

Crystallization of a three-dimensional electron gas

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The most important results of theoretical studies of crystallization of a three-dimensional electron gas are presented. The crystallization criterion, magnetic field effects, viscous-liquid model, and possibility of experimental realization of Wigner ordering are considered. The results of experimental studies in compensated semiconductors published during the last two decades are analyzed in detail using the concepts of Wigner crystallization. It is shown that there is not a single convincing experimental proof of the existence of Wigner crystallization, and that all known experimental results can be explained in a noncontradictory way by the magnetic-field-induced localization of electrons in potential wells created by randomly distributed impurities.

TABLE OF CONTENTS

Introduction	676
1. Electron-lattice stability	676
2. Charge density waves	678
3. Electron crystal in a magnetic field	679
4. On the possibility of experimental realization of Wigner ordering	682
4.1. Compensated semiconductors. 4.2. Magnetic insulators.	
5. Electron crystallization and activation conductivity	683
6. Electron condensation in InSb	685
7. Viscous-liquid model	686
8. Electron localization in $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$	687
8.1. Wigner condensation? 8.2. Viscous liquid? 8.3. Disorder.	
Conclusions	697
References	698

INTRODUCTION

More than 50 years ago E. Wigner demonstrated that under certain conditions a low density electron system must crystallize. Since, according to estimates, the required densities are much lower than the electron densities in real metals, this prediction of Wigner for many years was considered as an elegant, but unrealistic idea. A significant number of theoretical studies which made the criteria and conditions of electron crystallization more precise have appeared, but no experimental proofs were available.

In the last 10–15 years the interest in this problem has increased dramatically in connection with the study of one- and two-dimensional systems. In 1979 it was found that the electrons captured on the surface of liquid helium form a two-dimensional Wigner lattice. In the opinion of many researchers, there are convincing proofs of the existence of 2-D electron crystallization in inversion layers of metal-semiconductor interfaces, placed in a strong magnetic field. Approximately 20 years ago, there was expressed a possibility of crystallization of the ion plasma on the surface of white dwarfs and neutron stars, where magnetic fields reach $\sim 10^6$ and $\sim 10^{14}$ Oersted, respectively.

The problem of the existence of Wigner crystallization in a 3-D electron system was discussed for the first time already in the 60s. The first paper which attempted to explain some peculiarities of the Hall effect in a n -InSb semiconductor by assuming that a 3-D electron lattice is formed in a magnetic field was published in 1968. In 1979, the same assumption was made in regard to a n - $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ semiconductor. Since then, a number of publications have ap-

peared in which the assumption about the existence of electron ordering in n - $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ is supported and developed. The review paper of Ref. 1a and the collection of papers of Ref. 1b are devoted to the subject of Wigner crystallization.

This review describes and analyzes the most important results of theoretical and experimental studies of 3-D electron gas crystallization.

1. ELECTRON-LATTICE STABILITY

For sufficiently low concentrations, an electron gas will, as indicated for the first time by Wigner², spread in an orderly way against the background of homogeneous positive charge of ions (“jellium”), forming a nonconducting crystal lattice. Such an electron crystal is stable under the condition that the average potential energy of interelectron interaction is larger than the average kinetic electron energy. In fact, if the Coulomb repulsion energy exceeds the kinetic energy, the electrons will try to keep the largest possible distance from each other. Such a situation arises at sufficiently small concentrations of electrons n , i.e., for sufficiently large interelectron distances $r_0 = (3/4\pi n)^{1/3}$. If we express r_0 in units of the Bohr radius $a_B = \hbar^2/m_0e^2$, i.e., put $r_s = r_0/a_B$, then the Fermi energy of a free electron gas will be $\varepsilon_F = 3.68/r_s^2$ Ry and the average kinetic energy of electrons will be $\bar{\varepsilon} = (3/5)\varepsilon_F = 2.21/r_s^2$ Ry. The Coulomb repulsion energy of two electrons is $\varepsilon_{\text{pot}} = e^2/r_0 = 2r_s$ Ry. For sufficiently large r_s , the kinetic energy $\bar{\varepsilon} \sim r_s^{-2}$ becomes smaller than the potential energy $\varepsilon_{\text{pot}} \sim r_s^{-1}$, and cannot destroy the

ordered electron structure, the formation of which is favored by the electron repulsion.

It is obvious that the stability of an electron crystal is determined not only by the relative contribution of the potential ε_{pot} and kinetic $\bar{\varepsilon}$ electron energies, but also by the vibration amplitude of electrons near their equilibrium position. The zero-point vibration amplitude in an electron crystal is proportional to $r_s^{-3/2}$ and is small for sufficiently low concentrations. With the temperature rise to a value $\sim k_B T_M$ exceeding the energy of electron-electron interactions, the electron crystal "melts." ¹¹⁾

Obviously, one can obtain the stability criterion of an electron crystal without taking into account the lattice vibrations by equating the potential and kinetic energies. This imposes conditions on the critical values of the interelectron distance r_s^{cr} , or on the electron concentration n_{cr} . From the condition $\varepsilon_{\text{pot}} = \bar{\varepsilon}$, we obtain for a free electron gas

$$r_s^{\text{cr}} \approx 1.1, \quad n_{\text{cr}}^{1/3} a_B \approx 0.56. \quad (1)$$

In the Hartree-Fock approximation without taking into account the exchange and correlation terms, $\varepsilon_{\text{pot}} = 1.2/r_s$ Ry, and

$$r_s^{\text{cr}} \approx 1.84, \quad n_{\text{cr}}^{1/3} a_B \approx 0.34. \quad (2)$$

In the Hartree-Fock approximation with exchange ($\varepsilon_{\text{exc}} = -0.916/r_s$ Ry and correlation ($\varepsilon_{\text{cor}} = -0.88/r_s$ Ry) terms, $\varepsilon_{\text{pot}} = -0.596/r_s$ Ry and

$$r_s^{\text{cr}} \approx 3.7, \quad n_{\text{cr}}^{1/3} a_B \approx 0.17. \quad (3)$$

In the last two decades, a lot of attention has been devoted to the critical parameter of electron crystallization r_s^{cr} . Various methods have been used to calculate it, but the most reliable results are obtained only for the ground state calculations. A method used most often for the estimate of r_s^{cr} is the method based on the empirical Lindeman melting criterion. According to that criterion, the crystal lattice melts if the average displacement of an atom is $\langle u^2 \rangle^{1/2}$, i.e., the average amplitude of its vibrations, reaches a certain critical fraction δ of the interatomic distance R : $\langle u^2 \rangle^{1/2}/R = \delta$. The accuracy of determination of the crystal stability criterion depends, obviously, on the accuracy of the δ estimate. The parameter δ is not calculated theoretically, but must be estimated from the experimentally found melting temperature T_M . The value of δ turns out to be practically the same for metals of one class. Thus, for sodium and other alkali metals, an estimate from the formula, relating δ and the Debye and melting temperatures, to the interatomic distance and atomic weight³, gives $\delta \approx 1/4$. At $T = 0$ K, the amplitude $\langle u^2 \rangle^{1/2}$ of an electron crystal is determined by the zero-point vibrations of electrons. Assuming that only longitudinal vibrations with the frequency $\omega_p = (4\pi e^2 n/m_0)^{1/2}$ are important, Nozier and Pines⁴ have obtained

$$\frac{\langle u^2 \rangle^{1/2}}{R} = \frac{1}{r_s} \left(\frac{\hbar}{2m_0 \omega_p} \right)^{1/2} = \frac{1}{(12r_s)^{1/4}}.$$

From here it follows that an electron crystal must melt at

$$r_s^{\text{cr}} \approx 20, \quad n_{\text{cr}}^{1/3} a_B \approx 3.1 \cdot 10^{-2}. \quad (4)$$

Coldwell-Horsfall and Maradudin demonstrated⁵ that the value of r_s^{cr} is very sensitive to the choice of the parameter δ . From their formula, $r_s^{\text{cr}} = 0.405\delta^{-4}$, it follows that for

$\delta = 1/4$ $r_s^{\text{cr}} \approx 104$, and for $\delta = 1/2$ $r_s^{\text{cr}} \approx 6.5$. According to calculations of Kugler⁶, $r_s^{\text{cr}} \approx 490$ for $\delta = 1/3$ and $r_s^{\text{cr}} \approx 1540$ for $\delta = 1/4$. In the Ref. 6 it was shown also that the neglect of the phonon dispersion by the authors of Ref. 4 leads to a significant underestimate of r_s^{cr} .

Another method for the estimate of r_s^{cr} , suggested by de Wette⁷, starts with an assumption that the melting of an electron lattice occurs as a result of the disappearance of bound electron states. This approach is based, in effect, on one of the fundamental properties of a solid-state material, i.e., that the particles from which it is composed, occupy spatially localized potential wells which have at least one stable bound state. If the concentration of particles is high enough, and the radius of a potential well is small enough so it contains no bound states, a solid state cannot exist. It is assumed further that the cause of the formation of the bound electron states are electron lattice defects, namely, interstitial electrons and vacancies. According to de Wette, the potential V , acting on an electron is a cell with a radius r_s and explicitly depending on its coordinate r , is influenced by the lattice defects, located outside of the considered cell, i.e., at $r > r_s$. This means that V depends on the coordinates of other electrons implicitly (through r). Thus, the lattice defects located in the area $r > r_s$, determine whether the bound states in an electron potential well in the area $r < r_s$ are formed. The upper and lower limits of r_s^{cr} values were estimated for two sharply different forms of the potential in an elementary cell as follows:

$$50 \lesssim r_s^{\text{cr}} \lesssim 100, \quad 6.2 \cdot 10^{-3} \lesssim n_{\text{cr}}^{1/3} a_B \lesssim 1.2 \cdot 10^{-2}. \quad (5)$$

Van Horn⁸, who reconsidered de Wette's analysis, gives another estimate for r_s^{cr} . Unlike de Wette, Van Horn takes into account that not only interstitial, but also the site electron levels broaden forming an energy band. For that reason, not all site electrons can move to interstitial positions. Assuming that the potential in both site and interstitial positions can be approximated by the harmonic oscillator potential $V(r) = r^2/r_s^3$, and using the statistical Thomas-Fermi model, Van Horn found the proportion of electrons in sites and interstitial positions as a function of the Fermi energy ε_F and r_s . The Fermi energy was found from the condition that the sum of both contributions is equal to unity (i.e., that the total concentration of electrons is constant). It was taken into account that in a body-centered cubic lattice (the lowest ground state energy corresponds to such symmetry), an interstitial electron is either in the center of the side, or in the middle of the cube rib, and, therefore, an electron in a lattice site has six near neighbors located in interstitial positions. The critical parameter r_s^{cr} was calculated under the assumption that the lattice melts if half of all electrons moves into interstitial positions, i.e., if there is an equal probability to find an electron in a site or in an interstitial position

$$r_s^{\text{cr}} \approx 27, \quad n_{\text{cr}}^{1/3} a_B \approx 2.3 \cdot 10^{-2}. \quad (6)$$

An error can be due to the inaccuracy in the choice of the potential, in the energy of formation of an interstitial electron, and in the number of interstitial neighbors surrounding a site electron. In addition, if one assumes that the melting starts when not half but a quarter of electrons moves to interstitial positions, then $r_s^{\text{cr}} \approx 65$.

The most reliable values of r_s^{cr} have been obtained in

calculations of ground state energy, which enable one to establish the most precise relationship between the potential and kinetic energies. Carr *et al.*^{9,10} calculated the energy of the ground state of an electron crystal, ϵ_0 , with higher accuracy than Wigner,² by properly taking into account the lattice vibrations

$$\epsilon_0 = -\frac{1,79}{r_s} + \frac{2,66}{r_s^{3/2}} - \frac{0,73}{r_s^2} + O(r_s^{-5/2}). \quad (7)$$

The first term in (7) represents the energy of direct electrostatic interaction of electrons (Madelung energy) aligned into a regular lattice against the background of a uniform positive charge of ions (the "jellium" model). It turns out that the main contribution to this term is made by the interaction of the i th electron with the positive background in the same i th cell (the difference between the interaction energy of the i th electron with the other j electrons and the interaction energy of the same i th electron with the background in the j cells is small). Other terms in (7) can be found by expanding the potential energy in a Taylor series in powers of displacement of electrons from a lattice site. The second term describes energy of lattice vibrations. Other terms are anharmonic corrections to the energy. In addition, the energy ϵ_0 contains exponential terms $\sim \exp(-\text{const} \cdot r^{1/2})$, caused by the exchange interaction. However, at large $r_s > 6$, when an overlap of the wave functions is small, these terms are much smaller than the other terms in expression (7).

According to Refs. 9, 10, the critical parameter is

$$r_s^{\text{cr}} \approx 5 - 6, \quad n_{\text{cr}}^{1/3} a_B \approx 0.10 - 0.12. \quad (8)$$

However, this value of r_s^{cr} cannot be identified with the critical electron-electron distance at which the lattice melts, since it is equal to the limiting value of r_s above which the expression (7) for ϵ_0 converges.

Kugler⁶ calculated values of r_s^{cr} using the self-consistent harmonic approximation (RHA)²⁾, in which the anharmonic crystal is described by a set of harmonic oscillators. The main idea of the theory is reduced to the choice of the trial Hamiltonian for the oscillators. The best choice of the trial oscillators is sought with the help of a variational calculation of the free energy. The value of r_s^{cr} is calculated from the condition that the RHA approximation is not valid (i.e., that the RHA approximation does not have solutions):

$$r_s^{\text{cr}} \approx 22, \quad n_{\text{cr}}^{1/3} a_B \approx 2.8 \cdot 10^{-2}. \quad (9)$$

The inclusion of the anharmonicity effects not included in the RHA approximation leads to significantly larger values of r_s^{cr} :

$$r_s^{\text{cr}} \approx 700, \quad n_{\text{cr}}^{1/3} a_B \approx 8.9 \cdot 10^{-4}. \quad (10)$$

Unexpectedly large variations of r_s^{cr} values, found from the calculations of the binding energy ϵ_0 , are determined by the accuracy of calculations. We have already mentioned that the interaction between cells of an electron lattice related to long-range order contributes comparatively little to the energy ϵ_0 . For that reason, the violation of the long-range order during melting (r_s^{cr} characterizes the lattice melting) has a relatively small influence on the quantity ϵ_0 and, therefore, the reliability of the calculation of r_s^{cr} essentially depends on the accuracy of calculation of ϵ_0 . Besides, as is known, a significant error is related to the calculation of the

correlation energy ϵ_{cor} . In the last several years, quite reliable results have been obtained for ϵ_{cor} with the help of the variational Monte-Carlo methods (see Ref. 11).

Recently, Ceperley¹² basing his work on the previous calculations of ϵ_{cor} , used the Monte-Carlo method to calculate the critical parameter r_s^{cr} . According to these calculations, the parameter r_s^{cr} is slightly different for paramagnetic and ferromagnetic Fermi liquids. In the case of crystallization of a paramagnetic Fermi-liquid, we have

$$r_s^{\text{cr}} = 75 \pm 5, \quad n_{\text{cr}}^{1/3} a_B \approx (8.3 \pm 0.5) \cdot 10^{-3}. \quad (11)$$

For crystallization of a ferromagnetic Fermi-liquid it was found¹² that:

$$r_s^{\text{cr}} = 100 \pm 20, \quad n_{\text{cr}}^{1/3} a_B \approx (6.2 \pm 1.3) \cdot 10^{-3}. \quad (12)$$

At the present time it is generally accepted that these values of r_s^{cr} are the most reliable.

2. CHARGE DENSITY WAVES

Some idea about charge density waves (CDW) can be obtained by considering the simplest model of a metal, i.e., the "jellium" model, according to which the ion lattice is replaced by a uniformly distributed positive charge. It would seem that in this case the ground state of an electron system should correspond to a spatially uniform density of the negative charge. However, such an assumption turns out to be wrong, if the interelectron interactions are taken into account. Overhauser has shown¹³ that states of an electron system with a nonuniform charge distribution, i.e., CDW states, correspond to smaller energies, i.e., energetically are more favorable than states with a uniform distribution of a charge. The CDW state is the ground state in which the electron charge density ρ_- oscillates in space.

$$\rho_- = \rho_0 \{1 + \alpha \cos(\mathbf{K}\mathbf{r})\}; \quad (13)$$

here ρ_0 is the average electron charge density (which does not depend on the coordinates), and the wave vector \mathbf{K} and amplitude $\alpha < 1$ change with the temperature in such a way that the free energy remains at a minimum.

The oscillations of electron density must induce large electric fields in a metal, if the background of the neutralizing positive charge of ions has elastic rigidity. These electric fields will inhibit the formation of CDW's. The CDW states can be formed only in the case when the system of ions is deformable, i.e., if periodic displacements $\mathbf{u} = B \sin(\mathbf{K}\mathbf{r})$ can occur in the positive background. Then the local density of positive charge is $\rho_+ = -\rho_0(1 + \text{div } \mathbf{u})$, and the charge neutrality will be preserved everywhere if $\alpha = BK$. The question about whether the ground state of CDW can be formed in a specific metal depends on how well the ion lattice can be approximated by a deformable "jellium" model.

Spatial modulation of charge density can occur under the condition that each electron experiences the action of the oscillating potential $V(\mathbf{r}) = V_0 \cos(\mathbf{K}\mathbf{r})$. The potential $V(\mathbf{r})$ is caused by the exchange and correlation interaction of electrons.

Since V_0 is a function of the wave vector \mathbf{k} , the quantum-mechanical problem of finding the ground state of CDW includes the solution of the integral equation for $V_0(\mathbf{k})$. An approximate solution of the Schrödinger equa-

tion with the potential $V(\mathbf{r})$ has the form³⁾

$$\psi_{\mathbf{k}}(\mathbf{r}) \approx \exp(i\mathbf{k}\mathbf{r}) + \beta_+ \exp[i(\mathbf{k} + \mathbf{K})\mathbf{r}] + \beta_- \exp[i(\mathbf{k} - \mathbf{K})\mathbf{r}], \quad (14)$$

where β_{\pm} are functions of \mathbf{k} and differ from zero only if $V_0(\mathbf{k}) \neq 0$.

The wave function (14) is what leads to charge modulation, since

$$|\psi_{\mathbf{k}}(\mathbf{r})|^2 \approx 1 + 2(\beta_+ + \beta_-) \cos(\mathbf{K}\mathbf{r}).$$

The charge modulation amplitude α in expression (13) is given by the average value $2(\beta_+ + \beta_-)$ for the occupied states of the electron system. From expression (14) it can be seen that the CDW wave vector \mathbf{K} can be viewed as a vector of the reciprocal lattice of an electron crystal. Using a variational method (β_+ and β_- were viewed as variational parameters), it was found that $\beta_{\pm} = 0$ for the values of r_s from 0 to some value r'_s above which β_{\pm} increases with r_s .

