Origin of the magnetic fields of the earth and celestial bodies

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Various mechanisms for the generation of cosmic magnetic fields are analyzed from the standpoint of the possible application of these mechanisms to the earth and to specific celestial objects. The analysis focuses on nondynamo mechanisms. The generation of a field by a "battery" process at chemically peculiar Cp stars is analyzed. The important role played by meridional streams in the generation of fields at O and B stars is pointed out. The effect of chemical and temperature variations at the boundary between the earth's core and mantle on the generation of the terrestrial field is discussed. A thermomagnetic instability may play an important role in generating the field of white dwarfs and, possibly, neutron stars. An induction mechanism for field generation in close binary stars is examined. The distinctive features of the magnetic fields of the sun and of convective stars are discussed.

CONTENTS

1. INTRODUCTION

All celestial entities—the interstellar medium, the stars, the sun, the planets, and so forth—have magnetic fields. These fields are very important to the course of many astrophysical processes. For example, the fields of the intergalactic medium, $\sim 10^{-9}$ G, influence the behavior of matter near galaxies equally as strongly as the fields of neutron stars, $\sim 10^{12}$ G, influence the motion of the surrounding plasmas. The problem of the origin and evolution of celestial magnetic fields has yet to be finally resolved, despite its exceptional importance for astrophysics.

Since magnetic fields exist everywhere, it is tempting to think that there is some common, universal mechanism which generates them. A dynamo mechanism for field intensification is regarded as the basic mechanism in most recent studies. Dynamo theory is the subject of hundreds of papers and a number of monographs.[1–6] There is accordingly no need to present that theory or its application here, especially since the latest progress in this field is discussed by Ya. B. Zel'dovich and A. A. Ruzmaĭkin in this issue of *Uspekhi fizicheskikh nauk*. We will go into more detail on the difficulties which the dynamo theory runs into. The basic thrust of this review is to examine mechanisms other than dynamo mechanisms. We will attempt to show that under the actual conditions which exist in space there are a number of distinct and effective possibilities for the generation of magnetic fields, and the dynamo mechanism is only one of them.

We know that the operation of a dynamo mechanism requires plasma motions which are not highly symmetric.[7,8] Such motions are encountered fairly frequently under actual conditions, and the limitations which follow from the so-called antidynamo theorems (e.g., a dynamo is forbidden in the case of two-dimensional motions) apparently play no important role. The dynamo theory leads to both steady-state and time-varying solutions. It thus becomes possible to describe such time-varying phenomena as the solar cycle.

We should emphasize, however, that most of the existing solutions which permit comparison with observations are based on the linear approximation, in which the motions which generate the field are assumed to be given and not subject to effects of the field. Since the linear generation equations are homogeneous, the theory is not capable of calculating the strength of the field. It gives us only the growth rate of the field in the initial stage. In some cases, fields are estimated on the basis of qualitative considerations. Consequently, one can hardly say that there is quantitative agreement between the dynamo theory and observations. Furthermore, there are several entities for which the conditions required for the operation of the dynamo mechanism apparently do not hold, but these entities nevertheless have strong magnetic fields.

It is usually assumed[9,10] that the magnetic fields of hot peculiar stars and also of white dwarfs and neutron stars are of relic origin, i.e., that these fields survived and strengthened during the contraction of a protostellar cloud. Unfortu-

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nately, there is no theory for the evolution of the field during the formation of stars. The hypothesis that the field is of relic origin simply reverses the question, since there is the problem of explaining why many stars of the same spectral classes have no appreciable field. A low conductivity and intense mixing of the protostellar cloud would apparently prevent the retention of a strong field. At any rate, neither the terrestrial field nor the surface fields of the sun could be relic fields, since their decay time is far shorter than the lifetime of the system.

Stability is a very important question for any field generation theory, especially for the relic field theory, since instabilities can suppress a field far more rapidly than ohmic dissipation can. It has been shown[11,12] that neither a poloidal field nor a toroidal field separately would be stable; only a combination of the two would be stable. We recall that the toroidal field $\mathbf{B}_t$ in a spherical conducting body is azimuthal and exists only within the conducting body, while the poloidal field $\mathbf{B}_p$ also has a radial component and continues into the (possibly nonconducting) medium around the object. Fields should not give rise to motions of the medium which would suppress the fields. A toroidal field $\mathbf{B}_t$ is produced by a poloidal current $[\nabla, \mathbf{B}_t] = (4\pi/c)\mathbf{j}_p$. If this current does not flow along the field lines of the poloidal field $\mathbf{B}_p$, a force $\sim [\mathbf{j}_p \mathbf{B}_p]/c$ arises. As was shown in Refs. 11 and 12, the flows which arise here result in the vanishing of the component $\mathbf{j}_p \nparallel \mathbf{B}_p$, and we are left with only $\mathbf{j}_p = k\mathbf{B}_p$, where $(\mathbf{B}_p \nabla)k = 0$. The current $\mathbf{j}_p$ may not decay for a long time if it flows along those field lines of $\mathbf{B}_p$ which lie entirely in a region of high conductivity, i.e., which do not go into the surface layers of the star. Such a current could be sustained only by a deep toroidal field, so that there is no possibility of explaining surface fields as relic fields without appealing to the assumption of a continuous transport of deep fields to the surface by some mechanism. Consequently, Mestel[12] discussed some hybrid hypotheses in which one of the fields, say $\mathbf{B}_p$, was assumed to be a relic field, while the other field, $\mathbf{B}_t$, was sustained by some independent energy source.

There are also some other mechanisms for the generation of celestial magnetic fields, e.g., thermoelectric currents[13] and the Tolman effect.[14] It usually turns out, however, that the time required to reach the observed field strengths in these cases is greater than the lifetime of the object. For this reason, mechanisms of this sort are regarded as only the sources of a "seed" field, which is then intensified by motion of the medium.

2. FIELD GENERATION EQUATION

Ohm's law in a moving, unmagnetized plasma can be written[15]

$$\mathbf{j} = \sigma \left\{ \mathbf{E} + \frac{\nabla \mu_e}{e} + \left[\frac{\mathbf{v}}{c}, \mathbf{B}\right] - \eta \nabla T - \frac{[\mathbf{j}, \mathbf{B}]}{ecN_e} \right\}, \quad (1)$$

where $\mathbf{j}$ is the current, σ is the conductivity, $\mathbf{E} = -\nabla\varphi + (\dot{\mathbf{A}}/c)$ is the electric field, $\mathbf{v}$ is the plasma velocity, N_e is the electron density, μ_e is the chemical potential of the electrons, η is the thermoelectric coefficient, $\mathbf{B} = m\mathbf{H}$ is the magnetic induction, and $\nabla\mu_e = (\nabla P_e/N_e) - s_e \nabla T$, where P_e is the electron pressure, and s_e is the entropy per electron. Since the magnetic permeability m of a plasma is essentially unity, the quantity $\mathbf{B}$ is frequently called the "magnetic field." Here and below, we will make no distinction between $\mathbf{B}$ and $\mathbf{H}$.

From Eq. (1) and Maxwell's equations we easily find

$$\frac{\partial \mathbf{B}}{\partial t} = [\nabla, [\mathbf{vB}]] + \frac{c^2}{4\pi}\left[\nabla, \left(\frac{1}{\sigma}[\nabla, \mathbf{B}]\right)\right] - \left[\nabla, \frac{[\mathbf{jB}]}{eN_e}\right] - c[\nabla\eta, \nabla T]. \quad (2)$$

The first term on the right describes the change in the field during motions of the medium and is the primary term in the dynamo theory. The second term describes the ohmic decay of the field. The decay time may be very long for celestial objects, because of the large scale of the field and the high plasma conductivity. The third term reflects the Hall effect and is frequently smaller than the other terms.

In the surface layers of a star, which are not fully ionized, collisions of electrons with each other play a far smaller role than that played by collisions with ions and atoms. We can thus write $\eta = (k/e)[(\mu/kT) - 4]$, where we would have $\mu = kT\ln N_e + kT\ln[\sqrt{2}\pi\hbar^3(mkT)^{-3/2}]$ for a nondegenerate electron gas. From these expressions we find $[\nabla\eta, \nabla T] = (k/eN_e)[\nabla N_e, \nabla T] = (1/eN_e^2)[\nabla N_e, \nabla P_e]$. We see that the term $[\nabla\eta, \nabla T]$ in (2) is nonzero only if the gradient of the electron density is not parallel to the gradient of the electron pressure. The appearance of a magnetic field in the case $\nabla N_e \nparallel \nabla P_e$ is usually called the "battery effect." A particular case of the battery effect, which arises when the isotherms and isobars do not coincide in differentially rotating stars, is called the "Biermann effect."[16-18]

3. BIERMANN EFFECT

Let us examine the Biermann effect in a plasma consisting of electrons and ions of a single species. The condition for hydrostatic equilibrium for the electron component of the plasma is

$$m_e \frac{d\mathbf{v}_e}{dt} = -\frac{\nabla P_e}{N_e} + m_e \mathbf{g}_{eff} + e\mathbf{E}, \quad (3)$$

where $\mathbf{g}_{eff} = \mathbf{g} + [\boldsymbol{\omega}[\boldsymbol{\omega}\mathbf{r}]]$, $\mathbf{g}$ is the acceleration due to gravity, and $[\boldsymbol{\omega}[\boldsymbol{\omega}\mathbf{r}]]$ is the centrifugal acceleration. The equation for the ions is the same as (3) with the substitutions $e \to -e$, $m_e \to m_i$, $v_e \to v_i$, where m_e and m_i are the electron and ion masses, $\mathbf{v}_e$ and $\mathbf{v}_i$ are the respective velocities, and ω is the angular rotation velocity of the medium. If there are no plasma flows, we can write $m_e\dot{\mathbf{v}}_e = -m_i\dot{\mathbf{v}}_i$; adding the equations for the electrons and the ions, and substituting $\nabla P_e/N_e$ into the expression for $[\nabla\eta, \nabla T]$, we find

$$[\nabla\eta, \nabla T] = \frac{[\nabla N_e, \nabla P_e]}{eN_e^2} = -\frac{m_i}{2}[\nabla, \mathbf{g}_{eff}]. \quad (4)$$

If the rotation is differential, we have $(\nabla, [\boldsymbol{\omega}[\boldsymbol{\omega}\mathbf{r}]]) \neq 0$, and the term $[\nabla\eta, \nabla T]$ leads to a strengthening of the magnetic field. Estimating $|\nabla\omega|$ to be $(\omega/l)(\delta\omega/\omega)$, where $\delta\omega$ is the change in ω over a distance l, and solving (2) after discarding the first and third terms on the right side, we find the estimate $\sim 10^3$ G for the equilibrium value of the field for a star like the sun. This value, however, would be reached only after $5 \cdot 10^{10}$ yr, which is greater than the evolution time of the star. The Biermann effect is thus capable of producing only a seed field.

4. EFFECT OF CHEMICAL INHOMOGENEITIES

A situation in which ∇N_e and ∇P_e are not parallel may arise not only because of a rotation of a body but also because of an inhomogeneous distribution of chemical elements in it. At first glance it would appear that local chemical inhomogeneities, especially in stars, would dissipate very rapidly, even if they did somehow occur. However, this is by no means always the case. For example, the peculiar stars of type Ap are observed to have long-lived localized chemical anomalies in the form of distinct spots.[19,20] Composition inhomogeneities may also arise in the surface layers of neutron stars because of an asymmetry of the supernova explosions which result in the formation of the neutron stars.[21,22] Inhomogeneities may also appear in stars of other types in the course of development of instabilities. There is reason to believe that at the boundary between the earth's core and mantle there are also some large-scale structural inhomogeneities, which may play an important role in sustaining the earth's magnetic field.[23,24] We will discuss these examples and some other interesting cases here; we begin with hot magnetic stars.

5. MAGNETIC Cp STARS

Stars having anomalies in their chemical composition are called "peculiar Cp stars." Many of the Cp stars, especially those of spectral class Ap (more precisely, from B5 to FO), have strong magnetic fields, $\sim 10^2$–10^4 G. These are hot stars, with surface temperatures ranging from 8000° to 30 000°; their masses are several times the solar mass. At such temperatures, hydrogen is highly ionized. The outer convective zone is either very thin or absent altogether, and energy is transported by radiation since the temperatures are below the adiabatic temperature up to the surface. The rotation velocity of Cp stars with a strong field is usually two or three times lower than that of nonmagnetic stars of the same spectral class, although there are some exceptional cases. The weak convection and the slow rotation rule out an explanation of the magnetic field by a dynamo mechanism. The suggestion that a dynamo acts in an internal convective core[25,26] runs into the serious and unsolved problem of a large-scale transport of the field to the surface. As a rule, the observed dipole field of a Cp star is tilted with respect to the rotation axis. If the rotation is differential, the field twists up in such a way that regions with oppositely directed field lines come close together. The result is a rapid dissipation of the field.[27] Consequently, the large-scale field which is observed must be replenished at a rapid pace, either by a generation process or through the floating up of a relic field from the interior, if there is no differential rotation there. As a result, there are additional difficulties for both the hypothesis of a relic field and an α-ω dynamo. The hypothesis of a relic origin of the field runs into difficult stability problems.

