Influence of multiple scattering on the radiation of relativistic particles in amorphous and crystalline media

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The article sets forth the theory of the influence of multiple scattering on the radiation of ultrarelativistic particles in matter. The classical theory of the Landau-Pomeranchuk effect of suppression of the radiation of relativistic particles in an amorphous medium is presented both in the simple form given by the authors and in a refined form due to Migdal. It is shown that in investigation of the influence of multiple scattering on the radiation of fast particles in matter it is possible to use a functional integration method which permits construction of a quantitative theory of the effect in both amorphous and crystalline media. The question of the possibility of stochastic motion of fast charged particles in a crystal is discussed. The general regularities and distinctive features in processes of radiation of relativistic particles in amorphous and crystalline media and also in intense external electromagnetic fields are discussed.

CONTENTS

1. INTRODUCTION

Many electromagnetic processes which occur in the interaction of fast charged particles with matter, such as elastic scattering, radiation, and electron-positron pair production, develop in a large region of space along the particle momenta. The length of this region, which is called the coherence length, increases rapidly with increase of the particle energy. If this length is large in comparison with the average distance between the atoms in the medium, then essentially it is necessary to take into account the interaction of the incident particle with all atoms located within the coherence length. This was formulated clearly for the first time by Ter-Mikaelyan[1] in a study of the radiation of relativistic particles in crystals.

The importance of this remarkable and seemingly paradoxical phenomenon was soon evaluated. Landau and Pomeranchuk[2] showed on the basis of this result that in an amorphous material the increase of the coherence length with energy leads to a substantial decrease of bremsstrahlung (the Landau-Pomeranchuk effect). This was followed by discoveries of still other effects in electrodynamics and in the physics of strong interactions (see for example the reviews of Refs. 3–6 which are devoted to this subject, and references therein).

The interaction of a particle with the atoms within the coherence length can have both a regular nature and a stochastic nature.

A regular interaction is possible (see below) in passage of a particle through a crystalline medium. The influence of

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the periodicity of location of the atoms in the crystal on the radiation of an ultrarelativistic particle was first noted by Williams.[7] Correct criteria for the appearance of the effect and a rigorous and quantitative theory of it, however, were not given until the papers by Ferretti,[8] Ter-Mikaelyan,[1] and Überall.[9] In these studies it was shown that in motion of a relativistic particle in a crystal coherent and interference effects are possible in the radiation, and that as a result of these effects the radiation from a particle in a crystal can substantially exceed the radiation in an amorphous medium. The effect occurs in the case in which the particle is moving at a small angle to one of the crystalline axes and, in addition, if there are a large number of lattice atoms within the coherence length.

A stochastic nature of the interaction of a particle with the atoms of matter occurs in the case of passage of charged particles through an amorphous medium. The stochasticity is due in this case to the multiple scattering of the traversing particles by the atoms of the material. The influence of this scattering on the emission of radiation was first noted and studied in the papers of Landau and Pomeranchuk.[2] In these studies it was shown that multiple scattering of an ultrarelativistic particle in an amorphous medium within the coherence length can lead to a substantial decrease of the bremsstrahlung. The effect arises if the mean square angle of multiple scattering in the coherence length exceeds the square of the characteristic angle of radiation of the relativistic particle.

It should not be supposed, however, that stochasticity in the interaction process is a property characteristic only of amorphous media. Actually, even in the case of a crystal, and even in an ideal one at absolute zero temperature, the interaction can have a stochastic nature. The point is that in motion of the particle in the crystal its trajectory can be stochastic in spite of the absolutely ideal potential of the crystal lattice. This in turn is related to the number of integrals of motion of the particle in the crystal field (see Section 5).

Recently a number of studies[10-12] have been devoted to the fact that an effect similar to the Landau-Pomeranchuk effect can occur in the motion of a fast particle in a crystal. In this case, however, the coherent radiation of the relativistic particle, and not ordinary bremsstrahlung, is suppressed as a consequence of multiple scattering. Here it is important that in the motion in the crystal the conditions of appearance of the effect of radiation suppression are satisfied at substantially lower energies of the particles and in a substantially larger region of frequencies of the radiated photons than in an amorphous medium. For this reason new possibilities are opened up for study of the influence of multiple scattering on radiation by means of contemporary accelerators.

A number of reviews and books [5,13-21] have been devoted to the theory of the radiation from relativistic particles in matter. However, at the present time there is no review in which the theory of the radiation of relativistic particles in amorphous and crystalline media is presented from a unified point of view. There also are no expositions of the theory of the influence of multiple scattering on the radiation of fast particles in crystals, although as we have just mentioned there are at the present time new possibilities for investigation of this effect. The present review is devoted to a detailed discussion of these questions.

We shall begin with introduction of the concept of coherence length, which appears naturally in the theory of the radiation from relativistic particles in matter. Then we shall present the classical theory of the Landau-Pomeranchuk effect in amorphous media, both in its simple form given by these authors and in the refined form given by Migdal.[22]

In addition to the kinetic-equation method used in Ref. 22, for description of the Landau-Pomeranchuk effect it is possible to use also the method of functional integration.[23] The advantage of this method is the fact that it is possible by means of it in the framework of the original formulation of the problem of Landau and Pomeranchuk to construct in a unified way a quantitative theory of radiation both in amorphous and in crystalline media, and in addition to bring out the general regularities and distinctive features of the particle radiation processes in these cases and also of radiation in intense external electromagnetic fields. Sections 4.5, 4.6, and 6 of this review are devoted to this method and the results obtained with it.

