The hydromagnetic dynamo as the source of planetary, solar, and galactic magnetism

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The magnetism of most celestial bodies, i.e., planets, stars, and galaxies, is of hydromagnetic origin. The turbulent hydromagnetic dynamo is the principal mechanism whereby the magnetic field is amplified and maintained, and the theory of this phenomenon has advanced significantly in recent years. This review discusses applications of the theory of the turbulent dynamo to real objects, taking the Sun, the Earth, and the Galaxy as examples. Most of the discussion is concentrated on the large-scale magnetic field averaged over turbulent fluctuations. The average field is amplified and maintained by the average helicity of turbulent motion and large-scale shear flows such as differential rotation. The dynamo theory explains striking phenomena such as geomagnetic field reversal, the solar cycle, and the ring and bisymmetric structure of spiral galaxies.

CONTENTS

1. INTRODUCTION

The phrase *hydrodynamic dynamo* refers to the process whereby the magnetic field is amplified and maintained by the motion of a conducting fluid. The ability of hydrodynamic motion to simulate the action of the dynamo machine, but without wires and coils, was first noted at the physical level by J. Larmor[1] in an attempt to explain the origin of terrestrial and solar magnetism.

However, following the critical paper by Cowling,[2] attention shifted toward mathematical aspects, and attempts were made to establish the existence of the dynamo by constructing idealized examples and proving theorems. This culminated in considerable success, mostly in the kinematic formulation in which the reaction of the generated magnetic field on the motion producing it was not taken into account. Many examples of the dynamo have now been constructed for laminar flows.[3–5] Flows with stochastic current lines[6–8] are particularly interesting. Methods have been developed for deriving and solving the equations for the average magnetic field in turbulent flows,[4,9,10] and studies of the higher moments have revealed the nonuniform, intermittent character of the generated field, which is concentrated in individual braids and layers.[10,11]

Kinematic dynamos have been classified into fast and slow,[10,12,13] depending on the rate of growth of the field at high magnetic Reynolds numbers. A clear illustration of the slow dynamo is provided by the Alfven model,[10,14] in which *an initial closed tube of magnetic lines of force stretches to* twice its size and divides into two tubes after a close approach along one of the diameters and action of ohmic magnetic diffusion at the point of approach. The two tubes then combine into one, without rotation, and the resultant tube has twice the magnetic flux. The increase in the field in this model depends critically on magnetic diffusion, and stops when diffusion becomes negligible. The fast dynamo, in which the rate of growth does not depend on the low rate of magnetic diffusion, can be modeled by the well-known figure-of-eight, proposed by one of the present authors (see Refs. 10, 13, and 15).

It may therefore be concluded that, from the mathematical point of view, the existence of the hydromagnetic dynamo has been proved. However, the original question, relating to *physical* applications of the dynamo, still stands.

A moving conducting fluid (plasma) is a natural component of most celestial bodies. It is also used in large modern technological installations, such as MHD generators, breeder reactors with liquid-metal coolants, and metallurgical installations. It is common to characterize the relative contributions of motion capable of generating the field and magnetic diffusion ν_m by the dimensionless magnetic Reynolds number $\mathrm{Re}_m = lv/\nu_m$, where l and v are the spatial and amplitude scales of the velocity field. The Reynolds number is equal to the ratio of the rate at which magnetic energy is generated to Joule dissipation (intermediate scales are assumed absent). Table I lists some very approximate values of Re_m for a number of technological and cosmic objects. The magnetic Reynolds number is greater than unity for practically all cosmic plasmas, i.e., *motion predominates over* magnetic diffusion. This enables us to identify regions in which the magnetic energy exceeds kinetic and thermal en-

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TABLE I. Estimated parameters of some moving conducting fluids.

	l, m	v, cm/s	ν_m, cm^2/s	Re_m
MHD installation	10^2	$6\cdot10^4$	$1.5\cdot10^9$	$4\cdot10^{-3}$
Breeder reactor	$2.5\cdot10^2$	$5\cdot10^2$	$1.6\cdot10^3$	80
Earth's core	10^8	$4\cdot10^{-2}$	$2.6\cdot10^4$	160
Solar convective shell	$2\cdot10^{10}$	10^5	10^7	$2\cdot10^8$
Galactic disk	10^{21}	10^6	10^{21}	10^6

ergies and regions in which the magnetic energy is less than the critical value, so that the field can be amplified by the hydromagnetic dynamo.

The condition $\mathrm{Re}_m \gg 1$ is not sufficient for the dynamo to work. In general, topological complexity of the velocity field is also necessary. In laminar flows producing the dynamo effect, the velocity field is not trivial, e.g., the flow cannot be plane.[16] Turbulent flows are so tortuous that the condition of topological complexity is automatically satisfied. Usually, the sufficient condition for the generation of an average magnetic field is that the mean helicity $\langle \mathbf{v}\cdot\mathrm{curl}\,\mathbf{v}\rangle$ is not zero. Fields with zero average value are generated when the average helicity is zero.[17,18] When $\mathrm{Re}_m \gg 1$, a nonstationary random flow of a general form leads to an increase in the magnetic field.[10]

Thus, to determine whether the dynamo appears in a given plasma region, it is sufficient to estimate the magnetic Reynolds number and establish the nature of motion, e.g., to show whether it is turbulent. Although turbulence is typical, uncertainty about the parameters of a particular object often means that one cannot reliably demonstrate that its motion is turbulent. This is the situation in the case of the interior of the Earth, for which the estimated kinematic viscosity unfortunately "varies" between wide limits, amounting to nine orders of magnitude. Nor is there complete certainty about our knowledge of large-scale flows, e.g., differential rotation and meridional circulation in the core of the Earth. The turbulent nature of convective motion on the Sun is revealed by direct observations of granules and supergranules. Helioseismology has recently demonstrated that the angular velocity of the Sun depends on depth.[19] The determination of the velocity field in stars observed as point objects is a subtle problem but, in principle, it can be solved both observationally and theoretically.

The observational determination of the velocity field is complicated by the fact that the regions that are of interest for the dynamo are located in the interior of celestial bodies, most of which are remote from the observer. The only exception is the gaseous disk of the Galaxy. Here, we are in the interior of the dynamo region and have direct data on the distribution of rotational velocity along the radius of the disk and, at least in principle, on the random component of small-scale motion.

For the theory of the hydrodynamic dynamo, it is best to have theoretically calculated velocity fields. However, the solution of hydrodynamic problems that require the inclusion of gravitation, stratification, Coriolis forces, and other factors that are specific for celestial bodies is not as yet in a satisfactory state. Advances in hydrodynamics in this area are exceedingly important for the theory of the dynamo. We note the papers of Gilman[84] and Glatzmaier in this connection.[85]

The uncertainty about the velocity field is the main difficulty faced by applications of dynamo theory. Nevertheless, by using certain empirical, theoretical, and hypothetical flow parameters, it has been possible to obtain many results that are relevant for applications. The present review is concerned with the principal advances and remaining problems relating to applications of the hydrodynamic dynamo.

Some of the consequences of the theory of the dynamo in its modern form have been the subject of criticism. It is important to distinguish between difficulties that can be overcome (problems relating to the evolution of the theory) and the fundamental impossibility of explaining the origin of the magnetic field of certain objects in terms of the dynamo. We are persuaded that the principal problems of terrestrial, solar, and galactic magnetism can be solved only with the aid of dynamo theory. On the other hand, the magnetism of stars with chemical anomalies and, possibly, the magnetism of certain other objects, may be of different origin, e.g., chemical or thermal. The generation of magnetic fields by means other than the dynamo, e.g., by the battery mechanism, has been discussed in detail by A. Z. Dolginov in a review paper that we have already mentioned.[89] These mechanisms can operate in parallel with the dynamo. The important point is that the dynamo gives rise to an exponential increase in the field $[H\sim\exp(\gamma t)]$, whereas, in the battery effect, the magnetic field increases only as a certain power of the time. We note that the final field depends both on the dynamo and on the initial conditions. However, when $\gamma t \gg 1$, the field no longer depends on the initial state and, as a rule, begins to have a nonlinear effect on motion.

It is possible to combine the dynamo and battery mechanisms. This approach, referred to as the "semidynamo," was proposed and developed by E. M. Drobyshevski.[20] Moreover, the dynamo mechanism relies on the nucleating initial magnetic field, which must be produced by some other mechanism.

The hydromagnetic dynamo is not, therefore, an isolated mechanism for the generation of the magnetic field and, in the sense indicated above, cannot be regarded as universal. It is simply a physical process that is effective in the moving conducting medium that is a major component of celestial bodies.

