Heat conduction in bodies with linear thermoelasticity

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W. A. Day. *Heat Conduction with Linear Thermoelasticity.* Springer-Verlag, New York; Berlin; Heidelberg; Tokyo, 1985, p. 82 (Springer Tracts in Natural Philosophy/ Ed. C. Truesdale, V. 30).

Springer Publishing House has published within its series of tracts on Natural Philosophy the monograph by a member of the mathematics institute of Oxford University William Alan Day on the subject indicated in the title of this review. The book discusses the mathematical properties of the system generalizing the classical Fourier model to the case of thermal conductivity in a deformable solid. The author justifiably considers that for a comparative analysis it is more convenient to study fundamental problems using the example of one-dimensional linear processes. Moreover, he assumes rigid clamping of the ends of the solid, thereby specifying zero displacements. In such a formulation the problem is closest to the classical problem of heat conductivity in an absolutely rigid body. The equations of linear thermoelasticity reduce in this case to a single equation in partial derivatives of the fourth order for the temperature which is a generalization of the classical Fourier equation. In its dimensionless form the equation contains two parameters one of which reflects the interrelationships of the temperature and displacement fields, while the other characterizes the inertial property of mechanical displacements. If both parameters are set equal to zero we obtain the classical approximation. Another approximation—the coupled quasistatistical problem—is obtained if we set equal to zero the inertial parameter, but assume the coupling parameter to be different from zero (and positive). In this case the problem reduces to the solution of an integro-differential equation in partial derivatives. A discussion is given of the status of the classical and the coupled quasistatic approximations. Limitations are formulated imposed on the bounding functions which guarantee asymptotic adequacy of the approximate and exact solutions.

In the coupled quasistatic and dynamic cases violation of the maximum principle—one of the typical properties of parabolic systems—is noted. The solution of the classical equation of heat conductivity has this property to the fullest extent. But when more rigid limitations are imposed on functions describing the boundary conditions some weaker analogs of the maximum principle can be established also for the coupled quasistatic and dynamic cases. It is emphasized that the aforementioned deviations occur for arbitrarily small values of the coupling parameter.

The properties of the solutions of the coupled quasistatic approximation are investigated in considerable detail. The existence and uniqueness of the Cauchy problem are proved for the integro-differential equation in the case of homogeneous boundary temperatures. A solution of this problem in the form of an expansion in eigenfunctions is constructed. The trigonometric form of the solution is discussed which is convenient in the case when the boundary temperatures are almost periodic functions of the time. At low frequencies the averages of the squares of the temperature and the heat flux over time and over the dimensions of the solid can be represented in the form of converging series in terms of the index of differentiation of the squares of the differences and sums of the derivatives of the boundary temperatures. For time averages of the local values of the density of the total energy and the heat flux an equivalent of the maximum principle is derived.

The properties of the temperature solutions of the complete system of equations of thermoelasticity are discussed for the mixed boundary value problem with a fixed temperature at one boundary and a given heat flux the other. When the boundary heat flux vanishes a proof is given of the theorem concerning the asymptotic stability of the solution. It is also shown that under certain limitations on the rate of growth of the boundary heat flux with time the time average of the value of the local temperature is connected by a simple relationship with the average value of the boundary heat flux, and with time the instantaneous local temperature also tends to the same distribution.

On the whole the monograph collects together the scattered journal publications of the author and will be of interest for specialists in the field of equations of mathematical physics. In its applied aspect the monograph will allow one to approach in a more firmly based and rigorous manner to the use of approximate formulations for the solution of practical problems of calculating temperature stresses in deformable solids and construction elements.

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