EDITORIAL NOTE

The papers in the section "From the History of Physics" in this issue deal with quantum field theory, the 60th anniversary of the appearance of which is being marked this year, and the activity of one of its founders: the eminent English theoretical physicist P. A. M. Dirac (1902–1984).

P. A. M. Dirac and the formation of the basic ideas of quantum field theory

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This paper celebrates a double anniversary. In 1987, P. A. M. Dirac (1902–1984) would have been 85 years old and in addition it was 60 years ago that Dirac's paper on "The quantum theory of emission and absorption of radiation" appeared. That paper laid the foundation for modern quantum field theory. The appearance and evolution of the basic concepts and representations of quantum field theory are presented here more from a logical than a historical aspect. Special note is taken of the important role played by Dirac in this process.

CONTENTS

1. INTRODUCTION

It has now been 60 years since the appearance of P. A. M. Dirac's paper on "The theory of emission and absorption of radiation" [1] (1927b),[2] which laid the foundation for the modern theory of the interactions of microparticles. The author of this paper, one of the greatest contemporary theoreticians, whose highly unconventional approach to physical problems basically created the language which we use in any field of quantum theory, would have been 85 years old this year. The basic motivation which guided all of Dirac's work in physics seems to have been a deep-seated conviction that nature can be described in a simple and unified way. On the last page of the third edition of his great book *Principles of Quantum Mechanics* (1947) he wrote that a satisfactory theory should allow a simple solution for any simple physical problem. However, the simplicity for which he unflaggingly strived by no means appeared to him to be elementary. In a lengthy introduction to the article "Quantized Singularities in the Electromagnetic Field" (1931) Dirac wrote: "The steady progress of physics requires for its theoretical formulation a mathematics that gets continually more advanced. This is only natural and to be expected. What, however, was not expected by the scientific workers of the last century was the particular form that the line of advancement of the mathematics would take, namely, it was expected that the mathematics would get more and more complicated but would rest on a permanent basis of axioms and definitions, while actually the modern physical developments have required a mathematics that continually shifts its foundations and gets more abstract. Non-euclidean geometry and non-commutative algebra, which were at one time considered to be purely fictions of the mind and pastimes for logical thinkers, have now been found to be very necessary for the description of general facts of the physical world. It seems likely that this process of increasing abstraction will continue in the future and that advance in physics is to be associated with a continual modification and generalisation of the axioms at the base of the mathematics rather than with a logical development of any one mathematical scheme on a fixed foundation. There are at present fundamental problems in theoretical physics awaiting solution . . . which will presumably require more drastic revision of our fundamental concepts than any that have gone before. Quite likely these changes will be so great that it will be beyond the power of human intelligence to get the necessary new ideas by direct attempts to formulate the experimental data in mathematical terms. The theoretical worker in the future will therefore have to proceed in a more indirect way. The most powerful method of advance . . . is to employ all the resources of pure mathematics in attempts to perfect and generalise the mathematical formalism that forms the existing basis of theoretical physics, and after each success in this direction, to try to interpret the new mathematical features in terms of physical entities."

The reader will have to forgive us for this long quotation, but it would be difficult to devise a better formulation of a basic feature of actions of the type which are being taken

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today in fundamental theoretical physics.

Over the six decades of its existence, quantum field theory has changed in appearance several times. This process of evolution has touched not only the details but also, in a sense, the basic concepts. The process breaks up quite neatly into several successive stages.

In the first, which lasted about two decades, the basic problem was to extend the methods of quantum mechanics to relativistic systems with an infinitely large number of degrees of freedom. This was by no means the simple and nearly self-evident problem which it appears to be from the modern standpoint; it required the invention of many technical facilities which did not yet exist. It would be difficult to overestimate the role which Dirac played here. As R. Jost (1972) wrote: "Almost all important discoveries were made or independently also made by him." We might cite such elements of the "alphabet" of the modern theory as the δ-function (1927a), the general theory for transforming from one representation to another (1927a), Fermi-Dirac quantization (1926), second quantization (1927b), the relativistic wave equation for one particle in an external field (1928a, b), spinors (1928a), antiparticles (1930a, 1931), the multitime formalism, and the relativistically invariant form of writing the equations for a system of electrons interacting with an electromagnetic field (1932b).

However, the main obstacle along the path to transferring the methods of quantum mechanics to field systems was apparently not a matter of technical difficulties but the need to overcome the psychological barrier of the contrast between two forms of matter—particles and field—which are regarded as absolutely different entities from the classical standpoint. Extremely indicative in this sense is the following circumstance: While the idea of the fundamental concept of an operator-valued field came to Dirac as early as 1926, when he wrote: "It would appear to be possible to build up an electromagnetic theory in which the potentials of the field at a specific point x_0, y_0, z_0, t_0 in space–time are represented by matrices of constant elements that are functions of x_0, y_0, z_0, t_0" (1926)—and in 1927 Dirac subjected the variables describing a field to second quantization (1927b)—six years later he raised a decisive objection that Heisenberg and Pauli "regard the field itself as a dynamical system amenable to Hamiltonian treatment..... so that the usual methods of Hamiltonian quantum mechanics may be applied. There are serious objections to these views.... We cannot... suppose the field to be a dynamical system on the same footing as the particles ... The field should appear in the theory as something more elementary and fundamental" (1932a).

These 15–20 years were actually a time of an agonizing development of a fundamental new paradigm (and of becoming accustomed to it) in which classical particles and fields come to have completely equal rights as two different manifestations of a single unitary object: a quantized field. The new understanding of a basic organizational mechanism of nature was developed by various people in small pieces, which only gradually combined to form a unified picture. The method of second quantization of the amplitudes of an expansion in a Fourier integral which was developed by Dirac (1927b) in application to the electromagnetic field and by Jordan (1927) and Jordan and Klein (1927) in application to the field of electrons developed into a common theory of an arbitrary free quantum field. From this standpoint, Heisenberg and Pauli (1929, 1930) constructed a general scheme for the quantization of a field with an arbitrary Lagrangian (a canonical formalism) which is not explicitly relativisticaly covariant. For the interaction of electrons with an electromagnetic field, this deficiency was made up in the multitime formalism of Dirac, Fock, and Podolsky (1932b). At the same time, unitary views acquired some independent support from the physics of elementary particles. Fermi (1934) discussed the β-decay process as the production of an electron and a new particle: a neutrino. Yukawa (1935), using Tamm's (1934) and Iwanenko's (1934) idea regarding the exchange nature of nuclear forces proposed, in order to explain the forces of attraction between nucleons, to introduce new particles—mesons (which, as it was thought at the time, were soon discovered in cosmic rays). In this manner the number and variety of entities which came to be included in the concept of a quantized field increased rapidly.

Nevertheless, a systematic and final formulation of this new paradigm was in no hurry to appear. When we look at the reviews published at the end of this first stage of the evolution we see that Pauli (1941) was basically setting forth the theory of free quantized fields (in essentially its modern form). He limited himself to a brief mention of the results of some calculations on interaction processes, without even attempting to give a complete formulation of the problem. In Wentzel's book (1943) the problem of the interaction of electrons with an electromagnetic field was still being treated by a multitime formalism; only in one of the last sections did a second-quantized electron-positron field participate.

Practical calculations on real effects were carried out primarily by means of the perturbation theory developed by Dirac (1926, 1927b) for time-dependent perturbations. That theory corresponds to the method of the variation of constants in the theory of linear differential equations. After the appearance of the relativistic Dirac equation (1928a, b), calculations were carried out on several effects of electromagnetic interactions of electrons: the scattering of light by an electron (Klein and Nishina, 1929; Tamm, 1930) and the annihilation of an electron–positron pair [Dirac (1930b), Tamm (1930), and Oppenheimer (1930); the authors thought that they were carrying out calculations on the annihilation of electrons and protons]. Bethe and Heitler (1934) carried out calculations on the bremsstrahlung of electrons in the field of a nucleus and on the production of γ-ray pairs in the field of a nucleus. Calculations on the latter effects were also carried out by Racah (1934, 1936) and Nishina, Tomonaga, and Sacata (1934). The scattering of electrons by electrons was studied by Møller (1932).

In all these cases, the results found in lowest-order perturbation theory turned out to agree well with experimental data, thereby confirming that this new theory was sound. However, attempts to refine the predictions through calculations of higher-order approximations led to integrals which diverge at large momenta: *ultraviolet divergences.*

Pauli (1933) tells us that Ehrenfest noted, immediately after the appearance of a paper by Dirac (1927b), that it contained the concept of a point electron and would therefore lead to an infinite self-energy for the electron. The same point was emphasized by Dirac (1932a), who noted that in the classical problem of the interaction of an electron with a radiation field the equations "that determine the field produced by the electron . . . . are quite definite and unambiguous, but . . . (the equations) that determine the motion of the electron . . . . express the acceleration of the electron in terms of field quantities at the point where the electron is situated and these field quantities in the complete classical picture are infinite and undefined." In a sense, these pessimistic predictions were justified. It is true that Weisskopf (1934) managed to show that when a Dirac vacuum is taken into account the self-energy of an electron diverges only logarithmically, so that even with a cutoff at the Schwarzschild radius its increment in the "mechanical" mass remains small. However, any cutoff would have to violate the relativistic invariance of the theory.

In his Solvay report in 1933, Dirac (1934b) stated that external charges should polarize the vacuum in his theory, with the result that the electric charges which are normally observable for the electron, the proton, and other electrified particles are not the charges which are actually carried by these particles and which figure in the fundamental equations; they are instead smaller. A calculation which he carried out on this new physical effect again reduced to a logarithmically divergent integral, whose cutoff at momenta of the order of 100 *mc* (corresponding to the classical radius of an electron) yields a "radiation correction" to the charge of an electron, which reduced it by a fraction of about 1/137. Calculations on the "field" self-energy of a photon also led to an infinite result and again violated gauge invariance.

As early as the mid-1930s, there were suggestions (Weisskopf, 1936; Euler, 1936) that the infinities in higher orders for the observable effects were traces of these fundamental ultraviolet divergences and that they could be eliminated by subtracting from the infinite quantity for a bound electron the corresponding infinite quantity for a free electron (Kramers, 1938; Stückelberg, 1935, 1938). This was the basic idea of the renormalization method. However, the subtraction of certain infinities from others is such a delicate and at the same time not totally natural operation that its widespread acceptance had to await clear successes in explaining observable effects. A definite step in this direction was taken by the famous study by Bethe (1947), who calculated the so-called fine shift of the S level in hydrogenlike atoms, and also the study by Schwinger (1948), which contains several other results, including the radiation correction to the magnetic moment of the electron. Each of these effects had been reliably established experimentally not long beforehand through the use of microwave techniques in problems of atomic spectroscopy. An important role was played in the formation of the renormalization method by the appearance of a new and explicitly covariant form of the basic equations of quantum electrodynamics, which appeared at approximately the same time thanks to studies by Tomonaga (1946), Schwinger (1948, 1949), Feynman (1948a, b; 1949a, b), and Dyson (1949a, b), which marked the beginning of a new stage in the development of quantum field theory. Over the following decade, this theory evolved essentially completely into its modern form.

2. QUANTIZED FIELDS

A quantum (or quantized) field is a sort of synthesis of the concepts of a classical field of the electromagnetic type and of a probability field of quantum mechanics. According to the present understanding, it is the most fundamental and universal form of matter, underlying all specific manifestations of matter.

The concept of a classical field arose at the heart of the Faraday-Maxwell theory of electromagnetism, in the course of the rejection of an ether as a material carrier of electromagnetic processes. This rejection was forced by the negative results of Michelson's experiment and the derivation of the special theory of relativity. A fundamentally new point was that the field had to be regarded not as a form of motion of some medium but as a special form of matter with extremely unusual properties: In contrast with particles, a classical field is produced and annihilated without hindrance (it is emitted and absorbed by charges), has an infinite number of degrees of freedom, and is not localized at certain points but can instead propagate through space, transmitting the interaction (signal) from one particle to another at a finite velocity (which does not exceed the velocity of light, c).

From the logic standpoint, the unavoidability of the field concept follows directly from the impossibility of transmitting signals at a velocity greater than the velocity of light, which follows from the special theory of relativity. If we discard the Newtonian *actio in distans*, i.e., if we discard the idea of an instantaneous action of particles on each other at a distance, we find that we are forced to fill the space between the interacting particles with some agent which transmits this interaction from point to point: a relativistic field. For a mathematical description of a relativistic field we need to choose some representation of a Lorentz group whose "vectors" depend continuously on the spatial point and the instant of time.

The advent of quantum ideas led to a reexamination of the classical electromagnetic concepts regarding the mechanism for the emission and absorption of light, and it also led to the conclusion that these processes occur not continuously but through the emission and absorption of discrete portions of an electromagnetic field: photons. The contradictory picture which developed, according to which particles (photons) had to be associated with the electromagnetic field, while certain effects could be interpreted only in terms of electromagnetic waves, and others only in terms of particles, was called the "corpuscle-wave dualism." A resolution of this contradiction had to be sought in the direction of a systematic application of the rules of quantum mechanics to the electromagnetic field: the replacement of the dynamic variables of the electromagnetic field—the potentials **A** and φ and the fields **E** and **H**—by quantum-mechanical operators obeying corresponding commutation relations (1926).[3] This is what was done in Dirac's paper on "The quantum theory of the emission and absorption of radiation" (1927b), where a new method was developed for generating a quantum-mechanical description of an ensemble of identical systems. The essence of this "method of second quantization" was that the role of the dynamic coordinates would be played not by the coordinates of an individual system but by the numbers (the quantity) of systems in definite states, i.e., "occupation numbers." The application of this method to an expansion in oscillators of an electromagnetic field interacting with sources made it possible for the first time to calculate Einstein's A and B coefficients in a systematic way, not based on the correspondence principle, and to establish that "the Hamiltonian which describes the interaction of the atom and the electromagnetic waves can be made identical with the Hamiltonian for the problem of the interaction of the atom with an assembly of particles moving with the velocity of light and satisfying the Bose-Einstein statistics." The result was to resolve the problem of the corpuscle-wave dualism. The major reason for the importance of that study, however, is that a completely new physical entity emerged from it: a *quantized field*, This field satisfies the equations of classical electrodynamics but has its own values of the quantum-mechanical operators, which operate on a Schrödinger function, which in this case is frequently called a "state amplitude." It is for this reason that we regard the appearance of that paper as the birth of quantum field theory.

A second source of the general concept of a quantized field was the quantum-mechanical wave function of a nonrelativistic particle, which satisfies the famous equation proposed by Schrödinger. We recall that this is not an independent physical quantity but the state amplitude of a particle; probabilities for any physical quantities pertaining to the particle are written in terms of bilinear expressions in this amplitude. In quantum mechanics, a new wave field turned out to be associated with each material particle, although now, it is true, the field was a field of probability amplitudes.