The amplification of the magnetic field necessarily leads to a nonlinear situation in which the field itself affects motion. This is also indicated by observations of magnetic fields which usually reach a quasistationary (often oscillatory) state and evolve together with motion. The hydromagnetic dynamo must therefore operate in the nonlinear regime. Fortunately, the situation is not equivalent to the complete magnetohydrodynamic problem. The average magnetic energy is usually small in comparison with the kinetic energy, and the nonlinearity can be tackled in a simplified manner. For example, it is often possible to take into account the effect of the field on the flow helicity, to neglect its effect on rotation and turbulent diffusion, to take into account the suppression of Re γ at constant periodicity (determined by Imγ), and so on.

We shall consider applications of the theory of the hydromagnetic dynamo in three basic areas, namely, the nature of terrestrial magnetism, the solar cycle, and the amplification of the magnetic field in the Galaxy. Each topic has attracted many review papers and monographs (see, for example, Refs. 4, 9, 13, 21 and 36). However, since we are in the midst of considerable advances in cosmic magnetism and, especially, the alternative mechanisms discussed in Dolginov's review,[89] it is opportune to provide a brief qualitative discussion of the dynamo approach and note recent new results. Our decision to be brief has meant that we have had to confine our attention to the analysis of the average magnetic field, and leave on one side questions such as fluctuation fields and intermittency, even though they are of considerable interest for applications. Figure 1 shows schematically sections through the Earth, the Sun, and the gaseous disk of the Galaxy.

2. THE ORIGIN OF TERRESTRIAL MAGNETISM

The compass was invented in China in the second century and reached Europe in the thirteenth century. However, the turning of its needle was ascribed to a "force originating in the Polar star" and not to the geomagnetic field.[22] The psychological difficulty associated with the transition from an ordinary magnet to a large magnet such as the Earth was first overcome by Gilbert in 1600. In 1838, Gauss suggested that the Earth's magnetic field could be described by an expansion in terms of spherical harmonics, whose coefficients could be determined by direct measurement of the field on the surface (and now by satellite measurements, as well).

In the first approximation, the terrestrial magnet is a dipole inclined at 11° to the rotational axis, and produces a magnetic field of 0.3 G at the magnetic equator. The amplitudes of more than ten harmonics after the dipole have now been determined and have been found to decrease in accordance with a power-type law with a break at the eighth harmonic.

Are we justified in continuing with Gauss' formal expansion? The point is that the field distribution over the Earth's surface is, in fact, spotty. The residual field (total minus dipole) has a finite number of anomalies that occupy regions of size ranging from a few hundred to two thousand kilometers (we are ignoring local magnetic anomalies, e.g., the Kursk anomaly, which have a totally different origin). Empirically, this field is sometimes described by fifteen dipoles complementing the main dipole and distributed in the core or on the boundary between the core and the mantle.

The geomagnetic field is not static. The characteristic times of its variation range from about 10 years (more rapid variations are screened by the skin effect in the lower mantle) to the age of the Earth (4.6 billion years).

These variations are important in estimates of characteristic velocities in the Earth's core. For example, the westerly latitude drift of 0.18° per annum in the residual field corresponds to a velocity of 0.04 cm/s in the upper part of the core. However, additional information is necessary to enable us to identify this velocity with convection or differential rotation, or the velocity of some waves. Observations reveal the presence of random fluctuations in the direction of the dipole moment with characteristic times of 10^3–10^4 years. When the mean terrestrial dipole is averaged over these fluctuations, its direction is found to lie along the rotational axis. Consequently, rotation has a considerble effect on the evolution of the magnetic field. Inversion of the magnetic dipole occurs in a characteristic time of the order of 10^5 years. The process is random.[32] This is one of the most intriguing mysteries in the nonlinear magnetodynamics of the Earth. It would be too naive to view the inversion process as the rotation of the dipole magnetic moment vector, with its poles running over all the latitudes. A more likely scenario is that, during the inversion process, the magnetic field becomes more complicated, departs from the dipole configuration, and other harmonics assume comparable magnitudes. This is indicated by the fact that the amplitude of the resultant field decreases to 10% or less, whereas, in the normal state, the dipole mode accounts for about 90% of the field intensity.

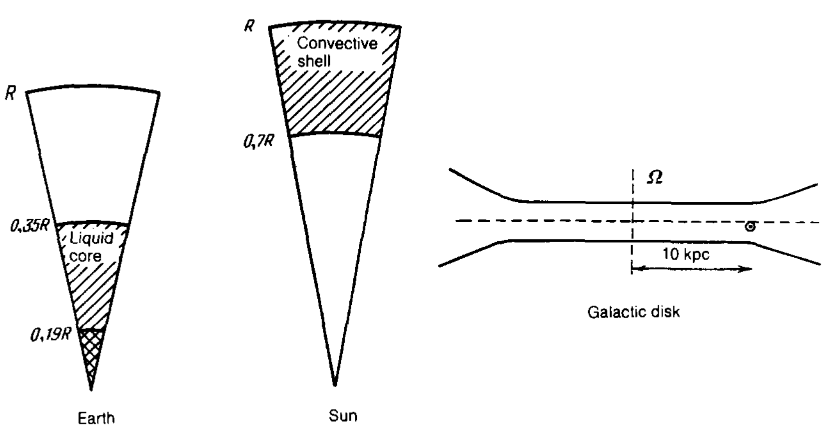

FIG. 1.

The principal source of motion in the Earth's core is probably the reduction in gravitational energy, due to the growth of the inner core $r < 0.19R \equiv R_1$, which occurs as the heavy fraction descends and the light fraction floats up.[23–25] Seismic data show that these fractions are iron and, probably, sulfur and oxygen ($\lesssim 10\%$ by mass). Convective stability is largely determined by variations in density with depth. Density is a function of three variables, namely, pressure, temperature, and mass concentration of the light fraction: $\rho = \rho(P, T, \xi)$. It has been shown[25] that variations in composition are more significant than temperature variations, and compositional rather than thermal convection is produced in the outer core.

The rate of laminar compositional convection has been estimated by Golitsyn[26] on the basis of similarity theory, taking into account the rapid rotation of the Earth:

$$v \approx C\left(\frac{Mg}{\Omega\rho}\right)^{1/2},$$

where $M = \langle \rho' v_r' \rangle$ is the flux of density deficiency, $\Omega \simeq 7\times 10^{-5}\ \mathrm{s}^{-1}$ is the angular velocity of the Earth, and C is a constant of the order of unity. The dependence on angular velocity is different in the case of turbulent convection. According to Stevenson,[27] $v \sim \Omega^{-1/4}$. However, numerical estimates of the rate of convection obtained for the parameters of the Earth's core are very similar and turn out to be of the order of 0.1–0.2 cm/s.

The nonuniform rotation of the Earth produces the inertial force $-\rho(\dot{\mathbf{\Omega}}\times\mathbf{r})$. The most interesting is the precession of the axis of rotation, for which $\dot{\Omega} \sim 2\times 10^{-16}\ \mathrm{s}^{-2}$ (the complete rotation of the axis over a cone of angle $23.5°\times 2$ occurs in 25 800 years). The angular velocity of this precession depends on the dynamic compression ε of the body which is different for the core and for the mantle. This difference between the angular velocities of precession brings into motion the liquid outer core filling the spherical cavity between the mantle and the solid inner core. The incompressible liquid in the cavity has a natural frequency $\Omega(1+\varepsilon)$. Hence, the effect of the induced force $\mathbf{F} = -\rho\mathbf{\Omega}_0\times(\mathbf{\Omega}\times\mathbf{r})$ of frequency $\mathbf{\Omega}$ is to produce high velocities of almost rigid-body rotation, $v_p \sim \varepsilon^{-1}\mathbf{\Omega}_p R_1 \sim 1$ cm/s,[28] at the same frequency Ω. Velocity deformations that are important for the generation of the magnetic field are produced by the transformation of precession energy into kinetic energy in the viscous boundary layer between the mantle and core. The efficiency of this transformation is low, which has given rise to scepticism in relation to the role of precession.[29,30] Further difficulties are noted in Ref. 27. However, the difficulty has not been resolved and precession continues to be discussed as a possible source of motion (dynamo) in the Earth and the planets.[31]

There is no doubt that nonthermal compositional convection is a flow capable of generating a magnetic field in the Earth's core. However, it is not clear whether the corresponding velocity field is laminar or turbulent. The difficulty relates to the uncertainty in kinematic viscosity[4]: $10^{-3} < \nu < 10^{9}\ \mathrm{cm^2/s}$. It is usually assumed that $\nu = 10^{-2}\ \mathrm{cm^2/s}$, which is close to the lower limit. The Reynolds number $\mathrm{Re} = lv/\nu$ for $l = 10^8$ cm and $v = 4\times 10^{-2}$ m/s is in excess of 10^6, and convection should be turbulent. Nevertheless, many investigators of the geomagnetic dynamo prefer stationary laminar flows. A detailed analysis of the numerous dynamo solutions for stationary velocity fields has been carried out by Yu. A. Brodsky[33] (see also Refs. 3–5). Because the velocity field is three-dimensional and asymmetric, the theory of the laminar dynamo is a relatively complicated mathematical construction.