The most striking feature of magnetic Cp stars is that the observed anomalies in chemical composition are distributed nonuniformly over the surface, in distinct large spots. Although the surface details cannot be resolved by a telescope, the existence of spots can be inferred from changes in the intensity of spectral lines which occur at the rotation period of the star. An intensity change might result not only from differences in the abundances of the elements but also from differences in excitation conditions, e.g., because of temperature inhomogeneities. However, a careful study of the series of lines of various elements points specifically to a difference in chemical composition. For example, the abundance of helium in some spots may be one-tenth the normal abundance, and the abundance of, say, europium may be 10^4 times as great. In some cases the abundance of certain elements will deviate substantially from their natural abundance over the entire star. Among the magnetic Cp stars there are both stars which are helium-poor and stars which are helium-rich, but there are apparently no entities with a normal abundance of helium. The particular chemical characteristics of Cp stars are described in detail in Refs. 19 and 20, among other places.

The magnetic fields of Cp stars can be observed in the Zeeman splitting of spectral lines. These are dipole fields, sometimes with an admixture of higher-order multipoles. The axis of the dipole is tilted with respect to the rotation axis of the star (and is sometimes displaced slightly away from the center of the star). No regular behavior has been found in the distribution of tilt angles. It is possible that the field has a complicated fine-scale structure (e.g., magnetic spots), but this structure lies beyond the resolution of existing instruments. There seems to be some correlation between the magnetic field structure and the chemical anomalies, but there are no clear data on this point.

The number of magnetic Ap stars among normal A stars is of the order of 10–20%. The Ap stars have been observed for many years, and over this time no change has been observed in the distribution of chemical anomalies. The lifetime of spots can be estimated from the facts outlined above; this estimate sets a lower limit $\sim 10^2$–10^3 yr on this lifetime. It is possible that spots persist for a much longer time. The Ap stars are also among the quietest of stars. They have no intense convection, no coronas, and no stellar wind; nevertheless, a small-scale turbulence with a velocity scale ~ 1 km/s and a length $\sim 10^3$ km is observed at the surface. The fact that chemical spots persist for a long time is thus surprising. Even a small-scale turbulence, but especially large-scale flows, should have spread out these spots over a time far shorter than the lifetime of the star. Furthermore, it would appear at first glance that local inhomogeneities should by themselves quickly sink or float up and spread out into the surrounding matter. However, a more careful analysis shows that chemically inhomogeneous structures which are equilibrium structures or at least long-lived ones are possible. Mestel[10] and Mestel and Moss[12,28,29] studied the nature of possible flows in a rotating star with an inhomogeneous distribution of the molecular weight $\mathfrak{M}$ and pointed out that a hydrostatic equilibrium $\nabla P = -\rho(\mathbf{g} = [\boldsymbol{\omega}[\boldsymbol{\omega}\mathbf{r}]])$, where $P = \mathfrak{R} T\rho/\mathfrak{M}$, and $\mathfrak{R}$ is the gas constant, can prevail if $T/\mathfrak{M}$ depends on the angles in a certain way. Flows of a medium caused by various factors and accompanied by a redistribution of the molecular weight may lead to a state in which an "$\mathfrak{M}$ barrier" arises, suppresses flows, and prevents the spreading out of inhomogeneities. However, those papers did not explain the reasons for the appearance of local chemical anomalies in Cp stars, and they did not discuss their observed characteristics.

In the "diffusion model" for the origin of chemical anomalies,[30–34] which is the model which has been developed to the greatest extent, it is assumed that the anomalies arise in the course of diffusion of ions and atoms driven by concentration and temperature gradients, the gravitational force, and radiation pressure. The particles of all elements other than hydrogen are treated as test objects, and their velocities with respect to the host plasma are determined. That approach leads to a qualitative explanation of the helium deficiency: The helium sinks after the star reaches the main sequence, and the excess abundance of certain other elements is attributed to their upward diffusion driven by radiation pressure. The formation of spots is linked with the circumstance that the magnetic field hinders the diffusion of ions across field lines.

Although the diffusion model has generated successful qualitative explanations for the anomalies in certain elements, its predictions contradict observations in several cases.[35] The dynamics of the stratification of elements at various depths depends strongly on the structure and characteristics of the levels of ions, about which we have inadequate data.

The most dubious point from the theoretical standpoint is the use of the approximation of test particles. In spite of their low abundance, heavy elements can greatly influence the opacity of a medium and thus the local temperature and associated flows.

Attempts have been made to explain the observed anomalies on the basis of the fallout on the star of interstellar matter,[36] or matter remaining after the formation of the star,[37] or comets like those of the solar system,[38] or even entire planets.[39] Since the matter near a star has already been subjected to chemical separation, and since furthermore the accretion of ions in the magnetic field of the star causes a further sorting of the ions, it is possible to see a qualitative explanation for the local anomalies of individual elements. There are data showing that some type A stars exhibit an excess IR emission, which is usually attributed to the presence of dust shells near these stars.[40] Unfortunately, we do not yet have any data on the composition of these shells. Furthermore, by no means all the observed anomalies can be explained by an accretion mechanism. The primary difficulty is in explaining both the deficiency and excess of helium without appealing to a mechanism for diffusion into the star or toward its surface. Little work of a quantitative nature has been done on the accretion hypothesis.

The reason for the distribution of anomalies in the form of spots is the most difficult theoretical question. The separation of elements by a magnetic field would require a significant difference in the ion diffusion coefficients along and across the field. For Cp stars, a difference of this sort could arise only right at the surface, where the optical thickness is less than 10^{-2}, with a field of 10^5 G. So far, observations have not provided any reliable data on the depths of the chemical anomalies, but data on the spectral lines of various elements lying on different sides of the Balmer discontinuity indicate that the anomalies stretch down to a depth greater than the optical thickness, $\sim 10^{-2}$ (Ref. 19). Furthermore, the magnetic fields observed at the surface are usually $\sim 10^3$ G, not 10^5 G. The long-term retention of ions would require a field with a very stable structure, with field lines running parallel to the surface. There are no data indicating the existence of such structures, correlated with the observed distribution of anomalies. Furthermore, magnetic confinement systems are usually imperfect, and instabilities may result in an escape of plasma in a time far shorter than the time $\sim 10^6$–10^8 yr required for magnetic separation. None of these questions has been studied theoretically.

In order to derive a theory it appears that we should abandon the approach of examining the diffusion of individual particles and take up the question of possible nonlinear hydrodynamic (or even hydromagnetic) instabilities of a hydrogen-helium plasma which might lead to the formation of local structures with a different molecular weight. We have in mind an analogy with soliton-like structures, but for a medium which is initially inhomogeneous in terms of molecular weight. We would then need to consider changes in transparency associated with a difference in the concentration of heavy elements and analyze the diffusion of these elements in the surrounding medium, which is inhomogeneous in terms of composition and temperature. Finally, we should consider the electric and magnetic fields which will unavoidably arise in a system with an inhomogeneous distribution of chemical elements.

However, regardless of the reason for the appearance of local chemical anomalies one can examine certain consequences of the very fact that they exist for a long time. Of primary interest are the spots of helium, which are exhibited by Ap stars in all cases where permitted by the sensitivity of the instruments. Surface anomalies of helium are apparently characteristic of all Ap stars, but it is not completely clear to what extent the overall abundance of helium in the star is anomalous. For a given effective temperature, a given gravitational force, and a given abundance of helium near the surface, stars with different overall abundances of helium are spectroscopically indistinguishable.[19]

We can show that the existence of long-lived helium anomalies in the form of large-scale spots leads to the appearance of a significant magnetic field.[38,41–44] Anomalies of other elements contribute to the field generation process in accordance with their concentration in the star and their degree of ionization.

We begin by determining the steady-state value which the field can reach. We assume that plasma flows are suppressed ($\mathbf{v}$ is small) and do not spread out the spots. We discard the terms $[\nabla,[\mathbf{vB}]]$ and the nonlinear terms $\sim [\mathbf{jB}]$ in (2). It is convenient to use a vector potential and to substitute current (1) into $\nabla^2\mathbf{A} = -(4\pi/c)\mathbf{j}$. Noting that the equation $\nabla\mu_e + e\mathbf{E} = m_e g_{\text{eff}}$ holds in the steady state, and using (1), we find

$$\nabla^2\mathbf{A} = -\frac{4\pi}{c}\sigma\left(\frac{m_e}{e}g_{eff} - \eta\nabla T\right). \tag{5}$$

We focus on the largest-scale variations in σ, η, and T; we set

$$\sigma\eta = (\sigma\eta)_0 + (\sigma\eta)_1 \sin\vartheta\cos\varphi + (\sigma\eta)_2 \sin\vartheta\sin\varphi + (\sigma\eta)_3\cos\vartheta, \tag{6}$$

where $(\sigma\eta)_i$ is a function of r alone. In a similar way, we write an expression for $T(r)$. There is no difficulty in finding an exact solution of (5), but we will not reproduce here the rather lengthy equations. We will restrict the discussion to the most important cases. Let us assume that the horizontal variations in the distributions of the conductivity and the thermoelectric coefficient are more prominent than the temperature variations. In calculating the toroidal field $\mathbf{B}_t$ we consider only the vertical gradient of the temperature. Furthermore, we omit the term $(m_e/e)g_{\text{eff}}$, which contributes less than $\eta\nabla T$ does. We then find the following expression for the toroidal field:

$$(B_t)_\varphi = \frac{4\pi}{3}\,[J_3 \sin\vartheta - (J_1 \cos\varphi - J_2 \sin\varphi)\cos\vartheta],$$
$$(B_t)_\vartheta = \frac{4\pi}{3}\,(J_1 \sin\varphi - J_2 \cos\varphi), \qquad (7)$$

$$J_i = \frac{1}{c}\Big[\frac{r}{R^3}\int_0^R \frac{\partial T_0}{\partial r}(\sigma\eta)_i r^2\,dr - \frac{1}{r^2}\int_0^r \frac{\partial T_0}{\partial r}(\sigma\eta)_i r^2\,dr - r\int_r^R \frac{\partial T_0}{\partial r}(\sigma\eta)_i \frac{dr}{r}\Big] \qquad (i = 1, 2, 3).$$

To estimate the toroidal field we assume that the helium variations stretch down to a depth $\sim L$. We set

$$(\sigma\eta)_i = Q_i \exp[(r - R)/L].$$

We adopt a $T_0(r)$ profile

$$T_0(r) = T_0(R) + (T_c/R)(R - r),$$

where T_c is the temperature at the center of the star. From (6) and (7) we then find the estimate

$$J_i \approx \frac{L}{Rc} Q_i T_c \left(1 - \frac{r}{R}\right) \exp\frac{r-R}{L}. \qquad (8)$$

The value of Q may be associated with the angular dependence of both the conductivity σ and the thermoelectric coefficient η. If the plasma is Lorentzian, then we have $\eta = (k/e)\ln[f(T)N_e]$. In the region of neutral helium we have $\ln N_e \approx \ln(\rho/m) - y$, where y is the helium concentration by weight. We adopt $y = 0.16 + 0.13\cos\vartheta$. If the horizontal variations in the temperature and conductivity are less pronounced than the variations in the helium distribution, we have $\eta_i \approx (k/e)y_i$ in the region of neutral helium and $-0.75\,(k/e)y_i$ in the He II region. As an example, we consider the case with $T_c = 2\cdot 10^7$ K. At a depth $r = 0.95R$ with $\sigma_0 = 10^{14}\,\mathrm{s}^{-1}$, we then find the estimate ~ 100 G for B_t.

If the dependence of $\sigma\eta$ on the angles ϑ and φ is smoother than as described in (6), the field may become fairly strong. We recall that the angles ϑ and φ in (6) are reckoned in the coordinate system of the chemical variation, which necessarily coincides with the system in which O_t is the rotation axis, and the angles are denoted by θ and φ. According to observations of spots on Cp stars, their temperature T differs from T of the surrounding regions by about 500°.

The poloidal field is zero except in the case in which both the product $\sigma\eta$ and T depend on the angles. A solution of (5) under assumption (6) yields the following expression for the dipole magnetic moment of the star:

$$\mathfrak{M}_i = \frac{2\pi}{c}\int_0^R r^2[(\sigma\eta)_k T_l - (\sigma\eta)_l T_k]\,dr \qquad (i, k, l \to x, y, z). \qquad (9)$$

Using the estimates of T_c and $(\sigma\eta)_k$ which were used above, we find from (9) that the dipole field at the surface is of the order of 1 G. This is far weaker than the observed field; we must either significantly change the accepted estimates of the characteristics of this star or seek some other mechanism to create or strengthen a poloidal field. This field can be strengthened considerably by poloidal plasma flows. Such flows might be, e.g., meridional circulations, which occur independently in each hemisphere and thus do not mix the matter of the two hemispheres; an asymmetry of these hemispheres will be preserved if it should arise for some reason. Since the "battery" toroidal field B_t is stronger than the poloidal B_p, we can use the equation $[\nabla, \mathbf{B}_p] \approx (4\pi\sigma/c^2)[\mathbf{v}\mathbf{B}_t]$. For example, if it is assumed that the poloidal flow occurs at a velocity $v_r = -u(r)(3\cos^2\theta - 1)$, $v_\theta = 3u(r)\sin\theta\cos\theta$ and that we have $v_\varphi = 0$, we would estimate the dipole field to be $B_{\mathrm{dip}} \sim (\sigma u/c^2)B_t$. In other words, comparatively low flow velocities $u \sim 10^{-3}$–10^{-4} cm/s would be sufficient to allow the poloidal field to reach a strength comparable to that of the toroidal field. A more accurate solution of (2) confirms this estimate.