The extension of the methods of quantum mechanics to problems containing not one particle but N particles made it necessary to examine the propagation of a field of probability amplitudes not in ordinary three-dimensional space but in a configuration space of $3N$ dimensions (or, correspondingly, $4N$ dimensions, in the relativistic case). The use of a description method of this sort leads to some rather cumbersome mathematical constructions which are not distinguished by their transparency. Again in this case, the use of the second quantization method proposed by Dirac (1927b), which had been extended by Wigner and Jordan (1928) to an ensemble of fermions, makes it possible to replace the field of amplitudes in a $3N$-dimensional space (if the N particles under consideration satisfy the indistinguishability principle) by a new field in ordinary 3-space which is an operator in the quantum-mechanical sense. In other words, even nonrelativistic quantum theory leads us in a natural way in multiparticle problems to the same concept of an operator-valued quantized field, although this field is nonrelativistic and conserves the number of particles.

Dirac's discovery (1928a, b) of a relativistic wave equation for the electron was an entire chapter in the new history of physics, whose description requires a separate paper. Here we will simply touch on those aspects of this discovery which pertain directly to field theory. As Dirac himself stated on more than one occasion, this was one of those cases—not all that rare in the history of science—in which an equation turned out to be far "smarter" than its inventor, and the true worth of its content went far beyond its original purposes. The problem which Dirac took up in formulating his equation was the fairly modest one of writing an equation which would correctly describe the behavior of a single relativistic electron in external force fields and which would yield a natural explanation for the spin of the electron. This problem was indeed solved, and exhaustively. In the process, however, three monumental discoveries were made; the importance of two of them would not be realized for some time.

First, irreducible representations of the Lorentz group of a new class—spinors—were rediscovered (not only for physicists but also for mathematicians interested in physics).[4] The second discovery, whose worth was immediately recognized, was the discovery that spin is—somewhat freely expressed—a "kinematic inevitability" for particles which are describable by a spinor representation. The third discovery was that the equation had, in addition to the "respectable" eigenstates with a positive energy, an identical spectrum but with energies $\leqslant -mc^2$. This discovery was originally perceived as a grave flaw in the theory; only after some false starts (1930a, b) was this discovery finally interpreted correctly, thanks primarily to the efforts of Dirac (1931, 1934b). In the first place, it is, strictly speaking, quite wrong to pose the problem of a single body in a relativistic theory. Particles—this may be the most characteristic distinction between relativistic quantum theory and the nonrelativistic theory—can be created and annihilated, just as photons of the electromagnetic field can be. Second, it turned out that for any relativistic charged particle there is necessarily a twin particle—an antiparticle—so that a pair can appear (and annihilate).

The last and most profound consequences of the Dirac equation becomes obvious when we apply the method of second quantization to it. Since electrons obey the Pauli principle and are described by Fermi statistics, we should use the method of Wigner and Jordan (1928). A corresponding study was carried out by Dirac (1934b) and Heisenberg (1934). As a result, the four-component operator-valued spinor field became part of the arsenal of the theory. This field now described electrons and positrons in a completely symmetric way.

Looking back, it is now easy to see that it would have been sufficient to recognize that an operator field which implements some local representation or other of the Lorentz group and which has a quantum-mechanical operator meaning would have to be associated with each species of relativistic particle. Such an operator field would be completely analogous to a quantized electromagnetic field, differing from it only, generally speaking, in the behavior under Lorentz transformations, possibly in quantization method, and in the values of the constants in the equations of motion. Like an electromagnetic field, it would be called upon to describe an entire set of indistinguishable particles (and indistinguishable antiparticles) of the given species.

In reality, however, as was mentioned in the Introduction, this new paradigm won general recognition only after a great deal of labor, and it took its final form perhaps only in the first series of studies by Schwinger (1948, 1949a, b). It was only then—and to a large extent due to the clarity of Feynman diagrams—that a picture of a common universal structure of all matter penetrated into the consciousness of physicists. Common physical entities—quantized fields in ordinary space-time—take the place of both the fields and the particles of classical physics; there is a single quantized field for each species of particle or field. With regard to interactions, we note that the elementary event is always an interaction of several fields of one or different species at a common space-time point or—in corpuscular terms—an instantaneous and local conversion of certain particles into others. On the other hand, the familiar interaction in the form of forces exerted by one particle on another is a secondary effect (Dirac, 1932a) which arises because two particles are exchanged as a result of sequential events of emission and absorption by third particles, generally of different species.

3. WAVE FIELDS AND CANONICAL QUANTIZATION

In accordance with the dual nature of quantized fields, a systematic exposition can be based on either corpuscular or field initial representations.

In the *field approach* it is first necessary to derive a theory for the corresponding classical field and then subject it to a canonical quantization in the procedure of Heisenberg and Pauli (1929, 1930). The final step is to develop a corpuscular interpretation for the resulting quantized field.

The primary initial concept here is the field $u^a(x)$, which is defined at each 4-point $x = (x^0 = ct, \mathbf{x})$ and which implements some fairly simple tensor or spinor representation of the Lorentz group (scalar, vector, bispinor, etc.). The index a specifies both the components of this representation and possible internal degrees of freedom. A corresponding covariant theory is constructed (whether this is done in the classical case or in the quantum case, through the use of the Heisenberg picture, is irrelevant) essentially automatically by means of a four-dimensional Lagrangian formalism in which the time and the spatial coordinates are treated in an absolutely symmetric way as independent variables (a mechanics with a finite number of degrees of freedom on a "four-dimensional time"). One chooses a local Lagrangian $L(x) = L(u^a(x), u^a_{,\mu}(x))$, ["local" here means that it depends on only the field components $u^a(\mathbf{x})$ and their first derivatives, $\partial u^a(x)/\partial x^\mu = \partial_\mu u^a(x) = u^a_{,\mu}(x)$, all taken at the same point x] and requires that it be invariant under the Poincaré group and the transformation group of the internal symmetries (if such exist). An integration over the 4-volume yields the action:

$$S = \int_R L(x)\, d^4\boldsymbol{x}. \tag{1}$$

One requires an extremum of this action, $\delta S = 0$, with respect to arbitrary variations $\delta u^a(x)$ which vanish at the boundaries of integration region R. As a result one finds the explicitly covariant equations of motion

$$\frac{\partial L}{\partial u^a(x)} - \frac{d}{dx^\mu}\frac{\partial L}{\partial u^a_{,\mu}(x)} = 0. \tag{2}$$

The next step is to appeal to the Nöther theorem (1918), according to which it follows from the invariance of action (1) for an arbitrary integration region R under a k-parameter continuous group of transformations of functions of the field $u^a(x)$ and independent variables x^μ that there exist k Nöther currents $J^\mu_i (i = 1, ..., k)$ with a vanishing divergence which are explicitly specified by the theorem. The future-oriented flux of each Nöther current across any spacelike hypersurface which goes off along spatial directions to infinity forms an integral quantity—the Nöther charge—which characterizes the field and which does not depend on the choice of hypersurface. It follows that: 1) Nöther charges are integrals of motion and 2) their behavior under transformations of the group is contravariant with respect to the transformation parameters.

In relativistic field theory, the action is always required to be invariant under the ten-parameter Poincaré group, so that ten Nöther charges are necessarily conserved. Dirac called them fundamental dynamic quantities: It follows from invariance under four displacements that four components of the energy-momentum vector P_μ are conserved. It follows from invariance under six rotations that six components of the four-dimensional angular momentum are conserved [three components of the three-dimensional angular momentum $M^i = \varepsilon^{ijk} M_{jk}/2$ and three Lorentz angular momenta ("boosts") $N_k = M_{0k}/c$].

The theory becomes richer in content if the action is also invariant when other continuous transformations (which are not part of the Poincaré group)—transformations of internal symmetries—are performed on the field under consideration. It then follows from the Nöther theorem that new conserved charges exist. It is frequently assumed, for example, that the field functions are complex: the Lagrangian is required to be Hermitian; and the action is required to be invariant under a gauge transformation of the first kind (a phase transformation), $u^a \to e^{i\alpha}u^a$, $u^{*a} \to u^{*a}e^{-i\alpha}$, where α is an x-independent phase. It then turns out that charge is conserved as a consequence of the Nöther theorem:

$$Q = i\int d\mathbf{x} \sum_a \left(\overset{*}{u}{}^a \frac{\partial L}{\partial \overset{*}{u}{}^a_{,0}} - u^a \frac{\partial L}{\partial u^a_{,0}}\right).$$

The complex functions $u^a(x)$ can therefore be used to describe charged fields. The same purpose can of course be served by expanding the region of values taken on by the superscript a in such a way that it also specifies the direction in isotopic space and by requiring that the action be invariant under rotations in it. Everything which has been said up to this point applies equally well to a classical field and a quantized field treated in the Heisenberg picture.

The *canonical quantization*, according to an interpretation of quantum mechanics developed by Dirac in his very first quantum-mechanical paper (1925), consists of the following for any classical dynamic system: All its dynamic variables, the observables A, B, ..., begin to be regarded not as ordinary c-numbers but as entities of a new algebra, q-numbers, which do not commute with each other. In order to retain the dynamic apparatus of Hamiltonian mechanics, in particular, all the properties of Poisson brackets, it is necessary that the quantum Poisson brackets of any pair of q-numbers be proportional to their commutator. In other words, it is necessary to make the substitution

$$A_c, B_c, \ldots; \{A_c, B_c\}, \ldots \Rightarrow A_q, B_q, \ldots; \{A_q, B_q\} = \frac{i}{\hbar}(A_q B_q - B_q A_q) \equiv \frac{i}{\hbar}[A_q, B_q], \ldots$$

The quantum Poisson brackets for canonically conjugate coordinates and momenta retain their classical values:

$$\{p_a, p_b\}_q = i\hbar^{-1}[p_a, p_b]_- = 0,$$
$$\{q_a, q_b\}_q = i\hbar^{-1}[q_a, q_b]_- = 0,$$
$$\{p_a, q_b\}_q = i\hbar^{-1}[p_a, q_b]_- = \delta_{ab}.$$

For a Hamiltonian treatment of a field system it is no longer possible to treat the time and the spatial coordinates in variational principle (1) as independent variables of equal status; i.e., it is necessary to violate the four-dimensional symmetry, retaining the time alone, while the spatial coordinates are adopted as continuous indices which number the degrees of freedom[5] (a mechanics with a continuous number of degrees of freedom on a one-dimensional time) and accordingly to write

$$S=\int \mathrm{d}t\mathscr{L}\left(u^a(\mathbf{x},\ t),\ u^a_{,i}(\mathbf{x},\ t);\ \dot{u}^a(\mathbf{x},\ t)\right),$$

where the Lagrange function (more precisely, the Lagrange functional) is

$$\mathscr{L}\left(u^a(\mathbf{x},\ t),\ u^a_{,i}(\mathbf{x},\ t);\ \dot{u}^a(\mathbf{x},\ t)\right)$$

$$=\int_{t=\text{const}} \mathrm{d}^3\mathbf{x}L\left(u^a(\mathbf{x},\ t);\ u^a_{,\mu}(\mathbf{x},\ t)\right)=\int \mathrm{d}^3\mathbf{x}L(\mathbf{x},\ t).$$

The generalized coordinates are now an infinite set of values of all of the field components $u^1,\ldots,u^A$ at all spatial points $\mathbf{x}$ at a certain time t; their canonically conjugate momenta are (functional) derivatives of the Lagrange function with respect to the generalized velocities,

$$\pi_b(\mathbf{x},\ t)=\frac{\delta\mathscr{L}(t)}{\delta\dot{u}^b(\mathbf{x},\ t)}=\frac{\partial L(\mathbf{x},\ t)}{\partial\dot{u}^b(\mathbf{x},\ t)}, \qquad (3)$$

and the Hamilton's function (Hamiltonian) is

$$\mathscr{H}(\pi,\ u)=\int \mathrm{d}^3\mathbf{x}\sum_a \pi_a(\mathbf{x},\ t)\dot{u}^a(\mathbf{x},\ t)-\mathscr{L}$$

$$=\int \mathrm{d}^3\mathbf{x}\left(\sum_a \pi_a\dot{u}^a-L\right).$$

It is understood here that all the generalized velocities $\dot{u}^a(\mathbf{x},t)$ are expressed by means of (3) in terms of the generalized coordinates $u^a(\mathbf{x},t)$ and momenta $\pi_b(\mathbf{x},t)$.

We can now apply the general method of canonical quantization to the field system, replacing the classical generalized coordinates and momenta by q-numbers which satisfy commutation relations (Heisenberg and Pauli, 1929, 1930).

$$u^a(\mathbf{x},\ t)\pi_b(\mathbf{y},\ t)\mp\pi_b(\mathbf{y},\ t)u^a(\mathbf{x},\ t)=i\hbar\delta(\mathbf{x}-\mathbf{y})\delta^a_b. \qquad (4)$$

The choice of sign, $-$ or $+$, corresponds to Bose-Einstein quantization or Fermi-Dirac quantization (more on this below).

It was assumed above that Eqs. (3) can be solved for all the generalized velocities. If it is not possible to determine all the velocities from these equations, the remaining relations are equations imposed on the generalized momentum and coordinates: constraints. A case of this sort had already been encountered in the quantization of the electromagnetic field, and it required no little inventiveness (Heisenberg and Pauli, 1929, 1920; Fermi, 1929, 1930) in order to avoid its contrived constructions. A regular method for constructing a Hamiltonian formalism and for carrying out quantization for systems with constraints was developed by Dirac beginning in 1950 (1950, 1951a, 1958, 1964). In later years, this method proved to be of decisive importance for constructing a theory of gauge fields (more on this below).

The Hamiltonian form of the theory, in which canonical commutation relations (4) are formulated, violates the explicit relativistic symmetry (as we have already stressed) because of the special role played by the time and by the appeal to a specific frame of reference. A special proof is thus required in order to retain relativistic invariance.[6] Furthermore, relations (4) tell us nothing about the commutation properties of the fields at 4-points separated by timelike intervals: The values of the fields at such points are related by causal dependences, and their commutation relations can be found only by solving the equations of motion jointly with (4).

4. FREE FIELDS

For free fields, for which the Lagrangian is quadratic in the field functions, and equations of motion (2) are linear, a problem of this sort can be solved in its general form. It becomes possible to establish—in a relativistically symmetric form—the commutation relations for fields at two arbitrary 4-points x and y:

$$u^a(x)u^b(y)\mp u^b(y)u^a(x)=-iP^{ab}\left(\frac{\partial}{\partial x}\right)\Delta_m(x-y). \qquad (5)$$

Here $\Delta_m(x)$ is the invariant commutation function of Jordan and Pauli (1928) (explicit expressions for it were studied by Dirac in 1934b), which satisfies the Klein-Gordon equation $(\Box-m^2)\ \Delta_m=0$, and P^{ab} is a polynomial—there is a specific one for each field—which makes the right side of commutation relations (5) satisfy equations of motion (2) in x and y.