Not surprisingly, the turbulent dynamo is simpler and easier to understand than the laminar dynamo. In the limit of high magnetic Reynolds numbers, there are now not only effective methods of obtaining and solving the equations for the mean field and the correlation function,[4,9,10,21] but it is even possible to deduce certain conclusions about the behavior of the true random magnetic field in turbulent flows.[10]

In applications to the geodynamo, it is usual to consider only the magnetic field averaged over fluctuations in the turbulent case and over the azimuth or time in the laminar case. It is striking that, in both cases, the sources of the dynamo are physically identical characteristics of the velocity field, namely, nonuniform (differential) rotation $\Omega(r,\theta)$ and mean helicity $\langle \mathbf{v}\cdot\mathrm{curl}\,\mathbf{v}\rangle$.

Differential rotation creates the toroidal magnetic field B_φ from the poloidal field $\mathbf{B}_\mathrm{p} = (B_r, B_\theta)$. The mechanism of this can be readily elucidated in terms of frozen-in magnetic lines of force, i.e., by neglecting dissipation. When $\nabla\Omega\mathbf{B}_\mathrm{p} \neq 0$, different segments of the lines of force of the field $\mathbf{B}_\mathrm{p}$ rotate with different velocities, so that the magnetic line is pulled out in the azimuthal direction (Fig. 2).

The mean helicity is a measure of the departure from mirror symmetry of the flux or, in other words, of preferential sense of twist. It is due to Coriolis forces in the stratified medium. The particular mechanism responsible for helicity is indicated in the next Section. The helicity $\langle \mathbf{v}\cdot\mathrm{curl}\,\mathbf{v}\rangle$ is a quadratic function of velocity. It appears in the averaged equations of electrodynamics, which are linear in the magnetic field because of the presence of the electric field $\mathbf{v}\times\mathbf{H}$ in Ohm's law for a moving medium. The electric field averaged over fluctuations is $\langle \mathbf{v}\times\mathbf{H}\rangle = \alpha\mathbf{B}$, where $\mathbf{B} = \langle \mathbf{H}\rangle$ is the mean magnetic field, $\alpha = -(\tau/3)\langle \mathbf{v}\cdot\mathrm{curl}\,\mathbf{v}\rangle$, and $\tau = l/v$ is the characteristic time of turbulent motion).

Approximating the generation region by a layer of thickness $2h$, which is much less than the radius of the outer core $R_2 \equiv 0.55R$, we find that the equation for the generation of an axially symmetric magnetic field can be written in the form

$$\frac{\partial A}{\partial t} = R_\alpha \alpha(r,\theta) B + \Delta A, \qquad (1)$$

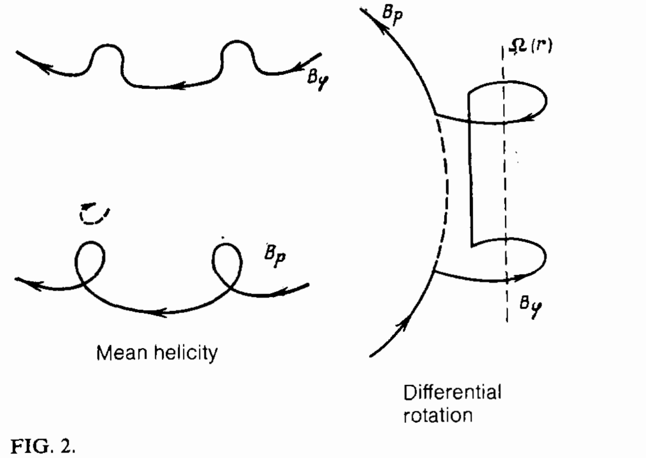

FIG. 2.

$$\frac{\partial B}{\partial t} = R_\Omega \nabla\Omega \cdot B_p + R_\alpha \, \mathrm{rot}_\varphi(\alpha B_p) + \Delta B, \quad (2)$$

where $B \equiv B_\varphi$ is the azimuthal component of the field, A is the azimuthal component of the vector potential of the poloidal field, and $\mathbf{B}_p = \mathrm{curl}\, A\mathbf{e}_\varphi$. The equations are written in the dimensionless form in which the relative strength of the sources on the right-hand side is represented by the numbers

$$R_\alpha = \frac{R_2 \,|\, \alpha_{max} \,|}{\beta}, \qquad R_\Omega = \frac{R_2^2 \Omega_{max}}{\beta}. \quad (3)$$

where R_2 is the radius of the outer core. Depending on the strength of the sources of helicity and differential rotation, it is possible to identify two limiting generation regimes in (2): in the first, we neglect the term containing the differential rotation (α^2-dynamo) and, in the second, the term R_φ curl $\alpha\mathbf{B}_p$ (α-Ω-dynamo). The criterion for a pure helicity dynamo is

$$R_\alpha^2 \gg R_\Omega$$

(see, for example, Ref. 13) and not the naive condition $R_\alpha \gg R_\Omega$. The reason for this is that the scale of the generated field is also a function of R_α. The amplitudes of the field components of the α^2-dynamo are of the same order:[36] $|B_\varphi| \sim |B_p|$.

Equations (1)–(2) have often been integrated numerically in the α^2-approximation (see, for example, Refs. 4 and 9) for spherical geometry and helicity of the form $\alpha(r)\cos\theta$. The angular dependence then represents the antisymmetry of the Coriolis force under reflection in the plane of the equator. The critical value R_α, after which self-excitation of the field takes place, is of the order of 10. Early researchers believed that the solutions were monotonic and the leading harmonic was the dipole term. However, more detailed analysis performed by the asymptotic WKB method, using the small parameter R_α^{-1}, showed that the growth rate had an imaginary component,[34] such that $\mathrm{Im}\,\gamma/\mathrm{Re}\,\gamma \sim 0.1 R\alpha^{-3}$. Assuming that, for low Rossby numbers, the helicity is of the order of the rate of convection, $\alpha \sim v = 4\times10^{-2}$ cm/s, we find that the period of the oscillations is $T \sim 10^5$ years, which is in agreement with the characteristic inversion time of the terrestrial dipole. It is interesting to consider how this result is affected when the nonlinear influence of the field on helicity is taken into account. Another interesting result of the analytic solution is its form. In this type of dynamo, the natural functions are not the dipole, quadrupole, and so on, but configurations approaching forceless helical structures.[35]

Nonlinear MHD α^2-dynamos (cf. the Soward and Busse models) have been constructed for cellular laminar convection and are described in detail in Ref. 4.

However, the α-Ω-approximation[36,37] is more suitable for the geodynamo. It is assumed that the dynamo is maintained by the Archimedes force that produces strong differential rotation and weak meridional circulation. The rotating system has a nonuniform density distribution and an axially symmetric field B_φ, and is unstable against excitation of waves propagating in the longitudinal direction. The magnetic, Archimedes, and Coriolis forces are of the same order, and the corresponding waves are therefore referred to as the MAC waves. MAC waves have helicity, so that the problem is, to some extent, self-consistent. However, because of mathematical difficulties, the discussion has so far been confined to the intermediate model, in which the helicity is not found, but is specified in advance in the form of a simple function. The next step forward as compared with the kinematic approximation is to take into account the magnetic forces in a self-consistent manner, and to determine the mean velocity, i.e., the differential rotation. The westerly drift of the nondipolar field and secular variations observed on the Earth's surface are related to the MAC waves. There is particular interest in the analysis of the spatial and temporal structure of the 20- and 60-year variations[38,39] that have been determined by direct measurements of the geomagnetic field.

Studies of the solar system by space probes are promising for the understanding of the nature of the magnetism of other planets. At present, great hopes are invested in theoretical advances. Attempts to construct a simple similarity (scaling) law relating the amplitude of the observed magnetic field to the characteristic parameters of a planet (radius, angular velocity, and so on) have not been adequately justified. The most successful of them (predicting correctly the field of Saturn) is the Dolginov similarity law, based on the precessional dynamo model.[31] The problem lies in the clear physical dissimilarity between, say, Mars and Jupiter.