In their original study[2] Landau and Pomeranchuk investigated radiation only in an unbounded medium. The coherence length for the radiation process at high energies can have a macroscopic size, so that the size of the target may be either greater or less than the coherence length. The thin-target case has not previously been investigated in detail. Section 6.3 of our review is devoted to the theory of the radiation from ultrarelativistic particles in a thin layer, in both amorphous and crystalline materials.

At the end of the review we discuss briefly experimental studies which are being performed of the interaction of high-energy particles with crystals.

2. COHERENCE LENGTH

2.1. Radiation formation length in the quantum theory

The radiation of a relativistic particle in matter develops in a large region of space along its momentum. In order to be convinced of this we recall that the radiation cross section is determined by a matrix element which contains under the integral over the space coordinates a factor $\exp(i\varkappa\mathbf{r})$, where $\varkappa$ is the momentum transfer, $\varkappa = \mathbf{p} - \mathbf{p}' - \mathbf{k}$, $\mathbf{p}$ and $\mathbf{p}'$ are the electron momenta before and after radiation, and $\mathbf{k}$ is the momentum of the radiated photon (here everywhere $\hbar = c = 1$).

The exponential $\exp(i\varkappa\mathbf{r})$ determines the effective values of $\mathbf{r}$ which contribute to the matrix element. In the relativistic region the radiation process develops mainly along the particle momentum $\mathbf{p}$ (the z axis). The effective region of z obviously is $z_{\text{eff}} \sim 1/\varkappa_\parallel$, and the effective region of distances perpendicular to $\mathbf{p}$ is $r_{\perp\,\text{eff}} \sim 1/\varkappa_\perp$. From the conservation of energy and momentum during the radiation

$$E = E' + \omega,$$
$$\mathbf{p} = \mathbf{p}' + \mathbf{k} + \varkappa \tag{2.1}$$

it follows that in the relativistic case when $\mathbf{p}$, $\mathbf{p}'$, and $\mathbf{k}$ are almost parallel to each other we have

$$\varkappa_\parallel = p - p' - k \approx \frac{\omega m^2}{2EE'}, \tag{2.2}$$

where E and E' are the electron energies before and after radiation, and m is the electron mass. It follows from this that $z_{\text{eff}} \sim 2EE'/m^2\omega$. The length

$$l_c = \frac{2EE'}{m^2\omega} \qquad (2.3)$$

is called the coherence length. The reason for this name will be made clear below.

The coherence length l_c increases rapidly with increase of the particle energy and with decrease of the frequency of the radiated photon, and for sufficiently large E and small ω it can reach macroscopic dimensions. For this reason in study of the radiation process it is not always possible to restrict the discussion to the interaction of the electron with one charged particle, in particular, with one charged nucleus. This means that if l_c is larger than the size of an atom, it is necessary to take into account the interaction of the incident electron not only with the nucleus of the atom, but with the atomic electrons. This interaction is taken into account as the screening effect in the Bethe-Heitler theory of bremsstrahlung[24,25]

However, if l_c exceeds the average distance between the atoms of the medium, then it is necessary to take into account the influence both of the atomic electrons and of many atoms on the particle radiation process.[1,2]

We note that the factor $\exp(i\varkappa\mathbf{r})$ enters not only into the matrix element of the radiation process, but also into the matrix elements of such processes as elastic scattering, electron-positron pair production and so forth. Therefore the concept of coherence length can be introduced not only for bremsstrahlung, but also for elastic scattering, for e^+e^- pair production, and so forth. Here the coherence lengths will be different for the different processes, since for these processes the specific expressions for conservation of energy and momentum are different. An important consideration is the fact that in the high-energy region the coherence lengths of all these processes rise with energy and at sufficiently high energies reach macroscopic dimensions.

2.2. Coherence length in classical electrodynamics

We have introduced the idea of coherence length in a discussion of the matrix element of the radiation process in quantum electrodynamics. However, this concept can be arrived at also in classical electrodynamics.

For this purpose let us consider the motion of a fast particle in a medium along a trajectory which is close to a straight line. This particle obviously will radiate. Here the phase difference $\Delta\varphi$ of the waves radiated by the particle at an angle θ to its momentum at moments of time t and $t + (l/v)$ will be

$$\Delta\varphi = \omega\frac{l}{v} - kl\cos\theta, \qquad (2.4)$$

where l is the path traveled by the particle in the interval of time $(t, t + (l/v))$.

We shall define the coherence length $l(\omega,\theta)$ as the distance in which $\Delta\varphi = 1$. Then it is easy to see that

$$l(\omega, \theta) = \left(\frac{\omega}{v} - k\cos\theta\right)^{-1}. \qquad (2.5)$$

In a medium the frequency ω is related to the wave vector $\mathbf{k}$ of the radiated wave by the expression $k = \omega\varepsilon^{1/2}$, where ε is the dielectric permittivity, and therefore the coherence length takes the form

$$l(\omega, \theta) = \frac{v}{\omega}(1 - v\varepsilon^{1/2}\cos\theta)^{-1}. \qquad (2.6)$$

We see that if $\cos\theta = 1/v\varepsilon^{1/2}$, then the coherence length goes to infinity. This case corresponds to Cherenkov radiation in the uniform motion of a charged particle.

