Angular momentum of electromagnetic waves

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Although comparatively little effort has been devoted to analyzing the angular momentum of radiation, studying it experimentally, or exploring the possibilities for making practical use of associated effects, research on these questions dates back many years and is definitely of interest.

Soon after the development of Maxwell's theory it was shown that ponderomotive forces arise in an alternating electromagnetic field. In his classic experiments, P. N. Lebedev showed that light exerts a pressure on an object which it strikes, i.e., that it has a momentum as well as an energy. A few years before Lebedev's experiments, in 1897, Sadovskiĭ, a university professor in Tartu, asserted[1] that circularly polarized light should have an angular-momentum flux. Sadovskiĭ's theory initially drew objections from Shaposhnikov,[2] who analyzed an infinitely extended plane wave. Later, however, it was shown in studies by Abraham,[3] Epstein,[4] and Ehrenfest[5] that a real, collimated, circularly polarized quasiplane wave should have an angular momentum. About forty years later, in 1935–1936, Holborn in England[6] and a group of physicists at Princeton University led by Beth,[7] working in consultation with Einstein, proved experimentally that circularly polarized light has an angular momentum. In Ref. 7, a circularly polarized wave was passed twice through a half-wave plate which could be rotated around the axis of the light wave. After the passage of the light, the plate acquired an angular momentum proportional to the number of photons which had passed per unit time. These exceedingly subtle experiments not only confirmed Sadovskiĭ's hypothesis but also made it possible to determine Planck's constant within $\sim 10\%$. In Beth's experiments the angular momentum was directed along the propagation direction of the light wave. From the methodological standpoint, however, nothing significant resulted in terms of an analysis of the angular momentum. While the first principles of the theory of the electromagnetic field are clear, in the detailed analysis of many aspects of this theory in the literature on angular momentum one runs into omissions, imprecision, and sometimes erroneous statements. In the well-known textbook by Pohl,[8] for example, we read the assertion that the angular momentum must coincide with the propagation direction under any conditions.

In the present note we analyze the properties of the angular momentum of radiation and the possibilities for an experimental study of this angular momentum. We are concerned primarily with time relationship between the nature of angular momentum and the particular features of the radiation source and of the wave process itself (the presence of plane or nonplane waves). The radiation may have both a spin angular momentum directed along the propagation direction and an orbital angular momentum. In particular, the angular-momentum flux may not coincide in direction with the Poynting vector **S** under certain conditions.

We first consider the classical treatment of the problem. The angular momentum of radiation with respect to a given point 0 or a given axis is defined by

$$\mathbf{M}=\frac{1}{c^2}\int [\mathbf{rS}]\,\mathrm{d}\tau, \tag{1}$$

where **r** is the radius vector, and $d\tau$ is a volume element. By definition, we have $M \perp S$, and for an unbounded plane wave propagating along the z axis there is no angular momentum: $M_z = 0$. For a real plane wave, which is spatially bounded (along the x and y axis), however, this is not the case. As was shown in Ref. 19 and 20, a finite contribution to M_z is made by the boundary region of the wave packet [expression (1) can be written as the sum of three integrals, one of which converts into a surface integral; it is this integral which determines M_z]. According to Heitler,[9] a circularly polarized plane wave which is nonvanishing only inside a cylinder with an arbitrary radius R and an axis along z has an angular-momentum component

$$M_z=\pm\frac{U}{\nu} \tag{2}$$

by virtue of the surface effect; here

$$U=\frac{1}{8\pi}\int (E^2+H^2)\,\mathrm{d}\tau$$

is the energy, and ν is the frequency of the radiation. In the classical case, an angular momentum can arise for a plane wave either along or opposite the direction of the Poynting vector. For spherical waves centered at the origin of coordinates one can also distinguish waves of two types, which correspond to two possible polarizations of the plane wave and which convert into each other when the replacements $E \to H$ and $H \to -E$ are made. These are waves of electric and magnetic multipole types. By virtue of the spherical symmetry, each wave type, like an atomic level, can be additionally characterized by an orbital quantum number l (the multipolarity of the radiation) and its projection m. The numbers l and m correspond to spherical harmonics Y_{lm}, which specify the angular distributions of the intensity. Multipole expansions in l and m are studied in detail in Ref. 9.