The presence of an oscillatory potential $V(\mathbf{r})$ in the Hamiltonian leads to the appearance of energy gaps in the electron spectrum $\varepsilon(\mathbf{k})$. The gaps are formed in the planes perpendicular to the vector \mathbf{K} at a distance $K/2$ from the point $\mathbf{k} = 0$. The CDW state will be the most favorable energetically if the contributions of all occupied states to the exchange and correlation potential $V(\mathbf{r})$ coincide in phase. This is guaranteed under the condition that the Fermi level ε_F is located in the gap; the wave vector of CDW in that case is $K \approx 2k_F$ ($\hbar k_F$ is the Fermi momentum). Since the magnitude of K is determined by the diameter of the Fermi surface $2k_F$, it is, generally speaking, incommensurable with an inverse lattice, i.e., there is no integer N , for which the magnitude of the inverse lattice vector $g = nK$ (the wavelength of CDW is about the same as the lattice period, but is incommensurable with it). Of course, the existence of commensurable CDS's is not forbidden. A commensurable structure of CDW can be realized, for example, in metallic compounds consisting of various ions. Thus, phase transitions between commensurable and incommensurable CDW structures have been observed in quasi-two-dimensional metals of the type TaS_2 .

Before the work of Overhauser, it was assumed that in the Hartree-Fock approximation, the lowest energy state Ψ of a degenerate electron system is the well-known Fermi sphere of occupied states described by plane waves. Such a function Ψ corresponds to the ground state of a gas of noninteracting fermions, for which the charge density is spatially uniform. Overhauser, in essence, has shown that for the one-electron states (14) with $K = 2k_F$ and properly chosen $\beta_{\pm}(\mathbf{k})$, the ground state Ψ , i.e., a CDW state with a modulated charge distribution, corresponds to a lower energy, than the solutions of the Hartree-Fock equations for the case when the exchange and correlation interactions with a uniform distribution of a charge are not taken into account.⁴⁾

Obviously, it is necessary to answer the question as to what is the relation between the electron crystal and the CDW. In a Wigner crystal the electrons occupy sites of a strictly periodic lattice, and for sufficiently large r_s the wave functions do not overlap. However, if the density of the electron system increases (r_s decreases), the wave functions start overlapping, the role of the exchange and correlation interactions increases, and spatial oscillations of the charge

density (13) and the CDW's may appear. A CDW state can be viewed, thus, as an intermediate phase between the Wigner crystal and the normal metal with a uniform charge distribution and $2 \leq r_s \leq 7$.

The possibility of formation of a CDW in the case of a three-dimensional gas essentially depends, as was already mentioned, on the degree of deformability of an ion lattice and its closeness to the model of a deformable "jellium." Overhauser holds the opinion that alkali metals are the most suitable three-dimensional objects for observation of CDW's, since the ion-ion interactions in them are weak, and the ions can easily occupy positions corresponding to a minimum of their potential energy. The existence of CDW's has been reliably established in quasi-two-dimensional systems.⁵⁾

3. ELECTRON CRYSTAL IN A MAGNETIC FIELD

The concept of CDW's turned out to be useful for studying electron crystals in a magnetic field. It was found that the magnetic field can induce a phase transition of a system of interacting electrons into the CDW or the Wigner crystal states.¹⁴⁻¹⁸ In a strong magnetic field, for which the cyclotron energy $\hbar\omega_c$ ($\omega_c = eH/m_0c$, H is the field strength) exceeds the average thermal energy $k_B T$ and the Fermi energy ε_F , the electron spectrum becomes quantized. The condition $\hbar\omega_c > \varepsilon_F$ can be written in the form $l < \lambda_F$, where $l = (c\hbar \approx / eH)^{1/2}$ is the cyclotron radius of a wave function, $\lambda_F \approx k_F^{-1}$ is the Fermi wavelength. The quantizing magnetic field reduces the Fermi energy $\varepsilon_F \sim H^{-2}$ and, thus, increases the ratio $\varepsilon_{\text{pot}}/\bar{\varepsilon}$, and this favors electron ordering. It is obvious that at larger electron concentrations one needs larger magnetic fields to satisfy the inequality $\varepsilon_{\text{pot}} > \bar{\varepsilon}$, which corresponds to a stable electron lattice. This follows directly from the crystallization criterion for a free electron gas which can be obtained by comparing the potential energy e^2/r_0 with the Fermi energy in a magnetic field $\varepsilon_F = 2\pi^4 \hbar^2 l^4 n^2 / m_0$:

$$l \lesssim r_0 \quad (\text{or } l \lesssim n^{-1/3}). \quad (15)$$

Obviously, the physical meaning of (15) is the same as the meaning of the crystallization criterion in the absence of the field: $a_B \lesssim r_0$ (or $a_B \lesssim n^{-1/3}$) (this relation is the result of the condition $e^2/r_0 \gtrsim \hbar^2(3\pi^2 n)^{3/2}/2m_0$).

A rather detailed study of the ground state of Wigner crystal and of the CDW states in a magnetic field was carried out by Kuramoto.¹⁶ He studied the lattice anisotropy and the charge density which arises as a result of the localization of a wave function in the direction perpendicular to a magnetic field within an internal region of the size $R \lesssim l$ and of the ordered alignment of electrons along the magnetic field, caused by Coulomb repulsion. The Wigner-Seitz cell was chosen in the form of an ellipsoid of rotation.⁶⁾ The anisotropy of the lattice is characterized by the ratio of the long and short ellipsoid axes $\gamma = c/a$ (c and a are the half-axis lengths of the ellipsoid). For a given electron density, the electrostatic Madelung energy reaches a minimum when the anisotropy parameter is $\gamma = 1$. This means that the direct Coulomb interaction favors an isotropic configuration of charges; however, the anisotropy of the kinetic energy caused by a magnetic field makes a Wigner lattice elongated in the direction of magnetic field more stable.

The wave function in a magnetic field can be written, as is known, in the form

$$\psi(\mathbf{r}) = [(2\pi)^{3/2} a_{\perp}^3 a_{\parallel}]^{-1/2} \exp\left(-\frac{x^2+y^2}{4a_{\perp}^2} - \frac{z^2}{4a_{\parallel}^2}\right), \quad (16)$$

where the variational parameters are

$$a_{\perp} = l \left(1 + \frac{24\gamma \varepsilon_{\text{B}}^*}{\omega_{\text{c}}^* r_{\text{s}}^*} f_{\perp}(\gamma)\right)^{-1/4}, \quad a_{\parallel} = a_{\text{B}}^* r_{\text{s}}^*{}^{3/4} (6\gamma f_{\parallel}(\gamma))^{-1/4},$$

$f_{\perp}(\gamma)$, $f_{\parallel}(\gamma)$ are some irrational functions of γ , $\omega_{\text{c}}^* = eH/mc$, $\varepsilon_{\text{B}}^* = \varepsilon_{\text{B}} m/m_0 \kappa^2$, $r_{\text{s}}^* = r_{\text{s}} a_{\text{B}}^*/a_{\text{B}}^*$, $\varepsilon_{\text{B}} = m_0 e^4/2\hbar^2$, $a_{\text{B}}^* = a_{\text{B}} m_0 \kappa/m$, m is the effective electron mass, κ is the permittivity. Using variational methods, the binding energy ε_0 and γ were calculated as a function of r_{s}^* for InSb. It was found that γ decreases with an increase in r_{s}^* . This result has a simple explanation. If the interelectron distance significantly exceeds the size of the charged area near the lattice site (i.e., the size of the wave function), the shape of an electron cloud does not affect the Coulomb interaction—only the spatial position of charges is important. For this reason, the anisotropy of a lattice decreases in order to increase the Madelung energy. On the other hand, the anisotropy of the charge density increases with an increase in r_{s}^* . This can be easily seen from a comparison of the dimensions of the wave function (16), a_{\perp} and a_{\parallel} : $a_{\perp} \approx l$ and depends on r_{s}^* very weakly, while a_{\parallel} increases significantly with an increase in r_{s}^* (for the parameters of InSb a_{\parallel} increases more than four-fold when r_{s}^* changes from 1 to 10).

With an increase of electron concentration, the overlap of the wave functions increases and the concept of a Wigner crystal becomes less and less valid. In fact, with a decrease of r_{s}^* , the importance increases of the exchange interaction, which is stronger in the direction parallel to the magnetic field than in the direction perpendicular to the field. This anisotropy of ε_{exc} is due to the fact that the extension of the wave function in the direction perpendicular to the field ($a_{\perp} \approx l$) is significantly smaller than the semi-axis a of the elementary cell ellipsoid, while the extension in the direction parallel to the field (a_{\parallel}) is close to the length of semi-axis c . This means that in the direction parallel to the field the overlap of the wave functions of electrons of neighboring cells is larger than the overlap of wave functions in the perpendicular direction. Calculations for InSb show that for $H = 10^5$ Oe ($l/a_{\text{B}}^* \approx 0.1$) and $r_{\text{s}}^* = 1$, the ratio $a_{\parallel}/c \approx 0.5$, while $a_{\perp}/a \approx 0.17$.

An analysis of the expression for the binding energy ε_0 shows that the zero-point vibrations are suppressed by a magnetic field in the plane perpendicular to \mathbf{H} . The energy of these vibrations becomes negligibly small if the field is sufficiently strong ($\omega_{\text{c}}^*/2\varepsilon_{\text{B}}^*)^2 r_{\text{s}}^*{}^3 \gg 1$).

The oscillations of the charge density in the direction parallel to the magnetic field, i.e., linear CDW's in the direction of \mathbf{H} , have been studied in Ref. 16. An electron system for that case was modeled in the form of a number of electron chains parallel to the field. Using a variational method and taking into account the exchange interaction between the electrons in each chain, the energy of the ground state, the amplitude of the CDW with a wave-vector $K = 2k_{\text{F}}$ and the distance between electron chains have been calculated.

The variation of the charge density in the direction of the magnetic field is given by an expression analogous to expression (13)

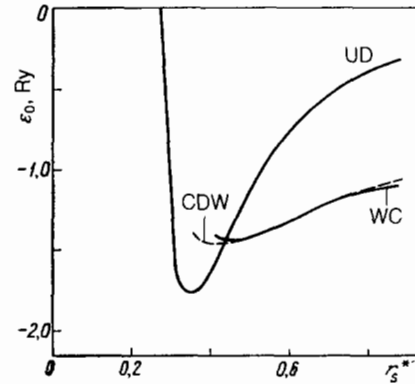


FIG. 1. The energy ε_0 of the ground state of an electron as a function of r_{s}^* for the cases of the Wigner crystal (WC), a CDW and a uniform distribution (UD) of electron density (Ref. 16). The parameters are for InSb; $H = 10^5$ Oe.

$$\rho(z) = \rho_0 [1 + A \cos(2k_{\text{F}} z)], \quad (17)$$

where $\rho_0 = n_z/L$, n_z is the number of electrons in a cylinder with the length L .⁷⁾ The amplitude A of a CDW decreases with an increase of the Fermi momentum $\hbar k_{\text{F}}$. For InSb and $H = 10^5$ Oe, A decreases with an increase of the parameter $a_{\text{B}}^* k_{\text{F}}$ from 1 to 3 by more than five-fold.

Figure 1 shows the dependence calculated in Ref. 16 of the ground state energy ε_0 on the interelectron distance r_{s}^* for a CDW, a Wigner crystal and a uniform distribution of electron density. The energy ε_0 for the uniform distribution was calculated by the standard Hartree-Fock approximation. From Fig. 1 it can be seen that ε_0 for a CDW and a Wigner crystal have very similar values for a rather wide range of distances r_{s}^* . The state with a uniform distribution of electron density is unstable even for such high concentrations for which $r_{\text{s}}^* = 0.5-1$.

Calculations¹⁶ show that even for high electron concentrations, when the exchange interaction between electrons becomes essential, the energy of a CDW state is slightly lower, than for case of a uniform distribution of charge. From this can be concluded that in strong magnetic fields at $T = 0$ K, the CDW phase must dominate in a wide range of electron densities. The author of Ref. 16 leaves open the question whether the transition of the chain phase into a three-dimensional CDW phase (taking place with an increase of electron density) occurs continuously or abruptly. Kuramoto supposes that the correlation interaction must not violate the stability of a CDW phase at high electron concentrations (this conclusion is based on some qualitative considerations).

Until now, in this section we discussed mainly the ground state energy of a Wigner crystal in a magnetic field. It is clear that this quantity, though very important in principle, cannot be directly compared with experimental data. The characteristics, which can be measured directly, are the threshold field H_{w} and the critical temperature T_{c} at which a state with uniform density becomes unstable towards the formation of a CDW or of a Wigner crystal. The critical temperature T_{c} and the threshold field H_{w} , as a function of electron concentration, were calculated in Refs. 15, 17, and 18.

Kleppmann and Elliott¹⁵ also calculated the electro-

TABLE I. Threshold field H_w for various electron concentrations n .

	$n, 10^{14} \text{ cm}^{-3}$	1	3	5	7	10
InSb	$H_w, \text{ kOe}$	2.4	5.7	8.8	12	16
Hg _{0.8} Cd _{0.2} Te	$H_w, \text{ kOe}$	3.7	9.2	14	19	26

static Madelung energy and compared the total energy of localized states with the energy of delocalized states for a low density electron system. The stability criterion of a Wigner lattice in a magnetic field at $T = 0 \text{ K}$ and for various values of n and H have been obtained from the condition of energy equality of localized and delocalized states. Table I shows the values of the threshold field H_w for several electron concentrations n , which we calculated using the approach of Ref. 15 for n -InSb ($m = 0.014 m_0, \kappa = 17$) and $n = \text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ ($m = 4.9 \cdot 10^{-3} m_0, \kappa = 17$).

The main qualitative result of Ref. 15 is that, as one could expect, the critical concentration n_{cr} , corresponding to the condensation of an electron system, increases with an increase in the magnetic field.

We consider now the results of calculations of Gerhardt's¹⁸ on the dependence of the critical temperature T_c on magnetic field and electron concentration. The calculations in Ref. 18 are carried out within the framework of the Hartree-Fock method for the extreme quantum limit of magnetic fields. It is assumed also that the cyclotron energy $\hbar\omega_c^*$ is larger than Coulomb energy (calculated per electron) which for an unscreened Coulomb potential $V(r) = e/\kappa r$ is equal to ϵ_B^*/r_s^* . Three inequalities of the extreme quantum limit

$$\hbar\omega_c^* > k_B T, \quad \hbar\omega_c^* > \frac{\epsilon_B^*}{r_s^*}, \quad \hbar\omega_c^* > \epsilon_F \quad (18a)$$

can be written in a different way (up to coefficients of the order of 1).

$$\left(\frac{a_B^*}{l}\right)^2 > \frac{T}{T^*}, \quad r_s^{*1/2} > \frac{l}{a_B^*}, \quad r_s^* > \frac{l}{a_B^*}. \quad (18b)$$

Here it was used that

$$\epsilon_B^* = k_B T^* = \frac{e^2}{2\kappa a_B^*}, \quad \frac{4\pi}{3} r_s^{*3} a_B^{*3} = n^{-1},$$

$$\frac{\hbar\omega_c^*}{2} = \epsilon_B^* \left(\frac{a_B^*}{l}\right)^2, \quad \epsilon_F = k_B T_F = \frac{\hbar^2 k_F^2}{2m}, \quad k_F = 2\pi^{1/2} l^2 n$$

(the last equality is equivalent to $\epsilon_F = (16/9)\hbar\omega_c^* \times (\epsilon_F^0/\hbar\omega_c^*)^3$).

For the semiconductor Hg_{0.8}Cd_{0.2}Te, for example with $a_B^* = 1.8 \cdot 10^{-5} \text{ cm}$, the conditions of the extreme quantum limit (18) are satisfied at temperatures below several degrees Kelvin, if the magnetic field exceeds several kilooersted.

Gerhardt's¹⁸ starts by considering an electron gas of uniform density, obeying in the absence of a magnetic field the Fermi-Dirac statistics. The problem is solved by a variational method. First, the free energy, including the effects of electron-electron interactions, is found. Then, the values of some parameters are determined, at which the state of an electron system with a uniform distribution of charge den-

sity becomes unstable, i.e., at which the CDW energy in a magnetic field becomes less than the energy of a uniform system. The critical temperature T_c is expressed in terms of the parameters which correspond to the transition of the system into a CDW state with an anisotropic distribution of electron density. In the calculations, the dispersion law $\epsilon(\mathbf{k})$ at $H = 0$, corresponding to the Hartree-Fock approximation, is replaced by the simple quadratic expression $\epsilon(k) = \hbar^2 k^2 / 2m$. Here the author of Ref. 18 starts from the unquestionably correct surmise that in the Hartree-Fock approximation one can hope to obtain correct results only qualitatively. In the limiting cases of strong and weak degeneration of an electron gas, one obtains the analytical expressions for T_c

$$T_c = \frac{8\beta}{\pi} T_F \exp\left[-\frac{\pi(T_F/T^*)^{1/2}}{\Phi}\right] \quad (T_c \ll T_F), \quad (19)$$

$$T_c = \frac{4\Phi}{\pi} (T_F T^*)^{1/2} \quad (T_c \gg T_F), \quad (20)$$

where $\beta = 1.78$ is the Euler constant, Φ is a dimensionless parameter, logarithmically dependent on H and n . Expression (19) refers to the extreme case of weak interelectron interaction $a_B^* k_F = (T_F/T^*)^{1/2} \gg 1$, i.e., to the weak magnetic field limit, and the expression (20)—to the case of strong interaction $a_B^* k_F \ll 1$, i.e., to the strong field limit. The area of intermediate magnetic fields (intermediate coupling) is calculated numerically. In numerical calculations the Coulomb potential $V(r)$ is replaced by a screened potential.

The results of calculations are given in Fig. 2. The esti-

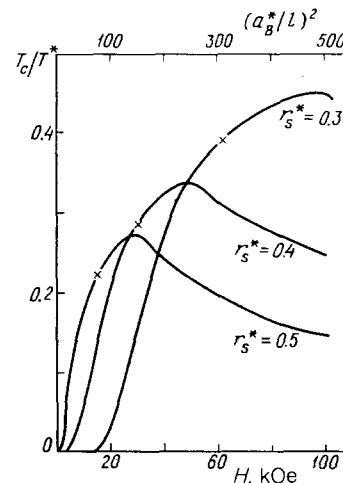


FIG. 2. The calculated critical temperature T_c , divided by the Bohr temperature T^* , as a function of magnetic field for n -Hg_{0.8}Cd_{0.2}Te ($T^* = 2.7 \text{ K}$).¹⁸ The curves are for $r_s^* = 0.5, 0.4$, and 0.3 correspond to electron concentrations $n = 3.1 \cdot 10^{14}, 6.0 \cdot 10^{14}$, and $1.4 \cdot 10^{15} \text{ cm}^{-3}$.

mates made by the author of Ref. 18, led him to conclude that for an unscreened Coulomb potential one obtains curves analogous to the curves in Fig. 2, but the T_c are slightly increased (approximately two times at the maximum), and the maxima of $T_c(H)$ are shifted towards lower magnetic fields (H'_{\max} decreases approximately by a factor of 0.7). The results of this estimate are physically clear, since the unscreened Coulomb potential is stronger and the transition takes place, thus, at a slightly higher temperature than the transition for a screened potential. The estimates for $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ show that for the concentrations $n \approx 3 \cdot 10^{14} - 1.5 \cdot 10^{15} \text{ cm}^{-3}$ and $H = 5 \cdot 10^4 \text{ Oe}$, the critical temperature is $T_c \lesssim 1 \text{ K}$.

