure.

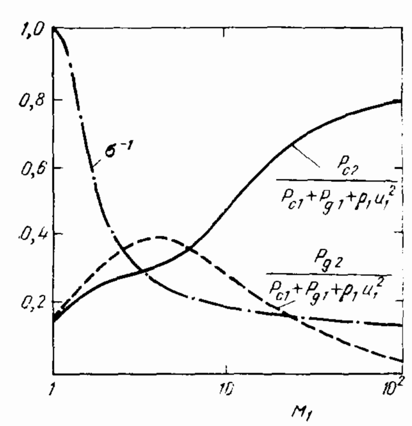


FIG. 5. The gas pressure P_{g2}, the cosmic-ray pressure P_{c2}, and the degree of compression σ behind a shock front as functions of the Mach number $\mathbf{Ma}_1$ (M_1) (Ref. 95).

$P_{c2}(P_{c1}+P_{g1}+\rho_1 u_1^2)^{-1}$ for the case with $K=0.5$ and $\beta \equiv 8\pi P_{g1}/B^2=1$. It can be seen from this figure that more than half of the pressure behind the shock front is that of accelerated particles when the Mach numbers are $\gtrsim 10$. Calculations were carried out for $\gamma = 4/3$, so the degree of compression, $\sigma = \rho_2/\rho_1$, approaches the limiting value $\sigma = 7$ at large Mach numbers.

The results of the calculations of Refs. 95 and 96, which are shown in Fig. 6 as plots of K as a function of **Ma**$_1$ for various values of the parameter β, show the particular values of the parameters K and **Ma**$_1$ for which there is a thermal front [the region under the k(**Ma**$_1$) curve] in the structure of the shock wave and for which values there is no such front [the region above the curve of k(**Ma**$_1$)]. With $K = 0.5$ and $\beta = 1$, for example, a solution with a thermal front prevails at essentially all Mach numbers under 50.

Analysis shows[102] that when viscosity effects are taken into account the discontinuous solutions of the system of MHD equations are found to be continuous solutions, in which the actual thickness of the thermal front is determined by the viscosity coefficients.

The MHD approach, which can be generalized to the case of a magnetic field of arbitrary orientation,[103] has thus revealed several important aspects of the acceleration of cosmic rays by shock waves. This approach has demonstrated that a substantial part (in some cases all) of the internal energy of the plasma in shock waves must be in the cosmic rays. This approach has also revealed features of the process by which Alfvén waves are excited and damped near a shock front and their effect on the acceleration efficiency.

On the other hand, the MHD approach has some drawbacks, as has been mentioned in the literature. This theory contains no information about the spectrum of the cosmic rays, $f(p)$. Furthermore—a particularly serious matter—it does not contain such an important parameter as the cutoff momentum p_m. This theory does answer the question of what fraction of the energy in a shock wave is carried by the cosmic rays, but it is of course incapable of resolving the question of whether this particular fraction would be possible at a specific finite value of p_m. The importance of this matter (which was pointed out, in particular, by Bulanov and Sokolov[104]) can be seen especially clearly in the following example. Magnetohydrodynamic calculations[92] show that at sufficiently large Mach numbers **Ma**$_1$ the pressure has a finite value at the shock front even if there are no cosmic rays ahead of the shock wave ($P_{c1}=0$). The meaning of this solution is that a vanishingly small number of cosmic rays may contain a finite amount of energy because of their hard spectrum, which stretches up to infinity. However, it is clear that a situation of this sort could not be realized at an arbitrarily large but finite cutoff momentum p_m. For this reason, the question of the efficiency of Fermi acceleration under the conditions prevailing in real systems, with a finite p_m, must be studied by approaches in which this quantity is explicitly present.

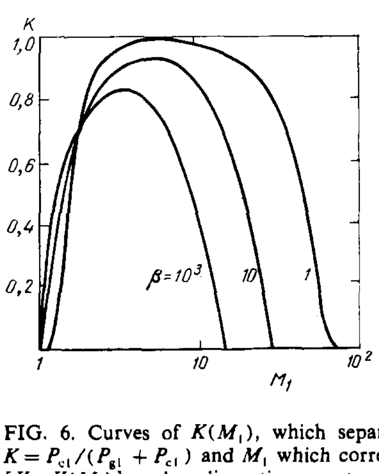


FIG. 6. Curves of $K(M_1)$, which separate regions in the plane of $K = P_{c1}/(P_{g1}+P_{c1})$ and M_1 which correspond to a smooth structure $[K > K(M_1)]$ and a discontinuous structure $[K < K(M_1)]$ of shock waves[95] (M_1 is the number **Ma**$_1$).

3.2. Kinetic model of Fermi acceleration

The collective nature of the processes by which particles are scattered in a collisionless plasma is the reason why the scattering of fast particles is a quasielastic process. The elementary event of scattering in the frame of reference moving with the scattering center occurs in an elastic manner. The physical reason for this circumstance is that the scattering center is a conglomerate of a large number of thermal particles, so the change in its energy in the course of the scattering is negligibly small.

In order to describe by a common approach not only the acceleration process but also the process in which the plasma particles are heated at the thermal front, followed by the injection of some fraction of these particles into the acceleration regime, we can assume that the nature of the motion of the thermal particles in a collisionless plasma is the same as that of the fast particles.[105-109] Although we could hardly claim anything approaching a rigorous basis for this assumption, it does allow an internally noncontradictory modeling of the process of Fermi acceleration in the case in which the injection occurs directly from the thermal distribution of particles, without a supplemental acceleration of these particles. For the sake of simplicity we can assume that the particle scattering events occur isotropically. In this case the propagation of plasma particles is described by the single parameter τ: the mean time between scattering events. This quantity is related directly to the level of plasma turbulence. Restricting the discussion (as before) to the case in which the regular magnetic field is perpendicular to the front, we can write a Boltzmann equation for the distribution of nonrelativistic particles ($v \ll c$) in the one-dimensional, steady-state case:

$$v\mu \frac{\partial f}{\partial x} = \mathrm{St}\, f, \tag{3.8}$$

where μ is the cosine of the angle between the particle velocity vector v and the x axis. Here we are ignoring possible charge-separation effects in the plasma, so the self-consistent structure of the shock wave is determined entirely by the pressure of the ions of the predominant species—protons in a space plasma. Using these assumptions regarding the nature of the scattering events, and ignoring the random motions of the scattering centers, which have a definite velocity $u(x)$ at each point x, we can write a collision integral in the form

$$\mathrm{St}\, f' = [\langle f' \rangle\, \theta(p'_m - p') - f']\, \tau^{-1}, \tag{3.9}$$

where the primes mean the local frame of reference, moving with the scattering centers, and the angle brackets mean an average over μ:

$$\langle F\rangle=\frac{1}{2}\int_{-1}^{1}F(\mu)\,d\mu.$$

Collision integral (3.9) shows that upon reaching the momentum p_m the particles leave the system. Transforming to the local frame of reference ($\mathbf{r}'=\mathbf{r}$, $\mathbf{v}'=\mathbf{v}-\mathbf{u}$) on the left side of Eq. (3.8), and dropping the primes, we find[105,106,112,113]

$$(v\mu+u)\left[\frac{\partial f}{\partial x}-\frac{du}{dx}\left(\mu\frac{\partial f}{\partial v}+\frac{1-\mu^2}{v}\frac{\partial f}{\partial\mu}\right)\right]$$

$$=[\langle f\rangle\theta(p_m-p)-f]\tau^{-1}. \qquad (3.10)$$

Collision integral (3.9) generally does not satisfy the necessary condition that the resultant momentum and the energy of the particles be conserved in each scattering event. It is not difficult to see that these conditions hold if[105,106]

$$\left\langle\mu\int_0^{p_m}\frac{fv}{\tau}p^2\,dp\right\rangle=0. \qquad (3.11)$$

This condition, as a supplement to kinetic equation (3.10), serves to find the self-consistent velocity profile $u(x)$.

Since the fast particles have a diffusion coefficient which increases with the energy,[1,2] $\varkappa=\tau v^{2/3}$, we will take a look at the case of a constant value of τ, which is the simplest case from the computational standpoint. In this case, self-consistency condition (3.11) is equivalent to the vanishing of the flux density of matter, j.

Multiplying Eq. (3.10) by $2\pi mp^2$, $2\pi(p\mu+m\mu)p^2$ and $\pi m[(v\mu+\mu)^2+v^2(1-\mu^2)]$ in succession; integrating over p, μ, and x, where m is the mass of the ions of the predominant species); and using (3.11), we find the integral relations

$$\rho u=\rho_1u_1-j_m, \qquad (3.12)$$

$$\rho u^2+P_{xx}=\rho_1u_1^2+P_1-q_m, \qquad (3.13)$$

$$\frac{\rho u^3}{2}+F+u(P_{xx}+E)=\frac{\rho_1u_1^3}{2}+\frac{\gamma_1}{\gamma_1-1}P_1-F_m, \qquad (3.14)$$

where

$$\rho=4\pi m\left\langle\int_0^\infty fp^2\,dp\right\rangle,$$

$$P_{xx}=2\pi\left\langle\mu^2\int_0^\infty fvp^3\,dp\right\rangle,$$

$$j=4\pi m\left\langle\mu\int_0^\infty fvp^2\,dp\right\rangle,$$

$$F=2\pi m\left\langle\mu\int_0^\infty fp^5\,dp\right\rangle$$

are the density of matter, a component of the pressure tensor, and the densities of the directed flux of matter and of energy, respectively. In addition,

$$j_m=\frac{4\pi m}{\tau}\int_{-\infty}^{x}dx\left\langle\int_{p_m}^{\infty}fp^2\,dp\right\rangle,$$

$$q_m=\frac{4\pi m}{\tau}\int_{-\infty}^{x}dx\left\langle\int_{p_m}^{\infty}(v\mu+u)fp^2\,dp\right\rangle,$$

$$F_m=\frac{4\pi m}{\tau}\int_{-\infty}^{x}dx\left\langle\int_{p_m}^{\infty}(v\mu+u)(v^2+2vu\mu+u^2)fp^2\,dp\right\rangle$$

are the fluxes of matter, momentum, and energy, respectively, which are carried out of the system by particles with a momentum $p>p_m$.

The adiabatic index γ in this nonrelativistic case must be set equal to 5/3. Since the thermal plasma and the accelerated particles are described by the same distribution function in this case, in contrast with the situation in a hydrodynamic description, we find that Eqs. (3.12)–(3.14) are the same as MHD equations (3.4). The only distinction is that in the MHD equations we use $j=0$ and $P_{xx}=P=P_g+P_c$, since the anisotropy of the thermal particles is ignored, the cosmic-ray pressure is assumed isotropic, and the contribution of the cosmic rays to the mass balance equation is assumed small because of the insignificant number of cosmic rays.

