G. P. Berman. *Some properties of quantum chaos.* In the study of stochasticity in classical Hamiltonian systems "nonlinear resonance" and "parameter of overlapping" of nonlinear resonances are effective and convenient concepts.[1] The transition from regular motion to chaotic motion is already possible in a system of two nonlinear resonances and arises when the interaction (overlapping) is strong enough. The report investigates the characteristic dynamics and spectral properties of systems of two interacting quantum nonlinear resonances, whose abbreviated Hamiltonian has the form

$$\hat{H} = -\gamma\hbar^2 \frac{\partial^2}{\partial\theta^2} + V_1 \cos(k_1\theta - \Omega_1 t) + V_2 \cos(k_2\theta - \Omega_2 t)$$

with periodic boundary conditions for the wave function $\psi(\theta + 2\pi, t) = \psi(\theta, t)$. The Hamiltonian $\hat{H}$ describes the interaction of radiation containing two frequencies with a nonlinear quantum system in the region of the quasiclassical population.[2–4] The parameters γ, $V_{1,2}$, and $\Omega_{1,2}$ are the renormalized anharmonicity constant and the amplitudes and frequencies of the external field, and $k_{1,2}$ are integers. The Hamiltonian $\hat{H}$ was constructed in the so-called approximation of moderate nonlinearity[2] and is identical to the Hamiltonian of a quantum rotator in the field of two waves. In the classical limit ($-i\hbar\partial/\partial\theta \to I$ is the action) $\hat{H}$ transforms into $H(I,\theta)$, and for the parameter of overlapping $K < K_c$ there exist two primary resonances centered in action at the points $I_{1,2} = \Omega_{1,2}/2\gamma k_{1,2}$. These primary resonances lead in the classical case to the appearance of resonances of a higher order. As a result the classical phase space exhibits renormalization properties.[5] In the quantum case a method of renormalization of the starting Hamiltonian $\hat{H}$ can also be developed.[6,7] This procedure is well defined, if the number of levels included in the primary quantum resonances is large, $\delta l = (4/\hbar\pi)(2V/\gamma)^{1/2} \gg 1$ ($V_{1,2} = V$, $k_{1,2} = 1$) which is the condition that the primary resonances are quasiclassical in nature. As a result, the renormalized Hamiltonian describing the behavior of the system between the nearest secondary resonances has the same form as the starting Hamiltonian. Such renormalization in the quantum case (unlike the classical case) is repeated a finite number of times until the width of the higher order resonances in the action is no longer equal to $\hbar$. The structure of the quasienergy functions and the spectrum of the quasienergies in the transitional region $K \sim 1$ ($V_{1,2} = V$, $-\Omega_1 = \Omega_2 = \nu$, $k_{1,2} = 1$) were studied numerically. For such values of the parameter K a transition occurs in the classical case to global chaos ($K > K_c \approx 0.71$), and there arises the question of the character of the restructuring of the quasienergy functions in this region. Figure 1 shows a diagram of the antisymmetric quasienergy functions, obtained by numerical diagonalization of the evolution operator as a function of the parameter K: $K = 0.625$ (a), $K = 0.8$ (b), $K = 1$ (c), $K = 1.176$ (d), l is the rms width of the quasienergy function and $\bar{n}$ is its "center of gravity" in the unperturbed basis ($V_{1,2} = 0$). Each point in the figure corresponds to a quasienergy function, the points on the horizontal line correspond to quasienergy states lying in the potential wells of the primary resonances

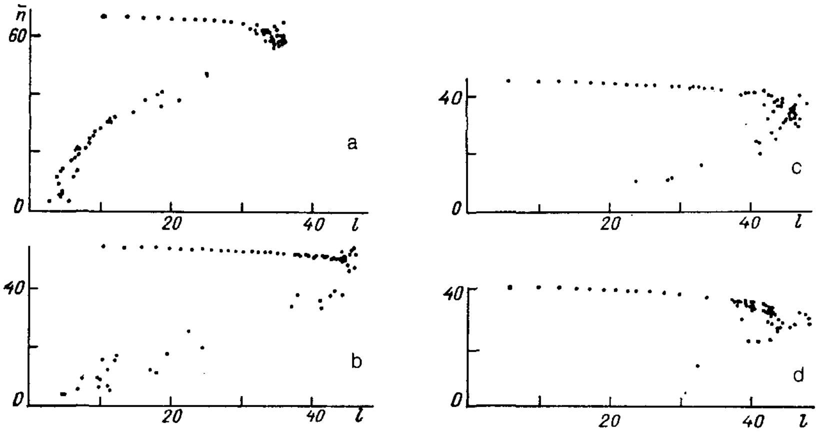

FIG. 1. Destruction of the lower branch of quasienergy functions in the (n,l) diagram as the parameter of overlapping K increases ($\delta l = 51$, $V = 20$; $\gamma = 2.5 \cdot 10^{-2}$; $h = 1$).

(the points lying above the horizontal lines are not shown). As the parameter K increases the secondary resonances are destroyed, which corresponds in the figure to a destruction of the lower branch—the quasienergy functions of the lower branch are restructured (delocalized, l increases) and are displaced into the region of the separatrices of the primary resonances, where they are distributed in an irregular fashion. The irregular character of the delocalization of the quasienergy functions is a quantum manifestation of classical chaos. The behavior of statistical distributions of the Fourier amplitudes of the quasienergy functions and the distances between the nearest quasienergy levels indicate the existence of substantial correlations even in the case of delocalized quasienergy functions. To clarify the degree to which the motion approaches the stochastic motion of the corresponding classical system the time-dependent correlation functions $\rho_{n,m}(t)$ were calculated numerically for different elements of the density matrix. For $K \sim 1$ the residual correlations decrease as δl increases to some finite level.

The dynamics of the system depends substantially on two parameters K and δl. When the resonances overlap ($K \sim 1$) an increase in δl leads to a characteristic (for a random process) rapid (at the initial stage) decay of the correlation functions for the components of the density matrix, and this enables the study of the motion at finite times on the basis of the statistical approach taking into account systematically the residual correlations (in the report the classical and quantum diffusion in energy are compared). The isolated quantum nonlinear resonance and the interaction of quantum nonlinear resonances can be observed, for example, in the case of interaction of coherent laser radiation with multilevel molecules. For $\delta l \gg 1$ and $K \sim 1$ the role of the stochastic component increases substantially, and this can be employed as one possible mechanism for exciting the system into the region of high-lying levels.

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