The origin of the magnetism of planets in the Earth group is examined with allowance for their thermal history in Ref. 40. Planetary bodies smaller than the Moon were never in the molten state. The residual magnetization of their rocks is a direct memory of the primordial field of the solar nebula. The magnetism of larger planets (Mercury, Mars, and Venus) is due to the residual magnetism produced during solidification in a magnetic field, which could have been the initial field or the field produced by the dynamo process during the evolutionary stage, say, prior to the solidification of the core (in the case of Mercury). It is only in the Earth that the dynamo continues to act, although the core is likely to solidify completely in the future. These ideas are not generally accepted in their entirety. For example, it has been suggested[27] that the dynamo is still operating in a thin layer in the core of Mercury, where it is maintained by chemical convection. In all cases, the dynamo is considered as the most probable source of magnetism at certain stages in the evolution of planets belonging to the Earth group.

In giant planets (Jupiter and Saturn), the dynamo is produced by effective thermal convection.[41] The energy source is provided by the gradual cooling of the planets from their initial hot state. Compositional convection due to the phase separation of helium from hydrogen and the descent of helium into the metal core is also possible on Saturn, which is cooler. Detailed analysis of convection can be found in the review article of Ref. 27 together with estimates of generated magnetic fields.

Thus that the hydromagnetic dynamo apparently operates in all planets with conducting liquid cores. The main task now is to obtain more detailed information on the nature of the motion and to construct specific models of the dynamo.

3. SOLAR CYCLE

The equilibrium of the Sun as a star is determined by the balance between the force of gravity and the pressure gradient. Solar activity is dominated by motion and magnetic fields. The magnetic field can be observed directly in active phenomena (spots, flares, and prominences) or is related to them (heating of the corona, coronal holes). Observations confirm the magnetic origin of solar activity.

However, this cannot be explained by the theory of the magnetism of a medium at rest. In actual fact, the conductivity of solar plasma in the uppermost layer is close to that of poor metals, and the characteristic dimensions of magnetic structures are relatively large. Hence, the effective time for a change in the magnetic field, which is proportional to the product of conductivity and the square of the dimensions, is significantly greater than the time observed for changing magnetic phenomena. The only natural cause of field variability is the hydrodynamic motion of plasma, especially rotation and convection. Hence, the principal mechanism of solar (and stellar) activity can be assumed to be the hydromagnetic dynamo.

The flow of the solar plasma is a combination of large-scale motion (differential rotation and meridional circulation) and stochastic motion (turbulent convection). The magnetic field in a medium of this type is a random quantity that can be found only by solving complicated stochastic equations. A more natural and simple approach is to find the smoothed, average parameters such as mean field, the mean square field, and other moments. The evolution of these quantities is described by simpler equations that *can* be solved. The principal advances in this theory of the solar dynamo are associated with the derivation and investigation of the equation for the mean magnetic field.

From the mathematical point of view, "mean" must be understood in the statistical sense. In practice, one deals with the magnetic field averaged over scales and times exceeding the scale and characteristic time of the main energy-bearing cells of turbulent motion, i.e., the supergranules. The transport of the mean magnetic field is determined by the mean flow parameters (Refs. 4, 9, 20, 21). The most important of these is differential rotation, whose effect on the poloidal field gives rise to the azimuthal component. Turbulent diffusion equalizes the mean field gradients. Random solar motion has a mean helicity whose effect on the azimuthal field results in the raising and rotation of the omega-shaped loops, and hence to the generation of the poloidal field. The mean helicity is a measure of the preferential handedness (left or right) of convective motion in the inhomogeneous rotating fluid. There is considerable interest in the observational confirmation of this mechanism on the Sun. This could be done, for example, by studying the motion of sunspots, taking all three velocity components into account. From the physical point of view, the existence of an average helicity is clear and natural: it is a consequence of the effect of Coriolis forces on convective elements that rise and descend in the inhomogeneous medium.

The three average characteristics of solar motion that we have mentioned (differential rotation, mean helicity, and turbulent diffusion) largely control the transport of the mean field, as was first demonstrated in Refs. 42–44. The effect of turbulent diffusion, the nonlinear influence of the field on helicity, and certain other effects were subsequently also noted.[45,46]

Of course, when we consider the average field, we can say nothing about small-scale or rapidly-varying fields. On the other hand, we are then in a position to explain large-scale and global variations of the magnetic field. The principal variation of this kind is the solar activity cycle (Fig. 3). The word "cycle" usually means repeating activity, e.g., the 11-year repetition of the sunspot pattern, or the 22-year repetition of the magnetic field configuration. On the other hand, this can also be related to the concept of the "Poincaré limit cycle" of the nonlinear solar magnetohydrodynamic system.

In reality, the phase portrait of the solar MHD system is even richer than this. It has been reported[48,49] that it is subject to secular modulation that may be associated with a two-frequency limit cycle. Specialists and amateurs have been shaken by the discovery of irregular global minima of solar activity that occurred in the past.[50] This is a direct indication of global stochasticity and of the existence of a "strange attactor" on the phase portrait of the solar MHD system.[13,47,51]

Recent years have seen the emergence of studies of the behavior of the magnetic field over long intervals of time, based on observations of prominences extending over more than 100 years.[52] The prominences "hang" on lines of separation between different magnetic-field polarities. They can therefore be used to construct lines of zero radial (poloidal) magnetic field. This method has been used to investigate the distribution of the poloidal field over the surface and the times of polarity reversal. Such studies can be complemented by investigations of the evolution of the magnetic flux associated with the azimuthal field, using data on sunspot areas that have been continuing over many years.[86]

3.1. Motion of the solar plasma

The transport of magnetic field in a moving medium is determined by magnetic diffusion (inversely proportional to the conductivity) and by deformations of the velocity field. Ohmic magnetic diffusion in the solar interior is so small that magnetic fields extending over scales exceeding the characteristic size of the granules ($\sim 10^8$ cm) can be regarded as frozen into the medium over long periods of time. The evolution of such fields is controlled by motion.

Magnetic lines of force are stretched by differential rotation. The latitude gradient of the solar angular velocity was established long ago by direct observations of the motion of sunspots and of the Doppler shift of spectral lines in surface layers.[53] Roughly speaking, the angular velocity difference between the equator and a pole is about 20%. Solar seismology is capable of determining the radial angular velocity gradient from the frequency splitting of oscillations localized at different depths. Analysis of early observations has shown that the radial angular velocity gradient is negative and several times greater than the latitude gradient.[54] Subsequent analysis of more extensive data on the splitting of the spectrum shows that the interior of the Sun ($\lesssim 0.2R_\odot$) rotates faster than the surface by a factor of not more than two.[19] However, the angular velocity at first decreases with depth (in the convective shell), but then increases again. Confirmation of this result on the radial angular velocity gradient in the convective shell after removal of observational uncertainties and contradictions would be of major importance for the theory of the solar dynamo. The sign of the product of the angular velocity gradient and the helicity defines the direction of propagation of the magnetic field wave (see below).

Standard models of internal structure show that the solar shell, which extends over about $0.3R$, is in a state of inten-

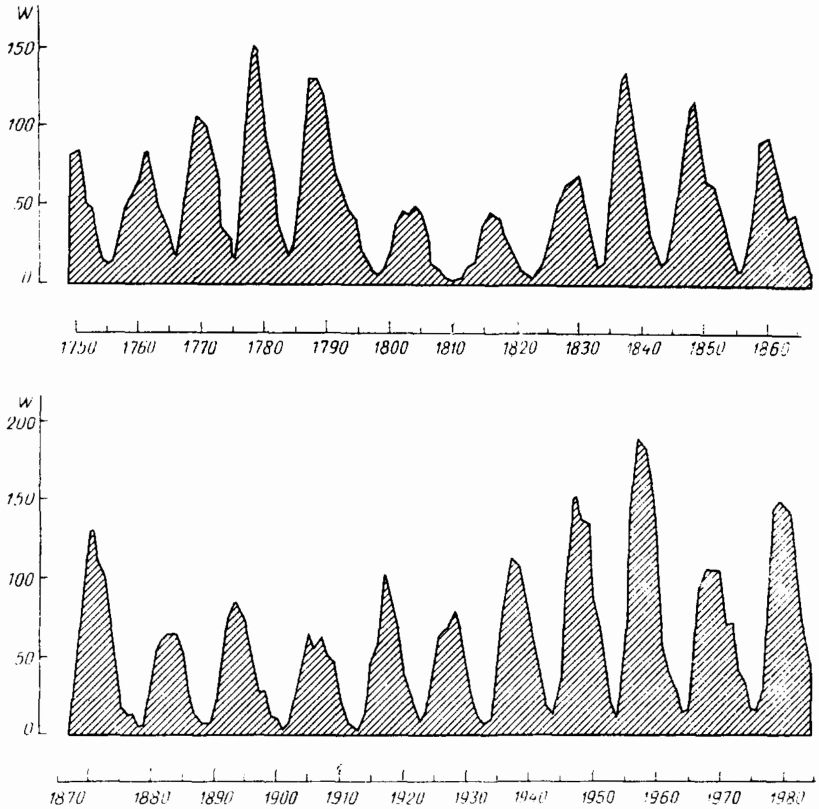

FIG. 3.

sive turbulent convection, in which the Rayleigh number is of the order of 10^{12} and the scale is determined by the height scale; the Prandtl number is very small ($\sim 10^{-9}$) because the transfer of heat is radiative. The turbulent nature of motion is usually determined by the Reynolds number. In the convective zone, $\mathrm{Re} = 5.5\times 10^3$ at a depth of $d_1 = 10^{-2}R_\odot$ and $\mathrm{Re} = 4\times 10^{12}$ at $d_2 = 0.3R_\odot$. Against the background of such intensive turbulence, the observed cellular picture, consisting of granulation and supergranulation, seems to be a surprising, synergetic phenomenon. We know that the typical scale of the granules is not very different from that of density variations. Simon and Weiss[55] have drawn attention to the fact that the density-stratified convective shell should naturally exhibit two further characteristic scales, one of which can be identified with supergranulation and the other with the giant cells that have not yet been directly observed.