According to (3.12)–(3.14), we thus have a shock wave with emission. A model of this sort is completely adequate for treating the cases of real waves of finite dimensions, which we discussed above, and in which the accelerated particles may rapidly leave the vicinity of the shock wave once they attain a certain momentum p_m. We also note that Eqs. (3.12)–(3.14) are balance equations and are general in nature. Their validity is totally independent of the degree of validity of the model approach which we are outlining here and which is based on the use of a collision integral in the form in (3.9). Only the specific form of the quantities j_m, q_m, and F_m is an attribute of this model.

As in any wave with emission, the degree of compression σ is determined by not only the Mach number $\mathbf{Ma}_1$ but also the fluxes j_{m2}, q_{m2}, and F_{m2}. Setting $x=\infty$ in Eqs. (3.12)–(3.14), we find the following expression for the degree of compression $\sigma=u_1/u_2$ in the case of large cutoff momenta, $p_m\gg mu_1$, in which the mass flux j_m and the momentum flux q_m carried off by the particles can be ignored[107,110,111,85]:

$$\sigma=(\gamma_2+1)\left\{\gamma_2\left(1+\frac{1}{\gamma_1\mathrm{Ma}_1^2}\right)-\left[\gamma_2^2\left(1+\frac{1}{\gamma_1\mathrm{Ma}_1^2}\right)^2+(\gamma_2^2-1)\left(\frac{2F_{m2}}{\rho_1u_1^3}-1-\frac{2}{(\gamma_1-1)\mathrm{Ma}_1^2}\right)\right]^{1/2}\right\}^{-1}. \qquad (3.15)$$

This expression holds in the general case of arbitrary values of γ_1 and γ_2 by virtue of the general applicability of Eqs. (3.14), which we have just noted.

For a numerical study of the structure of the shock wave on the basis of Eq. (3.10), it is convenient to reduce this equation to the integral equation[112,113]

$$f(x, p, \mu) = \int_{-\infty}^{\infty} \frac{\exp[-(x-x')/(v\mu+u)\tau]}{|v\mu-u|\tau}$$

$$\times \theta\left(\frac{x-x'}{v\mu+u}\right) \theta(p_m - p_*) f(x, p_*) dx', \tag{3.16}$$

where $p_* = m(v^2 + 2v\Delta u\mu + \Delta u^2)^{1/2}$, $\Delta u = u(x) - u(x')$, and $f(x,p) = \langle f(x,p,\mu)\rangle$. The plasma velocity profile in this expression is found from Eq. (3.13):

$$u(x) = u_1 - [P_{xx}(x) - P_1](\rho_1 u_1)^{-1}, \tag{3.17}$$

where we are ignoring the momentum loss $q_m \sim \rho_1 u_1^2 (mu_1/p_m)$, which is small, $q_m \ll \rho_1 u_1^2$, if the cutoff momentum is sufficiently large: $p_m \gg mu_1$. Specifying the Mach number **Ma**$_1$, specifying a Maxwellian distribution function $f_1(p)$ corresponding to a pressure P_1 for the plasma particles ahead of the shock front, and solving Eq. (3.16) jointly with (3.15) and (3.17) by iterative method, we can find a self-consistent velocity profile $u(x)$ and a distribution function $f(x,p,\mu)$ (Refs. 112 and 113). A model similar in physical content has been implemented by the Monte Carlo method.[107-109,111,114,115]

Some characteristic features of the self-consistent structure of a shock wave are illustrated by Fig. 7, which shows results calculated[112,113,116] on the velocity profile $u(x)$ and the particle density $n_2(p) = 4\pi p^2 f_2(p)$ of the plasma behind the shock front of a wave with a fixed degree of compression $\sigma = 3.5$. This value is realized at a Mach number **Ma**$_1 = 4.58$ if we ignore the energy flux (F_{m2}) carried off by particles escaping from the system. As in the case of the MHD description, the structure of the shock wave is characterized by two length scales. The extended region of a smooth variation in the velocity $u(x)$, over a distance $L \sim \varkappa(p_m)/u_1$—the prefront—stems from the pressure of the fast accelerated particles with velocities $v > u_1$. These particles penetrate in a diffusive process into the incoming flow and slow it down.[105-122] The extent to which the wave is modified by the pressure of the fast particles, which might be characterized by the plasma velocity drop and the prefront $(u_1 - u_S)/u_1$, and by the length L, increases with increasing cutoff momentum p_m. This tendency prevents an extreme increase in the pressure of the accelerated particles and is a factor which regulates the amount of energy which is expended on the acceleration of these particles.

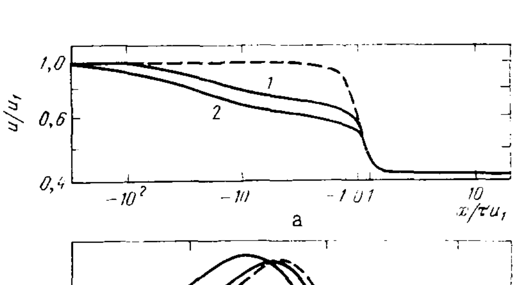


FIG. 7. a: Self-consistent profile of the plasma velocity in a shock wave, $u(x)$. b: Spectrum of plasma particles behind the shock front. 1—Cutoff momentum $p_m = 13\,mu_1$; 2—$p_m = 21\,mu_1$; Dashed lines—Non-self-consistent calculation of Z. The dashed line in part b corresponds to $p^{-2.2}$.

In addition to the prefront there is a region in which the velocity $u(x)$ varies sharply: a thermal front. In contrast with the MHD case, the thermal front here has a nonzero thickness $l \sim \tau u_1$. Most of the plasma particles are thermalized over this thickness.

The self-consistent spectrum of accelerated particles, $n_2(p)$, no longer has a universal shape over the entire momentum range $mu_1 \lesssim p \leqslant p_m$. The reason for this nature of the spectrum is that particles with momenta $p < p_m$ penetrate a distance $x(p) \approx \varkappa(p)/u_1 < L$ into the prefront region. They thus "feel" the velocity drop in the shock wave, $\Delta u = u(x(p)) - u_2$. Consequently, if the spectrum of accelerated particles is again written in a power-law form, $n \sim p^{-\gamma}$, the exponent will be a function of the momentum: $\gamma(p) = (\sigma_p + 2)/(\sigma_p - 1)$, where $\sigma_p = u(x(p))/u_2$ is the effective degree of compression for particles with momentum p. Only the very end of the spectrum ($p \approx p_m$) has a shape which is approximately universal.

The density of accelerated particles, $n_2(p)$, falls off with increasing cutoff momentum p_m. The energy density of these particles,

$$E_{c2} = \int_{mu_1}^{p_m} \varepsilon n_2(p)\, dp ,$$

remains approximately constant at about half the total internal energy of the plasma.

The distribution of impurity ions, whose contribution to the total pressure in the plasma is small because their density is small, can be found in the linear approximation with the velocity profile $u(x)$ determined by the pressure of the ions of the predominant species. Since the mean free path of the fast particles with respect to scattering, $\lambda = \tau v$, is an increasing function of the rigidity $R = p/Ze$ (Ze is the ion charge), ions with a large ratio A/Z (of the atomic weight A to the atomic number Z) will subsequently penetrate into the prefront region. As a result, their spectrum, plotted with the energy per nucleon, ε/A, as the independent variable, will become flatter. As a further consequence, as the shock wave propagates through a plasma in which the impurity ions are not fully ionized the spectrum of the accelerated particles will become enriched in the nuclei of heavy elements.[107,108,114,117] This is what is usually seen experimentally.

Since the amount of energy which is expended on the acceleration and thus the extent of the modification of the shock waves depend on the rate at which thermal particles are injected into the acceleration process, it is worthwhile to study this dependence. The reason is that the injection rate incorporated in this model (and according to which about 1% of the particles are accelerated) naturally does not reflect the entire variety of possibilities which may actually exist.

The extent to which a shock wave is modified by the accelerated particles (cosmic rays) can be studied as a function of the prespecified rate at which these particles are injected at the thermal front[85] by working from the diffusive transport equation for the cosmic rays, (2.1). The validity of this equation in this case results from the fairly large dimensions of the prefront.[118,105,106]

Solving this nonlinear problem becomes a substantially simpler process if the spatial distribution of the cosmic rays is described in the simplified form[110,123-125]:

$$f(x, p) = f_2(p)\,\theta\,[x + \bar{x}(p)], \tag{3.18}$$

where $\bar{x}(p) > 0$ is the distance over which cosmic rays with momentum p penetrate from the thermal front into the prefront region. The meaning of this approximation can be outlined as follows: At each point in the prefront $x < 0$, the dynamics of the medium is determined primarily by the pressure of the cosmic rays from a narrow interval of momenta near $p(x)$, in which we have $\bar{x}(p) \approx L(p) = |x|$. Specifically, cosmic rays with smaller momenta, for which the diffusion length is small ($L \ll |x|$), do not contribute to the cosmic-ray pressure at the point x. [Here we are assuming that the diffusion coefficient of the cosmic rays $\varkappa(p)$, and thus the diffusion length $L \approx \varkappa/u_1$ are increasing functions of the momentum.] The contribution of particles with large momenta, for which the relation $L(p) \gg |x|$, can also be ignored since the magnitude of the gradient of their pressure at the point x is far lower than that for particles with $L(p) = |x|$. The limiting expression of this "partitioning" is Eq. (3.18), which shows that at each point x in the prefront the cosmic-ray pressure gradient results from particles which have a definite momentum $p(x)$. Analysis shows that this approximation does not lead to any substantial errors if the diffusion coefficient of the cosmic rays increases rapidly with the energy.

If the cutoff energy ε_m is relativistic ($\varepsilon_m > mc^2$), the effective value of the adiabatic index γ_2 in expression (3.15) for the degree of compression σ is determined by the relation between the pressure of the relativistic particles, $P_{c2}^{(r)}$, and that of the nonrelativistic particles, $P_2 - P_{c2}^{(r)}$, behind the shock front:

$$\frac{\gamma_2}{\gamma_2 - 1} P_2 = \frac{5}{2} (P_2 - P_{c2}^{(r)}) + 4P_{c2}^{(r)}. \tag{3.19}$$

Here we have allowed for the circumstance that the adiabatic indices are 5/3 and 4/3 for the nonrelativistic and relativistic particles, respectively.

Since the diffusion approximation is valid for describing only sufficiently fast particles, the distribution function of the cosmic rays at some minimum energy ε_S must be joined with a thermal distribution. The joining procedure is equivalent to specifying the rate at which the particles are injected into the acceleration process at the thermal front. It is clear from general considerations that the energy of the injected particles, ε_S, should be a few times the characteristic thermal energy of the plasma particles behind the shock front. It can be taken to be mu_S^2, where u_S is that value of the plasma velocity which separates the prefront ($u_1 \geqslant u \geqslant u_S$) from the thermal front ($u_S \geqslant u \geqslant u_2$).