In the *corpuscular approach* to a relativistic quantum description of free particles, the states of the particle must form an irreducible representation of the Poincaré group. An irreducible representation of the group is characterized by specifying the values of the Casimir operators (operators which commute with all ten generators of the group: P^μ, M^i, N^j): for the Poincaré group, there are two such operators: the square of the mass, $m^2=P^\mu P_\mu$, and the square of the ordinary 3-spin (for a zero mass, the helicity operator plays the role of the second "casimir"). By specifying their values, we fix the representation, i.e., the "species" of the particle. The spectrum of the first of them is continuous: The square of the mass can take on any nonnegative values $m^2\geqslant 0$. The spectrum of the spin is discrete; it can take on only the integer or half-integer values 0, 1/2, 1, ... Furthermore, it is necessary to specify the behavior of an odd number of coordinate axes under reflection. If no other characteristics have to be specified, one says that the particle has no internal degrees of freedom, and it is "truly neutral." Otherwise, the particle has charges of some type or other.

In order to fix the state of a particle within a representation, it is necessary in quantum mechanics to specify the values of a complete set of commuting operators. For a free particle, it is convenient to choose the three components of the momentum $\mathbf{p}$ and the projection s of the spin l_s onto some direction. The state of a single free truly neutral particle is thus characterized completely by specifying the six numbers m, l_s, p_x, p_y, p_z, s. The first two of these numbers characterize the representation, while the last four characterize the state in it. For charged particles, one adds some other quantum numbers, which we denote by the single letter t.

A straightforward extension of these arguments to a system of n particles would result in the use of n sextets, one for each particle. In 1927 Dirac suggested that the state of an ensemble of n identical particles be characterized not by the state of each particle but by the number of particles,[7] $n_{\mathbf{p},s,t}$—occupation numbers—in each of the one-particle states (1927b). The "*interpretation*" *of a wave function which depends on such variables gives us not simply the expected numbers of particles in each state but the probability for any given distribution of particles among different states. This* probability is actually the square of the modulus of the normalized solution of the wave equation. It is in the probabilities of such distributions that Bose-Einstein [or Fermi-Dirac (1926)] quantum statistics deviates from classical statistics. This procedure is called *second quantization*, apparently because Dirac (1926, 1927b) carried out this procedure as a transition to a quantum-mechanical description of an electromagnetic field in the problem—already quantum-mechanical—of the interaction of electrons with this field.

In the occupation-number representation, a state $|n_{\mathbf{p},s,t}\rangle$ is written as the result

$$|n_{\mathbf{p},s,t}\rangle = (n_{\mathbf{p},s,t})^{-1/2}[a^+(\mathbf{p},s,t)]^{n_{\mathbf{p},s,t}}|0\rangle \qquad (6)$$

of an action on a *vacuum state* (i.e., a state in which there are no particles at all) of the *creation operators* $a^+(\mathbf{p},s,t)$. The creation operators a^+ and their Hermitian-conjugate *annihilation operators* a^-, were introduced by Dirac in the same paper (1927b), which we mark as the beginning of the age of quantum field theory.[8] They satisfy the commutation relations

$$[a^-(\mathbf{p},s,t),\ a^+(\mathbf{p}',s',t')]_{\mp} = \delta_{ss'}\delta_{tt'}\delta(\mathbf{p}-\mathbf{p}'), \qquad (7)$$

where the minus sign corresponds to Bose-Einstein quantization (Dirac, 1927b), and the plus sign to Fermi-Dirac quantization (Jordan, 1927; Wigner and Jordan, 1928). The occupation numbers themselves are the eigenvalues of the particle-number operators $\hat{n}_{\mathbf{p},s,t} = a^+(\mathbf{p},s,t)\,a^-(\mathbf{p},s,t)$. The state amplitude of a system containing precisely one particle with each set of quantum numbers $\mathbf{p}_1, s_1, t_1; \mathbf{p}_2, s_2, t_2; \ldots$, is then written

$$|\mathbf{p}_1, s_1, t_1; \ldots; \mathbf{p}_k, s_k, t_k; \ldots\rangle$$
$$= a^+(\mathbf{p}_1, s_1, t_1)\ldots a^+(\mathbf{p}_k, s_k, t_k)\ldots|0\rangle.$$

The creation and annihilation operators $a^\pm$ describe particles with definite momentum and spin values. To take the local properties into account, we need to put the $a^\pm$ in the coordinate representation. As transformation functions it is convenient to use the classical solutions of the equations of motion (2) of a suitable free field with tensor (or spinor) indices a and internal indices τ. The creation and annihilation operators are then written in coordinate space as

$$u^{a\tau(+)}(x) = (2\pi)^{-3/2}\int d^3\mathbf{p}e^{ipx}u^{a\tau}_{s,t}(\mathbf{p})\,a^{(+)}_{s,t}(\mathbf{p});$$
$$u^{a\tau(-)}(x) = (2\pi)^{-3/2}\int d^3\mathbf{p}e^{-ipx}\overset{*}{u}{}^{a\tau}_{s,t}(\mathbf{p})\,a^{(-)}_{s,t}(\mathbf{p}); \qquad (8)$$
$$p^0 = +(\mathbf{p}^2+m^2)^{1/2}.$$

These operators, however, are still not what we need to construct a local field theory: Their commutator or anticommutator does not vanish for spacelike x, y point pairs.[9] Accordingly, the values of (8) at different spatial points at the same time cannot be selected as Hamiltonian variables. The formal reason for this is that the δ-function can be constructed only from a complete set of solutions of Eq. (2), and such a set would contain, as can be seen in the example of the Dirac equation, both positive and negative frequencies, while each of the operators in (8) contains frequencies of only one sign. For the formation of a local field, it is thus absolutely necessary to construct a superposition of creation and annihilation operators (8).

For truly neutral particles this can be done directly, by defining a local Lorentz-covariant field which corresponds to such particles in the following way:

$$u^a(x) = u^{a(+)}(x) + u^{a(-)}(x). \qquad (9)$$

For charged particles, however, this approach cannot be taken: One of the operators a^+, a^- in (8) will increase the charge, while the other will reduce it, and their linear combination will not have definite properties in this sense. In order to form a corresponding local field it is thus necessary to pair the creation operators a_t^+ with the annihilation operators a_t^- not of the same particles but of some new particles (which we indicate with a superior bar), which realize the same representation of the Poincaré group, i.e., which have precisely the same mass and spin but which differ from the original particles only in the sign of the charge (of all charges). We write

$$v^{a\tau} = u^{a\tau(+)} + \bar{u}^{a\tau(-)}, \quad \overset{*}{v}{}^{a\tau} = \bar{u}^{a\tau(+)} + u^{a\tau(-)}. \qquad (10)$$

Simple calculations now show that for integer-spin fields whose field functions implement a single-valued representation of the Lorentz group, in the case of Bose-Einstein quantization, i.e., when we use the upper sign in commutation relations (7), the commutators $[u(x), u(y)]$ or $[v(x), v^*(y)]$ are proportional to the function Δ_m and vanish off the light cone. The same result is achieved for half-integer-spin fields which implement two-valued representations for the anticommutators $[u(x), u(y)]_+$ {or $[v(x), v^*(y)]_+$}, if we use a Fermi-Dirac quantization in (7) with a lower + sign.

In contrast, an attempt to quantize an integer-spin field by the Fermi-Dirac approach leads to a situation in which it is impossible to determine the local Hamiltonian variables. An attempt to quantize a half-interger-spin field by means of Bose-Einstein statistics leads to a situation in which it is impossible to construct a positive definite expression for the energy. These assertions are the content of the Pauli theorem (1940) regarding the relationship between spin and statistics.

Equations (8)–(10) express the relationship between the operators $a^\pm$, which create and annihilate free particles in stationary quantum-mechanical states, and the Lorentz-covariant field functions $u(x)$ or $v(x)$, $v^*(x)$, which satisfy linear wave equations (2). This relationship is an exact mathematical description of the corpuscle-wave dualism.

The new particles created by the operators $\bar{a}_t^+$—without which it would be impossible to construct local fields (10), are called *antiparticles*[10] with reference to the original particles. The unavoidability of the existence of an antiparticle for each particle is one of the principal results of the relativistic quantum theory of free fields.

Equations (9) and (10) lead to yet another very important conclusion. They show that in terms of local field functions the creation and annihilation operators are necessarily mixed. The same mixing accordingly occurs in the Lagrangian or Hamiltonian in which the local fields enter as a whole. As a result, in both places terms arise which contain unequal numbers of creation and annihilation operators for particles of a certain species.[11] Such terms will have nonzero matrix elements between states containing different numbers of particles. In relativistic quantum theory, particles can be *created and annihilated*—in precisely the same way as a classical field is produced and absorbed by charges. The charged particles are created and absorbed necessarily with conservation of total charge.

5. INTERACTION OF FIELDS

Solutions (8)–(10) of the free-field equations are proportional to the creation and annihilation operators of stationary states of particles; i.e., they can describe only those situations in which nothing happens to particles. In order to incorporate cases in which certain particles affect the motion of others or convert into others, we need to make the equations of motion nonlinear. In other words, we need to introduce in the Lagrangian terms L_{int} of higher powers in addition to the quadratic terms.

From the standpoint of the theory which we have seen so far, such interaction Lagrangians L_{int} might be any functions of the fields and their first derivatives, provided only that they satisfy some simple conditions:

1) Locality of the interaction. This requirement means that $L_{\text{int}}(x)$ depends on the various fields $u^a(x)$ and their first derivatives only at a single space-time point $x = (\mathbf{x}, t)$.

2) Relativistic invariance. To satisfy this requirement, L_{int} must be a scalar under Lorentz transformations.

3) Invariance under transformations from the group of *internal symmetries*, if such are present in the model under consideration. Included here, in particular, for theories with complex fields, is the requirement that the Lagrangian be Hermitian and invariant under gauge transformations of the first kind which are permitted in such a theory (i.e., global phase transformations).

In addition, one could require that the theory be invariant under certain discrete transformations, e.g., spatial reflection P, time reversal T, and charge conjugation C, which replaces particles by antiparticles. G. Lüders (1954) and W. Pauli (1955) proved the CPT theorem, according to which any interaction of fields which have been quantized in accordance with the Pauli theorem which satisfies conditions 1)–3) must necessarily also be invariant under a simultaneous imposition of these three discrete transformations.

The interaction Lagrangians which satisfy conditions 1)–3) are just as diverse as, for example, the Lagrangians which are allowed in classical mechanics. In the early 1950s it appeared that they were all of equal status and that the theory would give no hint about which of them would occur in nature or why. Consequently, immediately after the brilliant implementation in quantum electrodynamics of a program of renormalizations of divergences on the basis of a covariant perturbation theory, many attempts were undertaken to transfer this new method to other interactions. The results were rather discouraging: The renormalization procedure did not work in most other cases. In a report in 1950 (Pauli, 1953), Pauli constructed an entire table to show in which versions of the meson theory, in the calculation of which effects, and in which order of perturbation theory, renormalization ceased to be of assistance. The natural result was to fortify the skeptical opinions, according to which the renormalization procedure would do no more than "sweep the difficulties under a rug," while a real cure of the illness of divergences would be possible only through a radical modification of the theory on the basis of a "radical new idea."

With the passage of time, however, the views which prevailed were those of the investigators who—consciously or unconsciously—decided to turn the disadvantage of the method into an advantage. If, as it turned out, renormalizations did not work for many theoretical models, then we would not blame the renormalizations and would instead take the result as an indication that such—*unrenormalizable*—theories could not occur in nature. Arguments of this sort led to a condition of

4) Renormalizability. This condition turned out to be extremely restrictive, and tacking it on to 1)–3) left as the only possibilities interaction Lagrangians L_{int} in the form of polynomials of low degree in the fields under consideration. Fields of any even moderately high spins were ruled out entirely. Consequently, an interaction in a renormalizable quantum field theory does not allow—in striking distinction from the classical theory or from quantum mechanics—any arbitrary functions. Once a specific set of fields and their transformation properties have been chosen, the only latitude left in L_{int} is in a fixed number of coupling constants.

After choosing a specific set of fields and an expression for L_{int} which satisfies conditions (1)–(4), we fix the specific model of interacting fields. In the Heisenberg picture the complete system of equations for this model consists of equations of motion (2) which follow from the complete Lagrangian—a coupled system of partial differential equations with nonlinear interaction and self-effect terms—and canonical commutation relations (4). An exact solution for a problem of this sort can be found only in an extremely few cases, with little physical content (e.g., for certain models in a two-dimensional space-time). For this reason, the practical value of a direct quantization in the form in (4) is not great.

On the other hand, one can, as in ordinary quantum mechanics, transform by means of a unitary transformation $\Psi(t) = e^{i\mathscr{H}t}\Phi$ from the Heisenberg representation with constant state amplitudes to the Schrödinger representation, in which the state amplitude evolves in time in accordance with a Schrödinger equation,[12]

$$i\frac{\partial\Psi(t)}{\partial t} = \mathscr{H}\Psi(t), \tag{11}$$

and the field operators are constant.

In quantum field theory, a third representation proved to be most convenient. This was a representation which was introduced by Dirac as early as 1926; it was put in a relativistically invariant form by Tomonaga (1946) and Schwinger (1948). This representation is usually called the *interaction representation* (or, less frequently, the *Dirac representation*. In order to switch to that representation, one separates the complete Lagrangian of the system, L, into a free Lagrangian L_0, which is quadratic in the fields and their derivatives, and an interaction Lagrangian L_{int}. Accordingly, the complete Hamiltonian $\mathscr{H}$ converts into the sum of a free-motion Hamiltonian $\mathscr{H}_0$ and an interaction Hamiltonian $\mathscr{H}_1$. Now substituting a solution in the form

$$\Psi(t) = e^{-i\mathscr{H}_0 t}\Phi(t) \tag{12}$$

into (11), we find a Schrödinger equation for $\Phi(t)$, which is a state vector in the Dirac representation:

$$i\frac{\partial\Phi(t)}{\partial t}=H(t)\Phi(t), \quad (13)$$

where

$$H(t)=e^{i\mathscr{H}_0 t}\mathscr{H}_1 e^{-i\mathscr{H}_0 t} \quad (14)$$

is the Schrödinger Hamiltonian of the interaction representation. As can be seen from (14), it depends on the time. If we express it in terms of fields in the interaction representation,

$$u^a(\mathbf{x},\ t)=e^{i\mathscr{H}_0 t}u^a(\mathbf{x})\,e^{-i\mathscr{H}_0 t}, \quad (15)$$

we see that it depends on them in precisely the same way that $\mathscr{H}_1$ depends on the Schrödinger fields $u^a(\mathbf{x})$.