At first sight, the existence of convective cells with dimensions exceeding the height scale is inconvenient because, by virtue of the continuity of flux, slight motion in the dense lower part of a cell creates enormous velocities in its upper part. However, cells extending over many height scales are more effective in transporting heat as compared with cells extending over one height scale, because they have a smaller superadiabatic gradient for the same temperature difference. There is a characteristic cell size for which the second effect is more important, and the emergence of such cells is energetically convenient. These interesting and important estimates have not yet received quantitative justification. Of course, the Boussinesq approximation must be abandoned in numerical analyses. Although the Mach number is low, the fluid cannot be regarded as incompressible, so that the influence of the density gradient must be taken into account and, in particular, $\operatorname{div} \mathbf{v} = 0$ must be replaced with $\operatorname{div} \rho\mathbf{v} = 0$. The situation is not at all similar to laboratory turbulence, which is usually characterized by the two dimensionless numbers Re and **Ma**.

The effect of turbulent convection on the mean magnetic field is similar to that of magnetic diffusion or magnetic viscosity. Roughly speaking, turbulent diffusion is given by $\nu_T = \tau\langle v^2\rangle/3$, where τ is a correlation time, say, the lifetime of supergranules, and $\langle v^2\rangle$ is the mean square of velocity fluctuations. A more subtle point, first pointed out by Lebedinskiĭ[56] and Biermann,[57] is that this is the anisotropy of turbulent transport in the solar convective zone, due to the fact that the radial direction is special. The different transport of turbulent momentum in radial and cross-radial directions is an essential element of the mechanism maintaining the differential solar rotation.[58–60]

The average magnetic field must diffuse anisotropically, but this effect is usually ignored in models of the solar dynamo for simplicity.

Another feature of solar turbulence is associated with rotation. The Rossby number falls from $\mathrm{Ro} = 60$ at d_1 to 0.04 at d_2, so that rotation affects the shape of the convective cells directly only near the bottom of the conductive zone.[61] However, the shape of the cells is not the only point. An average helicity appears because of the rotation of the stratified turbulent convection.

We must now provide a qualitative explanation of the mechanism responsible for the average helicity. Let us suppose that a convective element rises (descends) in the radial direction. Because of the presence of the density gradient, it expands (contracts), i.e., acquires additional components v_θ, v_φ. The associated moment of Coriolis forces gives the element additional rotation. Roughly speaking,

$$\frac{d}{dt}\,\mathrm{rot}_r\,\mathbf{v} \approx -2\Omega\cos\theta\,\frac{v_r}{r}. \qquad (4)$$

The radial velocity is determined by the continuity equation: $v_r/r \sim v\nabla_r\,\rho/\rho \sim v/H$, where H is the density scale. In the upper part of the convective zone, the correlation time $\tau \sim l/v$ is small in comparison with the rotational period, $\mathrm{Ro} = v/l\Omega \gg 1$. Integration of (4) yields $\mathrm{curl}_r\,\mathbf{v} \sim \Omega\tau v\cos\theta/H$. In the lower part of the zone, $\mathrm{Ro} \ll 1$, i.e., the convective element can turn over several times in the time τ. We must therefore take the average value of $\mathrm{curl}_r\,\mathbf{v}$ over the rotation period. The helicity is defined as the product of the velocity v_r and the additional $\mathrm{curl}_r\,\mathbf{v}$. The equation for the generation of the mean magnetic field includes the quantity $\alpha = -\tau\langle\mathbf{v}\cdot\mathrm{curl}\,\mathbf{v}\rangle/3$, given by the following estimates:

$$\alpha \approx \begin{cases} \dfrac{l^2\Omega}{H}\cos\theta, & \mathrm{Ro} \gg 1, \\ \dfrac{l}{H}v\cos\theta, & \mathrm{Ro} \ll 1. \end{cases} \qquad (5)$$

The angular dependence is such that α vanishes on the equator and has different signs in the northern and southern hemispheres. According to existing models of the convective shell, $l^2\Omega/H$ increases in the inward direction and lv/H toward the surface. The function $\alpha(r,\theta)$ must therefore have a maximum at the point where $l\Omega \sim v(5\times10^9$ cm). The appearance of helicity signifies a departure from mirror symmetry. The pseudoscalar α is determined by the product of the pseudovector $\mathbf{\Omega}$ and the vector $\nabla\rho$. Hence, it is clear that rotation alone is not enough! We also need the vector $\nabla\rho$ or $\nabla\langle v^2\rangle$. We note that the inhomogeneity of velocity fluctuations leads to a further interesting effect, namely, diamagnetic transport of the mean magnetic field.[9,13,16]

The hydromagnetic dynamo is possible whenever motion overcomes the destructive effect of magnetic diffusion. This relationship is most simply characterized by the magnetic Reynolds number $\mathrm{Re}_m = lv/\nu_m$. This number is very large in the convective shell of the Sun, i.e., $\mathrm{Re}_m = 2\times10^7$ at d_1 and $\mathrm{Re}_m = 5\times10^9$ at d_2. The solar velocity field therefore operates similarly to a fast dynamo. The dynamo generates the average large-scale magnetic field and the intermittent structure at small scales.

The main sources of the mean field are differential rotation and average helicity. They operate against a background of strong turbulent diffusion that equalizes gradients and is responsible for diffusional transport and decay of the mean magnetic field. The reaction of the magnetic field on the helicity and differential rotation leads to a number of interesting nonlinear effects, including the stabilization and modulation of oscillations, torsional waves of meridional circulation, and global minima.

3.2. Main cycle

The magnetic hydrodynamics of the average field provides an explanation of the main (22-year) periodicity of the axially symmetric magnetic field. The basic equations for the mean field (1)–(2) in the convective solar shell can be written in the form

$$\frac{\partial A}{\partial t} = \alpha B + \beta\Delta A, \qquad (6)$$

$$\frac{\partial B}{\partial t} = ([\nabla\Omega,\ \nabla\mathbf{A}]\,r\sin\theta)_\varphi + \beta\Delta B; \qquad (7)$$

where β represents magnetic diffusion, including turbulent and molecular diffusion. To elucidate the nature of the solutions of (6)–(7), it is useful to put $\beta = 0$ initially, and then examine the role of magnetic diffusion. If we introduce the variable $\chi \equiv r\sin\theta A$, we obtain for it the somewhat unusual equation[62]

$$\frac{\partial^2\chi}{\partial t^2} = r\sin\theta\,(\alpha\,[\nabla\Omega,\ \nabla\chi])_\varphi, \qquad (8)$$

which resembles the equation of thermal conduction if we interchange t and r. The solutions of (8) take the form of waves of decreasing amplitude, propagating along $\Omega =$ const surfaces. The direction of propagation of the dynamo waves depends on the sign of the product $\alpha\nabla\Omega$. Let us suppose that Ω is a function of only the radial coordinate. The $\Omega =$ const surfaces are then spheres and

$$\frac{\partial^2\chi}{\partial t^2} = D\,\frac{\partial\chi}{r\partial\theta},$$

where $D = r\sin\theta\alpha\partial\Omega/\partial r$ is the local dynamo number. We shall seek a solution of the form $\chi \sim \exp(qt)$. We then have $q^2 + ikD = 0$, i.e., for $D < 0$,

$$\mathrm{Im}\,q \equiv \omega = \left(\frac{kD}{2}\right)^{1/2},$$

$$\mathrm{Re}\,q \equiv \gamma = \left(\frac{kD}{2}\right)^{1/2}.$$

Turning back to the field, we have a wave propagating from the pole to the equator:

$$\begin{aligned} A &\sim e^{\gamma t}\cos(\omega t - k\theta), \\ B &\sim e^{\gamma t}\cos\left(\omega t - k\theta + \frac{\pi}{4}\right). \end{aligned} \qquad (9)$$

The sign of $\partial\Omega/\partial r$ is the same in the northern and southern hemispheres of the Sun, but $\alpha \sim \cos\theta$ [see (5)], so that D changes sign. Hence it follows that, in the southern hemisphere, the dynamo waves propagate from the pole to the equator. When $D > 0$, the waves travel toward the poles. The phase velocity is $v_{\mathrm{ph}} = \omega/k = (D/2k)^{1/2}$ and the group velocity $v_g = d\omega/dk$ is smaller by a factor of two. The phase difference between B and A is $\delta = \pi/4$, so that B_r lags behind B_φ by $3\pi/4$.