The steady-state, one-dimensional transport equation for the cosmic rays can be used jointly with integral relations (3.12)–(3.14) (for the prefront region, the pressure $P_{xx} = P = P_c + P_g$ can be assumed to be isotropic, while the pressure of the thermal plasma, P_g, can be assumed to vary in accordance with the adiabatic law), expression (3.15), and relations (3.18) and (3.19) to determine a self-consistent profile of the plasma velocity in the prefront [$u_1 \geqslant u(x) \geqslant u_S$], the spectrum of the cosmic rays, and the fraction of the internal energy in the shock wave which is the energy of the accelerated particles (P_{c2}/P_2) at given values of the Mach number $\mathbf{Ma}_1$, the velocity of the shock front (u_1), and the injection rate.

The interrelationship between the acceleration efficiency and the injection rate is illustrated by Fig. 8, which shows the pressure of the relativistic accelerated protons, $P_{c2}^{(r)}$, behind the front of a strong shock wave as a function of the pressure of the injected particles, $\mathscr{P}_{inj}$ [the partial pressure $\mathscr{P} = dP/d\lg\varepsilon$ is related to the distribution function by $\mathscr{P} = (4\pi/3)p^3\varepsilon f)$], for $\mathbf{Ma}_1 = 10$, $u_1 = 3\cdot10^8$ cm/s, and cutoff energies of 10^{12} and 10^{15} eV (Ref. 85). We see that the pressure $P_{c2}^{(r)}$ increases rapidly with increasing injection pressure, and at $\mathscr{P}_{inj} > 10^{-4}\rho_1 u_1^2$ the relativistic particles are responsible for a substantial fraction of the total pressure. There are accordingly grounds for speaking in terms of two injection regimes: a "saturation" regime and a "nonsaturation" regime. The saturation regime refers to the injection which results in the transfer of a substantial fraction ($\gtrsim 10\%$) of the energy in the shock wave to accelerated particles. In the case above, the saturation regime corresponds to a pressure of the injected particles which is greater than $\mathscr{P}_{inj}^* = 10^{-4}\rho_1 u_1^2$. Although the definition of $\mathscr{P}_{inj}^*$ is slightly arbitrary, the introduction of this quantity does make it possible to formulate in a quantitative way a minimum requirement which an injection mechanism must meet if the Fermi acceleration is to be highly efficient. It is important to note that the quantity $\mathscr{P}_{inj}^*$ depends only weakly on the cutoff energy ε_m, as can be seen from Fig. 8. This circumstance is a manifestation of the self-regulating properties of the acceleration mechanism.

The shape of the cosmic-ray spectrum is shown in Fig. 9. This figure shows the exponent of the proton spectrum, $\gamma = -d\ln n/d\ln p$, at energies in the relativistic region, for protons which have been accelerated by shock waves with various Mach numbers.[111] The injection rate was determined by the circumstance that the spectrum of the acceler-

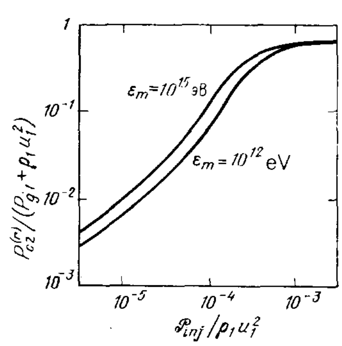


FIG. 8. The pressure ($P_{c2}^{(r)}$) of the relativistic particles accelerated by a strong shock wave ($\mathbf{Ma}_1 = 10$) as a function of the partial pressure of the injected particles.

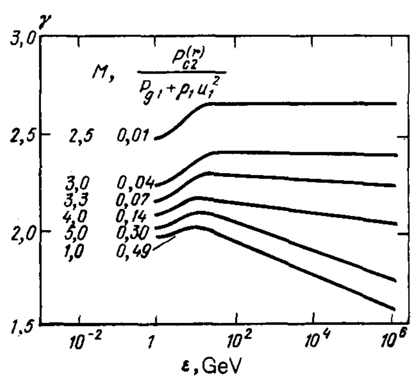


FIG. 9. Exponent in the differential energy spectrum of the accelerated protons, γ, as a function of the kinetic energy ε for shock waves with various Mach numbers $\mathbf{Ma}_1$ (Ref. 111).

ated particles was joined with the spectrum of the thermal particles calculated from Eq. (3.8). This situation corresponds to a partial pressure $\mathscr{P}_{inj} \approx 10^{-2}\rho_1 u_1^2$ of the injected particles,[116] so that we are in the saturation injection regime. It can be seen from this figure that for Mach numbers $\mathbf{Ma}_1 > 4$, which correspond to a degree of compression $\sigma > 4$, the exponent of the spectrum depends strongly on the energy. The spectrum is hardest at the highest energies, $\varepsilon \sim \varepsilon_m$, where we have $\gamma < 2$. The partial pressure of the cosmic rays, $\mathscr{P}_2$, is characterized by a minimum in the region $mc^2 < \varepsilon < \varepsilon_m$ in this case.

Moderate Mach numbers $\mathbf{Ma}_1 \approx 3$ correspond, as Fig. 9 shows, to an essentially constant spectral exponent over the entire energy range $\varepsilon > 10$ GeV. The acceleration efficiency increases with increasing Mach number; at $\mathbf{Ma}_1 = 10$ the relativistic accelerated particles contain half the total internal pressure behind the shock front. We can thus draw the important conclusion that the Fermi acceleration process is characterized by a high efficiency and by self-regulating properties—in the case of strong shock waves, a change in the injection rate over a broad range causes no significant change in the amount of energy which is transferred to the accelerated particles.

The self-consistent spectrum of cosmic rays and the modified plasma velocity profile $u(x)$ are established as the result of a competition among three physical effects. On the one hand, the cosmic rays produced by the shock wave increase the effective viscosity of the medium, ultimately increasing the thickness of the shock front. A thickening of the front reduces the efficiency of the acceleration of cosmic rays, as was shown in Subsection 2.1. On the other hand, the presence of relativistic particles among the cosmic rays reduces the effective adiabatic index of the medium and thereby increases the degree of compression of matter in the shock wave. The spectrum of the cosmic rays becomes harder, and the acceleration becomes more efficient. The same consequences result from the removal of energy from the system by the particles which reach the limiting energy ε_m.

In summary, the existing Fermi-acceleration models which incorporate the reaction of the accelerated cosmic rays on the structure of the shock wave are based on an extremely simple form of the description of the interaction of the cosmic rays with the medium. The problem of generating a detailed description of the turbulence near the shock front, which ultimately determines the nature of the motion of the cosmic rays, is thus relegated to a secondary place in these models. A microscopic description of the self-consistent turbulence which is generated by the cosmic-ray particles ahead of the shock front reveals the multifaceted nature of this problem[126–129] even in the quasilinear approximation. When the results of these two approaches will be combined in an organic way, we can expect substantial progress in research on the modification of a shock wave by the cosmic rays which it accelerates.

Nevertheless, we would like to emphasize that from this point of view the simplified nonlinear models of Fermi acceleration have revealed several important aspects of the acceleration process in its nonlinear stage which depend only slightly on the fine details of the interaction of the cosmic rays with the medium. We are thinking primarily of the nature of the modification of a plasma flow by the cosmic-ray pressure at various injection rates, the self-regulating property of the Fermi-acceleration process, and the particular features of the chemical composition of the accelerated particles. It is also important to emphasize that experiments can be carried out today to test theoretical predictions of this sort.

4. COSMIC RAYS AT SHOCK FRONTS

The discovery of this efficient particle-acceleration process—Fermi acceleration by shock waves—was a powerful stimulus to the development of theoretical research on various aspects of this process. Although the models which have recently been developed for this process contain some phenomenological elements, they do make quantitative predictions regarding this very complex problem. The extensive results available from experiments which have been carried out in interplanetary space make it possible to test the theoretical ideas.

4.1. Acceleration of cosmic rays by interplanetary shock waves

The numerous measurements which have been carried out in interplanetary space show that an increase in the intensity of particles with superthermal energies is essentially always observed near the fronts of shock waves. Experiments show that the spectrum of accelerated particles near a shock front can usually be described by a power law, and the particle intensity increases exponentially toward the front. Both these features are characteristic of the Fermi acceleration process, as we showed above. Exceptional cases are the increases in the intensity of fast particles which are observed near the fronts of quasiperpendicular shock waves. These events are characterized by a soft spectrum and a large anisotropy. They are caused by an acceleration of particles by the electric field[130,25] $\mathbf{E} = -[\mathbf{uB}]/c$.

Particularly extensive measurements have been carried out near the bow shock which arises as the supersonic streams of solar wind flow around the earth's magnetosphere.[131] A detailed comparison of the theory of Fermi acceleration derived for this case taking into account the shock wave geometry[107,109,120,132] with experimental results show that the set of observational facts—the spectrum of accelerated particles, their angular distribution, their spatial distribution, and the extent to which the shock wave is modified—is described well by the theory.

Analysis of several experiments has shown[69,133-136] that the spectra of the accelerated particles which are observed near the fronts of interplanetary shock waves formed by chromospheric flares at the sun have shapes corresponding to those predicted by the theory of Fermi acceleration. Further evidence for the validity of the theory comes from a comparison of calculations[137] of the spectral and spatial characteristics of the accelerated particles and of the MHD turbulence which they produce with experimental data. A typical proton spectrum measured at the front of an interplanetary shock wave[138] is shown in Fig. 10, which is taken from Ref. 111; this figure is a plot of the differential intensity $dJ/d\varepsilon$ as a function of the energy ε. This experiment shows that $\sim 1\%$ of the particles are subjected to acceleration. Although the maximum energy of the accelerated particles is low, these particles have about 25% of the total internal energy of the plasma. The smooth transition from the thermal part of the spectrum ($\varepsilon < 200$ eV) to the spectrum of accelerated particles ($\varepsilon > 200$ eV) indicates that the acceleration process begins directly from a thermal distribution. Specifically this principle is embodied in the kinetic model of Fermi acceleration.[105-116] A comparison of calculations carried out on the basis of this model[111] with experimental results reveals good agreement. This agreement indicates that although the structural description of the thermal front offered by the kinetic model is very schematic this model not only gives a correct description of the Fermi-acceleration process but also reflects the general features of the most typical aspects of the process by which particles are injected into the acceleration regime.

Fermi acceleration is also invoked as a possible mechanism for the generation of solar cosmic rays, which are emitted during solar chromospheric flares.[98,139-141] Although there are only limited possibilities for a detailed comparison of theory with experiment in this area, many features of the spectrum of solar cosmic rays can be described by the theory of Fermi acceleration.[141]

In summary, a detailed comparison of the theory with measurements carried out in interplanetary space shows that the mechanism of Fermi acceleration offers a good explanation for the generation of fast particles at the fronts of shock waves. The basic features which follow from this comparison—in particular—the high efficiency of the Fermi-acceleration process—can be utilized in problems involving the application of this process to remote objects, for which direct measurements are presently impossible.