The dependence of T_c on H and n is determined by the dependence of the Fermi temperature on these quantities, since $T_F \sim (n/H)^2$. For weak fields, T_c decreases rapidly with an increase in electron concentration. The minimal value of the field H , necessary for the induced transition to the CDW state at a fixed temperature, increases with an increase of n . For high values of H , the temperature T_c increases approximately linearly with an increase of n . For weak fields, T_c first increases with H , and then, at sufficiently large H , as can be seen from Fig. 2, T_c starts to decrease. These rules qualitatively agree, in general, with the results of Kleppmann and Elliott,¹⁵ if one assumes that an increase of T_c corresponds to an increase of the binding energy in a Wigner crystal. There is, however, one significant difference: according to Gerhardt, for sufficiently strong magnetic fields, T_c decreases with an increase in H , as, according to the Ref. 15, the binding energy increases monotonically with H .

For a fixed concentration of electrons, the Fermi-Dirac statistics valid for weak fields at high fields is replaced at the temperature T_c by Boltzmann statistics. Correspondingly, the chemical potential of the system $\xi = \varepsilon_F^n$ (in weak fields) becomes negative for high fields. The temperature T_c reaches its maximum value at $\xi = 0$, what corresponds to $T_c/T_F \approx 3.5$. In the limit of strong electron degeneration, the component K_z (which is parallel to the magnetic field $H = H_z$) of the wave vector K [see (14)], corresponding to the temperature T_c , is equal to $|K_z^c| = 2k_F$. With an increase in the magnetic field, the value of $|K_z^c|$ decreases and becomes equal to zero at $\xi/k_B T < 1.11$, or at $T_c/T_F \geq 1.10$. The crosses on Fig. 2 mark the points on a curve for which $T_c/T_F = 1.10$.

Two results of Gerhardt¹⁸ attract particular attention. In the region of intermediate magnetic fields, at a constant concentration n , a phase transition of the second kind from a uniform electron distribution phase into a linear CDW phase must take place; for high fields and a temperature $T_1(H) > T_c(H)$, a transition of the first kind into a CDW phase with hexagonal lattice symmetry must take place. Fukuyama¹⁷ also came to the conclusion that for sufficiently strong fields, a transition of the first kind must take place. If one lowers the temperature at a fixed value of the magnetic field strength, the CDW state must transform into a Wigner crystal either gradually, as Kuramoto¹⁶ thinks, or abruptly, i.e., experiencing one more phase transition, which is determined by the melting temperature T_M of a Wigner lattice. Such a phase transition from a CDW state into a Wigner crystal can occur if $T_M(H) < T_1(H)$. At the present time, however, there are no reliable data on the quantities T_1 and

T_M , and the question about transformation (continuous or abrupt) of a CDW state into a Wigner crystal state remains open.

4. ON THE POSSIBILITY OF EXPERIMENTAL REALIZATION OF WIGNER ORDERING

It is natural to pose a question: In what materials and under what conditions can one expect to discover the effects related to Wigner crystallization? Metals with high electron concentrations, for which $2 < r_s \leq 7$, are, obviously, unsuitable objects. Mott has assumed (see Ref. 19) that crystallization of a three-dimensional gas can be expected in strongly compensated semiconductors and magnetic dielectrics.

4.1. Compensated semiconductors

Consider an n -type semiconductor, InSb for example. By doping it with acceptors, it is possible to reduce significantly the concentration of free electrons and to create, in this manner, a material in which there are many (e.g., 30–100 donors per electron. Compensation allows one to model low-density electron systems. In this approach it is important that the ion background in a compensated semiconductor would be almost uniform. In fact, in the InSb or $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ type semiconductors, the Bohr radius a_B^* of an electron orbit is large and the orbit encompasses many doping centers. For example, in $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ with $n = N_D - N_A = 10^{14} \text{ cm}^{-3}$ and $(N_A/N_D) = 0.99$ an electron "sees" within its orbit (in its plane) approximately 300 impurity ions (for $n = 10^{15} \text{ cm}^{-3}$ an orbit covers ≈ 60 ions). Under these conditions, one can use the "jellium" model, on which all theoretical work on Wigner crystallization is based.

One must keep in mind, however, that the fluctuating electric field of randomly distributed impurity atoms, which is particularly strong in compensated semiconductors, can cause a breakdown of order in an electron system, i.e., lead to the destruction of a Wigner lattice. This fact must be taken into account in interpreting the results of measurements, since not even a minimally rigorous theory exists which would simultaneously treat electron correlation and effects caused by disorder. In any case, at the present time one should not expect quantitative agreement of experimental data with the results of theoretical studies of a low-density electron system, immersed in a "jellium" of ions. In an analysis of experimental data for n -type InSb or n -type $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ compensated semiconductors, one must also take into account the fact that a significant overlap of donor electron orbits leads to the formation of an impurity band. Let us make one more remark referring to the interpretation of transport processes in a magnetic field. It is usually ignored that in sufficiently strong magnetic fields the transverse radius of the Bohr orbit $a_1 \approx l$ becomes less than the distance between the impurity ions $r_i \approx N_i^{-1/3}$, and then the system of ions cannot be approximated by a uniform positive background. For $H \geq 10^4 \text{ Oe}$ we have $l < r_i$ with an impurity concentration of $N_i \lesssim 10^{18} \text{ cm}^{-3}$. This means that theoretical studies of electron crystallization based on the "jellium" model cannot be used, strictly speaking, for interpretation of experimental data in strong magnetic fields, for which $l < r_i$.

Compensated semiconductors have been proposed as objects for searching for Wigner crystallization because at

large a_B^* they simulate the conditions for the "jellium" model. At the same time, it is exactly in compensated semiconductors that the random potential of impurities can, most probably, prevent the formation of an electron lattice. For that reason, it is interesting to explore the question whether one can expect to find Wigner crystallization in uncompensated semiconductors with low concentration n , for which the requirement of the theory concerning a uniform positive background is not satisfied.

In uncompensated semiconductors of high purity (for example, $\text{Bi}_{1-x}\text{Sb}_x$), the concentration of electrons, which is equal to the donor concentration N_D , reaches 10^{12} – 10^{13} cm^{-3} . For such N_D , the distance between donors $r_D \approx N_D^{-1/3}$ exceeds the Bohr radius a_B^* , and the ion background is nonuniform. The electrons will "feel" the random distribution of donors. One neglects this and assumes that donors are distributed regularly, then even in that case one cannot expect to find ordering of electrons. The point is that the Wigner and the Mott conditions for electron localization in uncompensated semiconductors are similar. A rough criterion for Wigner crystallization $r_0 \gtrsim a_B^*$ can be obtained by comparing the electron interaction energy $\sim e^2/\kappa r_0$ with the electron kinetic energy $\sim \hbar^2 n^{2/3}/m$. The Mott transition criterion based on the comparison of the electron kinetic energy with the interaction energy of two electrons on the same site of a crystal lattice $\sim e^2/\kappa a_B^*$, can also be written in the form $r_0 \gtrsim a_B^*$ for $n = N_D$. Because of the qualitative coincidence of the two criteria, it is not clear how to distinguish experimentally the effects accompanying each of these transitions.

Using various physical considerations one can obtain for the critical impurity concentration N_M corresponding to a Mott transition, an equality which is more accurate than the rough condition $r_i \approx N_i^{-1/3} \approx a_B^*$:

$$N_M^{1/3} a_B^* \approx 0.25. \quad (21)$$

When a magnetic field is present, the following criterion is usually used instead of the rough condition $r_i \approx l$

$$(N_M a_{\perp}^2 a_{\parallel})^{1/3} \approx 0.25, \quad (22)$$

where $a_{\perp} = 2l_M = 2(c\hbar/eH_M)^{1/2}$, $a_{\parallel} = a_B^*/\ln(a_B^* l_M^{-1})^2$, H_M is the magnetic field at which the Mott transition takes place.

If one assumes that the expressions (11) and (12) ($n_{cr}^{1/3} a_B^* \approx 7 \cdot 10^{-3}$) for the Wigner ordering and expression (21) for the Mott transition are sufficiently accurate, then it is easy to see that in an uncompensated semiconductor electrons are localized on donors even for a concentration n , that exceeds by approximately 4 orders of magnitude the critical concentrations n_{cr} of Wigner crystallization. For $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$, for example, we have $N_M \sim 10^{12}$ cm^{-3} , and $n_{cr} \sim 10^8$ cm^{-3} . Thus, in uncompensated semiconductors it is practically impossible to discover electron condensation, since at concentrations $n_{cr} < n < N_M$ electrons remain localized on donors.

The decisive experiments for the discovery of Wigner ordering would be neutron diffraction studies of an electron lattice, but for electron concentrations $n < 10^{14}$ cm^{-3} such experiments at the present time are practically impossible.

4.2. Magnetic insulators

Let us assume that a crystal lattice has N sites and $n < N$ electrons. In order to move to an unoccupied ("irregular") site, an electron must acquire an energy of the order of $e^2/\kappa r_a$, where $r_a \approx N^{-1/3}$. This energy will decrease when more and more electrons are transferred to "irregular" sites. With a temperature rise, phase transitions of the first and second order can take place to a state in which all sites are filled randomly. Such order-disorder transitions have been discovered for the first time by Fervey in 1935 in the magnetic material Fe_3O_4 . In oxides with a mixed valency of the magnetite type, the effective mass of charge carriers increases because of the formation of polarons, as a result of which the kinetic energy becomes smaller than the potential energy. Under such conditions one can expect, generally speaking, formation of Wigner crystallization (see, for example, Ref. 19). However, taking into account the fact that in magnetic insulators, according to the Hubbard model, which considers only the short-range forces, energy-band splitting can take place for any number of electrons per atom, it is difficult to hope to separate effects which depend critically on Coulomb forces. To that one must add the uncertainty in the problem of magnetic properties (ferromagnetic or antiferromagnetic?) of an electron crystal.

5. ELECTRON CRYSTALLIZATION AND ACTIVATION CONDUCTIVITY

At $T = 0$ K, an electron crystal is an insulator, since there is an energy gap for one-particle excitations.²⁰ Motion of the lattice as a whole is ignored, i.e., it is assumed that the lattice is fixed. It is necessary to understand the nature of the conductivity mechanism in a Wigner crystal, i.e., the nature of excited states responsible for the charge transfer. It seems probable that these states are related to defects in a Wigner lattice. This surmise was offered by Care and March,²¹ who proposed to treat conductivity as a diffusion process analogous to diffusion of atoms in solids. Charge transport is accomplished via the electron vacancies, the states of which are separated from the ground state by an energy gap. The vacancies are formed as a result of electron transitions from sites into interstitials. Similar transitions take place at sufficiently high electron concentrations, i.e., for sufficiently narrow potential wells. The coefficient of electron diffusion via vacancies, $D \sim \exp(-\Delta\epsilon/k_B T)$, is related to the electron mobility μ by the Einstein relation $D = (k_B T/e)\mu$. The main contribution to the diffusion activation energy $\Delta\epsilon$ is made by the vacancy formation energy. Using a rather simple model, de Wette⁷ has obtained an estimate $\Delta\epsilon \approx 0.2/r_s$ Ry. At temperatures $T \sim \Delta\epsilon/k_B$ an electron lattice is not stable and decays. The concept of the Wigner crystal has meaning only under the condition $k_B T \ll \Delta\epsilon$.

From general considerations it is clear that in the absence of a magnetic field, the energy $\Delta\epsilon$ is of the same order of magnitude as the Madelung energy $\sim e^2/\kappa r_0$. At $H \neq 0$, the vacancy formation energy can vary. Assuming that this variation is proportional to the square of the electron displacement from a site, which for strong fields is of the order of the magnetic length l , Care and March²¹ came to the conclusion that $\Delta\epsilon \sim H^{-1}$. They affirm that their conclusions agree qualitatively with the experimental data of Somer-

ford.²² In connection with this let us turn to experimental results.

Somerford measured the conductivity σ in magnetic fields up to 10 kOe at temperatures $0.27 < T < 4.2$ K in strongly compensated samples of n -InSb (the compensation degree was $K = N_A/N_D \approx 0.8$, $N_D \approx (2-4) \cdot 10^{14} \text{ cm}^{-3}$). In samples of similar quality, Putley²³ measured $\sigma(H)$ and the Hall effect at $T \geq 1.3$ K. While at 4.2 K the conductivity decreases several-fold as the field increases to 10^2 Oe, at 0.27 K this decrease reaches six orders of magnitude²² from $\sigma \approx 10^{-1} \text{ Ohm}^{-1} \text{ cm}^{-1}$ to $\sigma \approx 10^{-7} \text{ Ohm}^{-1} \text{ cm}^{-1}$ (for $H \sim 10^2$ Oe, $\sigma \sim 10^{-1} \text{ Ohm}^{-1} \text{ cm}^{-1}$, for the studied range of temperatures, and for $H \sim 10^4$ Oe $\sigma \sim 10^{-2} \text{ Ohm}^{-1} \text{ cm}^{-1}$ at 4.2 K and $\sigma \sim 10^{-7} \text{ Ohm}^{-1} \text{ cm}^{-1}$ at 0.27 K). A sharp decrease in conductivity starts at fields $H \leq 2$ kOe. The Hall coefficient $|R|$ in fields $H < 10^3$ Oe changes little with temperature; with a field increase $|R|$ decreases with an increase in T , and for $H \sim 10^4$ Oe this decrease in the interval 1.5–20 K reaches almost two orders of magnitude, from $2 \cdot 10^7 \text{ cm}^3/\text{C}$ down to $\approx 2 \cdot 10^5 \text{ cm}^3/\text{C}$. According to Putley²³ for $H = 8.2$ kOe and in the temperature interval 1.3–4.2 K, $|R|$ decreases by a factor of ≈ 20 , and increases by a factor of ≈ 500 . This means that the electron mobility increases by a factor of ≈ 25 with a temperature rise, assuming that only one type of charge carriers participates in conduction.

The activation energy $\Delta\epsilon$, calculated from experimental curves of $R(T)$ (4 samples) and $\sigma(T)$ (2 samples) is almost one order of magnitude below the prediction of the theory of Jafet *et al.*²⁴ for the ionization energy of a hydrogen-like impurity center in the extreme quantum limit. With an increase of the magnetic field $\Delta\epsilon$ increases, but, according to the conductivity measurements of Ref. 22, that increase is significantly less than the increase found in the Hall effect data²³ (Fig. 3). Besides, the quantity $\Delta\epsilon$ itself, found from conductivity measurements, is smaller than the value of $\Delta\epsilon$ determined from the Hall effect measurement. An important feature of the Hall effect measurement is the presence of the threshold field $H_0^R \approx 3$ kOe, below which no activation energy is found. A sharp (but without abrupt changes) decrease in conductivity in Putley's experiments starts at ap-

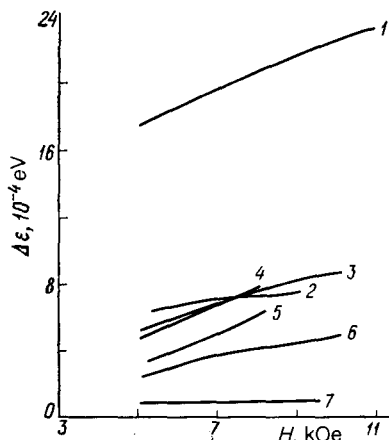


FIG. 3. Activation energy as a function of magnetic field.²² 1—Theory of Jafet *et al.*²⁴, 2—calculations of Durkan and March²⁵; 3–6—Hall effect data, Putley²³ with N_D from $4.5 \cdot 10^{14} \text{ cm}^{-3}$ and N_A from $3.8 \cdot 10^{14} \text{ cm}^{-3}$ (curve 3) to $10 \cdot 10^{14}$ and $9.8 \cdot 10^{14} \text{ cm}^{-3}$, respectively (curve 6), 7—conductivity data of Somerford.²²

proximately the same fields. At $T = 1.3$ K, the Hall coefficient $|R|$ increases ≈ 50 -fold when H increases from $3 \cdot 10^3$ Oe to $8 \cdot 10^3$ Oe.

A quick look at Fig. 3 is sufficient to become convinced that in none of the cases the activation energy decreases in a magnetic field, as predicted by Care and March.²¹ According to the Hall effect measurements, $\Delta\epsilon$ increases approximately linearly with the field H ; the increase of $\Delta\epsilon(H)$ is slower from the electrical conductivity data. This qualitative discrepancy between the experimentally found behavior of $\Delta\epsilon(H)$ and the one suggested by the authors of Ref. 21, allows one to affirm that the theoretical estimates of Ref. 21 are not suitable for interpretation of the experimental results being discussed.

A more realistic explanation of the behavior of $\Delta\epsilon(H)$ is given by Durkan and March.²⁵ First of all, it was necessary to understand the reason for the large difference between the activation energy values found in experiments with compensated n -InSb samples and the quantities ϵ_1 , calculated by Jafet *et al.*²⁴ The energy ϵ_1 of electron ionization from a donor to a conduction band is calculated in Ref. 24, while in the experiments of Refs. 22 and 23, in the opinion of Durkan and March, the observed activation energy is related to the transitions of electrons from the ground state of an impurity to the first excited state (i.e., the activation energy of the type ϵ_2). Durkan and March studied the behavior of $\Delta\epsilon(H)$ for that situation and took into account the screening of impurity ions by free electrons.²⁵ Taking screening into account leads to a significant shift of impurity levels, but the magnitude of the energy gap between the ground state and the first excited impurity state is not very sensitive to screening and, therefore, to the electron concentration n . The authors of Ref. 25 calculated the dependence of this gap on the magnetic field and found that it agrees satisfactorily with the experimental data (curve 2 in Fig. 3; the calculated activation energy increases in a magnetic field, as well as the ionization energy ϵ_1 , and the activation energy ϵ_2 of the hopping conductivity). Thus, according to Ref. 25, the activation energy $\Delta\epsilon$ determined from the Hall effect, is determined by the gap between the two lowest donor states.

