Physics of ϒ resonances: ten years later

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This paper reviews the present status of the physics of bottomonium: a bound system consisting of a heavy b quark and the antiquark $\bar{b}$. The basic experimental data on the levels of bottomonium are presented. Theoretical methods for describing the properties of these levels are discussed. Questions pertaining to the spectroscopy of bottomonium, including the fine and hyperfine splittings, radiative transitions between levels, and annihilation decays of the $b\bar{b}$ system, are discussed. Effects which are not describable by quantum-chromodynamics perturbation theory are taken into account. Transitions between bottomonium levels involving the emission of light muons are discussed. The possibilities of a search for hypothetical new particles and effects in the decays of ϒ resonances are also discussed.

CONTENTS

1. INTRODUCTION

In the summer of 1987 ten years will have elasped since Lederman's group published the first reports[1] of the observation of ϒ and ϒ′ resonances, with masses of about 9.5 and 10.1 GeV, in the mass spectrum of $\mu^+\mu^-$ pairs produced in the reaction $p + \text{nucleus} \to \mu^+\mu^- + X$. These resonances are levels of a bound system consisting of a b quark and its antiquark. The b quark is the heaviest quark known from experiments so far. It carries a new quantum number, usually called "beauty." [1)] In the ϒ resonances this is a hidden number, however, since there is a cancellation between b and $\bar{b}$. The lightest of the particles which contain a bare "beauty"—the B mesons—have the quark compositions $b\bar{u}$ (B^-) and $b\bar{d}$ (B^0) and respective masses 5.271 and 5.275 GeV (Ref. 2). In contrast with the levels of the ($b\bar{b}$) system, the beauty hadrons are produced only in pairs (e.g., $B\bar{B}$) in strong and electromagnetic interactions.

Beginning in 1978, and continuing to the present, research has been carried out on bottomonium at electron-positron storage rings—first at the DORIS installation in Hamburg (West Germany)[3] and later at some new storage rings: CESR at Cornell University in Ithaca (USA),[4,5] the modernized DORIS II,[6,7] and the VÉPP-4 in Novosibirsk (USSR).[8] Over this time, seven more levels of the $b\bar{b}$ system have been discovered below the threshold for the decay into the meson pair $B\bar{B}$, and at least three resonances have been discovered above this threshold.

A difference between the levels which lie above and below the $B\bar{B}$ threshold is that the former decay into a $B\bar{B}$ pair and thus have fairly large widths, from 20 to 110 MeV, while for the latter this decay is not possible, and their widths are smaller by a factor of about 1000 and are determined primarily by the annihilation of a quark pair $b\bar{b}$ into gluons (§4).

There is considerable interest in research on the bb system, since among the quark systems which have been observed experimentally it hold the records in terms of both the number of known (and expected) levels and the nonrelativistic nature of the motion of the quarks in bottomonium (these two properties are interrelated). The average quark velocity v in bottomonium is of such a magnitude that the parameter v^2/c^2, which describes relativistic effects, is of the order of 0.06. The complications caused by relativistic effects in an analysis of the dynamics of quarks are thus suppressed to a significant extent, and this system is our only "laboratory" for studying the strong interaction between a quark and an antiquark in essentially its pure form. (By way of comparison we note that the value of this parameter in charmonium is $v^2/c^2 \approx 0.2 - 0.25$, and it is of the order of unity in hadrons containing light quarks.) The annihilation of b and $\bar{b}$ quarks in narrow resonances occurs at distances of the order of the Compton wavelength of the b quark: $1/m_b \approx 0.4 \cdot 10^{-14}$ cm (we are using a system of units with $\hbar = c = 1$). These distances are in the region of asymptotic freedom of quantum chromodynamics (QCD), so for several quantities characterizing annihilation, e.g., for the relative probability for the decay of Υ into a hard γ ray and hadrons, it is possible to generate very definite predictions in terms of the coupling constant of quantum chromodynamics, α_s. Measurement of these quantities is thus one of the best ways to determine the value of α_s experimentally. Finally, the products of the annihilation of Υ resonances are all possible hadronic states. A study of the products of the decay of bottomonium levels is accordingly of considerable interest for other problems in hadron physics: searching for new resonances, including glueballs and various exotic states, and in general for studying the dynamics of hadrons.

Broad Υ resonances decay into $B\bar{B}$ pairs, as we have already mentioned, and accordingly constitute a B-meson "factory." The resonance Υ (10 575), which lies just above the $B\bar{B}$ threshold, is used particularly frequently for this purpose. The B mesons decay by a weak interaction, so that research on them can help us learn about the structure of the weak interaction of quarks.

Our purpose in this review, nearly ten years after the discovery of the first Υ resonances, is to draw a picture of the present status of experimental data on the levels of bottomonium and of the theoretical understanding of the internal dynamics of this system and of heavy quarkonium.

2. ACCELERATORS AND DETECTORS FOR STUDYING BOTTOMONIUM

Electron-positron storage rings turned out to be the most effective tools for studying the properties of this new family of particles. The advantages of colliding e^+e^- beams over extracted hadron beams are obvious. The $^3S_1$ resonances (Subsection 3.1), which have the quantum numbers of the photon, are produced quite well directly in the annihilation of the electron and the positron, and the width of the "narrow" resonances is determined exclusively by the energy spread of the beams in the storage rings. This energy spread is much better than the resolution in terms of the invariant mass of the leptons or hadrons in experiments using extracted beams with an energy of several tens of gigaelectron volts. Furthermore, in storage rings the center-of-mass frame of reference for the reaction is the same as the laboratory frame of the detector, and the energies and momenta of the secondary particles produced in the decays of various states of bottomonium are not high and can be measured quite accurately. A very important advantage of storage rings is the much lower background level, which allows a study of transitions between levels and the decays of levels under very "clean" conditions.

At present, three electron-positron storage rings are operating in the Υ energy range: DORIS II (West Germany), CESR (USA), and VÉPP-4 (USSR). The basic characteristics of these rings are listed in Table I.

The most important characteristic is the luminosity of the storage ring, which tells us the number of colliding particles per unit time and which has dimensions of $cm^{-2} \cdot s^{-1}$. However, the peak luminosity, given in Table I, does not give a comprehensive picture of the actual situation. The technical state and operating reliability of a storage ring may substantially reduce the integral luminosity, which, along with the acceptance of the detector, ultimately determines which level of the cross sections of the reactions of interest can be reached experimentally.

The experimental installations presently in operation at these storage rings can be classified in two groups. The first group consists of specialized detectors intended for studying processes involving γ rays in the final state (CUSB[5] at CESR and Crystal Ball[7] at DORIS II). There are compact installations, without a magnetic field, which use NaI(Ta) or BGO crystals. They contain rather simple systems of proportional chambers which can measure the directions of charged particles. In problems which do not require very precise measurements of the energies of the γ rays at a high γ detection efficiency, these installations are of course surpassed considerably by the general-purpose magnetic detectors (CLEO[4] at CESR, ARGUS[6] at DORIS II, and MD-1[8] at VÉPP-4).

The nature of the installations in use is illustrated in

TABLE I. Basic characteristics of the storage rings which have been optimized for studying the $b\bar{b}$ system.

Characteristic	CESR (Ithaca, USA)	DORIS II (Hamburg, FRG)	VÉPP-4 (Novosibirsk, USSR)
Circumference, m	770	300	380
Maximum beam energy, GeV	2×8	2×5,6	2×7
Energy spread, MeV (at E = 9.46 GeV)	3,8	8,2	4,4
Luminosity, $cm^{-2} \cdot s^{-1}$	$2 \cdot 10^{31}$	$3 \cdot 10^{31}$	$3 \cdot 10^{30}$
Experimental installations:			
Magnetic	CLEO	ARGUS	MD-1
Nonmagnetic	CUSB	Crystal Ball	OLYa

Fig. 1, which shows the layout of the ARGUS detector. This detector was constructed by an international collaboration including scientific centers from West Germany, the USA, Canada, Sweden, and the USSR (Institute of Theoretical and Experimental Physics, Moscow). The point at which the beams intersect, in a vacuum chamber at the center of the installation, is surrounded by a vertex drift chamber consisting of 600 cells, oriented parallel to the beam axis, in which the coordinates of charged-particle tracks can be measured within an error of about 50 μm.

The basic part of this installation—the cylindrical drift chamber[9]—is 2 m long and 0.9 m in radius and consists of 5940 cells, distributed in 36 layers. Half of the layers have wires running parallel to the axis of the beams, while the others are rotated through small angles for measuring the longitudinal coordinate. Each signal wire provides information on the coordinate of a track and on the ionization loss (dE/dx) in the cell. The error in the measurement of the momenta of the particles is about 1% at 1 GeV/c. The chamber is surrounded by 112 scintillation counters, in which the particle transit times are measured, and 1760 shower counters consisting of lead + scintillator sandwiches. The resolution in terms of the energy of the γ ray is about 10% at $E_\gamma = 1$ GeV. All these chambers and counters are in a solenoidal magnetic field of 0.8 T, directed parallel to the axis of the beams.

A more detailed description of the installations can be found in the original papers cited above.

Detectors of a new generation are already being developed: CLEO II at Cornell and SKIF at Novosibirsk.[10] These detectors combine a good momentum resolution with a very good resolution for γ rays, with the result that the list of problems which can be studied is lengthened considerably.

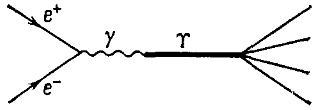

FIG. 2. Production and decay of a ϒ resonance in e^+e^- annihilation.

3. LEVEL SPECTRUM OF BOTTOMONIUM

3.1. Classification of levels

The levels of bottomonium are classified by analogy with positronium, as are the levels of charmonium (Refs. 11 and 12, for example). Each level is characterized by the resultant spin of the quark and the antiquark, S ($\mathbf{S} = \mathbf{s}_b + \mathbf{s}_{\bar{b}}$); the orbital angular momentum of the quarks, L; the total angular momentum (the spin of the resonance), $J(\mathbf{J} = \mathbf{L} + \mathbf{S})$; and the radial excitation number n_r (the number of zeros of the radial wave function). These quantities can be written in the compact form $(n_r + 1)^{2S+1}L_J$. For the values of L, the numerals are replaced by the letters used in atomic physics: S, P, D, F, ... The resultant spin S can take on only the two values 0 and 1. (The value of $2S + 1$ would then be 1 or 3.) States with $S = 0$ are "para-states," and those with $S = 1$ are "ortho-states." The values of L and S determine the spatial parity P and the charge parity C of the state: $P = (-1)^{L+1}$, $C = (-1)^{L+S}$. It is easy to see that the $^3S_1$ states have the quantum numbers of a photon, $J^{PC} = 1^{--}$, and are thus observed as resonances in the cross section for e^+e^- annihilation: the ϒ resonances, which are produced directly at electron-positron storage rings (Fig. 2). So far, six ϒ resonances, from ϒ(1S) to ϒ(6S), have been observed experimentally.[2] The masses of the first three levels lying below the threshold for the production of

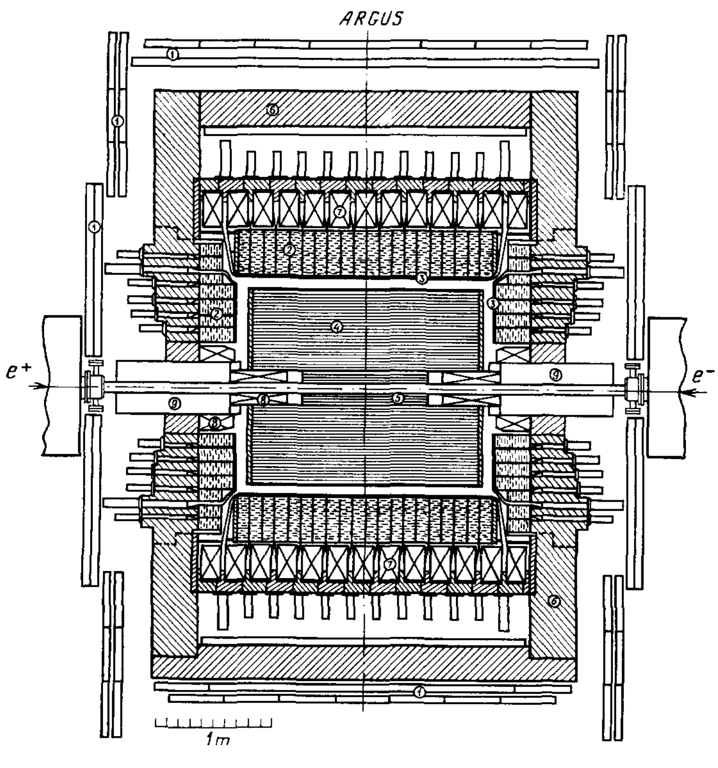

FIG. 1. Diagram of the ARGUS detector: section along the axis of the beams. 1—Muon chambers; 2—shower counters; 3—scintillation time-of-flight counters; 4—basic drift chamber; 5—vertex drift chamber; 6—magnetic yoke; 7—magnet windings; 8—compensating coil; 9—mini-beta-quadrupole lens.

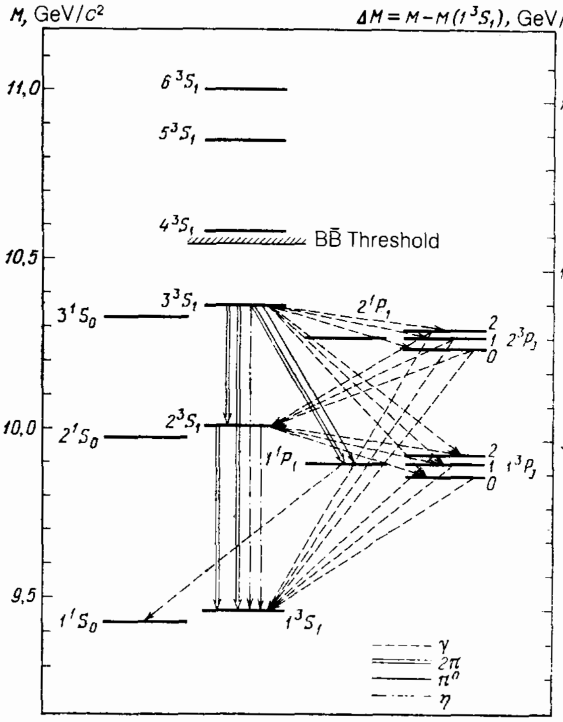

FIG. 3. Level scheme of bottomonium.

the meson pair $B\bar{B}$ have been measured with a very high accuracy,[8,13] much better than the energy resolution of the storage rings, thanks to a method of resonant depolarization of beams which was developed in Novosibirsk.[14] The other levels of bottomonium can be observed experimentally only as a result of transitions from Υ resonances. At present we know of two triplets of ^3P_J levels ($J = 0,1,2$), also called "χ_{bJ} resonances": the 1^3P_J levels, observed in transitions accompanied by the emission of a γ ray from $\Upsilon(2S)$ and $\Upsilon(3S)$, and the 2^3P_J levels, which are produced in γ transitions from $\Upsilon(3S)$ (Fig. 3). So far, levels of para-bottomonium have not been observed. Figure 3 shows the system of known and expected levels of bottomonium, along with the observed (and some of the expected) radiative and hadronic transitions between levels.

3.2. Potential model of quarkonium

Because of the large number of known levels in the $b\bar{b}$ system, a calculation of their positions is an important proving ground for models of quark dynamics. The most popular approach is that in which the interaction between the b and the $\bar{b}$ is described by a potential $V(r)$, and the levels are found by solving a nonrelativistic Schrödinger equation. In efforts to improve the accuracy, several studies[3)] have incorporated as perturbations the leading relativistic corrections and also the coupling with the $B\bar{B}$ channel. This coupling is particularly important in an examination of levels lying above the $B\bar{B}$ threshold.