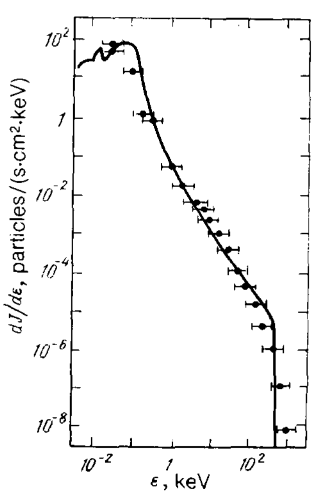


FIG. 10. Differential intensity of ions at the front of the interplanetary shock wave on 27 August 1978, with the kinetic energy as the independent variable.[111] Points—Experimental values[138]; line—calculated from a kinetic model.[111]

4.2. Low-energy galactic cosmic rays

The origin of high-energy galactic cosmic rays, with $\varepsilon \gtrsim$ 1 GeV/nucleon, which carry most of the total energy of these cosmic rays, is not the only unresolved question; the origin of the low-energy part of the spectrum of galactic cosmic rays which is observed near the earth, with $\varepsilon \leqslant 20$ MeV/nucleon, is also unknown. The suggestions of both galactic and solar origins for these particles run into significant difficulties (see Ref. 1 and the bibliography there). The idea that this part of the galactic cosmic rays might be generated in an interaction of the solar wind with the interstellar medium dates back a long way.[142-144] Since this region should contain a strong shock wave at a distance $R = 50$–100 AU from the sun, according to the present understanding,[52,78] the idea of a Fermi acceleration of background galactic cosmic rays is extremely attractive.[145,146] The reason for this attraction is that estimates show[145] that particles could be accelerated to energies $\varepsilon \sim 10$ MeV in this region. The analytic solution of this problem which we discussed back in Subsection 2.3 showed that particles which are moving away from a shock front, where they are accelerated, into the inner part of the heliosphere are subject to a modulating effect of the solar wind, in particular, an adiabatic slowing. Since the extent of this effect depends strongly on the particle diffusion coefficient, the only way to determine the role played by Fermi acceleration in the generation of low-energy galactic cosmic rays is to work from diffusion coefficients with values which do not go beyond the limits established by experiments.

The results of a numerical solution of steady-state transport equation (2.20) for a spherically symmetric model of the solar wind, $u(r<R) = u_1$, $u(r>R) = (u_1/4)(R/r)^2$, are shown in Fig. 11 as a plot of the expected proton intensity $dJ/d\varepsilon$ at the earth's orbit ($r = 1$ AU) as a function of the energy ε (Refs. 147 and 148). The value $u_1 = 500$ km/s was used for the velocity of the solar wind, and the value $R = 50$ AU was used for the radius of the shock wave. The rate of injection of protons with an energy $\varepsilon_0 = p_0^2/2m = 1$ keV at the shock front was chosen to make the energy density of the accelerated particles,

$$E_{c2} = \frac{2\pi}{m}\int_{p_0}^{\infty} f(R,\ p)\, p^4\, dp\,,$$

equal to 5% of the total internal energy of the plasma behind the shock front, $E_2 = (9/8)\rho_1 u_1^2$. In this case, the linear approximation can be used. The diffusion coefficient which was used,

$$\varkappa_1(r,\ p) = 6{,}5\cdot 10^{22}\left(\frac{p}{mc}\right)^{0,3}\left(\frac{r}{R}\right)^{0,2}\ \text{cm}^2/\text{s} \quad (\varkappa_2 \ll \varkappa_1),$$

has a spatial profile which does not contradict the results of observations taken in the region 1 AU$\leqslant r \leqslant$6 AU (Ref. 149), although the values of $\varkappa_1$ and $r = 1$ AU for energies $\varepsilon \lesssim 10$ MeV are slightly greater than the values observed in the quiet solar wind.[150]

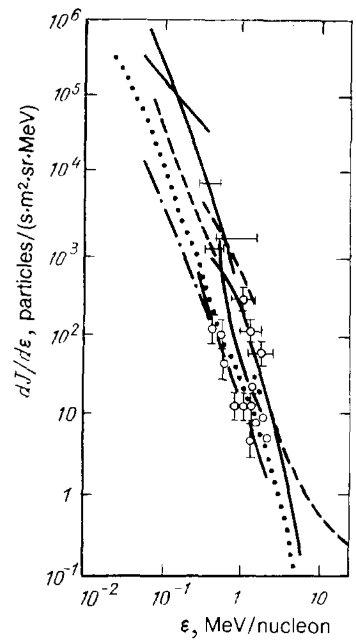


FIG. 11. Differential intensity of galactic cosmic rays near the earth's orbit, with the kinetic energy per nucleon as the independent variable. Points—Calculation[147] of the intensity of protons accelerated by a spherical shock wave of radius $R = 50$ AU; all other symbols—experimental values obtained during periods quiet in terms of solar activity.[151]

Shown for comparison in the same figure are the results of measurements of the intensity of galactic cosmic rays during periods quiet in terms of solar activity, as summarized in Ref. 151. Although the calculation was based on a highly simplified model of the solar wind, the relationship between theory and experiment here permits the conclusion that the Fermi acceleration of particles by a standing shock wave bounding the heliosphere can provide the observed intensity of galactic cosmic rays with energies $\varepsilon \lesssim 20$ MeV/nucleon. A final resolution of this problem will require further studies incorporating the actual geometry of the interplanetary magnetic field, which is one of the primary factors determining the nature of the propagation of cosmic rays in the heliosphere. On the theoretical side, it would be desirable to coordinate the solution of this problem with an identification of the role played by a shock wave as a modulating factor for galactic cosmic rays of higher energies, $\varepsilon \gtrsim 100$ MeV/nucleon.

4.3. Cosmic rays and supernovae

According to the present understanding, supernovae occur on the average once every 10–30 yr in our galaxy. In a supernova event, some 10^{50}–$3\cdot10^{52}$ erg is released in the form of the kinetic energy of the part of the star which is ejected as a result of the explosion. The result is the production of a strong shock wave in the surrounding interstellar medium. The shock wave propagates at a velocity $\sim 10^4$ km/s (Refs. 2, 40, 75, and 152).

Research on the acceleration of charged particles by the shock waves from supernovae is of considerable interest primarily because supernovae are regarded as the most likely source of the galactic cosmic rays on the basis of energy considerations.[75,2] They are capable of replenishing the loss of energy residing in the galactic cosmic rays due to their escape from the galaxy, $\sim 3\cdot 10^{40}$ erg/s, if the acceleration mechanism results in the transfer of at least 1% of the energy released in the explosion to accelerated particles. In this connection it is natural to attempt to apply the Fermi acceleration mechanism, with its high efficiency. Here are the basic problems which can be singled out in the study of the process of Fermi acceleration of particles in supernova remnants, regarded as a possible source of galactic cosmic rays: a) the shape of the energy spectrum of the accelerated particles which go into interplanetary space; b) the efficiency of the acceleration; c) the maximum energy of the accelerated particles.

In the linear approximation, the spectrum $N(p)$ (the total number of particles with momentum p) of the cosmic rays accelerated by a shock wave throughout its existence t_f can be found on the basis of the equations in Subsection 2.2 if we know the expansion law $R(t)$ and injection rate $N_0(t)$. Here the cosmic-ray density corresponding to the time t_f, at which the Mach number is $\mathbf{Ma}_1(t_f) = 1$, should be integrated over the volume occupied by the cosmic rays:

$$N(p) = 4\pi \int_0^\infty n(r, ap, t_f)\, a^{-2} r^2\, dr, \qquad (4.1)$$

where the factor $a(r)$ describes the change caused in the momentum of the cosmic rays by the relaxation of the medium behind the shock front to the state of the interstellar medium. If we assume that the relaxation for the cosmic rays occurs adiabatically, we find that this factor is determined by the ratio of $P_g(r, t_f)$, the pressure in the medium which is the location of the cosmic rays at the time t_f, and P_m — $a = (P_g/P_m)^{1/5}$, the pressure in the interstellar medium.[153-155] Studies of the shape of the cosmic-ray spectrum $N(p)$ which have been carried out by slightly different methods[153-155] with an expansion scaling law $R(t) = R_0(t/t_0)^{2/5}$ have demonstrated the following features of this spectrum: 1) The spectrum of cosmic rays generated by a shock wave during its existence is described by a power function $p^{-\gamma}$, where the exponent γ remains in the interval 2.1–2.3 for a wide momentum range $mc \leqslant p < p_{mf}$, regardless of the value of the injection momentum $p_0 < mc$ and regardless of the particular time dependence adopted for the injection power, $N_0(t)$ (calculations were carried out for $N_0 \sim R^{-d}$, where $d = 0$–2). 2) The early stages of the expansion, corresponding to Mach numbers $\mathbf{Ma} > 8$, contribute only insignificantly to the overall cosmic-ray spectrum $N(p)$ (except near p_m), because of geometric factors. For this reason, the contribution from the stage of free expansion ($R \sim t$), which preceeds the adiabatic expansion in the evolution of the shock wave from a supernova, can be ignored. 3) The later stages in the adiabatic expansion, which correspond to Mach numbers $\mathbf{Ma}_1 \lesssim 2$, contribute significantly to the overall cosmic-ray spectrum $N(p)$ only in a narrow region near the injection momentum p_0 because of the soft spectrum of the particles which are accelerated in this period. 4) The spectrum $N(p)$ is dominated by the stages corresponding to Mach numbers $2 < \mathbf{Ma}_1 \lesssim 5$ ($\sigma = 2.3$–3.6).