The angular momentum lost by the radiator (e.g., an atom) at the origin of coordinates is perceived as a spherical outgoing wave from the radiation source. For the composite system consisting of the atom and the field, angular momentum is conserved. A net angular momentum emitted by the atom crosses any spherical surface enveloping the source. Although the absolute angular-momentum density approaches zero with increasing sphere radius R, the integration volume simultaneously undergoes a corresponding increase. The magnitude of the total angular momentum

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which crosses the spherical surface is nonzero, and is given by expression (2), according to a direct calculation.[3,10] For a plane wave, there is no transverse component of the angular momentum (with $M_z = 0$) by virtue of the gauge invariance of the theory. Such a component would exist if the photon had a zero rest mass and thus three independent polarization states. In the general case of nonplane waves, however, in contrast with the case of a plane wave, it is generally necessary to consider the longitudinal component of the field. Let us examine this question in more detail.

The radiation from any source generally carries off fluxes of energy, momentum, and angular momentum. The particular features of the formation of the angular-momentum flux are conveniently studied in the example of the radiation from a rotator.

As was pointed out some time ago by Sommerfeld,[10] the problem of the radiation from a rotator reduces to one of ordinary dipole radiation. The field of a rotator may be treated as a superposition of the fields of two dipoles, which are oriented perpendicular to each other and which are radiating out of phase (or it may be treated as a superposition of the fields of three mutually perpendicular dipoles, in the case of a three-dimensional rotator). The equations for the dipole radiation in the general case of an arbitrary nonplane wave (in which case, the field can not be treated as a plane wave, even over a short distance) are given in Ref. 11:

$$\mathbf{H}_\omega = ik\,[\mathbf{d}_\omega \mathbf{n}]\left(\frac{ik}{R_0} - \frac{1}{R_0^2}\right)\exp(ikR_0), \tag{3}$$

$$\mathbf{E}_\omega = \left\{\mathbf{d}_\omega\left(\frac{k^2}{R_0} + \frac{ik}{R_0^2} - \frac{1}{R_0^3}\right) + \mathbf{n}(\mathbf{n}\mathbf{d}_\omega)\right.$$
$$\left.\times\left(\frac{k^2}{R_0} - \frac{3ik}{R_0^2} + \frac{3}{R_0^3}\right)\right\}\exp(ikR_0); \tag{4}$$

here $\mathbf{E}_\omega$ and $\mathbf{H}_\omega$ are Fourier components of the electric and magnetic fields, k is the wave vector, $\mathbf{d}$ is the dipole moment, $\mathbf{n} = \mathbf{R}/R$, and R is the radius vector to the point in space from the origin of coordinates, which is the position of the radiator. At distances small in comparison with the wavelength ($kR_0 \ll 1$), we can ignore terms $\sim 1/R_0$ and $1/R_0^2$; then

$$\mathbf{E}_\omega = \frac{1}{R_0^3}\,[3\mathbf{n}\,(\mathbf{d}_\omega \mathbf{n}) - \mathbf{d}_\omega]$$

corresponds to the static field of a dipole. There is no magnetic field in this approximation. In an analysis of electromagnetic radiation over distance large in comparison with the wavelength it is customary to ignore terms $\sim 1/R_0^2$ and $1/R_0^3$; in this approach, one finds the following standard expression for the wave in the wave zone:

$$\mathbf{E}_\omega = \frac{k^2}{R_0}\,[\mathbf{n}\,[\mathbf{d}_\omega \mathbf{n}]]\exp(ikR_0), \quad \mathbf{H}_\omega = -\frac{k^2}{R_0}\,[\mathbf{d}_\omega \mathbf{n}]\exp(ikR_0).$$