This approach to interpretation of the physical essence of the Cherenkov effect is due to Frank[26] (see also Ref. 27). An original feature of it is the discussion of phase relations for waves radiated by a particle from different portions of its trajectory. (The radiation formation length, i.e., the length in which the radiated waves add and reinforce each other, was called by Frank the Fresnel zone in analogy with diffraction theory.)

We emphasize that the concept of coherence length has a meaning independent of the quantity ε: it always determines the order of magnitude of the region of space for which interference effects are important in the radiation.

It is easy to see that Eq. (2.5) corresponds to Eq. (2.3) for the coherence length obtained in the preceding section in the case of radiation by a relativistic particle. For this it is sufficient in (2.5) to set $\theta = 0$ and $\varepsilon = 1$ and to replace the quantity $(1 - v)$ by $m^2/2E^2$. As a result we obtain the formula

$$l_c = \frac{2E^2}{m^2\omega}, \qquad (2.7)$$

in which there is no difference between the particle energies in the initial and final states, whereas in the quantum formula (2.3) the difference due to recoil on radiation is taken into account.

In obtaining Eq. (2.5) for the coherence length it is assumed that the particle trajectory is a straight line or departs very little from it. It is easy to take into account also the change of direction of the trajectory due to elastic collisions of the particle with the atoms of the medium. For this purpose it is necessary in Eq. (2.5) only to replace v by $v_\parallel = v\,\overline{\cos\vartheta_l}$, where ϑ_l is the multiple-scattering angle of the particle in the coherence length. Then we arrive at the following expression for the coherence length, which was obtained by Galitsky and Gurevich[28]:

$$l(\omega, \theta) = v\,\overline{\cos\vartheta_l}\,(\omega - kv\,\overline{\cos\vartheta_l}\cos\theta)^{-1}. \qquad (2.8)$$

We note that the average value $\overline{\cos\vartheta_l}$ in matter itself depends on the coherence length, and therefore Eq. (2.8) is actually an equation for determination of $l(\omega,\theta)$.

Taking into account the smallness of the characteristic angles of scattering and radiation of a relativistic particle in matter, we find for $\varepsilon = 1$

$$l(\omega, \theta) = l_c(1 + \gamma^2\overline{\vartheta_l^2} + \gamma^2\theta^2)^{-1}, \qquad (2.9)$$

where $\gamma = E/m$ is the Lorentz factor of the particle.

$$l(\omega, \theta) = l_c(1 + \gamma^2\theta^2)^{-1}. \qquad (2.10)$$

Equation (2.10) is general and does not depend on in what kind of medium the radiation occurs—amorphous or crystalline. This means that the coherence length itself does not yet determine the dynamics of the radiation process. In other words, on the basis only of the value of $l(\omega,\theta)$, we cannot say whether interference within the coherence length will lead to enhancement or weakening of the intensity of the radiation. In particular, for a uniform motion the formula (2.10) itself also has a meaning, although in this case for $\varepsilon = 1$ there is no radiation at all.

With increase of the energy E and with decrease of the frequency ω of the radiated wave, the length $l(\omega,\theta)$ increases. Here there is also an increase of the average values $\overline{\vartheta_l^2}$. Therefore with increase of $l(\omega,\theta)$ the condition $\gamma^2 \overline{\vartheta_l^2} \ll 1$ will be violated.

Equation (2.9) shows that for $\gamma^2 \overline{\vartheta_l^2} \gg 1$ the coherence length will decrease in comparison with the length determined by Eq. (2.10). We emphasize that this result indicates only that for $\gamma^2 \overline{\vartheta_l^2} \gg 1$ there is a decrease in the size of the region of space in which interference effects in the radiation are important, but as before on the basis only of the value of $l(\omega,\theta)$ we cannot say how the radiation process develops within the coherence length.

2.3. "Stripping" of a photon from the radiating particle

The coherence length can also be interpreted as the length in which stripping of a photon from the electron which radiates it occurs.[29,30]

Indeed, suppose that an electron, colliding with an atom, radiates an electromagnetic wave with frequency ω. For ultrarelativistic particles radiation occurs mainly at small angles $\theta \sim m/E$ to the momentum, and therefore in what follows we shall be interested in the radiation in the direction of motion of the particle.

It is clear that immediately after a collision with an atom the electromagnetic wave radiated by the electron (the wave packet) cannot be separated very far from it. The electron and the electromagnetic wave can be considered to be independent objects only if the wave moves away from the electron by a distance at least of the order of the length of this wave $\lambda = 1/\omega$. Since the electron and the wave radiated by it move in the same direction, the relative velocity at which they separate will be $v_{\rm rel} \approx 1 - v$ (see Fig. 1a). Therefore the interval of time in which the wave will be separated from the electron by a distance λ is

$$\Delta t = \frac{\lambda}{v_{\rm rel}} = \frac{2E^2}{m^2\omega}. \tag{2.11}$$

The distance traveled by the electron in a time Δt, $l_c = v\Delta t$, coincides with the coherence length introduced previously.

At large E and small ω, as has already been mentioned, the length l_c has macroscopic dimensions. If within this length the particle collides with many atoms of the medium, it is necessary to take into account the interference of the waves radiated in these collisions. However, if the collisions occur at distances greater than l_c, then the radiation events can be considered independent.