In another simple case, $\Omega = \Omega(\theta)$, the dynamo wave propagates in the radial direction. In contrast to the last case, this situation is not similar to the behavior of the field on the solar surface on which it propagates from high latitudes to the equator in the form of the well-known Maunder butterflies. However, waves propagating along the radial direction can be coupled when turbulent diffusion is taken into account.[62]

The inclusion of turbulent magnetic diffusion enables us, above all, to obtain a solution of constant amplitude because $\gamma \to \gamma - \beta k^2$. The dynamo number corresponding to $\gamma - \beta k^2 = 0$ is commonly referred to as the critical number for the generation of the mean field. Moreover, when $\beta \neq 0$, it is clear from (6) and (7) that there is a change in the phase difference between the poloidal and azimuthal components of the field. Studies of the dynamo equations, including diffusion and other effects, and of the correct geometry of the problem and the boundary conditions, are being carried out with the aid of computers. Numerical experiments based on the theory of the solar dynamo have been carried out, for example, in Refs. 44–46.

An asymptotic solution of the equations for the generation of the solar magnetic field was constructed and analyzed in Ref. 87. The variation of angular velocity with depth was assumed to be as indicated by helioseismology data, in which the radial angular velocity gradient in the conductive zone changes sign and its maximum value exceeds the latitude gradient. The mean helicity was taken from calculations based on the theory of mixing length. It was shown that three dynamo waves of the magnetic field were generated. The first wave was generated in the surface layer and had a maximum at a latitude of about 60°. Its activity took the form of migration toward the pole of the zone in which polar flares are formed. The second, less powerful, field wave was generated at the center of the convective zone and its activity manifested itself in the spot formation cycle. The third wave was generated at the bottom of the convective zone and was damped out rapidly as it propagated toward the surface. It could lead to multiple reversals of the polar magnetic field within the single main cycle.

In the linear theory, undamped or nonincreasing solutions are obtained only for a particular value of the dynamo number $d = D_{cr}$. When the reaction of the magnetic field on the turbulence is taken into account, a nonlinear solution can be constructed in the form of a limit cycle, at least for $D > D_{cr}$ (Fig. 4, Ref. 46).

The magnetic force has a stronger effect on the mean helicity than on the angular velocity. It has been estimated[64] that $(\delta\Omega/\Omega)/(\delta\alpha/\alpha) \sim (B_r/B_\varphi)v/\Omega r \ll 1$, where v is the characteristic amplitude of turbulent fluctuations. When the nonlinearity is small, it is natural to consider

$$\alpha_N = \alpha(r, \theta)(1 - \xi B^2). \qquad (10)$$

where ξ represents the nonlinear effect.

An undamped solution of the nonlinear problem may be sought in the form of an expansion in terms of the eigenvectors of the linear problem that correspond to complex eigenvalues $q_n = \gamma_n + i\omega_n$:

$$\begin{pmatrix} A \\ B \end{pmatrix} = \sum_{n=1}^{\infty} F^n(t) \begin{pmatrix} a_n \\ b_n \end{pmatrix}.$$

Replacing α with α_N in (6) and (7), we obtain a system of ordinary differential equations for $F^n(t)$ with cubic nonlinearity[65]:

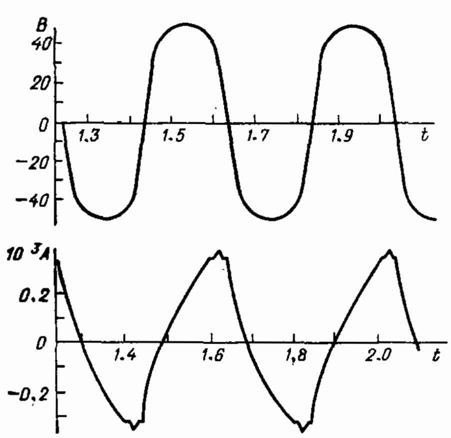

FIG. 4.

$$\frac{dF^n}{dt} = q_n F^n - \xi D \frac{dq_n}{dD} \sum_{l,m,s} K^n_{lms} F^l F^m F^s. \qquad (11)$$

where the coefficients K^n_{lms} are determined by the eigenfunctions a_n and b_m of the linear problem. In the first approximation in $D - D_{cr}$, it is sufficient to retain the two complex-conjugate modes $F^1 \equiv F, F^2 = F^*$ in (11), which corresponds to the dipole mode for the mean magnetic field. The stationary solution can be interpreted as a limit cycle in the form of a circle:

$$(\mathrm{Re}\, F)^2 + (\mathrm{Im}\, F)^2 = \mathrm{const} \cdot \frac{D - D_{cr}}{D}.$$

The local limit cycle in terms of the variables B and dB/dt has an elliptic shape.[47] It is important to note that, since the dynamo is nonconservative, this limit cycle cannot be given the usual energy significance of the kind assigned to a pendulum clock type dynamic system.

The periodic solution has the interesting symmetry property $F(t + T/2) = -F(t)$, i.e., it repeats at half the period, with the sign reversed.[65] It is clear even from (11) that its stability demands that $d\gamma/dD > 0$, i.e., solutions with a decreasing function $\gamma(D)$ are unstable.

The authors of Refs. 66 and 67 have drawn attention to an interesting property, namely, the alternation of amplitudes with even and odd numbers (Gnevyshev-Olya rule). They explain this in terms of the addition to the limit cycle of a quasiconstant mean-field component (which is added to the oscillating field for even numbers and subtracted for odd numbers). The origin of this component is something of a mystery. It is most naturally related to the stationary field of the solar core.

An increase in the dynamo number leads to the excitation of several modes. The simplest case of two interacting modes of dipolar and quadrupolar symmetry[65] may be of interest in connection with the secular modulation of the solar cycle.

3.3. Torsional waves

One of the striking events of the last few years has been the discovery of torsional waves that are superimposed on the overall differential rotation.[68] The period of the waves is close to 11 years, and their amplitude is of the order of 3–6 m/s. In each hemisphere, there are four zones of faster and slower rotation. A new zone appears near the pole and reaches the equator every 22 years. Studies of the motion of sunspots suggest that, in addition to torsional waves, there are apparently periodic meridional flows, as well.[69]

The good correlation between torsional waves and activity suggest that these waves can be explained in terms of the reaction of the magnetodynamo wave on differential rotation.[63,70–72]

3.4. Magnetohydrodynamic chaos

In reality, the solar cycle is not simply a limit cycle or a limit cycle with regular secular modulations imposed upon it. The discovery of the Maunder minimum and other global solar-activity minima has revealed the globally stochastic nature of solar activity. From the modern point of view, this picture is naturally related to a strange attractor.

In the first attempt to apply the idea of a strange attractor to the theory of the solar dynamo, the basic equations that included the reaction of the magnetic field on the helicity were reduced to the Lorentz system[73]

$$\dot{A} = -A + DB - CB,$$
$$\dot{B} = -\sigma B + \sigma A, \quad (12)$$
$$\dot{C} = -\nu C + AB;$$

where A and B are the azimuthal components of the vector potential and the mean field, C is the change in helicity due to the contribution of the magnetic helicity [for simplicity, the direct effect on α, such as (10), was ignored], and $\sigma \sim 1$ and $\nu \ll 1$ are constants. One of them characterizes the difference between the diffusion operators for A and B, and the other determines the rate of damping of magnetic helicity due to molecular magnetic diffusion, so that ν is very small in the solar convective zone.

