Intermittence and fractal dimensionality in multiple particle creation processes

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When two high-energy particles collide the most typical outcome is the creation of new particles—for the most part hadrons. The space-time evolution of such processes has always attracted much interest. Among the earlier investigations one should mention the papers by Heisenberg[1] and Wataghin,[2] but the most intensive discussion of the problem began after Fermi,[3] Pomeranchuk,[4] and Landau[5] published studies on the statistical and hydrodynamic descriptions of the evolution of such systems. In later years the space-time evolution of the multiple particle creation was studied within the framework of the multiperipheral model[1)] (originally involving single pion exchange;[6] subsequently using the parton description[7)]) and also of the quark-gluon plasma model (see, for instance, Ref. 8 and literature cited therein). The space-time evolution of the parton cascade in electron-positron annihilation and in the deeply inelastic hadron creation is also of great interest (see, for instance, Refs. 9, 10).

It should be emphasized at once that the problem incorporates two different, albeit closely related, aspects. The first is the space-time structure (topological, fractal, and so forth) that is formed during the collision of hadron matter cluster. The second is the nature of motion and interaction (with each other and with vacuum) of partons in the "boiling operator liquid (?)".

The problem of the hadron cluster structure has been discussed in terms of models employing a classical or quasiclassical description of the process, treating the cluster as a volume containing compressed, expanding hadron matter (see Refs. 1–5, 8). On the other hand, the quantum field approaches usually treat the development and evolution of the parton cascade.[6,7,9,10]

Particle physics is distinguished by the fact that the actual volumes in which interactions, defined by the size of the hadrons, take place are so small $\sim 1\ \text{fm}^3$ and the interaction time scales are so short $\sim 10^{-24}$ s, that it is possible to "observe" the process only via the final reaction products. The only methods of varying the "probes" measuring the hadron medium, i.e., of changing the space-time resolution of a given process, are changing the energies of the colliding particles or singling out certain classes of events associated with particular characteristics of secondary particles.

Generally speaking, both the properties of hadron matter and the evolution of the parton cascade may be quite irregular. Here we shall describe two recently advanced hypotheses regarding the possibility of extracting from experimental data the intermittence of the formed hadron matter cluster[11] and the fractal dimensionality of the internal random motion of partons.[12]

From the physical perspective, intermittence means the appearance of structures (generally of various scales) in the medium, such as eddies or deformations, or regions of inhomogenous densities in the liquid (which could have been initially featureless on a scale much greater than the minimal, say interatomic, length scale). From the mathematical perspective, intermittence is characterized by the presence of rare but pronounced peaks in the behavior of a random variable. The easiest method of distinguishing such behavior from its more regular counterpart is by considering the higher moments of the appropriate distributions. In an intermittent medium these will be markedly enhanced. The properties of intermittent media have been described, for instance, in Refs. 13 and 14, as well as the recent review article by Zel'dovich *et al.*[15]

In particle physics, the study of intermittence could shed new light on the nature of fluctuations in the space-time evolution of the hadron matter cluster during multiple particle creation, as well as on the internal structure of the cluster. Precisely with this goal in mind, the authors of Ref. 11 suggested investigating the way in which the factorial moment distributions (see (2) below) over the rapidity y depend on the rapidity scale—the width δy of the accessible rapidity range. Purely statistical fluctuations[2)] should yield no dependence whatever over a small range, whereas the existence of intermittence should lead to a power-law dependence of moments on the rapidity range δy. In particular, the transition region from one behavior to the other should indicate the (rapidity) scale of structural elements in the medium (see Fig. 1 below).

Let us note that the rare inelastic interaction events with strong peaks in the rapidity distribution of secondary particles have already been observed in high-energy experiments involving hadrons and nuclei.[16–23] Such events can yield information on the coherent emission of hadrons by the medium,[24–26] on the phase transition of quark-gluon plasma into hadrons,[27,28] and on minijet formation.[29,30] At the same time it should be emphasized immediately that intermit-

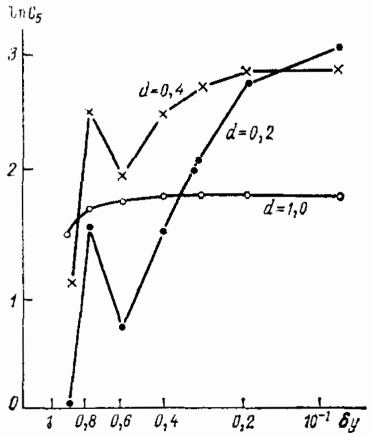

FIG. 1. Fifth order moment as a function of rapidity resolution δy for a smooth distribution (4). Evidently the behavior changes when $\delta y \sim d$.