The potential $V(r)$ is chosen on the basis of the following considerations. At small distances, where perturbation theory is applicable in QCD, the interaction of quarks is determined by one-gluon exchange (Fig. 4a). When the renormalization of the constant α_s due to radiation correc-

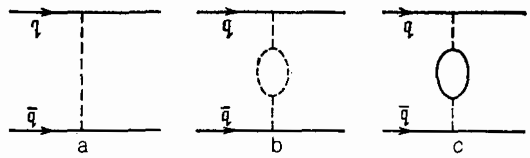

FIG. 4. The leading Feynman diagrams in a perturbation theory for the potential for one-gluon exchange. Dashed line—gluon; solid line—quark.

tions is taken into account (Fig. 4, b and c), this exchange leads to the potential

$$V_{\text{p.t}}(r) = -\frac{4}{3}\frac{\alpha_s(1/r)}{r}, \quad (1)$$

where $\alpha_s(1/r)$ falls off logarithmically as $r \to 0$ in accordance with the known expression

$$\alpha_s = \frac{2\pi}{b\ln(1/r\Lambda)} \quad (2)$$

(b is the first coefficient in the β function in QCD; in the distance interval of importance for the $b\bar{b}$ system we have $b = 9$; see the discussion in Ref. 16, for example). At large distances, use is made of the results of a QCD analysis on a space-time lattice (see, for example, the review in Ref. 17). That approach leads to a function which is linear in r for the energy $E(r)$ of a quark and an antiquark which are at rest and separated by a distance r. Since we have $E(r) = V(r)$ in the potential model, the following behavior is assumed for the potential at large distances:

$$V_{\text{n.p.t}}(r) = \text{const}\cdot r. \quad (3)$$

Clearly, this potential provides quark confinement. The primary distinctions among the various potential models in the literature lie in the procedure for joining potentials (1) and (3) [and in the procedure for dealing with the various perturbation-theory corrections to $V_{\text{p.t}}(r)$].

Incontestable advantages of the potential approach to the description of heavy quarkonium are its simplicity and transparency. This approach makes it possible to calculate (on the basis of a particular potential model which is adopted) the positions of levels, the widths of radiative transitions between levels, and the widths of annihilation decays. Some of the models reproduce the experimental data extremely closely (Table II).

3.3. QCD vacuum and the dynamics of quarkonium

A serious difficulty of the potential description is that it is actually not possible to construct a solid foundation for this description within the framework of QCD.[18] This circumstance can be seen (if only faintly) even in perturbation theory. Specifically, as was pointed out in Ref. 19 the contribution of diagrams like that in Fig. 5 to the scattering amplitude of a quark and an antiquark cannot be described by a two-particle interaction potential in a colorless state. This result is easy to explain: In these diagrams a real transverse gluon propagates along with the $b\bar{b}$ pair in the intermediate state. Therefore when such diagrams are taken into account, this contribution cannot be replaced by an effective potential in the two-particle sector, since the propagation of the gluon between emission and absorption introduces a temporal retardation in the interaction of the quarks, while the potential corresponds to an instantaneous interaction. The effect is

TABLE II. Spectrum of bottomonium levels below the $B\bar{B}$ threshold according to the potential model* (Ref. 19). [The (rounded) experimental values of the mass are given in parentheses for the states which have already been observed experimentally].

State	Mass, MeV	State	Mass, MeV	State	Mass, MeV	State	Mass, MeV
$1\,^3S_1\,(\Upsilon)$	9462 (9460)	$1\,^3P_1\,(\chi_{b1})$	9893(9892)	$1\,^3D_3$	10 167	$2\,^3D_1$	10 447
$1\,^1S_0\,(\eta_b)$	9427	$1\,^3P_0\,(\chi_{b0})$	9868(9860)	$1\,^3D_2$	10 162	$2\,^1D_2$	10 455
$2\,^3S_1\,(\Upsilon')$	10 013(10 023)	$1\,^1P_1$	9900	$1\,^3D_1$	10 155	$1\,^3F_4$	10 365
$2\,^1S_0\,(\eta_b')$	9994	$2\,^3P_2\,(\chi_{b2}')$	10 266(10 271)	$1\,^1D_2$	10 163	$1\,^3F_3$	10 364
$3\,^3S_1\,(\Upsilon'')$	10 355(10 355)	$2\,^3P_2\,(\chi_{b1}')$	10 252(10 255)	$2\,^3D_3$	10 459	$1\,^3F_2$	10 361
$3\,^1S_0\,(\eta_b'')$	10 339	$2\,^3P_0\,(\chi_{b0}')$	10 232(10 233)	$2\,^3D_2$	10 454	$1\,^1F_3$	10 364
$1\,^3P_2\,(\chi_{b2})$	9910(9913)	$2\,^1P_1$	10 258				

*The reason why the splittings δ_2 and δ_3 differ from those given in Subsection 3.6 is that the model of Ref. 29 predicts e^+e^- widths which are slightly on the high side: $\Gamma_{ee}(\Upsilon') = 0.62$ keV, $\Gamma_{ee}(\Upsilon'') = 0.42$ keV.

analogous to an effect in quantum electrodynamics (QED), which leads to the Lamb shift (Ref. 20, for example) of atomic levels by virtue of the emission and absorption of transverse photons by an electron. It is also not possible to describe the Lamb shift by any effective increment in the interaction potential in the atom.

Alternatively, this nonpotential effect could be regarded as the result of an interaction of a bound system with vacuum fluctuations of gauge fields (photons in QED or gluons in QCD). In perturbation theory this effect is small: The shift of the levels of quarkonium which it causes is proportional to α_s^5. The reason why this shift is small is that quarkonium effectively interacts only with fluctuations whose wavelengths are greater than or of the order of the size of the quarkonium. If we characterize the resultant intensity of such fluctuations by a mean square field strength tensor $\langle F_{\mu\nu}^2 \rangle$, the contribution of wavelengths longer than $\lambda_{\min}$ in perturbation theory is

$$\langle F_{\mu\nu}^2 \rangle_{\lambda > \lambda_{\min}} \propto \int_0^{1/\lambda_{\min}} p^3\, dp \propto \lambda_{\min}^{-4}. \tag{4}$$

For a Coulomb-like system we would have $\lambda_{\min} \sim (m\alpha)^{-1}$ (the first Bohr radius), so that the power to which the constant α_s is raised is large.

However, we know that in QCD long-wavelength vacuum fluctuations of the gluon field are not described by a perturbation theory and that the difference between the vacuum expectation value $\langle F_{\mu\nu}^a F_{\mu\nu}^a \rangle$ and that determined by this theory is nonvanishing (a gluon condensate)[4)] (Ref. 21):

$$\left\langle \frac{\alpha_s}{\pi} (F_{\mu\nu}^a)^2 \right\rangle = 0.012\text{–}0.018 \text{ GeV}. \tag{5}$$

In QCD, the effects of the interaction with the vacuum fluctuations are thus not small quantities of the order of the constant α_s.

The fluctuations which are responsible for the appearance of gluon condensate (5) have a fixed wavelength scale λ_g (this scale is independent of the parameters of quarkonium) and thus a fixed scale for frequencies, ω_g. This frequency scale, like other hadronic parameters, is determined by the infrared scale in QCD, Λ. If the characteristic frequencies of the quark system, ω_q, are greater than ω_g, the gluon field will not manage to keep up with the motion of the quarks, so that a retardation will arise and thus a deviation from a potential situation.[18]

For the same reason, the energy of a static quark and a static antiquark, $E(r)$, calculated on a lattice does not, in general, correspond to the potential in real quarkonium, since the revolution frequency of the quark system must be set equal to zero in a calculation of $E(r)$ (Ref. 22).

Under these conditions, a successful description of heavy quarkonium by potential models seems to be an extremely nontrivial matter. It is highly probable that this deviation from a potential situation will be manifested only at intermediate distances. At small distances, on the other hand, the Coulomb-like potential of one-gluon exchange operates, while at large distances (corresponding to small momenta of the quarks) a linear potential can be used. The nonpotential nature of the interaction at intermediate distances effectively reduces in the purely potential approach to a redefinition of the parameters of the potential, which are not calculated in this approach but instead found by fitting the experimental data.

In any case, the use of a potential description seems at present to be completely justified, particularly in those questions for which we have yet to find answers on the basis of the first principles of QCD.

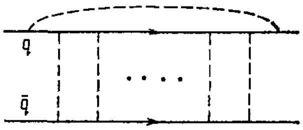

FIG. 5. Example of a Feynman diagram in a perturbation theory which leads to nonpotential effects in the interaction of quarks.

3.4. Superheavy quarkonium

The effect of a gluon condensate in the dynamics of heavy quarkonium can be dealt with in a very simple way if the condition $\omega_q \gg \omega_g$ holds and if the size of the quarkonium is much smaller than λ_g. This situation prevails for the deep levels of systems consisting of very heavy quarks. Specifically, if the quark mass m is sufficiently large then the first Bohr radius determined by potential (1), $r_0 = [(2/3) m\alpha_s]^{-1}$, is so small that quarks which are localized at this distance are insensitive to a difference between the long-range interaction and that described by (1). The dynamics of quarkonium will be determined primarily by potential (1). In this case the size of the quarkonium is of the order of r_0, and the characteristic quark revolution frequencies are of the order of the binding energy: $\omega_q \sim m\alpha_s^2$. Incidentally, we see that ω_q increases with increasing quark mass.

To incorporate the leading corrections for the interaction with nonperturbative vacuum fluctuations we can make use of the circumstance that under the conditions $r_0 \ll \lambda_g$, $\omega_q \gg \omega_g$ we can ignore the change in the field strength of the fluctuations in space over distances of the order of r_0 and in time over a time of order $\omega_q^{-1} \ll \omega_g^{-1}$. Here the problem of the level shift reduces to one of calculating a QCD analog of the Stark effect in a static (but random) chromoelectric[5)] field[18,23] $\mathbf{E}^a$ and then taking an average over vacuum fluctuations. When this average is taken, the part of the shift which is linear in $\mathbf{E}^a$ vanishes, since we have $\langle \mathbf{E}^a \rangle = 0$ by virtue of the rotational and color invariance of vacuum. The leading term is the quadratic term proportional to the expectation value:

$$\langle \mathbf{E}^a \mathbf{E}^a \rangle = -\frac{1}{4} \langle F^a_{\mu\nu} F^a_{\mu\nu} \rangle.$$

This equality is a consequence of the Lorentz invariance of vacuum, which leads to the relation[6)] (Ref. 18)

$$\langle (\mathbf{E}^a)^2 \rangle = -\langle (\mathbf{H}^a)^2 \rangle = -\frac{1}{4} \langle (F^a_{\mu\nu})^2 \rangle. \quad (6)$$

A direct calculation[22,23] leads to the following expression for the shift of an energy level with a principal quantum number $n = n_r + l + 1$ and an orbital quantum number l:

$$\frac{\delta E_{nl}}{|E_n|} = \left\langle \frac{\pi \alpha_s}{18} (F^a_{\mu\nu})^2 \right\rangle \frac{m^2}{[(2/3)\, m\alpha_s/n]^6}\, n^2 a_{nl}, \quad (7)$$

where E_n is the unperturbed "Coulomb" value of the binding energy, given by

$$E_n = -\left(\frac{2}{3} \frac{m\alpha_s}{n} \right)^2 m^{-1}, \quad (8)$$

and the dimensionless coefficients a_{nl} are quantities of the order of unity and are expressed in terms of n and l by

$$a_{nl} = \frac{1}{4n^3(2l+1)} \Big\{ (l+1) [F(n, l) - F(-n, l)] + l [F(n, -l-1) - F(-n, -l+1)] \Big\}, \quad (9)$$

where

$$F(n, l) = 2n[n^2 - (l+1)^2] + (n+l+2)(n+l+1) \times \left[\frac{(n-l)(n+l+3)}{9n+16} + \frac{4(2n-l)^2}{9n+8} \right].$$

It can be demonstrated that (7) cannot be reproduced by adding a potential perturbation of any sort (either local or nonlocal) to potential (1). For the 1S level we find the following value of the binding energy from (7)–(9):

$$E_{\text{bind}} = 2m - M_{1S} = \left(\frac{2}{3} m\alpha_s \right)^2 \times m^{-1} - 1.65 \left\langle \frac{\pi \alpha_s}{18} (F^a_{\mu\nu})^2 \right\rangle \frac{m}{[(2/3)\, m\alpha_s]^5}.$$

In the case of bottomonium we find the estimate

$$2m_b - M_\Upsilon \approx 190 \text{ MeV} - (65\text{–}100) \text{ MeV} \approx 90\text{–}125 \text{ MeV}$$

[where we have substituted the estimate (5) of the size of the gluon condensate, and we have used the values $m_b = 4.8$ GeV, and $\alpha_s = 0.3$ (see Subsection 3.5)]. This value of the binding energy agrees well with the estimate of 130 ± 50 MeV found from the sum rules (Subsection 3.5).

3.5. Sum rules for bottomonium

It can be seen from (7) that the relative magnitude of the shift is proportional to n^8. Consequently, an analysis of the interaction with fluctuations as perturbations quickly becomes inapplicable as n increases. In particular, for bottomonium expression (7) is poor even at $n = 2$. The reason is that the size of bottomonium is not sufficiently small in comparison with λ_g, and the characteristic frequencies are not large in comparison with ω_g. We are thus forced to seek alternative approaches to a description of the $b\bar{b}$ system. One such approach is based on sum rules.[11,12] Since the sum-rule method is set forth in detailed reviews,[11,12,24] we will skip the details here and simply point out the distinctions which are characteristic of bottomonium and list the results.

We recall that the sum rules are relations for integrals of the spectral state densities of quarkonium in channels with definite quantum numbers J^{PC}. For the channel with the quantum numbers of $^3S_1$ resonances, $J^{PC} = 1^{--}$, for example, the sum-rule method leads to predictions of the moments of the cross section for e^+e^- annihilation in a state containing heavy quarks:

$$M_n = \int R_b(s)\, (s^{n+1})^{-1}\, ds, \quad (8')$$

where s is the square of the total energy of the e^+ and e^- in the c.m. frame, and

$$R_b(s) = \frac{\sigma(e^+e^- \to \text{“}b\bar{b}\text{”})}{\sigma(e^+e^- \to \mu^+\mu^-)}. \quad (9')$$

The quotation marks in the expression $e^+e^- \to$ "$b\bar{b}$" means that we are not talking about an annihilation into a pair of free quarks but one into hadronic states containing a $b\bar{b}$: Υ resonances and $B\bar{B}$ meson pairs. We also recall that the contribution of the narrow resonance to $R(s)$ is expressed in terms of the width Γ_{ee} of its decay into e^+e^-, as follows:

$$R_{\text{res}}(s) = \frac{9\pi}{\alpha^2} \delta(s - M^2_{\text{res}})\, M_{\text{res}} \Gamma_{ee}. \quad (10)$$

Clearly, when n is sufficiently large the moment M_n will be dominated by the contribution of the low-lying resonance (in charmonium, the J/ψ resonance essentially saturates the moments even at $n \geqslant 4$; Ref. 11). Accordingly, if a theoretical expression calculated by the QCD methods is applicable at such values of n then the properties of the low-lying state in a channel with definite values J^{PC} can be determined by the sum-rule method.

In bottomonium, however, the smaller relative difference between the masses of the resonances has the consequence that the Υ meson and the ground states are dominant in the other channels only at $n \geqslant 20$. At such values of n, the expressions for the moments[25] which were originally found for charmonium are not directly applicable,[26] since the parameter describing the contribution of the QCD perturbation theory to the theoretical expressions is not α_s but

$\alpha_s n^{1/2}$. The reason for the appearance of this combination is that the nth moment is determined by the dynamics of the quarks at relative velocities $v \sim n^{-1/2}$. The perturbation theory for the Coulomb-like interaction (due to gluon exchange), in contrast, has the parameter $\alpha_s/v \sim \alpha_s n^{1/2}$. At large values of n it is thus necessary to deal with the Coulomb interaction exactly. Since we are talking about terms which are the leading terms $n \sim v^{-2}$, we can do this by a purely nonrelativistic method: by examining the Coulomb Green's function of the Schrödinger equation. We can illustrate the situation by writing the sum rules which arise as a result of that approach[27]:

$$\int R_b(s) \exp\left(\frac{4m_b^2 - s}{4m_b^2} n\right) ds \tag{11}$$

$$= \left(1 - \frac{16\alpha_s(m_b)}{3\pi}\right) m_b^2 \frac{9\sqrt{\pi}Q_b^2}{n^{3/2}} \left[\Phi_S(\gamma) - \left\langle \frac{\pi\alpha_s}{72}(F^a_{\mu\nu})^2 \right\rangle \frac{n^3}{m_b^4} X_S(\gamma)\right],$$

where m_b and Q_b are the mass and charge of the b quark,

$$\gamma = \frac{2}{3}\alpha_s n^{1/2} \tag{12}$$

is the Coulomb parameter, and the functions $\Phi_S(\gamma)$ and $X_S(\gamma)$ describe Coulomb effects. The first of these functions is given by

$$\Phi_S(\gamma) = 1 + 2\sqrt{\pi}\gamma + \frac{2\pi^2}{3}\gamma^2 + 4\sqrt{\pi}\sum_{n=1}^{\infty}\left(\frac{\gamma}{n}\right)^3 \times \exp\left[\left(\frac{\gamma}{n}\right)^2\right]\left(1 + \operatorname{erf}\frac{\gamma}{n}\right), \tag{13}$$

where

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)\,dt.$$

The function $X_S(\gamma)$ is an extremely complicated function, but in the region $\gamma \leqslant 1.5$, which is the region of interest for bottomonium, $X_S(\gamma)$ can be approximated by the simple expression

$$X_S(\gamma) \approx e^{-0,80\gamma}\Phi_S(\gamma).$$

In the limit of large n, the weight function in integral (11) agrees within a factor with that in (8):

$$s^{-n-1} = (4m^2)^{-n-1} \exp\left(\frac{4m^2 - s}{4m^2} \cdot n\right)(1 + O(n^{-1})).$$

We also see that (11) incorporates not only all terms of type $(\alpha_s n^{1/2})^k$ but also $\alpha_s(\alpha_s n^{1/2})^k$, since the first radiation correction, $1 - (16\alpha_s/3\pi)$, is included.

