Neutron stars and equation of state of nuclear matter

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Usp. Fiz. Nauk **152**, 683–689 (August 1987)

Neutron stars are compact objects whose properties depend strongly on the equation of state of the highly compressed nuclear matter. Analysis of the emission from several extraterrestrial sources whose activity is linked with neutron stars made it possible to refine the characteristics of these stars. This refinement in turn made it possible to obtain additional information on the properties of nuclear matter.

Neutron stars (with a mass M of the order of the solar mass $M_{\odot} \approx 2\cdot 10^{33}$ g and a radius $R\sim 10$ km) are quite rightly regarded as a laboratory of modern physics. Specifically, stars of this sort, which had been predicted theoretically back in the 1930s, are today the only observable objects in which the macroscopic density of matter reaches 10^{14}–10^{15} g/cm^3 (these densities are at the level of the density of nuclear matter and above) and whose magnetic fields B are in the range 10^{11}–10^{13} G (close to the so-called critical magnetic field $B_0 = m^2c^3/e\hbar = 4.4\cdot 10^{13}$ G, at which the energy corresponding to a transition between two adjacent Landau levels, $\hbar eB/mc$, is comparable to the rest energy of the electron, mc^2). Their gravitation fields are at such a level that effects of the general theory of relativity become important (the radius of a neutron star with the solar mass would be only a few times greater than the gravitational radius $R_g = 2GM/c^2$, so that the characteristic values of the dimensionless gravitational potential at the surface of the star, $\Phi/c^2 = R_g/R$ would be 0.2–0.4).[1,2] Such parameter values have not yet been achieved under terrestrial conditions[1)] (Ref. 3), of course, so the possibility of utilizing the properties of neutron stars to extract information on the characteristics of matter in such extreme states has been under discussion for a long time—since long before these stars were observed experimentally.[4,5] The discovery of several new extraterrestrial sources, with an activity which is believed to be determined specifically by the processes which occur near compact neutron stars, has attracted even more interest to this problem.[1,2,6] In particular, the theory of neutron stars based on the theory of the nucleon-nucleon interaction, the theory of the superfluidity of a neutron liquid, and solid state theory has also been developed further.[7–10]

Historically the first to be discovered, in 1967, were radio pulsars: sources of pulsed radio emission from space. Later, in 1971, x-ray pulsars were observed. Sources of γ bursts were observed in 1973, and 1975 brought "bursters," i.e., sources of x-ray bursts. (Admittedly, the association of γ bursts with neutron stars is not yet completely established.) For other sources we already have some fairly reliable models which are capable of explaining many characteristics of the observed radiation. For example, the radio pulsars are linked with single rotating neutron stars whose activity results from a loss of the kinetic energy of rotation.[1] Other sources are identified with neutron stars which are members of close binary systems. Their activity results from the energy of the matter which has flowed from the companion star to the compact neutron star.[11]

What information can we obtain by analyzing the radiation from such sources? Apparently one of the most interesting directions here is based on the circumstance that observations can yield estimates of the characteristics of real neutron stars, which can therefore be compared with theoretical predictions. In turn it becomes possible in principle to draw some completely definite conclusions regarding the internal structure of such stars, the equation of state of neutron matter, and thus the nature of the nucleon-nucleon interaction.

Most models for the structure of neutron stars are based on a solution of the Tolman-Oppenheimer-Volkoff equation[4]

$$\frac{dP}{dr} = \frac{G}{r^2}\frac{[\rho(r)+(P(r)/c^2)]\,[m(r)+(4\pi r^3P(r)/c^2)]}{1-(2Gm(r)/rc^2)};$$
$$\frac{dm}{dr} = 4\pi r^2\rho(r),$$