In both limiting cases, of small and large values of R_0, terms $\sim 1/R^2$ in $\mathbf{E}_\omega$ are ignored. As Heitler pointed out,[9] these terms play a fundamental role, determining the angular momentum in the case if a spherical wave. We wish to stress that for spherical waves with a singularity at the origin the electric vector has a component E_R along the direction of the Poynting vector. In this case we have

$$M_z = \frac{1}{4\pi c}\int d\tau R\,(E_R H_z - E_z H_R).$$

Expression (2) remains in force. A finite contribution to M_z is made by the intermediate region with $E_R \sim 1/R_0^2$ and $H \sim 1/R_0$ (for a magnetic multipole we would have $E \sim 1/R_0$,

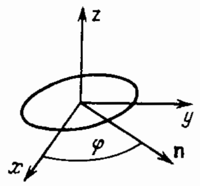

FIG. 1. Radiation in the plane of a rotator.

Accordingly, if we ignore terms of higher order in $\mathbf{H}_\omega$ ($\sim 1/R_0^2$) and in $\mathbf{E}_\omega$ ($\sim 1/R_0^2$) in accordance with the discussion above, we are left with terms $\sim 1/R_0$ in $\mathbf{H}_\omega$ and $\sim 1/R_0$ and $\sim 1/R_0^2$ in $\mathbf{E}_\omega$.

The flux of the Poynting vector dS through an area $R^2\,d\Omega$ (Ω is the solid angle) is expressed in terms of $\mathbf{E}$ and $\mathbf{H}$:

$$d\mathbf{S} = \frac{c}{4\pi}\,2\,\mathrm{Re}\,[\mathbf{E}\mathbf{H}^*]\,R^2\,d\Omega. \tag{5}$$

The flux of angular momentum across an area $R^2\,d\Omega$ is also expressed in terms of these fields:

$$d\mathbf{M} = \frac{1}{4\pi}\,2\,\mathrm{Re}\,[\mathbf{R}\,[\mathbf{E}\mathbf{H}^*]]\,\mathbf{R}^2\,d\Omega = \frac{1}{c}\,[\mathbf{R}\mathbf{S}]\,R^2\,d\Omega. \tag{6}$$

Substituting $\mathbf{E}$ and $\mathbf{H}$ from (3) and (4) into (5) and (6), and retaining only the terms listed above, we find

$$d\mathbf{S} = \frac{c}{4\pi}\,2\,\mathrm{Re}\left\{-k^4\,|[\mathbf{d}^*\mathbf{n}]|^2\,\mathbf{n} + \frac{2ik^3}{R}\,[\mathbf{d}^*(\mathbf{n}\mathbf{d})\right.$$
$$\left.-(\mathbf{n}\mathbf{d})(\mathbf{n}\mathbf{d}^*)\,\mathbf{n}]\right\}d\Omega, \tag{7}$$

$$d\mathbf{M} = \frac{k^3}{\pi}\,\mathrm{Re}\,\{i\,(\mathbf{n}\mathbf{d})(\mathbf{n}\mathbf{d}^*)\}\,d\Omega. \tag{8}$$

We consider a rotator in the x, y plane (Fig. 1). For a rotating dipole we can write the expression

$$\mathbf{d} = \mathbf{d}_0\,(\mathbf{e}_x \cos\omega t + \mathbf{e}_y \sin\omega t); \tag{9}$$

where $\mathbf{e}_x, \mathbf{e}_y$ (and $\mathbf{e}_z$) are unit vectors. We assume $d = d_0\exp(-i\omega t)$; we then have

$$\mathbf{d} = d_0\,(\mathbf{e}_x + i\mathbf{e}_y). \tag{10}$$

For clarity we will calculate $d\,\mathbf{S}$ and $d\,\mathbf{M}$ for $\mathbf{n}$ lying in the x, y plane (Fig. 1). Substituting (10) into (7), (8), we find