Just how important an exact account of the Coulomb interaction is can be seen from expression (13): with $\gamma = 1$ (and, correspondingly, $n \approx 25$) we find $\Phi_S(\gamma) \approx 50$; i.e., Coulomb effects change the answer by a factor of tens.

Analysis of (11) yields the following conclusions[27]: The constant α_s which appears in the Coulomb effects (at distances of the order of 0.2 fm) should be 0.30 ± 0.03; the mass separation of the quark threshold, $2m_b$, and the mass of the Υ should be $2m_b - M_\Upsilon = 130 \pm 50$ MeV, so that we have $m_b \approx 4.8$ GeV; and, finally, the width of the decay of the Υ into the e^+e^- pair should be $\Gamma_{ee}(\Upsilon) = 1.15 \pm 0.2$ keV, in extremely good agreement with the present experimental value,[7)] $\Gamma_{ee}(\Upsilon) = 1.22 \pm 0.05$ keV.

The most important difficulty in applying the sum-rule method to bottomonium is in dealing with relativistic corrections, i.e., in dealing with the next term in the expansion in $1/n$ (Ref. 28). Here a summation must be taken over all terms of the type $(\alpha_s n^{1/2})^k n^{-1}$ or, equivalently, $\alpha_s^2(\alpha_s n^{1/2})^k$ (since we have $\alpha_s n^{1/2} \sim 1$). These terms contain, along with purely relativistic effects (which are described by a Breit-Fermi Hamiltonian), radiation corrections of the order of α_s^2. So far, no such calculations have been carried out.

This difficulty is manifested extremely sensitively in, for example, the calculation[27] of the difference between the masses of the 1P and 1S levels of bottomonium. Using sum rules analogous to (11), but for the P-wave states of bottomonium, we find the estimate $M(1P) - M(1S) = 370 \pm 30$ MeV (the "error interval" here reflects the uncertainty in the parameters α_s and $\langle (F^a_{\mu\nu})^2 \rangle$, which is allowed by the consistency of sum rules (11) with experimental data on Υ resonances). Taking account of the spin of the quarks, we find that the P level splits into three ^3P_J resonances ($J = 0,1,2$) and a $^1P_1$ level, while the 1S level splits into 1^3S_1 [$\Upsilon(1S)$] and 1^1S_0 [$\eta_b(1S)$]. Experimentally, the mass differences are (in MeV) $M(\chi_{b0}) - M(\Upsilon) = 400$, $M(\chi_{b1}) - M(\Upsilon) = 432$, $M(\chi_{b2}) - M(\Upsilon) = 453$. The "center of gravity" of the 1P levels lies above Υ, by about 440 MeV. The nonrelativistic estimates are naturally compared with the last of these numbers, since the fine splitting—a relativistic effect—is completely absent from these estimates. We see that the experimental number lies outside the limits of the nonrelativistic estimate. On the other hand, the difference is of the same order of magnitude as the fine splitting of the ^3P_J levels, which may serve as a measure of the relativistic corrections (we might note that the mass of χ_{b0} falls within the predicted interval). Accordingly, this discrepancy which arises from the underestimate of the relativistic corrections in Ref. 27 by no means discredits the sum-rule method in comparison with the potential models (some of which have yielded a more accurate prediction of the masses of the P states). It simply illustrates the need to improve the accuracy of the sum rules by incorporating relativistic effects, although this is an extremely difficult problem from the technical standpoint, because of the factors described above.

3.6. Fine and hyperfine splittings

Forces which depend on the spin and orbital angular momentum in heavy quarkonium arise as relativistic corrections to the basis interaction. These forces—spin-spin, spin-orbit, and tensor—lead to a splitting of levels with different values of S and J. In the potential model, the question of the Hamiltonian of these forces reduces to the question of the Lorentz structure of the interaction potential. The potential is usually treated as a mixture of a vector potential V_V term (γ_μ matrices appear in the quark vertices) and a scalar V_S term (unit matrices appear in the vertices):

$$V(r) = V_V(r) + V_S(r). \tag{14}$$

It is thus a straightforward matter to use the standard procedure (Ref. 20, for example) to find the spin-dependent part of the Breit-Fermi Hamiltonian.[11,13] This part is the sum of a spin-orbit interaction (V_{LS}), a tensor interaction (V_T), and a spin-spin interaction (V_{SS}):

$$V_{\rm LS}=\frac{1}{2m^2r}\left(3\,\frac{dV_{\rm V}}{dr}-\frac{dV_{\rm S}}{dr}\right)\mathbf{LS},$$
$$V_{\rm T}=\frac{1}{6m^2}\left(\frac{1}{r}\,\frac{dV_{\rm V}}{dr}-\frac{d^2V_{\rm V}}{dr^2}\right)\left(3\,\frac{(\mathbf{Sr})(\mathbf{Sr})}{r^2}-\mathbf{S}^2\right),$$
$$V_{\rm SS}=\frac{2}{3m^2}\,(\mathbf{s}_1\mathbf{s}_2)\,\Delta V_{\rm V}, \quad (15)$$

where the operator $\mathbf{S}=\mathbf{s}_1+\mathbf{s}_2$ is the resultant spin of the quark and the antiquark.

The potential for the one-gluon exchange (1), is undoubtedly of the vector type (the radiation corrections in α_s are also calculated in perturbation theory[29]). The linear part of potential (3), on the other hand, is usually taken to be of the scalar type.

It can be seen from expression (15) that in this case the spin-spin interaction $V_{\rm SS}$, to which only $V_{\rm V}$ contributes, appears to be the simplest. If $V_{\rm V}$ is determined exclusively by the perturbative Coulomb-like potential (1), then we can write

$$V_{\rm SS}=\frac{32\pi}{9m^2}\,\alpha_s\delta(\mathbf{r})\,(\mathbf{s}_1\mathbf{s}_2). \quad (16)$$

(The distance dependence of α_s should be taken into account along with other radiation corrections; the result of a corresponding calculation will be presented below.) The presence of the δ-function means that the SS interaction occurs over distances of the order of m^{-1}, which appear as pointlike distances for a nonrelativistic system. According to (16), the spin-spin splittings in states with $L\neq 0$ (splittings between the states ${}^1L_{J=L}$ and the center of gravity of the ${}^3L_J$ levels) should not occur, since the wave function of such states vanishes at $r=0$. For the splitting between the n^3S_1 and n^1S_0 levels we have

$$M(n\,{}^3S_1)-M(n\,{}^1S_0)=\frac{32\pi}{9m^2}\,\alpha_s\,|\psi_{nS}(0)|^2. \quad (17)$$

The value of $|\psi(0)|^2$ is related to the width of the decay of the ${}^3S_1$ level into an e^+e^- pair by virtue of the process shown in Fig. 6, by the well-known formula

$$\Gamma(n\,{}^3S_1\to e^+e^-)=\frac{4\pi}{m^2}\,\alpha^2Q^2\,|\psi_{nS}(0)|^2\left(1-\frac{16\alpha_s}{3\pi}\right), \quad (18)$$

where Q is the charge of the quark in units of $|e|$ ($Q_b=-1/3$). The radiation correction in QCD has also been incorporated. As a result, we find the following expression for the hyperfine splitting of the S level [the radiation correction[30] to (16) is taken into account]

$$\delta_n=M(n\,{}^3S_1)-M(n\,{}^1S_0)=\frac{8\Gamma_{ee}}{9Q^2}\,\frac{\alpha_s(m)}{\alpha^2}\left(1+6.1\,\frac{\alpha_s}{\pi}\right). \quad (19)$$

With $\alpha_s(m_b)\approx 0.17$ (more on this below) and with the experimental values of the e^+e^- widths of the $\Upsilon(1S)$ through $\Upsilon(3S)$ resonances, we find

$\delta_1\approx 40$ MeV, $\delta_2\approx 18$ MeV, $\delta_3\approx 14$ MeV.

In the approach described in Subsection 3.4, which is not based on a phenomenological confinement potential, there is an additional contribution to the splitting from the interaction of spins (chromomagnetic moments) of quarks with fluctuations of the chromomagnetic component of the gluon field (an analog of the quadradic Zeeman effect in a random magnetic field). In particular, this contribution to δ_1 is[31]

$$\tilde{\delta}_1=\frac{1{,}12}{m\,[(2/3m\alpha_s)]^2}\left\langle\frac{\pi\alpha_s}{18}\,(F^a_{\mu\nu})^2\right\rangle. \quad (20)$$

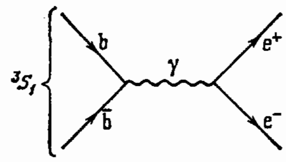

FIG. 6. Feynman diagram for the annihilation of b and $\bar{b}$ quarks into an e^+e^- pair in a Υ meson (the decay $\Upsilon\to e^+e^-$).

The constant α_s in the denominator must be normalized to the size of the 1S state for $\Upsilon(1S)\langle r\rangle\approx 1$ GeV^{-1}: $\alpha_s\approx 0.3$. The correction $\tilde{\delta}_1$ for bottomonium would be about 2 MeV in this case; this figure is much smaller than δ_1, so that the differences between the masses of the Υ and the η_b should be 35–45 MeV (we are taking account of the uncertainties of the parameters used here). Interestingly, for charmonium expression (19) describes a splitting between J/ψ and η_c of about 60 MeV; this figure is much smaller than the experimental value of 116 MeV. When we take contribution (20) into account, on the other hand, we find a value close to the experimental value, although the assumption that the motion of the quarks is of a Coulomb nature—the assumption under which expression (20) was derived—is totally inapplicable to charmonium.

The question of the fine splitting of the levels of orthoquarkonium is a more complicated one, since interaction (15) contains spin-orbit and tensor forces. That both terms are important can be seen from the experimental value of the ratio of the splittings of the 1^3P_J levels in the $b\bar{b}$ system:

$$\rho_1=\frac{M(1\,{}^3P_2)-M(1\,{}^3P_1)}{M(1\,{}^3P_1)-M(1\,{}^3P_0)}=0.67\pm 0.04. \quad (21)$$

For the pure spin-orbit interaction $V_{\rm LS}$ this ratio would be 2 (according to the Landé interval rule, which is well known in atomic physics), while for a purely tensor interaction $V_{\rm T}$ it would be 2/5.

For Coulomb-like interaction (1), $V_{\rm LS}$ and $V_{\rm T}$ are proportional to each other [see (15)]:

$$V_{\rm LS}=\frac{2\alpha_s}{m^2r^3}\,\mathbf{LS},\quad V_{\rm T}=\frac{2\alpha_s}{3m^2r^3}\left(3\,\frac{(\mathbf{Sr})(\mathbf{Sr})}{r^2}-\mathbf{S}^2\right). \quad (22)$$

The expectation value of the expression $\alpha_s/(m^2r^3)$ over the P-state wave function cancels out in a ratio ρ of the type in (21), and we find

$$\rho_n=\frac{4}{5}=0.8 \text{ for arbitrary } n; \quad (23)$$

this value is close to the value of ρ_1 in (21) but slightly larger. The difference between the "Coulomb" number (23) and the experimental number can be explained in the following way. In the potential model which leads to expression (15), the term $V_{\rm S}$, linear in r, makes a small negative contribution to the spin-orbit potential, according to (15). This contribution is $\delta V_{\rm LS}=-(\text{const}/r)\mathbf{LS}$; it slightly offsets the perturbative LS interaction and reduces the value of ρ. [The same effect results from the radiation corrections to the potentials $V_{\rm LS}$ and $V_{\rm T}$, given in (22).] In the picture of the effect of vacuum fluctuations, the interaction with the field of fluctuations gives rise to a small negative contribution (a few MeV) only to the LS interaction. It does not give rise to corrections to the tensor forces. Accordingly, again in this picture the value of ρ is slightly smaller than 0.8 and does not lead to contradictions.

It is thus natural to expect that the ratios of the splittings of the P levels of the type in (21) must always be smaller than 0.8—and to an extent which increases with increasing size of the P state, since the nonperturbative effects become more important in this case.[8)] In charmonium (which is much larger in size than bottomonium) we have $\rho_1(c\bar{c}) = 0.48$. For the 2^3P_J levels of $b\bar{b}$, the ratio ρ_2 is known only crudely at this point ($\rho_2 = 0.73 \pm 0.25$). On the basis of this discussion we might expect that this ratio should be smaller than ρ_1, because of the large size of the 2P state.

In conclusion we would like to call attention to Table II, taken from Ref. 29, where radiation corrections to the perturbative Breit-Fermi potential were taken into account, and calculations were carried out in a potential model. Those calculations rank among the most successful in terms of the agreement with the calculated positions of the levels of bottomonium with the experimental information available. The positions which Gupta *et al.*[29] predicted for the still unknown D and F levels can probably serve as guidelines for seeking these states experimentally.

4. RADIATION TRANSITIONS IN BOTTOMONIUM

Transitions between levels of the $b\bar{b}$ system accompanied by the emission of a photon are very important from the practical standpoint, since they make it possible to observe bottomonium states which are not produced directly in e^+e^- annihilation. From the theoretical standpoint these transitions are completely analogous to transitions in atoms or in positronium. For example, transitions between $^3S_1$ levels (Υ resonances) and ^3P_J levels are of the E1 (electric dipole) type according to the standard classification. The widths of these transitions are described by (see, for example, Refs. 11 and 12)

$$\Gamma(n\,^3S_1 \to m\,^3P_J + \gamma) = \frac{4}{27}(2J+1)\,\alpha Q_b^2 \omega^3\,|I_{mn}|^2, \quad (24)$$

$$\Gamma(m\,^3P_J \to n'\,^3S_1 + \gamma) = \frac{4}{9}\,\alpha Q_b^2 \omega^3\,|I_{mn'}|^2, \quad (25)$$

where ω is the energy of the photon, and I_{mn} is the matrix element of the radius between the $|mP\rangle$ and $|nS\rangle$ states, given by

$$I_{mn} = \int r R_{mP}(r)\, R_{nS}(r)\, r^2\, dr \quad (26)$$

[$R(r)$ is the radial part of the wave function].

Experimentally, the 1^3P_J and 2^3P_J levels have been reliably observed,[32] and the average positions of these levels and the transition probabilities are given in the Appendix. The most accurate data on the position of the 1^3P_J levels were obtained by the ARGUS group. Figure 7 shows the spectrum of inclusive photons which have converted into an e^+e^- pair in the decay of the $\Upsilon(2S)$ meson. We clearly see three peaks, which correspond to the transitions

$$\Upsilon(2S) \to \gamma 1\,^3P_J \qquad (J = 0,\ 1,\ 2),$$
$$\gamma \to e^+e^-.$$

So far, we have no direct data on the spins of these states. However, if we assume $J = 0$ for the level with the smallest mass and $J = 2$ for that with the largest mass, and if we then divide the measured transition probabilities by a factor of $\omega^3(2J+1)$—which follows from (24)—we find, for the 1^3P_J levels, for example, ratios which are close to unity:

$(1.00 \pm 0.26) : 1 : (1.03 \pm 0.19)$.