i.e., a solution of the equation of the hydrostatic equilibrium of a cold, spherically symmetric star incorporating effects of the general theory of relativity. The pressure $P(r)$ and the density $\rho(r)$ must be related by the equation of state $P = P(\rho)$; it is in the determination of this equation of state that the basic difficulties arise. Crudely speaking, these difficulties arise because various models which agree with "terrestrial" limitations, i.e., with data on the scattering of free nucleons and on the experimental determination of the energy and equilibrium density of nuclear matter, can be constructed for the nucleon-nucleon interaction potential (which is now known quite well).[12] The primary reason for this situation is that determining the properties of dense matter ($\rho\sim\rho_{nucl}\approx 2\cdot 10^{14}$ g/cm^3) is a nontrivial problem in many-body theory.[2)] As a result, we presently have about 20 models, ranging from "soft" equations of state (derived from models in which the average interaction energy corresponds to an attraction at densities of the order of the nuclear density) to "hard" equations of state (obtained for models in which there is a repulsion already at densities below the nuclear density). The harder equations of state have higher pressures at a given density of matter.

The gravitational mass M is plotted against the central density ρ_c and the radius of the neutron star, R, for various equations of state in Fig. 1 (Ref. 2). We see that the harder equations of state predict a larger maximum mass, a larger maximum radius, and a lower central density for the star. The maximum masses are in the interval (1.4–2.7) $M_{\odot}$, and the corresponding radii R are 7–10 km.

The greatest uncertainties arise in the case of fairly mas-

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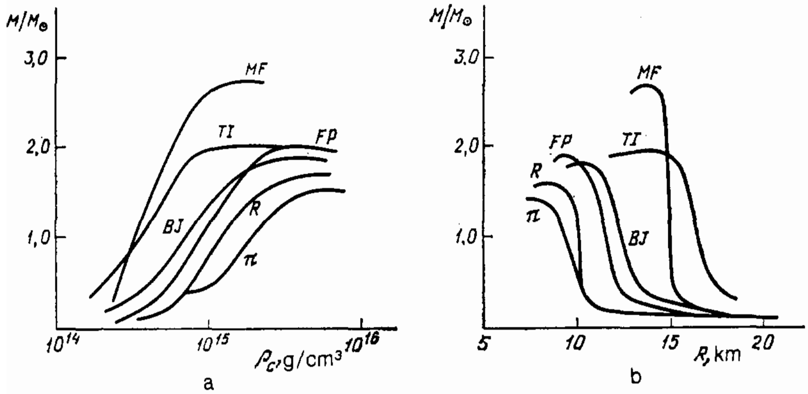

FIG. 1. Gravitational mass of a neutron star as a function of its radius R and its central density ρ_c. The curve labels correspond to various equations of state, whose properties are discussed in detail in Refs. 2, 7, and 8. The hard equations of state (TI, MF) lead to smaller central densities ρ_c and larger radii R for the star than are found from the soft equations of state (R, π).

sive stars, for which the central density should be well above the nuclear density, as can be seen in Fig. 1. One reason for these uncertainties is that at these densities various phase transitions can occur, and the characteristics of these transitions are presently known only within an error of the order of 100%. Among these phase transitions are the following:

1. crystallization of nuclear matter[13];
2. pion condensation[14];
3. transition to a quark-gluon plasma.[15]

Since the various models, corresponding to different equations of state, lead to a fairly broad spectrum of characteristics of a neutron star, as we have seen, one might hope that an accurate determination of these characteristics would also make it possible to find specific results on the actual equation of state of the neutron matter—on the nature of the nucleon-nucleon interaction. Unfortunately, it has not yet been found possible to obtain reliable estimates of even the basic characteristics of neutron stars. For example, the uncertainty in the radius R is 50–100% on the average.[12] There is a similar uncertainty in cases in which it has been found possible to estimate the moment of inertia of the star.[16]