$$d\mathbf{S} = \frac{c}{4\pi}\,k^2 d_0^2\mathbf{n}\,d\Omega, \tag{11}$$

$$d\mathbf{M} = \frac{k^3}{\pi}\,d_0^3\mathbf{e}_z\,d\Omega. \tag{12}$$

We see $d\,\mathbf{S}$ is directly along $d\,\mathbf{M}$ here, and $d\,\mathbf{M}$ is directly along $\mathbf{e}_z$; i.e., in this case these vectors form a right angle with each other. Relation (2) also holds in this case ($d\,\mathbf{S}$ and $d\,\mathbf{M}$ do not depend on φ, and an integration can be carried out over φ). In a similar way, we can calculate $d\,\mathbf{S}$ and $d\,\mathbf{M}$ for an arbitrary direction of $\mathbf{n}$ (Fig. 2). In the general case we find

$$d\mathbf{S} = \frac{c}{2\pi}\,k^2 d_0^2\,(1 + \cos^2\theta)\,d\theta\cdot\mathbf{n}, \tag{13}$$

$$d\mathbf{M} = \frac{k^3}{\pi}\,d_0^3\sin^2\theta\,d\theta\cdot\mathbf{e}_z. \tag{14}$$

Integrating over the solid angle, we find the following results

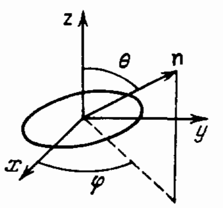

FIG. 2. Radiation from a rotator in an arbitrary direction.

for the total flux: $S = 4ck^2d_0^3/3$, $M_z = 4k^3d_0^3/3$. In other words, general relation (2) holds. The fluxes of angular momentum and energy can thus make an angle with each other in any part of a spherical wave generated by a rotator. This assertion applies not only to the near field but also the far field. We do wish to stress, however, that in the latter case the spherical wave is assumed to be propagating in some constant solid angle. If, with distance from the source, the radiation detection area remains constant, so that the solid angle decreases, we will ultimately arrive at the case of an ordinary plane wave, which has no longitudinal component of the polarization, and no angle can arise between **M** and **S**.

We turn now to the quantum-mechanical treatment of the problem of the angular momentum of radiation. As in the classical case, there is no point in examining an unbounded plane photon, since the wave vector is normalized to an arbitrarily large but finite volume. In quantum mechanics the momentum of a photon and its angular momentum are described by noncommuting operators. This statement means that states of a photon with a definite angular momentum do not have a definite momentum, and vice versa. A plane wave can thus have only a spin angular momentum. According to Ref. 9, a bounded, plane, circularly polarized wave which is propagating along the axis (the z axis) of a cylinder of arbitrary radius carries an angular momentum $M_z = \pm U/\nu$. If $E = h\nu$, then we have $M_z = \pm h$; i.e., a spin of 1 corresponds to each plane circularly polarized photon. The spin component is directed along the wave vector (and has a magnitude of ± 1). There is no transverse component of the spin by virtue of gauge invariance. If a plane wave is characterized by the quantities P_x, P_y, P_z, then a cylindrical wave and a spherical wave are characterized by P_x, M_x, U and M^2, M_x, and U, respectively.

For a spherical wave we need to consider the orbital angular momentum in addition to the intrinsic angular momentum of the photon. In the quantum-mechanical case, the relations $M^2 = hl(l+1)$, $M_z = mh$ hold for the angular momenta of spherical photons. In the problem of the radiation from a rotator, the angular momentum along the axis arises specifically because the wave is spherical, rather than planar. In such a wave the momentum is not determined. The part of this wave which is cut out by the solid angle $\Delta\Omega$ carries angular momentum only to the extent that this wave differs from a plane wave. Under these conditions there may be an angle between **S** and **M**. This angle ultimately arises because there is a longitudinal component of the electromagnetic field, which we must take into account to the extent to which there is a deviation from a plane wave. On the whole, there is a complete correspondence between the quantum-mechanical and classical descriptions of the angular momentum of radiation by virtue of the integer value of the spin of the photon.