The evolution of the fields $u^a(\mathbf{x}, t)$, on the other hand, is described by Heisenberg equations of motion for the free field,

$$i\frac{\partial u(\mathbf{x},\ t)}{\partial t}=[u(\mathbf{x},\ t),\ H_0]_-,$$

since

$$H_0=e^{i\mathscr{H}_0 t}\mathscr{H}_0 e^{-i\mathscr{H}_0 t}=\mathscr{H}_0, \quad (16)$$

which are the same as the linear, explicitly relativistically covariant equations of the Lagrangian description, (2). Consequently—and this is a very important advantage of the interaction representation—no difficulties arise in imposing a quantization of a covariant type. All the quantities in the theory are expressed in terms of free fields (15), whose commutation relations are written in the explicitly covariant form in (5).

The general solution of (13) can be written in the form $\Phi(t)=S(t, t_0)\Phi(t_0)$, where the evolution operator $S(t, t_0)$ satisfies the same equation [Eq. (13)] in terms of t and can be written as a chronological exponential function:

$$S(t,\ t_0)=T\left\{\exp\left(-i\int_{t_0}^{t}H(t')\,\mathrm{d}t'\right)\right\}. \quad (17)$$

For a comparison with experiment, the most interesting problem is that of scattering, for which we need an evolution operator over an infinite time interval, which transforms a stationary state $\Phi_{-\infty}$, in which the system is before the scattering, at $t\to-\infty$, into a stationary state $\Phi_{+\infty}$, which the system reaches after the scattering, at $t\to+\infty$:

$$\Phi_{+\infty}=S\Phi_{-\infty}. \quad (18)$$

S is the scattering matrix (Heisenberg, 1943). Probabilities for transitions from a given initial state $\Phi_{-\infty}$ to some final state Φ_f', i.e., the effective cross sections for scattering or for other processes, are expressed in terms of the squares of its matrix elements:

$$M_{\mathrm{fi}}=\langle\overset{*}{\Phi}{}_f' S\Phi_{\mathrm{i}}\rangle. \quad (19)$$

Taking the limit $t\to+\infty$, $t_0\to-\infty$ in (17), and expressing the Hamiltonian $H(t)$ in terms of a spatial integral of the interaction Lagrangian[13]

$$H\ (t)=\ -\int_t \mathrm{d}^3\mathbf{x}L\,(x)^*)$$

(here and below, we omit the subscript "int"); where it is to be understood that the interaction Lagrangian is written not in terms of Heisenberg fields but in the form of the same function of the fields (15) in the Dirac representation, we find a compact expression for the scattering matrix:

$$S=\mathrm{T}\left\{\exp\left(-i\int_{-\infty}^{\infty}\mathrm{d}tH(t)\right\}=\mathrm{T}\left\{\exp\left(i\int_{\infty}\mathrm{d}^4xL(x)\right)\right\}. \quad (20)$$

This expression is explicitly relativistically invariant.

The scattering matrix can be used to find probabilities for physical processes without plunging into the details of the time evolution, described by the amplitude $\Phi(t)$. We found it by integrating Eq. (13) (Tomonaga, 1946; Schwinger, 1948, 1949a, b), but this is not the only possible approach. Feynman (1949a, b) found expressions for the successive terms in a series expansion of the exponential functions in (20) by working in the framework of a Lagrangian form of quantum mechanics which he had developed (Feynman, 1948) in elaborating the ideas which Dirac had formulated back in 1933. A third method was pointed out by Stückelberg (Stückelberg and Rivier, 1949; Stückelberg and Green, 1951), who suggested constructing a scattering matrix without resorting to equations of motion but instead by using explicitly formulated physical requirements. Bogolyubov's causality condition (1955) played a decisive role in this program of research. That condition made it possible to develop (Bogolyubov and Shirkov, 1955a, b, 1957) a systematic scattering-matrix theory which includes Eq. (20). This direction laid the foundation for an axiomatic field theory.

We need to stress, however, that expression (20), despite its elegant form (or because of it), is not a ready-made solution for further use; it would be described more accurately as only a compact symbolic equation. This can be seen if only from the circumstance that a straightforward and automatic calculation of matrix elements (19) requires writing the scattering matrix in the form of a normal, rather than a chronological, product. The problem of transforming one product into another is the actual difficulty. So far, it has been solved only by approximate methods.

6. PERTURBATION THEORY

It is accordingly necessary to resort to the assumption that the interaction is weak and to assume that the interaction Lagrangian L_{int} is proportional to a small interaction constant g. It then becomes possible to expand the chronological exponential function (20) in a power-law perturbation-theory series:

$$S=1+\sum_{n\geqslant 1}g^n S_n.$$

The matrix elements in (19) for each order of the perturbation theory are expressed in terms of the matrix elements of chronological products of the corresponding number of interaction Lagrangians:

$$\int\langle\Phi_f^*\mathrm{T}\{L(x_1)L(x_2)\ \dots\ L(x_n)\}\Phi_{\mathrm{i}}\rangle\,\mathrm{d}x_1\ \dots\ \mathrm{d}x_n;$$

i.e., it is necessary to transform to normal form not an exponential function but simple polynomials of a specific type.

The basis for this transformation is the formula

$$\mathrm{T}\ \{u\ (x_1)\ u\ (x_2)\}\ =\ :u\ (x_1)\ u\ (x_2):\ +\ \underbrace{u\,(x_1)\,u}\,(x_2), \quad (21)$$

which expresses the T-product of two field operators in terms of their normal product: . . . : and a chronological convolution or the Stückelberg-Feynman propagator

$u(x_1)u(x_2) = -i\Delta^c(x_1 - x_2)$. The generalization of (21) to the T-product of an arbitrary number of operators (Wick, 1950) amounts simply to writing and adding on the right-hand side normal products with all possible numbers and arrangements of convolutions.

A practical calculation of matrix elements and of integrals over $x_i, \ldots, x_n$ of these elements is carried out by a technique proposed by R. Feynman in 1949. This technique includes the famous Feynman diagrams and correspondence rules. Each quantized field $u_a(x)$ is characterized by a corresponding propagator $\Delta^c_{aa'}(x-y)$, which is expressible in terms of Feynman diagrams by a (internal) line which connects vertices with which the fields involved in the convolution are associated. Each interaction, represented on the diagram by a vertex, is characterized by a coupling constant and by a matrix factor from the corresponding $L(x)$. A sum over all combinations of convolutions corresponds now to a sum over all possible diagrams.

Feynman's rules won popularity because of their clarity as well as their simplicity of use. The diagrams make it possible in a sense to see with one's own eyes the propagation process (the lines) and the mutual conversions (vertices) of particles—real ones, in the initial and final states represented by external lines, and virtual ones, in those represented by internal lines. We have touched upon the widely used concept of a virtual particle. In ordinary quantum-mechanical perturbation theory, "virtual" is a label put on intermediate states in the course of a transition to which the energy is not necessarily conserved, because of the quantum-mechanics energy-time uncertainty relation and the brief time spent in these states. In the invariant perturbation theory which is used in field theory, this uncertainty is transferred from the energy to the mass for the conservation of relativistic symmetry. The Stückelberg-Feynman propagator is the Green's function of the equation $\hat{L}_{ab}u^b(x) = 0$, which is satisfied by the field $u^a(x)$; i.e., it is determined by the equation

$$\hat{L}_{ab}\Delta^c_{bb'}(x) = (\Box - m^2)\,\Delta^c(x) = -\delta(x). \qquad (22)$$

Its Fourier transform thus contains a pole $(k^2 - m^2 + i\varepsilon)^{-1}$, not a $\delta(k^2 - m^2)$-function; i.e., it is nonzero even in the case $k^2 \neq m^2$. With this stipulation, Feynman diagrams may indeed by regarded as perhaps the best possible method for describing quantum-field processes in classical language.

Particularly simple expressions are found for the matrix elements of any process in the first nonvanishing order of perturbation theory, which correspond to so-called tree diagrams, which have no closed loops: There are no integrations over the momenta at all in them. For the basic processes of quantum electrodynamics, matrix elements of this sort were constructed back in the early 1930s (as we have already mentioned), and they turned out to agree reasonably well with experiment (the discrepancy was at the level of 10^{-2}–$10^{-3} \sim \alpha$).

However, attempts to calculate radiation corrections to these expressions, e.g., to the Klein-Nishima-Tamm formula for Compton scattering, ran into some extremely specific difficulties. Corrections of this sort correspond in the language which we would use today to diagrams with closed loops containing integrals over momenta of virtual particles. In most cases, these integrals diverge in the ultraviolet region. Consequently, not only are the corrections themselves formally not small—they are in fact infinite.

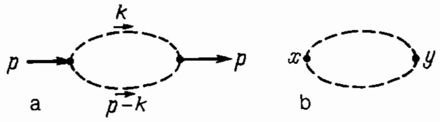

FIG. 1. Single-loop Feynman diagram with two scalar lines. a—In the momentum representation; b—in the coordinate representation.

For example, the Feynman integral

$$I(p) = \frac{i}{\pi^2}\int \frac{d^4k}{(m^2 - k^2 - i\varepsilon)\,[m^2 - (p-k)^2 - i\varepsilon]}, \qquad (23)$$

which corresponds to an extremely simple single-loop diagram with two scalar lines (Fig. 1a), is divergent. We see that in this case the divergence is logarithmic, so that if we introduce a cutoff at the upper limit of the integration, $|k| \to \infty$, we can write

$$I_\Lambda(p) = \frac{i}{\pi^2}\int\limits_{|k|\leqslant\Lambda} \frac{d^4k}{(m^2 - k^2 - i\varepsilon)\,[m^2 - (p-k)^2 - i\varepsilon]} \xrightarrow[\Lambda\to\infty]{} \ln\Lambda^2 + I_{\text{fin}}(p), \qquad (24)$$

where I_{fin} is a finite expression.

To determine the nature of the divergence which arises we note that the integral (23) is proportional to the Fourier transform of a diagram written in the coordinate representation, which is equal (according to Fig. 1b) to the square of the propagator of a scalar field, $\Delta^c(x\text{-}y)$. Near the light cone, this propagator has a singularity:

$$\Delta^c(x-y) \sim \frac{1}{4\pi}\,\delta(\lambda) - \frac{i}{4\pi^2}\,\frac{1}{\lambda}, \quad \lambda = (x-y)^2. \qquad (25)$$

We see that it is a generalized function. The operation of multiplication is not defined for such objects, as can be seen from the divergence of the integral $I(p)$. According to the complementarity relation of the Fourier transformation, large values of the momentum variable k correspond to small values of the 4-interval λ. The physical sources of the ultraviolet divergences of quantum field theory thus lie in the concept that the interaction is of a local nature. We might say that such divergences are quantum-field analogs of the infinite self-energy of the electromagnetic field of a point electron in classical electrodynamics.

7. DIVERGENCES AND RENORMALIZATIONS

As we mentioned in the Introduction, the problem of ultraviolet divergences arose at the very birth of quantum electrodynamics and was solved—at least from the standpoint of deriving unambiguous final expressions for most physical quantities of interest—in the late 1940s on the basis of the renormalization idea.

The essence of the renormalization method used to eliminate the ultraviolet divergences is that the infinite effects of quantum fluctuations which correspond to integrations over closed loops of diagrams can be separated out into additive structures [like the first term in (24)] which, as was shown, reduce to corrections to the original values of the electron mass m_0 and charge e_0. In other words, the mass m and the coupling constant $\alpha = e^2$ change because of the interaction with quantum vacuum fluctuations; i.e., they are, as we say, renormalized:

$$m_0 \to m = m_0 + \Delta m = m_0 Z_m, \quad (26)$$

$$\alpha_0 \to \alpha = \alpha_0 + \Delta\alpha = \alpha_0 Z_\alpha. \quad (27)$$

Because of the divergences, both the radiation corrections Δm and $\Delta\alpha$ and the multiplicative renormalization factors Z_m and Z_α are singular.

Most constructive implementations of the renormalization program take the approach of introducing an auxiliary regularization, similar to the momentum cutoff used above. The use of a regularization makes it possible to avoid dealing with meaningless expressions such as (23) and to carry out intermediate calculations on the basis of finite "regularized" approximations analogous to (24). After the calculations are completed, the regularization is removed in order to return to the real case (e.g., the momentum cutoff Λ is allowed to go to infinity).

In addition to cutoffs, quantum field theory uses other forms of regularization, e.g., the Pauli-Willars regularization (1949), an analytic regularization, and a dimensional regularization. The latter consists of replacing an integration over a 4-momentum manifold by some symbolic operation which corresponds at a formal level to an integration in a momentum space with a noninteger number of dimensions, $D = 4 - 2\varepsilon$ (less than four). The infinitely small parameter of this regularization, ε, is allowed to go to zero at the end of the calculations. In recent years the dimensional regularization introduced by 't Hooft and Veltman (1972) has been adopted widely in calculations in quantum-field models with gauge symmetry. The point is that in general the introduction of a regularization disrupts the symmetry properties of the original theory; e.g., the introduction of a momentum cutoff disrupts relativistic invariance. From the technical standpoint it is convenient to work with a regularization which causes a "minimal" disruption of the various invariance properties, among which gauge invariance is of importance these days.

After the introduction of a regularization, expressions of the type Δm, Δg, and Z_i turn out to depend explicitly on the regularization parameter, and they become singular only in the limit in which the regularization is removed. In practical calculations, the divergences are usually removed by taking the approach of introducing some additional terms—"counterterms" in the original Lagrangian. For this purpose one expresses the seed masses m_0 and coupling constants g_0 in terms of the physical properties m and g by means of formal relations which are the inverses of (26) and (27). Expanding them in series in the physical interaction parameter,

$$\begin{aligned} m_0 &= m + gM_1 + g^2 M_2 + \dots, \\ g_0 &= g + g^2 G_1 + g^3 G_2 + \dots, \end{aligned} \quad (28)$$

one chooses coefficients M_l and G_l (which are singular in the limit in which the regularization is removed) in a way designed to cancel the divergences in the succeeding terms of the expansion of the scattering matrix.

That class of models of quantum field theory for which a program of this sort can be carried out systematically in all orders of perturbation theory, and in which all the ultraviolet divergences, without exception, can be put aside into counterterms (i.e., ultimately into renormalization factors for the masses and coupling constants), is called the "class of renormalizable theories." In such theories, all the matrix elements and Green's functions are expressed, after the completion of the renormalization, in a nonsingular way in terms of the physical values of the masses and coupling constants and also of kinematic or space-time arguments.