Roughly speaking, activity turns off because (12) has a zero solution of the form $(A, B, C) = (0,0,0)$, i.e., in principle, the Sun can survive even without the magnetic field. For low dynamo numbers $D < 1$, this zero-order solution is stable. When $D > 1$, there are two additional singular points $S_{\pm} = \pm [\nu(D-1)]^{1/2}, \{\pm [\nu(D-1)]^{1/2}, D-1\}$. For a particular range of parameter values, i.e., in the solar context $\sigma \to 1$, $\nu \ll 1$, $D \to 4(\sigma - 1)^{-1}$, the trajectories of (12) in (A, B, C) space execute quasiperiodic transitions from the neighborhood S_+ to the neighborhood S_- and back, and in the course of this process they sometimes (randomly) reach the neighborhood of the singular point (0, 0, 0). The time spent by the mapping point in this neighborhood is $\tau_M \sim \nu^{-1}(D - D_{cr})^{-1/2}$ (Ref. 47), i.e., it is not very long. An approximate derivation of the equation for $\dot{C}$ with small ν is given in Ref. 64.

The simplified set of dynamo equations given by (12) has an important defect: it does not contain the dynamo waves that are present in the initial MHD equations for the mean field (see Section 3). (We note that, when (12) is used, the main 22-year cycle must be understood as a transition from S_+ to S_- and back. The frequency of this transition is of the order of $(\sigma D)^{1/2}$.)

A more realistic simplification of the mean-field equations (6) and (7), that includes the dynamo waves, leads to complex equations for A and B. This approach was developed in Refs. 51 and 74, in which the reaction of the magnetic field, not on the mean helicity but on differential rotation, was taken into account. The result was a set of equations of order 6 for the complex variables $A(t)$, $B(t)$, and $\Omega(t)$:

$$\dot{A} = -A + 2DB,$$
$$\dot{B} = -B + iA - \frac{i}{2}\Omega A^*, \quad (13)$$
$$\dot{\Omega} = -\tilde{\nu}\Omega - iAB,$$

which is the complex generalization of the Lorentz equations given by (12). The parameters $\tilde{\nu}$ and D are real and positive. When $\tilde{\nu} \ll 1$, the sequence of bifurcations increases with increasing D, and eventually leads to the stochastic behavior of the system, with episodes of highly suppressed magnetic activity.

The trivial solution (0, 0, 0) is stable so long as $D < 1$. The first bifurcation for $D_1 = 1$ leads to the appearance of an oscillating solution corresponding to the dynamo wave. Moreover, when $D \gg 1$, the set of equations given by (13) has an exact periodic solution

$$B = |B| e^{ipt}, \quad A = |A| e^{i(pt+\varphi)}, \quad \Omega = |\Omega| e^{2ipt},$$

where $p \sim 4D/(2+\tilde{\nu})$, $|A| \sim 8D/(2+\tilde{\nu})$, $|B| \sim |\Omega| \sim 16D/(2+\tilde{\nu})^2$. This periodic solution is a limit cycle in the 6-dimensional phase space (A, B, Ω). On the $(\mathrm{Re}\, A, \mathrm{Re}\, B)$ plane, the limit cycle takes the form of two ellipses. Numerical calculations for $\tilde{\nu} = 0.5$ show that this solution becomes unstable for $D_2 = 2.07$. After this bifurcation, the solution becomes doubly periodic with two distinct frequencies (torus in phase space). The doubly periodic solution, in turn, becomes unstable for $D_3 = 3.47$ and the three-frequency solution appears. After $D_4 = 4.81$, we obtain the Feigenbaum cascade of solutions with period doubling, which converges to $D_{\text{chaos}} = 4.84$. For large dynamo numbers, the solutions become random in time and there are episodes with highly suppressed amplitude. An example, taken from the paper by Weiss *et al.* (1984) for $\mathrm{Re}\, B$ is shown in Fig. 5($D = 17$).

It is important to note that, in contrast to the parameter $\nu \ll 1$ in (12), the parameter $\tilde{\nu}$ is of the order of $h/H > 1$ in the convective zone, where h is the depth of the zone and H the height scale for the density variation. This is so because this parameter is determined by the reciprocal of the angular velocity damping time due to turbulent viscosity. This emphasizes once again the model character of this example. Its value lies in the indication it provides of the fundamental possibility of global MHD chaos in the solar dynamo and in estimates of the orders of magnitude of the various quantities.

4. GALACTIC MAGNETIC FIELD

The magnetic field of galaxies is weak in comparison with the solar and terrestrial fields. It is measured in microgauss. However, the galactic field has a record-setting spatial scale (kpc). The creation of a magnetic field of this order requires an emf (BRh/ct) of the order of 10^{10} acting over 10^{10} years, where $B \simeq 2\,\mu$G, $R \simeq 15$ kpc, and $h \simeq 400$ pc. The last two quantities are, respectively, the radius and half-thickness of the galactic gas disk. Hoyle[75] was the first to make this estimate, and was forced to conclude that the ga-

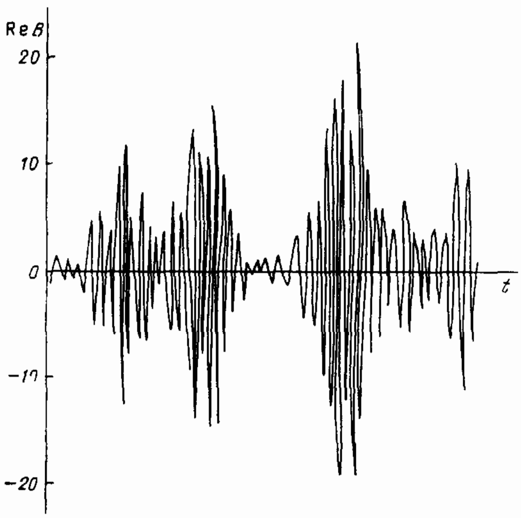

FIG. 5.

lactic field was the initial, primordial field.

In reality, the motion of the ionized galactic gas acts as the hydromagnetic dynamo. The dynamo produces an exponential increase in the very weak initial field, which can be created by the battery-type effect or is the by-product of ejection from stars. The characteristic field rise time is of the order of 5×10^8 years (see later), which is much shorter than the lifetime of the Galaxy (10^{10} years). The field then settles down because of the reaction of motion. The steady field is maintained by an emf equal to $\int \mathbf{E},d\mathbf{l} \sim c^{-1}(\mathbf{v}\times\mathbf{B})h$, where $\mathbf{v}$ is the characteristic velocity of the random motion of the gas (of the order of 10 km/s). Random motion also leads to turbulent diffusion and dissipation of the field. However, its component due to helicity is of the order of $0.1v\sim 1$ km/s, which is entirely sufficient to create 10^{10} V.

The theory of the hydromagnetic dynamo was first applied to the explanation of the galactic mechanism in Refs. 76 and 77, which reported simultaneous but independent work. The main sources of the large-scale mean magnetic field were again assumed to be differential rotation and mean helicity. However, in contrast to planets and stars, the azimuthal field B_φ, which is even relative to the central plane, and the corresponding quadrupole field (B_ρ, B_z), are more readily excited in the relatively thin galactic disk. This, and some of the subsequent work summarized in Refs. 13 and 21, have elucidated the conditions for the enhancement of the magnetic field in the local solar neighborhood. The useful feature of the galactic dynamo is that we find ourselves inside the dynamo region.

A. M. Shukurov and one of the present authors[78] have constructed a dynamo model that takes into account the distribution of angular velocity and disk thickness along the radius. The unexpected result of this work is that the large-scale field of the Galaxy and of the similar Andromeda galaxy is generated only at the center and in an annular region that includes the solar neighborhood. The result was confirmed experimentally soon after.[79]

Recent years have seen the advent of intensive radioastronomical studies of large-scale magnetic fields in the nearest spiral galaxies. Apart from ring-shaped magnetic structures (the Galaxy, M31, IC342, and M81), bisymmetric configurations resembling a double helix have been discovered.[80]

To determine the radial field distribution and to compare the results with observations, the theory of the dynamo in a thin rotating layer had to be developed further.[81,88]

The theory of the generation of the mean magnetic field can be formulated as an eigenvalue problem for (1) and (2) with zero conditions on the boundaries. The distribution of the field in space is described by a linear combination of eigenfunctions, and the eigenvalues determine the rate of growth $\Gamma\equiv d\ln B/dt$ of different field modes. Let ρ, φ, z be cylindrical coordinates, and let us ignore, for the moment, the dependence on φ. In terms of dimensionless variables, the generation equations then assume the form

$$\Gamma A=\alpha B+\frac{\partial^2 A}{\partial z^2}+\lambda^2\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho A, \qquad (14)$$

$$\Gamma B=-DG\frac{\partial A}{\partial z}+\frac{\partial^2 B}{\partial z^2}+\lambda^2\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho B; \qquad (15)$$

where z is measured in units of the characteristic half-thickness h_0, ρ in units of the disk radius R, the time in units of h_0^2/β, and $G=\rho d\Omega/d\rho$ is a measure of the differential rotation. The characteristic values $G_0\sim\Omega_0\sim10^{-15}\,\mathrm{s}^{-1}$, $\alpha_0\sim10^5$ cm/s, $\beta\sim10^{26}\,\mathrm{cm}^2/\mathrm{s}$ can serve as estimates of differential rotation, mean helicity, and turbulent diffusion in the solar neighborhood of the Galaxy. The quantity $D=R_\alpha R_\Omega=\alpha_0 G_0 h_0^3/\beta^2$ is the dimensionless dynamo number and $\lambda^2=h_0^2/R^2\sim10^{-3}$ is a small parameter that appears because of the considerable difference between the vertical and horizontal dimensions of the disk. This difference ensures that the magnetic field diffuses relatively rapidly in the z-direction, at right-angles to the plane of the disk ($h_0^2/\beta\sim5\times10^8$ years), and slowly along the radius ($R^2/\beta\sim5\times10^{11}$ years). The age of the galaxies does not exceed 10^{10} years.