So far, observations give us no information on such flows in the surface layers of Cp stars, so this mechanism for the strengthening of the battery field ranks only as one possibility.

The time required to reach a nearly steady state can be estimated from Eq. (2). Using the same approximations as above, for a star with the same characteristics, we estimate this time to be of the order of 10^6–$5\cdot 10^6$ yr, or far shorter than the evolution time of the star.

At any rate, we can say that if large-scale variations in helium distribution in Cp stars persist for more than 10^5–10^6 yr, than the magnetic field which they produce should be taken into consideration in a theory for these stars.

An interesting possibility was taken up in Refs. 12, 28, and 29: The poloidal field of Cp stars may be a relic field, while the toroidal field is generated by a battery effect associated with helium spots. In the same studies it was shown that there could be a self-consistent picture, in which the helium variations are distributed in such a way that they tend to suppress flows which might lead to the spreading out of these variations. A toroidal field is necessary for the stability of the overall resultant field of the star.

What can the battery mechanism predict regarding other entities? To answer this question, we begin with the most important of these entities: the earth.

6. MAGNETIC FIELDS OF THE EARTH AND OTHER PLANETS

The magnetic field of the earth has existed for more than $(3$–$4)\cdot 10^9$ yr at about the same average level, varying in magnitude and even in sign over intervals of 10^5–10^6 yr. The conductivity of the earth's core is $\sim 3\cdot 10^{15}\,\mathrm{s}^{-1}$, and that of the lower mantle is smaller by a factor of 10^3–10^4. Ohmic dissipation would thus annihilate the earth's field in $\sim 10^5$ yr unless it were sustained by some mechanism.

According to the dynamo theory, the field is produced by convective motions in the earth's liquid core. These motions are twisted primarily in one direction by Coriolis forces, in the manner of cyclones in the earth's atmosphere. The cyclonic convection leads to a strengthening of the field. The theory has been derived in only the linear approximation, however, so it does not tell us the strength of the field. It also does not explain why the axis of the dipole is tilted ($\sim 11.5°$) or other characteristics of the field.

The question of the nature of the convection in the earth's core and even the question of the existence of this convection have yet to be resolved unambiguously. If the suggestion of a chemical and thermal equilibrium at the boundary between a solid inner core and a liquid core of the earth is correct, than a stratification of the liquid core should be stable, and no thermal convection should arise.[45,46] In recent years, however, the model of a core which is cooling off at a rate of ~ 50 K/10^9 yr has been winning progressively wider recognition.[47] In the course of this cooling, an inner core forms and continues to grow. In this connection, the question of the contributions of various sources of the earth's heat is being reexamined. Several investigators[48,49,47] believe that along with the decay of radioactive elements an important role is being played by the energy liberated during gravitational separation: the precipitation of heavy substances, primarily iron, at the lower boundary of the liquid core and of light substances on its upper boundary. This process generates a convection in the liquid core, which would be required for dynamo action.[50,51] The convection rate and thus the possibility of a dynamo depend very strongly on the composition and state of the matter of the core. These characteristics are not known very well. The convection rate is usually determined by requiring that it be capable of sustaining the observed magnetic field by the dynamo mechanism. Flows with a velocity $\sim 10^2$–10^3 cm/s would be required to replenish the energy which the field loses by the ohmic mechanism ($\sim 3\cdot 10^{17}$ erg/s).

The light fraction, which floats up to the upper boundary of the core, apparently contains, in addition to iron, a slightly larger fraction of such elements as S, O, and Si. Examination of the Fe–S phase diagram shows that as a heavier fraction precipitates on an inner core a fraction with a relatively high sulfur concentration will float up in the form of separate liquid drops.[52]

Calculations based on core models[47,51] indicate that there may exist a layer (~ 60 km thick) right at the boundary with the metal which is stable with respect to convection. According to seismic data, there is a rather diffuse layer at the boundary of the solid core which seems to consist of a mixture of solid and liquid phases. At the lower boundary of the mantle there is apparently a hot layer with a reduced viscosity, in which the molten and solid materials separate. Hot jets arise in this layer and melt narrow tubes (~ 10–20 km) through the lower and upper mantle, all the way to the earth's surface. Within the tubes, the viscosity is greatly reduced by the high temperature of the matter. This matter rises under the influence of the Archimedes force. The existence of hot tubes which penetrate through the entire mantle would make it possible to explain the composition of the matter ejected by volcanoes.[53]

Although the mantle is solid, flows are possible, although they would be exceedingly slow (~ 0.1–0.5 cm/yr); there can even be a convection.[54] The seismic data indicate that the upper mantle is highly inhomogeneous in terms of density and chemical composition. It is not clear, however, to what extent the lower mantle is inhomogeneous. The relief irregularities at the lower boundary of the mantle are apparently small, of the order of 1 km over a horizontal distance of 10^3 km (Refs. 2 and 55). However, the variations in the density and chemical composition can be great.

There are indications[55–57] of a correlation between the large-scale gravitational and magnetic anomalies. The only possible source of magnetic anomalies which would correlate with gravitational anomalies would be regions with a high conductivity, i.e., regions inside the earth's core or right at its boundary.

Lateral variations of the velocity field of elastic waves propagating through the lower mantle (2200–2900 km) have been observed.[23,58] The change in velocity reached 50 m/s with an average value of 13.6 km/s. Under the assumption that the observed changes in velocity are linked with temperature variations, Kalugin *et al.*[24] determined the electrical conductivity of the lower mantle, taking it to be of the nature of an intrinsic electronic conductivity of a semiconductor and dependent on the temperature. The conductivity distribution which was found had positive anomalies in the regions of Australia, Central Africa, and South America and negative anomalies in the central parts of Asia, the Atlantic Ocean, and the Pacific Ocean. The size of these anomalies reached 40–50% of the mean value. The conclusion that there are variations in the electrical characteristics remains in force if the changes in velocity are due to the chemical composition, rather than the temperature (Fig. 2).

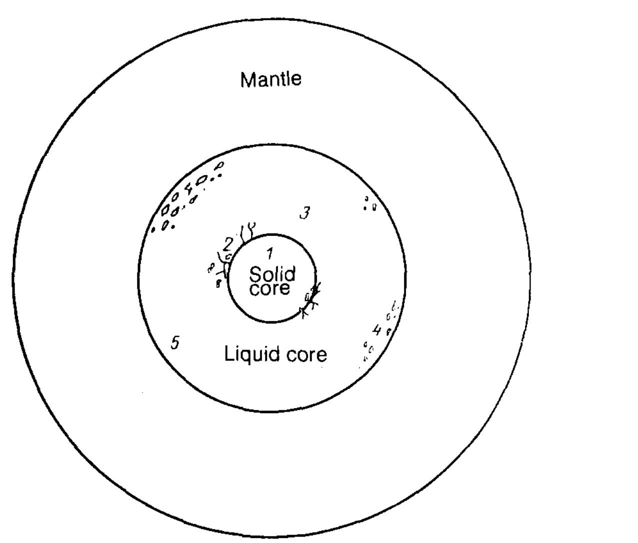

FIG. 1. Schematic diagram of the internal structure of the earth.[47,50,51] 1—Solid inner iron core; 2—"soft" shell of the inner core with liquid inclusions (a dendritic structure); 3—outer liquid iron core with O, S, and Si impurities; 4—mixture of liquid and solid phases at the freezing point; 5—outer region of core, where convection is apparently suppressed. The distributions of the density, the temperature, and the electrical conductivity at the mantle-core boundary are nonuniform.

Consequently, there is reason to believe that large-scale variations in composition or temperature exist at the boundary between the mantle and the core (and also at the boundary between solid and liquid cores). The matter of the mantle penetrates to some extent into the core, and vice versa. Even though we have no direct, unambiguous data on the structure of the boundary regions, we can answer the question of just what these chemical and temperature variations would have to be in order to produce a battery field comparable to the observed field of the earth.

The battery mechanism is one particular manifestation of a thermogalvanoelectric effect.[15] The various versions of these effects pertinent to the earth and other planets have been studied in several places.[13,59,60] The existing models, however, have not won acceptance because (a) they frequently predict fields far weaker than the observed fields, (b) they have no foundation in the form of reliable data on the interior of the earth, and (c) they can not explain changes in the magnetic field over $\sim 10^4$–10^5 yr, since the thermal regime of the earth and its structure change over a vastly longer time. These objections will be countered to some extent if the predictions regarding inhomogeneities of

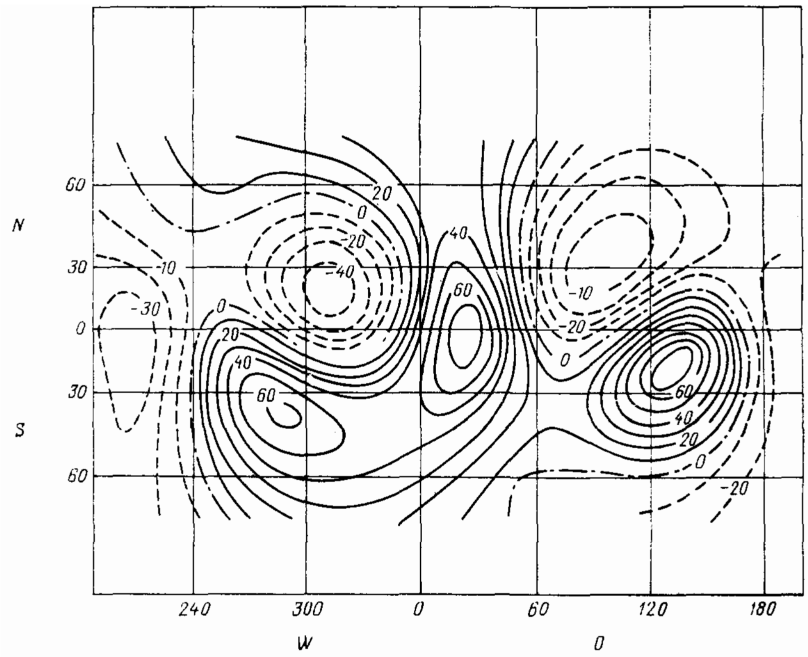

FIG. 2. Deviation from a spherically symmetric distribution of the electrical conductivity of the lower mantle at depths $2200 < h < 2900$ km. These results were found in Ref. 24 on the basis of the data of Refs. 23 and 58 on the velocities of seismic waves. The contour lines are labeled with the percent deviation from the accepted mean value.

the lower mantle and a prediction regarding the existence of a hot transitional layer between the mantle and the core proved to be correct. Such a transitional layer, with a high conductivity, and not involved in convection, may be inhomogeneous in composition and temperature. Slow flows, where they are nevertheless possible, may be responsible for variations in the magnetic field.

If we use the very simple assumption (6) regarding the distributions of the conductivity and the thermoelectric coefficient, we can use (7) for the toroidal field and (9) for the magnetic moment. The explicit expression for the thermoelectric coefficient and estimates of this coefficient depend strongly on the assumptions made regarding the composition and state of the core-mantle boundary region. We find a minimum value of η by assuming a purely metallic conductivity for this region: $\eta = (\pi^2 k/e)(kT/\epsilon_F)$, where ϵ_F is the limiting Fermi energy. For iron at a density ~ 10 g/cm^3 we find the estimate $\eta \approx 6 \cdot 10^{-8}$. As usual, we adopt a conductivity $\sigma \approx (3-4) \cdot 10^{15}$ s^{-1}. Assuming that the horizontal inhomogeneities in $\sigma\eta$ at the core-mantle boundary span a depth interval ~ 70 km (i.e., $L \approx 0.02R$, where $R = 3.48 \cdot 10^8$ cm is the radius of the core), and assuming that $\sigma\eta$ varies $\sim 50\%$ from one pole to the other, we find from (7) an estimate of the toroidal field: $B_t \sim (2\text{–}3) \cdot 10^{-3}$ G. This value does not contradict any facts, but it disagrees strongly with the predictions of the α-ω dynamo.

A poloidal field arises if, in addition to the variations in $\sigma\eta$, there are temperature variations which do not coincide with them. Assuming that the temperatures differ $\sim 500°$ between the poles at the core-mantle boundary, we find from (9) an estimate of the magnetic moment which is lower by a factor of about 50 than the observed value, $8 \cdot 10^{25}$ G$\cdot$cm^3. If the thermoelectric coefficient η at the mantle boundary is determined not by variations in the characteristics of a metal but by variations of a metal-to-semiconductor type, then the estimate of η and thus that of $\mathfrak{M}$ might be significantly larger. A seed field is required for the action of any dynamo mechanism. Even if the field produced by variations inside the earth turns out to be weak, it could contribute substantially to the operation of the dynamo. There is the possibility that a nondipole component of the field, which apparently correlates with gravitational anomalies[55,56] and with the velocities of seismic waves at the core boundary,[23,58] is determined specifically by composition variations.