With regard to the mass of a neutron star, we have only a single reasonably accurate estimate at this point. That estimate was based on an analysis of the effects of the general theory of relativity in the close binary system containing the radio pulsar 1913 + 16 (Ref. 17). The mass of the pulsar was found to be $M = (1.42 \pm 0.06)\ M_{\odot}$, and the mass of its invisible companion was approximately the same: $M_c = (1.41 \pm 0.06)\ M_{\odot}$ (Ref. 18). Incidentally, these values agree surprisingly well with a theoretical estimate of the mass of a neutron star based on an analysis of stellar evolution. Specifically, the mass of a neutron star should be close to the so-called Chandrasekhar limit $M_{\rm Ch} = 1.4\ M_{\odot}$ [so that we have[19] $M_{\rm theo} = (1.4 \pm 0.2)\ M_{\odot}$], regardless of whether the star forms as the result of a collapse of the dense core of a massive star or as the result of a collapse of a white dwarf which has reached the limiting stable mass $M_{\rm Ch}$ as the result of an accretion of matter. It can be seen from Table I that all the other (less accurate) estimates of the masses of neutron stars, based on analysis of the emission of x-ray pulsars, agree with this theoretical conclusion.[19]

In summary, the neutron stars which are actually observable have not yet made it possible to choose among the existing models, aside from drawing the rather obvious conclusion that a model of free neutrons is unrealistic (in such a case the maximum mass of a stable neutron star would be only $0.7M_{\odot}$). This result, in particular, is a consequence of the fact that the mass of a neutron star is less convenient than, say, the radius R for determining the equation of state. To explain this point, we note that Fig. 1 shows that all the equations of state which have been considered allow a stable configuration of a neutron star with a mass $M \approx 1.4\ M_{\odot}$. Unfortunately, estimates of the characteristics of neutron stars based on analysis of several other observable effect, e.g., the abrupt changes in period which occur in the cases of certain radio pulsars,[1] have been based to a large extent on a specific mechanism for the effects; i.e., they have been model-dependent.[12] At present these estimates are incapable of unambiguously determining the actual equation of state of neutron matter.

Be that as it may, it has become clear that we need a simultaneous determination of at least two characteristics of a neutron star, e.g., its mass and radius. Only in such a case could we hope to obtain unambiguous information on the actual equation of state of neutron matter. In this connection, attempts have been made to carry out multifaceted studies of this sort in several recent analyses of the radiation from very different sources.[20–25]

There seem to be several reasons for these studies. First, the theory for the radiation from certain sources (e.g., x-ray bursts) has reached a level at which the mass and radius of the neutron star can be determined fairly reliably from the observed radiation. Second, the discovery of rapidly rotating radio pulsars (three of which have periods $P < 10$ ms, which correspond to several hundred revolutions per second) has stimulated the development of a theory of rotating neutron stars and, in particular, a theory for the stability of such rotating stars. Finally, as we have already mentioned, the theory of stellar evolution is today capable of giving us a

TABLE I. Masses of some x-ray pulsars.[19]

x-ray pulsar	Her X-1	Cen X-3	4U1538-52	SMC X-1	4U0900-40	LMC X-4
$M/M_{\odot}$	$1.45^{+0.35}_{-0.40}$	$1.07^{+0.63}_{-0.60}$	$1.87^{+1.33}_{-0.87}$	$1.05^{+0.40}_{-0.30}$	$1.85^{+0.35}_{-0.30}$	$1.70^{+1.90}_{-1.00}$

fairly reliable value for the mass of a neutron star which forms as the result of gravitational collapse. This capability can also be utilized in the analysis of observations.

Let us examine some specific studies in which attempts have been made to determine the characteristics of observable neutron stars.

1. The spectrum of many γ bursts is known[26] to contain an emission line at 400–500 keV. If this line is assumed to correspond to the 511-keV line of the two-γ annihilation of electron-positron pairs, with a gravitational red shift

$$z=\frac{\Delta\lambda}{\lambda}=\frac{GM}{c^2R}, \tag{1}$$

then we could estimate the ratio M/R for the neutron star, as can be seen from (1).

Liang's analysis[23] of 40 sources showed that the red shifts are $z = 0.30 \pm 0.05$. If the masses of the actual neutron stars are assumed to be close to their "evolutionary" value $(1.4 \pm 0.2)M_\odot$, then it turns out that the observed red shifts z correspond to models constructed on the basis of soft equations of state. The harder equations of state lead to values $z = 0.1$–0.2, below the observed values. Consequently, soft equations of state are preferable according to this interpretation.