The solution of (14)–(15) will be sought in the form

$$A=Q(\rho)\,a(\rho,z),\qquad B=Q(\rho)\,b(\rho,z),$$

where a and b satisfy the set of one-dimensional equations

$$\gamma(\rho)\,a=\alpha(\rho,z)\,b+\frac{\partial^2 a}{\partial z^2},\qquad \gamma(\rho)\,b=-DG(\rho)\frac{\partial a}{\partial z}+\frac{\partial^2 b}{\partial z^2}. \qquad (16)$$

The radial function is found to satisfy the Schrödinger-type equation

$$\lambda^2\frac{d}{d\rho}\left(\frac{1}{\rho}\frac{d}{d\rho}\rho Q\right)+(\gamma(\rho)-\Gamma)\,Q=0,$$

in which the rate of growth, $-\gamma(\rho)$, in the problem defined by (16) for the vertical distribution plays the part of the potential. This approach is similar to the adiabatic approximation in quantum theory, which is valid for $\Delta\Gamma\ll\Delta\gamma$, where $\Delta\Gamma$ is the separation between the corresponding eigenvalues. For the galactic disk, $\Delta\gamma\sim1$ and $\Delta\Gamma\leqslant0.4$, i.e., this condition is satisfied.[81]

The solution of (16) for a given helicity that is antisymmetric in z and for $G(\rho)$ determined from the observed rotation curve, has been carried out numerically for the simplest vacuum boundary conditions[82]

$$\frac{\partial a}{\partial z}(\pm h)\approx0,\qquad b(\pm h)=0.$$

The dynamo number calculated from parameter values typical for the solar neighborhood ($D\simeq-10$) is such that the lowest field mode with a symmetric azimuthal field and the corresponding meridional quadrupole field can be excited (Fig. 6).

To find the radial field distribution, we must know the potential $-\gamma(\rho)$, determined by the radial dependence of the dynamo number and the disk half-thickness. Observations show that the thickness of the gas disk increases with distance from the center of the Galaxy. The following ap-

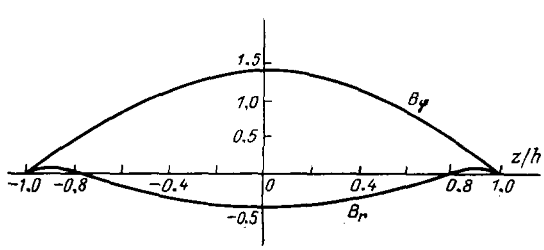

FIG. 6.

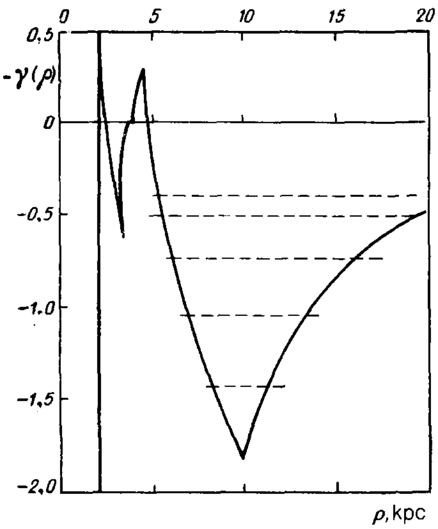

FIG. 7.

proximation was used in Ref. 81 for the half-thickness of the ionized layer: $h(\rho) = 0.37(1 + \rho/0.4\ \text{kpc})^{1/2}$. The function $\gamma(\rho)$, calculated by solving (16), is shown in Fig. 7. The solution was based on the observed rotational curve and mean helicity of the form $\delta(z-0.5) - \delta(z+0.5)$.

The magnetic field is generated in regions with $\gamma(\rho) > 0$. For the Galaxy, this is the central part and the ring in the solar neighborhood around $\rho = 10$ kpc. An analogous picture is obtained for Andromeda M31. In galaxies with single-hump or monotonic rotational curves, the large-scale field can be generated only at a certain distance from the center.

The growth rate Γ_n of the radial modes is most simply found in the WKB approximation:

$$\int_{u_n}^{w_n} (\gamma(\rho) - \Gamma_n)^{1/2}\, d\rho = 2\pi\lambda \left(n + \frac{1}{2}\right),$$

where the turning points are determined by the conditions $\gamma(u_n) - \gamma(w_n) = \Gamma_n$. The lowest mode $n = 0$ grows with the characteristic time $\Gamma_0^{-1} \simeq 3 \times 10^8$ years.

Let us now consider the generation of nonaxially-symmetric φ-dependent magnetic fields,[83] such that the initial field has a component in the plane of the disk. The differential rotation twists this field into a two-arm spiral (Fig. 8). The separation between the turns of the spiral in which the fields have opposite directions decreases rapidly with time, $\Delta\rho \sim \rho_0/\Omega_0 t$, where Ω_0 is the characteristic angular velocity and ρ_0 is the scale of the variation in this velocity. There is a corresponding reduction in the characteristic time for the diffusion of the magnetic field. For the mode $B \sim \exp(im\varphi)$,

$$\tau_d \sim \frac{(\Delta\rho_m)^2}{\beta},$$

where $\Delta\rho_m = \rho_0/m\Omega_0 t$ and β is the turbulent diffusion coefficient. This time must be compared with the characteristic time for the generation of the field by the dynamo mechanism:

$$\tau_g \approx \frac{h_0^2}{\beta}\frac{1}{\gamma_0}(R_\alpha R_\Omega)^{-1/2},$$

where $R_\alpha = h_0\alpha/\beta$ and $R_\Omega = h_0^2\Omega_0/\beta$ are dimensionless numbers that determine the strength of the generation sources, $R_\alpha R_\Omega = D$, and $\gamma_0 \simeq 0.4$. The above estimate of τ_g refers to the local neighborhood ρ and is therefore independent of m (for small m). The excitation condition $\tau_g < \tau_d$ yields

$$m \leqslant \frac{\rho_0}{h_0}\gamma_0^{2/3} R_\Omega^{-1/4} \approx 2,$$

where $\rho_0 = 3$ kpc. It is thus clear that the lowest nonaxially-symmetric modes with $m = 1, 2$ can definitely be excited in spiral galaxies. It is also clear that the $m = 0$ mode is excited first, and is followed by the $m = 1$ mode, which corresponds to the observed bisymmetric structures. This estimate has stimulated the development of a numerical model of the nonaxially symmetric dynamo in a thin disk.[88]

Thus, already the analysis of the mean magnetic field shows that the theory of the hydrodynamic dynamo leads to informative applied results. Still greater prospects are opened up by taking into account higher statistical moments of the field.

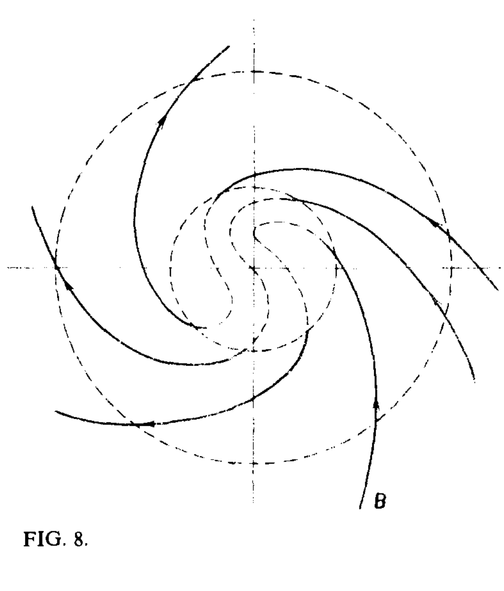

FIG. 8.

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