Changes in the magnitude and even the sign of the field (determined from the magnetization of ancient rocks) over a time $\sim 10^5$–10^7 yr might be associated both with large-scale changes in composition, because of an exchange of material at the mantle-core boundary, and changes in the nature of poloidal flows.

Our knowledge of the conditions prevailing in the interiors of the other planets is incomparably poorer than that about the earth. Why, for example, does Venus not have any appreciable field? Will its slow rotation and the essential absence of tidal flows lead to more nearly uniform chemical and temperature conditions at the boundaries of a liquid core? Alternatively, is the absence of a field associated with the slowness of poloidal flows in a core? Further research is required to answer these questions.

Again, there is little we can say about why little Mercury, rotating slowly (56 days), has a magnetic field with a dipole moment of $5.2 \cdot 10^{22}$ G$\cdot$cm^3, while Mars, rotating far more rapidly (24 h 37 min), has either a very weak field or none at all. There are no reliable data on the cores of these planets, their sizes, the flow velocities, chemical or temperature variations, and so forth.

Strong magnetic fields are possessed by the giant planets Jupiter, Saturn, and Uranus. Each rotates rapidly and has many satellites, so that the tidal flows are strong and asymmetric. The interiors of these planets apparently have a region with liquid metalized hydrogen. The conductivity of this region is $\sigma \approx 10^{17}$ s^{-1}.

A possible convection in the interiors of the giant planets, combined with their rapid rotation, creates favorable conditions for a dynamo. It is not clear why all these planets have a dipole field. For the existing differences in physical conditions, these planets should have fields of different multipolarities, according to the dynamo theory.[1–6]

The high conductivity of metallic hydrogen and the solubility of helium in it make it possible that the giant planets might have variations in their helium distribution which could lead to the generation of a field. Further research on the roles played by various mechanisms is required.

Analysis of lunar rocks has shown[46,61,62] that they solidified $\sim 3\cdot 10^9$–$4\cdot 10^9$ yr ago in the presence of a field $\sim 5\cdot 10^{-2}$–10^{-2} G. The radius of a possible lunar core could not exceed 300–400 km. In order to produce a field $\sim 10^{-2}$ G at the surface, the field at the core would have to be ~ 10 G. If the field were caused by dynamo action, the velocities in the lunar core would have to be far larger than those in the earth's core. We have no direct data on the convection velocities in the earth's core or (especially) in the lunar core. At any rate, field generation is possible only in regions with a high conductivity. An alternative to the dynamo mechanism is the suggestion that the lunar field might have been induced by the earth if the moon had once been far closer to the earth, $(5–10)R_{\text{Earth}}$ and if the earth's field had been far stronger than it is today (~ 10–50 G). Another alternative is that there were large chemical and temperature variations in regions with a high conductivity inside the moon, which might have produced a field by the "battery" effect.

7. MAGNETIC FIELD OF THE SUN

The magnetic field of the sun contains a hierarchy of structures which correspond roughly to the stages of convection cells. The small-scale structures which are observed appear as a multitude of distinct thin filaments ($\sim 10^2$–$3\cdot 10^2$ km) with a strong magnetic field ($\sim 1.5\cdot 10^3$ G). These filaments are concentrated in intergranulation gaps and cover 20% of the entire area. Between the filaments the field is comparatively weak (~ 10–100 G) and random. The granulations at the surface consist of small-scale cells. A single cell exists for 5–10 min and then disappears. The magnetic flux, on the other hand, does not disappear as rapidly. The filaments move over the surface under the influence of convective fluxes, around these fluxes. The time scale for the coalescence of filaments seems to be longer than the lifetime of the granules.[63–65]

The toroidal magnetic field of the sun apparently is of the form of magnetic tubes which are stretched out parallel to the surface of the sun. Observations of changes in the polarity of active regions on the sun indicate that the directions of the toroidal field are opposite in the two hemispheres. The directions change from one solar cycle to the next. In some cases, supergranulation convection cells ($\sim 10^4$–$5\cdot 10^4$ km) carry off parts of tubes to the surface, in the form of huge loops. The field in a tube is 10^3–$5\cdot 10^3$ G. In those regions where the most intense tubes emerge at the surface, there is a pronounced change in the heat transfer, which leads to the appearance of a sunspot. The field associated with the supergranulation is regarded as a field of intermediate scale.

Large regions on the sun are occupied predominantly by fields of one polarity. The field averaged over these areas is regarded as a large-scale field. This field (~ 1–2 G) is far weaker than a small-scale field. In the polar regions it appears as a poloidal dipole field. The regions of identical polarity coincide with the hemispheres only on the average. This result means that in addition to the dipole component there are other components. The effective heliomagnetic equator is not a circle but a wavy line. This line changes in shape over the course of the cycle. Large-scale field structures are carried off by the solar wind into interplanetary space. At minimum solar activity, one observes four magnetic sectors in the solar wind, while at activity maximum only two sectors are observed. The sector structure reflects the shape of the heliomagnetic equator.[66] The dynamo theory suggests the following picture for the solar cycles. The differential rotation of the sun stretches and twists the poloidal field, creating a toroidal component. The inverse process—the formation of a poloidal field from a toroidal one, a necessary part of the dynamo mechanism—is not so simple. There are a variety of possibilities for this process. The most popular is the suggestion offered by Parker,[1] that a poloidal field forms during the coalescence of magnetic loops which are stretched out from the toroidal field by a cyclonic convection. This picture can be described in the magnetohydrodynamics of mean fields, proposed and developed in Refs. 1–6 and 48. It has been shown that gyrotropic motions of plasma at any scale (in the case at hand, the gyrotropy is created by the rotation of the sun) could create both toroidal and poloidal fields.

If the motions required for a dynamo do in fact occur on the sun, it becomes possible to explain several important characteristics of the magnetic solar cycle. Dynamo waves, propagating in succession from the pole to the equator, would implement the change in the polarity of the toroidal and poloidal field components. The observed migration of active regions toward the equator (the "Maunder butterfly") can also be explained as a consequence of the propagation of a dynamo wave. Disruptions of the periodicity of the cycles (the "Maunder minimum") could be interpreted as the results of changes in the convection regime. In addition to the large-scale structures, the dynamo theory has been used to study small-scale structures.[1–6,67] It has been shown that a field intensification is also possible at small scales.

However, we should not ignore the questions and weak points of this theory. The existing linear theory does not determine the strength of the field on either the large or small scale.

An attempt to explain the shape of the heliomagnetic equator on the basis of the α-ω model runs into fundamental difficulties associated with the rapid dissipation of axially asymmetric field components.[27]

The solar field is well organized in the form of filaments and tubes with a strong field—from huge sunspots to small facular points. The magnetic tubes under the photosphere and the convective flows apparently form two coexisting structures. The convection occurs between the tubes, without substantially deforming them. The situation could be understood qualitatively as the result of a displacement of magnetic fields from regions spanned by intense stochastic motions and the suppression of these motions in regions with a sufficiently strong field. The theory as it exists today does not give us a quantitative picture of the separation of the medium into magnetic and convective phases.

Zel'dovich *et al.*[68] have shown that the magnetic field generated by random gyrotropic flows has the property of "intermittency": Against the background of comparatively uniform fluctuations of the field, some spatially narrow but intense bursts appear at times. Kleorin *et al.*[69] interpret the narrow magnetic filaments on the sun as a manifestation of a peculiar structure of field correlators. These studies are very promising, although this approach is still in its infancy.

The behavior of the large-scale field, which is the average of the small-scale field, is correlated over large distance and even between hemispheres. Consequently, the magnetic flux tubes have large dimensions. The regularity in the distribution of magnetic field polarities in large active regions indicates that the subsurface field is directed latitudinally and is ordered, i.e., that there exists a field component which does not exhibit characteristics of chaos. These circumstances are difficult to explain under the assumptions (which are customary in the dynamo theory) that the field is distributed over the entire convection zone. In order to protect the field structure from destruction by convective flows, we would have to assume that the field inside the tubes corresponds to an equal distribution of the energy between the field and the convection (or we would have to make even further assumptions). Such a field becomes buoyant and, according to Ref. 1, floats up to the surface of the sun over a time shorter than the period of the cycle. In order to avoid the rapid loss of magnetic flux due to instabilities driven by the buoyancy of tubes, several investigators[70,71] have suggested that the horizontal flux is concentrated in a narrow layer at the bottom of the convection zone or at the upper boundary of the radiative zone. The interaction of the convection with the magnetic field would not be as strong in such a layer. Schussler[70] and Van Ballegojer[71] analyzed the possibility that magnetic field tubes would be present at the bottom of the convection zone in a region in which convection penetrates into the radiative region—the so-called overlap zone. This zone is defined as the region in which the mean temperature gradient is lower than the adiabatic gradient, but where the convective motion still persists. The properties of this overlap zone were studied in Refs. 70 and 71. In this region the convection penetrates slightly into the radiative zone, making the positions of the magnetic tubes unstable and provoking their floating up to the surface. Under the assumption that the horizontal field consists of a large number of narrow, nearly parallel tubes which are separated from each other by field-free regions and which contain a field strong enough to withstand deformation caused by convection, it was shown[71] that a nearly stable position of tubes with a field $\sim 10^4$ G requires a horizontal plasma flow at a velocity ~ 5–10 m/s along the tube.

There are many effects which might lead to an entrainment of magnetic tubes bringing them to the bottom of the convection zone.[4,72,73] The bottom of the convection zone or the top of the radiative zone would appear to be the most suitable region for tube confinement.

The α-ω version of the dynamo theory, which is the version applied to the sun most frequently, uses the assumption that the angular velocity of the differential rotation increases with distance into the sun. This result agrees with observations of a nearly rigid-body rotation of large-scale magnetic structures, which overtakes the differential rotation at a given latitude, but on the other hand it contradicts the differential-rotation theory of Ref. 74, which predicts that the angular velocity will decrease with distance into the interior of the sun.

Observations and analysis of vibrations of the solar surface yield information on the differential rotation of the interior regions of the sun.[75–78] The results of Refs. 75–78 show that the rotation is a nearly rigid-body rotation down to depths $\sim 0.6R_{\odot}$, slowing down slightly with the depth. This behavior contradicts the customary assumptions of the α-ω dynamo model (Fig. 3).

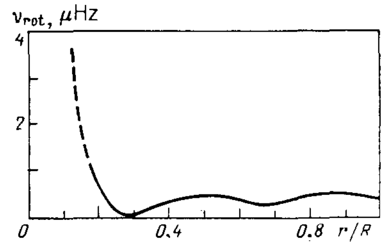

FIG. 3. Depth profile of the rotation frequency of the inner regions of the sun.[76] The depths are expressed as fractions of the radius.

The choice of values and functional dependence for such quantities as the helicity, the angular velocity, and the turbulent and magnetic viscosities is usually made more or less arbitrarily. For this reason, a detailed analysis and the acquisition of experimental data are in many regards more important than a detailed analysis of approximate models. Many years of observations of large-scale motions in the surface layers of the sun based on an analysis of the Doppler broadening of the Fe II spectral line with $\lambda = 5250.2$ Å have shown[79–81] that in addition to the differential rotation there are torsional vibrations, with a period of 11 yr, in the surface layers. Four zones are observed in each hemisphere: two with a slow rotation and two with a fast one. The mean amplitude of the torsional velocities is 3–5 m/s. The pattern of vibrations drifts from the pole toward the equator. A wave which arises at the pole reaches the equator in 22 yr, from the north and south at nearly the same times.

Adherents of the dynamo theory interpret this result as a confirmation of their theory, which explains the vibrations as resulting from the Lorentz forces of the 22-yr dynamo wave of the large-scale magnetic field.[82,83] The opponents focus on the following difficulties[79–81]: a) The amplitude of the torsional wave remains nearly constant to latitudes $\pm 75°$, while the magnetic fields of the solar cycle are at a maximum at ± 15–$20°$ and are very weak at latitudes $> \pm 45°$. b) The strength of the magnetic field varies markedly during the cycle, while the magnitude of the torsional velocities remains nearly constant. c) The angular rotation velocity apparently does not increase with depth.

There is the possibility that a torsional wave is a primary natural oscillation mode of the sun, which is excited by, for example, convective flows, and which serves as a trigger for the solar cycle. Torsional vibrations can exist only in an elastic medium. The elasticity which is necessary may be provided by the magnetic field, if the field structure is maintained over a time much longer than the oscillation period. According to the observations which have been carried out,[79–81] the high-latitude magnetic flux moves toward the poles, rather than toward the equator, during the cycle, and this process is not a diffusion but instead more of the nature of a regular motion in some meridional flow. The observed drift of active zones toward the equator over a cycle may be interpreted not only as the result of the propagation of a dynamo wave but also as a displacement of a region in which magnetic tubes emerge from the interior at the surface. This displacement could be associated with a torsional wave or some other factors.