For samples with higher concentration of dopants than those studied in Refs. 22, 23 for which the electron concentration was $n \sim 10^{16} \text{ cm}^{-3}$, the screening effect in strong magnetic fields can, according to Ref. 25, significantly lower the ionization energy of donors. At the same time, for uncompensated samples of InSb with $n \sim 10^{16} \text{ cm}^{-3}$ in magnetic fields up to 200 kOe, the good agreement of the dependence of ionization energy on the field²⁶ with the Jafet theory is established.²⁴ One can expect that for compensated samples with concentrations $n \approx 10^{14} \text{ cm}^{-3}$ in strong fields $H \gg 10^4$ Oe, for which the impurity band narrows significantly, the values of $\Delta\epsilon$ found from the Hall effect must be close to the values predicted by Jafet *et al.*²⁴ To our knowledge, such measurements have not been made.

The reasons why the values of $\Delta\epsilon$ found from the conductivity and the Hall effect measurements^{22,23} are different, and why their dependences on the magnetic field are different, are unclear. Care and March²¹ try to explain this by the fact that for fields below some critical field ($H_0 \approx 3-4$ kOe), conductivity and the Hall effect are determined by the electrons from the lowest impurity band, which diffuse through a Wigner lattice. For fields higher than the critical one, the

donor functions are compressed so strongly that the Hall conductivity can be realized only by the activation of electrons to an excited impurity band (which, possibly, overlaps with the conductivity band). But in that case it remains unclear why for conductivity to exist there is no need for the activation of electrons to excited states in fields $H > H_0$. Possibly, the reason is more in the insufficient reliability of the $\Delta\varepsilon$ values found from electron conductivity measurements, as Somerford himself recognizes.²² The analysis of the measurements of σ is complicated by the fact that at such small values of $\Delta\varepsilon$, the variation in the probability of scattering in a magnetic field can mask a comparatively small increase in the activation energy $\Delta\varepsilon(H)$. In the case of one type of carrier this must affect the magnetoresistivity, but not the Hall effect.

It is useful to look more carefully at the weak anisotropy of conductivity observed by Somerford²² upon rotation of the magnetic field. The absence of anisotropy which one can expect in the case of electron crystallization in a magnetic field (see Section 3) allows one to conclude that the ordering of the electron system has not been found. It is appropriate to remind here that in the region of conductivity via impurities, σ_{xx} and σ_{zz} differ little in sufficiently strong magnetic fields, so that the weak anisotropy of magnetoresistivity can be associated with the impurity conductivity.

6. ELECTRON CONDENSATION IN InSb

The existence of a threshold magnetic field H_0 below which no activation energy was found in the Hall effect and the conductivity measurements is considered by the authors of Ref. 21 as a demonstration of electron condensation. Since the electron concentration in InSb samples used in Refs. 22 and 23 is small, the screening effect (which depends on the magnetic field) can be ignored. Care and March²¹ assume that for the fields $H < H_0$ conductivity is realized by electrons from the lower impurity band. With an increase of the field above the threshold H_0 the overlap of the donor wave functions decreases to such an extent that the system goes into a nonconducting state, i.e., the Wigner electron crystallization takes place. A rough estimate of the field H_w corresponding to the Wigner ordering can be obtained from the condition $l \approx r_0$. For $n = 4.9 \cdot 10^{13} \text{ cm}^{-3}$ this condition is achieved in the field $H_w \approx 230 \text{ Oe}$ which is significantly lower than the threshold field $H_0^R \approx 3\text{--}4 \text{ kOe}$ found from Hall effect measurements.

The main objection to the interpretation of experiments of Refs. 22 and 23 on the basis of concepts of electron crystallization is that one is dealing with electrons from the impurity states which are exposed to a significant influence of a random potential. It is in this direction that one must look for the causes of the special features in the Hall effect and conductivity measurements discovered by Putley and Somerford.

First of all, one has to note the fact that the Hall electron mobilities $\mu = |R| \sigma$ for samples of InSb from Ref. 23 at $T = 4.2 \text{ K}$ indicate strong scattering: $\hbar/\tau > \varepsilon_F$ (τ is a relaxation time). This inequality becomes stronger with an increase of the field H . For example, for a sample with $n = 4.9 \cdot 10^{13} \text{ cm}^{-3}$ and the compensation degree $K \approx 0.9$ at $T = 4.2 \text{ K}$ and $H \approx 5 \cdot 10^2 \text{ Oe}$ we have $\hbar/\tau\varepsilon_F \approx 5$, and for $H \approx 8 \cdot 10^{13} \text{ Oe}$ we have $\hbar/\tau\varepsilon_F \approx 35$. Therefore, the conduc-

tivity is indeed realized via the impurity states, while the "band" nature of the conductivity becomes ever weaker as the field increases. Thus, one is dealing with donor-band electrons. The strengthening of the inequality $\hbar/\tau > \varepsilon_F$ in a magnetic field corresponds to an increasingly larger electron localization in the wells of the potential pattern created by the random distribution of donors and acceptors in a crystal.

For transitions caused by the disorder of impurities one can obtain only a relatively rough estimate for the criterion for the existence of an electron transition (which we shall call the Mott-Anderson transition), similar to the condition (21). Let us consider the transition from a dielectric state into a metallic state in the absence of a magnetic field.

Fluctuations of the impurity distribution form a random potential pattern with an average well width $\bar{\gamma}$. The quantity $\bar{\gamma} < \varepsilon_p$ where ε_p is the energy of a flow level. The conductivity will have a metallic character, i.e., electrons will be delocalized, when $\varepsilon_F \geq \varepsilon_p$. The Fermi energy ε_F is determined by the concentration n of delocalized electrons, $n = N_D - N_A = N_D(1 - K)$, where $K \leq 1$. All impurities are ionized, i.e., $N_D = N_D^+$, $N_A = N_A^-$. We shall call following condition

$$\bar{\gamma} \leq \varepsilon_F. \quad (23)$$

the criterion of the Mott-Anderson transition. The typical size of the fluctuation wells is equal to the screening radius $R_0 = a_B^*/2(na_B^*)^{1/6}$. In the strong doping limit, $N_i a_B^* \gg 1$ ($N_i = N_D + N_A$), each fluctuation well contains a sufficiently large number of impurities $\bar{N}_i = N_i(4\pi/3)R_0^3 \gg 1$. In that case, $\bar{\gamma}$ can be estimated as $\bar{\gamma} \approx (e^2/\kappa R_0)(4\pi N_i R_0^3)^{1/2}$.²⁷ This expression can be transformed to the form

$$\bar{\gamma} \approx 5\varepsilon_B^* \frac{(N_i a_B^*)^{1/2}}{(n a_B^*)^{1/12}} = 5\varepsilon_B^* (N_i a_B^*)^{5/12} \frac{(1+K)^{1/2}}{(1-K)^{1/12}}. \quad (24)$$

The Fermi energy is

$$\varepsilon_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} = \frac{\hbar^2}{2m} (3\pi^2 N_i)^{2/3} \left(\frac{1-K}{1+K} \right)^{2/3} = \varepsilon_B^* (3\pi^2 n a_B^*)^{2/3}. \quad (25)$$

From (23)–(25) we find

$$n^{1/3} a_B^* \geq 0.42 \left(\frac{1+K}{1-K} \right)^{2/3}, \quad \text{or} \quad r_s^{*cr} \leq 1.5 \left(\frac{1-K}{1+K} \right)^{2/3}. \quad (26)$$

In the absence of compensation ($K = 0$ and $n = N_D = N_i$), one obtains from (26) $N_i^{1/3} a_B^* \geq 0.42$, which is noticeably different from the Mott criterion (21). The reason for the difference is related, probably to the fact that the expression for $\bar{\gamma}$ from Ref. 27 is by its meaning a result of the law of large numbers and is valid in the limit of the strong inequality $N_i a_B^* \gg 1$ (for example, $N_i a_B^* \geq 10\text{--}100$). Besides, one should not forget that instead of the condition $\varepsilon_F \geq \varepsilon_p$ the condition $\varepsilon_F \geq \bar{\gamma}$ was used. Nevertheless, qualitatively the criterion (26) reflects correctly the dependence of the concentration n (or N_i) on the degree of compensation K . For InSb samples studied by Putley²³ $K \approx 0.9$ and, according to expression (26), $r_s^{*cr} \leq 0.2$. As analysis, for example, that of Gerhardt's¹⁸ has shown for the case of $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ crystals with $n \approx 10^{14} \text{ cm}^{-3}$, for r_s^{*cr} of the same order of magnitude, a Wigner transition should take place (the value of the Bohr radius $a_B^* = 6.4 \cdot 10^{-6} \text{ cm}$ for InSb and $1.8 \cdot 10^{-5} \text{ cm}$ for

$\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$, i.e., they differ by only a factor of three). It is obvious, therefore, that under such conditions the electron transitions caused by Wigner crystallization are difficult to detect against the background of effects related to localization of electrons on impurities.

In a quantizing magnetic field $\hbar\omega_c^* > \varepsilon_F$, the Fermi energy decreases ($\varepsilon_F \sim H^{-2}$) and at the same level H_0 , the Fermi level in a doped compensated semiconductor will be below the flow level (the energy ε_p changes with the field slower than ε_F). When the Fermi level descends into the wells of the potential pattern, the electrons become localized. The threshold field H_0 of electron localization in potential wells is given for the case of classical screening (the electron wavelength λ is less than the screening radius R_0) by the expression²⁸:

$$H_0 \approx \pi^2 \left(\frac{12}{\pi} \right)^{1/6} \frac{c\hbar}{e} \frac{a_B^{*1/2} n^{7/6}}{N_D^{1/3}}. \quad (27)$$

An estimate of H_0 from expression (27) for the samples studied by Somerford²² and Putley²³ leads to the following results. For a sample with $n = 2 \cdot 10^{14} \text{ cm}^{-3}$ and $N_D = 1.8 \times 10^{15} \text{ cm}^{-3}$ ($K = 0.8$) (Ref. 22), $H_0 \approx 0.9 \text{ kOe}$; for a sample with $n = 4 \cdot 10^{14} \text{ cm}^{-3}$ and $N_D = 3.6 \cdot 10^{15} \text{ cm}^{-3}$ ($K = 0.8$) (Ref. 22), $H_0 \approx 1.5 \text{ kOe}$. These values of H_0 are in agreement with the values of the threshold fields H_0^{xx} , found by Somerford in his conductivity measurements, i.e., $H_0^{xx} \leq 2 \text{ kOe}$. For a typical sample in Putley's experiments with $n = 4.9 \cdot 10^{13} \text{ cm}^{-3}$ and $N_D \approx 4 \cdot 10^{14} \text{ cm}^{-3}$ ($K \approx 0.9$), expression (27) gives $H_0 \approx 200 \text{ Oe}$, which is approximately one order of magnitude below the experimentally determined value of H_0^R . A discrepancy between the values of H_0 and H_0^R is, possibly, due to the rather rough estimate of N_D and N_A in Ref. 23, to which expression (27) is rather sensitive.

For fields $H > H_0$, conductivity has an activation character, i.e., for sufficiently high temperatures $k_B T \gtrsim \varepsilon_p - \varepsilon_F = \varepsilon_2$ electrons are excited above the flow level and become delocalized. The activation energy increases, obviously, with an increase in the magnetic field. Besides, conductivity can contain a nonactivation component, if there are many donors and the overlap of the donor wave functions is sufficiently large. Therefore, as a result of random distribution of dopants, conductivity by activation can arise at low temperatures, with the activation energy increasing in a magnetic field.

What arguments are given in support of the suggestion that the experiments of Putley and Somerford give some evidence of electron crystallization? Durkan and March²⁵ express the opinion that the evidence for Wigner crystallization in Putley's experiments is given by the rather sudden (i.e., observed only at certain H) appearance of the activation change in the Hall coefficient. If one studies carefully the curves from Putley's work, it is difficult to agree that the activation energy appears in a strictly fixed magnetic field. There was no reason to expect that, just as, incidentally, there was no reason to expect the same also for other electron transitions. Magnetic-field-induced localization of electrons in potential wells must also take place, according to the criterion $\varepsilon_F = \varepsilon_p$, at a well defined value of H . In practice, however, the transition from a delocalized state to a localized one is always noticeably "smeared" so that one cannot distin-

guish using the "sharpness" characteristic (even if such were to exist) between a Wigner crystallization and a localization appearing as a result of disorder.

Care and March²¹ assert that the assumption about localization of electrons caused by a random distribution of dopants does not allow one to explain experimental data of Somerford,²² who found in his electric conductivity measurements the activation energy up to temperatures of 0.27 K. The authors of Ref. 21 express the opinion that since, according to Mott, the impurity conductivity by activation must be replaced, at sufficiently low temperatures, by the conductivity with a variable hopping length $\sigma \sim [\exp(-T_0/T)]^{1/4}$, Somerford's results do not correspond to the conclusions of the theory of electron localization resulting from disorder. This opinion of the authors does not seem convincing, since it is well known how difficult it is to distinguish the activation exponential dependence from the Mott exponential (see Section 8.2). It is not at all obvious that Somerford's data at low temperatures cannot be described by an exponential of the form $\exp(-T_0/T)^{1/4}$ (unfortunately, Somerford himself did not attempt to do that). Besides, it is quite possible that the temperature in Somerford's experiments is still insufficiently low for a transition to a conductivity with a variable length of hopping. In our opinion, there is no support for thinking that experiments of Putley and Somerford give some evidence of Wigner ordering in electron systems.

7. VISCOUS-LIQUID MODEL

Adkins²⁹ made an attempt to explain qualitatively the influence of randomly distributed defects on the conductivity of an inversion layer, in which the electron system is ordered due to correlation effects. The model suggested in Ref. 29 is valid, in the opinion of its author, also for a three-dimensional case. Considerations of Adkins can be summarized as follows.

When the interelectron correlation dominates, the electrons, in the absence of a structure (and, first of all, of a disorder) of the positive background, become localized relative to each other, forming a regular lattice. If such⁹ Wigner crystal, for any reason, occupies only a part of the solid's volume, then in the case of translational motion it will participate in charge transport. However, the disorder in the background fixes an electron lattice in space and violates its translational symmetry, since the position of each electron is determined not only by correlation, but also by disorder. The electron system in the presence of disorder becomes similar to a polycrystal: A lattice is broken up into small crystallites, inside which the regularity of the electron lattice is preserved. The position and orientation of each crystallite are fixed in space as a result of influence of the background disorder and neighboring crystallites. Such a description of an electron system is obviously oversimplified. In reality, there will be no well defined regions with a rigid crystal order, separated by sharp boundaries. An electron lattice will be deformed and strained, and the correlation in the position of electrons will decrease smoothly with distance. Such a structure is similar to a supercooled liquid. The distance over which the spatial correlation in the liquid remains significant, corresponds to the size of a crystallite in the polycrystal description of the system.

At $T = 0$ K, this electron liquid becomes frozen as a glass. At $T \neq 0$ K, the energy of thermal motion causes a restructuring of the system; an electron diffusion analogous to the diffusion of molecules in a normal liquid arises. The electron mobility in this model must change with temperature in an activation manner, similar to the changes of fluidity in the theory of a viscous liquid. Unlike the case of the classical liquid theory, according to which a certain amount of energy is needed to cause relative motion of atoms, in the electron liquid model electrons must overcome the potential barriers created by the background disorder. Practically all the electrons participate in the flow of the liquid, since they are forced to do that by Coulomb forces. One-particle processes are suppressed, since the energy required for the excitation of an electron from a state localized in the liquid to a delocalized state, is rather large, of the order of $e^2/\kappa r_0$. Therefore, the main features of the Adkins viscous-liquid model near the transition of an electron system from a dielectric into a metallic state are as follows: Practically all the electrons participate in conductivity, and their mobility varies with temperature in an activated manner.

The following assumptions are made for an estimate of conductivity: 1) each microscopic activation act is accompanied, as a result of interelectron correlation, by the motion of a whole group of electrons; 2) the disorder in the position of impurities, acting on each electron separately, fixes it in a particular place, but the correlation interaction favors the fixation in space also of neighboring electrons. For that reason, when an electron is thermally excited from its fixed position, the whole group of electrons moves; 3) the structure of the electron group is preserved in the process of sequential hopping (this point represents a rather rigid requirement). For a group of z electrons, performing thermally activated hopping over a barrier of height W , the diffusion coefficient is

$$D \approx L^2 \nu \approx L^2 \frac{k_B T}{h} \exp\left(-\frac{W}{k_B T}\right), \quad (28)$$

where L is the hopping length, ν is the hopping frequency. Since the surrounding liquid prevents large displacements of a group of electrons, the most probable are hops over a distance L , of the order of the interelectron distance r_0 . For that reason, L^2 in formula (28) can be expressed in terms of the electron concentration: $L^2 \approx n^{-2/3}$. Using the Einstein relationship for mobility $\mu = (ze/k_B T) \cdot D$, we can find an estimate for conductivity $\sigma = (n/z)ze\mu = en\mu$ (n/z is the number of electron groups per unit volume, ze is the charge of each group):

$$\sigma \approx \frac{ze^2 n^{1/3}}{2\pi\hbar} \exp\left(-\frac{W}{k_B T}\right). \quad (29)$$

The activation energy W depends on the nature of disorder and on electron concentration n , decreasing to zero when the entire system goes into a metallic state. The parameter z characterizes the relative proportion between the influence of correlation and the influence of the background disorder. According to Adkins, one can expect that with an increase of n , the parameter z will increase, and the barrier height W will decrease. The barriers disappear, obviously, and the electrons become delocalized when the kinetic energy of electrons (the energy of plasma vibrations) exceeds the binding energy at a site.

The described model of an electron liquid is qualitative and cannot serve as a basis for a quantitative analysis of conductivity. This refers also to the low temperature region, where conductivity with a variable hopping length dominates, and W is a function of temperature.

In Ref. 29 a qualitative picture of the formation of the Hall effect is also studied. It is assumed that the correlation is sufficiently strong to force all the electrons to move in external fields. A small number of sites can exist at which the electrons are tightly bound, and where the correlation forces are not sufficient to force them to move. The liquid will flow around such sites. Since electrons do not tunnel, but move classically above the barriers, they are exposed to the influence of a classical Lorentz force. At the same time, it is not at all obvious that the classical Hall effect must be in evidence in an electron liquid, since almost all the electrons are not moving. The classical Hall effect, however, must arise, according to Adkins, in an electron liquid because the liquid can transmit pressure.