Note that not only the radiation process, but also a number of other electrodynamic processes at high energies such as electron-positron pair production, ionization loss of clusters, electromagnetic showers, and so forth develop in a large region of space along the particle momenta, and therefore for such processes also it is necessary to take into account interference effects in the interaction. We shall make clear why this is so.

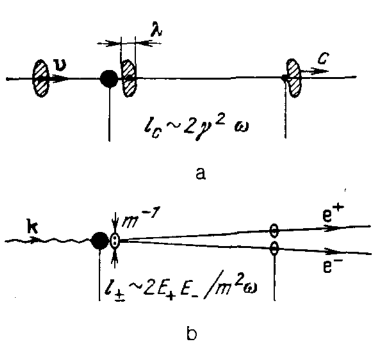

FIG. 1. Formation lengths of processes of radiation (a) and production of an electron-positron pair (b) at high energies.

Let us consider first the production of an electron-positron pair by an energetic photon in the field of a nucleus. The electron and positron can be considered to be free particles only in the case in which these particles separate from each other by a distance greater than $2/m$. At high energies they are emitted mainly at small angles to the photon momentum $\vartheta_\pm \sim m/E_\pm$, where E_- and E_+ are the electron and positron energies, and therefore before the electron and positron separate by a distance $2/m$ they will traverse a path equal in order of magnitude to $l_\pm = 2E_+E_-/m^2\omega$ (Fig. 1b). The quantity $l_\pm$ is the length in which the electron-positron pair is formed. Since the characteristic values of the energies E_+ and E_- of the particles produced are in order of magnitude $E_+ \sim E_- \sim \omega/2$, we have $l_\pm \sim \omega/2m^2$ and consequently for sufficiently large ω this length can become arbitrarily large.

We note that the length $l_\pm$ has the same structure as the coherence length l_c in the radiation process: the only difference is that the initial and final energies of the electron in the radiation process are replaced here by the energies of the electron and positron.

The relative velocity with which the electron and positron separate in production of an electron-positron pair $v_{\rm rel} \sim v(\vartheta_+ + \vartheta_-)$ is small in comparison with the velocity of translation of the particles v, and therefore the electron and positron move for an extended period of time close to each other. Here the ionization loss by the pair particles will be decreased in comparison with the case in which the particles have been separated far from each other.[31] This is due to the fact that the main contribution to the ionization loss of a charged particle is from the region of impact parameters $v/\omega_p \gtrsim \rho \gtrsim 1/m$, where ω_p is the plasma frequency. Therefore if the electron and positron are at a distance s from each other smaller than v/ω_p, then the Coulomb electromagnetic fields of the electron and positron will cancel, as a result of which in this case the main contribution to the ionization loss will be from the region of impact parameters $s \gtrsim \rho \gtrsim 1/m$. It is clear that the ionization loss of the system consisting of the electron and positron will increase with separation of the particles.

For $s > v/\omega_p$ the electron and positron will lose energy to ionization as independent particles.

A similar effect occurs in passage of fast molecules through a thin layer of matter.[32] The ionization loss of the atoms formed as the result of breakup of a molecule turns out in this case to be greater than the ionization loss in the case in which these atoms are separated by a large distance. This effect is due to the coherent addition of the Coulomb fields of the separating atoms of the molecule.

The examples given show that if fast particles are close together for an extended period of time, the efficiency of their interaction with the atoms of the medium can differ from the efficiency of interaction in the case in which the particles are far apart. Here interference effects in the interaction are important, as a result of which the yield of various reactions can either increase or decrease in comparison with the yield of these reactions in the case in which the particles are far apart.

3. RADIATION BY A RELATIVISTIC PARTICLE IN CLASSICAL ELECTRODYNAMICS

3.1. The field of a moving electron

As we have already pointed out, the coherence length itself does not determine the dynamics of the radiation process, but determines only the size of the region in which interference effects in the radiation are important. In order to find the consequence of the interference it is necessary to know the evolution of the field created by a moving electron. This question from the classical point of view will be discussed in the present section.

We recall that the potential $\mathbf{A}(\mathbf{r},t)$ of the field of a moving electron is given by the wave equation

$$\left(\nabla^2-\frac{\partial^2}{\partial t^2}\right)\mathbf{A}(\mathbf{r},\ t)=4\pi e\mathbf{v}(t)\,\delta(\mathbf{r}-\mathbf{r}(t)), \tag{3.1}$$

where e is the charge of the electron, $\mathbf{v}(t)$ and $\mathbf{r}(t)$ are the velocity and trajectory of the electron in the external field, and $\delta(\mathbf{r})$ is a delta function.