For an arrangement of levels of this sort we can find the position of the center of gravity of the levels (M_{cg}) from the formula

$$M_{cg} = \frac{1}{9}\sum_{J=0}^{2}(2J+1)\,M_J,$$

where M_J is the mass of level J. For the 1^3P_J levels we find

$$M_{cg} = 9900.2 \pm 0.6\ \text{MeV}.$$

Matrix elements (26) for the transitions $3S \to 2P$ and $2S \to 1P$, and thus the widths of the transitions, are described satisfactorily by the potential models.[15] (The experimental widths are reproduced within a factor of 1.5–2.) For the transitions $2S \to 1P$ and $1P \to 1S$, however, it is possible to establish limitations which follow from the general principles of quantum mechanics. Specifically, from the fundamental commutation relation $[p_i, r_k] = -i\delta_{ik}$, follow nonrelativistic Thomas-Reich-Kuhn and Wigner nonrelativistic sum rules for the quantities $m\omega|I|^2$, summed over the

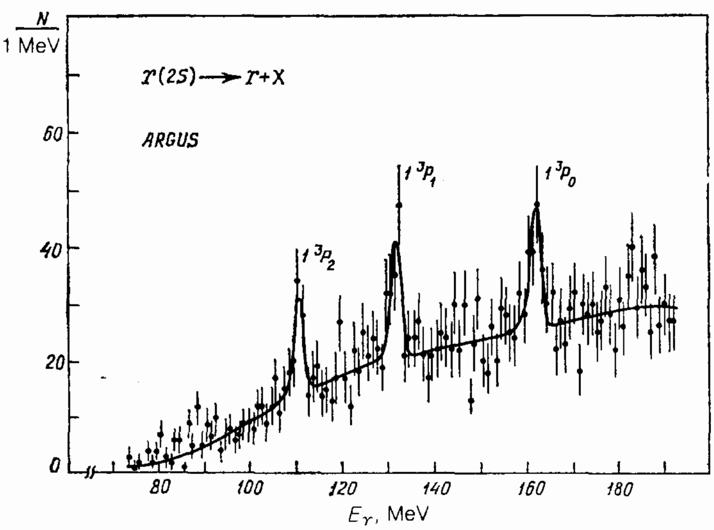

FIG. 7. Inclusive spectrum of photons from the $\Upsilon(2S)$ resonance in the ARGUS experiment.[32] The line is the best fit with three photon lines of the shape set by the energy resolution of the detector for the converted photons, plus a smooth background described by fifth-degree Legendre polynomials.

various transitions (see the derivation and discussion in Refs. 11, 12, and 33; see also Ref. 34 in connection with transitions in bottomonium). In particular, the first of these sum rules is

$$\sum_i m(E_i - E_n)\,|I_{in}|^2 = 3 \quad \text{for arbitrary } n. \tag{27}$$

(The existence of relations of this sort might be expected on the basis of no more than the following estimate: $\langle p\rangle\langle r\rangle \sim 1$, $|\langle p\rangle| = m\omega\langle r\rangle$, and thus $m\omega\langle r\rangle^2 \sim 1$.) For the transitions $2S \to 1P$ the sum rules predict upper limits

$$m\omega\,|I_{12}|^2 \leqslant 2. \tag{28}$$

in charmonium this value for $2S \to 1P$ transitions is

$$m_c\omega\,|I_{12}|^2 = 1 \pm 0{,}3.$$

If we assume that this value changes only slightly when we switch to bottomonium, we find the following results for γ transitions from the 2^3S_1 level to the 1^3P_J level, with the experimental values of the energy release being:

$$\Gamma(\Upsilon(2S) \to \chi_{bJ} + \gamma) = (1 \pm 0{,}3)\cdot\begin{cases} 0.7\ \text{keV} < 1.4\ \text{keV}, J = 0, \\ 1.2\ \text{keV} < 2.4\ \text{keV}, J = 1, \\ 1.5\ \text{keV} < 3.0\ \text{keV}, J = 2 \end{cases} \tag{29}$$

[the limitations which follow from inequality (28) are also given here]. The experimental widths found from the values of $\Gamma_{tot}(\Upsilon(2S))$ and $B(\Upsilon(2S) \to \chi_{bJ} + \gamma)$ are 1.3 ± 0.4, 2.0 ± 0.5, and 2.0 ± 0.5 keV, respectively. We see that the central values of these numbers exceed estimate (29) and are probably close to the upper limits in (29). This result may mean that the quantity $\Gamma_{tot}(\Upsilon(2S))$ given in the tables of Ref. 2, and found from the value of $\Gamma_{ee}/B_{\mu\mu}$, is slightly too high.

For the $1P \to 1S$ transitions, the quantum-mechanical inequalities are

$$1 \leqslant m\omega\,|I_{11}|^2 \leqslant 3. \tag{30}$$

Actually, the lower limit can be increased to $2^{13}/3^8 \approx 1.25$, which corresponds to the limiting value as $m \to \infty$, which is realized in a purely Coulomb system. The widths of the transitions $1^3P_J \to 1^3S_1\gamma$ can therefore be fixed within a factor of 1.5:

$$\Gamma(\chi_{bJ} \to \Upsilon(1S)\,\gamma) \approx 30\ \text{keV}. \tag{31}$$

Measurements of these widths require a determination of the absolute values of the total widths of the χ_{bJ} levels. (This problem has not been solved completely, even for the corresponding levels in charmonium.)

The widths of magnetic-dipole (M1) transitions between ortho- and para-levels are the simplest properties to find theoretically. The reason is that the magnetic-moment operator does not have a coordinate dependence, so that only transitions with $\Delta n_r = 0$ and $\Delta L = 0$, for which the overlap integral is unity, are allowed in the nonrelativistic limit. In this case the transition widths are given by

$$\Gamma(n\,{}^3L_J \to n\,{}^1L_L + \gamma) = \frac{4}{3}\,\alpha Q_b^2\,\frac{\omega^3}{m^2}. \tag{32}$$

Because of the small energy release ω, these widths are extremely small. The largest value, $\omega \approx 35$–40 MeV, is expected for the transition $\Upsilon(1S) \to \eta_b$. The transition width here should be[34]

$$\Gamma(\Upsilon(1S) \to \eta_b\gamma) \approx 2\text{—}3\ \text{eV}. \tag{33}$$

This width is very small for an observation of this transition in the near future.

With regard to the forbidden M1 transitions with $\Delta n_r \neq 0$, we note that their amplitudes contain a small factor $\sim v^2/c^2$ and that the widths of these transitions are also very small, despite the large energy release.

5. ANNIHILATION DECAYS

5.1. One-photon annihilation of $^3S_1$ states

The production of Υ resonances in e^+e^- annihilation results from the process illustrated in Fig. 6. Some of the decays of these particles also involve the same mechanism. The virtual photon formed in the annihilation of b and $\bar{b}$ may then decay into a lepton pair (e^+e^-, $\mu^+\mu^-$, or $\tau^+\tau^-$) or into a quark pair $q\bar{q}$ (q = u, d, s, c). The total width and relative probability for the decay by this mechanism are therefore

$$\Gamma_{1\gamma} = \Gamma_{ee}(R+3), \quad B_{1\gamma} = B_{\mu\mu}(R+3), \tag{34}$$

where R is the known ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons}}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

when the total e^+e^- energy is equal to the mass of the $b\bar{b}$ resonance (actually, R is measured along with the resonance). Experimentally we have $R \approx 3.6$ and, for example, $B_{\mu\mu}(\Upsilon) = 2.8 \pm 0.2$ % in this energy region. Accordingly, about 18% of the decays of the Υ resonance occur by the one-photon mechanism. We might also point out that all the characteristics of events with such decays of Υ resonances are identical to the characteristics of events in the continuum. As a particular example we note that events with a decay into a $q\bar{q}$ pair should exhibit the well-known two-jet structure.

5.2. Hadronic annihilation

The greatest fraction ($\sim 80\%$) of annihilation decays of bottomonium is annihilation into hadrons due to an interaction of b quarks with gluons. Because of the large mass of the b quark, the annihilation occurs at short range and can be dealt with in QCD perturbation theory.[35] In perturbation theory, the $b\bar{b}$ states with quantum numbers $J^{PC} = 1^{--}$ annihilate into three gluons (Fig. 8). The width of this decay is described by

$$\frac{\Gamma(^3S_1 \to 3g \to \text{hadrons}}{\Gamma(^3S_1 \to e^+e^-)} = \frac{10}{81\pi}\,\frac{\pi^2-9}{Q_b^2}\,\frac{\alpha_s^3(m_b)}{\alpha^2}. \tag{35}$$

[Both widths are proportional to the quantity $|\psi(0)|^2$, which cancels out in ratio (35).] The radiation correction[36] to this formula is a factor $(1 \pm 0.5\alpha_s/\pi)$ if the constant α_s in

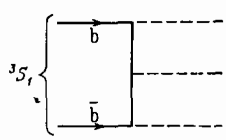

FIG. 8. Feynman diagram describing the annihilation $(b\bar{b})_{^3S_1} \to$ gluons (the decay $\Upsilon \to$ hadrons).

(35) is determined in an $\overline{\text{MS}}$ renormalization scheme at a momentum $q = 0.48 M_\Upsilon \approx m_b$.

It can be seen from (35) that the width for the direct decay is proportional to α_s^3, so measurements of this quantity would constitute an extremely sensitive tool for determining the strong-interaction constant α_s. The ratio on the left side of (35) for the Υ resonance can be found from the value of $B_{\mu\mu}$ since the total width of the Υ is the sum of $\Gamma_{1\gamma}$, Γ_{3g}, and the width for the decay into a photon + hadrons ($\gamma + 2g$): $\Gamma_{\gamma gg}$ (see the following subsection of this paper; numerically, we have $\Gamma_{\gamma gg} \approx \Gamma_{\mu\mu}$). We thus find

$$\frac{\Gamma_{3g}}{\Gamma_{\mu\mu}} = \frac{1}{B_{\mu\mu}} - \left(R + 3 + \frac{\Gamma_{\gamma gg}}{\Gamma_{\mu\mu}}\right) \approx 28.1 \pm 2.5, \qquad (36)$$

which leads, according to (35), to

$$\alpha_s(m_b) = 0.168 \pm 0.005 \qquad (37)$$

(this value corresponds to the infrared QCD parameter $\Lambda_{\overline{\text{MS}}}$, determined in the two-loop approximation: $\Lambda_{\overline{\text{MS}}} = 160 \pm 40$ MeV).

One might ask just how well the characteristics observed experimentally for Υ-resonance decay events correspond to the picture of three-gluon annihilation. Let us look at a result found by the ARGUS group.[37] After analyzing the statistical characteristics of the hadronic decays of an Υ resonance—the sphericity,[38] the thrust,[39] and the second Fox-Wolfram moment[40] (a distribution of events in this moment is shown in Fig. 9)—the group established that the extent to which the fraction of two-jet hadronic decays (i.e., decays into $q\bar{q}$ pairs) of the Υ meson exceeded the expected contribution from one-photon annihilation ($RB_{\mu\mu} \approx 10.1\%$) was no greater than 5.3% (at a 90% confidence level). At a qualitative level, we might note that a sphericity of Υ decay events significantly larger than in the continuum had been recognized experimentally a long time ago and has frequently been utilized in experimental analysis as a criterion for selecting events for suppressing the contribution of the "pedestal" under the resonance.

5.3. Annihilation into photon + hadrons

If we replace one of the gluons by a photon in the diagram in Fig. 8, we obtain a diagram which describes the decay of $^3S_1$ states into a hard photon (often called a "direct photon") and hadrons. The ratio of widths is given by[41]

$$\frac{\Gamma_{\gamma gg}}{\Gamma_{3g}} = \frac{36}{5} Q_b^2 \frac{\alpha}{\alpha_s} = \frac{4}{5} \frac{\alpha}{\alpha_s}. \qquad (38)$$

The radiation correction to this formula vanishes (within $\pm 0.6\alpha_s/\pi$) if the constant α_s in (38) is normalized in the $\overline{\text{MS}}$ scheme with $0.27 M_\Upsilon \approx m_b/2$ (Ref. 42). With $\alpha_s \approx 0.17$, ratio (38) should be 0.035, which corresponds to $B(\Upsilon \to \gamma + 2g) \approx 2.8\%$.

Of interest in addition to the total width $\Gamma_{\gamma gg}$ is the photon spectrum in this decay. In the decay into $\gamma + 2g$, bottomonium serves as a nearly point source of gluons. The invariant mass (M) of the gluon system is then expressed directly in terms of the photon energy: $M^2 = M_\Upsilon^2 (1 - x)$, where $x = 2E_\gamma / M_\Upsilon$, $x < 1$. The region $0.5 \lesssim x < 1$ thus can be used to study the fragmentation of gluons into hadrons in the invariant-mass interval 45 GeV$^2 \lesssim M \lesssim 90$ GeV^2 (at $x \lesssim 0.5$ it is essentially impossible to distinguish direct photons because of the background of secondary γ rays). A

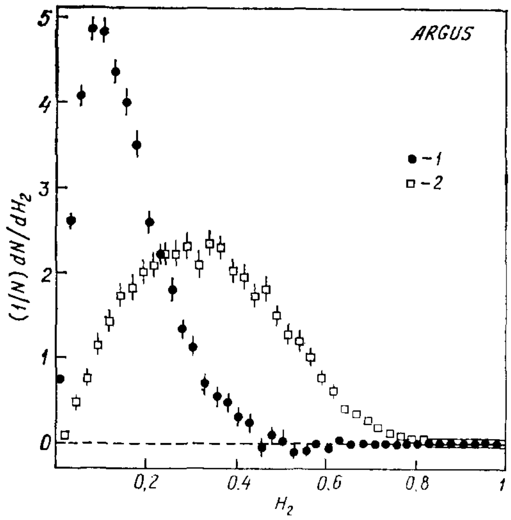

FIG. 9. Distribution of the number of events in the second Fox-Wolfram moment[40] for direct decays of the Υ meson (1) and for the continuum with $s^{1/2} = 9.98$ GeV (2) in an experiment by the ARGUS group.[37]

study of this sort for gluon physics is analogous to measuring the ratio R for quark physics in e^+e^- annihilation, where the virtual photon serves as a point source of quarks.

With regard to R we know that this quantity reaches the perturbation-theory value calculated from free quarks at $M^2 \gtrsim 2$ GeV^2. For gluon sources there are theoretical grounds[43] for expecting that the asymptotic behavior of perturbation theory will set in for these sources at far larger values of M^2: $M^2 \gtrsim 20$–25 GeV^2. We might thus expect that the spectrum of photons in the decay $\Upsilon \to \gamma +$ hadrons should differ from that predicted by perturbation theory at $x \gtrsim 0.7$–0.8.

So far, the experimental data from the study of direct photons are rather contradictory, apparently reflecting significant difficulties in the subtraction of the very large background from secondary γ rays. For example, the spectrum of direct photons found by the CUSB group[44] can be described well by the curve calculated in QCD perturbation theory up to values $x \approx 0.9$ with a branching fraction $B(\Upsilon \to \gamma + 2g) = 2.99 \pm 0.59\%$. On the other hand, the CLEO group[45] reports a direct-photon spectrum with a maximum at $x \approx 0.7$, which is described poorly by the QCD curve, and for which the branching fraction is $B(\Upsilon \to \gamma + 2g) = 1.88 \pm 0.22\%$.

With regard to a search for glueballs and new resonances in general in these decays at energies up to 2 GeV, we note that in comparison with the corresponding decays of the J/ψ mesons the Υ meson would seem to have no advantages. On the contrary, it is more likely to be at a disadvantage. A search of this sort would be difficult here since it would require a substantially better relative resolution in terms of the photon energy. Furthermore, the branching fraction of the decay is smaller than that for J/ψ because of the square of the quark charge in expression (38) and also, and primarily, because of the suppression of the fraction of each exclusive hadronic decay channel of the Υ meson.