There is a definite correlation between the even and odd 11-yr solar-activity cycles, but there is no apparent correlation between the 22-yr magnetic cycles. The periodicity of the solar cycles is inexact and is reminiscent of the periodicity of the fall of drops from a partly open water faucet. A certain amount of magnetic energy accumulates in the sun, floats up to the surface, and escapes from the sun, making room for a new portion. In some cases, the magnetic flux tubes of a new cycle appear against the background of the flux from tubes of the preceding cycle, which has not yet been completed. The tubes of the new cycle have a polarity opposite to that of the preceding cycle.

Gudzenko and Chertoprud[85] have pointed out that the cyclic solar activity is similar to the behavior of an autonomous relaxation oscillator. Our water faucet falls in this category. There is no feedback in an oscillator of this sort. Energy is not converted from one form into another, but it does reach a certain level and leave the system. A relaxation oscillator maintains the oscillation phase well even without locking by some external source. The solar cycle is analyzed quantitatively as an example of a relaxation oscillator in Refs. 85 and 86.

Is a dynamo the only possible mechanism for the generation of a field inside the sun? Large temperature gradients (and, possibly, chemical gradients) exist in various parts of the photosphere, e.g., sunspots, quiet regions, and active regions. The short lifetime of these inhomogeneous structures in the convection zone rules out the possibility that a significant field is produced by the battery effect. At the bottom of the convection zone, however, the lifetime of these structures may be very long, since there are no fast motions there. The quiet conditions and the sharp change in the heat transfer in this region are reminiscent of the situation at the surface of a Cp star (§5), so it is important to analyze possible instabilities which might create inhomogeneous structures at the bottom of the convection zone. There has been no such analysis previously.

Indications were found in Ref. 87 that the phases of the solar cycle correlate with the intensity of the flux of solar neutrinos. This surprising result, if correct, would completely destroy the present understanding of the origin of the solar cycle. The implication would be that the solar cycle is regulated by processes which occur deep in the interior of the sun. Before we adopt such a radical conclusion, we must test the reality of this correlation.

The possibility of observing, along with the sun, a large number of stars differing in spectral type, size, mass, rotation velocity, multiplicity, etc., is a major advantage for the theoretical work. Stars with convection shells belong to late spectral types. These stars have well-developed chromospheres, coronas, and stellar winds. In many regards they are similar to the sun, but most of them have a higher activity. Dark spots have been observed on convective stars. The spots usually cover ~ 0.2–0.6 of the surface of the star, or a much larger fraction than in the case of the sun ($\sim 4\cdot 10^{-3}$). The temperature of the spots is of the order of 3700–3900 K. This temperature is nearly independent of the effective temperature of the star, which ranges from 4100 K to 6100 K. It is also nearly independent of the effective gravitational force, which ranges from $(2$–$4)\cdot 10^4$ cm/s^2 for dwarfs to 10^2–10^3 cm/s^2 for giants. The magnetic field is strong in a spot, about $(1.5$–$2)\cdot 10^3$ G, and nearly independent of the gravitational force and of the temperature.[88]

The activity of convective stars is cyclic. The periods of the cycles of rapidly rotating stars are shorter than for the slowly rotating stars. All stars with rotation periods longer than 8 days appear to have a nearly identical period of cyclic activity, ~ 10 yr. If the region from which the magnetic tubes float up is the bottom of the convection zone we might expect that stars of small mass, which are apparently totally convective, should not have a strong field. Indeed, some very late M dwarfs exhibit no chromospheric emission.[88,89]

8. HOT O AND B STARS

The stars of spectral classes O and B have high surface temperatures, $\sim 5\cdot 10^4$–$2\cdot 10^4$ K. Hydrogen is fully ionized from the center to the surface, and the temperature gradient is lower than the adiabatic level (except in regions near the center). Consequently, no external convection zone arises. However, these stars are observed to have hot chromospheres and coronas and an intense stellar wind, which has a velocity $\sim 10^8$ cm/s and which ranges out to distances ~ 1–3 pc. The mass carried off by this wind can reach $\sim 10^{-7}$–10^{-5} $M_\odot$/yr. The plasma of the stellar wind stretches out the field lines and carries off magnetic flux with itself. Nevertheless, many O and B stars are observed to have a global magnetic field $\sim 10^2$ G. If this is a relic field, then magnetic tubes must be floating up continually, replenishing the flux loss. If the field is instead generated by flows, these flows must be fairly intense.

Since O and B stars should not have heat convection, the presence of coronas and a stellar wind is usually explained in terms of a radiation pressure exerted by the light. The radiation pressure exerted on heavy ions is stronger than the gravitational force. These ions transfer their momentum to the surrounding matter, causing a heating of this matter and its outflow from the atmosphere.[90]

In addition to these flows in rapidly rotating stars, which the O and B stars are, there is an intense meridional circulation. The conditions for a hydrostatic equilibrium and for a thermal equilibrium cannot be satisfied simultaneously if nonspherical forces set up by the rotation or a magnetic field are acting in the star. As a result, a circulatory flow arises. The meridional flows in regions with an optical depth $\tau > 1$ have been analyzed in detail by many investigators.[10,91–93] The region $\tau < 1$ was studied in Refs. 92 and 93.

An equation determining the circulation velocity can be found from (12) in the form $\mathbf{V}(\rho c_p \nabla T - \nabla P) = -\nabla \mathbf{F}$, where $\mathbf{F} = \int \mathbf{n} I(r,\mathbf{n}) d\mathbf{n}$ is the radiation flux, and $I(r,\mathbf{n})$ is the radiation intensity at point r in direction $\mathbf{n}$. The intensity $I(r,\mathbf{n})$ is determined from the radiation transport equation (Ref. 94, for example). If the isotherms in the star do not coincide with the isobars because of the rotation of the star, the condition $\nabla\mathbf{F} \neq 0$ holds locally, although on the average over a constant-potential surface we have $\langle \nabla \mathbf{F} \rangle = 0$.

Calculations of the meridional circulation velocities carried out in Refs. 92 and 93 show that the radial velocity V_r reaches a maximum at a depth where the optical thickness τ is close to unity. At $\tau < 1$, the quantity ρV_r falls off with decreasing density, while at $\tau > 1$ it increases. This result means that the tangential velocities V_ϑ are directed oppositely at $\tau > 1$ and $\tau < 1$; i.e., the circulation fluxes are closed in the photosphere.

Estimates[92,93] for $\tau \lesssim 1$ yield

$$v_r \approx \frac{L\Omega^4 R^5 \varkappa(\tau)\tau}{G^3 M^3 (\rho\varkappa)_{\tau=1}}, \qquad v_\vartheta \approx \frac{R}{h} v_r, \tag{10}$$

where $\varkappa(\tau)$ is the opacity at depth τ, h is the scale height of a homogeneous atmosphere, and L is the luminosity of the star.

The circulation velocities can reach the velocity of sound at depth $\tau \sim 1$ for O and B stars with a typical angular velocity $\Omega \sim 10^{-4}\ s^{-1}$. The velocity gradients are also very large, so the onset of turbulence is very probable. For a star with $L = 5\cdot 10^{37}$ erg/s, $M = 5M_\odot$, $R = 5\cdot 10^{11}$ cm, and $\Omega = 10^{-4}\ s^{-1}$ in the region $\tau \sim 1$, for example, we find $V_{r\max} \approx 10^3$ cm/s and $V_\vartheta \approx 10^7$ cm/s. The intensity and velocity of the turbulent motions depend strongly on Ω. In the steady state, in which the energy acquired by the turbulence is dissipated by viscosity, we estimate the turbulent velocity to be $v_t \approx V_\vartheta (l_t/h)$, where l_t is a length scale of the turbulence.

Since l_t is usually smaller than h by a factor of several units, we find $v_t \sim 10$ km/s. These values agree well with the observed velocities of microscopic turbulence in stars of early types.[95] Stars with the same values of M, R, and the surface temperature T_S, but with different angular velocities, may have different levels of microscopic turbulence. It is well known[96] that turbulent motions can excite sound waves. The chromospheres and coronas in stars of an early type may arise because of a dissipation of sound waves. The power of the emission of acoustic waves per 1 cm^3 of a turbulent medium is $\rho(v_t^3/l_t)(v_t/c_S)^5$ in order of magnitude, where c_S is the sound velocity.[96,97] The energy radiated per second by the stellar atmosphere is therefore given in order of magnitude by

$$L_* \sim 4\pi R^2 \rho v_t^3 \frac{h}{l_t}\left(\frac{v_t}{c_S}\right)^3. \tag{11}$$

In the surface layers of O and B stars we have $c_S \approx 2\cdot 10^6$ cm/s and $v_t \sim c_S$. Assuming $h \sim 10 l_t$, we easily find $L_* \sim 10^{33}$–10^{34} erg/s. A significant fraction of this energy is radiated by the chromospheres and coronas in the UV and x-ray ranges; another part of it may be transferred to the stellar wind. Consequently, at low rotation velocities (if $v_t < c_S$) the luminosity of the corona, L_*, for O and B stars increases very rapidly with increasing Ω. If $\Omega R \gtrsim (GM/R)^{1/2}(GMh\rho c_S/L)^{1/4}$, the circulation velocity V and the turbulent velocity v_t are of the order of the sound velocity, i.e., of the order of 10–30 km/s. The velocities V and v_t cannot exceed c_S, even if the star is rotating very rapidly. The luminosity of the corona of a rapidly rotating O or B star is of the order of $4\pi R^2 \rho c_S^2$ and is nearly independent of Ω. Since we have $\rho \sim T^{5/4} g^{1/2}$ and $c_S \sim T_S^{1/2}$, we easily find $L_* \sim T^{9/4} R^2 g^{1/2} \sim L^{0.7} M^{0.5} R^{-0.4}$, where L is the optical luminosity. Coronas can then therefore become hot enough to emit a stellar wind.

By themselves, meridional circulations, being axisymmetric, do not excite a magnetic field. However, the Coriolis forces acting on turbulent flows can give rise to a differential rotation of surface layers.[98] The resultant flow which arises from meridional circulations and the differential rotation is three-dimensional, has a helicity, and can therefore generate a field. Unfortunately, quantitative estimates are very uncertain in this case, since we have no direct observations of flows in O or B stars. In fact, opinion is divided regarding the reason for the appearance of the coronas of these stars.

9. THERMOMAGNETIC INSTABILITY OF WHITE DWARFS

The most important distinguishing feature of white dwarfs (which comprise about 10% of all stars) is their high density, $\rho \approx 10^5$–10^7 g/cm^3. The pressure in a white dwarf is maintained by degenerate electrons, which also determine the thermal conductivity to a significant extent. Fields $\sim 10^6$–10^8 G have been observed for several tens of white dwarfs. In the remaining cases, fields have not been observed. That statement does not at all mean, however, that most dwarfs do not have fields. It would be difficult to observe fields weaker than $\sim 10^4$–10^5 G because of the strong gravitational force, the rapid rotation, and the intense flows in the surface layers, which give rise to a pronounced broadening of spectral lines and keep the integral circular polarization small.[99]

There are various points of view regarding the origin of the observed fields. If a white dwarf formed (for example) as the core of a red giant after a significant fraction of its mass had been ejected, then the field of the dwarf might have been produced in a preceding stage of the evolution and preserved in the dwarf. Intense turbulent convective flows are observed near the surfaces of many dwarfs. On the one hand, these flows oppose the preservation of a relic field but on the other they promote the dynamo mechanism. Neither of these possibilities has been studied quantitatively as it applies to white dwarfs.

Let us examine yet another possibility, which is unrelated to any assumption regarding a relic field or a dynamo mechanism. We will show that a "thermomagnetic" instability[100] unavoidably arises in the degenerate electron gas of a white dwarf and causes an effective strengthening of the field. The instability proceeds in the following way. We assume that there is a temperature gradient ∇T_0 in the system, maintained by an external source. If even a very weak magnetic field $\mathbf{B}_0$ arises in the system because of some factor, a heat flux $\mathbf{q}_0$ will arise in the direction perpendicular to both ∇T_0 and $\mathbf{B}_0$. Thus the slightly "hotter" electrons moving toward the surface will be deflected by the field to a slightly greater extent than the "colder" electrons, moving toward the center, will be. In addition, the deflection of the hotter electrons will be in the opposite direction. The flux $\mathbf{q}_0$ gives rise to a temperature gradient ∇T_1, which creates a solenoidal thermoelectric field $\mathbf{E}$ in the direction of $\mathbf{q}_0$. The field $\mathbf{E}$ in turn gives rise to a magnetic field $\mathbf{B}_1$, which is in the same direction as $\mathbf{B}_0$ and intensifies it (Fig. 4).