The early methodological difficulties and errors in the analysis of the angular-momentum problem stemmed from ascribing to unbounded plane waves a kind of absolute nature; later on, the difficulties stemmed primarily from the separate analyses of the near and far zones. As has been mentioned already, the intermediate zone plays an important role in the formation of the angular momentum. For a systematic analysis of the angular momentum of radiation, we should thus either consider the overall system consisting of the source and the radiation or directly consider a longitudinal component of the field which stems from a deviation from a plane wave.

Analysis of nonplane waves requires generalizing the ordinary concept of the Stokes parameter in an analysis of polarization states. Roman[12] and Barakat[13] were the first to point out the need for a generalization of this sort for nonplane waves. In general, a description of the properties of electromagnetic waves by means of Stokes parameters is inadequate. For a plane wave, the Stokes parameters s_0, s_1, s_2, s_3 completely specify the polarization state and can easily be determined experimentally. For a quasimonochromatic plane wave propagating along the z direction we would have

$$\begin{aligned} &s_0 = \langle a_1^2\rangle - \langle a_2^2\rangle, && s_1 = \langle a_2^2\rangle - \langle a_2^2\rangle, \\ &s_2 = 2\langle a_1a_2\cos(\varphi_1 - \varphi_2)\rangle, && s_3 = 2\langle a_1a_2\sin(\varphi_1 - \varphi_2)\rangle, \end{aligned} \tag{15}$$

where a_1 and a_2 are the instantaneous amplitudes of the two mutually perpendicular components E_x and E_y, and φ_1 and φ_2 are their phases. The Stokes parameters are determined experimentally by measuring the intensity $I(\chi, \delta)$ in the direction of a light wave which makes an angle χ with the x axis (δ is the retardation of the y component):

$$\begin{aligned} &s_0 = I(0^\circ, 0) + I(90^\circ, 0), \\ &s_1 = I(0^\circ, 0) - I(90^\circ, 0), \\ &s_2 = I(45^\circ, 0) - I(135^\circ, 0), \\ &s_3 = I\left(45^\circ, \frac{\pi}{2}\right) - I\left(135^\circ, \frac{\pi}{2}\right). \end{aligned} \tag{16}$$

The parameter s_0 corresponds to the total intensity of the light, while s_3 corresponds to the difference between (a) the intensity of the light which is transmitted through an instrument which transmits oscillations with the right-hand circular polarization and (b) the intensity of light which is transmitted through an instrument which transmits oscillations with a left-hand circular polarization (i.e., the angular momentum of a plane wave). The Stokes parameters satisfy the relation $s_0 = s_1 + s_2 + s_3$. The polarization state of a wave is imaged by a point on a sphere $s_1 + s_2 + s_3 = \text{const}$ (a Poincaré sphere). The Stokes parameters can also be written directly in terms of the projections (E_x and E_y) of the complex vector of the electromagnetic wave:

$$\begin{aligned} &s_0 = \langle E_x^2\rangle + \langle E_y^2\rangle, && s_1 = \langle E_x^2\rangle - \langle E_y^2\rangle, \\ &s_2 = 2\,\mathrm{Re}\,\langle E_xE_y^*\rangle, && s_3 = 2\,\mathrm{Im}\,\langle E_y^*E_y\rangle. \end{aligned} \tag{17}$$

For a covariant description of the polarization properties of a plane light beam it is convenient to replace the four real Stokes parameters by the 2×2 matrix (a coherence matrix)

$$R_2 = \left\| \begin{matrix} \langle E_xE_x^*\rangle & \langle E_xE_y^*\rangle \\ \langle E_yE_x^*\rangle & \langle E_yE_y^*\rangle \end{matrix} \right\| = \frac{1}{2}\sum_{i=0}^{3} s_i\sigma_i, \tag{18}$$

where σ_i are the Pauli matrices. For an analysis of the polarization characteristics by means of a coherence matrix, one uses rotation groups (SU_2) and the Lorentz group.