2. The problem of rapidly rotating neutron stars has been analyzed in more detail. In particular, a detailed analysis of their stability has been carried out, and specific models have been constructed for these rotating stars.[20,22,27] It has been found that they have large radii R and smaller central densities ρ_c than nonrotating stars of the same mass (Fig. 2). These results are not surprising, since rotation reduces the effect of gravity.

It has also been shown that the process primarily responsible for the loss of stability of a rotating neutron star is gravitational radiation, which arises because of nonaxisymmetric perturbations of the surface of a pulsar.[22,27] The limiting period P_{min}, at which the star becomes unstable, is close to the "Kepler" period P_K, at which the velocity at the equator reaches the first cosmic velocity:

$$P_{min}\approx P_K^*=\alpha\left(\frac{R^3}{GM}\right)^{1/2}.$$

According to Newtonian mechanics, we would have $\alpha = 11.8$ for a uniform Maclaurin spheriod.[28] For the real rotating neutron stars, for which we need to consider both the effects of the general theory of relativity and the nonuniformity of the density (which furthermore depends on the specific equation of state of the matter), we would have[20,27] $\alpha = 11.5$–13.7.

It turns out that for the hardest equations of state the limiting rotation period of a neutron star with a mass $M = 1.4\ M_\odot$ would be[27] 1.65 ± 0.15 ms; i.e., this period would be essentially the same as the period of the fastest millisecond pulsar, PSR 1937 + 214, with $P = 1.56$ ms. Turning to models of neutron stars based on soft equations of state, we note that they would allow an even faster rotation, up to $P_{min} = 0.6$ ms. Figure 2 shows the mass as a function of the radius of a neutron star rotating with a period P_K for one of the equations of state.[27]

As we have seen, the determinations of P_{min} have not been very accurate. One thus cannot rule out the possibility that the hard equations of state actually contradict observations, so that preference should be given to soft equations of state. This conclusion was reached in Ref. 22.

There is, however, another interpretation of the existing results. The idea here is that rapidly rotating radio pulsars may have been driven into a rapid spin as the result of an accretion of matter which has flowed from a companion star to the neutron star.[29] Two pulsars with millisecond periods are indeed observed in binary systems.[30] After the accretion stage has played out, the period of such pulsars should remain essentially constant, since the observed slowing rates dP/dt are exceedingly small. It might therefore be suggested that the observed period $P = 1.56$ ms corresponds to precisely that period P_{min} at which the entire angular momentum carried by the accreting matter has been expended not on a further increase in the angular rotation velocity of the star but on the emission of gravitational waves. In this interpretation, preference would instead be given to hard equations of state.[27]

3. Finally, important information has been extracted from an analysis of the emission of x-ray bursts. We recall that their activity stems from a fusion burning of matter which has accreted on a neutron star which has a weak magnetic field. Because of this weak field, the matter accumulates over the entire surface of the star, in contrast with (for example) the case of x-ray pulsars, where all the active processes occur near the magnetic poles.[11]

After a certain time (a few hours for most observed sources) the temperature of the surface layers reaches 10^7–10^8 K, at which a fusion reaction begins. This reaction usually last a few seconds, and the emission spectrum is close to that of a blackbody with some temperature T.

Figure 3 shows the typical behavior of the temperature T and of the radiation flux F which would be received at infinity. We can distinguish two stages reliably. In the first

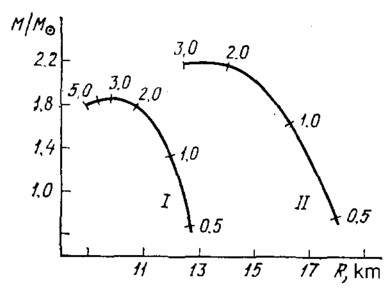

FIG. 2. Plot of the mass as a function of the equatorial radius according to the BJ model.[2,7,8] I—Nonrotating neutron star; II—star which is rotating at the limiting angular velocity. The numbers are the values of the central density ρ_c expressed in units of 10^{15} g/cm^3 (Ref. 27).

