Some episodes

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Dirac's response to a paper of Tamm on the Dirac-Kapitza effect (inverse Compton effect) is discussed. A remark of Skobel'tsyn on Dirac's "Recollections of an exciting era" is given.

The question of the physical meaning of the solutions to the relativistic electron equation put physicists in a difficult situation. Since the only positively charged particle known at the time was the proton, it was difficult to propose the existence of an antielectron—the positron—that had in no way manifested itself.

In this connection, it is interesting to give some extracts from correspondence between Dirac and I. E. Tamm from 1930. Dirac had become acquainted with Tamm already in Göttingen at the beginning of 1927. The acquaintanceship became friendship, and the two theoreticians made more than one walking tour in the Caucasian mountains.

The year 1927 was marked by an important event. Dirac published the paper that laid the foundation of quantum electrodynamics.[1] In particular, this paper gave a theoretical derivation of the Einstein coefficients and, therefore, of spontaneous emission. The theory now made it possible to obtain expressions for the absorption and emission of light without any additional hypotheses.

Tamm was attracted by the new ideas and occupied himself with theoretical investigation of the Compton effect—the scattering of light by electrons. This effect had been studied experimentally in detail in 1923 by Compton, who, using a Bragg spectrometer in his investigations, had succeeded in measuring the change $\Delta\lambda$ in the wavelength of the light as a result of scattering by an electron.[2,3] Compton had also analyzed the kinematics of the process. Using the notion of a photon)this was apparently the first practical use of the concept of photon momentum in kinematics)Compton obtained an expression for $\Delta\lambda$:

$$\Delta\lambda = \frac{2h}{mc}\sin^2\frac{\varphi}{2},$$

where φ is the scattering angle in the system in which the electron is initially at rest.[3] In fairness, it should be noted that Debye obtained the same formula at almost exactly the same time.[4]

Tamm obtained an expression for the scattering cross section and sent a paper to the *Zeitschrift für Physik*, in which it was published. The paper was called: "On the interaction of free electrons with radiation in accordance with the Dirac theory of the electron and in accordance with quantum electrodynamics."[5]

Even before this paper had appeared, Tamm had written to Dirac about the formula. It should be mentioned that a few months earlier the same journal had published a paper of the Swedish physicist Waller[6] that contained practically the same conclusion as Tamm had obtained. Dirac knew of Waller's calculations.

But neither the paper of Waller nor the paper of Tamm was the first. In 1929, the *Zeitschrift für Physik* published a famous paper of the Swedish theoretician Oskar Klein and his young Japanese collaborator Nishina with a formula that became known as the Klein-Nishina formula.[7]

However, Tamm's paper (like Waller's) was not simply a repetition. It contained an important improvement of the method.

Klein and Nishina had used a semiclassical method that at the time was generally accepted. It had been used by Dirac[8] himself in 1926, and by Gordon[9] in 1927 to derive an expression for the cross section of the Compton effect for a particle without spin (of course, in those years nothing was known about spin). To calculate the scattering of an electromagnetic wave by a charged particle, Maxwell's classical equations were used, but the transition current was substituted for the classical current on their right-hand side. The justification of this operation was that if in the ordinary formula for the current the wave function and its conjugate were replaced by their expansions in Fourier integrals the resulting double integral could be interpreted as the sum (integral) of all possible contributions (with different frequencies) to the total radiation.

This semiclassical method in no way took into account the quantum nature of the electromagnetic field and by its own nature did not involve calculations with intermediate states, the Compton effect being treated in a certain sense as a first-order effect.[1)] This fact was fortunate for the authors, since they did not encounter a paradox that surprised Tamm and Waller.

We know now that the quantum nature of the electromagnetic field is manifested in the higher approximations of perturbation theory in calculations of the influence of vacuum polarization and of level shifts (Lamb effect). Therefore, without the work of Tamm (and Waller) the further development of quantum electrodynamics would have been impossible. The entire scheme of calculations developed in these papers survived in the literature for many years.

As we have already said, Tamm acquainted Dirac with his, at that time, still unpublished results. In his answer of February 20, 1930, Dirac wrote as follows to Tamm:[2)]

"I think it very remarkable that you should have found a different formula for the scattering of radiation by a free electron. I have recently been looking into this question myself, and my work confirms the Klein-Nishina formula. I think you should still get this formula when you use the method of quantisation of waves. Are you sure you have not made a mistake?"

The problem of an error was clarified in a following letter of March 21 of the same year:

"I see on reading your letter again that v is volume and not velocity, which puts it right." But the point of interest is not this curious misunderstanding. Dirac was disturbed by something else. Believing that the proton corresponds to the states with negative energy, Dirac wrote:

"I do not understand why you say the m in the formula should be the mass of the electron and not of the protron. I

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should think it would be some sort of mean, as the theory is symmetrical between the electron and proton, and this would give the right energy for cosmic radiation. The theory at present predicts that an electron and protron should have the same mass, and is therefore inaccurate and unreliable in all questions where one has to take the different masses into account. I suppose the reason why the Klein-Nishina formula is right is because the process concerns only one particle, and not two interacting particles."

Although Tamm did note the difficulty with the interpretation of the negative-energy states in his paper, he exhibited confidence in the logical elegance of the theory and revealed no hesitation about the validity of formulas in which m denotes the electron mass.

To be more confident about the validity of the Dirac equation than the very author of the equation required at that time not a little courage. It is true that Tamm was eight years older than Dirac, but in the years during which the quantum theory was created age was, if anything, a disadvantage.

Tamm's formulas not only agreed with the Klein-Nishina formulas; they also enabled one to recognize particular properties of the interaction of the Dirac electron with the electromagnetic field. Tamm discovered the paradox, following the details of the transition to the old classical formula of Thomson.