In contrast with the plane wave, which has only two transverse components of the electric field, E_x and E_y, in the general case of a nonplane wave we would need to consider a longitudinal component E_z. The 2×2 coherence matrix describing a plane wave would then be replaced by the 3×3 matrix

$$R_3=\left\| \begin{matrix} \langle E_xE_x^*\rangle & \langle E_xE_y^*\rangle & \langle E_xE_z^*\rangle \\ \langle E_yE_x^*\rangle & \langle E_yE_y^*\rangle & \langle E_yE_z^*\rangle \\ \langle E_zE_x^*\rangle & \langle E_zE_y^*\rangle & \langle E_zE_z^*\rangle \end{matrix} \right\| = \sum_{i=1}^{8} r_i\rho_i. \qquad (19)$$

The nine quantities r_i are generalized Stokes parameters. The 3×3 matrices ρ_i are basis matrices—analogs of the Pauli matrices. Explicit expression (for them are given in Ref. 12. The generalized Stokes parameters can be expressed in terms of the instantaneous amplitudes a_i and the phases φ_i of the components E_i

$$\begin{gathered} r_0=2\langle a_2^2\rangle,\quad r_1=\langle a_2^2\rangle-\langle a_3^2\rangle,\quad r_2=2\langle a_1a_2\cos(\varphi_1-\varphi_2)\rangle, \\ r_3=2\langle a_1a_2\sin(\varphi_1-\varphi_2)\rangle,\quad r_4=\langle a_1^2\rangle+\langle a_3^2\rangle-2\langle a_2^2\rangle, \\ r_5=2\langle a_1a_2\cos(\varphi_1-\varphi_2)\rangle-2\langle a_2a_3\cos(\varphi_2-\varphi_3)\rangle, \qquad (20) \\ r_6=2\langle a_1a_2\sin(\varphi_1-\varphi_2)\rangle-2\langle a_2a_3\sin(\varphi_2-\varphi_3)\rangle, \\ r_7=2\langle a_1a_3\cos(\varphi_1-\varphi_3)\rangle,\quad r_8=2\langle a_1a_3\sin(\varphi_1-\varphi_3)\rangle. \end{gathered}$$

These nine quantities, which generalize the Stokes parameters, are generally independent and form a complete system. The longitudinal component E_z is a special case; **E** and **H** are nonequivalent according to (3)–(4). For **H**, terms $\sim 1/R_0$ have been taken into account, and for **E** terms $\sim 1/R_0$ and $\sim 1/R_0^2$ have been taken into account. In the general case of a quasimonochromatic nonplane wave, a derivation of the complete polarization characteristic would therefore in principle require an experimental measurement of all nine generalized Stokes parameters. In constrast with plane waves, for which polarization measurements are very simple in nature, according to (16), the problem in this case is not a trivial one. The problem is particularly involved for measurements in the low-frequency range, where the plane-wave approximation breaks down. To derive explicit expressions for the matrix elements of the coherence matrix, one can use measurements of the angular momentum and the angle which it makes with the Poynting vector. As has already been pointed out, the Stokes parameter s_3 corresponds to the angular momentum; correspondingly, the parameters r_3, r_6, and r_8 can be determined from the angular momentum. The net result is that it is worthwhile to take up the problem of determining the possibilities and effectiveness of measurements of the angular momentum for nonplane waves.