In renormalizable models (among which quantum electrodynamics is one of the theories of physical interest, along with the pseudoscalar model of the pion-nucleon interaction of the Yukawa type—now sanctified by tradition), it thus turned out to be possible in this manner to abstract completely away from the "seed" parameters of the original Lagrangian and also from the ultraviolet divergences, considered separately, and to characterize the results of the calculations completely through the specification of a finite number of physical values of the masses of the particles and of the coupling constants.

The case for this point of view appeared as a result of a careful analysis of the mathematical nature of the quantum-field infinities on the basis of the theory of generalized Sobolev-Schwartz functions. From this standpoint, ultraviolet divergences are a reflection of the indefiniteness in the products of propagators (which are generalized functions) when the values of their space-time arguments coincide. On this basis, Bogolyubov and his students (Bogolyubov and Parasyuk, 1955; Bogolyubov and Shirkov, 1955b; Parasyuk, 1956; Stepanov, 1965) developed a technique for redefining the products of causal propagators in such a way that the scattering matrix turns out to be finite in arbitrary orders of perturbation theory. The concluding assertion is the content of the Bogolyubov-Parasyuk theorem (1955, 1957) regarding renormalizations. The "formula" part of this assertion—the so-called R-operation—is the practical basis for finding finite and unambiguous results without resorting to counterterms.

The renormalizable models, which are usually based on Lagrangians with dimensionless coupling constants, are characterized by the logarithmic nature of the divergent contributions to the renormalization of the coupling constants and masses of the fermions and by quadratically divergent radiation corrections to the masses of scalar (or pseudoscalar) particles.

On the other hand, in the unrenormalizable models, as examples of which we might cite Fermi's formulation (1934) of a weak interaction on the basis of a four-fermion Lagrangian—a formulation which has by now faded away into the past—and the quantum theory of gravitation, it is not possible to collect all the divergences into groups which renormalize a small or at least a finite number of constants. In other words, in the language of the R-operation it is not possible to express all the matrix elements in terms of corresponding physical values of the masses and charges. A renormalization is of no assistance to unrenormalizable theories.

8. ULTRAVIOLET ASYMPTOTIC BEHAVIOR AND THE RENORMALIZATION GROUP

The ultraviolet divergences in quantum field theory are closely related to the high-energy asymptotic forms of the renormalized expressions. For example, corresponding to the logarithmic divergence $\ln \Lambda^2$ of the regularized Feynman integral (24) is the logarithmic asymptotic expression

$$\ln(\mu^2 p^{-2}) + \text{const} + O(m^2 p^{-2}) \quad \text{at} \quad p^2 \gg m^2 \quad (29)$$

of its finite part $I_{\text{fin}}(p)$ and thus of the corresponding renormalized expression. The numerical coefficient of the ultraviolet momentum logarithm in the asymptotic expression (29) is precisely equal to the coefficient in front of the divergent logarithm in the regularized integral (24).

Since the divergences are generally logarithmic in renormalizable models with dimensionless coupling constants, the ultraviolet asymptotic expressions of the more-complex Feynman integrals have here the typical structure of polynomials in powers of ultraviolet logarithms, $l = \ln(p^2/\mu^2)$, where p is a large 4-momentum, and μ is some parameter with the dimensionality of mass, which arises in a natural way in the course of the subtraction of infinities. As an example we consider the integral

$$\left(\frac{i}{\pi^2}\right)^2 \int \frac{dq}{(m^2-q^2)\,[m^2-(p+q)^2]} \int \frac{dl}{(m^2-l^2)\,[m^2-(q+l)^2]}\,, \tag{30}$$

which corresponds to the two-loop diagram in Fig. 2.

It is not difficult to see that the "internal" integral over the 4-momentum l is the same as the single-loop integral in (23), which we discussed above. If we first carry out its regularization and subtraction, we find for (30)

$$\frac{i}{\pi^2} \int \frac{dq}{m^2-q^2} \frac{I_{\rm fin}(q)}{m^2-(p+q)^2}.$$

Since $I_{\rm fin}(q)$ has the logarithmic asymptotic expression (29) at large values of q, we see that this two-loop integral $I^2(p)$ as a whole diverges as the square of a logarithm, $\ln^2 \Lambda$. Accordingly, after the subtraction, its finite part will have an ultraviolet asymptotic expression of the l^2 type. In the general case, a ν-loop Feynman diagram can be represented by a polynomial of degree ν after a renormalization of the divergences in the ultraviolet region:

$$I^{\nu}(p) \approx a_{\nu} l^{\nu} + b_{\nu} l^{\nu-1} + \ldots + z_{\nu} + O\left(\frac{m^2}{p^2}\right).$$

We now consider a set of diagrams with a "given kinematics," i.e., with a fixed number of external lines and corresponding 4-momentum arguments (e.g., of the type shown in Fig. 3). Such a set forms a strongly coupled or one-particle-irreducible vertex function, in terms of which we can express the matrix element of a suitable physical process.

The ultraviolet asymptotic expression $G(p)$ is determined by the following sum over ν:

$$G(p;\ q) = g \sum_{\nu} a_{\nu} (gl)^{\nu} + g^2 \sum_{\nu} b_{\nu-1} (gl)^{\nu-1} + \ldots\ . \tag{31}$$

The first term on the right-hand side is the sum of the "leading" logarithms; the second term is the sum of the "next-to-leading" logarithms; and so forth. For sufficiently large values of p^2, it is clear that the growth of logarithm l compensates for the small value of the perturbation-theory parameter g. As a result, to determine the asymptotic form

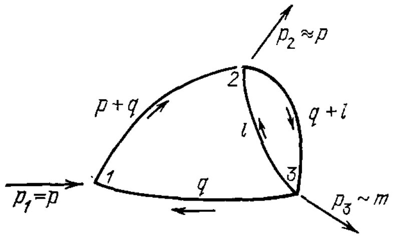

FIG. 2. Two-loop diagram in the φ^4 scalar model, considered in the asymptotic case $|p_1| \sim |p_2| = p \gg m$, $p_3 \sim m$.

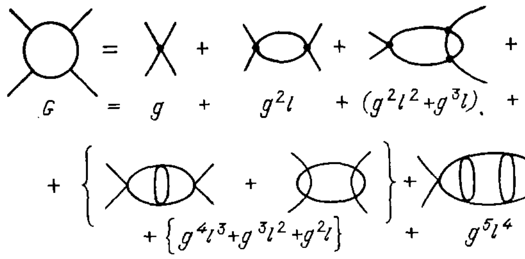

FIG. 3. Schematic representation of ultraviolet asymptotic contributions to a 4-tail vertex function.

of G we need to be able to determine sufficiently high terms in the sums which arise and to carry out an infinite summation. A summation of leading logarithms in quantum electrodynamics was first carried out by Landau, Abrikosov, and Khalatnikov (1954) through the solution of a system of approximate integral equations for the propagators and a vertex function. The renormalization group provides a general method for determining the leading and next-to-leading logarithmic contributions.

This method is based on the property of the invariance of the renormalized matrix elements and of the complete Green's functions ("complete" here meaning that radiation corrections are incorporated), including vertex Green's functions, under the so-called "renormalization" transformations. The renormalization group was discovered by Stückelberg and Petermann (1953). It is formed by continuous single-parameter transformations of a coupling constant (or constants) and the accompanying transformations of propagators and vertex functions.

The mathematical apparatus of the renormalization group can be formulated either with the help of some functional equations which were first written for quantum electrodynamics by Gell-Mann and Low (1954) and which express a group composition law, or in terms of corresponding differential equations introduced by Bogolyubov and Shirkov (1955c, d). A central role is played in this formalism by a special function: the "effective" or "invariant" coupling constant $g(k^2)$. This function is the quantum-mechanical analog of the numerical constant g which appears in the original Lagrangian. We can explain the essence of the situation by looking at the example of quantum electrodynamics. In this case vacuum fluctuations give rise to a spatial screening of the electric charge of an electron, e, which is similar to the screening of an external charge which is introduced into a polarizable medium in electrostatics. To measure the charge of an electron, one should place it in an electromagnetic field and "feel" it by means of the quanta of this field. It turns out that the quanta or "feelers" may, on their way to the electron, undergo a virtual dissociation into an electron-positron pair. This pair forms an effective dipole of the quantum-field vacuum, which generates the screening effect, which is a function of the distance from the electron. Because of vacuum fluctuations, a numerical parameter—the electron charge e—is thereby converted into an effective-charge function $\bar{e}(r)$. It is customary to use the square of the Fourier transform of this function, $\bar{\alpha}(k^2)$, which is an increasing function of its argument, in qualitative agreement with the picture of classical screening.

A comparison with experiment now makes it possible to determine the value of the function α for any fixed value of its argument, $k^2=\mu^2$, so that we have $\overline{\alpha}(\mu^2)=\alpha_\mu$. The parameter μ, with the dimensionality of mass, characterizes the 4-momentum of a photon (or the frequency of the electromagnetic field) which is used to measure the charge of the electron. The method customarily used in classical electrodynamics to determine a charge is to choose $\mu=0$, so that we would have $\overline{\alpha}(0)=1/137$. This "Millikan" value is also ordinarily used for the parametrization of physical results in quantum electrodynamics. In general, however, this parametrization could be carried out with the help of any pair of values μ and α_μ physically equivalent to the Millikan value [i.e., the values must lie on the same curve $\overline{\alpha}(k^2)$ in the k^2, α plane which is fixed by the condition $\overline{\alpha}(0)=1/137$]. Different parametrizations are of completely equal validity and the physical quantities which are measured experimentally (the probabilities for processes) do not depend on the particular choice of one or another of these parametrizations. This latitude in choosing a parametrization corresponds to renormalization invariance. The renormalization-group transformation corresponds to a transformation from one of the parametrizations, $[\mu_1, \overline{\alpha}(\mu_1^2)=\alpha_1]$, to some other parametrization, $[\mu_2, \overline{\alpha}(\mu_2^2)=\alpha_2]$.

The renormalization-group method makes it possible to combine effectively information from perturbation theory with the properties of renormalization-invariance. Technically, this is done on the basis of differential group equations. One ultimately finds expressions which, on the one hand, have the appropriate group property, and, on the other, correspond to the particular perturbation-theory terms which are used.

In quantum electrodynamics, for example, the effective charge $\overline{\alpha}$ is given in lowest-order perturbation theory by the expression

$$\overline{\alpha}(x, \alpha)=\alpha+\alpha^2(3\pi)^{-1}\ln x, \quad x=-p^2\mu^{-2}\gg 1. \qquad (32)$$

The single-loop renormalization-group approximation based on it,

$$\alpha_1(x, \alpha)=\frac{\alpha}{1-(\alpha/3\pi)\ln x}, \qquad (33)$$

takes the form of a geometric progression, which is the result of a calculation of the sum of the first term on the right-hand side of (31).

Expression (33), which was originally derived in the mid-1950s (Landau *et al.*, 1954), before the development of the renormalization-group method, acquired widespread fame since it was regarded by certain authors (e.g., Landau and Pomeranchuk, 1955, and Landau, 1955) as evidence of an internal difficulty in local quantum electrodynamics. It can be seen that this expression has a "ghost" pole at $p^2=-\mu^2\exp(3\pi/\alpha)$, whose position and the sign of whose residue contradict several general properties of a local quantum field theory. Closely related to the possible existence of a pole of this sort is the so-called zero-charge problem, i.e., the vanishing of the renormalized charge for any fixed value of the seed charge in the original Lagrangian. The ghost-pole difficulty was interpreted in the mid-1950s as proof of an internal contradiction of quantum electrodynamics, and the generalization of this result to renormalizable traditional pion-nucleon models of strong interactions was interpreted as an indication of the contradictory nature of the overall local quantum field theory.

However, radical conclusions of this sort, based solely on the equations of the leading-logarithm approximation, turned out to be a bit hasty. If we work from the two-loop approximation,

$$\overline{\alpha}(x, \alpha)=\alpha+\frac{\alpha^2}{3\pi}l+\frac{\alpha^3}{9\pi^2}l^2+\frac{\alpha^3}{12\pi^2}l, \quad l=\ln x, \qquad (34)$$

i.e., if we consider the next-to-leading logarithmic contributions, we find that the renormalization-group method leads to the expression

$$\overline{\alpha}_2(x, \alpha)=\frac{\alpha}{1-(\alpha/3\pi)l+(3\alpha/4\pi)\ln[1-(\alpha/3\pi)l]}, \qquad (35)$$

which is the sum of the first two infinite sums from the right-hand side of (8.3). We see that as the product αl increases the two-loop correction in the denominator in (8.7) becomes important, and it shifts the position of the ghost pole. As can be shown in the renormalization-group method, the range of applicability of expressions (33) and (35) is limited by the condition $\overline{\alpha}\ll 1$. Consequently, the paradox of the ghostly-pole phenomenon or the vanishing of the charge turns out to be illusory: The only way in which we could decide whether this difficulty actually appears in the theory would be to have obtained some unambiguous results in the strong-coupling region, $\overline{\alpha}\gtrsim 1$. So far, we have only the conclusion that in application to spinor electrodynamics perturbation theory is not a logically closed theory, despite the fact that the expansion parameter α is unquestionably small.

For quantum electrodynamics, incidentally, this problem might be regarded as purely academic (Landau, 1955) since according to (33) even at the huge energies $\sim 10^{15}$–10^{16} GeV which are being considered in current attempts to unify the interactions the condition $\overline{\alpha}\ll 1$ is not violated. The situation in quantum mesodynamics appears far more serious: This is the theory of the interaction of pseudoscalar meson fields with fermion nucleon fields, which appeared at the beginning of the 1960s to be the only candidate for the role of a renormalizable model of strong interactions. In it, the effective coupling constant was large at ordinary energies, and a clearly incorrect analysis by perturbation theory led to the same zero-charge difficulties.

As a result of all these studies, the outlook for the future prospects of renormalizable quantum field theories seemed a bit gloomy. It seemed that the qualitative diversity of renormalizable quantum field theories was negligible: For any renormalizable model, the only possible effects of interactions—for small coupling constants and moderate energies—were unobservable changes in the constants of free particles and the occurrence of quantum transitions between states with such particles. The lower-approximation probabilities for these transitions could now be supplemented with calculations of the (small) corrections of higher-order approximations. The existing theory—again, regardless of the specific model—was inapplicable to such large coupling constants or asymptotically high energies. Quantum electrodynamics remained the only (although brilliant) application to the real world which met these requirements. All hopes for new results were perhaps pinned on the development of non-Hamiltonian (e.g., axiomatic) methods or methods which did not use an expansion in a coupling constant. Much hope was placed on dispersion relations and a study of the analytic properties of the S-matrix. Many physicists began to look for a way out of the difficulties along noncanonical paths: nonlinear, nonlocal, indefinite, and so forth.

The source of the new views of the situation in quantum field theory was the discovery of new theoretical facts associated with non-Abelian gauge fields.