In the case of noninteracting charged particles the Lorentz force, acting only on the moving particles, is balanced by the Hall field force, and the Hall coefficient R is determined by the concentration of mobile particles. The Lorentz force in an electron liquid arises only when the hopping of a group of electrons takes place. The arising Hall field leads to the formation of a pressure gradient in the liquid, and all the electrons, even those that do not move, participate in the transfer of pressure. Thus, the effect of the Lorentz force on a group of electrons during hopping is transmitted to the surrounding liquid and is balanced mostly by the constant action of the Hall field \mathbf{E}_H on the liquid as a whole. From the comparison of these two forces $[\mathbf{j} \times \mathbf{H}] = en\mathbf{E}_H$ (j is the current density, n is the total concentration of electrons) we find the Hall effect coefficient $|R| = 1/en$.

Thus, in the limiting case of strong interelectron correlation, the Hall effect depends on the total concentration of electrons (and mobility in that case increases in an activation manner with increasing temperature. In the other (known) limiting case of noninteracting particles, the Hall coefficient reflects the activation-induced increase in electron concentration taking place with a temperature rise (mobility in that case is assumed constant or changing slowly with temperature). With increasing z , more and more electrons become bound in sites, the Hall coefficient increases and, finally, begins to increase exponentially with increasing temperature. In the intermediate (between the two extreme cases) situation, the Hall coefficient can be approximated, according to Adkins, by the expression

$$R = \frac{1}{en} z(1-z). \quad (30)$$

8. ELECTRON LOCALIZATION IN $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$

8.1. Wigner condensation?

In the last several years an extensive discussion has developed on the nature of the metal-dielectric transition induced by a magnetic field in an $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ semiconductor. Some authors³⁰⁻³⁵ assert that in a strong magnetic field an $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ electron gas condenses into a Wigner lattice or into a CDW state. According to other reports³⁶⁻³⁹ a viscous liquid is formed. Experimental data are reported demonstrating that the metal-dielectric transitions are caused

by the "freezing-out" of band electrons into impurity levels.^{40,41} The suggestions were expressed that the electron phase transition takes place within the impurity donor band, separated from the conduction band.^{42,43} Finally, proofs are given of electron localization in the wells of the potential pattern created by a random distribution of impurities.⁴⁴⁻⁴⁷ Let us discuss the main results of the cited references.

Nimtz *et al.*³⁰ studied the magnetoresistance (ρ_{xx} and ρ_{zz}) and the Hall effect in $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ crystals with electron concentrations of $n = 4.5 \cdot 10^{14} \text{ cm}^{-3}$ and $n = 6.4 \cdot 10^{14} \text{ cm}^{-3}$ at temperatures of 1.5 K and 4.2 K, in magnetic fields up to $8.5 \cdot 10^4$ Oe. The compensation K in these samples estimated from electron mobility is, respectively, about 0.5 and 0.7. It was found that below a certain magnetic field $H = H'$, ρ_{xx} and ρ_{zz} increase almost by two orders of magnitude, and for the fields $H > H'$ this increase becomes significantly smaller. At 42 K, H' is ≈ 36 kOe for the first sample, and ≈ 67 kOe for the second sample.

The authors of Ref. 30 assert that in the region $H < H'$, the electron system is ordered. Conductivity in that phase is realized by the motion of a Wigner lattice as a whole through the sample from the cathode, where it is formed, to the anode where it is destroyed. According to the authors of Ref. 30, the reasons for the conclusion that electron crystallization has been observed in these experiments are the following: a) the presence of a threshold field H' ; b) its increase with an increase of concentration n ; c) a sharp growth of ρ_{xx} and ρ_{zz} at $H < H'$.

These arguments in favor of Wigner crystallization are based on an obvious misunderstanding. It is correct that in case of electron ordering the resistance has to increase sharply in a magnetic field, and that there must be a threshold field separating the electron gas and electron crystal phases. It is also correct that this field is larger for higher electron concentrations. However, Wigner condensation has to arise in magnetic fields *exceeding* the threshold field. Resistance in the condensed phase should then be not only larger, but also increasing with the field faster than for the electron gas phase.

There are no principally new results in the work of Nimtz *et al.*³¹ as compared with Ref. 30. It is emphasized there that the temperature below which the curves $\rho_{xx}(H)$ have a breaking point, is significantly higher than the critical temperature T_c , which corresponds, according to the calculations of Refs. 15, 16, and 18 to the beginning of electron crystallization. The values of H' for which the curves of $\rho_{xx}(H)$ have breaking points, differ by a factor of 4-6 from the critical field H_w for the Wigner transition calculated in Ref. 15. The reason for the discrepancies between the experimental results and the results of calculations of Ref. 31 are not clear, but they suggest that the breaking points of the curves $\rho_{xx}(H)$ are related to electron ordering.

In Ref. 31 it is noted that at low temperatures at which the curves $\rho_{xx}(H)$ have breaking points, the anisotropy of magnetoresistance is very small: ρ_{xx} differs from ρ_{zz} only by $\sim 10\%$, and this, supposedly, agrees with the theoretical concepts about electron crystallization. However, as we have seen in Section 3, an electron gas, crystallizing in a magnetic field, becomes one-dimensional, as a result of which the anisotropy of magnetoresistance must sharply increase, and this is, in fact, observed in layered conductors. A weak anisotropy of magnetoresistance in the low tempera-

ture region, as well as a decrease in the slope of the curves $\rho_{xx}(H)$ and $\rho_{zz}(H)$ for fields $H > H'$ are typical for impurity-type conductivity.

The discussion of the charge transfer by a Wigner lattice as a result of its motion as a whole through the crystal is not consistent with the concept of an electron condensate-dielectric, if one does not have in mind the viscous liquid model (see Section 7). All the above-mentioned allows one to assert that the interpretation of the authors of Refs. 30 and 31 is incorrect.

From the measurements of time dependence of the hot-electron current j in $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ samples with $n \sim 10^{14} \text{ cm}^{-3}$, Nimtz *et al.*^{33,37,48} determined the energy relaxation time τ_e . Knowing τ_e it is possible, using the energy balance equation $jE\tau_e = C_e \Delta T_e$, to find the thermal capacity C_e of an electron system for the values known from measurements of j , of the electric field E and of the temperature increase $\Delta T_e = T_e - T_l$ in an electric field. The curve of $C_e(T_e)$, as well as that of $\tau_e(T_e)$, has a maximum at $T_e \approx 1.8$ K ($H = 40$ kOe) and $T_e \approx 2.3$ K ($H = 60$ kOe) (the lattice temperature is $T_l = 1.5$ K). The authors of Ref. 33 interpret the rise of τ_e and, therefore, of C_e with a decrease of T_e from 3.5 to 2.5 K as a transformation of an electron gas into an electron liquid, for which the thermal conductivity is larger than the conductivity of the gas. A decrease of C_e with a decrease in T_e below 2.3 K is caused, according to Nimtz, by the formation of a Wigner crystal.

The curves of $\tau_e(T_e)$ were determined also for $n\text{-InSb}$ in Ref. 48 ($n = 1.2 \cdot 10^{14} \text{ cm}^{-3}$) and in Ref. 49 ($n = 7.7 \cdot 10^{13} \text{ cm}^{-3}$). The curves $\tau_e(T_e)$ and $C_e(T_e)$ are analogous to the curves for $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$, i.e., they have a maximum at some value of T_e . The authors of Ref. 49, however, interpret the results of measurements on the basis of a one-electron approach, without involving interelectron correlations. The decrease of τ_e with a decrease of T_e from $T_{e,\text{max}} \approx 6$ K to ≈ 1.5 K is explained by a change in the probabilities of scattering by the piezoelectric and deformation potentials, while a decrease of τ_e with an increase of T_e above 10 K is associated with an increase of scattering by polar phonons. This interpretation seems well-founded and is therefore, quite probably, valid also for $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$, an analog of $n\text{-InSb}$.

Rosenbaum *et al.*³⁴ studied the standard galvanometric phenomena in $n\text{-Hg}_{0.76}\text{Cd}_{0.24}\text{Te}$ crystals at $0.01 \leq T \leq 0.7$ K for fields $H \leq 75$ kOe (electron concentration $n = 1.4 \cdot 10^{14} \text{ cm}^{-3}$, mobility at 77 K $\mu = 1.5 \cdot 10^5 \text{ cm}^2/\text{v}\cdot\text{sec}$, and the compensation estimated from mobility, $K \approx 0.4$). A sharp increase of $\rho_{xx}(H)$, $\rho_{zz}(H)$ and the Hall resistance $\rho_{xy}(H)$ was found for the fields $H \geq H_0^{xx}$, H_0^{zz} , and H_0^{xy} .

Analyzing the Hall effect measurements, Rosenbaum *et al.*³⁴ exclude the possibility of electron freezing-out to isolated donors as a possible reason of the $\rho_{xy}(H)$ increase, since in that case the electron concentration would have to decrease exponentially with the temperature decrease [$n \sim T^{1/2} \times \exp(-\varepsilon_D/k_B T)$, $\varepsilon_D(H)$ —is the ionization energy of a donor], and the slopes of the curves $\ln \rho_{xy}(H)$ would have to be different at different temperatures. This does not agree with the results of Ref. 34, according to which the curves $\rho_{xy}(H)$ are practically parallel for $H > H_0^{xy}$. Later measurements of Shayegan *et al.*^{39,40} have shown that the slope of the curves $\ln \rho_{xy}(H)$ at $H > H_0^{xy}$ increases with a temperature decrease from 1.3 to 0.5 K, as well as in the case of $n\text{-InSb}$. If one assumes, as the authors of Ref. 34 do, that

carriers of only one type contribute to conductivity, then the slope increase with decreasing temperature indicates a decrease of n , i.e., a freezing-out of electrons.

In the opinion of the authors of Ref. 34 localization of the Anderson type, caused by disorder, also cannot be responsible for the increase of $\rho_{xy}(H)$, since then the field H_0^{xy} would not depend on T , but the measurements show that H_0^{xy} increases from 6.5 kOe to ≈ 9 kOe with a temperature increase from $T = 0.01$ K to $T = 0.7$ K. The authors of Ref. 34 associate the temperature dependence of H_0^{xy} with electron crystallization. Later we discuss briefly questions of possible influence of disorder on this dependence.

Rosenbaum *et al.*³⁴ see the reason for the increase of $\rho_{xy}(H)$ for $H > H_0^{xy}$ in the transition of an electron system at $H = H_0^{xy}$ to a CDW or a Wigner crystal state. Under these conditions, one should expect that the critical temperature of transition, T_c depends on the critical field H_0^{xy} . However, the observed increase of H_0^{xy} with temperature T does not agree with the theoretical predictions of Refs. 17 and 18; according to Ref. 18 for strong magnetic fields $T_c \sim H_w^{-1}$ (Fig. 2). In this situation the authors of Ref. 34 decided to use an estimate of the melting temperature T_M of a Wigner lattice suggested by Kleppmann and Elliott.¹⁵ An approximate relation between T_M and the binding energy ε_0 of atoms in crystals of inert gas (Ne, Ar, Kr, etc.) $k_B T_M \sim \varepsilon_0/8$ (Ref. 3) was used for calculation of T_M in an electron crystal (in this case ε_0 has the meaning of the electron binding energy). Such an estimate gives $dT_M/dH \approx 0.3$ K/kOe. The experimentally determined value is 0.22 K/kOe. This agreement, which is the main argument of the authors of Ref. 34 in favor of the hypothesis concerning electron ordering at $H > H_0^{xy}$ seems to us an illusory one. First of all, the empirical formula for T_M does not take into account the dominant influence of a magnetic field, which significantly changes the conditions for the formation of a Wigner crystal. Secondly, it is not at all clear whether the Wigner lattice must necessarily transform at the melting temperature into a uniform electron gas (a sharp transition) or to an intermediate CDW phase (a more or less "smeared" transition).

The authors of Ref. 34 assume that for the studied conditions the random distribution of impurities is not essential since the radius of the electron wave function is large. An estimate, however, shows that for $n = 1.4 \cdot 10^{14}$ cm⁻³, the Bohr radius $a_B^* = r_0 = n^{-1/3}$ and, therefore, for $K \approx 0.4$, a_B^* only slightly exceeds the interimpurity distance $\approx N_i^{-1/3}$. In this case, the very existence of a Wigner crystal is quite questionable, and any estimates using the formulas of the "jellium" model are even more unreliable.

Now let us return to the question about the temperature dependence of the field H_0^{xy} . An entirely acceptable explanation is offered by Shayegan *et al.*^{39,40} They associate the shift of H_0^{xy} with temperature with the influence of a screening effect on the magnetic-field-induced Mott transition. As Fenton and Haering have shown⁵⁰ (see the discussion in Sec. 5 of the experiments of Refs. 22 and 23), the screening decreases the electron binding energy to a donor. With a temperature increase, some bound electrons become delocalized, and, as a result, the screening of the potential of impurity ions increases. This, in turn, must cause a further generation of free electrons and suppression of the dielectric state. For that reason, one needs larger critical fields H_0 for the metal-dielectric transition to take place. The tempera-

ture dependence of H_0 under such conditions must be determined by the details of the structure of the impurity band and the nature of the change in the binding energy with increasing screening.

One can offer also another explanation of the form of $H_0^{xy}(T)$, which also is not related to Wigner crystallization and is based on the model of overlapping band and impurity states⁵¹ (this model will be discussed later during the discussion of Refs. 36–38). As the temperature rises more and more electrons become delocalized and their average energy $\bar{\varepsilon}$ increases. Therefore, one needs larger magnetic fields in order to displace the electrons closer to a band edge and practically localize them. Quantitative estimates of hybridized band and impurity states are complicated in that case, but such an examination gives the qualitatively correct tendency of the change of H_0 with temperature.

The interpretation of experiments on hot electrons in n -Hg_{0.76}Cd_{0.24}Te suggested by Field *et al.*³⁵ seems rather probable at first glance. They measured the volt-ampere characteristics in samples with $n = 1.4 \cdot 10^{14}$ cm⁻³ at $T = 10$ –100 mK in a magnetic field $H = 64$ kOe. It was found that for weak electric fields $E \lesssim 1$ mV/cm deviations of the volt-ampere characteristics from linearity begin to take place. Nonlinear conductivity is small; even at the lowest temperature of 10 mK it does not exceed 10% of the linear conductivity. With a temperature rise to 300 mK, the nonlinearity disappears. For magnetic fields 20 kOe $\gtrsim H \gtrsim H_0^{xy} \approx 8$ kOe, the threshold electric field E_d , at which deviation from Ohm's law begins to take place, decreases and at $H > 20$ kOe, E_d increases linearly with H . The authors correctly note that for the ionization of electrons, bound on donors, one needs electric fields exceeding E_d by several orders of magnitude.

Field *et al.*³⁵ relate the nonohmic dependence of $j(E)$ to the collective motion ("sliding") of the Wigner crystal domains, which have been pinned by impurities before an electric field has been turned on. This effect is analogous to a collective motion of CDW's in quasi-one-dimensional and layered structures, for example, in NbSe (see Ref. 52) or in graphite in a phase induced by a magnetic field $H \gtrsim 210$ kOe.⁵³ In these materials the nonlinearity of $j(E)$ also starts at fields $E_d \sim$ mV/cm. The temperature dependence of the field E_d [a decrease of $E_d(T)$] is, according to the measurements of Ref. 35, analogous to the $E_d(T)$ dependence in materials of the NbSe₃ type, which, incidentally, until now has not found an unambiguous explanation. However, unlike the case of layered materials, nonlinear conductivity in Hg_{0.8}Cd_{0.2}Te is, as has been noted, unusually small: In an electric field $E \sim$ mV/cm, σ increases only by $\sim 10\%$, while in crystals of the NbSe₃ type it increases severalfold.

It is natural that the low values of the threshold fields E_d might suggest a thought concerning the motion of the CDW domains, as is the case in quasi-one-dimensional metals NbSe₃, etc. However, in three-dimensional structures, the fluctuating impurity potential can easily destroy the ordered electron system. One can offer another explanation of the origin of the nonlinear behavior of $j(E)$ characteristics taking place in electric fields \sim mV/cm.

For an intermediate compensation (we recall that for the samples studied in Ref. 35, $K \approx 0.4$) the state density has a Coulomb gap. Usually, an energy gap is $\Delta \lesssim 1$ meV. This means that for electric fields $E \lesssim 1$ mV/cm, electrons can be

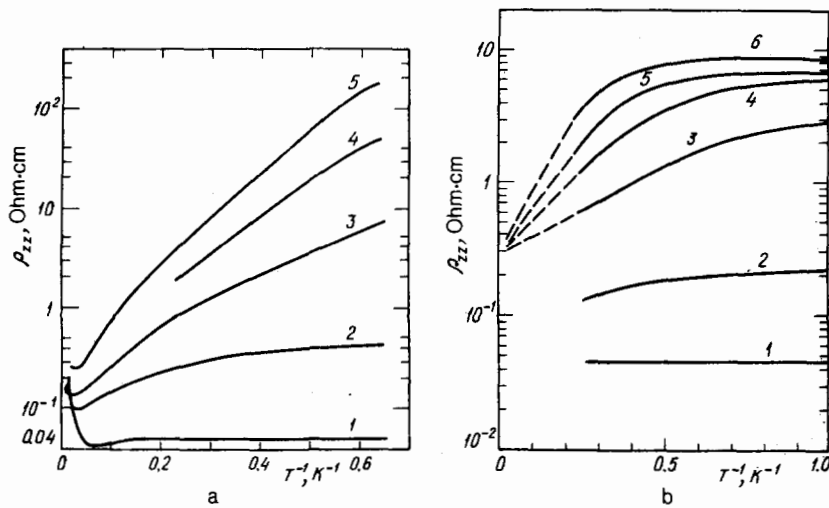


FIG. 4. Temperature dependence of longitudinal magnetoresistance for $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$.³⁶ a: $n = 3.4 \cdot 10^{14} \text{ cm}^{-3}$, $\mu = 4.6 \cdot 10^5 \text{ cm}^2/\text{Vsec}$; b: $n = 3.0 \cdot 10^{14} \text{ cm}^{-3}$, $\mu = 5.5 \cdot 10^5 \text{ cm}^2/\text{Vsec}$; H (Oe): 0 (1), $1 \cdot 10^4$ (2), $3 \cdot 10^4$ (3), $5 \cdot 10^4$ (4), $7 \cdot 10^4$ (5) and $11 \cdot 10^4$ (6).

excited through the Coulomb gap with the result that the volt-ampere characteristic becomes nonlinear. Obviously, with a temperature increase one needs lower electric fields for ionization of electrons through the energy gap, since the thermal "smearing" of the Fermi level increases. As far as the $E_d(H)$ dependence, mentioned in Ref. 35 is concerned, its cause within the framework of the suggested explanation is still not clear, since the theory of conductivity in a magnetic field for the case when the Fermi level is at a minimum of the state density is not available.