The retarded solution of Eq. (3.1) has the form

$$\mathbf{A}(\mathbf{r},\ t)=e\int\frac{d\tau}{|\mathbf{r}-\mathbf{r}(\tau)|}\,\mathbf{v}(\tau)\,\delta(\tau-t+|\mathbf{r}-\mathbf{r}(\tau)|). \tag{3.2}$$

This expression can be rewritten in the form of a Fourier expansion

$$\mathbf{A}(\mathbf{r},\ t)=\frac{e}{2\pi^2}\,\mathrm{Re}\int\frac{d^3k}{k}\left\{\frac{\mathbf{v}(t)}{k-\mathbf{k}\mathbf{v}(t)}\exp[i\mathbf{k}(\mathbf{r}-\mathbf{r}(t))]\right.$$
$$\left.-\mathbf{I}(\mathbf{k},\ t)\exp[i(\mathbf{k}\mathbf{r}-kt)]\right\}, \tag{3.2'}$$

where

$$\mathbf{I}(\mathbf{k},\ t)=\int_{-\infty}^{t}d\tau\exp[i(k\tau-\mathbf{k}\mathbf{r}(\tau))]\,\frac{d}{d\tau}\,\frac{\mathbf{v}(\tau)}{k-\mathbf{k}\mathbf{v}(\tau)}. \tag{3.3}$$

For uniform motion of the charge the second term in Eq. (3.2′) vanishes, and the first term determines the Coulomb field of the electron

$$\mathbf{A}(\mathbf{r},\ t)=\mathbf{A}_v=e\mathbf{v}\left[(z-vt)^2+\frac{\rho^2}{\gamma^2}\right]^{-1/2}, \tag{3.4}$$

where $\gamma=(1-v^2)^{-1/2}$, the z axis is parallel to $\mathbf{v}$, and $\boldsymbol{\rho}$ is a radius vector in the plane orthogonal to $\mathbf{v}$.

In the presence of acceleration, the second term in Eq. (3.2′) determines the radiation field of the particle. The spectral and angular density of the radiation is given by the quantity $\mathbf{I}(\mathbf{k},\infty)$:

$$\frac{d\mathscr{E}}{d\omega\,do}=\frac{e^2}{4\pi^2}\,|[\mathbf{k}\mathbf{I}(\mathbf{k},\ \infty)]|^2,\qquad |\mathbf{k}|=\omega. \tag{3.5}$$

In the following we shall be interested in the motion and radiation of an electron in matter. In this case the electron velocity changes as the result of collisions with atoms of the medium. At high energies the change of velocity occurs during small time intervals in comparison with the radiation formation time, so that one can assume that the electron velocity changes in jumps in collisions with the atoms. Therefore we should discuss first of all the case in which the electron velocity in the time interval $(-\infty,0)$ is equal to $\mathbf{v}$, while in the time interval $(0,+\infty)$ it is equal to $\mathbf{v}_1$.[30,33] With this motion of the particle before scattering ($t<0$) the vector potential $\mathbf{A}(\mathbf{r},t)$ is the vector potential of the electron Coulomb field (3.4). After the scattering ($t>0$), according to Eq. (3.2),

$$\mathbf{A}(\mathbf{r},\ t)=\frac{e}{2\pi^2}\,\mathrm{Re}\int\frac{d^3k}{k}\,e^{i\mathbf{k}\mathbf{r}}\left\{\frac{\mathbf{v}_1}{k-\mathbf{k}\mathbf{v}_1}\,[1-e^{-i(k-\mathbf{k}\mathbf{v}_1)t}]\right.$$
$$\left.\times e^{-i\mathbf{k}\mathbf{v}_1t}+\frac{\mathbf{v}}{k-\mathbf{k}\mathbf{v}}\,e^{-ikt}\right\}. \tag{3.6}$$

This formula can also be written in the form

$$\mathbf{A}(\mathbf{r},\ t)=\Theta(r-t)\,\mathbf{A}_v+\Theta(t-r)\,\mathbf{A}_{v_1}, \tag{3.7}$$

where $\Theta(x)$ is the Heaviside step function ($\Theta(x)=0$ if $x<0$ and $\Theta(x)=1$ if $x\geqslant 1$) and $\mathbf{A}_{v_1}$ is the vector potential of the Coulomb field of an electron moving uniformly with a velocity $\mathbf{v}_1$.

Similar results are easily obtained for the retarded potential of the electric field of the electron.

Equations (3.6) and (3.7) show that before scattering the electromagnetic field which surrounds the electron is a Coulomb field moving with the electron with velocity $\mathbf{v}$. The main contribution to this field is from Fourier components with wave vectors $\mathbf{k}$ whose directions are close to the direction of the particle velocity $\mathbf{v}$; specifically, the angle between the effective values of $\mathbf{k}$ and $\mathbf{v}$ is of the order m/E. After scattering ($t>0$) the Coulomb field is stripped from the electron and continues to advance with velocity $\mathbf{v}$ in the direction of initial motion of the particle, gradually being reconstructed into the radiation field. This means (as is shown by the function $\Theta(r-t)$ in the first term of Eq. (3.7)) that in the region $r>t$ which the signal of the collision has not yet reached the field will be Coulomb in spite of the fact that there is no electron at the point $z=vt$; after scattering it moves in the direction of $\mathbf{v}_1$. For $r<t$ according to Eq. (3.7) there is present only the second term, which consists of the Coulomb field of a particle moving in the direction of $\mathbf{v}_1$. Therefore at $r=t$ there is a rearrangement of the electron field, as the result of which radiation occurs.