The thermomagnetic instability in a nondegenerate plasma with a predominant electron heat transfer has been studied in detail experimentally.[101,102] The experiments were carried out in the following way. Pellets of a material $\sim 10^{-3}$ cm in size were bombarded with an intense laser beam, which created a high temperature gradient ∇T_0 in the material. The measurements showed that in a time of only $\sim 10^{-9}$ s, before the pellet had time to evaporate, it acquired a field of the order of several megagauss. A theory for this phenomenon was proposed in Refs. 102–105. A generalization was made to the case of a degenerate electron plasma

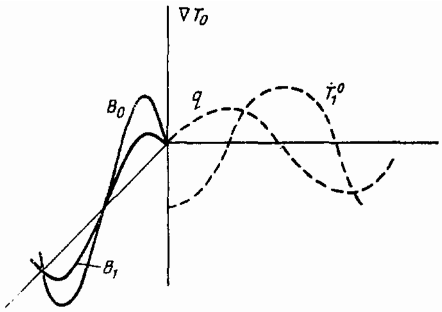

FIG. 4. Schematic diagram of perturbations of the temperature (T_1), the heat flux ($\mathbf{q}$), and the magnetic field which grow in the course of a thermomagnetic instability.[106,107]

and applied to the case of white dwarfs in Refs. 106 and 107. In a degenerate plasma, the equations of motion and the continuity equations are of the same form as in a nondegenerate plasma.[108] Viscous tensions can be ignored in the equations, since the rise of the instability of interest here turns out to be much longer than the mean free time of the ions for white dwarfs. In addition, possible flows within the dwarf in the unperturbed state (e.g., a meridional circulation) can be ignored, since the incorporation of these flows does not have any substantial effect on the estimates regarding the onset of the instability. Writing all quantities as sums of unperturbed quantities P_0, T_0, V_0, ... and small perturbations P_1, T_1, V_1, ..., one can linearize the hydrodynamic equations. In addition, we use Ohm's law as in (1). Noting that the magnetic field renders the medium anisotropic, we use the thermal-emf tensor $\hat{\eta}_{ik}$, and we write the electric field produced by this force as $\hat{\eta}\nabla T = \eta_{\parallel}\nabla_{\parallel}T + \eta_{\perp}\nabla_{\perp}T + \eta_B[\mathbf{b}\nabla T]$, where the $\parallel$ and $\perp$ mean that we need to take the component of the vector ∇ which is parallel to or perpendicular to the magnetic field. Here $\mathbf{b} = \mathbf{B}/|\mathbf{B}|$. The electrical conductivity σ_{ik} is also a tensor. To calculate η_{ik} and σ_{ik}, we could use, for example, Ref. 108. The electric current density is given by $\mathbf{j} = e\int(\mathbf{u}-\mathbf{v})f(\varepsilon)d\mathbf{p}$, where $\mathbf{u}$ and $\mathbf{p}$ are the velocity and momentum of an electron, $f(\varepsilon)$ is the energy distribution of the electrons, normalized to the number of particles: $\int f(\varepsilon)d\mathbf{p} = N_e$. The function $f(\varepsilon)$ is found from the solution of the kinetic equation; in our case it is sufficient to use the "relaxation approximation." Furthermore, in the degenerate core of a white dwarf, provided that the field does not exceed 10^9 G there, the inequality $\omega_B\tau \ll 1$ holds, where $\omega_B = eB/m_e c$ is the electron cyclotron frequency, and τ is the scale time between Coulomb collisions of electrons with ions.

It is convenient here to use the energy transport equation in the form

$$\rho c_P \frac{dT}{dt} + \left(\frac{\partial \ln \rho}{\partial \ln T}\right)_P \frac{dP}{dt} = -\nabla\mathbf{Q} + \mathbf{j}\left(\mathbf{E} + \left[\frac{\mathbf{v}}{c}\mathbf{B}\right]\right), \quad (12)$$

where c_p is the specific heat (per unit mass), and the vector $\mathbf{Q} = \int \varepsilon \mathbf{v} f(\varepsilon)d\mathbf{p}$ is the energy flux density. For small deviations from equilibrium we can write

$$\mathbf{Q} = -\frac{\mu}{e}\mathbf{j} - T\hat{\eta}\mathbf{j} - \hat{\varkappa}\nabla T, \quad (13)$$

where the tensor $\hat{\varkappa}$ is the thermal conductivity, which is given explicitly in Ref. 108, among other places.

Now linearizing the Navier-Stokes equations, the continuity equation, and Eqs. (2) and (12) jointly with the Poisson equation for the gravitational potential, we find a linear system of equations whose solution determines the behavior of small perturbations in the degenerate core of a white dwarf.

We first consider perturbations with a wavelength λ much shorter than the length scale (L) of the variation of the unperturbed quantities. We can ignore the change in the gravitational potential in this case, since it contributes less to the changes in the density and the pressure. For simplicity we assume that the unperturbed functions depend on only the one Cartesian coordinate z. As usual, we assume that the perturbed quantities are proportional to $\exp(i\omega t - i\mathbf{k}\cdot\mathbf{r})$, where $\mathbf{k}$ is the wave vector. Substituting this functional dependence and the explicit equation of state of a nonrelativistic degenerate plasma into out linearized system of equations, we find a dispersion relation from the condition for the compatibility of this system of equations. This dispersion relation determines ω as a function of $\mathbf{k}$. The quantity $\omega = \omega_1 - i\omega_2$ turns out to be complex. Its imaginary part determines the rise time of the instability, $t_0 = 2\pi/\omega_2$, while its real part determines the velocity of the spatial drift of these instabilities, $\mathbf{v}_d = \partial\omega_1/\partial\mathbf{k}$. An analytic solution of the dispersion relation and an explicit expression for t_0 were derived in Refs. 106 and 107.

The region near the boundary of the degenerate core is the most favorable region for the onset of a small-scale thermomagnetic instability, since dT_0/dz is large in this region, while the density is not yet overly high. Perturbations with $k_z = 0$ grow most rapidly. Instabilities which arise at some depth drift toward the surface at a velocity of the order of

$$v_d \approx \frac{1}{4\pi^2}\frac{\lambda}{l_0}\frac{\lambda}{t_0}\frac{k_z}{k}, \quad (14)$$

where $\lambda = 2\pi/k$ is the wavelength of the perturbation. An instability traverses a distance $\sim l_0$, over which there are significant variations in the unperturbed quantities, in a time $\sim l_0/v_d = (l_0/\lambda)^2 t_0 \gg t_0$, i.e., an instability has time to grow dramatically before it leaves the degenerate core of the dwarf. Perturbations with $k_z \approx 0$ undergo essentially no drift.

For nonradial vibrations of a star, which may arise for various reasons,[109] or under the influence of tidal forces, if the star is part of a binary system, large-scale perturbations arise, spanning a large area at the surface of the degenerate core. For definiteness, we consider perturbations of the type $Y_{lm}(\vartheta\varphi)\exp(i\omega t - ikr)$, where $Y_{lm}(\vartheta\varphi)$ is a spherical harmonic. We assume that the perturbations have a comparatively small radial dimension, i.e., that the condition $kl_0 \ll 1$ holds, and the dimensions in the meridional and azimuthal directions can be arbitrary. This case was studied in Refs. 106 and 107. The estimates of t_0 and V_d turn out to be the same as in the planar case.

The thermomagnetic instability thus intensifies perturbations of any scale in the magnetic field, the velocity, the temperature, the density, and so forth. A condition for the onset of instability is that the quantity t_0 be positive. Furthermore, the time t_0 must be shorter than the time scale for the ohmic dissipation of the field. These two conditions lead to the inequalities

$$T_7 > 0.55\frac{A^{1/5}Z\Lambda\mathfrak{M}^{1/5}}{\mathfrak{R}^{2/5}T_{e4}^{4/5}}, \qquad \rho < 2.2\cdot 10^2\frac{\mathfrak{R}^{12/5}T_{e4}^{24/5}}{A^{1/5}Z\mathfrak{M}^{6/5}}; \quad (15)$$

where Λ is the Coulomb logarithm; $\mathfrak{M} = 10^{-33}\ M$ g; $\mathfrak{R} = 10^{-9} R$ cm; $T_7 = 10^{-7} T_0$ K; $T_{e4} = 10^{-4} T_e$ K; M and R are the mass and radius of the white dwarf; and A and Z are the atomic number and the charge of the ions of the plasma of the star. We see that as the white dwarf becomes hotter and less massive the upper boundary of the instability region sinks deeper. In a dwarf consisting of light elements, the instability region is wider than in a dwarf consisting of heavy elements.

In addition to the thermomagnetic instability, a convective instability can occur in a white dwarf.[110] At modest surface temperatures, the lower boundary of the convection zone may descend, even into the degenerate region. As the surface temperature rises, however, the convection zone becomes progressively thinner, and its lower boundary rises into a region of low density. According to Ref. 110 for example, at $T_{e4} = 3$ a convection arises only in a region with $\rho < 4 \cdot 10^{-4}$ g/cm^3. For the thermomagnetic instability, we have the opposite situation. While at $T_{e4} = 1$ the lower boundaries of the convection zone and of the zone of the thermomagnetic instability are roughly coincident, as T_e increases the thermomagnetic instability comes to span a progressively wider region inside the degenerate nucleus, and its lower boundary descends rapidly into a region with a high density. The boundary value of ρ is proportional to $T_e^{4/5}$. For hot white dwarfs, the regions in which these two instabilities occur thus do not overlap.

Under conditions (15) we find the following estimate of the rise time of the instability[106,107]:

$$t_0 \approx 5.6 \cdot 10^{-3} \frac{T_7 (\rho Z/A)^{10/3}}{\Lambda T_{e4}^8}. \tag{16}$$

Near the boundary of the degeneracy zone with $\Lambda = 2$, $T_{e4} = 3$, and $T_7 = 1$–2, the growth time is about $3 \cdot 10^3$ yr if $\rho Z/A \approx 10^3$ g/cm^3 or $6 \cdot 10^6$ yr if $\rho Z/A \approx 10^4$ g/cm^3. For a hot white dwarf with $T_{e4} = 6$, $T_7 = 4$, and $\Lambda = 2$, the instability grows considerably more rapidly: in $t_0 = 6 \cdot 10^4$ yr with $\rho Z/A = 10^4$ g/cm^3. If, on the other hand, we have $T_{e4} = 10$, then in the region with $\rho Z/A = 10^4$ g/cm^3 and $T_7 = 10$ the growth time is of the order of $6 \cdot 10^3$ yr.

Would it be possible to explain the observed fields of white dwarfs by the mechanism of a thermomagnetic instability? It is obvious that we cannot directly apply our conclusions to a description of the geometry and strength of the observed field, since that would require solving a nonlinear self-consistent problem. Nevertheless, it is possible that the fields at the surface of white dwarfs arise specifically from internal fields of thermomagnetic origin. The internal fields may be transported to the surface in a variety of ways: by flows, by convection, etc. At any rate, the fact that magnetic fields should be produced in the interior of a degenerate dwarf and can reach a high level must be taken into consideration both in analyzing the heat transfer and mixing of matter and in explaining the observed fields at the surface.

10. THERMOMAGNETIC INSTABILITY AND EFFECT OF CHEMICAL INHOMOGENEITIES IN NEUTRON STARS

Neutron stars are usually thought of as examples of systems in which a magnetic field is preserved and intensified in the course of the formation of the system. However, there are indirect indications that the field may not only decay but also intensify during the subsequent life of the star.[111] The age of a star found, for example, from observations of the envelope of the supernova whose explosion caused the formation of a neutron star, frequently turns out to be far greater than the age estimated from the slowing of the rotation, which is usually linked with the magnetic dipole emission of the star. For example, the remnant of the supernova which apparently resulted in the formation of the pulsar MSH-15-52 has an age of 10^4 yr, while its rate of slowing due to magnetic dipole emission yields an age of only $\sim 1.6 \cdot 10^3$ yr; i.e., it appears that the star began to emit as a pulsar long after it was formed.

These facts justify a search for field generation mechanisms in the stage in which a star has already formed. There has been absolutely no study of the field generation process in the formation stage.

Blandford *et al.*[111] studied the possibility that a thermomagnetic instability would develop in the solid skin of a neutron star. A fundamental distinction from the case of white dwarfs is that here it is not necessary to consider flows, and appropriate kinetic coefficients must be used. Otherwise, all the results of the preceding section of this review remain in force.

The ions in the skin of a neutron star form a Coulomb crystal. The crystallization temperature is $T_{cr} = 3.6 \cdot 10^5 \rho^{1/3}$. The depth of the phase boundary is $Z_{cr} = 1.5 \cdot 10^{15} \rho^{1/3} g$, where g is the acceleration due to gravity. The kinetic coefficients calculated in Ref. 108 are $\sigma = 3.3 \cdot 10^{15} \rho^{2/3} T$ s, $\varkappa = 4.9 \times 10^{11} \rho^{2/3}$ erg/(cm·s·deg), and $\tau = 8.02 \cdot 10^{11}$ [s/deg]$\cdot T^{-1}$ s. In the calculations one can make use of the circumstance that the time required for relaxation to thermal equilibrium is far shorter in this case than the ohmic dissipation time. At $T = 10^8$ K, at depths with 10^7 g/cm$^3 \leqslant \rho \leqslant 10^{11}$ g/cm^3, there is a solid skin, whose conductivity results in the decay of the field over several million years.

Linearizing (12), we can find an eigenvalue equation, and from the eigenvalues we can determine the field growth rate. Calculations show that the field near the surface reaches $\sim 10^{12}$ G in $\sim 10^4$ yr if it was originally of the order of 10^8 G.