At first glance, it could appear natural that the negative -energy levels should not play any role in the scattering process if the energy of the radiation is low. Thus, in this case one could apparently ignore the negative-energy states.

This was what was done in the calculations of Dirac and Gordon (mentioned above). For scalar particles, the problem with negative energies did not arise at all. The Waller-Tamm paradox revealed the deep meaning hidden in the relativistic Dirac equation.

The roots of the Waller-Tamm paradox lie in the fact that for a particle with half-integral spin the sign of the energy does not commute with the current operators and with the sign of the mass.

In the Dirac representation in which the energy matrix is diagonal, the current operators α and the mass (sign) operator β are not diagonal.

It is well known that this last circumstance has as consequences effects such as the Klein paradox)the production of pairs in a strong static field)and the so-called *zitterbewegung*—oscillations with frequency $2mc^2/\hbar$ of the wave function of an electron at rest.

In the Dirac representation, the current $\psi^*\alpha\psi$ is proportional to the expectation value $\langle\sigma(\sigma q)/(E+m)\rangle$ in the state with the given spin projection; q is the momentum transfer, and E is the energy of the electron. The current couples states with $m>0$ (first pair of components of the wave function) and states with $m<0$ (second pair) and becomes $\langle\sigma(\boldsymbol{\sigma}\cdot\mathbf{n})\tanh(a/2)\rangle$, where $\mathbf{n}$ is the unit vector of the direction of the rapidity of the electron in the final state, and a is its magnitude (velocity $v=\tanh a$).

This phenomenon is absent for particles with integral spin. The appearance of the states with negative mass in the Compton effect *shows that hidden properties, i.e., properties* not contained explicitly in the original assumptions are located in the solutions of the Dirac equation.

In connection with the Compton effect, it is also helpful to recall the paper that Dirac published together with Kapitza in 1933.[10] It was called: "The reflection of electrons from standing light waves." The paper was based on the beautiful idea of the part played by stimulated emission. A significant fraction of this short paper is devoted to a discussion (unfortunately with pessimistic conclusions) of the experimental possibilities of optics in those years. The creation of sufficiently strong electromagnetic fields became realistic only with the discovery of the laser. At the beginning of 1987, Bucksbaum *et al.*[11] published the results of the first demonstration of the Dirac-Kapitza effect.

The collision of an electron with an electromagnetic wave is described by the Klein-Nishina formula (with corresponding change of the initial and final states) only when the field intensity is low. If, in contrast, a laser electromagnetic field has such a high intensity that the number of photons in a vibrational mode is $N\gg 1$, then stimulated emission comes into play. A photon emitted by an electron cannot choose its direction (and, accordingly, final energy) arbitrarily, and the electron must, by virture of the stimulated nature of the radiation, emit a photon with the same wave vector as is characteristic of the standing wave. Representing the plane standing wave as a superposition of two waves with wave vectors $\mathbf{k}$ and $-\mathbf{k}$, one can describe the process of scattering of the electron by such a wave as the absorption of a photon from one wave and emission of a photon coherent with the second wave.

Thus, the process reduces effectively to transfer of a photon from one wave to the other.[3)] At the same time, the electron changes the component of its momentum normal to the wave front by the amount $2k$. This is none other than the Bragg condition for scattering of electrons by a diffraction grating. Therefore, the scattering can be described as the diffraction of the quantum wave of the electron by the classical (periodic) potential formed by the standing electromagnetic wave.

Thus, in the limit $N\to\infty$ the scattering of the electron can be equally well described in the classical picture and in the quantum picture and provides a good example of Bohr's correspondence principle.

In the experiments of Ref. 11, electron scattering by a traveling wave, and not diffraction by a standing wave, was observed. The Dirac-Kapitza effect (the role of stimulated emission) was demonstrated rather convincingly.

Finally, it is helpful to add an explanation to a statement on p. 145 of Dirac's Recollections[4)]. In discussion with me, D. V. Skobel'tsyn told me the following[5)]:

"... With regard to Dirac's recollections, the facts about which he speaks are in themselves correct. At Cambridge, he was present at the seminar (led, it appears, by Cockroft) at which I attempted to explain to the hearers the results of my experiments (made in collaboration with E. Stepanova) on the emission of positrons by a radioactive source. From what Dirac writes it is clear that he paid attention to my words. But he understood them quite differently from what I intended (I probably spoke in French). The main thing is that the event, which Dirac recalls perfectly correctly, occurred in 1934 (during the International Physics Congress at London), *and not in 1926–1927*, as Dirac said ... Dirac does not mention my name, referring to the words of the lecturer. There is no possibility that he could be referring to anybody but myself, since at that time I was a "monopolist" in experiments of such kind; nobody else was making experiments with α (or with β) rays in a Wilson chamber in a magnetic field."

[1)]The Klein-Nishina method did not in any way take into account the difference between spontaneous and stimulated emission and was, strictly speaking, incorrect. In Dirac's quantum electrodynamics, spontaneous emission occurred as a consequence of the commutation relations between the operators of the electromagnetic field.

[2)]*Editor's Note*: The author of this article kindly supplied to Sov. Phys. Usp. the actual wording of the extracts quoted in the article.

[3)]The electron can, in principle, also absorb two, three, or more photons, and this would lead to additional maxima in the cross section. Experimentally, they have not yet been observed.

[4)]See the reference to Dirac's "Recollections" under the heading "Other articles in this issue" in the Table of Contents.

[5)]Academician D. V. Skobel'tsyn has kindly allowed me to publish his remarks.

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[11]P. A. Bucksbaum, M. Bashkansky, *et al.*, Phys. Rev. Lett. **58**, 349 (1987).

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