At the scattered electron the Coulomb field associated with it does not appear immediately. According to Eq. (3.6) for an interval of time $\Delta t\lesssim(k-\mathbf{k}\mathbf{v}_1)^{-1}$ the Fourier components of the potential $\mathbf{A}(\mathbf{r},t)$ which have a wave vector $\mathbf{k}$ actually do not exist. Since the main contribution to the potential $\mathbf{A}_{v_1}$ is from wave vectors $\mathbf{k}$ whose directions are close to the direction of the velocity $\mathbf{v}_1$ (the angle between the effective values of $\mathbf{k}$ and $\mathbf{v}_1$ is of the order m/E), the time interval during which the Fourier components of the Coulomb field of the scattered electron are essentially absent will be of the scale

$$\Delta t\sim\frac{2E^2}{m^2\omega}.$$

Figuratively speaking, we can say that after the first collision the electron for a time interval $\Delta t\sim 2E^2/m^2\omega$ is in a "semibare" state, i.e., substantially without its Coulomb field.[1)]

In this time the electron travels a distance $l_c=\Delta v\Delta t$, which coincides with the coherence length introduced previously.

The results obtained in this section are illustrated in

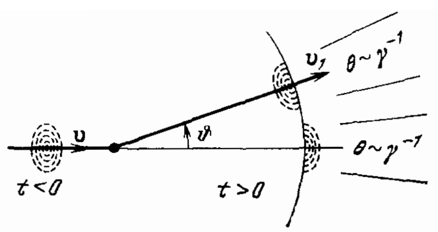

FIG. 2. The field surrounding an electron before ($t<0$) and after ($t>0$) scattering by an atom at an angle ϑ.

Fig. 2, in which we have shown schematically the equipotential surfaces of the electron field before and after scattering.

3.2. Radiation in the case of many collisions of an electron in the coherence length

In the preceding section we showed that after a collision an electron is for a considerable length of time in a semibare state, i.e., substantially without its Coulomb field. During this time interval the electron can undergo many collisions with the atoms of the medium. Therefore we shall investigate now what happens in this case with the electric field which surrounds the electron.[30]

Let us discuss first of all the case of two collisions.

If at $t = t_1 = 0$ as the result of a collision with an atom the velocity of the electron changes from a value $\mathbf{v}$ to a value $\mathbf{v}_1$, and at $t = t_2$ it changes from $\mathbf{v}_1$ to $\mathbf{v}_2$, then according to Eq. (3.2) the vector potential at $t > t_2$ will be given by the formula

$$\mathbf{A}(\mathbf{r}, t) = \frac{e}{2\pi^2} \operatorname{Re} \int \frac{d^3k}{k} e^{i\mathbf{kr}} \times \left\{ \frac{\mathbf{v}_2}{k - \mathbf{kv}_2} \left[1 - e^{-i(k-\mathbf{kv}_2)(t-t_2)}\right] e^{-i\mathbf{kr}(t)} + \frac{\mathbf{v}_1}{k - \mathbf{kv}_1} \left(e^{i(k-\mathbf{kv}_1)t_2} - 1\right) e^{-ikt} + \frac{\mathbf{v}}{k - \mathbf{kv}} e^{-ikt} \right\}, \tag{3.8}$$

where $\mathbf{r}(t) = \mathbf{r}_2 + \mathbf{v}_2(t - t_2)$ and $\mathbf{r}_2$ are the coordinates of the point at which the second collision occurred.

The terms in this expression which contain the vectors $\mathbf{v}$ and $\mathbf{v}_2$ have the same structure as the corresponding terms in Eq. (3.6)—they are determined by the evolution of the field in the directions of the initial and final motions of the particle. The term which contains $\mathbf{v}_1$ determines the evolution of the field in the direction of the intermediate motion of the particle.

If the second collision occurs at a moment of time $t_2 \ll (k - \mathbf{kv}_2)^{-1}$, then the electron at the moment of the collision is in a semibare state. Here according to Eq. (3.8) radiation of waves with wave vectors $\mathbf{k}$ close in direction to $\mathbf{v}_1$ will be suppressed in comparison with the case in which the second collision occurs at a moment of time $t_2 \gtrsim (k - \mathbf{kv}_1)^{-1}$.

Let us consider now the case in which in the coherence length there are many ($N > 2$) collisions at moments of time $t = t_j \geqslant 0$, where $j = 1,2,\ldots,N$. Then the vector potential at $t > t_N$ will be given by Eq. (3.2) with

$$\mathbf{I}(\mathbf{k}, t) = \mathbf{I}(\mathbf{k}) = \sum_{j=1}^{N} \left(\frac{\mathbf{v}_j}{k - \mathbf{kv}_j} - \frac{\mathbf{v}_{j-1}}{k - \mathbf{kv}_{j-1}} \right) \exp\left[i(k - \mathbf{kv}_{j-1}) t_j\right]. \tag{3.9}$$

The characteristic angles of scattering of a fast particle in matter are small: $|\mathbf{v}_j - \mathbf{v}| \ll v$.

If

$$(k - \mathbf{kv}_{j-1}) t_j \ll 1, \tag{3.10}$$

the electron in all collisions will be in a semibare state. In this case

$$\mathbf{I}(\mathbf{k}) = \frac{\mathbf{v}_N}{k - \mathbf{kv}_N} - \frac{\mathbf{v}}{k - \mathbf{kv}}. \tag{3.11}$$

We see that the value of $\mathbf{I}(\mathbf{k})$ will depend only on the initial and final velocities of the particle and will not depend on its intermediate velocities.