We note, however, that all these studies[153-155] have ignored an important circumstance: the presence of runaway particles. As was shown in Subsection 2.3, ahead of the shock front at each instant there are particles with momenta $p_m(t) \leqslant p \leqslant p_m(t_0)$ in addition to the particles with momenta

$p_0 \ll p \lesssim p_m(t)$. The latter particles are being accelerated in the given stage. The former particles were accelerated in earlier stages of the expansion, and the velocity of their diffusive propagation exceeds the velocity of the shock front, $\dot{R}(t)$, at the time t. The overall spectrum of these particles, $N(p)$, can be calculated by using expression (2.37), but there is a simpler approach. The number of runaway particles can be found from the expression

$$N(p) \approx Vn(p = p_m(t_p), t_p), \tag{4.2}$$

where n is the density of particles at time t_p, when the cutoff momentum p_m has the value p, and $V \sim R^3$ is the volume occupied by the particles with a momentum p_m. Since the density of accelerated particles at large Mach numbers $\mathbf{Ma} \gg 1$ is $n \sim N_0(t)p^{-2}$, where $N_0(t)$ determines the rate of injection at the shock front [see (2.35)], we find $N(p) \sim N_0(t_p)R^3p^{-2}$. We thus see that if the particle injection rate varies in accordance with $N_0 \sim R^{-3}$ the spectrum of the runaway particles will have the universal form[84,85] $N(p) \sim p^{-2}$. An injection law $N_0 \sim R^{-3}$ has a completely clear physical meaning: For an adiabatic expansion law ($R^{-3} \sim \dot{R}^2$) it corresponds to a situation in which the accelerated particles acquire a fixed fraction of the energy $\rho_1\dot{R}^2/2$ of the flow of the medium which is incident on the shock front. In this case, generalizing the results of Refs. 153–155, we note that the spectrum of cosmic rays generated by the shock wave from a supernova has a power-law form $N(p) \sim p^{-\gamma}$ with an exponent which lies in the interval $2 < \gamma \lesssim 2.3$ for the momentum region $mc \ll p \lesssim p_m$, where the maximum momentum p_m corresponds to the initial stage of the adiabatic expansion and is found from the relation $\varkappa_1(p_m) \times R(t_0)\dot{R}(t_0))^{-1} = 1$. This shape of the cosmic-ray spectrum, considered along with the energy dependence of the time which these cosmic rays spend in the galaxy, corresponds well to the observed spectrum of cosmic rays in the energy range 10^{10}–10^{15} eV (Ref. 2, for example).

Although we know almost nothing about the actual injection processes which can operate at shock fronts in the interstellar medium, it is interesting to examine the resultant cosmic-ray spectrum $N(p)$ which would be formed under conditions of a saturated injection. Only under this condition can a substantial fraction of the total energy of the shock wave be transferred to the cosmic-ray particles. Such a substantial transfer would be necessary if we are taking supernovae to be the primary source of the galactic cosmic rays. Furthermore, the results of measurements carried out in interplanetary space make the saturation injection regime look extremely realistic.

In the saturation injection regime, the spectrum of accelerated particles at the shock front at each instant does not have a power-law shape $p^{-\gamma}$ with a single value of γ, in contrast with the linear case, as was shown in Subsection 3.2. The problem of determining the resultant cosmic-ray spectrum $N(p)$ accordingly becomes slightly more complicated. However, one can assume that the same geometric factors which operate in the linear case will cause the cosmic-ray spectrum $N(p)$ to be dominated by those stages in the evolution of the shock wave which correspond to a degree of compression $\sigma \approx 3.6$, except at low momenta $p < mc$ and at large momenta $p \sim p_m$. Calculations carried out on the basis of a kinetic model (Fig. 9) show that with $\sigma \approx 3.5$ and $\varepsilon_m = 10^{15}$ eV the saturation injection regime corresponds to the accelerated-particles spectrum $n(\varepsilon) \sim \varepsilon^{-\gamma}$ in which the exponentremains nearly constant over the entire range of relativistic energies at the value $\gamma \approx 2.3$. About 7% of the total internal pressure corresponds to relativistic particles. It was also shown that at large Mach numbers $\mathbf{Ma}_1 \gg 1$ the partial pressure $\mathscr{P}(p_m)$, of the particles with the maximum momentum p_m, varies only slightly with p_m. This result means that the density of these particles can be described by $n(p_m) \sim \rho_1\dot{R}^2p_m^{-2}$. It then follows from expression (4.2) that the spectrum of runaway particles is a power-law spectrum $N(p) \sim p^{-\gamma}$ with $\gamma \approx 2$. Shock waves from supernovae can thus provide a spectrum of galactic cosmic rays with the necessary shape and the necessary amplitude.

The maximum momentum p_m in the resultant spectrum of cosmic rays generated by a shock wave from a supernova is determined by the initial stage of the adiabatic expansion and can be found from the relation $\varkappa_1(p_m) = R(t_0)\dot{R}(t_0)$. As usual, it is being assumed here that the value of the diffusion coefficient in the internal region does not impose any restriction on the value of p_m, i.e., that we have $\varkappa_2 < \varkappa_1$ because the medium behind the shock front is highly perturbed. Using typical values[40,152,155] for the initial velocity, $\dot{R}(t_0) = 4\cdot 10^8$ cm/s, and for the radius of the initial stage of the adiabatic expansion, $R(t_0) = 30$ pc, for the shock wave of a type II supernova propagating through a homogeneous medium with a density $N_g = 3\cdot 10^{-3}$ cm^{-3}, we find the value $\varkappa_1(p_m) = 3.6\cdot 10^{28}$ cm^2/s for the diffusion coefficient corresponding to the maximum momentum of the cosmic rays. Assuming a diffusion coefficient

$$\varkappa_d(p) = 5\cdot 10^{27}\left(\frac{pc}{3\ \text{GeV/nucleon}}\right)^{0.3}$$

for the galactic disk, as is predicted by the diffusion model[2,156] for the propagation of cosmic rays in our galaxy, on the basis of measurements of the chemical composition of the galactic cosmic rays, we find $p_m c = 2\cdot 10^{12}$ eV/nucleon (Ref. 157).

Near the shock front, where the cosmic rays generate an MHD turbulence,[31] the diffusion coefficient $\varkappa$ should be less than the mean value $\varkappa_d$. Theoretically, the lower limit on $\varkappa$ is $\varkappa_{min} = \rho_B v/3$. This value corresponds to the maximum momentum $p_m c = 3\cdot 10^{15}$ eV/nucleon if we take $B = 3\cdot 10^{-6}$ G as a typical value of the interstellar magnetic field. There has been no detailed study of the extent to which the generation of an MHD turbulence by cosmic-ray particles ahead of a shock front might lower the diffusion coefficient. The estimates which have been made[158] are based on the assumption that the pressure of the cosmic rays at the shock front, P_{c2}, does not exceed the pressure of the interstellar magnetic field, $B^2/8\pi$. Such a situation would be unlikely, since in the interstellar medium we would have[2] $P_{c1} \approx B^2/8\pi$, even ahead of the shock front.

The limitation imposed on the maximum momentum p_m by the quasilinear theory of Alfvén turbulence can be found by using the results of Ref. 31. Bell[31] solved the problem of the acceleration of particles (cosmic rays) by a parallel plane shock wave on the basis of a self-consistent diffusion coefficient

$$\varkappa = \frac{4}{3\pi}\rho_B v\frac{B^2/8\pi}{E_w},$$

whose value is determined by the energy density (E_w) of the

Alfvén waves which are interacting resonantly with the cosmic-ray particles. These waves are in turn excited in the region ahead of the shock front as a result of the anisotropic angular distribution of the cosmic rays. In the quasilinear approximation, the corresponding growth rate is

$$\Gamma=\frac{4\pi}{3}\,p^4 v\,\frac{c_a}{E_w}\,\frac{\partial l}{\partial x}.$$

The self-consistent distribution of cosmic rays ahead of the shock front ($x<0$) is then described by the expressions

$$f_1(x,\ p)=f_2(p)\left(1-\frac{x}{x_0}\right)^{-1},$$

$$x_0=\frac{4}{3\pi}\,\frac{B^2/8\pi}{\mathscr{P}_2(p)}\,\frac{v}{c_a}\,\rho_B, \qquad (4.3)$$

where $\mathscr{P}=(4\pi/3)p^4 vf(p)$ is the partial (in terms of $\ln p$) pressure of the cosmic rays (in Subsection 3.2 we used a pressure which was partial in terms of $\ln\varepsilon$; in the relativistic region, these quantities are the same). We thus see that a self-consistent turbulence can lead to an acceleration of cosmic rays by a shock wave of nonzero dimensions R if the condition $x_0 \ll R$ holds. Although expressions (4.3) were derived in the approximation linear in the cosmic-ray pressure, it can also be used for estimates in cases in which the cosmic rays contain a substantial fraction of the total energy in the shock wave, since the modification of the shock wave by the cosmic-ray pressure could not substantially influence the excitation of Alfvén waves. Under conditions of unsaturated injection in a strong shock wave, as was shown in Subsection 3.2, the partial pressure of the relativistic accelerated particles goes through a minimum at a certain energy. If the shock wave is strong, it is a simple matter to find an approximate expression for the minimum pressure: $\mathscr{P}_{\min}\approx(6/\sigma)\pi^2\rho_1 u_1^2/\ln^2(p_m/mc)$. Using this expression, we can rewrite the condition $x_0\ll R$ as

$$p_m c \ll \frac{9\pi^3}{\sigma}\,eBR\,\frac{u_1^2}{c_a c}\,\ln^{-2}\frac{p_m}{mc},$$

Using some typical values for the interstellar medium [$c_a\approx 10^7$ cm/s, $u_1=4\cdot 10^8$ cm/s, $R(t_0)=10^{20}$ cm, $\sigma=4$, and $B=3\cdot 10^{-6}$ G], we find $p_m c\ll 2\cdot 10^7\ mc^2$. We thus see that the quasilinear theory does not impose any restrictions on the maximum momentum of the cosmic rays in comparison with the momentum which would correspond to the minimum permissible diffusion coefficient, $\rho_B c/3$. At the same time, this circumstance means that under conditions of saturated injection the Alfvén turbulence would become definitely nonlinear, and we would need to seek more reliable estimates on the basis of a nonlinear theory.

To what extent could the maximum energy of the cosmic rays be increased by a collective supplemental acceleration of these cosmic rays by the ensemble of all the shock waves from supernovae which simultaneously exist in the galaxy? To answer this question, we seek the time scale of the collective acceleration according to expression (2.39), which in this case takes the form

$$\tau_a = V\,\frac{4\pi}{3}\int\limits_{t_0}^{t_f}\nu\,\frac{\sigma-1}{\sigma+1}\,R^2\dot R\,dt.$$

Since we do not know the actual time dependence of the degree of compression σ, which is determined by the cosmic-ray pressure in addition to everything else, we ignore the difference between the factor $(\sigma-1)/(\sigma+1)$ and unity, finding

$$\tau_a=\frac{2V}{5V_{SN}(t_f)\,\nu}, \qquad (4.4)$$

where $V_{SN}(t_f)=(4\pi/3)R^3(t_f)=(4\pi/3)R^3(t_0)\mathbf{Ma}^3(t_0)$. The maximum energy of the cosmic rays can be estimated from $\tau_a=\tau_e(p_m)$. We use the results of the diffusion model for the propagation of cosmic rays in the galaxy. That model, which on the basis of measurements of the chemical composition of the galactic cosmic rays, predicts[159,2] that the volume occupied by the galactic cosmic rays is $V=5\cdot 10^{68}$ cm^3 and that the time spent in the galaxy by galactic cosmic rays with an energy $\gtrsim 3$ GeV/nucleon is

$$\tau_e = 3\cdot 10^{8}\left(\frac{3\ \text{GeV/nucleon}}{pc}\right)^{-0.3}\ \text{yr}.$$

Also adopting $\nu=1/10$ yr^{-1} as the frequency of supernovae, we find $p_m c=3\cdot 10^4$ GeV/nucleon. Using some results from the homogeneous model for the propagation of galactic cosmic rays,[160,159] $V=2.5\cdot 10^{67}$ cm^3 and $\tau_e=2\cdot 10^7$ (3 GeV/pc)$^{-0.3}$ yr, we find $p_m c=10^5$ GeV/nucleon.