9. GAUGE FIELDS[14]

Non-Abelian gauge fields were introduced by C. N. Yang and R. L. Mills in 1954 on the basis of an analogy with the electromagnetic field. Their arguments went roughly as follows.

If a theory is invariant under some group of global gauge transformations of the fields $u^a(x)$ which figure in it (e.g., if it is invariant under rotations in isotropic space, under Lorentz rotations of the coordinate system, etc.), then this global invariance can always be converted into a local invariance; i.e., we can allow our own rotations at each 4-point x if we everywhere replace the ordinary derivatives by "elongated" derivatives,

$$\partial_\mu u^a(x) \to (D_\mu u)^a \equiv \partial_\mu u^a - igB^{ab}_{0\mu}u^b, \qquad (36)$$

adding a "purely gradient" compensating field to the set of fields of the system,

$$B^{ab}_{0\mu}(x) = i(\Lambda^{-1}(x))^{ac}\partial_\mu\Lambda^{cb}(x). \qquad (37)$$

For the elogated (covariant) derivatives, the same transformation law as for the fields themselves, $u^a(x)$, thus holds if the field B_0 transforms in accordance with

$$B'^{ab}_{0\mu}(x) = (S^{-1})^{ac}B^{cd}_{0\mu}(x)S^{db} + i(S^{-1})^{ac}\partial_\mu S^{cb}, \qquad (38)$$

which generalizes the gauge transformation of the electromagnetic potential. Such a procedure can always be carried out, and it has no physical consequences of any sort.

A new effect arises if we expand the concept of a gauge field, switching from a "purely gradient" field B_0 to some field B which depends in an arbitrary way on the coordinates:

$$B_{0\mu}(x) \Rightarrow B_\mu(x).$$

This field is not necessarily representable in form (37), but it has the same quantum numbers and the same transformation law (38). That this expansion is not a trivial one is seen in the circumstance that the covariant derivatives in (36) cease to be commuting, and their commutator forms a gauge-covariant entity which transforms under an associated representation of the group G—the stress tensor of the gauge field:

$$F_{\mu\nu} = -i(D_\mu D_\nu - D_\nu D_\mu)$$
$$= \partial_\nu B_\mu - \partial_\mu B_\nu + ig(B_\mu B_\nu - B_\nu B_\mu) \qquad (39)$$

[it is easy to see that it is zero for the "purely gradient" field in (37)]. This expansion alters the theoretical scheme and can be justified (or refuted) only by its agreement or disagreement with experiment, which offers us examples of both cases.[15]

The dynamics of the interaction of material fields $u^a(x)$ with a gauge field is characterized completely by the circumstance that ordinary derivatives are replaced in their Lagrangian by elongated derivatives D_μ, and an invariant Lagrangian of a free gauge field is added:

$$L(B) = -\tfrac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}). \qquad (40)$$

A potential $B^{ab}_\mu(x)$ cannot be inserted in the Lagrangian, since it would violate gauge invariance. Consequently, the quanta of the Yang-Mills field, like those of the electromagnetic field, are massless.

In contrast with the Abelian electromagnetic case, the Yang-Mills field is not diagonal in the group index, and the emission (or absorption) of a Yang-Mills quantum is accompanied by a change in the "charge" state of the emitting particle. The primary distinction, however, is that the tensor of the non-Abelian field $F^{ab}_{\mu\nu}$ is expressed in a nonlinear way in terms of the potential B^{ab}_μ, so that already the equations of motion of the "free" field turn out to be nonlinear in the absence of matter: The quanta of a Yang-Mills field are themselves charged. Consequently, they have no solutions of the customary plane-wave type which describe free particles, which are the customary starting points in a quantization.

On the other hand, the well-known difficulty regarding the singularity of the Lagrangian in the case of an electromagnetic field goes over entirely into the Yang-Mills theory. It can be seen from (39) and (40) that the generalized momenta $\pi^{ab}_0(x)$, which are the canonical conjugates of the component $B^{ab}_0(x)$, of the potential, vanish identically. Consequently, a theory which includes gauge fields is a system with constraints and thus cannot be quantized by the standard methods.[16] This circumstance, like the apparent absence from nature of massless vector particles—aside from photons—limited the interest in non-Abelian fields; for more than a decade they were regarded basically as elegant trinkets which were irrelevant to the real world.

The situation changed in the second half of the 1960s, when it became possible to quantize a Yang-Mills field by the path-integration method in a generalized Hamiltonian dynamics which had been developed by Dirac (1950, 1951a, 1958, 1964, 1966). Here we learned the procedure required for going over to a quantum description for systems with constraints. This progress became possible after it was learned that both a pure massless Yang-Mills field and such a field which interacts with fermions are renormalizable.

The path integral for the formulation of quantum dynamics was introduced by Feynman (1948a), who worked from an idea which Dirac had expressed back in 1933: That the time evolution of a quantum system over a finite time interval could be represented as a composition of a large number of evolutions over small time intervals. A finite-transformation function in this case appears in the form of a multiple integral of the product of a large number of "elementary" transformation functions over the possible values of the dynamic variables at intermediate time intervals. By shrinking the small time intervals and by letting the number of intermediate integrations increase without bound, we arrive at a new representation of quantum amplitudes in terms of integrals of infinite multiplicity. In the quantum mechanics of systems with few degrees of freedom, these integrals are known as path integrals, and in quantum field theory they are known as functional integrals. In quantum field theory, a path integral is a functional integral since the integration is carried out over all possible values of the field functions over the entire space-time. It turned out that the new entity had many properties of an ordinary integral; for example, it could be integrated by parts, and the integration variables could be changed. Because of specifically these circumstances, the path-integral formalism proved convenient for studying effects of the transformation of field functions. This convenience explains the role which it plays in the quantization of gauge fields.

Immediately after the pure Yang-Mills was quantized (Popov and Faddeev, 1967, and also De Witt, 1967) it became possible to solve the problem of the "soft" introduction of a mass to the quanta of a gauge field without violating gauge symmetry. This approach uses the idea of a "spontaneous symmetry breaking" which Bogolyubov (1961) developed for problems in quantum statistics. In this approach, asymmetric solutions of problems having a definite symmetry can be realized under certain conditions. For a quantum-field implementation of the mechanism of spontaneous symmetry breaking it turned out to be necessary to resort to the use of an additional scalar field which disrupted the stability of the symmetric lower state of the system of fields. This field is called a "Higgs field," and the approach which furnishes a mass to the quanta of gauge fields is the "Higgs mechanism" (1964).

By the late 1960s or the early 1970s, it was also established ('t Hooft, 1971, and Slavnov, 1972) that the Higgs mechanism does not violate the properties of renormalizability of gauge quantum interactions. As a result, it was possible as early as the late 1960s to construct a unified renormalizable theory of weak and electrointeractions: the "Salam-Weinberg-Glashow model." [17)]

The symmetry group of this model is the direct product of U(1) and SU(2), so it contains two coupling constants,[18)] g_1 and g_2. The mediators of the weak interaction in this model are heavy vector bosons: the quanta of a non-Abelian gauge field of the SU(2) group. For a decade and a half, this model was well supported by experiment. The moment of triumph was the experimental discovery of the intermediate vector $W^{\pm}$ and Z^0 particles, with masses ~80–90 GeV, which had been predicted by the theory. A question which remains unresolved at this point is the observation of the quanta of a Higgs field: the so-called Higgs scalar mesons.

Immediately after the derivation of a unified SU(2)×U(1) theory in the early 1970s, it was discovered that non-Abelian quantum fields have a remarkable property: asymptotic freedom. If we treat the nonlinear self-effect terms of a Yang-Mills field as a small perturbation in the "linear" plane-wave approximation, on the basis of which a quantization is carried out, then we find that the radiation corrections which arise when this self-effect is taken into account have some unusual properties. The signs of the leading-logarithm contributions from these corrections are opposite to the sign of the corresponding corrections in quantum electrodynamics (and in other known renormalizable models). In the gauge theory of the SU(3) group, for example, with n types of fermion fields of matter, the single-loop approximation of perturbation theory for the effective coupling constant is

$$\bar{\alpha}_s(k^2, \alpha_s) = \alpha - \alpha^2\beta(n)\ln\frac{k^2}{\mu^2}, \quad \beta(n) = \frac{33-2n}{12\pi}.$$

Corresponding to this case we have the renormalization group formula

$$\bar{\alpha}_s(k^2) = \frac{\alpha_s}{1+\alpha_s\beta(n)\ln(k^2/\mu^2)}. \qquad (42)$$

We see that for sufficiently small values of n the numerical constant $\beta(n)$ is positive, and we can go to the ultraviolet limit without the difficulty of a ghost pole. In the limit $k^2 \to \infty$, the effective $\bar{\alpha}_s$ vanishes.

This phenomenon of the self-switching-off of the strong quark-gluon interaction at short range, which was discovered by Gross and Wilcheck and also by Politzer in 1973, and which became known as "asymptotic freedom," furnished an explanation for the parton structure of hadrons which had been established in the late 1960s in experiments on deep inelastic lepton-hadron scattering. As a result, it became one of the cornerstones in the erection of the modern theory of strong interactions on a chromodynamics foundation.

The symmetry foundation of quantum chromodynamics (QCD) is the SU(3) transformation group in the space of internal ("color") variables. The hypothesis of a new quantum number was introduced (Bogolyubov, Struminskiĭ, and Tavkhelidze, 1965; Han and Nambu, 1965) in order to solve the problem of the correspondence between the composite quark model of hadrons and the Pauli exclusion principle. According to this hypothesis, each quark must exist in three modifications; the "combining rules" for the new degree of freedom are reminiscent of the rules for combining colors in a spectrum, as was later noticed. In terms of this analogy, three basic colors are associated with quarks, and the observable hadrons, all of which are singlets of the color group, are colorless or white. Color is thus not directly observable.

The hypothesis of a gauge mechanism of quark interactions leads to a new gauge vector field whose quanta—gluons—mediate the interaction. Gluons are like quarks in that they carry a color charge, but they come in eight varieties. The new gauge mechanism displaced the Yukawa mechanism as the basis of strong interactions.

A more detailed comparison with experiment at momentum-transfer values $Q^2 \gtrsim 100$ GeV^2 showed that in this region we have $\alpha_s \lesssim 1/5$. The coupling is thus weak enough that we can use a renormalized perturbation theory. In place of (42) here it is customary to write

$$\bar{\alpha}_s(Q^2) = \frac{1}{\beta\ln(Q^2/\Lambda^2)}, \quad \Lambda^2 = \mu^2\exp\left(-\frac{\beta}{\alpha}\right). \qquad (43)$$

The so-called scale parameter of QCD, which characterizes the region in which strong coupling arises, turns out to be about $\Lambda \approx 200$–100 GeV. We leave the weak-coupling region as Q^2 decreases to about ~10 GeV^2; this figure corresponds to distances ~10^{-14} cm. Here we enter the region of traditional hadron concepts, where quark-gluon degrees of freedom are not manifested, as we know quite well.

The new theory, which has already given us a quantitative description of strong interactions in the region of large momentum transfer, is thus faced with the problem of explaining the basic properties of hadron physics and also the phenomenon of the unobservability of quarks and gluons: confinement. So far, we have no reliable information about what QCD will give us in the region of small momentum transfers and large distances. Here we are forced to appeal to the hypothesis that the nonlinear effects of gluodynamics lead to such an increase in the attractive forces between colored entities with increasing distance that quarks and gluons cannot move apart to a distance greater than 10^{-14} cm.

The urgency of the strong-coupling problems resulted in the development of methods for studying gauge fields which do not make use of the concept that the interaction is weak and of perturation theory. Such a nonperturbative approach to QCD was constructed on the basis of the path-integral concept.

Important results were obtained by means of a procedure which had been developed a bit earlier for quantum-statistics problems for evaluating a path integral by the method of functional steepest descent, analogous to the method of steepest descent in the theory of functions of a complex variable. It was shown by this method for several rather simple models that quantum-field quantities, regarded as functions of the coupling constant g, have a singularity of characteristic form $e^{-1/g}$ near the point $g = 0$ and that, completely consistently, the coefficients f_n in the power-law expansions $\Sigma_n f_n g^n$ of perturbation theory increase factorily at large values of n: $f_n \sim n!$. In this way, the hypothesis of the nonanalyticity of the theory in terms of charge, which had been expressed back in the early 1950s, was effectively confirmed.

Analytic solutions of nonlinear classical equations of motion for field functions which are of a localized nature (solitons and—in the Euclidean version—instantons) play an important role in calculations of this sort. These solutions, which provide the action functional with a minimum, are analogs of stationary points in the ordinary steepest-descent method. In expanding the field variables of integration near these "steepest-descent" solutions one reduces the problem to linearized field equations whose solutions are required for an approximate evaluation of a path integral along the "line of steepest descent" in function space.

In quantum-mechanical terms, we are dealing with the structure of the vacuum state of a gauge system of nonlinear fields—states which correspond to nonvanishing vacuum expectation values of products of field operators (see, for example, the reviews by Vaĭnshteĭn *et al.*, 1977, 1982). These condensate expectation values are nonperturbative effects, which depend in a nonanalytic way on the coupling constant.

Indirect information about effects of this sort comes from the sum rules and also in the representation of a functional integral by means of the special methods of so-called contour dynamics, in which the vector field functions $B_\mu(x)$, which depend on the point x in the 4-dimensional space-time, are replaced by new dynamic variables (a Wilson loop), which depend functionally on the values of B_μ on some closed contour Γ which lies in a space-time manifold. This approach makes it possible to reduce by one the dimensionality of the set of independent variables, and in a number of cases it becomes possible to simplify significantly the formulation of a quantum-mechanical problem.

A number of successful studies has recently been carried out through a numerical evaluation of path integrals, represented approximately as repeated integrals of high multiplicity. For this representation, one introduces a discrete lattice in the original space of (configuration) variables. Lattice calculations of this sort, as they are called, require the use of particularly powerful computers for realistic models, so it is only very recently that they have begun to be accessible. In particular, the Monte Carlo method has yielded some encouraging calculations of the masses and anomalous magnetic moments of hadrons on the basis of QCD concepts.

A study of the field model proposed by Yang and Mills thus revealed that models satisfying the requirement of renormalizability might have a totally unexpected richness of content. In particular, there was a shattering of the naive faith that the spectrum of an interacting system would be qualitatively similar to that of a free system, differing from it only in a shift of levels and perhaps the appearance of a small number of bound states. It turned out that the spectrum of a system with an interaction (hadrons) may have nothing in common with the spectrum of the free particles (quarks and gluons) and might therefore give us not even a hint about which types of fields would have to be included in the elementary microscopic Lagrangian.