5.4. Annihilation of P levels

In the perturbation-theory picture, the $^3P_0$ and $^3P_2$ levels annihilate into two gluons, while the $^3P_1$ state annihilates primarily into a gluon and a $q\bar{q}$ pair (Ref. 46; see Fig. 10 of the present paper). The widths of these decays are

$$\Gamma(^3P_0 \to 2g) = 6\alpha_s^2\, |R'_P(0)|^2\, m_b^{-4},$$
$$\Gamma(^3P_2 \to 2g) = \frac{4}{15}\,\Gamma(^3P_0 \to 2g),$$
$$\Gamma(^3P_1 \to g + q\bar{q}) \approx \frac{20}{9}\,\frac{\alpha_s}{\pi}\,\ln(mR)\,\Gamma(^3P_2 \to 2g), \qquad (39)$$

where $R'_P(0)$ is the derivative of the radial part of the wave function of the P level at the origin, and R is a characteristic radius of the P state (the last expression is of logarithmic accuracy). An estimate[27] of $R'_P(0)$ by the sum-rule method yields the following expected widths:

$$\Gamma(^3P_0) \approx 360\,\text{keV},\ \Gamma(^3P_2) \approx 100\,\text{keV},\ \Gamma(^3P_1) \approx 25\,\text{keV} \qquad (40)$$

(similar values are predicted by the potential models).

The measurement of absolute widths is a fairly complicated experimental problem. It is considerably simpler to extract ratios of widths from experiments. The reason is that the width of radiative transitions from 1^3P_J levels to $\Upsilon(1S)$ differ by only a ratio of the factors ω^3, which is easily dealt with. Accordingly, the ratio of the measurable quantities $B(\chi_{bJ} \to \Upsilon(1S) + \gamma)$ can be used to find the ratio of the total widths of the χ_{bJ} levels. From the data available we find the following estimates by using this method:

$$\frac{\Gamma(\chi_{b1})}{\Gamma(\chi_{b2})} = 0.54 \pm 0.16, \quad \frac{\Gamma(\chi_{b0})}{\Gamma(\chi_{b2})} > 2. \qquad (41)$$

While the second of these ratios does not contradict estimates (40) at least qualitatively, the first is higher than the expected value. If this discrepancy does not fade away as the experimental data become more accurate, the meaning will be that either the nonlogarithmic terms in $\Gamma(^3P_1)$ are large or nonperturbative effects are important.

5.5. Nonperturbative effects in hadronic annihilation

An indication of the presence of nonperturbative effects—an indication which is more definite experimentally than that which we have just discussed—follows from a comparison of the constants $\alpha_s(m_b)$ and $\alpha_s(m_c)$ which can be extracted with the help of (35) from data on the Υ and J/ψ widths, through the use of the well-known expression for the dependence of α_s on the normalization point. Specifically, armed with value (37) for $\alpha_s(m_b)$, we can calculate $\alpha_s(m_c)$ from the expression from the single-loop approximation:

$$\alpha_s(m_c) = \alpha_s(m_b)\left(1 - \frac{25\alpha_s(m_b)}{12\pi}\ln\frac{M_\Upsilon^2}{M_{J/\psi}^2}\right)^{-1} \qquad (42)$$
$$= 0.224 \pm 0.007.$$

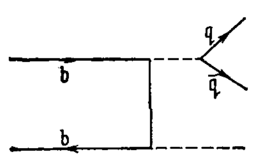

FIG. 10. Decay of the $^3P_1$ level into a gluon and a $q\bar{q}$ pair.

With this value of $\alpha_s(m_c)$, we find $\Gamma_{3g}(J/\psi)/\Gamma_{ee}(J/\psi)$ according to (35), while the experimental ratio is 9.2 ± 1.5. Incorporating the two-loop contribution into the relation between $\alpha_s(m_b)$ and $\alpha_s(m_c)$ simply deepens the contradiction since it accelerates the "running" of the constant α_s [accordingly, $\alpha_s(m_c)$ turns out to be slightly larger than in (42)]. As another example we might cite the ratio of ratios Γ_{3g}/Γ_{ee} for ψ' and J/ψ; experimentally, it has a value of 2.1 ± 0.5, while perturbation theory evidently predicts a value of unity.

The physical reason for the appearance of nonperturbative effects in the hadronic annihilation of quarkonium is[47] the existence of nonperturbative, long-wavelength vacuum fluctuations of the gluon field, which substantially modify the spectral density of soft gluons in comparison with that from perturbation theory. This difference is parametrized by the vacuum expectation value of gluon operators of the type in (5) and of higher dimensionality. Another way to clarify the picture of the effect in the example of the annihilation of the $^3S_1$ state is as follows: as it interacts with a component of the fluctuation field of the magnetic (or electric) type, the $^3S_1$ state converts into a $^1S_0$ (^3P_J)-color state, which annihilates into two gluons. (The effect is analogous to the well-known conversion of ortho-positronium into para-positronium in an external magnetic field.)

Qualitatively it is clear that this effect should be significantly weaker in the $b\bar{b}$ system than in charmonium, because the $b\bar{b}$ system is smaller than the $c\bar{c}$ system (for identical quantum numbers). Qualitatively, the effect is amenable to estimates[47] only for very heavy quarkonium in the Coulomb limit (by analogy with the calculations described in Subsection 3.4). Several distinctive features of the nonpeiturbative corrections are found here. First, the parameters which render vacuum expectation value (5) dimensionless are the size of quarkonium and its binding energy, not the mass of a quark. Second, the corrections are not expressed in terms of the value of the wave function (or of its derivative, in the case of the P state) at the origin. Accordingly, the ratios $\Gamma_{3g}(n^3S_1)/\Gamma_{ee}(n^3S_1)$ are not identical for different values of n, when the corrections are made. Third and finally, the relative magnitude of these corrections is substantially different in the cases of the annihilation of S and P states. For P states, the relative magnitude of these corrections is $(v/c)^{-4}$ times as large as that for $^3S_1$ states. The fact that the corrections are not universal can be understood easily: The interaction with the soft gluon fields does not occur at distances small in comparison with the size of quarkonium, so that the effect does not reduce to an expression proportional to the wave function at the origin.

General formulas for these corrections are given in Ref. 47. In particular, the correction to (35) for the 1^3S_1 ground state is

$$\frac{\delta(\Gamma_{3g}/\Gamma_{ee})}{\Gamma_{3g}/\Gamma_{ee}}(1\,^3S_1) = -4.0\,\frac{\pi^2\langle\alpha_s(F_{\mu\nu}^a)^2\rangle}{2^9\cdot 9\,(\pi^2-9)\,\alpha_s(m)\,[(2/3)\,m\alpha_s(r^{-1})]^4}. \qquad (43)$$

An extrapolation of this formula down the quark mass scale to Υ yields a correction to Γ_{3g}/Γ_{ee} of about -0.4% and a correction for J/ψ of about -5%. The numerical factor of 4.0 in (43), however, arises from the subtraction of two large numbers (22.9–18.9), which correspond to the interaction with the electric and magnetic components of the gluon fluctuation field. Since the dynamics of the quarks at the Υ and J/ψ resonances is extremely different from the Coulomb dynamics, this cancellation probably does not occur, and the effect may be several times as large. In this case, the correction in the Υ annihilation would be at the percent level, while the annihilation of J/ψ into hadrons would be suppressed by a matter of tenths; the result would be to eliminate the contradiction discussed above, which arises in a comparison of the Υ and J/ψ widths with the expression for the evolution of α_s.

As was mentioned earlier, the correction has different values for the n^3S_1 levels with different values of n. In this connection it would be very interesting to compare the values of Γ_{3g}/Γ_{ee} for Υ, Υ', and Υ'' at an accuracy level of a few percent. However, this accuracy level has not yet been achieved[48] in the measurements of $B_{\mu\mu}$ for the Υ' and Υ'' resonances, and we can only hope for an improvement in the experimental data in the future.

We might also note that the nonperturbative effect which we have been discussing might also be responsible for the disruption of the similarity between the probabilities for exclusive decays of various $^3S_1$ levels. Such examples are known for J/ψ and ψ'. The most obvious one is $B(\psi'\to\rho\pi)/B(J/\psi\to\rho\pi)<6\cdot10^{-3}$, while one might naively expect that the relative probabilities for each decay channel would be proportional to $B_{\mu\mu}$ [we recall that we have $B_{\mu\mu}(\psi')/B_{\mu\mu}(J/\psi)\approx0.26$].

In the case of P levels, the correction is extremely large, and an extrapolation of the Coulomb formulas[47] yields $\delta\Gamma/\Gamma(\chi_{b0})\approx0.35$, so there may be significant deviations of the annihilation widths of the P levels from the perturbation-theory predictions.

The nonperturbative correction has its smallest relative value in the ratio $\Gamma_{\gamma gg}/\Gamma_{3g}$ for the $^3S_1$ states (it differs by a factor of $-1/3$ from the correction to Γ_{3g}/Γ_{ee}). One might therefore expect that a calculation of ratio (38) by perturbation theory would be valid within no worse than 1%, so a determination of the constant α_s from measurements of this ratio appears theoretically to be reliable.

6. HADRONIC TRANSITIONS

6.1. Multipole expansion and transitions between $^3S_1$ levels

Transitions between quarkonium lvels accompanied by the emission of light mesons (π, η) differ from radiative transitions in that they have no direct analog in atomic physics. However, there is a remote analogy, and it can be used to advantage in describing these transitions. The analogy is that the emission of gluons by heavy quarks underlies hadronic processes. One-gluon transitions are of course impossible, since the initial and final states of quarkonium are colorless. The minimum number of gluons is thus two, so that the analogy is with two-photon transitions in atoms. Because of "color confinement," gluons convert into hadrons: π and η mesons. In a hadronic transition, two processes thus occur: the emission of gluons by heavy quarkonium and the conversion of gluons into light mesons (Fig. 11).

The wavelengths of gluons are large in comparison with the size of quarkonium (a consequence of the nonrelativistic nature of the system), so that a multipole expansion can be used to analyze the interaction of quarkonium with a gluon field, as in the case of radiative transitions.[49] For example, the leading term in this expansion—the Hamiltonian of the E1 interaction—is

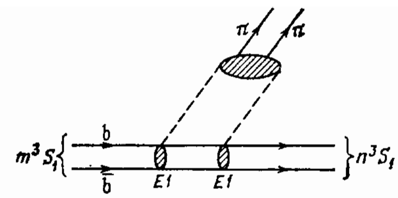

FIG. 11. Feynman diagram describing the transition $m^3S_1\to n^3S_1+\pi\pi$ in heavy quarkonium. The hatched regions on the quarkonium line represent the E1 interaction with the gluon field (the dashed lines); the hatched region at the top represents the production of pions by the gluon field.

$$H(\mathbf{E}_1)=-\frac{1}{2}\,g\xi^a\mathbf{r}\mathbf{E}^a(0), \tag{44}$$

where $g=(4\pi\alpha_s)^{1/2}$, the vector $\mathbf{r}$ is the relative position of the quark and the antiquark, and $\xi^a=t_1^a-t_1^a$ is the difference between the color generators which are acting on the quark (t_1^a) and on the antiquark (t_2^a).

The gluon system which is emitted in second order in Hamiltonian (44), in a transition between $^3S_1$ states, has the quantum numbers of the $\pi\pi$ system. It has therefore been suggested that transitions $(n^3S_1)\to(m^3S_1)+\pi\pi$ occur in second order in $H(\mathrm{E1})$. The expression[50]

$$A_{\pi\pi}\equiv A(n\,^3S_1\to m\,^3S_1\pi\pi)=\langle\pi\pi\mid\pi\alpha_s\mathbf{E}^a\mathbf{E}^a\mid0\rangle(\psi'\psi)\,A_0, \tag{45}$$

can be easily derived for the transition amplitude; here ψ' and ψ are the polarization amplitudes of the initial and final states, and A_0 is the quarkonium matrix element

$$A_0=(24)^{-1}\,\langle mS\mid\xi^a r_i G(E)\,r_i\xi^a\mid nS\rangle, \tag{46}$$

where $G(E)$ is the Green's function of the quark-antiquark pair. Because of the color operators ξ^a, expression (46) contains a Green's function which describes the propagation of a quark pair in a color-octet state. The energy E at which $G(E)$ comes in is equal to the residual energy of the quark pair after the application of a single operator $H(\mathrm{E1})$. It is not possible to calculate A_0, since we do not know just which states saturate the color Green's function. In any case, it is reasonable to suggest that at energies in the mass region of the narrow resonances a color state of the $b\bar{b}$ pair will arise for only a short time (short in comparison with the time scale $\omega_q\sim\Delta E$, where ΔE is the level spacing). (Above the $B\bar{B}$ threshold, a color state of the quark pair $b\bar{b}$ may be realized in a colorless $B\bar{B}$ hadronic state.) In this case, $G(E)$ should depend only weakly on the energy over an interval of the order of ΔE, and it can be replaced by a constant. The emission of the entire gluon system corresponds effectively to a point interaction, and the operators $\mathbf{E}^a$ in the amplitude $\langle\pi\pi|\pi\alpha_s\mathbf{E}^a\mathbf{E}^a|0\rangle$, which describes the conversion of gluons into π mesons, come in a single space-time point. For the local operator $\alpha_s(\mathbf{E}^a)^2$ we can write

$$\alpha_s(\mathbf{E}^a)^2=-\frac{\alpha_s}{4}(F^a_{\mu\nu})^2+\alpha_s\frac{(\mathbf{E}^a)^2+(\mathbf{H}^a)^2}{2}. \tag{47}$$

Both of the terms on the right side are related to the energy-momentum tensor $\theta_{\mu\nu}$. The first term is proportional to the trace $\theta_{\mu\mu}$ in the limit in which the masses of the u, d, and s quarks are zero:

$$-\frac{\alpha_s}{4}(F^a_{\mu\nu})^2=\frac{2\pi}{b}\,\theta_{\mu\mu}, \qquad (48)$$

where $b = 9$ is the first coefficient of the Gell-Mann–Low function in QCD with three light quarks. [Equation (48) is an expression of the so-called conformal anomaly (Ref. 51; see also Ref. 16).] The second term in (47) is proportional to the energy density of the gluon field θ^G_{00}:

$$\alpha_s\,\frac{(\mathbf{E}^a)^2+(\mathbf{H}^a)^2}{2}=\alpha_s\theta^G_{00} \qquad (49)$$

(the superscript G means that we are considering only the gluon part of $\theta_{\mu\nu}$).

It can be seen from (48) and (49) that the terms on the right side of (47) differ in their order in α_s [the anomaly actually "eats α_s out" from $\alpha_s(F^a_{\mu\nu})^2$]. Consequently, the term (49) can be ignored in general in a certain approximation. The matrix element $\langle\pi\pi|\theta_{\mu\mu}|0\rangle$ is determined by current algebra in the low-energy limit. In the chiral limit (with $m^2_\pi = 0$) the relation

$$\langle\pi^+\pi^-\mid\theta_{\mu\mu}\mid 0\rangle=q^2, \qquad (50)$$

holds[50] where $q = p_+ + p_-$ is the total 4-momentum of the $\pi\pi$ system. The amplitude for the $\pi\pi$ transition can thus be written as follows

$$A_{\pi^+\pi^-}=2\pi^2b^{-1}(q^2-C)(\psi'\psi)A_0, \qquad (51)$$

where the phenomenological constant C describes deviations from the chiral limit as well as the contribution of the term (49) and other possible corrections. Expression (51) agrees very well with the experimental distributions in the invariant mass of the two-pion system in the transitions $\psi'\to J/\psi\pi^+\pi^-$ (Ref. 52) and $\Upsilon'\to\Upsilon\pi^+\pi^-$ (Ref. 53; see Fig. 12 of the present paper). The constant C is not large: $C = (4.6 \pm 0.2)m^2_\pi$ for the first transition and $C = (3.3 \pm 0.2)m^2_\pi$ for the second. Corresponding data on the decay $\Upsilon'\to\Upsilon\pi\pi$ were obtained in Refs. 6, 54, and 55.

This picture of the hadronic transitions between $^3S_1$ levels can be tested well in a case in which the initial state is polarized. Since the E1–E1 transition dominates the multipole expansion during the emission of two soft pions, the spin quantum numbers of the final $^3S_1$ state must be the same as the quantum numbers of the initial state; the initial polarization must therefore be conserved. An experiment of this type, with transversely polarized Υ' mesons, has been carried out by the ARGUS groups.[53] The angular distributions of the two leptons in the exclusive decay $\Upsilon'\to\Upsilon\pi^+\pi^-\to\pi^+\pi^-\mu^+\mu^-(e^+e^-)$ turned out to be similar to the distributions of muon pairs in the decay $\Upsilon'\to\mu^+\mu^-$ (Ref. 53; see Fig. 13 of the present paper). The degree of polarization of the Υ mesons turned out to be the same as that of the original Υ' mesons.