In fact, measurements of this sort were carried out a long time ago. Particularly noteworthy are the experiments by Gorozhankin[14,15] and Lertes.[16] Gorozhankin used magnetic dipoles supplied with an alternating current with a frequency of 0.5 MHz and a phase shift a 90°. The rotating electromagnetic field created by these dipoles drove a rotor mounted in jeweled bearings into rotation. Lertes produced a rotating electromagnetic field by means of two electric dipoles formed by two capacitors with field lines which ran perpendicular to each other in the horizontal plane. An alternating voltage with a phase shift of 90° was applied to the capacitors. In plan view, the capacitor plates formed a square of a sort. Experiments were carried out at frequencies $\omega=4.62\cdot10^7$, $\omega=1.75\cdot10^8$ and $\omega=4.48\cdot10^8$. A glass cell with a volume ~ 100 cm^3 was suspended on a thin filament between the capacitor plates. This cell was filled with various polar liquids. When a voltage was applied to the capacitor plates, the cell rotated through a certain angle. That experiment was carried out to test Debye's theory of polar liquids.

Those investigators did not call attention to the circumstance that in their experiments the energy flux of the rotating electromagnetic field was propagating in a direction not parallel to the rotation axis. Those experiments essentially demonstrate the presence of a component of the angular momentum which is perpendicular to the momentum in the near field.

The following experiment might be carried out to study the angular momentum of radiation with a wavelength of the order of 1–1.5 cm (Fig. 3).

The angular momentum is detected by a light metal ring 1, which is a good absorber for radiation at these wavelengths. The ring is suspended on a thin quartz filament 2. At the center of the ring are two dipoles, 3′ and 3″, which are crossed at right angles and which are supplied with power from a common oscillator with a phase shift of $\pm 90°$. The quartz filament is attached to the ring by three branch filaments 2′, 2″, and 2‴. A tiny mirror 4 attached to the quartz filament is used to observe and measure the angle through which the ring rotates. The sensitivity of this apparatus might be improved, first (as in Beth's experiments), by making use of a resonance effect, by changing the phase shift from $+90°$ to $-90°$ at the frequency at which the ring is driven or, second, by using a photooptic method to measure small rotation angles of the mirror.[17] This approach would make it possible to detect rotations of the mirror through a small fraction of an arc second. Noise of any sort could be eliminated by placing the entire apparatus in vacuum; radiometric effects should be eliminated as a result. The annular shape of the detector should rule out an effect of radiation pressure on the rotation of the detector.

The experimental methods for determining the angular-momentum vector which we have discussed here yield data on the generalized Stokes parameters for nonplane waves. At low frequencies the angular momentum at a given

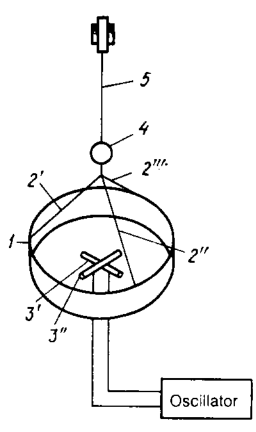

FIG. 3. Schematic diagram of measurements of angular momentum in the UHF range. 1—Receiving ring, which is the detector of the angular momentum; 2, 2′, 2″, 2‴—suspension of the ring; 3′, 3″—dipoles; 4—tiny mirror for observing a rotation of the detecting ring; 5—quartz filament.

power will be higher than at high frequencies. In other words, the effects which we have been discussing here would occur in an ac electric motor, where the rotating electromagnetic field set up by the stator transmits its own angular momentum to the rotor. Consequently, questions pertaining to research on the angular momentum of radiation may bear directly on, for example, research on the effects of low-frequency radiation on biological objects, research on wave propagation through gyrotropic media, and the analysis of long-wave radiation in the magnetosphere, with frequencies between a fraction of a hertz and hundreds of hertz. For such frequencies, the entire earth is the near zone. There is also the question of developing specific experimental procedures for measuring the generalized Stokes parameters (or, in other words, for measuring the generalized polarization states for nonplane waves).[19,20] On this basis one could obtain additional information on the source—information beyond that which can be extracted from measurements of the polarization of plane waves. The analysis above and the experimental arrangements discussed here show that measurements of the angular momentum of electromagnetic waves should play an important role here.[18]

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