8.2. Viscous liquid?

We now discuss the results of papers in which some special features of the transport phenomena are considered as evidence in favor of a viscous-liquid model (see Section 7). Nimitz *et al.*^{31-33,36-38} give the results of measurements of magnetoresistance, photoconductivity and volt-ampere characteristics of $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ samples. The parameters of the studied samples are approximately the same as in Ref. 30: at 4.2 K, $n = 3 \cdot 10^{14} \text{ cm}^{-3}$, $\mu = (4.5-5.5) \cdot 10^5 \text{ cm}^2/\text{Vsec}$. For samples with $n = 3.0 \cdot 10^{14} \text{ cm}^{-3}$, $\mu = 5.5 \cdot 10^5 \text{ cm}^2/\text{Vsec}$ and $n = 3.4 \cdot 10^{14} \text{ cm}^{-3}$, $\mu = 4.6 \cdot 10^5 \text{ cm}^2/\text{Vsec}$, the compensation is $K = 0.4$ and $K = 0.5$, respectively. For the temperature range 1.4–10 K, ρ_{xx} and ρ_{zz} increase sharply in a magnetic field—for $T = 1.4 \text{ K}$ and $4 \lesssim H \lesssim 100 \text{ kOe}$, ρ_{xx} and ρ_{zz} increase by 3–4 orders of magnitude. For fields $H \gtrsim 10^4 \text{ Oe}$ and in the interval from 1.5–2 to 5–10 K (Fig. 4), ρ_{zz} decreases with increasing T . This decrease can be described by the exponent $\rho_{zz} = \rho_0 \exp(\Delta\epsilon/k_B T)$. In the region of lower temperatures $1 < T < 2 \text{ K}$, ρ_{zz} has a weaker dependence on T , and for $H \gtrsim 5 \cdot 10^4 \text{ Oe}$, $\rho_{zz}(T)$ becomes saturated.

Nimitz *et al.*^{33,36-38} explain the dependence of resistance on activation by the increase of electron mobility at constant concentration, in agreement with the viscous-liquid model. Since, according to Refs. 30–33, the Hall coefficient changes rather weakly in a magnetic field, the authors of Refs. 37, 38 have performed measurements of photoconductivity and volt-ampere characteristics with the aim of finding out whether the dependence of resistance on activation is caused by changes in concentration or mobility.

In the experiments on photoconductivity, electron-hole

pairs were generated using a pulsed laser with a frequency exceeding the forbidden band. It was observed that the dark conductivity $\sigma = en\mu_e$ increased by $\Delta\sigma = e(\Delta n\mu_e + \Delta p\mu_h)$, with $\Delta n = \Delta p$. Since in HgCdTe $\mu_e \gg \mu_h$, the relative change in conductivity is

$$\frac{\Delta\sigma}{\sigma} \approx \frac{\Delta n}{n}. \quad (31)$$

Measuring σ and $\Delta\sigma$ as functions of temperature and of the electric field E , it is possible to obtain information about variation of n with T and E , assuming that the concentration of photocarriers Δn is constant. The measurements of $\sigma(T)$ and $\Delta\sigma(T)$ dependences in a longitudinal field $H = 5.5 \cdot 10^4 \text{ Oe}$ from 1.4 to 4.2 K have shown that $\Delta\sigma$ increases with the temperature approximately as σ . The same is true for the curves $\sigma(E)$ and $\Delta\sigma(E)$ for fields E up to 2 V/cm at $H = 5.5 \cdot 10^4 \text{ Oe}$ and 1.4 K. Based on the equality (31), the authors of Refs. 37, 38 conclude that the temperature and electric field have practically no effect on electron concentration, and the dependences $\sigma(T)$ and $\sigma(E)$ are caused by changes in the electron mobilities $\mu_e(T)$ and $\mu_e(E)$.

This conclusion is based on the assumption that during the measurement process the concentration of photocarriers Δn does not change with the temperature (or with the field). Because of the different equilibrium mobilities of the thermal and photoexcited hot electrons the measurements had to be carried out over time periods $t \gg \tau_e$ (τ_e is the energy relaxation time). However, during these time intervals t , the initial concentration of minority carriers Δn must change as a result of recombination. The authors of Ref. 38 assert that they succeeded, using a certain fitting procedure, to determine those values of Δn which correspond to an initial time period $t = 0$. Therefore, in their opinion, Δn is approximately constant, and, therefore, $n \approx \text{const}$, and the electron mobility increases with the temperature rise from 1.4 to 4.2 K. This conclusion about the constancy of concentration does not seem to be convincing. First of all, there is no confidence that one can reliably determine the initial value of Δn by approximating the slowly varying part of a photosignal by functions exponentially decreasing with time. Secondly, in the presence of compensating impurities creating a fluctuating potential, one cannot exclude the possibility of capture of

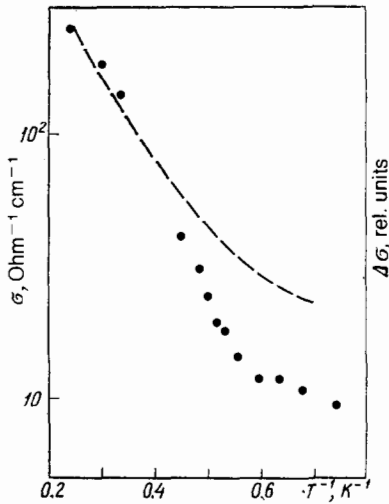


FIG. 5. Conductivity σ (dashed lines) and photoconductivity $\Delta\sigma$ (dots) as a function of temperature for $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ with $n = 3.4 \cdot 10^{14} \text{ cm}^{-3}$, $\mu = 5 \cdot 10^5 \text{ cm}^2/\text{Vsec}$ at $H = 5.5 \cdot 10^4 \text{ Oe}$.³⁸

minority carriers by the defects during short intervals of time smaller than the measurement time t . In that case, $\Delta n \neq \text{const}$, and $n = n(T)$.

Starting from the fact that the conductivity changes with the temperature in an activation manner, Nimtz *et al.*³⁸ tried to describe experimental results for $\sigma(T)$ and $\Delta\sigma(T)$ using exponential functions. It turned out, however, that as T decreases below 2–3 K, one starts observing a significant deviation from the exponential (Fig. 5), both for $\sigma(T)$ and $\Delta\sigma(T)$. The curves $\sigma(T)$ and $\Delta\sigma(T)$ can be described by an expression of the type

$$\sigma(T) = \sigma_1 \exp\left(-\frac{\Delta\varepsilon}{k_B T}\right) + \sigma_2(T), \quad (32)$$

where σ_2 is a rather weak function of temperature.

In the opinion of the authors of Ref. 38, the tendency of $\sigma(T)$ to saturation (in the field $\mathbf{H} \parallel \mathbf{j}$) with an increase of T , as well as the saturation of $\rho_{zz}(T)$ (Ref. 36), is related to a shunting effect of a surface layer of high conductivity, which does not depend on the temperature. However, such an explanation is not valid for photoconductivity, since the magnitude of $\Delta\sigma$ depends only on the excess charge carriers in a sample independently of the spatial inhomogeneity of the conductivity σ . A conjecture was expressed, therefore, that at $T \leq 2 \text{ K}$ and $H = 5.5 \cdot 10^4 \text{ Oe}$ the electron mobility μ_e becomes comparable with the mobility of holes μ_h , and both μ_h and μ_e do not depend on T . The relationship (31) does not hold, then, and one cannot expect an activation dependence of photoconductivity. As we can see, in order to explain the deviation of dependences $\sigma(T)$ and $\Delta\sigma(T)$ from exponential dependences, physically different assumptions have to be made.

Thus, Nimtz *et al.*^{36–38} assert that if one ignores the shunt effects, which become noticeable at $T \leq 2\text{--}3 \text{ K}$, the dependence $\sigma(T)$ can be described by an exponential caused by an activation change in electron mobility. In the opinion of the authors of Refs. 37, 38 such a dependence corresponds to Adkins' model of the flow of viscous liquid of correlated electrons through a crystal containing randomly distributed defects.

One can suggest a more realistic, in our opinion, interpretation of the experimental data of Nimtz *et al.*^{32–33,36–38} The critical concentration of impurities corresponding to a Mott transition is, according to expression (21), $N_M = n_M = 2.7 \cdot 10^{12} \text{ cm}^{-3}$ for uncompensated $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ samples [$n = 6 \cdot 10^{12} \text{ cm}^{-3}$ for samples with $K = 0.4$ according to (26)]. Therefore, for donor concentrations $N_D > 10^{14} \text{ cm}^{-3}$ the Fermi level is above the flow level in the region of delocalized valence band states, and the electrical conductivity has a metallic character (see Fig. 4, curves 1).

It is easy to see that the impurity states do indeed merge with the conduction band, if the width of the impurity band W is estimated, let us say, in the strong binding approximation. For a typical sample from Refs. 36–38 with $n = 3.4 \cdot 10^{14} \text{ cm}^{-3}$, the interelectron distance r_0 is approximately $0.9 \cdot 10^{-5} \text{ cm}$. In compensated samples, the distance between donors $r_D < r_0 < a_B^*$ ($= 1.8 \cdot 10^{-5} \text{ cm}$). For a simple cubic lattice $W = 6\hbar^2/mr_D^2 > 10 \text{ meV}$ (for a fcc lattice, $W > 15 \text{ meV}$). These values of W significantly exceed the ionization energy of an isolated donor $\varepsilon_D = \hbar^2/2ma_B^{*2} \approx 0.2 \text{ meV}$.

A typical property of impurity-type conductivity is the strong electron scattering, for which $\hbar^2/\tau > \varepsilon_F$ (see Section 6). For typical samples from Refs. 36–38 $\varepsilon_F = \hbar^2(3\pi^2n)^{2/3}/2m \approx 2.9 \text{ meV}$ is larger than $\hbar/\tau \approx 0.5 \text{ meV}$, i.e., scattering is weak for $H = 0$, and conductivity is realized over band states. The picture changes in a magnetic field. In the extreme quantum limit $\hbar\omega_c^* > \varepsilon_F$, which is achieved at $H \gtrsim 4 \text{ kOe}$, $\varepsilon_F \sim H^{-2}$. At $H = 10^4 \text{ Oe}$, $\varepsilon_F \approx 7.5 \cdot 10^{-2} \text{ meV}$, and this is by almost an order of magnitude less than \hbar/τ , even if one does not take into account a decrease of τ in a magnetic field. Thus, for the fields $H \gtrsim 10^3 \text{ Oe}$, scattering becomes strong, and this indicates conductivity via impurity states. The Fermi level descends from the region of delocalized into the region of localized states.

Now it is easy to understand the reasons for the decrease in ρ_{zz} with increasing temperature T and the appearance of activation conductivity. As the temperature increases a step of the Fermi distribution function becomes "smeared," the average electron energy $\bar{\varepsilon}$ increases, and more and more electrons with energies $\bar{\varepsilon} \gtrsim \hbar/\tau$ with higher mobility appear. As a result of the increase in the number of electrons with higher mobilities, ρ_{zz} decreases with an increase in T . A similar situation exists for $n\text{-Ge}$ (Ref. 51) near the metal-dielectric transition for $n \gtrsim n_M$. The difference consists of the fact that in $n\text{-Ge}$ with $n \gtrsim n_M$, the Fermi level is located at $H = 0$ in the region of localized states, where $\hbar/\tau > \varepsilon_F$, while for samples of $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ at $H = 0$, $n \gtrsim n_M$ and the Fermi level lies above the flow level, $\varepsilon_F > \hbar/\tau$. In magnetic fields $H \gtrsim 10^3 \text{ Oe}$, $\hbar/\tau > \varepsilon_F$ in $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$, and the change in $\rho_{zz}(T)$ is completely analogous to the change in $\rho(T)$ at $H = 0$ in $n\text{-Ge}$.

The decrease in $\rho_{zz}(T)$, as well as the decrease in $\rho(T)$ in $n\text{-Ge}$, cannot be described by a single exponential or power function law. Some sections of the $\rho_{zz}(T)$ curves can be approximated by exponential dependences of the type of $\exp(\Delta\varepsilon/k_B T)$ or $\exp(T_0/T)^\alpha$ with $1/4 \leq \alpha \leq 1/2$, or even by power functions. For example, experimental points from Ref. 36 (see Fig. 4) for $H = 2 \cdot 10^4 \text{ Oe}$ fall on the straight line $\ln \rho_{zz}(T^{-1/2})$ in the interval 1.5–8 K, and for $H = 3 \cdot 10^4 \text{ Oe}$ in the entire interval 1.5–10 K where $\rho_{zz}(T)$ is decreasing.

The impossibility to approximate $\rho_{zz}(T)$ by a single law indicates a deep similarity of the situations in $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ and Ge. This is supported also by the fact that the experimental temperature dependence of conductivity $\sigma_{zz}(T)$ (see Fig. 5) is described by a function of the type (32), typical for transport of the charge via the mixed band and impurity states near the metal-dielectric transition.⁵¹

With an increase in the magnetic field, a pseudogap in the density of states between the localized ($\hbar/\tau < \epsilon_F$) and delocalized ($\hbar/\tau > \epsilon_F$) states increases as a result of compression of the wave functions, and the Fermi level descends below the flow level. As a result of thermal excitation of electrons from the Fermi level above the flow level, conductivity acquires an activation character. In sufficiently strong fields, the impurity states are separated from the band states—a gap is formed (the Mott transition). With an increase of H the range of temperatures increases where $\sigma_{zz} \sim \exp(-\Delta\epsilon/k_B T)$. We must note that the values of $\Delta\epsilon(H)$ (≈ 0.1 – 1.5 meV) and σ_{zz} (≈ 0.01 – $0.1 \Omega^{-1} \text{cm}^{-1}$) at $T \lesssim 10$ K are typical for the impurity-type conductivity. The closeness of values of ρ_{xx} and ρ_{zz} in magnetic fields up to 10^5 Oe indicates the same. In the region of temperatures corresponding to the transition from the impurity-type conductivity to band conductivity, the Hall coefficient $|R(T)|$ has, as a rule, a maximum. Such a maximum in a magnetic field at $H \approx 5 \cdot 10^2$ Oe was observed by Nimtz *et al.*⁵⁴ in a sample of $n = \text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ with $n \approx 2 \cdot 10^{15} \text{cm}^{-3}$ and $\mu \approx 3 \cdot 10^4 \text{cm}^2/\text{Vsec}$. Unfortunately, for cleaner samples of $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ (Refs. 36–38) there are no data on the measurements of $|R(T)|$ in the region of magnetic fields, where a maximum should be observed.

The studies of the authors of Refs. 44 and 45 support the correctness of the above interpretation of the work of Nimtz *et al.* On the curves of $\sigma_{zz}(T)$ for $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ crystals with $K \approx 0.7$ – 0.8 in the temperature interval 1.7–30 K, one can select areas where $\sigma_{zz}(T)$ increases exponentially. $\sigma_{zz}(T)$ can be described by an exponential in the region $T = 3$ – 10 K with $\Delta\epsilon \approx 1$ meV; below 3 K, the dependence is weaker, and above 10 K—stronger. However, in the entire interval of growth of $\sigma_{zz}(T)$ 1.7–30 K the curves cannot be approximated by a single law, nor can this be done for the curves of $\sigma_{zz}(T)$ for the samples from Refs. 36–38. The Hall

coefficient $|R|$ decreases in magnetic fields exceeding a certain threshold field H_0^R , with the rate of decrease of $|R(H)|$ diminishing as the temperature increases (see Fig. 8 in Section 8.3). An analysis shows that the concentration of light electrons, i.e., electrons activated above the flow level, increases at higher temperatures. The picture of temperature dependences of $\sigma_{zz}(T)$ and $R(T)$ convincingly demonstrates the changes in electron concentration and is not consistent, therefore, with the viscous-liquid model.

Concluding the discussion of the work of Nimtz and coauthors we cannot agree with their unqualified assertion that “the magnetically induced Wigner condensation of electrons behaving like a viscous liquid is established). Dependences $\rho_{zz}(T, H, E)$ and $\rho_{xx}(T, H)$ (see Fig. 4 and 5) can be explained based on the model of overlapping band and impurity states between which, in sufficiently strong magnetic fields H , there is formed a gap, increasing with increasing H .

We turn now to the special features of the dependences $\rho_{xx}(H, T)$ and $\rho_{xy}(H, T)$ discussed by Shayegan *et al.* in Ref. 39. They consist of the following: 1) below a certain temperature T_0 and in fields $H_0^{xx} \leq H \leq H_0^{xy}$, ρ_{xx} changes more with temperature than ρ_{xy} ; 2) for the field interval, $H_0^{xx} < H < H_0^{xy}$ at $T < T_0$, the curve of $\rho_{xy}(H)$ has a small dip (Fig. 6). At $T > T_0$, the dip disappears, Shayegan *et al.*³⁹ consider this relatively small dip as a serious anomaly, which they associate with the state of viscous electron liquid. The authors of Ref. 39 assert that their attempts to describe ρ_{xx} and ρ_{xy} with the help of the two-band model was not successful. According to Ref. 39, the first peculiarity in the behavior of $\rho_{xx}(T)$ and $\rho_{xy}(T)$ can be explained if one assumes that there is only one type of electron with constant concentration n , and the mobility changes with temperature in accordance with the viscous-liquid model. As far as the dip in $\rho_{xy}(H)$ at $T < T_0$ is concerned, it is, according to the authors of Ref. 39, caused by an increase in n as H increases. It is assumed that as H increases either the number of electrons localized on impurities decreases as a result of an increase in interelectron correlation (the correlations release electrons from the potential wells), or the velocity of the flow of a correlated liquid around the localized states increases “in order to compensate the deficiency of delocalized states.”