We might also note that according to (51) a system of two pions should be a purely S-wave system; i.e., the π mesons should be emitted in a totally isotropic fashion. Experimentally, this behavior is found, again highly accurately. The admixture (ε) of a D-wave amplitude in the transition $\Upsilon'\to\Upsilon\pi\pi$ was recently measured[53] and was found to be only $|\varepsilon| = 0.018^{+0.108}_{-0.009}$.

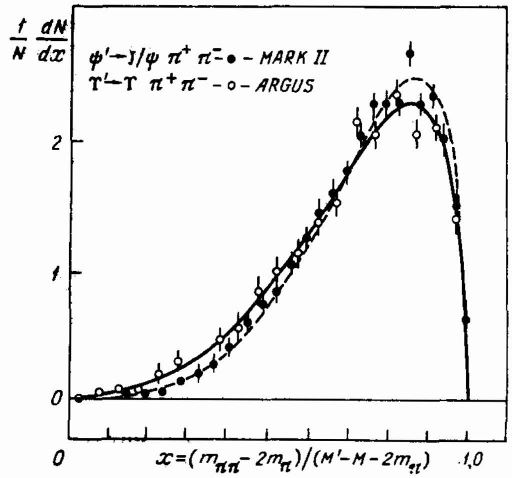

FIG. 12. Normalized invariant-mass distribution of the $\pi\pi$ system according to data obtained by the MARK II group[52] and by the ARGUS group.[53] The curves are best fits by expression (51). The parametrizations of Refs. 56 and 57 lead to curves which are not distinguishable at the accuracy of this figure.

The contribution to the amplitude of term (49) was analyzed in Ref. 56. It has two consequences: first, the appearance of a D wave, whose contribution to the probability should be of the order of 1%; second, a correction term, which is written in (51) as the constant C but which is actually a small quantity which depends on q^2,

$$C\to\varkappa(\Delta E)^2\left(1+\frac{2m^2_\pi}{q^2}\right)+O(\varkappa^2), \qquad (52)$$

where $\varkappa$ is related to the fraction (ρ^G) of the π-meson momentum which is carried by gluons by

$$\varkappa=\frac{9}{6\pi}\,\alpha_s\rho^G\approx 0.15.$$

It was predicted[56] that the effective value of C in an expression like (51) should decrease as we go from $\psi'\to J/\psi\pi\pi$ to $\Upsilon'\to\Upsilon\pi\pi$.

So far, the experimental data are not capable of distinguishing between a constant value of C and the form described by (52); nor is it possible to draw a distinction with the predictions of certain other models,[57] in which the form

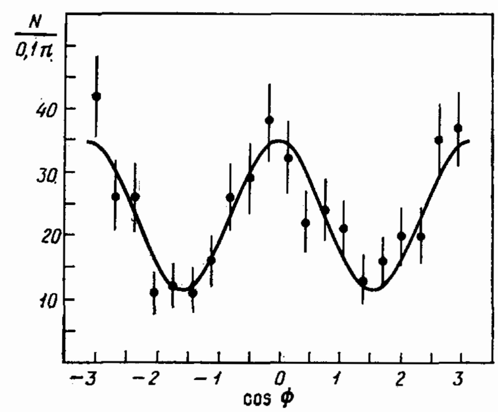

FIG. 13. Distribution in the azimuthal angle ϕ of electrons and muons in the exclusive decay $\Upsilon'\to\Upsilon\pi+\pi^-\to\mu^+\mu^-(e^+e^-)\pi^+\pi^-$ in an ARGUS experiment.[53] The curve is the best fit, with $\alpha = P_+P_- = 0.75 \pm 0.10$, where P is the degree of polarization of the beams. The corresponding value for muons from the decay $\Upsilon'\to\mu^+\mu^-$ is $\alpha = 0.68 \pm 0.02$.

of the correction term is different. There are also models[58] which are at odds with the existing data on the shape of the invariant-mass spectrum of the $\pi\pi$ system.

6.2. Transitions accompanied by the emission of a single η or π^0 meson

Since the amplitude A_0 in (50) is not amenable to calculation, we cannot verify whether this expression agrees with the absolute value of the width of $\pi\pi$ transitions (we do note, however, that rough estimates of the ratio of the values of A_0 for the transitions $\Upsilon'\to\Upsilon\pi\pi$ and $\psi'\to J/\psi\pi\pi$ have yielded a correct prediction[34,57] of the width of the $\Upsilon'\to\Upsilon\pi\pi$ transition). Nevertheless, a further test of the validity of the approach described in the preceding subsection can be made by studying the ratio of the widths of $\pi\pi$ and η transitions. The quantum numbers of the η meson are shared by the gluon system which is emitted in the transition $n^3S_1\to m^3S_1$ as a result of an interference of the E1 interaction, (44), and the M2 interaction described by the Hamiltonian

$$H\,(\mathrm{M2}) = -(4m)^{-1}\, gS_j\xi^a r_i D_i H_j^a\,(0), \qquad (53)$$

where m is the mass of a quark, the operator $\mathbf{S}=\mathbf{s}_1+\mathbf{s}_2$ is the total spin, and D_i is a covariant derivative. It is easy to see that the E1 and M2 interactions depend on r in an identical way, so that the amplitude for the η transition is expressed in terms of the same coordinate matrix element A_0 (Ref. 50):

$$A_\eta \equiv A\,(n\,^3S_1\to m\,^3S_1\eta)$$
$$= i\,(\partial_k\,\langle\eta\mid \pi\alpha_s E_k^a H_j^a\mid 0\rangle)\, m^{-1}\varepsilon_{ijm}\psi_i'\psi_m A_0. \qquad (54)$$

The matrix element for the production of η by the gluon operator $\mathbf{E}\cdot\mathbf{H}$ which appears in this expression is determined by[59] the SU(3) flavor symmetry and by an anomaly in the divergence of the axial current in QCD:

$$\langle\eta\mid \pi\alpha_s E_k^a H_i^a\mid 0\rangle = \frac{1}{3\sqrt{6}}\,\pi^2 f_\pi m_\eta^2\delta_{ik}, \qquad (55)$$

where f_π is the constant of the decay $\pi\to\mu\nu$, having a value $f_\pi\approx 132$ MeV. For A_η we thus find

$$A_\eta = \frac{\pi^2}{9}\cdot\left(\frac{3}{2}\right)^{1/2} f_\pi m_\eta^2 m^{-1}\,[\varepsilon_{ijk}\psi_i\psi_j'\,(\mathbf{p}_\eta)_k]\,A_0, \qquad (56)$$

where $\mathbf{p}_\eta$ is the momentum of the η meson. As a result, the unknown constant A_0 cancels out in the ratio of the widths of the η and $\pi\pi$ transitions, and the ratio is determined from (50) and (56) in terms of exclusively known quantities[50]:

$$\frac{\Gamma\,(n\,^3S_1\to m\,^3S_1\eta)}{d\Gamma\,(n\,^3S_1\to m\,^3S_1\pi^+\pi^-)/dq^2}$$
$$= 16\pi^2\left(\frac{b}{9}\right)^2 f_\pi^2\left(\frac{p_\eta}{M}\right)^2\frac{p_\eta}{|\mathbf{q}|}\left(\frac{m_\eta^2}{q^2-C}\right)^2\left(1-\frac{4m_\pi^2}{q^2}\right)^{-1/2}, \qquad (57)$$

where M is the mass of the quarkonium, and $p_\eta=|\mathbf{p}_\eta|$. The corrections to this formula can be expected to be at a minimum in the case $q^2=m_\eta^2$, since in this case the quarkonium experiences an identical recoil. Interestingly, a ratio of two quantum anomalies appears in (57): one in the axial current and one in the medium of the energy-momentum tensor. In this sense we could say that the η and $\pi\pi$ transitions are associated with some extremely profound, and genuinely quantum-mechanical, properties of quantum chromodynamics.

For transitions between ψ' and J/ψ, we find from (57), after an integration over q^2, the ratio of total widths $\Gamma(\psi'\to J/\psi\eta)/\Gamma(\psi'\to J/\psi\pi^+\pi^-)$. It turns out to be 0.10 if $M=M_\psi$, or 0.14 if $M=M_{J/\psi}$. (The difference between the masses of the levels is of the order of v^2/c^2 and is ignored in a nonrelativistic treatment of heavy quarkonium.) Experimentally, this ratio is 0.082 ± 0.013; this value can be judged to be in good agreement with the theoretical number, since the expected error in relation (55) is of the order of 30% [the typical accuracy of SU(3) relations].

The width of the transition $\Upsilon'\to\Upsilon\eta$ is proportional to p_η^3 and thus very sensitive to the exact value of the difference between the masses of Υ' and Υ, which is close to m_η. If we take this mass difference to be[13] 563.3 MeV, and if we take $C=3.2m_\pi^2$, we find from (57)

$$\Gamma\,(\Upsilon'\to\Upsilon\eta)\approx 5\cdot10^{-3}\,\Gamma\,(\Upsilon'\to\Upsilon\pi^+\pi^-).$$

Since the branching ratio of the latter decay is about 20%, we conclude that $B(\Upsilon'\to\Upsilon\eta)$ should be about 0.1%; this is half the upper limit on this quantity which has been established in an experiment by the CUSB group[55] (see the Appendix). A search for the decay $\Upsilon'\to\Upsilon\eta$ would thus be an extremely interesting and feasible experimental project.

We have a few comments about transitions in which a π^0 meson is emitted instead of an η. These transitions occur because of a breaking of the isotopic symmetry by the masses of the u and d quarks; their amplitudes are proportional to m_d-m_u. Because of the mass difference, the gluon operator $\mathbf{E}^a\cdot\mathbf{H}^a$ produces a π^0 mesons[60]:

$$\frac{\langle\pi^0\mid\mathbf{E}^a\mathbf{H}^a\mid 0\rangle}{\langle\eta\mid\mathbf{E}^a\mathbf{H}^a\mid 0\rangle} = \sqrt{3}\,\frac{m_d-m_u}{m_d+m_u}\,\frac{m_\pi^2}{m_\eta^2}. \qquad (58)$$

For the ratio of the widths of the π^0 and η transitions we thus find[61]

$$\frac{\Gamma\,(n\,^3S_1\to m\,^3S_1\pi^0)}{\Gamma\,(n\,^3S_1\to m\,^3S_1\eta)} = 3\left(\frac{m_d-m_u}{m_d+m_u}\right)^2\frac{m_\pi^4}{m_\eta^4}\,\frac{p_\pi^3}{p_\eta^3}. \qquad (59)$$

According to the analysis of Ref. 62, the ratio $(m_d-m_u)/(m_d+m_u)$ is about 0.3. With this value, we find the ratio of the decay widths in charmonium to be

$$\frac{\Gamma\,(\psi'\to J/\psi\pi^0)}{\Gamma\,(\psi'\to J/\psi\eta)}\approx 22\cdot10^{-3},$$

in reasonable agreement with the experimental value $(37\pm12)\cdot10^{-3}$.

6.3. The problem of the decay $\Upsilon''\to\Upsilon\pi\pi$

As was discovered in the first experiments[63] in 1982 and confirmed by new data,[64] the invariant-mass distribution of the $\pi\pi$ system in the transition $\Upsilon''\to\Upsilon\pi\pi$ is totally unlike the spectra in the transitions $\Upsilon'\to\Upsilon\pi\pi$ and $\psi'\to J/\psi\pi\pi$ and is not described by (51) (Fig. 14).

The reason for this feature is not known; various possible explanations could be discussed. We should apparently reject without further consideration an explanation based on a deviation of matrix element (50) from linearity in q^2 at large values of q^2 because of $\pi\pi$ resonances or for some other reasons. (We recall that the difference between the Υ'' and Υ masses is $\Delta\approx 895$ MeV.)

There are at least two objections to this explanation.

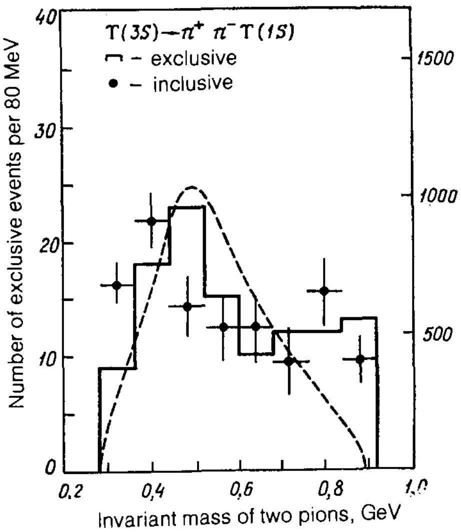

FIG. 14. Experimental data of the CLEO group[64] on the distribution in the invariant mass of the two pions in the decay

$\Upsilon'' \to \Upsilon\pi^+\pi^-$
$\quad \hookrightarrow$ всё anything

(inclusive events) and

$\Upsilon'' \to \Upsilon\pi^+\pi^-$
$\quad \hookrightarrow \mu^+\mu^-(e^+e^-)$ exclusive events).

The dashed line is the distribution which arises in the case in which an exotic resonance X with a mass of 10 245 MeV is dominant (see the discussion in the text proper).

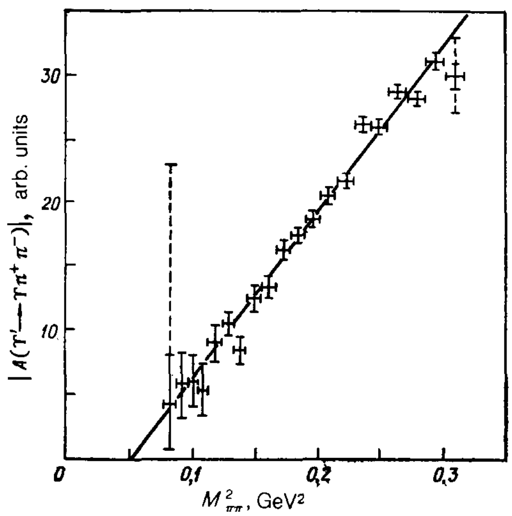

FIG. 15. $M^2_{\pi\pi}$ dependence of the matrix element for the decay $\Upsilon' \to \Upsilon\pi^+\pi^-$ extracted from data of the ARGUS group.[53] The dashed lines are the systematic errors which occur at the two extreme points because of the edge of the phase volume and the experimental resolution in the invariant mass.

First, the spectrum in the decay $\Upsilon'' \to \Upsilon\pi\pi$ differs from that described by (51) even at values of q which are observed in the transitions $\pi' \to J/\psi\pi\pi$ and $\Upsilon' \to \Upsilon\pi\pi$: $(q^2)^{1/2} \lesssim 550$ MeV (actually, just above the threshold $q^2 = 4m_\pi^2$ there is a significant number of $\Upsilon'' \to \Upsilon\pi\pi$ decay events; this situation is not observed in the two other transitions). Second, a singularity in q^2 which disrupts the linearity of matrix element (50) is expected[65] theoretically at a mass above 1 GeV, while experimentally the possible contribution of the next term in the expansion in $q^2(q^4/M^2)$ in the decay $\Upsilon' \to \Upsilon\pi\pi$ corresponds to $M > 1$ GeV at a 90% confidence level, according to an analysis of the q^2 dependence of the matrix element, which can be found from the data of Ref. 53 (Fig. 15).

One possible explanation might be based on the circumstance that the quarkonium matrix element A_0 is very small for this decay. (The width of the transition is much smaller than that for $\Upsilon' \to \Upsilon\pi\pi$, despite the fact that the phase volume is many times as large.) This suppression may be lifted by effects of the recoil of the b quarks, which varied over the phase volume. The amplitude A_0 may depend strongly on q^2. This explanation might be tested by searching for the transition $\Upsilon'' \to \Upsilon\eta$, whose width in this case is determined by (57) with $q^2 = m_\eta^2$. As can be seen from Fig. 14, some 10–15% of the $\Upsilon'' \to \Upsilon\pi\pi$ decay events fall in the mass interval of the dipion system, 520–600 MeV. Hence we can estimate the ratio of widths from (57):

$$\frac{\Gamma(\Upsilon'' \to \Upsilon\eta)}{\Gamma(\Upsilon'' \to \Upsilon\pi\pi)} \approx 0.017-0.025,$$

or, correspondingly, $B(\Upsilon'' \to \Upsilon\eta) \approx 0.1\%$.