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tence of rapidity distributions only determines the size of structures (clusters) on the rapidity scale, and not in actual space-time.

The method of calculating factorial moments using experimental data on multiple creation processes suggested in Ref. 11 is extremely simple. Let the full range of rapidities Δy accessible at a given energy be broken up into M segments of length δy, i.e., $\Delta y = M\delta y$. Consider events with multiplicity n. For each of them, calculate the quantities

$$M^{i-1}\sum_{m=1}^{M}\frac{k_m!\,(n-i)!}{(k_m-i)!\,n!} \tag{1}$$

(where k_m is the number of particles in the mth segment) for different integers $i > 1$.[3)]

Averaging over all observed events we obtain the ith factorial distribution moment:

$$F_i = M^{i-1}\left\langle \sum_{m=1}^{M}\frac{k_m!\,(n-i)!}{(k_m-i)!\,n!}\right\rangle . \tag{2}$$

A remarkable property of the factorial moments F_i is that for even arbitrarily high multiplicities they coincide with ordinary rapidity distribution moments C_i calculated in the usual manner. Let us dwell on this for a moment. The probability of finding k_m particles in the mth segment p_m makes real sense only when averaged over an infinite number of realizations. Only then can one formulate the probability distribution $P(p_1,\dots p_M)$ and define its moments as

$$C_i = M^{i-1}\sum_{m=1}^{M}\int\prod_j \mathrm{d}p_j P(p_1,\ \dots,\ p_M)\,p_m^i . \tag{3}$$

It turns out that the quantities F_i calculated from formula (2) at finite multiplicities n coincide with the C_i moments (see Ref. 11): that is, factorial moments to a great extent suppress the effect of statistical errors produced by the finite number of observed events. Later we shall demonstrate this assertion within the framework of a concrete model. For now let us consider the properties of these moments.

First, it is obvious that the higher the moment number i, the more pronounced is the strongest peak in the distribution.

Second, it is fairly simple to prove generally for regular distributions (see Ref. 11) that when the rapidity range is divided into sufficiently small segments ($\delta y \to 0$) the moments C_i are independent of δy.

Third, the moments C_i calculated for regular rapidity distributions begin to depend on δy only when the magnitude of δy begins to approach the characteristic scale over which the distribution changes significantly. If δy is further increased the behavior of the moments C_i becomes unstable and irregular. This is illustrated in Fig. 1 where we plot the fifth order moment C_5 as a function of δy for a model of a regular rapidity distribution consisting of a plateau featuring a Gaussian cluster

$$h(y) = 1 + 5\exp\left(-\frac{y^2}{d^2}\right). \tag{4}$$

By plotting C_5 for different cluster widths $d = 0.2$, 0.4, and 1.0, Fig. 1 demonstrates that the moment is independent of δy when $\delta y \ll d$, begins to decrease when $\delta y \gg d$, and then behaves irregularly as δy increases further. Interestingly, this behavior of distribution moments is preserved when the individual events obey distribution (4) but the cluster center is randomly scattered over the entire rapidity range. In this last case the total distribution consists of a level plateau and exhibits no structure, but the distribution moments reveal the underlying cluster structure of individual events.

Fourth, when intermittence is present, the moments C_i exhibit a power-law dependence on δy. The power is determined by the way in which the particles are distributed after successive division into ever smaller rapidity ranges, i.e., by the scale of the structural elements in the studied medium. In order to characterize the successively diminishing scale it is useful to define a method of dividing the rapidity range Δy into ever smaller segments. To this end, let us divide the range into some number λ of segments, then divide each of the segments again into λ segments, and repeat this procedure ν times to eventually obtain M segments of length δy. Clearly

$$\lambda^\nu = M = \frac{\Delta y}{\delta y} . \tag{5}$$

In the above prescription, the numbering m of each concrete segment among the total number of segments M $\{m = 1, 2,\dots, M\}$ can be replaced by an equivalent numbering ν of indices α_j $\{\alpha_1,\dots,\alpha_\nu\}$ which label the successive divisions, and the probability of turning up in segment m can be replaced by product of probabilities of turning up in the appropriate segment α_j at each successive division:

$$p_m = \prod_j^{\nu} w_{\{\alpha_j\}} = \frac{1}{M}\prod_j^{\nu} W_{\{\alpha_j\}}, \tag{6}$$

where $W\{\alpha_j\} = \lambda w\{\alpha_j\}$ and $\overline{W} = 1$. If the normalized quantities $W\{\alpha_j\}$ are random and independent, the distribution (6) describing their product will be intermittent,[11,15] and hence C_i will depend on δy according to the power law

$$C_i = M^{\varphi_i} \tag{7}$$

where the power

$$\varphi_i = \frac{\mathrm{d}\ln C_i}{\mathrm{d}\ln \delta y}, \tag{8}$$

depends on i and $W\{\alpha_j\}$. This is easy to demonstrate for a concrete model. Let each division of the expanding medium (string, plasma, etc.) produce with a low probability β a higher density fluctuation on the rapidity scale ($W\{\alpha_j\} = 1 + a$), compensated by the higher probability $1-\beta$ of producing a less dense fluctuation ($W\{\alpha_j\} = 1 - b$).