According to the customary terminology, the collective mechanism is a version of interstellar acceleration since the cosmic rays are subjected to the acceleration throughout the time which they spend in the galaxy. It has been assumed that the existing results from measurements of the chemical composition of the galactic cosmic rays refute such models for the origin of galactic cosmic rays.[161-163,2] However, it was recently established[164] that a systematic analysis of the inhomogeneities in the distribution of the sources of cosmic rays and of the interstellar gas in the galaxy leads to the opposite conclusion: that an interstellar acceleration does not contradict observations.

It can thus be concluded that under some assumptions regarding the nature of the injection of particles into the acceleration regime and the propagation of the galactic cosmic rays in the galaxy—whose validity of course requires further refinement—the process of acceleration of cosmic rays by shock waves from supernovae can result in generation of galactic cosmic rays at energies up to 10^{14}–$3\cdot 10^{15}$ eV/nucleon. We note that in this connection that a galactic origin of the cosmic rays, at least at energies up to 10^{17} eV, causes no particular doubt today. Accordingly, the estimate of the upper limit on the energy given here, which should probably be regarded as on the optimistic side in view of the uncertainties in the parameter values which we have used, forces us to conclude that the Fermi mechanism does not meet the requirements which it would have to meet in order to explain the generation of cosmic rays at the very highest energies. At present it is totally unclear whether this difficulty can be overcome through a further development of the theory or whether the mechanism of Fermi acceleration must be rejected as a possible source of the galactic cosmic rays, despite all its attractive features.

We have restricted the discussion above to those cases in which the present state of the theory and the present state of the experimental work allow a comparison of the two in a most comprehensive way; the list of possible applications of the Fermi acceleration process is of course much longer. The shock waves which arise as matter accretes on compact astrophysical objects,[165] the shock waves at the boundary between the galactic wind and the intergalactic medium,[166] and the shock waves in quasars and in the cores of active galaxies[167]—these are all examples of entities in which Fermi acceleration could play an important role. The necessary research here is still in an initial stage, and we do not rule out the possibility that some of these objects will contribute substantially to the observed spectrum of high-energy galactic cosmic rays.

5. CONCLUSION

Studies of the Fermi acceleration of charged particles by shock waves clearly constitute progress along the path toward an understanding of the acceleration processes which operate in space plasmas. At the moment, Fermi acceleration is the best-developed mechanism. It is capable of making substantive predictions which can be tested by observations. The development of the theory of Fermi acceleration has made it possible to explain a long list of phenomena observed in interplanetary space in which charged particles are accelerated near the fronts of shock waves. The quantitative predictions of the theory agree well with the existing measurements.

The results of research on astrophysical applications of this process, despite the existing difficulties, can also be regarded a bit optimistically as a step down the path to the solution of the important problem of the origin of the galactic cosmic rays. As has been shown, the Fermi acceleration of particles by shock waves from supernovae would be capable of forming a spectrum of cosmic rays with the necessary shape and the necessary amplitude over a wide energy range. Further progress in this direction can be expected in the solution of two important problems which are of fairly general significance for the theory of Fermi acceleration.

The first of these problems, and also the more complicated, and the one which has been studied less, is that of the injection of thermal particles into the acceleration process. At the moment we have nothing in the way of a solidly based theoretical prediction of the extent to which particles could be accelerated directly from their thermal distribution behind a shock front without a supplemental acceleration. Furthermore, we have no corresponding predictions of which plasma parameters determine the number of injected particles. Here it would be particularly important to determine the possibilities of the saturated injection regime. In this case, when the rate at which particles are injected into the acceleration process exceeds a certain minimum value the accelerated particles will acquire a substantial fraction (about half) of the total energy of the plasma in the shock wave, by virtue of the self-regulating properties of the Fermi-acceleration process. Judging from the information which has been obtained in interplanetary space, the saturated-injection regime can be regarded as realistic, although it is not clear to what extent it can occur in the interstellar medium. Another circumstance which makes this point so important is that it is apparently only under the condition of saturated injection that acceleration by shock waves from supernovae would be capable of providing an energy at the scale required for the galactic cosmic rays.

The second important problem is generating a systematic description of the development of plasma turbulence ahead of a shock front by accelerated particles under conditions such that these particles carry a substantial fraction of the total energy in the shock wave. This problem is complicated, in particular, by the circumstance that the MHD turbulence can reach a nonlinear level, according to estimates. In addition to the Alfvén turbulence, which is the type usually discussed, there might be an excitation of long-wavelength perturbations of other types in the prefront region.[168-170,84] A study of the dynamics of plasma turbulence, which ranks along with the cosmic rays as an important factor in determining the structure of a shock wave, would also be of importance to the problem of the origin of cosmic rays. An increase in the extent to which the medium ahead of the shock front is perturbed results in a decrease in the cosmic-ray diffusion coefficient in this region. Under the conditions corresponding to shock waves of nonzero dimensions, this decrease in the diffusion coefficient would promote an increase in the maximum energy of the accelerated particles.

In conclusion we wish to stress that a study of the Fermi-acceleration process is also important for the physics of collisionless shock waves, since the high efficiency of this process qualifies the accelerated particles as a factor which would substantially influence the dynamics and structure of the shock wave.

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[1)]The term "thermal front" which we are using here reflects the fact that in this case the discontinuity bears the same relationship to a thermal plasma as the front of a shock wave bears to an ordinary gas (but this analogy does not extend to such more particular concepts as an isothermal density jump[20]).

[1]I. N. Toptygin, Cosmic Rays in the Interplanetary Magnetic Fields (in Russian), Nauka, M., 1983.

[2]V. S. Berezinskiĭ, S. V. Bulanov, V. L. Ginzburg, V. A. Dogel', and V. S. Ptuskin, Cosmic-Ray Astrophysics (in Russian), Nauka, M., 1984.

[3]Ya. L. Blokh, L. I. Dorman, and N. S. Kaminer, in: Proceedings of the Sixth International Conference on Cosmic Rays, Moscow, 1959 (in Russian), Vol. 4, p. 178.

[4]E. P. Ney, I. R. Winckler, and P. S. Freier, Phys. Rev. Lett. **3**, 183 (1959).

[5]A. N. Charakhch'yan, V. F. Tulinov, and T. H. Charakhch'yan, Zh. Eksp. Teor. Fiz. **38**, 1031 (1960) [Sov. Phys. JETP **11**, 742 (1960)].

[6]A. N. Charakhch'yan, Usp. Fiz. Nauk **83**, 35 (1964) [Sov. Phys. Usp. 7, 358 (1964)].

[7]K. A. Andersen and D. C. Enmark, J. Geophys. Res. **65**, 1034 (1960).

[8]W. I. Axford and G. C. Reid, J. Geophys. Res. **67**, 1692 (1962); **68**, 1793 (1962).

[9]E. N. Parker, Phys. Rev. **109**, 1328 (1958).

[10]L. I. Dorman and G. I. Freĭdman, in: Questions of Magnetohydrodynamics and Plasma Dynamics (in Russian), Zinatne, Riga, 1959, p. 77.

[11]F. Hoyle, Mon. Not. R. Astron. Soc. **120**, 338 (1960).

[12]V. P. Shabanskiĭ, Zh. Eksp. Teor. Fiz. **41**, 1107 (1961) [Sov. Phys. JETP **14**, 791 (1962)].

[13]P. D. Hudson, Mon. Not. R. Astron. Soc. **131**, 23 (1965).

[14]B. A. Tverskoĭ, Zh. Eksp. Teor. Fiz. **52**, 483 (1967) [Sov. Phys. JETP **25**, 317 (1967)].

[15]B. A. Tverskoĭ, Zh. Eksp. Teor. Fiz. **53**, 1417 (1967) [Sov. Phys. JETP **26**, 821 (1968)].

[16]I. I. Alekseev and A. P. Kropotkin, Geomagn. Aeron. **10**, 953 (1970). [Geomagn. Aeron. (USSR) **10**, 755 (1970)].

[17]D. A. Bryant, T. L. Cline, V. D. Desai, and F. B. McDonald, J. Geophys. Res. **67**, 4983 (1962).

[18]S. N. Vernov, E. V. Gorchakov, and G. A. Timofeev, Geomagn. Aeron. **9**, 961 (1969). [Geomagn. Aeron. (USSR) **9**, 775 (1969)].

[19]L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, Addison-Wessley, Reading, Mass., 1959 [Russ. original, Gostekhizdat, M., 1953].

[20]Ya. B. Zel'dovich and Yu. P. Raizer, *Physics of Shock Waves and High Temperature Hydrodynamic Phenomena*, Academic Press, N.Y., 1966, [Russ. original, Nauka, M., 1957].

[21]B. B. Kadomtsev, Collective Phenomena in Plasmas (in Russian), Nauka, M., 1976.

[22]R. Z. Sagdeev, in: Reviews of Plasma Physics, Vol. 4, p. 23, Consultants Bureau, New York, 1966 [Russ. original, Atomizdat, M., 1964].

[23]L. A. Artsimovich and R. Z. Sagdeev, Plasma Physics for Physicists (in Russian), Atomizdat, M., 1979.