We should stress here that both the establishment of these extremely important qualitative features and the carrying out of the vast majority of quantitative calculations in QCD are based on a combination of perturbation-theory calculations with the requirements of renormalization-group invariance. In other words, the renormalization-group method has occupied a place along with renormalized perturbation theory as one of the basic calculation facilities in modern quantum field theory.

10. CONCLUSION

Looking back, we might say that a stage of progressive development for quantum field theory which lasted some 30 years was followed in the mid-1950s by a "time of confusion." At this time, various semiphenomenological and even mutually exclusive approaches became rather widespread, and the terms "elementary particle theory" and "quantum field theory" had different meanings for most physicists. During this period, however, some profound changes were occurring at the core of the theory, unseen by a superficial observer. Since these profound changes were detached from the urgent physical problems of the day, they acquired the official status of a "theoretical theory" at the time. Ultimately, these changes led to some fundamental advances, associated primarily with the creation of a quantum theory of gauge fields, with the result that the situation started to change in the opposite direction in approximately the late 1960s. Quantum field theory, supplemented with the principle of gauge symmetry and based on a new (quark) foundation, quickly returned to the forefront, having seized some strong positions not only in electrodynamics but also in the theory of weak and strong nuclear interactions. Thereby the spiral of the development of particles and their interactions in a sense reversed, rising to a new qualitative level.

From the conceptual standpoint, the primary result reduces to the discovery of a simple and universal mechanism for constructing a dynamics of relativistic quantum fields, based on gauge symmetry: "the symmetry underlies the dynamics." The realization of this thesis substantially simplified the logical basis of the theory for the interaction of fields, giving it features of universality. However, this progress, arose on a rather complicated foundation.

An explicit formulation of gauge dynamics, which is organically related to the group nature of symmetry transformations, draws heavily from the language of the theory of continuous groups. In addition, the development of a formalism for the quantization of gauge fields led to a more widespread use of the representation of a functional integral. Consequently, the mathematical apparatus of the theory of Lie groups and also the methods of functional analysis have evolved into an every day working tool in the field of particle theory.

At the same time, the simplicity of the gauge mechanism is based on the some rather complicated physical concepts. We have in mind two completely different factors. One stems from the general thrust of local gauge transformations, which appeals to the unobservability of the phase of a complex field. The second is associated with the impossibility of directly observing the color degree of freedom, which underlies the gauge symmetry which leads to QCD and thus to the fact that quarks and gluons are observable only indirectly: through an interpretation of what is happening inside hadrons.

We can thus discern two tendencies. One of them, in this context the governing one, is the progressive simplification of the logic of particle theory, with decreasing numbers of independent basic physical assumptions and parameters. This simplification is occurring against the background of a torrential growth of observational facts and laws (including the number of particles, quantum numbers, and approximate symmetries). The second, and accompanying, tendency is a significant hindrance to the dissemination of new ideas and their perception by wide circles of physicists: the increasing complexity of the mathematics and the increasing degree of physical abstraction. Quantum field theory is becoming progessively more mediated. Adding to the inadequacy of the concept of the trajectory of an electron is the impossibility (noted above) of an asymptotic—in the free state—observation of the quanta of the fields of strong interactions and the "color" quantum number. The idea of a vacuum as a composite state having little in common with a physical void is becoming progressively stronger. It might be said that in the modern theory of the microworld we are dealing with a level of abstraction of the fundamental physical concepts which is higher than in quantum mechanics. This continually changing picture is remarkably similar to Dirac's foresights which we recalled at the beginning of this paper.

There is yet another important aspect of the situation which should be brought up here. A systematic increase in the level of mathematization has been a characteristic feature of theoretical physics throughout its history. The leading branches of physical theory have always drawn upon new mathematical methods and in some cases have stimulated their development deep in the realm of mathematics itself. In the recent past this role has been played by the theories of electromagnetism, the kinetic theory of matter, the theory of relativity, and quantum mechanics. Since grabbing the baton for its leg of this relay race, quantum field theory has made extensive use of generalized functions, the theory of continuous groups, and the functional integral. It should be noted that quantum field theory is not only "assimilating" new branches of mathematics at a rapid pace but is also promptly "transmitting" newly developed theoretical methods to other branches of physical theory.

A well-known example is the technique of Feynman diagrams. Another is the associated concept of the functional integral. Both were used in quantum statistics back in the 1950s. They were subsequently used in some other fields of theoretical physics which dealt with systems with a large number of degrees of freedom.

The method of the renormalization group is very illustrative. Three decades after its discovery in quantum field theory, it was converted into a useful method for research in the theory of critical phenomena, in turbulence theory, in noncoherent transport theory, in percolation theory, and in other fields of theoretical physics, quite remote from each other in terms of the physics involved.

Yet another example might be the practice of using modern high-speed computers to carry out algebraic transformations and analytic and symbolic calculations. This field of application of computers, which dates back to about the middle of the century, was used successfully in the 1970s in the problem of analytic calculations of Feynman diagrams in higher-order perturbation theory. Special software systems were developed for this purpose. Later, thanks to a large extent to the efforts of field theoreticians, software systems for analytic calculations of a more general nature appeared. These general-purpose systems have found a variety of applications in various fields of theoretical physics, mechanics, and mathematics.

It is thus fair to say that quantum field theory is playing a pioneering role as the founder and disseminator of the newest "mathematical technology" in the exact sciences. To a significant extent, it is determining the mathematical level of natural science today.

At the same time, as we have already mentioned, the modern theory of particles and their interactions is playing the role of a generator of progressively more complex physical concepts and models, which are finding progressively greater use in related fields, e.g., astrophysics and nuclear physics. They are coming to the attention of physicists in other specialities and then of an even wider audience.

Returning to the heart of the present state of quantum field theory, we note that despite some extremely impressive progress there are still serious problems to be resolved.

In the standard theory of electroweak interactions, the problem of the Higgs boson, which is still eluding experimental observation, is looming progressively more disconcerting. In this connection we should state that the Higgs mechanism itself, which imparts a mass to W and Z bosons through a "soft" breaking of gauge symmetry, violates the general thesis that the dynamics is determined by the symmetry, despite the elegance of this mechanism. On the whole, it appears as a rather contrived approach.

In the theory of strong interactions it has not been possible to find a convincing explanation for the fundamental property of the confinement of quarks and gluons by working from the basic equations of quantum chromodynamics. Only very recently have qualitative indications appeared that it may be possible to find the confinement effect on the basis of QCD calculations carried out by the Monte Carlo method on a lattice. In general, the problem of reaching an understanding of the structure of the vacuum state of non-Abelian gauge fields is still far from resolution.

We need to stress that the substantial progress which has been achieved over basically the last 15 years has dealt almost exclusively with the mechanism for the interaction of fields and particles. With regard to a quantitative description of the properties of the particles (e.g., the values of the masses) on the basis of first principles, on the other hand, the progress has so far been more modest.

Progress in the observation of the properties of particles and resonance states has given the theoreticians copious material, which has in turn led to the observation of new quantum numbers (isotopic spin, strangeness, charm, and so forth). It has also resulted in the construction of corresponding so-called broken symmetries and corresponding systematics. These developments have in turn motivated searches for a substructure of numerous hadrons and resonance states. As a result, such "1950s elementary particles" as nucleons and π mesons have ceased to be elementary, but we have yet to see a really convincing quantitative explanation of their characteristics. The first attempts to calculate the masses and magnetic moments of hadrons on the basis of QCD lattice calculations, which we mentioned above, will, if they are ultimately successful, make it possible to express these parameters theoretically in terms of the masses of quarks and the value of the quark-gluon interaction constant.

A good illustration of the situation is the extent of the breaking of isotopic spin, which is manifested in the difference ΔM between charged and neutral mesons and baryons (e.g., p and n, $K^{\pm}$ and K^{0}). The initial understanding (which looks somewhat naive today)—that this difference was of an electromagnetic origin (because of the numerical relation $\Delta M/M \sim \alpha$)—has given way to the conviction that the difference stems from a difference Δm between the masses of u and d quarks. Even if a quantitative implementation of this idea does prove to be successful, we see that the problem will not be completely solved: It will only be pushed down from the hadron level to the quark level. The formulation of the old puzzle of the muon is evolving in a similar way: Why do we need a μ meson, and why should it, so similar to the electron, be 200 times heavier? Moved to the quark-lepton level, this question has become quite general, now referring not to a pair of particles but to three "generations" of particles, without undergoing any essential change.

What are the immediate prospects for the development of quantum field theory? What are the most urgent problems facing this theory?

Two directions, associated with a "grand unification of interactions" and with supersymmetry, arose in the 1970s and have been developed significantly.

The first is based on the idea of unifying the strong interaction of quantum chromodynamics with the unified electroweak interaction at energies $|Q| \sim M_X \approx 10^{15}$ GeV and above. The starting point here was the circumstance that, according to the renormalization-group equations [see (42) above], the effective coupling parameter of QCD, $\bar{\alpha}_3(Q^2)$, like the second effective charge $\bar{\alpha}_2(Q^2)$ of the unified theory of electroweak interactions, decreases with increasing Q^2, while $\bar{\alpha}_1$ increases [this quantity corresponds approximately to $\bar{\alpha}_{\rm QED}$ at sufficiently low energies; see (41)]. The increase is of such a nature that if we extrapolate the quantitative behavior available to ultrahigh energies we find that these functions $\bar{\alpha}_i$ ($i = 1,2,3$), which diverge by an order of magnitude at the energies of today's experiments, become comparable to each other at energies of the order of M_X. The corresponding values turn out to be $\bar{\alpha}_i(M_X^2) \sim 1/40$. Comparison of this fact with the circumstance that all the known broken symmetries become progressively more accurate as the energy is increased suggests that this equality of effective constants is not simply a fortuitous matter and that instead there is some higher symmetry at energies above M_X. This symmetry would be described by a group G which would split up into the symmetries which are observable today, $SU(2)\times U(1)$ and $SU_c(3)$, by virtue of mass terms. The masses which break the symmetry would be equal to M_X in order of magnitude.

Other considerations in favor of the existence of a unified symmetry group are related to the values of the electric charges of the elementary particles or, as one might prefer to say, to the question of why nature has a minimal electric charge of such a nature that we have $\bar{\alpha}_{\rm QED}(Q^2=0) = 1/137$. The advent of quantum mechanics raised the hope that a systematic quantum-mechanical theory of the electromagnetic field would provide some answer to this question. For example, we recall that specifically this hope inspired Dirac's paper (1931) in which he introduced a monopole. The result, however, was the establishment of not the magnitude of the electric charge but only of the relation between the magnitudes of the electric and magnetic charges. Such attempts were also undertaken subsequently, e.g., by Heisenberg (1953–1959) in his well-known series of studies on the derivation of a nonlinear theory.

The entire situation changed substantially, however, when quarks unambiguously took the place of protons and neutrons in our understanding of the basic structural elements of matter. It turned out that the charge of an electron *is not* a minimal "elementary charge" and that instead other elementary particles could have *other* (and smaller) charges. Consequently, any hope for finding the value of the elementary charge from the internal logic of a "future electrodynamics" had to be acknowledged as futile: Electrodynamics allows the existence of different charges for different elementary particles. This is an experimental fact. The question itself did not disappear, however; it simply moved to a different plane: If electrodynamics is compatible with different charges of elementary particles, then why do the charges of leptons and quarks form simple rational ratios? We find it difficult to believe that there could be any explanation other than the explanation that leptons and quarks must be related by some symmetry transformations (which become explicit only at sufficiently high energies), so the ratios of electric charges are the "Clebsch-Gordan coefficients" of the corresponding group.[19] Such considerations of course tell us nothing about the energies at which the symmetry is restored.

In certain versions of grand unification, e.g., that with $G = SU(5)$, there is the possibility of transitions between quarks and leptons. As a result, there is a nonconservation of baryon charge; in particular, the proton is unstable. However, an intense search has uncovered no indications of any sort for the existence of such an effect.

The second direction is based on the symmetry under transformations which entangle boson fields of integer spin with fermion fields of half-integer spin. This symmetry is called "supersymmetry." Such transformations form a group which is a nontrivial expansion of the Poincaré group. The corresponding algebra of generators, which contains spinor generators and also their anticommutators along with the ordinary (vector and tensor) generators of the Poincaré group, was discovered by Gol'fand and Likhtman (1971). The first supersymmetric quantum-field models were constructed in the early 1970s.

The field representations of the supersymmetry group—the so-called superfields Φ—are specified on manifolds which include, in addition to the four ordinary coordinates x_μ, an even number of special algebraic entities: The generators of a nilpotent Grassmann algebra with an involution θ_i, $\bar{\theta}_j$ of elements which anticommute exactly with each other and which are two-component spinors (Weyl spinors) under transformations of the Poincaré group. The superfields $\Phi(x,\theta,\bar{\theta})$ can always be represented as polynomials in θ, $\bar{\theta}$ which contain a small number of terms, by virtue of the nilpotency. The coefficients $\varphi(x)$, $\psi(x)$, . . . , of these expansions, which are fields in the usual sense, are called the "constitutent fields." From the standpoint of the Poincaré group, a single superfield Φ is equivalent to a certain set of a finite number of Bose and Fermi fields. The superfield model with an interaction (or self-effect) is equivalent to a series of interactions of the constituent fields: interactions whose constants are related to each other in a fixed way.

Some particularly interesting models contain non-Abelian gauge fields as constitutents. These models, which have both gauge symmetry and supersymmetry, are called "supergauge models." In supergauge models one observes the remarkable fact that ultraviolet divergences cancel out. Models have been seen in which the interaction Lagrangian, expressed in terms of the constituent fields, is a sum of expressions each of which is separately renormalizable and which generates a perturbation theory with logarithmic divergences. However, the divergences corresponding to a sum of Feynman diagrams with contributions from different terms of a virtual superfield cancel each other out. This property of a complete cancellation of divergences might be considered in connection with the known fact that the degree of an ultraviolet divergence of the self-mass of an electron in QED decreases when we switch from the original noncovariant calculations of the late 1920s to an actually covariant perturbation theory which incorporates positrons in intermediate states. The analogy is strengthened by the possibility of using supersymmetric Feynman rules, in which case the canceling divergences do not appear at all.

The fact that ultraviolet divergences cancel out completely in arbitrary orders of perturbation theory—a fact which has been established for several supergauge models—has raised the hope of a possible theoretical superunification of the elementary interactions. In other words, this unification, incorporating supersymmetry and unifying all four interactions, including the gravitational interaction, would be of such a nature that not only do the unrenormalizable effects of "ordinary" quantum gravitation disappear but also the completely unified interaction turns out to be free of ultraviolet divergences. The physical arena of superunifications would be at length scales of the order of the Planck length ($Q\sim 10^{19}$ GeV, $R_{\rm Pl}\sim 10^{-33}$ cm).