Evidently,

$$\overline{W} = \beta(1 + a) + (1 - \beta)(1 - b) = 1, \tag{9}$$

whence it follows that $b \ll a$ given $\beta \ll 1$.

The distribution moment takes the form

$$C_i = [\beta(1+a)^i + (1-\beta)(1-b)^i]^\nu \equiv M^{\varphi_i}, \tag{10}$$

$$\varphi_i = \ln\left\{\frac{ab}{a+b}\left[\frac{(1-b)^i}{b} + \frac{(1+a)^i}{a}\right]\right\}\frac{1}{\ln\lambda}. \tag{11}$$

This model may be used to compute the fifth order moments taking into account the statistical fluctuations $\widetilde{C}_5$, suppressed fluctuations C_5, and factorial moments F_5 as functions of δy[4)]. We simulated 1000 events with five successive divisions ($\nu = 5$) and parameters $\lambda = 2$, $\beta = 0.1$, $a = 0.27$, $b = 0.03$ in equations (9)–(11). The results of the

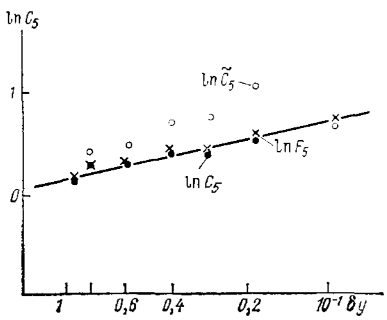

FIG. 2. The role of statistical fluctuations in the moment analysis method is apparent in the difference between moments incorporating such fluctuations (ln$\tilde C_5$—circles) and moments with fluctuations suppressed (lnC_5—points). Factorial moments (lnF_5—crosses) practically coincide with ordinary, fluctuation-free moments and are well described by the analytic prediction of a straight line from (9)–(11).

calculations are plotted in Fig. 2. It is evident that statistical fluctuations markedly change the moment—the difference between $\tilde C_5$ (circles) and $\tilde C_5$ (points) is large. On the other hand, in the calculation of factorial moments F_5 (crosses) are statistical fluctuations suppressed and indeed $F_5 \approx C_5$, as anticipated (see discussion of formula (2)). The numerical values of F_5 and C_5 fall very nearly onto a straight line predicted by theoretical computation (formula (11) yields the value of the slope $\varphi_5 = 0.14$). Consequently, corrections due to the finite multiplicity n incorporated in the calculation of factorial moments (2) are indeed important if the results are to make physical sense.

In order to apply this method to analyzing experimental data one must observe a sufficiently large number of events with fairly large multiplicities (since all formulae for moments are averaged over distributions). Such an analysis has not been attempted to date. Instead, Bialas and Peschanski[11] attempted to employ the proposed method to a single event with an exceptionally large number of secondary hadrons—the Si–AgBr interaction studied by the JACEE collaboration.[22] They concluded[5)] that such an event cannot be described in terms of statistical fluctuations only. This is illustrated in Fig. 3.

Points and circles in Fig. 3 refer to the values of the moment F_5 calculated for slightly shifted rapidity ranges; crosses label factorial moments when experimental data is made to fit a smooth, intermittence-free distribution.[31] The discrepancy between the factorial moments calculated from experimental data (points, circles) and those calculated for a smooth distribution (crosses), as well as the agreement of the former with the intermittence model indicate that there

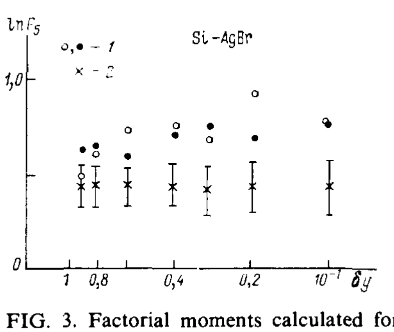

FIG. 3. Factorial moments calculated for the Si–AgBr experimental event[22] are fairly well described by the intermittence model (9)–(11) and obviously differ from smooth distribution moments.[31]

exist fluctuations on different scales that are not purely statistical. A word of caution: even with a multiplicity $n = 1000$ the error in a fifth-order moment can reach 15–20% (see Ref. 11). As usual, the error is inversely proportional to the square root of the number of events and decreases with increasing multiplicity.