[24] C. F. McKee and D. J. Hollenbach, J. Ann. Rev. Astron. Astrophys. **18**, 219 (1980).
[25] M. E. Pesses, Thesis, Univ. of Maryland, College Park, 1979.
[26] G. M. Webb, W. I. Axford, and T. Terasawa, Astrophys. J. **270**, 573 (1983).
[27] I.R. Sagdeev, R. Z. Sagdeev, V. D. Shapiro, V. I. Shevchenko, and K. Szegö, Preprint KFKI-1984-75, Central Research Institute for Physics, Budapest, 1984.
[28] A. S. Arefiev, M. E. Gedalin, V. V. Krasnoselskikh, and J. G. Lominadze, in: Proceedings of Course and Workshop on Plasma Astrophysics, ESA SP-251, Sukhumi, 1986, p. 243.
[29] G. F. Krymskiĭ, Dokl. Akad. Nauk SSSR **234**, 1306 (1977) [Sov. Phys. Dokl. **22**, 327 (1977)].
[30] W. I. Axford, E. Leer, and G. Skadron, in: Proceedings of the Fifteenth International Cosmic Ray Conference, Plovdiv, 1977, Vol. 11, p. 132.
[31] A. R. Bell, Mon. Not. R. Astron. Soc. **182**, 147 (1978).
[32] A. R. Bell, Mon. Not. R. Astron. Soc. **182**, 443 (1978).
[33] R. D. Blandford and J. R. Ostriker, Astrophys. J. **221**, L29 (1978).
[34] V. N. Tsytovich, *Theory of Turbulent Plasma*, Plenum, N. Y., 1974 [Russ. original, Atomizdat, M., 1971].
[35] V. K. Elshin, G. F. Krymskiĭ, S. I. Petukhov, Yu. A. Romashchenko, and I. A. Transkiĭ, Geomagn. Aeron. **19**, 793 (1979) [Geomagn. Aeron. (USSR) **19**, 411 (1979)].
[36] F. C. Michel, Astrophys. **247**, 664 (1981).
[37] J. A. Peacock, Mon. Not. R. Astron. Soc. **196**, 135 (1981).
[38] G. M. Webb, L. C. O'Drury, and P. Bierman, Astron. Astrophys. **137**, 185 (1984).
[39] L. C. O'Drury, Rep. Prog. Phys. **46**, 973 (1983).
[40] I. S. Shklovskiĭ, Supernovae and Related Problems (in Russian), Nauka, M., 1976.
[41] I. N. Toptygin, Space Sci. Rev. **26**, 157 (1980).
[42] W. I. Axford, in: Proceedings IAU/IUPAP Symp. No. 94, D. Reidel, Dordrecht, 1980, p. 339.
[43] W. I. Axford, in: Proceedings of the Seventeenth International Cosmic Ray Conference, Paris, 1981, Vol. 12, p. 155.
[44] R. D. Blandford, in: Proceedings of Workshop on Particle Acceleration in Astrophysics, La Jolla. AIP Conference Proceedings, Vol. 56, 1979, p. 335.
[45] E. Fermi, Phys. Rev. **75**, 1169 (1949); Astrophys. J. **119**, 1 (1954).
[46] E. G. Berezhko, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 416 (1981) [JETP Lett. **33**, 399 (1981)].
[47] E. G. Berezhko and G. F. Krymskiĭ, Pis'ma Astron. Zh. 7, 636 (1981) [Sov. Astron. Lett. 7, 352 (1981)].
[48] G. F. Krymskiĭ, Geomagn. Aeron. **4**, 977 (1964) [Geomagn. Aeron. (USSR) **4**, 763 (1964)].
[49] E. N. Parker, Planet. Space Sci. **13**, 9 (1965).
[50] A. Z. Dolginov and I. N. Toptygin, Zh. Eksp. Teor. Fiz. **51**, 1771 (1966) [Sov. Phys. JETP **24**, 1195 (1967)].
[51] G. F. Krymskiĭ, Modulation of Cosmic Rays in Interplanetary Space (in Russian), Nauka, M., 1969.
[52] E. N. Parker, *Interplanetary Dynamical Processes*, Interscience, N. Y., 1963 [Russ. transl., Nauka, M., 1965].
[53] L. Y. Gleeson and A. J. Webb, Astrophys. Space Sci. **60**, 335 (1978).
[54] E. G. Berezhko, Geomagn. Aeron. **24**, 714 (1984) [Geomagn. Aeron. (USSR) **24**, 584 (1984)].
[55] G. F. Krymskiĭ, A. I. Kuz'min, P. A. Krivoshapkin, I. S. Samsonov, G. V. Skripin, I. A. Transkiĭ, and N. P. Chirkov, Cosmic Rays and the Solar Wind (in Russian), Nauka, Novosibirsk, 1981.
[56] V. N. Vasil'ev, I. N. Toptygin, and A. G. Chirkov, Geomagn. Aeron. **18**, 415 (1978) [Geomagn. Aeron. (USSR) **18**, 279 (1978)].
[57] G. M. Webb, Astron. Astrophys. **124**, 163 (1983).
[58] L. C. O'Drury, W. I. Axford, and D. Summers, in: Proceedings of the Seventeenth International Cosmic Ray Conference, Paris, 1981, Vol. 2, p. 327; Mon. Not. R. Astron. Soc. **198**, 833 (1982).
[59] G. F. Krymskiĭ, V. K. Elshin, Yu. A. Romashchenko, and I. P. Bezrodnykh, Izv. Akad. Nauk SSSR. Ser. Fiz. **42**, 1070 (1978) [Bull. Acad. Sci. USSR Phys. Ser. **42**(5), 153 (1978)].
[60] V. K. Elshin, G. F. Krymskiĭ, S. I. Petukhov, A. A. Turpanov, and Yu. A. Romashchenko, Geomagn. Aeron. **21**, 781 (1981) [Geomagn. Aeron. (USSR) **21**, 587 (1981)].
[61] V. N. Vasil'ev, I. N. Toptygin, and A. G. Chirkov, Izv. Akad. Nauk SSSR. Ser. Fiz. **42**, 984 (1978) [Bull. Acad. Sci. USSR Phys. Ser. **42**(5), 77 (1978)].
[62] V. N. Vasil'ev, I. N. Toptygin, and A. G. Chirkov, Kosm. Issled. **18**, 556 (1980) [Cosmic Res. (USSR) **18**, 401 (1980)].
[63] V. L. Prishchep and V. S. Ptuskin, Astron. Zh. **58**, 779 (1981) [Sov. Astron. **25**, 446 (1981)].
[64] M. A. Forman and L. C. O'Drury, in: Proceedings of the Eighteenth International Cosmic Ray Conference, Bangalor, 1983, Vol. 2, p. 267.
[65] G. F. Krymsky, A. I. Kuzmin, S. I. Petukhov, and A. A. Turpanov, in: Proceedings of the Sixteenth International Cosmic Ray Conference, Kyoto, 1979, Vol. 2, p. 38.
[66] P. O. Lagage and C. J. Cesarsky, in: Proceedings of International School and Workshop on Plasma Astrophysics, Varenna, 1981, p. 317.
[67] P. O. Lagage and C. J. Cesarsky, Astron. Astrophys. **118**, 223 (1983).
[68] B. A. Tverskoĭ, in: Proceedings of the Tenth Leningrad Seminar on Space Physics (in Russian), Leningrad, 1978, p. 137.
[69] B. A. Tverskoĭ, Geomagn. Aeron. **23**, 353 (1983) [Geomagn. Aeron. (USSR) **23**, 291 (1983)].
[70] G. M. Webb, Astrophys. J. **270**, 319 (1983).
[71] S. V. Bulanov and V. A. Dogel', Pis'ma Astron. Zh. **5**, 521 (1979) [Sov. Astron. Lett. **5**, 278 (1979)].
[72] H. J. Völk, G. E. Morfill, and M. A. Forman, in: Proceedings of the Sixteenth International Cosmic Ray Conference, Kyoto, 1979, Vol. 2, p. 38.
[73] H. J. Völk, G. E. Morfill, and M. A. Forman, Astrophys. J. **249**, 161 (1981).
[74] G. F. Krymsky, A. I. Kuzmin, and S. I. Petukhov, in: Proceedings of the Sixteenth International Cosmic Ray Conference, Kyoto, 1979, Vol. 2, p. 44.
[75] V. L. Ginzburg and S. I. Syrovatskiĭ, Origin of the Cosmic Rays [in Russian], Izv. Akad. Nauk SSSR, Moscow, 1963.
[76] B. A. Trubnikov, in: Reviews of Plasma Physics, Vol. 1, p. 105 (ed. M. A. Leontovich), Consultants Bureau, N. Y. 1965 [Russ. original, Atomizdat, Moscow, 1963, Vol. 1, p. 98].
[77] C. F. McKee and J. P. Ostriker, Astrophys. J. **218**, 148 (1977).
[78] V. B. Baranov and K. V. Krasnobaev, Hydrodynamic Theory of Space Plasmas (in Russian), Nauka, M., 1977.
[79] G. M. Webb, W. I. Axford, and M. A. Forman, in: Proceedings of the Seventeenth International Cosmic Ray Conference, Paris, 1981, Vol. 2, p. 309; in: Proceedings of the Eighteenth International Cosmic Ray Conference, Bangalore, 1983, Vol. 2, p. 263.
[80] M. A. Forman, G. M. Webb, and W. I. Axford, in: Proceedings of the Seventeenth International Cosmic Ray Conference, Paris, 1981, Vol. 2, p. 313.
[81] L. I. Dorman, M. E. Kats, S. F. Nosov, Yu. I. Fedorov, and B. A. Shakhov, Geomagn. Aeron. **22**, 705 (1982) [Geomagn. Aeron. (USSR) **22**, 585 (1982)].
[82] G. F. Krymskiĭ and S. I. Petukhov, Pis'ma Astron. Zh. **6**, 227 (1980) [Sov. Astron. Lett. **6**, 124 (1980)].
[83] E. G. Berezhko and G. F. Krymskiĭ, Izv. Akad. Nauk SSSR. Ser. Fiz. **46**, 1656 (1982) [Bull. Acad. Sci. USSR Phys. Ser. **46**(9), 7 (1982)].
[84] E. G. Berezhko, in: Proceedings of Course and Workshop on Plasma Astrophysics, ESA SP-251, Sukhumi, 1986, p. 271.
[85] E. G. Berezhko, V. K. Elshin, G. F. Krymskiĭ, and A. A. Turpanov, Izv. Akad. Nauk SSSR. Ser. Fiz. **51**, 1790 (1987) [Bull. Acad. Sci. USSR Phys. Ser. **51**(10), (1987)].
[86] A. N. Tikhonov and A. A. Samarskiĭ, *Equations of Mathematical Physics*, Pergamon, Oxford, 1964 [Russ. original, Nauka, M., 1972].
[87] E. G. Berezhko and G. F. Krymsky, in: Proceedings of the Eighteenth International Cosmic Ray Conference, Bangalore, 1983, Vol. 2, p. 255.
[88] E. G. Berezhko and G. F. Krymsky, in: Abstracts of the Twenty-fifth Plenary Meeting of COSPAR, Graz, Austria, 1984, p. 128.
[89] A. M. Bykov and I. N. Toptygin, Izv. Akad. Nauk SSSR. Ser. Fiz. **43**, 2552 (1979) [Bull. Acad. Sci. USSR Phys. Ser. **43**(12), 79 (1979)].
[90] A. M. Bykov and I. N. Toptygin, Izv. Akad. Nauk SSSR. Ser. Fiz. **44**, 2574 (1980) [Bull. Acad. Sci. USSR Phys. Ser. **44**(12), 102 (1980)].
[91] A. M. Bykov and I. N. Toptygin, Izv. Akad. Nauk SSSR. Ser. Fiz. **45**, 474 (1981) [Bull. Acad. Sci. USSR Phys. Ser. **45**(4), 23 (1981)].
[92] L. C. O'Drury and H. J. Völk, Astrophys. J. **248**, 344 (1981).
[93] W. I. Axford, E. Leer, and Y. F. McKenzie, Astron. Astrophys. **111**, 317 (1982).
[94] Y. F. McKenzie and H. J. Völk, Astron. Astrophys. **116**, 191 (1982).
[95] H. J. Völk, L. C. O'Drury, and Y. F. McKenzie, Astron. Astrophys. **130**, 19 (1984).
[96] H. J. Völk, Preprint MPI H-1984-V19, Max-Planck-Institut für Kernphysik, Heidelberg, 1984.
[97] J. A. Skilling, Astrophys. **170**, 265 (1971).
[98] A. Achterberg and C. A. Norman, Astron. Astrophys. **89**, 353 (1980).
[99] A. Achterberg, Astron. Astrophys. **76**, 195 (1981).
[100] V. S. Ptuskin, Astrophys. Space Sci. **76**, 265 (1981).
[101] M. A. Lee and H. J. Völk, Astrophys. Space Sci. **24**, 31 (1973).
[102] S. V. Bulanov and I. V. Sokolov, Astron. Zh. **61**, 882 (1984) [Sov. Astron. **28**, 515 (1984)].
[103] G. M. Webb, Astron. Astrophys. **127**, 94 (1983).
[104] S. V. Bulanov and I. V. Sokolov, Pis'ma Astron. Zh. **10**, 594 (1984) [Sov. Astron. Lett. **10**, 247 (1984)].
[105] G. F. Krymskiĭ, in: Proceedings of the Seventh European Symposium on Cosmic Rays (in Russian), Leningrad, 1980, p. 200.
[106] G. F. Krymskiĭ, Izv. Akad. Nauk SSSR. Ser. Fiz. **45**, 461 (1981) [Bull.