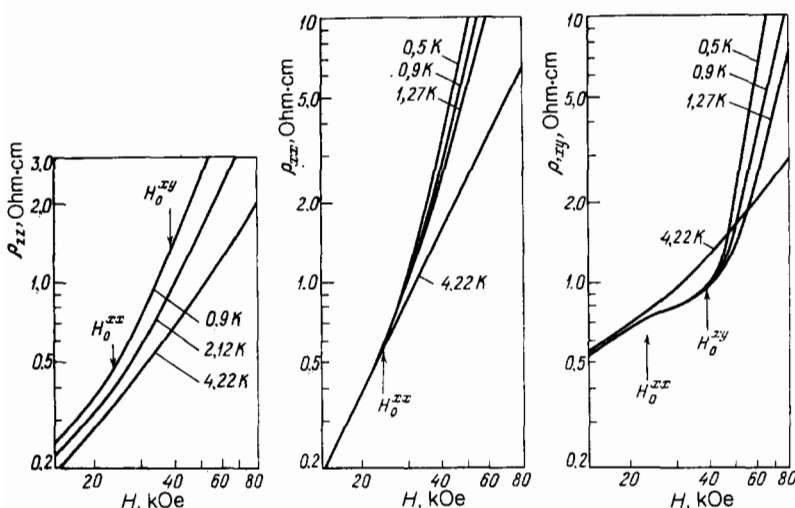


FIG. 6. Longitudinal, transverse and Hall resistance as a function of magnetic field for a $n\text{-Hg}_{0.79}\text{Cd}_{0.21}\text{Te}$ sample with $n = 1.2 \cdot 10^{15} \text{cm}^{-3}$.³⁹

This interpretation is rather artificial and not convincing. Peculiarities of $\rho_{xy}(H, T)$ and $\rho_{xx}(H, T)$, observed by the authors of Refs. 44 and 45 can be rather easily explained within the framework of the two-band model, when both light electrons (concentration n_1) and heavy electrons (concentration n_2) participate in transport phenomena. Approximately one can assume that the former are band electrons with energies $\varepsilon > \varepsilon_p$, and the latter—electrons with $\varepsilon < \varepsilon_p$, moving via the impurity states. Assuming constancy of the relaxation time, one obtains for arbitrary magnetic fields

$$\rho_{xy} = \frac{H}{ec} \frac{n_1 \mu_1^2 + n_2 \mu_2^2 + (n_1 + n_2) (\mu_1^2 \mu_2^2 H^2 / c^2)}{(n_1 \mu_1 + n_2 \mu_2)^2 + (n_1 + n_2)^2 (\mu_1^2 \mu_2^2 H^2 / c^2)},$$

$$\rho_{xx} = \frac{n_1 \mu_1 + n_2 \mu_2 + \mu_1 \mu_2 (\mu_1 n_2 + \mu_2 n_1) (H^2 / c^2)}{(n_1 \mu_1 + n_2 \mu_2)^2 + (n_1 + n_2)^2 (\mu_1^2 \mu_2^2 H^2 / c^2)}.$$

An analysis shows that for the typical values of the parameters $n_1 \approx 10^{14} \text{ cm}^{-3}$, $\mu_1 \approx 10^5 \text{ cm}^2/\text{Vsec}$, $n_2 \approx 10^{15} \text{ cm}^{-3}$, $\mu_2 \approx 10^3 \text{ cm}^2/\text{Vsec}$ for field intervals from $8 \cdot 10^3$ to $3 \cdot 10^4$ Oe, there must be a change in the slope of the curve $\rho_{xy}(H)$ (a dip), while for $H \gtrsim 3 \cdot 10^4$ Oe ρ_{xy} increases monotonically in complete agreement with the data of Refs. 34, 39, 40, 43–47. At temperatures $T > 4.2$ K, n_1 increases, n_2 decreases, and the dip disappears.⁸⁾

From the expressions given above it also follows that for $H < H_0^{xy}$, the dependence of $\rho_{xx}(T)$ on T in the low-temperature region must be stronger than that of $\rho_{xy}(T)$. Thus, for $H \approx 10^4$ Oe an increase of n_1 with an increase of T from $1 \cdot 10^{14}$ to $3 \cdot 10^{14} \text{ cm}^{-3}$ and a corresponding decrease of n_2 from $1 \cdot 10^{15} \text{ cm}^{-3}$ to $8 \cdot 10^{14} \text{ cm}^{-3}$ leads to a decrease of $\rho_{xy}(T)$ by a factor of 1.5 and of $\rho_{xx}(T)$ by a factor of 5 (it is assumed that the mobilities $\mu_1 = 10^5 \text{ cm}^2/\text{Vsec}$, $\mu_2 = 10^3 \text{ cm}^2/\text{Vsec}$ vary negligibly little with T). For stronger dependences of $n_1(T)$ and $n_2(T)$ on T the difference between $\rho_{xy}(T)$ and $\rho_{xx}(T)$ is even larger. Our interpretation of the peculiarities of the curves $\rho_{xy}(H, T)$ and $\rho_{xx}(H, T)$ is, obviously, an approximate one, since for the conditions discussed it is not possible, strictly speaking, to separate electron states into localized and delocalized ones. However, based on the more general model of mixed band and impurity states, it is possible to explain qualitatively the peculiarities of $\rho_{xy}(H, T)$ and $\rho_{xx}(H, T)$.

It is interesting to consider the dependences of the fields H_0^{zz} , H_0^{xy} and H_0^{xy} on n for $\text{Hg}_{0.79}\text{Cd}_{0.21}\text{Te}$ samples from Ref. 39 (Fig. 7). Together with experimental results, Fig. 7 gives the calculated values of the critical fields H_M and H_W for the Mott and Wigner transitions, respectively, and for the beginning of the extreme quantum limit region ($\hbar\omega_c^* = \varepsilon_F$) H_Q . The authors of Ref. 39 have determined the field H_M from the equality, similar to expression (22)

$$na_{1a}^2 \approx (0.25)^3. \quad (33)$$

Let us note that, based on the meaning of the Mott transition criterion, expression (33) must contain not the electron concentration, but the donor concentration [see (22)]. The field H_W was estimated from the formulas of the work of Kleppmann and Elliott.¹⁵ One must emphasize that the error in the estimates of H_W from the formulas of Ref. 15, which, incidentally, are valid for $T = 0$ K, is rather large and can reach an order of magnitude and more. On the other hand, the estimates of H_M , too, cannot claim greater accuracy because of some uncertainty in the value of the right-hand

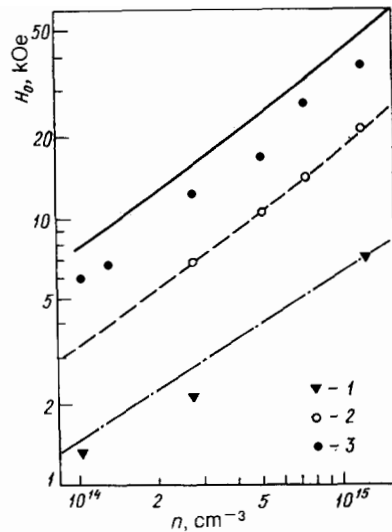


FIG. 7. Critical fields as a function of electron concentration for $\text{Hg}_{0.79}\text{Cd}_{0.21}\text{Te}$.³⁹ 1— H_0^{zz} ; 2— H_0^{xx} ; 3— H_0^{xy} . The continuous line corresponds to H_M (the Mott transition), the dashed line to H_W (Wigner transition), dot-and-dash line to H_Q —the extreme quantum limit.

side of (33) and, what is more important, because the compensation, which, as we have seen [see expression (26)], can change the value of N_M , is not taken into account.

At the same time, the experimental points H_0^{xy} lie rather close to the curve $H_M(n)$, and this indicates that a Mott transition may be taking place at $H = H_0^{xy}$: the impurity states become separated from the band states. The authors of Ref. 39, 40 who give the following arguments are inclined to the same conclusion. If the energy of the ground state of an isolated impurity center ε_D exceeds the binding energy of electrons in a Wigner lattice $\varepsilon_0 \approx (e^2/\kappa r_0)$, the system of electrons undergoes a Mott transition to a dielectric state for fields $H > H_M$. According to Fenton and Haering,⁵⁰ ε_D is always larger than ε_0 for any reasonable value of the right-hand side of expression (33). The proposition that at $H = H_0^{xy}$ a Mott transition is taking place is developed by the authors of Refs. 39, 40 in Refs. 42, 43, which we shall discuss in Sec. 8.3.

Summarizing the discussion of the experimental results of Refs. 39, 40, one must note that these papers do not provide any proof of the existence of a viscous electron liquid.

8.3. Disorder

We turn now to articles 40–47, in which the special features of galvanomagnetic phenomena in $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ crystals, similar to the ones observed in Refs. 39, 40, are explained by localization of electrons on impurities. The authors of Refs. 39, 40 (see Sec. 8.2) came to the conclusion that the increase of $\rho_{xy}(H)$ for fields $H > H_0^{xy}$ is caused, most probably, as for the case of $n\text{-InSb}$, by a Mott transition. Rather convincing proofs of the freezing-out of electrons onto impurities in $n\text{-Hg}_{1-x}\text{Cd}_x\text{Te}$ ($0.18 < x < 0.35$) are given by Raymond *et al.* (Ref. 41) and de Vos *et al.*⁵⁵

In Ref. 41 samples with $n = 10^{14} - 10^{16} \text{ cm}^{-3}$ and the compensation K from 0.3 to 0.95 have been studied for fields up to 200 kOe. Assuming that only band electrons (n_1, μ_1) and partially localized electrons of impurity states (n_2, μ_2) participate in charge transport, with $\mu_1 \gg \mu_2$, they took into

account the contribution of heavy electrons (n_2, μ_2) to the diagonal components of the conductivity tensor $\sigma_{xx}(H)$, $\sigma_{yy}(H)$, and neglected that contribution into the Hall component $\sigma_{xy}(H)$. Then, the concentration n_1 in the magnetic fields $\mu_1 H/c \gg 1$ and $\mu_2 H/c \ll 1$ can be expressed in terms of the measured quantities R and σ_{xx} in the following manner:

$$n_1 = \frac{\sigma_{xy} H}{ec} = \frac{1}{ec} \frac{RH^2}{(RH)^2 + \rho_{xx}^2}$$

In fields $H < 200$ kOe, the term ρ_{xx}^2 cannot be neglected, and, therefore, the simple formula $n = 1/ecR$ cannot be used for the determination of n_1 .

Measurements have shown that n_1 depends on H only very weakly until a certain value of the field, and then starts decreasing sharply. Thus, for a $\text{Hg}_{0.75}\text{Cd}_{0.25}\text{Te}$ sample with $n_1 = 10^{16} \text{ cm}^{-3}$ and $K = 0.5$ at 4.2 K, n_1 decreases sharply for fields $H > 70$ kOe (at $H \approx 180$ kOe, $n_1 = 10^{14} \text{ cm}^{-3}$). The critical concentration of a Mott transition for this sample [expression (21)] is $N_M = 10^{14} \text{ cm}^{-3}$, so that at $H = 0$ all the electrons are in the conductivity band. For fields $H > 70$ kOe, the impurity states are separated from the band states, and the created gap increases as H increases. The dependence $n_1(1/T)$ for a $\text{Hg}_{0.66}\text{Cd}_{0.34}\text{Te}$ sample with the impurity content $N_i = 4.5 \cdot 10^{14} \text{ cm}^{-3}$, which is somewhat smaller than $N_M \approx 10^{15} \text{ cm}^{-3}$ for this composition, shows that the activation energy increases with H , in accordance with the theory of Yafet *et al.* (Ref. 24) for an isolated ion.

For a sample with $x = 0.18$ and $n_1 \approx 8 \cdot 10^{14} \text{ cm}^{-3}$ ($K = 0.6$), Raymond *et al.* (Ref. 41) have obtained for $H \gtrsim 30$ kOe the same activation dependence of $n_1(1/T)$ as for samples with higher doping, but, unlike the latter, n_1 in this sample starts decreasing already in relatively weak magnetic fields. The last fact the authors explain by the presence of a resonance level, lying at $H = 0$ above the Fermi level and associated not with a minimum of the conductivity band at $\mathbf{k} = 0$, but with a side minimum. With an increase of H , the last Landau level intersects the resonance level, and the largest part of band electrons is "frozen out" into it. A similar hypothesis about the presence, high in the conduction band, of a resonance level, associated with Te vacancies, was suggested by Nimtz *et al.*³⁷, but with an opposite purpose—to explain the absence of the electron freezing-out at $H = 0$ into shallow donor levels which, in their opinion, are absent in $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$ with $x > 0.16$. The authors of Ref. 37 see some support to their assumption in the work of Swartz *et al.*⁵⁶ who calculated the position of a donor level formed by vacancies of Te, and came to the conclusion that it is located high in the conductivity band. The results of calculations of Ref. 56 seem unconvincing since they are carried out for an unrealistic localized potential of an impurity (Slater-Koster approximation), and the calculations use 21 (!) fitting parameters.

The direct proof of the existence of shallow donor levels in $n\text{-Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ and of electrons bound on these donors in sufficiently strong magnetic fields, has been obtained in studies of the cyclotron resonance on impurities.^{42,43} The infrared transmission spectra with $\hbar\omega \approx 10.5$ meV in fields $H = 3\text{--}10$ kOe at $1.4 < T < 6$ K of samples with $n = 3 \cdot 10^{13}$ and $6 \cdot 10^{13} \text{ cm}^{-3}$ have been studied. The transmission spectra, depending on the magnitude of the magnetic field, show two minima—one caused by the cyclotron resonance of band electrons; and the other, caused by cyclotron resonance of

impurity electrons. In the case of impurity resonance electrons are transferred from the ground bound state to an excited state with the same spin. The dependence of the energy difference between the peaks of cyclotron resonance for impurity and band electrons $\hbar\omega_{CI} \sim \hbar\omega_{CC}$ on the magnetic field H agrees, with an accuracy to about 20% (as for InSb), with the results of calculations (including the effects of non-parabolicity) for the model of a hydrogenlike donor. The observation of the impurity cyclotron resonance in $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ gives unambiguous evidence that in sufficiently strong magnetic fields electrons are bound on shallow donors in the ground state.

Of interest is the fact that impurity cyclotron resonance has been discovered in fields smaller than the field H_M at which a Mott transition is taking place. In the opinion of Shayegan *et al.*,^{42,43} "even on the metal side of the metal-dielectric transition, the delocalized electrons are in the donor-band states which are separated from the states of the true conduction band." The authors of Ref. 43 also explain the dip on the curve of $\rho_{xy}(H)$ by metallic conductivity via an impurity band (separated from the conduction band), which is treated as an infinite metallic donor cluster, existing in a crystal together with isolated donors. The increase of $\rho_{xy}(H)$ at $H > H_0^{xy}$ is caused by the "macroscopic" metal-dielectric transition, which, according to Ref. 43, is caused by the breakdown of a cluster.

The interpretation of the unquestionably interesting experimental data of Ref. 42 is obviously inconsistent. The authors surmise, undoubtedly with justification, that at $H \approx H_0^{xy} \approx H_M$ a metal-dielectric transition is taking place, i.e., a separation of impurity states from the band states. However, the explanation of the impurity cyclotron resonance for the fields $H < H_M$ based on the assumption concerning the coexistence in a crystal of an infinite cluster of donors and isolated donors seems rather artificial. The experiment can naturally be explained within the framework of the model of merging band and impurity states. In such mixed states, which are realized with high probability in compensated samples (the samples used in Refs. 42, 43 are clearly compensated: according to estimates, for a sample with $n = 3 \cdot 10^{13} \text{ cm}^{-3}$, $K \approx 0.7$ and for a sample with $n = 6 \cdot 10^{13} \text{ cm}^{-3}$, $K \approx 0.8$), the impurity electrons preserve, to some extent, their "individuality" (Ref. 51), as a result of which, together with cyclotron resonance of band electrons impurity cyclotron resonance is also observed.

ρ_{xx} , ρ_{zz} and ρ_{xy} were studied at $1.4 < T < 30$ K in samples with compensation $K = 0.6\text{--}0.9$ in papers of Refs. 44–47. It was found that for samples with $n = (2\text{--}5) \cdot 10^{14} \text{ cm}^{-3}$, $\rho_{xx}(H)$ and $\rho_{zz}(H)$ increase sharply in fields $H > H_0^{xx}$ and $H > H_0^{zz}$, and with a temperature rise, the magnitude and slopes of the curves $\rho_{xx}(H)$ and $\rho_{zz}(H)$ decrease rapidly. The resistivities ρ_{xx} and ρ_{zz} change with the field and the temperature to a much greater extent than had been predicted by the theory of scattering of electrons by impurity ions in the extreme quantum limit,⁵⁷ and are not described by one exponential law. The Hall coefficient $|R|$ decreases in fields H exceeding a certain field $H_0^R \approx 1.5\text{--}4$ kOe, but this decrease is much slower than the increase of magnetoresistivity: for fields up to 20 kOe, ρ_{xx} and ρ_{zz} increase by 1–2 orders of magnitude, while $|R|$ decreases for the same fields by not more than a factor of 2–3. Figure 8 shows the dependence on the magnetic field of the resistivity $R(H)$ for a n -

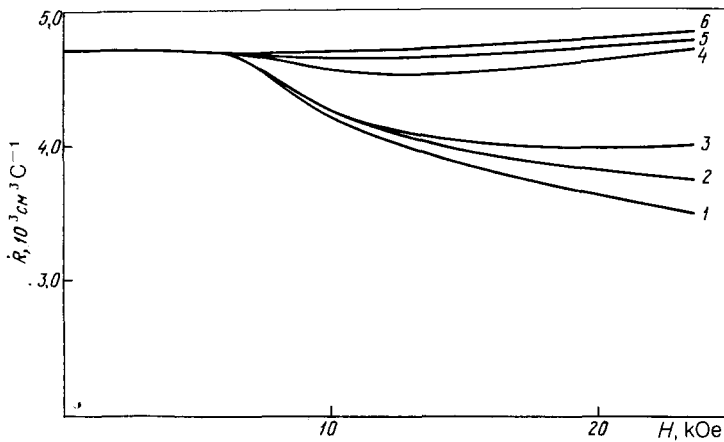


FIG. 8. R as a function of H for $n\text{-Hg}_{0.79}\text{Cd}_{0.21}\text{Te}$.⁴⁵ $n = 1.1 \cdot 10^{15} \text{ cm}^{-3}$, $\mu = 1.3 \cdot 10^5 \text{ cm}^2/\text{Vsec}$, $K \approx 0.6$, $T(K) = 1.7$ (1), 3.6 (2), 4.2 (3), 12.3 (4), 16.7 (5) and 85 (6).

$\text{Hg}_{0.79}\text{Cd}_{0.21}\text{Te}$ sample. Such a falling-off of the curves $R(H)$ is typical for a situation when the electrons participating in conductivity have significantly different mobilities $\mu_1 \gg \mu_2$.

The analysis of the $R(H)$ and $\rho_{xx}(H)$ dependences within the framework of the two-band conductivity model allowed one to establish that for fields $H > H_0^R$, the concentration of light electrons n_1 decreases, and the concentration of heavy electrons n_2 increases (the sum $n_1 + n_2$ remains constant). At $H \geq 10 \text{ kOe}$, the decrease of $n_1(H)$ slows down. Such a variation of $n(H)$ is similar in appearance to freezing-out onto donors. In order to establish whether, in fact, this effect is related to freezing-out of electrons, we use expression (22) to estimate the critical field H_M at which the impurity states become separated from the conduction band. For samples with $N_D \approx (2-6) \cdot 10^{15} \text{ cm}^{-3}$ studied in Refs. 44, 45 the values of H_M are in the range of 60–130 kOe, which is larger than the measured values of H_0^R by one or two orders of magnitude. This means that the established decrease of $R(H)$ and, correspondingly, of $n_1(H)$, cannot be explained by the freezing-out of electrons onto donors.