Another, more exotic, possible explanation[66] is that the spectrum in this decay is distorted because a resonance X in the $\Upsilon\pi$ (or, equivalently, $\Upsilon''\pi$) system lies close to the physical decay region. The mass of X here would have to be slightly greater (by something of the order of tens of MeV) than the mass difference $M(\Upsilon'') - m_\pi = 10\,216$ MeV. The decay amplitude is then dominated by the pole of the X resonance which corresponds to the process shown in Fig. 16. Here X lies nearly on the mass shell. [The possibility that X is lighter than $M(\Upsilon'') = m_\pi$, and that a real decay $\Upsilon'' \to X\pi$ occurs, is ruled out since in this case the pion spectrum would consist of two essentially monoenergetic lines.] The shape of the pion spectrum depends on the quantum numbers (the spin and parity) of resonance X. If $J^P = 1^+$, the transitions $\Upsilon'' \to X\pi$ and $X \to \Upsilon\pi$ would be S-wave transitions, and the vertex would be proportional to the pion energy (according to the partial conservation of axial current). For the other waves, a power of the pion momentum should be involved. The theoretical curve in Fig. 14 is drawn for the contribution of an X pole in the case of an S-wave vertex with $M_X = 10\,245$ MeV. The maximum on this curve corresponds to the case in which one of the pions (that emitted at the $\Upsilon'' \to X\pi$ vertex) predominantly has an energy close to the minimum possible, $\varepsilon_1 = m_\pi$, i.e., is nearly at rest. In this case the second pion would necessarily have an energy close to $\varepsilon_2 = \Delta - m_\pi$. A configuration of this sort corresponds to $q^2 = 2m_\pi\Delta \approx (500\text{ MeV})^2$. Clearly, this feature should be seen more obviously in the pion energy spectrum (which has

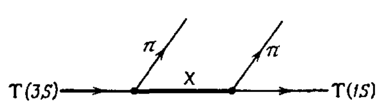

FIG. 16. Hypothetical mechanism for the decay $\Upsilon'' \to \Upsilon\pi\pi$ by virtue of an exotic resonance X in the $(\Upsilon\pi)$ channel.

not yet been plotted by the experimentalists) than in the q^2 distribution. There is some excess of events at large q^2 in comparison with a purely pole contribution in Fig. 14; the reason may be a small contribution of an amplitude proportional to q^2.

The resonance X might be manifested in the decay $\Upsilon(4S)\to X\pi$, whose signature is a monoenergetic pion. Unfortunately, however, it is exceedingly difficult to predict the probability for this transition. If we assume that the vertices representing the transition of all Υ resonances into $X\pi$ are identical, then we can work from the $\Upsilon''\to\Upsilon\pi\pi$ widths to estimate the magnitude of this vertex; taking this approach, we find $\Gamma(\Upsilon(4S)\to X\Pi)\approx 0.1$ MeV. This figure corresponds to a branching fraction of about 0.5%. However, this estimate may be incorrect, even in order of magnitude—in either direction.

A resonance X with these hypothesized properties would evidently have to be an isovector resonance, so it should consist of four quarks ($b\bar{b}q\bar{q}$, where q = u or d). Four-quark systems of this sort, consisting of heavy and light quarks, have been discussed in the literature from time to time (Ref. 67, for example). However, the previous candidates have also been amenable to a standard interpretation as quark-antiquark systems. We do not rule out the possibility that a first indisputably four-quark resonance will be found in connection with the decay $\Upsilon''\to\Upsilon\pi\pi$. In concluding this subsection we note that a resonance X might also dominate the transition $\Upsilon''\to\Upsilon'\pi\pi$, but the small phase volume in this decay (the energy release is only about 53 MeV) would make it extremely difficult to establish a difference between different functional dependences of the decay amplitude on the kinematic variables.

6.4. Transitions from Υ'' to the 1^1P_1 level

Because of the negative C parity, $^1P_1$ states cannot be produced in γ transitions from Υ resonances. However, pion transitions are allowed for precisely the same reason. The transition $\Upsilon''\to {}^1P_1\pi\pi$ has been proposed[56] as a source of 1^1P_1 resonances in an experiment. (The 1^1P_1 level subsequently decays primarily through a transition to $\eta_b\gamma$ so that η_b can also be observed.)

This transition results from an interference of the E1 and M1 interactions with gluons. In Ref. 56, gluons were treated as free (the parton picture), and the branching fraction of the transition $\Upsilon''\to 1^1P_1\pi\pi$ was estimated to be at the level of 1%. However, at an energy release of only about 455 MeV, the parton picture for gluons seems totally inapplicable. The reason is that the matrix element $\langle\pi^+\pi^-|E_i^aH_k^a|0\rangle$, which describes the conversion of gluons into π mesons in this transition, is determined[55] by that fraction (ρ^G) of the pion momentum which is carried by gluons:

$$\langle\pi^+\pi^-\mid\pi\alpha_s E_i^a H_k^a\mid 0\rangle=\frac{1}{2}\pi\alpha_s\rho^G\varepsilon_{ikm}(\varepsilon_1 p_{2m}+\varepsilon_2 p_{1m}),\qquad(60)$$

where $\varepsilon_{1,2}$ and $\mathbf{p}_{1,2}$ are the energies and momenta of the pions. If we compare the square of this amplitude ($\pi^0\pi^0$ states are also taken into account) with the square of the corresponding parton (gluon) amplitude, summed over the color and polarization states of the gluon, we find

$$\frac{\sum|A_{\pi\pi}|^2}{\sum|A_{gg}|^2}=\frac{(3/4)(\rho^G)^2(\varepsilon_1^2\mathbf{p}_2^2+\varepsilon_2^2\mathbf{p}_1^2)}{16(\omega_1^2\mathbf{k}_2^2+\omega_2^2\mathbf{k}_1^2)}\approx\frac{1}{80}\,\frac{\varepsilon_1^2\mathbf{p}_2^2+\varepsilon_2^2\mathbf{p}_1^2}{\omega_1^2\mathbf{k}_2^2+\omega_2^2\mathbf{k}_1^2}\qquad(61)$$

($\rho^G\approx 1/2$, and $\omega_{1,2}$ and $\mathbf{k}_{1,2}$ are the energies and momenta of the gluons). The small numerical factor here arises primarily because the number of gluon states in the denominator is larger than in the case of pions. An additional factor of about 0.22 stems from the integration over the phase volume when the mass of the π mesons is taken into account (we recall that gluons are massless). As a result it turns out[68] that the width of the $\pi\pi$ transition should be smaller than the parton estimate by a factor of nearly 400; i.e., it should be less than 10^{-4} of the total Υ'' widths.

As a result of this suppression, the transition involving the emission of a single π^0 meson, which violates isospin, should be an order of magnitude more probable than a $\pi\pi$ transition. Working from an estimate of the conversion of gluons into π^0 on the basis of (58), we find the following prediction of the ratio of widths:

$$\frac{\Gamma_{\pi^0}}{\Gamma_{\pi\pi}}=\frac{1120}{(\alpha_s\rho^G)^2\,\varphi\Delta^6}\left(\frac{\pi^2}{3\sqrt{2}}\,\frac{m_d-m_u}{m_d+m_u}\,f_\pi m_\pi^2\right)^2\approx 20\qquad(62)$$

($\varphi\approx 0.22$ is the factor—mentioned above— by which the integral over the phase volume of the two pions is suppressed).

The CLEO group recently reported data[64] on the spectrum of the invariant missing mass for the two pions in Υ'' decays. In this spectrum there is a small peak at a mass of about 9900 MeV: the expected mass of the 1^1P_1 level. The number of events in the peak corresponds to a branching fraction of $0.37\pm 0.15\%$ for this decay. However, the statistical significance of this peak is low (about 2.5 standard deviations). If this peak is confirmed at this level of the decay branching fraction as the statistical base is improved, then we will be able to conclude from the discussion above that we are dealing here with a transition caused not by a multipole emission but by some other mechanism. We do not rule out the possibility that again in this case the situation is dominated by the X resonance discussed in the preceding subsection.

7. SEARCH FOR NEW PARTICLES AND EFFECTS IN THE DECAYS OF Υ RESONANCES

Heavy quarkonium is a very compact blob of energy. Its decays may result in the production of new particles still in the hypothetical realm and also the production (under unusual conditions) of ordinary known hadrons, for which it will be possible to test various models of hadron dynamics.

7.1. Higgs boson

One of the most interesting possibilities is a search for the Higgs boson H in the decay[69] $\Upsilon\to H\gamma$. If the relationship between H and the b quarks is the same as in the minimal model of the electroweak interaction with a single Higgs doublet,

$$-(\sqrt{2}G)^{1/2}\,m_b H(\bar{b}b),\qquad(63)$$

then the following expression[70] will hold for the branching fraction for this decay, where the QCD radiation correction has been made:

$$\frac{\Gamma(\Upsilon\to H\gamma)}{\Gamma(\Upsilon\to\mu^+\mu^-)}=\frac{Gm_b^2}{\sqrt{2}\pi\alpha}\left(1-\frac{m_H^2}{m_\Upsilon^2}\right)\left(1-\frac{4\alpha_s}{3\pi}a(x)\right)$$

$$\approx 0.008\left(1-\frac{m_H^2}{m_\Upsilon^2}\right)\left(1-\frac{4\alpha_s}{3\pi}a(x)\right).\qquad(64)$$

Here $a(\varkappa)$ is a function of the ratio $\varkappa = m_H^2/m_\Upsilon^2$ which has a fairly complicated form,[70] but in the interval $0 \leqslant \varkappa \lesssim 0.7$ this function varies only slightly around a value $a \approx 6$. At an m_H mass close to m_Υ, it has an ordinary Coulomb singularity $(1-\varkappa)^{-1/2}$. We see that the radiation correction is large: It reduces the prediction for the branching fraction by a factor of nearly two (under these conditions, it would generally be necessary to consider higher-order perturbation theories, but technically that is a rather complicated matter).

A search for this decay is the best way to find a Higgs boson with a mass $m_H < m_\Upsilon$. In hadronic decays of Υ this boson is produced about an order of magnitude more poorly,[71] and events with an H boson do not have such a noticeable signature as the monochromatic photon in the decay $\Upsilon \to H\gamma$. The absence of a noticeable signature also hinders a search for H in hadron reactions which are unrelated to Υ resonances.

An experimental search[72,73] for monochromatic photons in Υ decays yielded a negative result. The best limitation on the decay branching fraction, according to preliminary data of the CUSB group,[73] is lower than that predicted by expression (64), even without the radiation corrections, at 1–2 GeV $< m_H <$ 4–5 GeV. However, the correction for α_s brings prediction (64) under the upper limit found.

The CLEO group studied the decay $\Upsilon \to \gamma\pi^+\pi^-$. That group rules out the presence of monochromatic photons at a level $B(\Upsilon \to \gamma\pi^+\pi^-) < 3\cdot 10^{-5}$ at invariant $\pi^+\pi^-$ masses from 0.5 to 8.5 GeV, except in three narrow regions, one of which is near the mass of the ρ meson. A Higgs boson with a mass of less than $2m_K$ should decay primarily into $\pi\pi$ pairs (Ref. 71); not into $\mu^+\mu^-$ pairs, as was assumed some time ago[74]). The theoretical predictions in this mass region is therefore $B(H \to \pi^+\pi^-) \gtrsim 50\%$, and we have

$$B(\Upsilon \to H\gamma)\, B(H \to \pi^+\pi^-) \gtrsim 6\cdot 10^{-5}. \qquad (65)$$

The data of Ref. 75 thus rule out an H boson with a mass between 0.5 and 1 GeV (except in the region near m_ρ) which has the standard value of the coupling with quarks.

The coupling of the H boson with leptons and quarks is rigidly fixed by the theory only in the minimal model. Even in the theory with two doublets a very wide spectrum of possibilities appears, and the interaction of Higgs bosons with various fermions could, generally speaking, be intensified or weakened to various extent. It is thus clearly worthwhile to pursue the search for the decay at any attainable accuracy level, and in various channels for the H decay (e.g.,[76] $H \to \tau^+\tau^-$).

7.2. Axion

Another hypothetical particle is the axion, a, which was invoked in an effort to solve the problem of the natural conservation of CP invariance in QCD.[77] In contrast with the scalar H boson, the axion is a pseudoscalar and very light particle (a typical mass is $\lesssim 50$ keV). If we write the coupling of the axion with the b quark in the form

$$-ia(\sqrt{2}G)^{1/2}\, x_b m_b\, (\bar{b}\gamma_5 b),$$

where x_b is a dimensionless number, the decay probability is given by an expression of the type[78]

$$\frac{\Gamma(\Upsilon \to a\gamma)}{\Gamma(\Upsilon \to \mu^+\mu^-)} = x_b^2 \frac{Gm_b^2}{\sqrt{2}\pi\alpha}\left[1 - \frac{4\alpha_s(m_b)}{3\pi}\left(\frac{\pi^2}{8} + 2\ln 2\right)\right]$$

$$\approx 0.006 x_b^2 \qquad (66)$$

(the correction for α_s here is extremely important, as it is in the case of H emission).

If $m_a < 2m_e$, the axion decays into two photons over a macroscopic time, so that the decay does not occur inside a detector. The upper limits which have been found[79–81] on $B(\Upsilon \to a\gamma)$ in the case of a long-lived axion of this sort range from[80] $3\cdot 10^{-4}$ to[81] $8\cdot 10^{-4}$; these figures correspond to $x_b \lesssim 1.6$. For an axion which decays rapidly into an e^+e^- pair inside the detector (such a decay is evidently possible if $m_a > 2m_e$), we find limits on $B(\Upsilon \to a\gamma)$ at the level of[81] $3.1\cdot 10^{-4}$ or[82] $5\cdot 10^{-4}$ (the limit of Ref. 81 also applies to the case in which the a decays into 2γ in the detector). At an intermediate lifetime ($\sim 4\cdot 10^{-12}$ s), and at a mass of 1–3 MeV, the efficiency of a search is slightly poorer, and the best limit is[81] $1.3\cdot 10^{-3}$.

7.3. The decay $\Upsilon \to$ "nothing"

Searches for the axion make use of "tagged" Υ mesons, produced in the decay $\Upsilon' \to \Upsilon\pi^+\pi^-$, in order to distinguish from the background caused by the process $e^+e^- \to \gamma\gamma$. The same "tagged" Υ can be utilized to search for the decay of Υ into invisible particles. Such a decay would be seen experimentally as the vanishing of the Υ or the decay of the Υ into "nothing." Decays of this sort are predicted by several models. For example, the standard model has the decay $\Upsilon \to \nu\bar{\nu}$, for which the probability, for each type of neutrino is

$$B(\Upsilon \to \nu\bar{\nu}) = \frac{9}{64}\frac{G^2 m_\Upsilon^4}{\pi^2\alpha^2} B(\Upsilon \to \mu^+\mu^-) \approx 6\cdot 10^{-6}, \qquad (67)$$

so that an experiment of this sort could in principle be used to limit the number of neutrino species. In the supersymmetric models, the decay of an Υ into a photino and gravitino[83] would look the same. For this decay, one can find a limitation on the mass scale of breaking of supersymmetry; alternatively, one could find a lower limit on the mass of the gravitino. In reality, however, at the accuracy attainable a search for the decay $\Upsilon \to$ "nothing" cannot compete in these areas with the limitations which have been found from a search for the process $e^+e^- \to \gamma +$ "nothing" at higher energies.[84] Nevertheless, the limit established by the ARGUS group,[81]

$$B(\Upsilon \to \text{"nothing"}) < 2.3\% \text{ (90\% confidence level)}, \qquad (68)$$

gives us the best limitation on the oscillations of "our" matter into mirror matter in the model of so-called mirror particles.