In conclusion, we can state that the proposed method[11] has already suggested the existence of intermittence in multiple creation processes. Its application to high-energy data promises many interesting new results. In particular, it is hoped that by separating out the purely statistical fluctuations this method will allow us to distinguish intermittence[15] from fixed-scale dynamical peaks[24–26] associated with the finite-wavelength gluon emission by the medium caused by the passage of the incident particle.

Let us now turn to the parton evolution in multiple creation processes. Obviously it will be determined by the nature of these processes and particularly by the structure of the formed cluster. Recall that in the simple multiperipheral cascade model the partons execute Brownian motion[6,7] between the points at which particles are created. The mean square impact parameter $\bar\rho^2$ (distance in the transverse plane), which determines the inclination b of the diffraction cone in the differential elastic scattering cross section, is proportional to the number of steps, i.e., the average multiplicity n. Each break in the trajectory corresponds to the creation of a particle:

$$b \sim \overline{\rho^2} \sim \bar n, \tag{12}$$

—in the multiperipheral model both $\bar n$ and b increase logarithmically with energy. In fact they increase differently, with $\bar n$ growing rather more quickly. The simple linear connection between the cone inclination and average multiplicity can be violated even in the multiperipheral model by taking into account rescattering and parton transformations. The parton diffusion then becomes more complicated, and the parton diagrams begin to resemble branching electrical networks or fractals.[32–34]

Here an obvious question arises: what is the internal dimensionality of the parton random walks in the system? This dimensionality is denoted by $D'_{\rm W}$ and is defined as[34]

$$b \sim \overline{\rho^2} \sim \bar n^{2/D'_{\rm W}}, \tag{13}$$

with

$$D'_{\rm W} = 2 + \theta, \tag{14}$$

where θ is the anomalous diffusion coefficient indicating that the diffusion coefficient depends on distance according to

$$\varkappa(\rho) = \varkappa \rho^{-\theta}. \tag{15}$$

From (13) we obtain

$$D'_{\rm W} = 2\,\frac{d \ln \bar n}{d \ln b}. \tag{16}$$

Thence it follows that experimental data on the average multiplicity and inclination of the diffraction cone can yield the internal dimensionality of the parton random walks. The corresponding results for proton-proton interactions are plotted in Fig. 4. In a first approximation they are described by a straight line whose slope yields

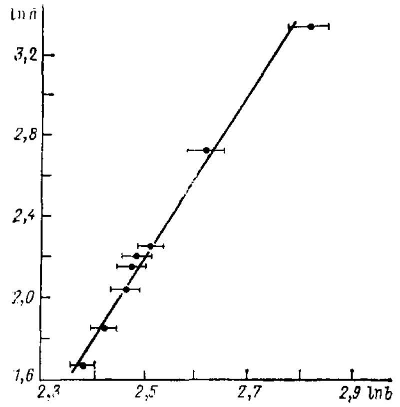

FIG. 4. Experimental data on the average multiplicity n and diffraction cone inclination b for pp- and p$\bar{\text{p}}$-interactions (points) determine the dimensionality of the internal random walks of the partons. From the slope of the drawn line we find $D'_{\rm W} \approx 7.5$, i.e., either the parton random walks are tangled with repeated returns to the origin or the number of partons is large.

$$(D'_{\rm W})_{\rm p} \approx 7.5 \pm 1.5. \qquad (17)$$

Such a high (recall that for Brownian motion $\theta = 0$ and $D'_{\rm W} = 2$) internal dimensionality of parton random walks implies that a parton repeatedly diffuses back to the origin many times, i.e., its trajectory is quite tangled.

Such diffusion may take place in fractals.[34,35] Solving the appropriate diffusion equations yields the probability of finding a parton with impact parameter ρ:

$$P(\rho) = \frac{(\varkappa D'^{2}_{\rm W}\bar{n})^{-D/D'_{\rm W}}}{\Gamma((D/D'_{\rm W})+1)} \exp\left(-\frac{\rho^{D'_{\rm W}}}{\varkappa D'^{2}_{\rm W}\bar{n}}\right), \qquad (18)$$

where D is the Hausdorff dimensionality of the fractal cluster which determines its spatial structure. It is not clear whether the nature of parton random walks is directly connected with the internal spatial structure of the hadron matter cluster. Experimental data on πp-interactions at lower energies yield $(D'_{\rm W})_\pi \approx 4$.