Knowing $\mathbf{I}(\mathbf{k})$, we can according to Eq. (3.5) find the spectral and angular distribution of the radiation. The spectral density of the radiation will be given by the formula

$$\frac{d\mathscr{E}}{d\omega} = \frac{2e^2}{\pi} \left\{ \frac{2\xi^2 + 1}{\xi(\xi^2+1)^{1/2}} \times \ln\left[\xi + (\xi^2+1)^{1/2}\right] - 1 \right\}, \tag{3.12}$$

where $\xi = \gamma\vartheta/2$ and $\vartheta = |\mathbf{v}_N - \mathbf{v}|/v$ is the scattering angle of the particle ($\vartheta \ll 1$).

We note certain features of the radiation process if the condition (3.10) is satisfied.

The first term of Eq. (3.11) determines waves propagated in a direction close to the direction of the final motion of the particle $\mathbf{v}_N$, and the second term determines waves which propagate in a direction close to $\mathbf{v}$. Radiation occurs mainly into cones with opening angles of the order m/E (Fig. 2). The phases of the waves which propagate near directions $\mathbf{v}_N$ and $\mathbf{v}$ are opposite, and therefore depending on the relation between the scattering angle ϑ and the opening angle of the radiation cone m/E various interference patterns will occur.

If $\vartheta \ll m/E$, then there will be a strong cancellation of the fields associated with the two terms in Eq. (3.11). In this case the quantity $|\mathbf{I}(\mathbf{k})|$ will be proportional to ϑ, and the spectral distribution of the radiation will be proportional to ϑ^2:

$$\frac{d\mathscr{E}}{d\omega} \approx \frac{2e^2}{3\pi} \gamma^2\vartheta^2. \tag{3.13}$$

For scattering angles which exceed the opening angle of the radiation cone, $\vartheta \gg m/E$, cancellation of the fields practically does not occur. Here the spectral distribution of the radiation will depend weakly on the scattering angle:

$$\frac{d\mathscr{E}}{d\omega} \approx \frac{4e^2}{\pi} \ln \gamma\vartheta. \tag{3.14}$$

Thus, we see that for $\gamma\vartheta \ll 1$ and $\gamma\vartheta \gg 1$ the nature of the radiation is substantially different. In the first case there is strong interference of the waves radiated in the directions of the initial and final motions of the particle; in the second case there is practically no interference.

3.3. Radiation in distances greater than the coherence length

The formulas obtained above can be used when the path traveled by an electron in matter is less than or of the order of the coherence length. In other words, it is required that the thickness of the material not exceed the coherence length. However, if the target thickness T is greater than the coherence length l, then to find the spectral density of the radiation a special calculation is necessary. This calculation will be carried out in the following section. Here we shall give only simple estimates of the spectral density of radiation in this case and shall establish conditions under which multiple scattering substantially changes the nature of the radiation of a particle in matter.

To estimate the spectral density of the radiation for $T \gg l$ it is obviously necessary to break down the target into layers whose thickness is equal to the coherence length l and to sum the fields which arise in each of the layers. Interference of the radiation from the individual layers need not be taken into account here. Then in the n-th portion of the path the spectral and angular density of the radiation will be given by Eqs. (3.5) and (3.11). The combined spectral and angular density of the radiation in this case will be

$$\frac{d\mathscr{E}}{d\omega\, do} \approx \frac{e^2}{4\pi^2} \sum_n \left| \left[\mathbf{k} \left(\frac{\mathbf{v}_n}{k-\mathbf{k}\mathbf{v}_n} - \frac{\mathbf{v}_{n-1}}{k-\mathbf{k}\mathbf{v}_{n-1}} \right) \right] \right|^2, \quad (3.15)$$

where $\mathbf{v}_n$ is the velocity of the particle at the end of the n-th portion and the summation is carried out over all portions of the path traveled by the particle in the target material.

The main contribution to the integral over the angles of radiation is given by values $\theta \sim \max(\gamma^{-1}, |\mathbf{v}_n - \mathbf{v}_{n-1}|/v)$ in each term of Eq. (3.15). Therefore, using Eq. (3.12) and taking into account that the number of layers is T/l, we eventually obtain

$$\frac{d\mathscr{E}}{d\omega} \approx \frac{T}{l} \frac{d\mathscr{E}_l(\gamma\vartheta_l)}{d\omega}, \quad (3.16)$$

where $d\mathscr{E}_l/d\omega$ is given by Eq. (3.12), ϑ_l is the scattering angle on the coherence length l, and l is determined from the relation (2.9) for $\theta \sim \max(\gamma^{-1}, \vartheta_l)$.

In derivation of Eq. (3.16) no specific law of motion of the particle was used, and therefore this formula can be used for evaluation of the radiation spectrum in both amorphous and crystalline media and also in motion of the particle in external electromagnetic fields.

In the case of amorphous materials the mean square angle of multiple scattering of the particle per unit length is given by the formula[5,34]

$$q = nv \int d^2\vartheta\, \vartheta^2 \sigma(\boldsymbol{\vartheta}), \quad (3.17)$$

where n is the density of atoms and $\sigma(\boldsymbol{\vartheta})$ is the differential cross section for elastic scattering of the particle by an individual atom of the medium at an angle $\boldsymbol{\vartheta}$.

Note that the integral entering into Eq. (3.17) diverges logarithmically in the region of large ϑ. In multiple-scattering theory the upper limit in this integral is of the order[34] $\vartheta_{\max} \sim 1/E/R_n$, where R_n is the radius of the nucleus of the atom. In problems involving radiation, the upper limit of the integral should be set equal to $\vartheta_{\max} \sim m/E$ (see Section 19 of the book by Ter-Mikaelyan[5]).