The increase in the field is limited by the decrease in the temperature and the decrease in the temperature gradient. Estimates of the cooling rate depend strongly on the particular model adopted. A star cools off particularly rapidly if its field exceeds 10^{13} G, i.e., if it becomes quantizing. If the core is a superfluid, the central temperature will decrease to 10^9 K in 10 yr and to $\sim 3 \cdot 10^7$ K in 10^5 yr, while the surface temperature will decrease to $3 \cdot 10^6$ and $3 \cdot 10^5$ K, respectively. If the core is not a superfluid, then it will cool off much more slowly. In all cases, the field will no longer be increasing after 10^5–10^6 yr.

Blandford *et al.*[111] studied the instability only in the solid layers of a neutron star, since in using the equation of hydrostatic equilibrium, rather than the Navier-Stokes equation, to describe the plasma they reached the erroneous conclusion that a thermomagnetic instability does not occur in the liquid phase. However, it is specifically in the liquid phase, until crystallization sets in, that the temperature gradient is highest, so that the conditions are most favorable for field generation. The problem for the liquid phase was solved by Urpin *et al.*[112] In contrast with the case of white dwarfs (see §9), Urpin *et al.*[112] considered both azimuthal and radial perturbations of arbitrary scale with a time dependence $\exp(\gamma t)$. Furthermore, in the case of rapidly rotating neutron stars they allowed for the effect of Coriolis forces.

Using the same method as in Refs. 106 and 107 (§9), Urpin *et al.*[112] derived an eigenvalue equation for ω, from which they determined the field growth rate. A numerical solution showed that in a liquid shell in layers with $\rho < 10^{11}$ g/cm^3, consisting of electrons and Fe^{56} nuclei, a thermomagnetic instability sets in at temperatures $T > 3\cdot 10^6$ K. For example, if the surface temperature of a neutron star is $T \sim 3\cdot 10^6$ K, a perturbation with a wavelength ~ 80 m, at a depth ~ 40 m, grows most rapidly, with a time scale ~ 70 days. The magnetic field of the star in this case is a set of "magnetic spots" with dimensions of 70–100 m. A poloidal dipole field might arise from this configuration as a result of nonlinear effects or as a result of poloidal flows. Although these possibilities have not yet been explored, the results which have been obtained[112] are important to theories of neutron stars.

The supernova explosion which gives rise to a neutron star is probably rendered asymmetric by the onset of various instabilities. Evidence for this assertion comes from the fact that many neutron stars have high proper velocities, apparently because of recoil in the course of the explosion. The observed precessional motion of neutron stars which are parts of binary systems can also be explained in terms of an asymmetry of the explosion.[22] Even a slight initial asymmetry should increase dramatically in the initial stage of the contraction of the star. Also causing deviation from a spherical contraction are the centrifugal forces, which can reach $> 1\%$ of the gravitational force at the equator of a neutron star. Finally, the initial magnetic field of a supernova star may strongly influence the symmetry of the explosion.

A deviation from a spherical contraction leads to difference in the optical thickness in different directions for neutrinos which arise during the conversion to a neutron star. These variations in optical thickness in turn lead to variations in the yield of nuclear reactions involving neutrinos and thus to an asymmetric distribution of the chemical elements in the surface layers of the neutron star.

Both the change in chemical composition itself and the associated changes in thermal conductivity and thus in the temperature of a given region may lead to changes in the pressure and to the appearance of flows which tend to spread out the inhomogeneities. This spreading out, however, does not necessarily occur, since factors which influence the pressure may cancel each other out. In §5 we examined the possibility of a cancellation of this sort for chemically peculiar stars. In the case at hand, the inhomogeneities may survive until the beginning of crystallization, and the crystallization will fix them. Magnetic fields of relic, thermomagnetic, or other origin may also hinder mixing.

What field could produce the hypothesized inhomogeneities in the composition? In a magnetic field, the thermal emf $\hat{\eta}$ is a tensor. For degenerate electrons, the component parallel to the field is

$$\eta_{\parallel}\nabla_{\parallel}T = \frac{\pi^2 k^2 T \epsilon_F}{3ec^2 p_F^2}\left[6 - 2\frac{V_F^2}{c^2} - \frac{1}{\Lambda_{ei}}\left(1 - \frac{V_F^2}{c^2} + \frac{V_F^4}{c^4}\right)\right]\nabla_{\parallel}T, \tag{17}$$

$$\Lambda_{ei} = \ln\left[(4Z)^{1/3}\left(1 + \frac{2akT}{Z^2e^2}\right)\right] - \frac{V_F^2}{2c^2}, \qquad a^3 = 3\,(4\pi N_i)^{-1}.$$

Here $p_F = \hbar(3\pi N_e)^{1/3}$ is the Fermi momentum; $\epsilon_F^2 = m_e^2c^4 + c^2p_F^2$; Λ_{ei} is the Coulomb logarithm; N_i is the density and Z the charge of the ions; and $V_F = c^2p_F/\epsilon_F$. The component of $\hat{\eta}$ in the direction $[\mathbf{B}, \nabla T]$ differs from $\eta_{\parallel}$ by a factor of $c\omega_B\tau/2e$, where ω_B is the electron cyclotron frequency, and τ is the time scale of electron-ion collisions.

The quantity $-c[\nabla\hat{\eta}, \nabla T]$ appears in Eq. (2). It follows from (17) that we have $-c[\nabla\eta_{\parallel}, \nabla T] = \beta T[\nabla N_e, \nabla T]$, where we would have $\beta \approx 1.3\times 10^{15} N_e^{-2/3}$ in the nonrelativistic limit, while in the extreme relativistic limit we would have $\beta \approx 1.1\cdot 10^5 N_e^{-1/3}$.

To estimate the field growth rate from (2), we assume that there are no flows in the layer under consideration, and we ignore the small terms. We then have the approximate equation $\partial\mathbf{B}/\partial t \approx -c[\nabla\hat{\eta}, \nabla T]$. As an example we consider a layer at a depth with $\rho = 3.3\cdot 10^6$ g/cm^3 at a temperature $\approx 4\cdot 10^7$ K. The temperature gradient in this case is very large, $|\nabla_r T/T| \sim 10^{-3}$ cm^{-1} (Refs. 112 and 113). An estimate of the field then yields $B_t \approx 5\cdot 10^{14}|\nabla_\vartheta N_e/N_e|t$, where t is the time in years. The field reaches $\sim 10^{12}$ G in $5\cdot 10^4$ yr if $|\nabla_\vartheta N_e/N_e| \sim 10^{-7}$ cm^{-1}, i.e., if the electron density varies 10% between the pole and the equator. A study of the field and temperature distributions at the surfaces of neutron stars may yield information on the actual distribution of composition inhomogeneities.

11. INDUCTION MECHANISM FOR FIELD GENERATION IN CLOSE BINARY STARS

What new possibilities for field generation arise in binary stellar systems? Observations show that $\sim 60\%$ of all stars are components of binary or multiple systems. Allowing for observational selection, we could expect[114] this number to be far greater, reaching 70–80%. Among binary systems, ~ 10–20% are close binaries, for which the distance between the stars is no more than a few tens times the radius of one of the stars. Falling in the category of close binaries are many novae and novalike stars, symbiotic stars, U-twins, etc. It is not rare to find matter flowing from one star to the other in systems of this sort. In such a case, and also if there is a sufficiently intense stellar wind, a plasma cloud with a density which can reach 10^6–10^{11} particles/cm^3 will arise between and around the stars.

An induction mechanism for field generation in such systems was proposed in Refs. 115–117. That mechanism can be summarized as follows. If one of the stars has some seed field, even if a weak one, then as the star rotates around its axis and revolves in its orbit it induces a current in the neighboring star. This current in turn induces a magnetic field in the first star, which adds to and intensifies the original field. This mechanism is reminiscent of Herzenberg's dynamo.[118] Herzenberg analyzed a model of two conducting spheres which are rotating with an identical angular velocity around nonparallel axes in a conducting medium which is at rest. It was found that for certain parameter values this system has a steady-state magnetic field. However, the actual process by which the field was generated was not analyzed, nor were the necessary conditions for generation or the field growth time. Herzenberg's ideas were pursued by Gaĭlitis.[119,120] It was found in particular that a steady-state solution also exists if the spheres are replaced by any solids of revolution. Wilkinson and Lowes[121] carried out an experiment to test Herzenberg's results. They found that in a system of two conducting cylinders which are rotating around nonparallel axes and which are immersed in a conducting medium a magnetic field arises spontaneously and intensifies. Unfortunately, in order to arrange the required conductivity these investigators were forced to use ferromagnetic materials. That choice seriously complicates an effort to interpret the experiment unambiguously.

Let us go back to the case of binary stellar systems. For simplicity we restrict the discussion to the case in which one of the stars is far more massive than the other. We use a nonmoving coordinate system, which is rigidly fixed in the more massive star. We place the origin of coordinates at the center of this star. We denote the radii of the stars by a_1 and a_2, and their angular velocities by ω_1 and ω_2. We assume that the rotation of the stars is a rigid-body rotation, and we assume that the orbit is circular with a radius $R \gg a_1$ and a_2. The orbital angular velocity is Ω.

Close binaries are frequently surrounded by plasma clouds with dimensions much greater than the dimensions of the system (for example, RSOph TCrB). The plasma velocities in such clouds are usually low, ~ 10 km/s, while the density is fairly high, 10^8–10^{12} cm^3. The conductivity of such a medium can not be ignored in an analysis of field generation. Furthermore, the stars themselves may be surrounded by spherical or disk-shaped envelopes, rapidly rotating around the corresponding star. Both the envelopes and the plasma clouds are sustained by an influx of matter from the same star which is filling its Roche cavity or intensely omits plasma in the form of a stellar wind.

Both the surface layers of the stars and the plasma near them are usually spanned by small-scale turbulent motions. It is thus necessary to use the turbulent conductivity coefficients, which are far smaller than those for a quiet medium. We will also consider the repulsion of the field from regions with a high conductivity (the "diamagnetic" effect[8,122]).

We find an equation for the magnetic field in a form convenient for our problem by taking the curl of the expression of Ohm's law and eliminating the current through the use of Ampere's law:

$$\left[\boldsymbol{\nabla}, \left(\mathbf{E} + \left[\frac{\mathbf{v}}{c}\mathbf{B}\right]\right)\right]$$
$$= \frac{c}{4\pi}\left[\boldsymbol{\nabla}, \left\{\frac{(1+\chi)^{1/2}}{\sigma}[\boldsymbol{\nabla}, (1+\chi)^{1/2}\mathbf{B}]\right\}\right], \qquad (18)$$

where v is the large-scale velocity of the medium, $\chi = \sigma V_t l_t / c$ is the magnetic Reynolds number, V_t and l_t are a velocity scale and a length scale of the turbulence, and σ is the turbulent conductivity. The factor $(1+\chi)^{1/2}$ reflects the "diamagnetism" of the medium.

For a solution it is convenient to use an integral form of Eq. (18):

$$\mathbf{B} = \frac{1}{4\pi}\frac{\sigma}{1+\chi}\left\{\int d\mathbf{r}'\left(\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}(\mathbf{B}\boldsymbol{\nabla}')\frac{1+\chi}{\sigma}\right.\right.$$
$$\left.+\left[\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}(1+\chi)^{1/2}[\mathbf{B}\boldsymbol{\nabla}']\frac{(1+\chi)^{1/2}}{\sigma}\right]\right)$$
$$\left.-\frac{1}{c}\int d\mathbf{r}'\left[\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\left(\mathbf{E}+\left[\frac{\mathbf{v}}{c}\mathbf{B}\right]\right)\right]\right\}. \qquad (19)$$

Equation (19) can be simplified considerably by noting that the quantities $\boldsymbol{\nabla}\chi$ and $\boldsymbol{\nabla}\sigma$ are large only in a thin surface layer of the stars and also noting that at the boundary between media with different turbulence levels the normal components of the magnetic field should be equal, while the tangential component ouside the star should differ from that inside the star by a factor of $(1+\chi_1)^{1/2} \times (1+\chi_0)^{-1/2}$. The subscript 0 refers to the medium between the stars, while 1 and 2 refer to the stars. We restrict the discussion to the slowly varying part of solution (19), since a rapidly varying field will fall off rapidly with distance because of a skin effect. We can thus average (19) over a time much shorter than the field rise time but much longer than the rotation periods of the stars. Assuming ω_1 and $\omega_2 \gg \Omega$, and ignoring the change in the field over the time required for one star to complete one revolution in its orbit, we can carry out an averaging over the orbital motion. In place of (19) we then find

$$\mathbf{B}(\mathbf{r}) = -\frac{1}{c^2}\frac{\sigma_0}{1+\chi_0}\left\langle \int \left[\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}[\mathbf{v}(\mathbf{r}')\,\mathbf{B}(\mathbf{r}')]\right] d\mathbf{r}'\right\rangle. \qquad (20)$$

Here $\langle \ldots \rangle$ represents this averaging over the time; the integration is carried out over the volumes of the stars; and σ_0 and χ_0 refer to the medium between the stars. In a coordinate system at rest, the integrand and the integration region depend on the time.