The $|R(H)|$ and $n_1(H)$ dependences can be explained by taking into account the fact that the samples studied in Refs. 44, 45 are strongly doped $N_D a_B^{*3} \approx 10-30$ and strongly compensated. In the extreme quantum limit the Fermi level ε_F , which at $H = 0$ is located above the flow level ε_p , descends below the flow level, and the electrons become localized in the potential well created by randomly distributed impurities.²⁸ Conductivity, which at $H = 0$ has a metallic character, at $k_B T \gtrsim \varepsilon_p - \varepsilon_F$ acquires an activation nature. If the overlap of the donor wave functions is large, then the low-mobility electrons, tunneling between wells under barriers, contribute to conductivity. Such a situation is realized in the crystals studied in Refs. 44, 45.

From the condition of equality $\varepsilon_F(H) = \varepsilon_p(H)$, one can find the threshold field H_0 required for localization of electrons. Comparison of experimental results with the results of calculations should be carried out for the least compensated samples for which $\hbar/\tau < \varepsilon_F$. This is due to the fact that the compensation degree K is determined with the help of the Brooks-Haering formula for μ , which, of course, is not valid for the case of strong scattering $\hbar/\tau > \varepsilon_F$.

For samples with $K \approx 0.7-0.8$ and $\varepsilon_F > \hbar/\tau$, the electron wavelength λ is larger than the screening radius R_0 . For that reason, one should use the following expression for the case

of quantum screening²⁸:

$$H_0 \approx \frac{3\pi^2 c \hbar}{e} \frac{a_B^* n^{\frac{1}{3}}}{N_D} = \frac{3\pi^2 c \hbar}{e} a_B^* n \frac{1-K}{1+K}. \quad (34)$$

Fig. 9 compares the results of calculations for H_0 carried out by the formula (34) with the experimentally found values of H_0^R for several samples of $n\text{-Hg}_{1-x}\text{Cd}_x\text{Te}$ with $x \approx 0.2$, $K = 0.7-0.8$ and $n = 10^{14}-10^{15} \text{ cm}^{-3}$. One can see from Fig. 9 that there is rather satisfactory agreement between theory and experiment. For comparison, the curve of $H_w(n)$ corresponding to a Wigner transition, is taken from Fig. 7 to Fig. 9. Realizing that the values of H_w calculated in various papers and, particularly, in the paper of Kleppmann and Elliott,¹⁵ give only approximate values, we still want to recall the results of Gerhardt's¹⁸ carried out for $\text{Hg}_{0.8}\text{Cd}_{0.2}\text{Te}$ (see Fig. 2). According to Ref. 18, for samples with $n = 10^{14}-10^{15} \text{ cm}^{-3}$ one can expect a Wigner condensation at temperatures $T < 1 \text{ K}$, and for fields H_w of several tens of kilooersted. This is very different from the experimental results for H_0^R shown in Fig. 9. For that reason we emphasize once again that the explanation of peculiarities of

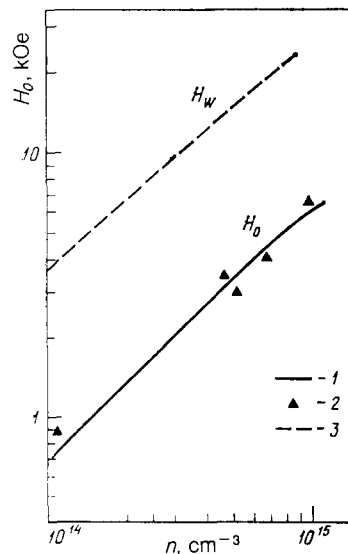


FIG. 9. The critical field H_0 as a function of electron concentration calculated by formula (34), and of the field H_0^R found from Hall effect measurements. 1— H_0 ; 2— H_0^R ; 3— H_w .

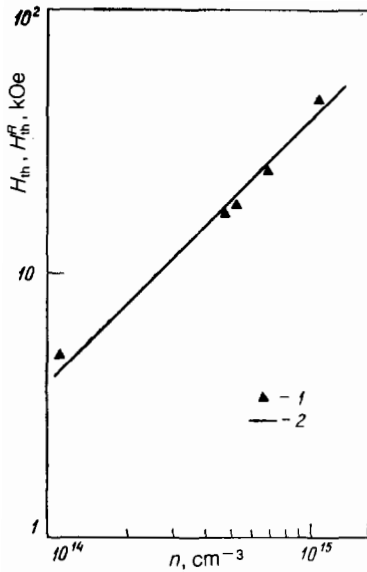


FIG. 10. Critical field H_{0h}^R (1) as a function of electron concentration and the result of calculations of H_{0h} with the help of formula (34) (2).

kinetic effects observed in the temperature range $T > 1$ K and for magnetic fields of the order of $H \sim 10^3$ Oe using the concepts of Wigner crystallization (Refs. 30–32, 36, 37) does not seem convincing.

Since the samples studied in Refs. 44, 45 have a slightly different content of cadmium (and, therefore, also the Bohr radii) and the degree of compensation, we show in Fig. 10 the dependence on n of the reduced value of the threshold field

$$H_{0r}^R = \frac{1+K}{1-K} \frac{a_B^*(x=0,2)}{a_B^*(x)} H_0^R.$$

As can be seen from Fig. 10, the dependence of $H_{0r}^R(n)$ on n is practically linear and agrees well with the results of calculations for

$$H_{0r} = \frac{1+K}{1-K} \frac{a_B^*(x=0,2)}{a_B^*(x)} H_0$$

carried out by the formula (34).

Aronzon *et al.*^{46,47} studied ρ_{xy} and ρ_{xx} in strongly doped and compensated $n\text{-Hg}_{1-x}\text{Cd}_x\text{Te}$ samples with $x < 0.2$ at $T = 14\text{--}30$ K and for fields H up to 80 kOe. They also studied the propagation of microwaves. From the measured phases and amplitudes of microwaves one could measure the dependences of σ_{xx} and σ_{xy} and the permittivity κ on the frequency of radiation ω and on H . It was established that for each semiconductor sample, there exists a threshold field H_0 near which the kinetic coefficients and the permittivity have irregular behavior: 1) at $H = H_0$ $\sigma_{xx} \approx \sigma_{xy}$, 2) σ_{xy} and σ_{xx} decrease sharply with an increase in H , and in the temperature dependence $\sigma_{xy}(T)$ at $H > H_0$ there appear exponential terms; 3) at $H \gtrsim H_0$ the permittivity $\kappa(\omega)$ undergoes a jump; 4) at $H \lesssim H_0$, $\sigma_{xy}(\omega) = \sigma_{xy}(0)$, $\sigma_{xx}(\omega) = \sigma_{xx}(0)$, and at $H > H_0$ the equalities are not satisfied. All these special features can be explained in the same way as in Refs. 44, 45, namely, that at magnetic fields close to H_0 , the transition from metallic conductivity to activation conductivity takes place as a result of localization of elec-

trons in the wells of a potential pattern, created by impurities.

We would like to call attention to a confusion (in essence a terminological one) in the question concerning what should one regard as a metal-dielectric transition induced by a magnetic field—the onset of activation conductivity at the field $H = H_0^R$ occurring as the Fermi level descends below the flow level, or the appearance of a gap between the impurity and band states at $H = H_0^{xy}$? For uncompensated crystals, $H_0^R \approx H_0^{xy}$, and there is no confusion. For compensated crystals, at $H_0^{xy} > H \gtrsim H_0^R$ ($\epsilon_F \lesssim \epsilon_p$), the conductivity σ becomes activation, although there is also a “metallic” component of σ via the impurities (at sufficiently low temperatures this can be the hopping conductivity). At $H \gtrsim H_0^{xy}$ the impurity states become separated from the band states by a gap $\Delta\epsilon$, and at low temperatures, $k_B T \ll \Delta\epsilon$, the conductivity becomes of purely impurity type (diffusion- or hopping-type). In our opinion, it is more logical (although it is not of fundamental significance) to think that the metal-dielectric transition occurs at the field H_0^R , where the activation component of conductivity appears.

Speaking of the metal-dielectric transition, it is necessary to emphasize that the condition $\sigma_{xx} = \sigma_{xy}$ used in Refs. 46, 47 as the criterion of the transition, is only an approximate one. This condition can be obtained from simple considerations. For band electrons ($\epsilon_F \gg \epsilon_p$) in strong magnetic fields, $\sigma_{xy} \gg \sigma_{xx}$; for localized electrons ($\epsilon_F \ll \epsilon_p$) $\sigma_{xy} \ll \sigma_{xx}$ and, therefore, the transition ($\epsilon_F = \epsilon_p$) has to take place somewhere at $\sigma_{xy} \sim \sigma_{xx}$. However, it is not necessary that the exact equality $\sigma_{xy} \sim \sigma_{xx}$ must correspond to the condition $\epsilon_F = \epsilon_p$ (in particular, because the band and impurity components of σ_{xy} and σ_{xx} vary with H in a different way). Numerous experimental results support this.

In conclusion we shall mention the papers of Kasuya *et al.*⁵⁸ and Burns *et al.*⁶⁴ who studied transport phenomena in SmB_6 and $n\text{-Ge}$, respectively. In Ref. 58 it was found that in the interval $0.015 \leq T \leq 0.1$ K, the conductivity of SmB_6 is constant and by 2–3 orders of magnitude below the minimal metallic conductivity σ_{\min} .¹⁹ In order to explain the comparatively small values of σ , the authors of Ref. 58 suggested a modification of the Wigner crystallization model, similar to the viscous-liquid model.²⁹ The difference from the latter model is that a group of electrons, localized by a random potential, tunnel (and do not hop over the barrier) from one configuration to another one with the same energy. Therefore the conductivity is of a metallic rather than an activation nature. Without entering into a detailed discussion of this model, we note only that the experimental fact $\sigma < \sigma_{\min}$, which forced the authors of Ref. 58 to involve the complicated scheme of tunneling groups of correlated electrons, does not require for its explanation any additional hypotheses. The point is that, as is commonly accepted now (see, for example, Ref. 59), there is no minimal metallic conductivity for a metal-dielectric transition, at least, in the absence of a magnetic field. The quantity σ can be much smaller than the values given by the expression for σ_{\min} , which Mott suggested in 1972.⁶⁰

Burns *et al.*⁶⁴ have measured ρ_{xx} , ρ_{zz} and ρ_{xy} in a $n\text{-Ge}$ sample with $n = 6 \cdot 10^{17} \text{ cm}^{-3}$ and $\mu \approx 350 \text{ cm}^2/\text{Vsec}$ at $0.5 < T < 4.2$ K for fields $H \leq 1.5 \cdot 10^5$ Oe, parallel to the $\langle 111 \rangle$ axis. A uniaxial pressure P , sufficient to change the flow of electrons from the four valleys into one valley, was applied in

the same direction. ρ_{zz} and ρ_{xx} change with H slowly in a compressed sample ($P \neq 0$) at $T = 4.2$ K, and at $T < 1$ K, ρ_{zz} increases sharply for fields $H > H_0^z \approx 1.1 \cdot 10^5$ Oe; at 0.55 K, ρ_{zz} increases by a factor of $\sim 10^4$ in the field interval $(1.1 - 1.5) \cdot 10^5$ Oe, while ρ_{xx} increases only by a factor of 10. In the absence of deformation the increase of ρ_{zz} and ρ_{xx} starts at lower fields $H_0^z \approx H_0^x \approx 4-5 \cdot 10^4$ Oe. Therefore, at $H = 1.5 \cdot 10^5$ Oe, the anisotropy ρ_{zz}/ρ_{xx} reaches $\sim 10^3$. The authors of Ref. 64 explain the large anisotropy of $\rho(H)$ by the formation of a collective electron state—the spin-density-waves (SDW). They start with the fact that because of the small g -factor of electrons in n -Ge, both spin states in fields of $\sim 10^5$ Oe are populated. When the states of a degenerate electron gas with opposite spins are occupied, then, according to Ref. 65 the SDW states are formed creating a gap in the electron spectrum. Since the Fermi level lies in this gap the diagonal components of the conductivity tensor σ_{xx} and σ_{zz} disappear as $T \rightarrow 0$, and the nondiagonal component σ_{xy} retains its classical value. Finding the inverse of the tensor σ_{ij} , Burns *et al.*⁶⁴ come to the conclusion that $\rho_{zz} \rightarrow \infty$, and $\rho_{xx} \rightarrow 0$, which is in qualitative agreement with the observed large anisotropy ρ_{zz}/ρ_{xx} . This explanation of the anisotropy is, in our opinion, incorrect. For the n -Ge sample from Ref. 64 $\sigma_{xx} > \sigma_{xy}$, i.e., $\mu H/c < 1$ right up to fields $H \approx 2.9 \cdot 10^5$ Oe, while according to Ref. 65, the inverse condition should hold: $\sigma_{xx} \ll \sigma_{xy}$.

Burns *et al.*⁶⁴ note that impurities must destroy the long-range order of the SDW states. This may lead also to the formation of domains of a size much larger than the period of SDW. The conductivity of the sample will then be determined by the hopping of electrons between the domains, similarly to what is assumed in the viscous-liquid model.

The interpretation of experiments of Ref. 64, taking into account conduction via impurities seems to be more probable. It is well known that the strong increase of resistivity $\rho(H)$ at low temperatures is caused by the transition from band to impurity-type conductivity. $\rho(H)$ increases particularly sharply-exponentially, in the region of hopping conductivity. In this case, for an ellipsoidal Fermi surface the exponent depends on the ratio of the longitudinal and transverse masses. For n -Ge, this ratio is ≈ 20 , and, therefore, $\rho_{zz}(H)$ must change much faster than $\rho_{xx}(H)$.²⁷ The results of measurements support this explanation. In the case of uniaxial compression along the $\langle 111 \rangle$ axis ε_F increases (by a factor of ~ 2.5), and higher magnetic fields are required for localization of electrons and a transition to hopping conductivity. The authors of Ref. 64 have observed exactly such a shift of the threshold field H_0^z towards higher fields. If for fields $1.1 \cdot 10^5 \leq H \leq 1.5 \cdot 10^5$ Oe the extreme quantum limit were reached ($\mu H/c \gg 1$, $\hbar\omega_c^* \gg \varepsilon_F$), the field H_0^z would be increased in the case of uniaxial compression by a factor of $\approx 2.5^2 \approx 6$ since $\varepsilon_F \sim H^{-2}$. For the samples from Ref. 64, $\mu H/c < 1$, and ε_F decreases slower (approximately as H^{-1}) than in the extreme quantum limit.

CONCLUSIONS

In the last sections of this review we have presented the main results of experimental papers in which the special features of the dependences of kinetic coefficients n -InSb and n -Hg_{0.8}Cd_{0.2}Te on the temperature, and the magnetic and

electric fields are explained by the ordering of an electron system in magnetic fields, exceeding some threshold field H_0 . The results of papers in which the authors relate the corresponding peculiarities to localization of electrons in the potential wells of the pattern created by the random distribution of impurities are also presented.

The activation character of the variation of the Hall effect and of the n -InSb conductivity in fields $H > H_0$ was the main reason for the statement²¹ that in the field $H = H_0$ a system of impurity-band electrons undergoes a transition to a dielectric state, i.e., Wigner crystallization takes place. Here, the dominant role played by the random potential created by impurities is completely ignored.

Quite numerous measurements of the Hall effect, magnetoresistivity, photoconductivity and volt-ampere characteristics in n -Hg_{0.8}Cd_{0.2}Te crystals are interpreted by the authors of Refs. 30–39 as a proof of the formation of a Wigner lattice, or of SDW, or of a viscous liquid. The analysis given in Sec. 8 shows that these conclusions do not have a solid foundation. It seems that often the desirable is claimed to be the reality.

The conclusions of Refs. 40–47 in which it is proven that the peculiarities of kinetic coefficients are caused by a magnetic-field-induced localization of electrons on impurities seem quite convincing. It is worthwhile to mention here, in particular, the papers of Refs. 42, 43 whose authors observed impurity cyclotron resonance in n -Hg_{0.8}Cd_{0.2}Te. This can serve as an unambiguous indication of the fact that in sufficiently strong magnetic fields electrons are bound on shallow donors.

Summarizing the foregoing one can conclude that none of the published experimental facts (see Refs. 30–47, 58) gives any smallest evidence of demonstrating a transition of an electron system of 3-D semiconductors into the state of a Wigner crystal (or, a CDW, or a viscous liquid).

Returning to the question asked in Sec. 4, i.e., under what conditions can one expect an experimental demonstration of the crystallization of a three-dimensional electron gas, we have to admit that we do not know the answer as yet. In uncompensated semiconductors the crystallization practically cannot be observed since the electrons are localized on donors (the Mott transition) at concentrations exceeding, by several orders of magnitude, the critical concentration for a Wigner crystallization. In compensated semiconductors, for which one could expect the realization of the conditions for the theoretical “jellium” model, a random potential of impurities prevents the formation of an electron lattice (this is supported by all known experimental data).

¹¹As calculations show, a three-dimensional electron crystal melts at the temperature $k_B T_M$, which is significantly below $\varepsilon_{pot} \neq e^2/r_0$ (see the end of this section).

²²In English—renormalized harmonic approximation.

²³The wave function (14) represents a well-known solution in the empty lattice approximation of the energy band theory.

²⁴Many researchers are critical of Overhauser's conclusion concerning the existence of CDW's in metals with a spherical Fermi surface (alkali metals), since correlation effects can suppress the CDW state. A number of experimental facts which, according to Overhauser, prove the existence of CDW in alkali metals, admit also another interpretation. At present it is generally accepted (and supported experimentally) that CDW's originate in those cases when there are parallel (congruent) regions on the Fermi surface. Overhauser still holds the opinion that there are proofs of the existence of CDW in alkali metals. This problem is still not resolved.

²⁵CDW's were experimentally found in quasi-one-dimensional chalcogen-

- ides of the transition metals TaS₂, NbSe₃, etc. (see Refs. 52, 61, 62). At some critical temperature, such a system goes into a low temperature phase, in which a spontaneous distortion of the lattice periodicity arises (a superlattice is formed). The distortion is stabilized by the modulation of the electron charge density, i.e., a CDW. The CDW is formed because the electrons of congruent regions of the Fermi surface strongly interact with the phonons with a wave vector equal to $Q = 2k_F$. A decrease in the electron energy caused by the formation of a gap (the gap arises because of the $2k_F$ periodicity of distortions) stabilizes the new phase, in which both the electron density and the position of ions are modulated (see Ref. 63). The CDW's were detected directly, for example, in electron-microscopic measurements.^{52,61}
- ⁶In other papers (for example, Ref. 14), the elementary cell, which in the situation being discussed is a six-sided prism, was replaced by a cylinder of the same volume.
- ⁷Here an ellipsoid with a large ratio of its axes has been replaced by a cylinder.
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