Acad. Sci. USSR Phys. Ser. **45**(4), 11 (1981)].
[107]D. C. Ellison, Thesis, Catholic University, Washington, 1981.
[108]D. C. Ellison, F. C. Jones, and D. Eichler, J. Geophys. **50**, 110 (1981).
[109]D. C. Ellison, J. Geophys. Res. Lett. **9**, 911 (1981).
[110]D. Eichler, Astrophys. J. **277**, 429 (1984).
[111]D. C. Ellison and D. Eichler, Astrophys. J. **286**, 691 (1984).
[112]E. G. Berezhko, V. K. Yelshin, G. F. Krymsky, and Yu. A. Romaschenko, in: Proceedings of the Eighteenth International Cosmic Ray Conference, Bangalore, 1983, Vol. 2, p. 259.
[113]E. G. Berezhko, V. K. Elshin, G. F. Krymskiĭ, and Yu. A. Romaschenko, Izv. Akad. Nauk SSSR. Ser. Fiz. **48**, 2221 (1984) [Bull. Acad. Sci. USSR Phys. Ser. **48**(11), 155 (1984)].
[114]D. C. Ellison, J. Geophys. Res. **90**, 29 (1985).
[115]D. C. Ellison, F. C. Jones, and D. Eichler, in: Proceedings of the Eighteenth International Cosmic Ray Conference, Bangalore, 1983, Vol. 2, p. 271.
[116]E. G. Berezhko, V. K. Yelshin, G. F. Krymsky, and A. A. Turpanov, in: Proceedings of the Nineteenth International Cosmic Ray Conference, La Jolla, 1985, Vol. 3, p. 152.
[117]D. Eichler and K. Hainebach, Phys. Rev. Lett. **47**, 1560 (1981).
[118]D. Eichler, Astrophys. **229**, 419 (1979).
[119]R. D. Blandford, Astrophys. J. **238**, 410 (1980).
[120]D. Eichler, Astrophys. J. **244**, 711 (1981).
[121]A. F. Heavens, Mon. Not. RAS **204**, 699 (1983).
[122]A. Achterberg, R. Blandford, and V. Periwal, Astron. Astrophys. **132**, 97 (1984).
[123]G. F. Krymsky, Adv. Space Res. **4**, 175 (1984).
[124]D. Eichler, Astrophys. J. **294**, 40 (1985).
[125]D. C. Ellison and D. Eichler, J. Geophys. Res. **3**, 124 (1985); Phys. Rev. Lett. **51**, 2735 (1985).
[126]A. A. Galeev, Zh. Eksp. Teor. Fiz. **86**, 1655 (1984) [Sov. Phys. JETP **59**, 965 (1984)].
[127]B. É. Gribov, R. Z. Sagdeev, K. Sëge, V. D. Shapiro, and V. I. Shevchenko, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 230 (1986) [JETP Lett. **43**, 291 (1986)].
[128]E. N. Kruchina, R. Z. Sagdeev, and V. D. Shapiro, Zh. Eksp. Teor. Fiz. **88**, 789 (1985) [Sov. Phys. JETP **61**, 464 (1985)].
[129]S. L. Musher, A. M. Rubenchik, and I. Ya. Shapiro, Zh. Eksp. Teor. Fiz. **90**, 890 (1986) [Sov. Phys. JETP **63**, 519 (1986)].
[130]E. T. Sarris and J. A. Van Allen, J. Geophys. Res. **79**, 4157 (1974).
[131]S.-I. Akasofu and S. Chapman, *Solar-Terrestrial Physics*, Oxford Univ. Press, 1972 [Russ. transl., Nauka, M., 1974].
[132]M. A. Lee, J. Geophys. Res. **87**, 5063 (1982).
[133]M. F. Scholer, F. M. Ipavich, G. Glochler, and D. Hovestadt, J. Geophys. Res. **88**, 1977 (1983).
[134]P. van Nes, R. Reinhard, T. R. Sanderson, and K.-P. Wenzel, J. Geophys. Res. **89**, 2122 (1984).
[135]B. T. Tsurutani and R. P. Lin, J. Geophys. Res. **90**, 1 (1985).
[136]T. R. Sunderson, R. Reinhard, P. van Nes, and K.-P. Wenzel, J. Geophys. Res. **89**, 19 (1985).
[137]M. A. Lee, J. Geophys. Res. **88**, 6109 (1983).
[138]Y. T. Gosling, J. R. Asbridge, S. J. Bame, W. C. Feldman, R. D. Zwickl, G. Paschmann, N. Sokopke, and P. J. Hunds, J. Geophys. Res. **86**, 547 (1981).
[139]M. A. Lee and L. A. Fisk, Space Sci. Rev. **32**, 205 (1982).
[140]T. Bai, H. S. Hudson, R. M. Pelling, R. P. Lin, R. A. Schwarts, and T. T. Van Rosenvinge, Astrophys. J. **267**, 433 (1983).
[141]D. C. Ellison and R. Ramaty, J. Geophys. Res. **4**, 6 (1985); Astrophys. J. **298**, 400 (1985).
[142]I. V. Dorman and L. I. Dorman, Izv. Akad. Nauk SSSR. Ser. Fiz. **31**, 1239 (1967) [Bull. Acad. Sci. USSR Phys. Ser. **31**, 1266 (1967)].
[143]J. R. Jokipii, Astrophys. J. **152**, 799 (1968).
[144]M. F. Bakhareva, Geomagn. Aeron. **15**, 393 (1975) [Geomagn. Aeron. (USSR) **15**, 327 (1975)].
[145]S. I. Petukhov, V. S. Nikolaev, and A. A. Turpanov, in: Proceedings of the Seventeenth International Cosmic Ray Conference, Paris, 1981, Vol. 3, p. 460.
[146]D. Eichler, M. E. Pesses, and J. R. Jokipii, in: Proceedings of the Seventeenth International Cosmic Ray Conference, Paris, 1981, Vol. 3, p. 463.
[147]S. I. Petukhov, A. A. Turpanov, and V. S. Nikolaev, Izv. Akad. Nauk SSSR. Ser. Fiz. **48**, 2066 (1984) [Bull. Acad. Sci. USSR Phys. Ser. **48**(11), 1 (1984)].
[148]S. I. Petukhov, A. A. Turpanov, and V. S. Nikolaev, J. Geophys. Res. **4**, 196 (1985).
[149]D. C. Hamilton, J. Geophys. Res. **82**, 2157 (1977).
[150]R. D. Zwickl and W. R. Webber, J. Geophys. Res. **83**, 1157 (1978).
[151]M. A. Zel'dovich and Yu. I. Logachev, Kosm. Issled. **21**, 803 (1983) [*Not* translated in Cosmic Res. (USSR) **21** (1983), but see Izv. Akad. Nauk SSSR Ser. Fiz. **47**, 1785 (1983) [Bull. Acad. Sci. USSR Phys. Ser. **47**(9), 109 (1983)].
[152]R. A. Chevalier, Ann. Rev. Astron. Astrophys. **15**, 175 (1977).
[153]R. D. Blandford and J. P. Ostriker, Astrophys. J. **237**, 793 (1980).
[154]T. Y. Bogdan and H. J. Völk, Astron. Astrophys. **112**, 129 (1983).
[155]H. Moraal and W. I. Axford, Astron. Astrophys. **125**, 204 (1983).
[156]V. L. Ginzburg, Ya. M. Khazan, and V. S. Ptuskin, Astrophys. Space Sci. **68**, 295 (1980).
[157]V. L. Ginzburg and V. S. Ptuskin, in: Proceedings of the Seventeenth International Cosmic Ray Conference, Paris, 1981, Vol. 2, p. 336.
[158]V. N. Fedorenko, Astrophys. Space Sci. **96**, 25 (1983).
[159]V. L. Ginzburg and V. S. Ptuskin, in: Scientific and Technological Progress. Astronomy Series (in Russian), VINITI Akad. Nauk SSSR, M., 1983, Vol. 24, p. 94.
[160]M. Garcia-Munoz, G. M. Mazon, and J. A. Simpson, Astrophys. J. **217**, 859 (1977).
[161]S. Hayakawa, Cosmic Ray Physics, J. Wiley, New York, 1969.
[162]R. Cowsik, Astrophys. J. **241**, 1195 (1980).
[163]D. Eichler, Astrophys. J. **237**, 809 (1980).
[164]I. Lerche and R. Schlickeiser, J. Geophys. Res. **3**, 226 (1985).
[165]R. Cowsik and M. A. Lee, in: Proceedings of the Seventeenth International Cosmic Ray Conference, Paris, 1981, Vol. 2, p. 318.
[166]J. R. Jokipii and G. E. Morfill, J. Geophys. Res. **90**, 29 (1985).
[167]D. Kazanas and D. C. Ellison, J. Geophys. Res. **90**, 128 (1985).
[168]J. F. McKenzie and G. M. Webb, J. Plasma Phys. **31**, 275 (1984).
[169]A. P. Zank and J. F. McKenzie, J. Geophys. Res. **3**, 111 (1985).
[170]E. A. Dorfi and L. C. O'Drury, J. Geophys. Res. **3**, 121 (1985).

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