The calculations carried out in Refs. 116 and 117 show that the field in a binary system increases with the time in accordance with $\sim \exp(\gamma t)$, until nonlinear effects associated with the inverse reaction of the field on the motion stop the process. Two modes, with the following growth rates, grow most rapidly:

$$\gamma_1 \approx 0.1\frac{\omega_1 a_1^2}{D_0}\frac{\Omega R^2}{D_0}\left(\frac{a_1 a_2}{R^2}\right)^3\frac{D_1}{a_1 a_2}\left[2\left(\frac{1+\chi_0}{1+\chi_1}\right)^{1/2}+1\right]$$
$$\times\left[\left(\frac{1+\chi_0}{1+\chi_1}\right)^{1/2}-1\right](\mathbf{e}_\Omega\mathbf{e}_1)\,[\mathbf{e}_\Omega[\mathbf{e}_1\mathbf{e}_2]]^2,$$
$$\gamma_2 \approx -\frac{\omega_1 a_1^2}{D_0}\frac{\omega_2 a_2^2}{D_0}\left(\frac{a_1 a_2}{R^2}\right)^2\frac{D_1}{a_1 a_2}\left[\frac{3}{2}\left(\frac{1+\chi_0}{1+\chi_1}\right)^{1/2}+1\right]^2$$
$$\times(\mathbf{e}_\Omega\mathbf{e}_1)\,(\mathbf{e}_\Omega\mathbf{e}_2)\,[\mathbf{e}_\Omega[\mathbf{e}_1\mathbf{e}_2]]^2. \qquad (21)$$

The signs of γ_1 and γ_2 depend on the relative orientation of the vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_\Omega$, which are directed along the rotation axes of the stars and perpendicular to the orbit, respectively. Expression (21) is not suitable in the case $\chi_0 = \chi_1$, since the calculations omitted a small term which becomes important at $\chi_0 = \chi_1$. That case, however, is of no interest for a description of close binaries. It was assumed in (21) that we have $\chi_1 = \chi_2$ and $D_1 = D_2$, where $D = c^2(1+\chi)/4\pi\sigma_t$ is the magnetic viscosity coefficient of a turbulent medium.

Expression (21) can be understood at a qualitative level without any difficulty. Over a time $\sim a^2/D_0$, required for a field near a star to penetrate a distance of the order of the radius of the star, new turns of field lines numbering $\sim \omega a^2/D_0$ will appear. The quantity $\Omega R^2/D_0$ has a corresponding meaning for the orbital motion. The field which arises is toroidal. It falls off with distance as r^{-2}. The response of the first star to the field which it excites in the second star is therefore proportional to r^{-4}. The factor $\sim D_1/a_1 a_2$ describes the penetration of the field into the interiors of the stars.

As in Herzenberg's model,[118] the effect disappears if the rotation axes of the stars are parallel. The tidal friction tends to synchronize the intrinsic rotation of the stars and their orbital motion, to bring the axes into a parallel arrangement, and to make the orbit circular. On the other hand, there are factors which act in the opposite direction, especially in systems in which there is an outflow of matter from the stars. The process by which the tilting of the axes of these systems changes has not been studied theoretically. Observations show that even extremely close binary systems are not always synchronized.[123] It may happen that along the way to synchronization a star is captured into a resonant orbit. Resonant orbits emerge when the orbital and axial periods are related in a certain way. The orbit of Mercury is an example in the solar system. The rotation of Mercury around its axis is 3/2 times as fast as its revolution around the sun. There are accordingly grounds for assuming that in some cases the rotation axes of the stars do not become absolutely parallel.

Let us consider as an example a close binary system of a red dwarf and a white dwarf. New stars are some typical examples of such systems. Matter flows away from the cold red dwarf to the white dwarf, forming around it a fairly dense, disk-shaped, conducting envelope, which is itself surrounded by a lower-density plasma cloud. The flareup of a nova is a thermonuclear explosion of matter which has fallen onto the white dwarf. For definiteness we assume that the radius of the red dwarf is $a \sim 10^{10}$ cm, that of the white dwarf is 10^8 cm, and that the distance between them is $\sim 5\cdot 10^{10}$ cm. The usual estimates of the turbulence at the surface of the red dwarf[88,95] correspond to a basic length scale $l_t = 10^{-2}a = 10^8$ cm and a velocity scale $V_t \approx 10^4$ cm/s, i.e., $D_1 \approx 10^{12}$ cm^2/s. The estimates of the parameter values of the plasma surrounding the star are less certain. We can apparently take $l_t = 10^{10}$ cm, since the source of turbulence in the cloud is the orbital motion of the stars. The velocity scale is of the order of the sound velocity; i.e., (at $T = 10^4$ K), $V_t = 10^6$ cm/s, and we have $D_0 = 10^{16}$ cm^2/s. For typical novae we would have $\Omega \sim 3\cdot 10^{-4}$ s^{-1}, $\omega \sim 10^{-3}$ s^{-1}. Hence the field rise time is $\tau \sim 10^3$ yr. If we instead have $l_t = 10^9$ cm, then we would have $\tau \sim 10$ yr.

There exist binary systems with a short orbital period (<3 h) which consist of a white dwarf and a red dwarf the emission from which has a large circular polarization ($\sim$10–14%). Such a polarization is evidence of a strong magnetic field, $\sim 10^5$–10^7 G, in the emitting region. It is possible that the field is due to an induction mechanism. Unfortunately, the use of the linear approximation, which makes it possible to establish the fact of a rapid increase in the field, does not allow us to determine its final equilibrium value.

Among close binaries there are systems in which the radiation emitted by one of the stars strikes the other and creates a nonuniform temperature distribution over the surface. In this case, the thermal emf $\eta\nabla T$ can give rise to a significant magnetic field.

Tidal flows exist in the constituent stars of binary systems. If the rotations around the axes and the revolution along the orbit are not synchronized, and if the axes of the stars are not parallel, these flows will have a complicated three-dimensional structure. The trajectories of tidal flows were found in Refs. 124–126 for the case in which the stars are not overly close together, and the analysis can be restricted to quadrupole deformations. In the case of an incompressible medium with a free surface, the flows are potential flows. A compressibility, a dissipation, and a differential rotation will give rise to a solenoidal component. This component is required for the generation of a field.

Tidal deformations themselves are nearly reversible, and the object acquires its previous configuration after a comparatively short period; the energy is returned to gravitational form. This does not happen if there is a differential rotation or a strong dissipation, which shifts the tidal hill. Although the figures of the bodies in binary systems take on their original features, the elements of matter which are participating in the tidal flows do not return to their previous positions. Their trajectories become progressively more entangled, with the result that the magnetic field lines which are frozen in them stretch out, so that the field is intensified.

The precession of a liquid in a nonspherical envelope also gives rise to flows similar to tidal flows and thereby permits the intensification of a field. The possible role of precession in generating the earth's magnetic field was pointed out by Malkus and Proctor.[127] However, they treated the precession only as an energy source of gyrotropic turbulent motions, which actually produced the field. The efficiency of that mechanism is extremely small—too small to explain the generation of the observed field of the earth.[128]

The field in a binary system may be produced directly by tidal flows. An explicit analytic expression for the velocities of these flows was found in Refs. 124–126. In close systems, these velocities may be high. For example, for a system of two white dwarfs $\sim 5\cdot 10^9$ cm apart, with masses equal to ~ 0.3 of the solar mass, with radii $\sim 10^9$ cm, with a period of 100 s for the rotation around the axes, and with an inclination of the axis of 45° with respect to the orbital plane, the velocities of the potential tidal flows would reach 20 m/s, and the height of the tidal hill would exceed 100 m.

The solenoidal component of the tidal velocities is far smaller than the potential component, and the eddy flows associated with a possible convection or differential rotation are apparently large. The resultant flow has a helicity and is favorable for field generation. The field growth rate depends strongly on the particular characteristics of the object. A rough estimate yields a time scale of the order of $3(R/l)\delta L/\delta v$ for the field growth, where R is the radius of the star, l is the amplitude of the tidal flows, and $\delta v/\delta L$ is the change in the solenoidal component of the velocity over a length scale δL.

12. MAGNETIC FIELDS OF GALAXIES

In galaxies one frequently observes, in addition to the very complex and nonuniform small-scale field (~ 1 pc) and intermediate-scale field (~ 100 pc), a well-ordered large-scale structure ($\sim 10^3$–10^4 pc). The average field is $\sim 3\cdot 10^{-6}$ G. The global and local structures seem to have different origins. The local structure is more closely associated with local sources. The small-scale field may be supplied to a galaxy by stellar winds from hot O and B stars and also along with matter ejected during supernovae explosions. For example, the Crab Nebula—the remnant of the explosion SN 1054—has a size ~ 1 pc and a field $\sim 3\cdot 10^{-4}$ G. During the subsequent expansion to a size ~ 10 pc, the field decreased to $\sim 3\cdot 10^{-6}$ G, corresponding to the average galactic field. Over the lifetime of a galaxy, one supernova occurs in a region with dimensions ~ 6 pc. This frequency is sufficient to provide all the magnetic flux at small and intermediate scales. Stars of types O and B contribute comparatively little. They lose matter in the form of a stellar wind ($\sim 10^{-8}$–10^{-5} of a solar mass per year), which takes up a region with dimensions $\sim$ 1–3 pc. The azimuthal component of the field decreases in inverse proportion to the distance from the star. If the field near the surface of the star, say at a distance $\sim 10^{12}$ cm from the center, is 30 G, then it will decrease to $\sim 3 \cdot 10^{-6}$ G at a distance of 3pc.

The contribution from the stellar winds of the stars of the later spectral classes is slightly smaller than that from O and B stars. All stars, however, intersect a cloud of interstellar gas quite frequently. The dimensions of these clouds are $\sim$ 8–10 pc, and the average distance between them is $\sim$40–50 pc. Over the time $\sim 10^5$–10^6 yr required to cross a cloud, the stars heat and ionize the closest gas regions. An inhomogeneous temperature distribution in a cloud leads to a thermoelectric current and an associated magnetic field. The increased field observed in cloud regions illuminated by stars ($\sim 10^{-5}$ or even $\sim 10^{-4}$ G) may be a consequence of this field generation process.

In addition to the small-scale random field, one observes magnetic bubbles.[129,130] These entities are of the nature of envelopes with a size of 100–300 pc and a field $\sim 10^{-6}$–10^{-5} G. Near the sun ($\sim$ 3 kpc), four such bubbles have been observed in an arm of Orion. It is usually assumed that these huge bubbles arise when hot stars of early classes, which emit a stellar wind, cluster together. The combined effects of the individual stellar winds create a common cavity, which surrounds the cluster of stars.

The differential rotation of a galaxy stretches out the field lines, but this rotation could hardly transform a small-scale field into a large-scale field. The global field is not a relic field, since the field decay time at the scale of the spiral structure of a galactic disk is only $\sim 5 \cdot 10^8$ yr.

A large-scale azimuthal field has been observed in our own galaxy. In the arms, where the density is high, the field direction deviates slightly from azimuthal, and the magnitude of the field becomes slightly greater. Most papers[1–6,131–133] explain the large-scale galactic field on the basis of the α-ω dynamo theory: as the result of a differential rotation and of gyrotropic flows in the interstellar medium. The theory makes use of observational data on the galactic differential rotation and a string of assumptions regarding the structure of the field and its behavior during the evolution of the galaxy. The structure and gyrotropy of the motions of the galactic plasma are assumed to be known. The theory predicts that the axisymmetric even mode of the α-ω dynamo theory grows fastest of all. This mode creates a toroidal azimuthal field which is symmetric with respect to the galactic plane[1–6,131–133] and also a quadrupole poloidal field. The field of our own galaxy, according to the theory of Refs. 131 and 132, correlates with the observed radial profile of the differential rotation.

There are observational data which indicate that in many spiral galaxies (possibly our own) the field has a "bisymmetric" spiral structure. The magnetic field lines are stretched out along local spiral arms. They enter from one side of the galactic disk and emerge from the diametrically opposite direction, satisfying the condition $\nabla \cdot \mathbf{B} = 0$ (Refs. 134–136).

Baryshnikova *et al.*[133] have shown that attempts to explain such a structure as a relic structure[137] seem to have no basis. A far more plausible explanation is that it is an axially asymmetric mode of an α-ω dynamo, although the growth rate of this mode is slightly lower than that for the symmetric mode. Perhaps other modes should also be taken into consideration. In particular, there is an odd mode which corresponds to an azimuthal field directed oppositely on the two sides of the plane of the galactic disk. Such a structure might also appear bisymmetric to an external observer. The poloidal field should be a dipole field in this case. Unfortunately, the poloidal component is far weaker than the toroidal component as has not yet been observed.

Vallee[138] has pointed out that large satellite galaxies have been observed near spiral galaxies with a bisymmetric magnetic structure in all cases studied (M 33, 1C 342, M 81, M 51, N 6946), while no such satellites have been observed near galaxies with a purely azimuthal magnetic field (M 31, N 253, and Voie Lactee). According to Vallee,[138] this result is evidence that the field structure is determined by tidal interactions. However, no quantitative calculations have been carried out to support that idea.

At present we have nothing in the way of a well-developed theory to explain the origin and structure of the large-scale field of galaxies, other than the dynamo theory. Some specific applications of the dynamo theory for describing the magnetic fields of celestial bodies, including galaxies, are given in a paper by Ya. B. Zel'dovich and A. A. Ruzmaĭkin in this issue of Soviet Physics-Uspekhi.

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