For an implementation of this idea, studies are being made of supergauge models based on superfields constructed in such a way that the maximum spin of the constituent ordinary fields is two. The corresponding constituent field is identified with the gravitational field. Models of this sort are called "supergravity models."

Although we do not have space here to attempt anything in the way of a detailed discussion of the work in this rapidly developing and extremely young field,[20)] we would like to point out that attempts presently being made to construct finite supergravities make use of concepts of Minkowski spaces with more than four dimensions and also concepts of strings and superstrings. In other words, the local quantum field theory which is "familiar to us" converts at distances below the Planck length into a quantum theory of one-dimensional extended entities embedded in a space with a higher number of dimensions.

We should add that the "superphysics" strategy (supersymmetries, unifications and superunifications, and superstrings) has so far been based exclusively on internal and purely theoretical motivations. No experimental evidence of any sort indicating a need for a superphysics has so far been found. If such evidence is found, however, we will be witnessing a triumph of a methodological construct of Dirac (1939b) which he formulated most laconically at a 1956 meeting of the Department of Theoretical Physics at Moscow State University: "Physical law should have mathematical beauty."

[1)]Weisskopf (1980) refers to the publication of that paper as the birth of quantum electrodynamics. The eminent theoretician Jost (1972) wrote that that paper by Dirac, dated 2 February 1927, contains the foundations of quantum electrodynamics and the invention of second quantization. It is the nucleus from which quantum field theory developed.

[2)]A year given in parentheses after the author's name is a reference to the literature.

[3)]In certain cases, we will use this convention to cite papers only by Dirac.

[4)]Curiously, Dirac apparently assigned so little value to it that in his first paper (1928a) he did not even write out explicitly the transformation law for his wave functions under a Lorentz transformation. In the best mathematical style, he restricted his paper to the proof of a theorem of the existence of a linear transformation of the components ψ of such a nature that the equation assumes its previous form in the new frame of reference after the transformation. The word *spinor* itself was dreamed up by Ehrenfest, who in perplexity addressed the following question to B. Van der Waerden in the summer of 1929: Does a spinor analysis equivalent to a tensor analysis exist in a form accessible to study? (Van der Waerden, 1960). The answer took the form of Van der Waerden's paper "Spinor analysis" (1929). Just a few years later it was found that corresponding quantities had been discovered 16 years earlier in pure mathematics by E. Cartan (1913).

[5)]In the special theory of relativity, the partitioning of the 4-world into space and time is ambiguous: As the space at a given instant of time one could adopt a spacelike hypersurface from any single-parameter family of such a nature that a single surface passes through each point x. In 1949 and 1962, Dirac studied the possibilities which would arise if such surfaces were chosen to be different from coordinate planes.

[6)]Proof of the invariance of commutation relations (4) was offered by Rosenfeld (1930). According to Wentzel (1960), Pauli used to say this about the proof: "Ich warne Neugierige" ("I caution the curious").

[7)]We shall no longer write out the indices which characterize the representation as a whole.

[8)]A historically curious point is that in this paper Dirac was constantly attempting to go over from the variables a and a^*, which had already arisen naturally in the classical stage of his work, to a canonical action-angle pair, but in each case the attempt was awkward, and he was forced to go back. It can now be seen that his work would only have benefited from the removal of all these "Brownian motions" associated with the canonical variables.

[9)]It is proportional not to the Pauli-Jordan function Δ_m but to the functions $\Delta_m^{(\pm)}(x-y)$, which do not vanish off the light cone.

[10)]Conversely, the original particles t are the antiparticles of the antiparticles $\tilde{t}$: Particles and antiparticles enter the description on absolutely equal footings (Dirac, 1934b; Heisenberg, 1934). In this ideology, truly neutral particles are their own antiparticles.

[11)]Exceptional cases are quadratic Hamiltonians of the free field: With a reasonable choice of variables marking the particles, terms in them which do not conserve the number of particles cancel.

[12)]In his 1964 and also 1966 papers, Dirac emphasized that such a transformation is unfounded: In his lectures on quantum field theory (1966), he wrote that the argument in favor of the equivalence of the Heisenberg and Schrödinger pictures "is valid only if $e^{i\mathscr{H}t}$ exists and can be applied to one state vector to give another, and for the Hamiltonians one meets with in quantum field theory there is good reason to believe that this is not so, because of convergence difficulties, and so the two pictures are not equivalent."

[13)]In certain cases, such a transformation requires a refinement of the meaning of a chronological product (Medvedev *et al.*, 1972).

[14)]Questions regarding the early history of the appearance of the concept of a gauge field in physics are covered in the review by Okun' (1984). We would especially like to mention the work by Fock (1926), to whom the competing term "gradient transformation" can be credited.

[15)]Electrodynamics has long given us some positive experience. A negative example might be, say, the conservation law of baryon charge, which does not correspond to any massless gauge field, as far as we can see.

[16)]In the electromagnetic case, this difficulty is circumvented by an artificial approach which was first used by Fermi (1929, 1932). For a Yang-Mills field this approach leads to a violation of unitarity (Feynman, 1963).

[17)]Beginning at roughly this point we will cite the original papers only sporatically. For a more comprehensive picture we direct the reader to review publications listed at the end of our bibliography section.

[18)]The electromagnetic constant is expressed as a combination of these constants:

$$e = g_1 g_2 (g_1^2 + g_2^2)^{-1/2}. \tag{41}$$

[19)]The opposite assertion—that the absence of simple rational ratios between the charges of leptons and quarks would imply that all the grand unification models being discussed in the literature are inadequate—was established by Okun', Voloshin, and Zakharov (1983).

[20)]Essentially an entire issue of *(Uspekhi fizicheskikh nauk (Soviet Physics Uspekhi)* was devoted to supersymmetry precisely two years ago. The December 1986 issue has some papers on superstrings.

ANNOTATED BIBLIOGRAPHY

No claim is made that the list below is comprehensive to any extent. Furthermore, the brief notes accompanying the citations are not offered as an exhaustive reflection of the contents. We have attempted only to point out what was of interest to us, for our purposes. We have not attempted to compile a detailed list for the present stage, which dates back roughly to the 1960s, because of the overly long list of important original papers, which furthermore have frequently been published in parallel and independently. Accordingly, we prefer to direct the reader interested in gaining a historical perspective on the latest developments in quantum field theory to the reviews and books at the end of this bibliography.

1913

Cartan, E., Bull. Soc. Math. de France **41**, 53.

The discovery of what we now call spinor representations of groups.

1918

Nöther, E., Nachr. Ges. Wiss. Gött Math.-Phys. Kl., p. 235 [Russ. transl., Fizmatgiz, variational principles of mechanics, M., 1959, p. 611 (cited below as VPM [1918]).

The Nöther theorems.

1925

Dirac, P. A. M., The fundamental equations of quantum mechanics, Proc. R. Soc. London, Ser. A **109**, 642–653 [Russ. transl., Usp. Fiz. Nauk **122**, 611–621 (1977)].

A quantum algebra which differs from a classical algebra only in noncommutativity is introduced. The commutator is shown to be a quantum analog of the Poisson brackets. These are the *only* changes required for making the transition from classical mechanics to quantum mechanics.

1926

Dirac, P. A. M., On the theory of quantum mechanics, Proc. R. Soc. London, Ser. A **112**, 661–677.

It is shown that two classes of solutions are permitted for a system of many identical particles. One class consists of solutions which are symmetric in terms of the indices of the particles and which lead to Bose-Einstein statistics. The other class consists of antisymmetric solutions which lead to a new statistics (Fermi-Dirac), which is constructed. A time-dependent perturbation theory is constructed. A distinction is *drawn between "identities" and "equations": a rudimentary form of the* distinction between operator equalities and the auxiliary conditions which are satisfied only after multiplication by the wave function from the right.

Fock, V. A., Über die invariante Form der Wellen- und der Bewegungsgleichungen für einen geradenen Massenpunkt, Z.Phys. **39**, 226.

To the best of our knowledge, the first introduction of a complete gauge transformation in which the addition of a 4-gradient to the electromagnetic potential (a gradient transformation of type II) is offset by the multiplication of the wave function of a charged material point by a coordinate-independent phase (a gradient transformation of type I).

1927

Dirac, P. A. M.:

a) The physical interpretation of the quantum dynamics, Proc. R. Soc. London **113**, 621–641.

The δ-function and all the rules for dealing with it. Matrices with continuously indexed rows and columns. Various representations and the general theory of transformations. Schrödinger functions as functions of a transformation from a q representation to a representation in which the Hamiltonian is diagonal.

b) The quantum theory of the emission and absorption of radiation, Proc. R. Soc. London **114**, 243–265.

A second-quantization method, which is introduced, is used to convert the theory for the interaction of a material system with a classical transverse electromagnetic field into a theory for an interaction with a new physical entity: a quantum field. The representation of a point interaction is carried over into the quantum field theory. An interaction picture is used. Time-dependent perturbation theory is developed further. Multiplication by a finite phase space is introduced.

c) The quantum theory of dispersion, Proc. R. Soc. London **114**, 710–728.

A Hamiltonian including a quantum field—a vector potential—is written out explicitly for the first time. The scattering process is studied. For this purpose, a perturbation theory of first order in the term $\mathbf{A}^2$ in the Hamiltonian and of second order for terms linear in $\mathbf{A}$ is taken into account. The first indications of divergences arise.

Jordan, P., Zur Quantenmechanik der Gasentartung, Z. Phys. **44**, 473.

Two methods for second quantization. It is possible to construct a quantum-mechanical theory of matter in which the electrons are represented by quantum waves in ordinary 3-space.

Jordan, P., and Klein, O., Z. Phys. **45**, 751.

Quantization of an electron field; inclusion of the Coulomb interaction.

1928

Dirac, P. A. M.:

a) The quantum theory of the electron, Proc. R. Soc. London, Ser. A **117**, 610–624 [Russ. transl., Proc. Inst. Hist. Sci. Tech. Acad. Sci. USSR **22**, 34–52, Izd. Akad. Nauk SSSR, M., 1959].

The Dirac equation. Linear Hamiltonian; matrices; proof of relativistic invariance. Extraneous solutions. Spin. Motion in a central field (Postnonrelativistic approximation).

b) The quantum theory of the electron II, Proc. R. Soc. London Ser. A **118**, 351–361 [Russ. transl., as in a) pp. 53–68, Izd. Akad. Nauk SSSR, M., 1959].

Theorem of charge conservation; vanishing of the divergence of the 4-current. Selection rules. Applications to the Zeeman effect.

Jordan, P. and Pauli, W., Z. Phys. **47**, 151 [Russ. transl., in Pauli W. Papers on Quantum Theory, Nauka, M., 1977 (cited below as Pauli W. [1928][20)])].

Quantum electrodynamics without charges. Permutation of the fields **E** and **H**. Operator-valued functionals. Invariant Δ-function for zero mass as a relativistic generalization of the Dirac δ-function.

Wigner, E. and Jordan, P., Z. Phys. **47**, 631.

Second quantization of fermions.

1929

Fermi, E., Sopra l'electrodinamica quantistica, Rend. Acad. Lincei **9**, 881 [Russ. transl. in Fermi E. Scientific papers, V. 1, p. 302, Nauka, M., 1971 (cited below as Fermi E. [1929])].

Electromagnetic field potentials which satisfy a wave equation in the Lorentz gauge are written as expansions in oscillators. A Hamiltonian and equations of motion are written for a field interacting with nonrelativistic charges in configuration space.

Heisenberg, W. and Pauli, W., Zur Quantendynamik der Wellenfelder, Z. Phys. **56**, 1 [Russ. transl., Pauli W. [1928], p. 89].

A general Lagrangian and Hamiltonian form of the c-field equations and energy and momentum conservation are described. A canonical quantization (either Bose or Fermi) is carried out. Quantum canonical equations of motion are written. Energy and momentum conservation laws are written. It is shown that the commutation relations are invariant under an infinitesimal Lorentz transformation (see p. 6 of the text). A complete system of equations of spinor electrodynamics is then written. A small non-gauge-invariant increment $\varepsilon(\partial Q_\alpha/\partial x_\alpha)^2$, $\varepsilon \to 0$, is added to deal with the vanishing of p_4 at the origin. A method for calculating effects by perturbation theory is developed (after the Dirac model).

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Spinor calculus.

Klein, O. and Nishina, Y., Z. Phys. **52**, 853.
Scattering of light by an electron: the Klein-Nishina formula.

1930

Dirac, P. A. M.:

a) A theory of electrons and protons, Proc. R. Soc. London, Ser. A **126**, 360–365 [Russ. transl. Usp. Fiz. Nauk **10**, 581–591 (1930)].
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b) On the annihilation of electrons and protons, Proc. Cambridge Philos. Soc. **26**, 361–375.
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c) Note on exchange phenomena in the Thomas atom, Proc. Cambridge Philos. Soc. **26**, 376–385.
Density matrix.

d) The Principles of Quantum Mechanics, Clarendon Press, Oxford, 1930, 1935, 1947, 1958 [Russ. transl., GTTI, M., L., 1932; ONTI, 1937; Fizmatgiz, M., 1960; Nauka, 1974, 1979].

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1932

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b) Dirac, P. A. M., Fock, V. A., and Podolsky, B., "On quantum electrodynamics," Phys. Z. Sowjetunion **2**, 468–479 [Russ. transl., Fock V. A., Papers on quantum field theory, Izd. Leningr. Univ., L., 1957, pp. 70–82, cited below as Fock V. A. [1932]].
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The density matrix describing the distribution of electrons among positive and negative levels is redefined in such a way that only the contributions from filled positive and vacant negative levels are retained. In other words, only the contributions from real electrons and positrons are retained. Explicit expressions for, and singularities of, the functions Δ and Δ_1 with a nonzero mass are analyzed in detail.

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A representation of the solution of the Dirac equation with an electromagnetic field in the form of an integral over the proper time is proposed (the Fock proper-time method).

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b) A general theory of the S matrix and renormalization.

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Axiomatic perturbation theory for the scattering matrix based on Bogolyubov's causality condition.

b) Usp. Fiz. Nauk **57**, 2.

Use of the R-operation in low-order perturbation theories for quantum electrodynamics.

c) Dokl. Akad. Nauk SSSR **103**, 203.

Functional renormalization-group equations of quantum electrodynamics are derived for the general case. The relationship between the work by Stückelberg and Petermann (1953) and Gell-Mann and Low (1954) is established. Differential group equations are constructed for the first time, and a program for systematically improving the results of ordinary perturbation theory is formulated.

d) Dokl. Akad. Nauk SSSR **103**, 391.

The differential renormalization-group equations are used to derive a single-loop sum and a previously unknown two-loop sum of ultraviolet logarithms for the effective charge in quantum electrodynamics. In addition, a single-loop ultraviolet asymptotic behavior and an infrared asymptotic behavior are derived for the electron propagator in a transverse gauge.

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