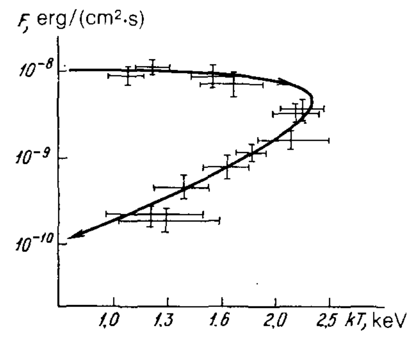

FIG. 3. Evolution of the temperature T and the energy flux F during an x-ray burst. The curve is theoretical, and the points are observations.[25]

stage—the heating—the temperature T rises, but this rise is not accompanied by an increase in the luminosity $L = 4\pi d^2 F$, where d is the distance from the source. Such a constancy of the flux agrees with a theory in which the luminosity L in this stage should be close to the so-called Eddington luminosity

$$L_E = 10^{38} \frac{M}{M_\odot} \text{ erg/s}, \quad (2)$$

which depends only on the mass of the neutron star.[28,31,32] In the second stage—cooling—the luminosity and the temperature conform well to the law

$$L = 4\pi R^2 \sigma T^4, \quad (3)$$

which corresponds to the cooling of a blackbody.

The actual equations from which calculations are carried out must be more complicated, since they must incorporate effects of the general theory of relativity, the unavoidable distortion of the thermal radiation by Comptonization, and the uncertainty which exists in the chemical composition of the matter [on which the Eddington luminosity (2) depends], a possible anisotropy of the burst, and so forth.[25] In general, however, as can be seen from relations (2) and (3), a comparison of the observed characteristics of x-ray bursts with the theory yields simultaneous estimates of the mass and radius of the neutron star.

For example, analysis of two bursts observed from the source 4U 1746-37 on the European satellite EXOSAT shows that the mass and radius of the neutron star lie in the following ranges[25]:

$$M = (1.0 \pm 0.8)\ M_\odot,$$

$$R = 6 \pm 3 \text{ km}.$$

It can be seen from Fig. 1a that such values correspond to softer equations of state. It we instead follow Fujimoto and Taam,[24] we find an even greater accuracy for the source MXB 1636-536, which was observed on the Japanese satellite TEHMA. According to Ref. 24 (see also Ref. 33) we would have

$$M = (1.46 \pm 0.19)\ M_\odot,$$

$$R = 10.2 \pm 1.1 \text{ km}.$$

These values agree with only one of the equations of state which had been proposed earlier: the model of Friedman and Pandharipande.[27] This model occupies an intermediate position in the sense that it lies between hard and soft equations of state (Fig. 1).

The examples which we have discussed here show that we are fairly close to being able to determine the actual characteristics of neutron stars from observations of several sources. At the moment, however, we would generally have to give preference to the soft equations of state, as we have seen.

This cannot be regarded as a final conclusion in any way, of course, since the results found from analysis of the radiation from sources of this type still depend to a large extent on their interpretation. In particular, the nature of the emission lines observed in the spectra of γ bursts may be totally unrelate to the annihilation line. Nevertheless, there is the hope that further developments of the theory and improvements in experimental accuracy will make it possible finally to choose among the equations of state which have been proposed and therefore to draw specific conclusions about the basic properties of highly compressed neutron matter in the near future.

I wish to thank V. L. Ginzburg, A. V. Gurevich, and D. A. Kirzhnits for useful discussions.

[1)]The important point here is not the density of matter itself (densities greater than the density of nuclear matter can be reached in collisions of heavy ions) but the remoteness of neutron matter from the stability valley.

[2)]An equation of state cannot as of now be derived also in quantum chromodynamics.

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Translated by Dave Parsons