According to the hypothesis of mirror particles,[85] there is a separate world of particles which interact with each other by means of "their own" vector fields (the photon, etc.). The signs of the breaking of P and CP parity in the mirror world are opposite to those in "our" world (so that left-right symmetry is, in a sense, restored). It has been found[85] that among the known interactions the only one which could be common to "our" particles and the mirror particles is the gravitational interaction. However, new and sufficiently weak interactions might also be shared[86]; in particular, they might send "our"neutral particles into the mirror particles. If the mirror world is a precise copy of our own, then the mass of the Υ_m (the subscript "m" specifies a mirror particle) is degenerate with the mass of the Υ, and the presence of a vector interaction

$$\frac{G_X}{\sqrt{2}}(\bar{b}\gamma_\mu b)(\bar{b}_m\gamma_\mu b_m)$$

would lead to $\Upsilon \leftrightarrow \Upsilon_m$ oscillations. After oscillating into an Υ_m, an Υ meson would decay into mirror particles, which would be invisible to us.[86] The probability for this process is described by

$$B(\Upsilon \to \Upsilon_m \to \text{mirror particles})$$

$$= \frac{729 G_X^2 m_\Upsilon^4}{64\pi^2\alpha^4}[B(\Upsilon \to \mu^+\mu^-)]^2 = 0.3\left(\frac{G_X}{G_F}\right)^2. \qquad (69)$$

A comparison of this expression with the experimental limitation in (68) gives us a limit on the interaction constant G_X:

$$G_X \leqslant 0.3 G_F. \qquad (70)$$

7.4. Gluino

The presently popular models based on supersymmetry require the existence of superpartners of gluons: gluinos. Gluinos are Majorana particles with spin 1/2 and form a color octet. Because of the latter circumstance, the interaction of gluinos with gluons is slightly more intense than that of quarks with gluons. In particular, if a gluon $\hat{g}$ is sufficiently light, the decay $\chi_{b1} \to \hat{g}\hat{g}g$ could occur by the mechanism shown in Fig. 10, when the $q\bar{q}$ pair is replaced by $\hat{g}\hat{g}$. The ratio of the widths of the decays into $\hat{g}\hat{g}$ and $q\bar{q}$, if the q and $\hat{g}$ have the same mass, is determined exclusively by a color factor:

$$\frac{\Gamma(\chi_{b1} \to \hat{g}\hat{g}g)}{\Gamma(\chi_{b1} \to q\bar{q}g)} = 3 \qquad (71)$$

(here we are also incorporating the factor of 1/2 which arises from the identity of the Majorana gluinos). As a result the branching fraction for the decay $\chi_{b1} \to \hat{g}\hat{g}g$ may be about[46,87] 20%. Using $B(\Upsilon' \to \chi_{b1} + \gamma) = 7\%$, we would expect that a gluino pair would be present in about 1.5% of the Υ' decays, provided that the mass of the gluino did not exceed about 4 GeV.

The decay properties of gluinos depend on the particular model. In the most popular models, a photino, $\hat{\gamma}$—the superpartner of the photon—is lighter than a gluino, and the latter decays into $\hat{\gamma}q\bar{q}$ through the exchange of a scalar quark $\hat{q}$. The lifetime depends on the masses of the $\hat{g}$ and $\hat{q}$ and is given by

$$\tau_{\hat{g}} \approx 1.2\cdot 10^{-11}\left(\frac{m_{\hat{q}}}{100\ \text{GeV}}\right)^4\left(\frac{1\ \text{GeV}}{m_{\hat{g}}}\right)^5 \text{s}. \qquad (72)$$

The ARGUS group carried out a search for secondary vertices in the detector caused by the decay of gluinos.[88] The negative result of that search rules out gluinos with a mass between 1 and 4.5 GeV and a lifetime between 10^{-11} and 10^{-9} s.

However, there is the possibility, although it is improbable, that the photino is heavier than the gluino.[89] In such a case the gluino either would be stable in general—if it is the lighest superparticle— or would decay into a gluon and a gravitino over a very long time: months, years, etc. In this case, the decay of gluinos would of course not occur in the detector. However, some of the events should give rise to the production of a pair of heavy charged particles which would be stable at the scale of the detector and which would cause an anomalous ionization. Indeed, the lightest meson-like hadron formed by gluinos might be[89] ($\hat{g}g$): a glueballino, which is neutral. However,the lightest Coulomb-like hadron should be the state[90] ($\hat{g}uud$): a gluebarino, which has a charge. Tracks with an anomalous ionization in Υ' decays have been sought in an experimental search[91] for free quarks. If we assume that the probability for the fragmentation of a gluino into a gluebarino is not less than 10^{-2} (this figure is considerably smaller than the ratio p/π, i.e., considerably smaller than the probability for the fragmentation of a light quark into a baryon, and certainly smaller than the ratio $\Lambda_c/D \sim 1$—which is the same probability for a heavy c quark in e^+e^- annihilation), then we would conclude that the negative result of the experiment of Ref. 91 rules out the presence of a (quasi)stable gluino of this sort with a mass up to 4 GeV.

7.5. Hadronic states in the decays of bottomonium

In contrast with other hadronic processes, in the annihilation of bottomonium all the energy of the hadrons comes from a very compact region. This distinguishing feature could in principle lead to the production, in the decays of bottomonium, of hadronic states which either are not produced in other hadronic reactions or are produced there at a low probability. Such unusual conditions might serve as a "laboratory" for testing various models of hadron dynamics. In particular, various hadronic resonances could form in the decays of the Υ meson.

The research on the hadronic decays of the Υ is at present essentially limited to the study of only general, integral correlation characteristics such as the sphericity, the thrust, and the second Fox-Wolfram moment, which were mentioned in Subsection 5.2. Investigators are becoming convinced that these characteristics agree well with a three-gluon annihilation mechanism. However, today's detectors, with their good resolution and good identification of particles, are capable of studying subtler details of the observable events. For example, the decays of Υ have revealed an unexpectedly large number of events with baryons and even with double baryon-antibaryon pairs. There is the possibility of studying the correlation of the polarizations of the Λ hyperons in the decays $\Upsilon \to \Lambda\Lambda$ + anything, which may show whether Λ hyperons are produced independently. In particular, if a $\Lambda\Lambda$ resonance does exist, it may be manifested in such a correlation.

Our purpose here is to draw the attention of specialists in hadron dynamic models, who might suggest some specific questions for study in the decays of Υ and Υ' resonances in specific models. We do not rule out the possibility that answers to many such questions can be found in the statistical base of bottomonium decay events which has already been accumulated.

8. CONCLUSION

Today, a decade after the discovery of the first levels, bottomonium remains a field with many opportunities for experimental and theoretical research. The problems of the physics of bottomonium which have essentially been solved at this point are only some of the problems "lying on the surface." Suffice it to say that we still do not know the branching fraction for the decay of an Υ resonance in even one exclusive hadronic channel. Several of the problems which we believe are of foremost interest have been discussed in this review. Let us list them again: 1) refining the values of $B_{\mu\mu}$ for the excited Υ resonances; 2) measuring the spectrum of direct photons in the decay $\Upsilon \to \gamma +$ hadrons and measuring the probability for this decay; 3) observing and measuring the probabilities for the transitions $\Upsilon' \to \Upsilon\eta$ and $\Upsilon'' \to \Upsilon\eta$; 4) determining the dynamics of the $\Upsilon'' \to \Upsilon\pi\pi$ transition. These problems also are actually lying on the surface; it is only a matter of time before they will be solved at the existing accelerator installations with existing detectors.

Subtler problems such as observing the bottomonium ground state η_b or measuring the total widths of the χ_{bJ} levels will probably require new and more sophisticated experimental apparatus. As time goes by, of course, new problems may arise and, as usually happens, come to be regarded as the most important ones.

With regard to the theoretical problems pertaining to bottomonium physics, an assessment of their comparative importance is unavoidably a subjective procedure. It does appear to be beyond dispute, however, that the central question has been and remains that of generating a competent description of the interaction of quarks within the framework of QCD. As we have already mentioned, the existing methods solve this problem only partially. The potential description uses phenomenological parameters (the slope of the linear part of the potential, the nature of its joining with the Coulomb-like part, and the hypothesized Lorentzian structure of the potential) which are fitted to the experimental data, rather than calculated from the original theory: QCD. Furthermore, because of the factors discussed in Subsection 3.3, it is not clear to what extent we can expect the potential approach to be applicable at all in QCD. On the other hand, the sum rules, although a direct consequence of the well-grounded operator relations in QCD, have clearly inadequate predictive powers; for example, the properties of the excited states in a channel with definite quantum numbers J^{PC} are still completely inaccessible to study by this method. Furthermore, as we discussed in Subsection 3.5, the accuracy which is technically attainable in calculations of the coefficients in an expansion in vacuum expecta-

TABLE III.

Particle	Mass M, MeV	Total width Γ, MeV	Decay modes	Probability, % (upper limits on the 90% confidence level)
Υ (9460), 1^3S_1	$9460{,}0^a \pm 0{,}2$	$0.043 \pm 0{,}003$	e^+e^-	$2.8 \pm 0{,}3$ [b]
			$\mu^+\mu^-$	2.8 ± 0.2 [c]
	$\Gamma_{ee} = 1{,}22 \pm 0.05$ keV		$\tau^+\tau^-$	3.2 ± 0.4
			$\rho\pi$	< 0.21
			$J/\psi + X$	2
			"nothing"	< 2.3 [61]
			γgg	$2{,}99 \pm 0.59$ [44]
				$1{,}88 \pm 0{,}14 \pm 0.17$ [45]
χ_{b0} (9860), 1^3P_0	$9859.8 \pm 1{,}3$		Υ (9460) γ	< 6
χ_{b1} (9895), 1^3P_1	$9891.9 \pm 0{,}7$		Υ (9460) γ	35 ± 8
χ_{b2} (9915), 1^3P_2	$9913.3 \pm 0{,}6$		Υ (9460) γ	22 ± 4
Υ (10 023), 2^3S_1	$10\,023.4 \pm 0{,}3$	$0{,}030 \pm 0{,}007$	$\mu^+\mu^-$	$1.8 \pm 0{,}4$
			$\tau^+\tau^-$	1.7 ± 1.6
	$\Gamma_{ee} = 0{,}54 \pm 0{,}03$ keV		Υ (9460) $\pi^+\pi^-$	$18{,}7 \pm 1.0$ [d]
			Υ (9460) $\pi^0\pi^0$	8.6 ± 1.1 [e]
			Υ (9460) η	< 0.2
			χ_{b0} (9860) γ	4.3 ± 1.0
			χ_{b1} (9895) γ	6.7 ± 0.9
			χ_{b2} (9915) γ	6.6 ± 0.9
			γgg	3.37 ± 1.14 [44]
χ_{b0} (10 235) 2^3P_0	$10\,233 \pm 5$		Υ (9460) γ	Seen
			Υ (10 023) γ	»
χ_{b1} (10 255), 2^3P_0	$10\,255 \pm 2$		Υ (9460) γ	»
			Υ (10 023) γ	»
χ_{b2} (10 270), 2^3P_0	$10\,271 \pm 2$		Υ (9460) γ	»
			Υ (10 023) γ	»
Υ (10 355), 3^3S_1	$10\,355{,}5 \pm 0{,}5$	$0.012 {+0{,}010 \atop -0{,}004}$	$\mu^+\mu^-$	3.3 ± 1.5
			Υ (9460) $\pi^+\pi^-$	4.5 ± 0.8 [f]
			Υ (10 023) $\pi^+\pi^-$	3 ± 2 [g]
	$\Gamma_{ee} = 0{,}40 \pm 0{,}03$ keV		χ_{b1} (10 255) γ	15.6 ± 4.2
			χ_{b2} (10 270) γ	12.7 ± 4.1
Υ (10 575), 4^3S_1	$10\,577 \pm 4$	24 ± 2		
	$\Gamma_{ee} = 0{,}24 \pm 0.05$ keV			
Υ (10 860), 5^3S_1	$10\,865 \pm 8$	110 ± 13		
	$\Gamma_{ee} = 0.31 \pm 0.07$ keV			
Υ (11 020), 6^3S_1	$11\,019 \pm 9$	79 ± 16		
	$\Gamma_{ee} = 0.13 \pm 0.03$ keV			

[a] $M(\Upsilon(9460)) = 9460{,}59 \pm 0{,}12$ MeV [13a].
[b] $B(\Upsilon(9460) \to e^+e^-) = 2{,}42 \pm 0.14 \pm 0.14\%$ [53].
[c] $B(\Upsilon(9460) \to \mu^+\mu^-) = 2.30 \pm 0{,}25 \pm 0.13\%$ [53], $2{,}53 \pm 0.17 \pm 0.16\%$ [92].
[d] $B(\Upsilon(10\,023) \to \Upsilon(9460)\,\pi^+\pi^-) = 18{,}1 \pm 0{,}5 \pm 1.0\%$ [53].
[e] $B(\Upsilon(10\,023) \to \Upsilon(9460)\,\pi^0\pi^0) = 9{,}5 \pm 1{,}9 \pm 1.9\%$ [53].
[f] $B(\Upsilon(10\,355) \to \Upsilon(9460)\,\pi^+\pi^-) = 3{,}47 \pm 0.34\%$ [64].
[g] $B(\Upsilon(10\,355) \to \Upsilon(10\,023)\,\pi^+\pi^-) = 2.1 \pm 0.5\%$ [64].

tion values of operators in QCD has so far been inadequate to find satisfactorily the positions of the P levels in the $b\bar{b}$ system.

Another question which seems important is that of calculating the nonperturbative effects in the annihilation of heavy quarkonium, both in the total widths and in the spectra, in particular, in the spectrum of direct photons in the decay of an Υ resonance near the maximum photon energy.

Yet another interesting field for research is the interaction of heavy quarkonium with gluons, light quarks, and the light mesons made up of them. Here we mean hadronic transitions between levels of quarkonium and the role played by an admixture of a pair of light quarks in heavy quarkonium. In particular, if the resonance X discussed in Subsection 6.3 does exist, then a pair of light quarks in it will be in an isotriplet state. At present we know of no useful approximations for dealing with the dynamics of such four-quark systems.

We have listed here only certain types of problems which bear directly on the questions which we have discussed in this review. This list of course does not come close to exhausting the total list of questions. For example, we have said absolutely nothing about the production of bottomonium in processes other than e^+e^- annihilation or the scattering of bottomonium by hadrons, primarily because of the paucity of experimental data and of theoretical studies in this field. We have also said nothing about theoretical models for exclusive hadronic decays of levels of the $b\bar{b}$ system, because it seems to us that this field is still extremely far from that stage of development in which it could be regarded as one of the "advances" in the physics of heavy quarkonium.

In summary, the number of experimental and theoretical problems is large, and one does not need any great predictive powers to assert that the physics of bottomonium will remain the subject of significant experimental and theoretical research for many years yet and will permanently remain of interest for reaching an understanding of the dynamics of quarks and gluons.

9. APPENDIX

We have added a summary of experimental data on the levels of bottomonium. Most of these results were reproduced from the tables of Ref. 2. For the most part, these are numbers averaged over the results of several experiments, published through 1 December 1985. We have also included in Table III a few results pertaining to the probabilities of certain inclusive decay modes of Υ and Υ' resonances (data of this sort are usually not cited in the tables of Ref. 2), and we have added in the form of notes some new data published after the compilation of the tables of Ref. 2.

[1)]The symbol "b" has two interpretations: a romantic one, "beauty" (the Russian word is "krasota" or "prelest' ") and a mundane one, "bottom" (the Russian work is "niz"). The latter interpretation stems from the fact that the b quark is the lower component of a weak doublet (t, b) (t meaning "top"; the t quark has yet to be observed experimentally, because its mass is even larger than that of the b quark). For the $b\bar{b}$ system there is a second interpretation; it is called "bottomonium" by analogy with positronium (e^+e^-) and charmonium ($c\bar{c}$).

[2)]The primes on the radially excited states have recently started to be replaced by excitation numbers; the Υ resonance is designated $\Upsilon(1S)$, Υ' is $\Upsilon(2S)$, etc. We will be making frequent use of this notation below.

[3)]A detailed review of the potential models, with an exhaustive bibliography, was published a few years ago in this journal[15a] (see also Ref. 15b).

[4)]The smaller of the values shown here corresponds to that extracted from a comparison of the sum rules of charmonium with experimental data[21]; the larger one was found for bottomonium[27] (Subsection 3.5). When we allow for the theoretical uncertainties in the extraction of vacuum expectation value (5), we conclude that values in the interval given here do not contradict an analysis of the two systems.

[5)]In the leading nonrelativistic approximation, the quarks interact with only an electric (not magnetic) component of the field, in complete analogy with the interaction in electrodynamics.

[6)]It should not be surprising that it follows from this result and from (5) that $\langle (E^a)^2 \rangle$ has a negative value, since, as was stipulated in connection with (5), this quantity is the difference between the total vacuum expectation value and its value in perturbation theory.

[7)]The experimental data used in the text are given in the Appendix.

[8)]In a potential model, an increase in nonperturbative effects corresponds to the situation in which the relative contribution of the linear part of the potential increases with increasing size of the system.

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