Another interpretation is possible for these results. It may be that what increases is the number of partons, rather than the complexity of their trajectories. The cone inclination reflects the average distance to the parton. Then the number of Brownian motion steps executed by a parton must equal $\sim \bar{n}^{2/D'_{\rm W}}$, i.e., the number of partons is large $\sim \bar{n}^{1-2/D'_{\rm W}}$.

Regardless of the interpretation, experimental data confirm the utility of statistical-hydrodynamic models of inelastic hadron collisions and indicate that equilibrium in the transverse direction may be established in such systems.

It is of interest to evaluate the internal dimensionality of parton random walks in multiple processes caused by different colliding particles (for instance, electron-positron annihilation). Unfortunately, no such convenient quantity as the inclination of the diffraction cone is available in such events. The clearest picture of the parton cascade evolution in electron-positron annihilation is provided by the Altarelli–Parisi equations,[36] which we can complement by introducing the dynamical cutoff of the parton cascade evolution.[37,38] Here the parameter analogous to the square of the distance is inverse 4-momentum squared $1/k^2$ of the parton. It is then reasonable, by analogy with expression (16), to define the internal dimensionality of parton random walks as

$$D'_{\rm W} = -2\,\frac{d\ln\bar{n}}{d\ln k^2}. \qquad (19)$$

Without going into the computational details (see Ref. 12) let us formulate the final conclusion. Using expressions for average parton cascade multiplicity and experimental data on its growth with energy (see, for instance, Ref. 39), it is a simple matter to show that the internal dimensionality of parton random walks (19) is always close to unity. This indicates that during the parton cascade evolution in electron-positron annihilation the partons deviate only slightly from rectilinear motion. In this the parton motion in electron-positron annihilation differs markedly from the tangled paths executed by partons in hadron processes.

Further development of the above-described approaches could help us understand how the complex nature of parton random walks is related to the structure of the hadron medium not only in terms of rapidities but spatially, to the interactions of the partons with each other and with vacuum (in particular, with the walls of the "bag"), to the intermittence of rapidity distributions, and so on. One hopes that future studies will shed light on these and many other questions.

GLOSSARY OF TERMS AND DEFINITIONS

Hadrons—strongly interacting particles consisting of quarks and gluons.

Rapidity $y = (1/2)\ln[(E+p_L)/(E-p_L)]$—a kinematic variable characterizing the motion of a secondary particle along the collision axis of the primary particles (E and p_L are the energy and the longitudinal component of the particle momentum).

Deeply inelastic hadron creation—the process by which hadrons are created (due to a large momentum transfer) when a high-energy lepton (electron, muon, neutrino) collides with a hadron.

Diffraction cone—the sharply peaked forward distribution of elastically scattered particles which results in the elastic scattering cross section resembling a narrow cone centered about the collision axis.

Inclusive distribution—the distribution of particles over some variable (for instance rapidity) which is obtained when a single (random) particle of a given type is recorded in a given event.

"Bag"—a model in which a hadron is pictured as a bag enveloping its constituent quarks and gluons.

Multiperipheral cascade—the Feynman diagram of the multiperipheral model in which particles (or partons) are sequentially emitted by the exchanged particle (parton). The motion of the exchanged parton in the plane perpendicular to the collision axis is analogous to ordinary Brownian motion.

Multiperipheral model—a model in which hadrons interact by peripherally exchanging a single meson (or parton).

Parton—a constituent of a hadron (quark, gluon).

Parton cascade—the evolution (multiplication) process of high-energy partons.

Si–AgBr event—the interaction of a high-energy Si nucleus with a photoemulsion observed by a Japanese-American collaboration studying cosmic rays.

Electron-positron annihilation—the interaction between an electron and a positron in which both are annihilated (at high energies usually leading to the creation of many hadrons).

[1)]Since this article may be of interest to specialists in other fields of physics who are unfamiliar with particle physics terminology, a glossary of terms and definitions is appended.

[2)]The term "statistical fluctuations" refers to fluctuations in distributions that are due to the finite statistics (number of events) in a given experiment.

[3)]Segments with $k_m < i$ do not contribute to the sum since $k_m!/(k_m - i)! = k(k_m - 1)\ldots(k_m - i + 1)$ equals zero for $k_m < i$.

[4)]It appears that the difference between moments C_5 and C_5 (F_5) is comparable to the difference between spatial and statistical moments in spatial intermittence (see Ref. 40); the study of their dependence on the magnitude of the rapidity range also has an analog in the spatial intermittence picture.[13,14,41]

[5)]Takagi reached the same conclusion using a different approach.[31]

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