Substituting the relation (3.17) into Eq. (2.9), we find that if $\gamma\vartheta_l \ll 1$, then $l \simeq 2\gamma^2/\omega$, while if $\gamma\vartheta_l \gg 1$, we have $l \simeq (2/\omega q)^{1/2}$. Using these expressions for l, we obtain according to Eq. (3.16)

$$\frac{d\mathscr{E}}{d\omega} \approx \begin{cases} \dfrac{2e^2}{3\pi}\gamma^2 qT, & \gamma\vartheta_l \ll 1, \quad (3.18\text{a}) \\ \dfrac{2e^2}{\pi}(2\omega q)^{1/2} T \ln(\gamma\vartheta_l), & \gamma\vartheta_l \gg 1. \quad (3.18\text{b}) \end{cases}$$

The first of these formulas coincides with logarithmic accuracy with the Bethe-Heitler formula,[24,25] in which the influence of multiple scattering on the radiation is not taken into account. The second formula corresponds to the case in which multiple scattering plays an important role in the radiation (it differs only by a numerical factor from the result of Landau and Pomeranchuk[2]).

In motion in a crystal at a small angle ψ to one of the crystallographic axes (the z axis) the mean square angle of the particle in scattering by the ordered atoms of the lattice can considerably exceed the mean square scattering angle in an amorphous medium (see Section 5). Here over a wide range of angles ψ ($R/d \gg \psi \gg \psi_c$) the relation between these quantities per unit length is given by the formula

$$q_c \sim \frac{R}{4\psi d} q, \quad (3.19)$$

where R is the screening radius of the atom, d is the distance between atoms in the lattice along the z axis, and ψ_c is the critical angle of axial channeling. Accordingly for small and large scattering angles the spectral density of radiation in the crystal will be given by Eq. (3.18) with the only difference that the quantity q in Eq. (3.18) must be replaced by q_c. The first of the formulas (3.18) in this case will differ only by a numerical factor of the order of unity from the corresponding result of the theory of coherent radiation of a relativistic electron in a crystal.[5,17] The second formula will correspond to the case in which multiple scattering has a significant effect on the radiation of a particle in a crystal.

The relations presented therefore permit us to estimate on the basis of a unified approach the spectrum of radiation of a fast particle in both amorphous and crystalline media. These relations make clear not only the reason for the change in the nature of the radiation of a particle in the medium at $\gamma\vartheta_l \sim 1$, but also the reason for the difference in the radiation spectra in a crystal and in an amorphous medium for $\gamma\vartheta_l \ll 1$. This difference is due to the difference in the average values of the scattering angles in these cases.

3.4. Spectral density of radiation

In Section 3.1 a general formula (3.5) was obtained for the spectral and angular density of radiation of a particle moving with a certain acceleration. We made use of this formula to estimate the radiation spectrum, breaking down the path traveled by the particle in the material into portions equal to the coherence length. Here interference effects in the radiation were taken into account in each of the portions, but interference between neighboring portions was not taken into account. We shall now remove this limitation.

We shall start as before from the general formula (3.5) in which we shall understand $\mathbf{I}(\mathbf{k})$ to mean the expression

$$\mathbf{I}(\mathbf{k}) = \int_{-\infty}^{\infty} dt\, \mathbf{v}(t) \exp[i(\omega t - \mathbf{k}\mathbf{r}(t))], \quad (3.20)$$

where ω and $\mathbf{k}$ are related as $k = \varepsilon^{1/2}\omega$. As a result of this the influence of polarization of the medium on the radiation, which was not considered in the previous section, is taken into account.

Following Landau and Pomeranchuk,[2] we carry out in (3.5) integration over the angles of radiation. The spectral density of radiation obtained in this way can be represented in the form

$$\frac{d\mathscr{E}}{d\omega}=\frac{e^2k}{\pi}\int_{-\infty}^{\infty}dT\int_{-\infty}^{\infty}d\tau e^{-i\omega\tau}\left(\mathbf{v}(T+\tau)\mathbf{v}(T)-\frac{1}{\varepsilon}\right)\times\frac{\sin k\,|\mathbf{r}(T+\tau)-\mathbf{r}(T)|}{|\mathbf{r}(T+\tau)-\mathbf{r}(T)|},\qquad(3.21)$$

where

$$\mathbf{r}(T+\tau)-\mathbf{r}(T)=\int_{T}^{T+\tau}dt\,\mathbf{v}(t).$$

The characteristic scattering angles of a fast particle in matter are small, and therefore in Eq. (3.21) we can set

$$\mathbf{v}(T+\tau)\approx\mathbf{v}(T)\left[1-\frac{1}{2}\boldsymbol{\vartheta}^2(\tau)\right]+\boldsymbol{\vartheta}(\tau),\qquad(3.22)$$

where $\boldsymbol{\vartheta}(\tau)$ is the angle of scattering in the time interval $(T,T+\tau)$, and $\vartheta\ll1$.

Using this expression, we find with accuracy to terms of order ϑ^2

$$\frac{d\mathscr{E}}{d\omega}\approx-\frac{e^2k}{\pi}\int_{-\infty}^{\infty}dT\int_{-\infty}^{\infty}\frac{d\tau}{\tau}e^{-i\omega\tau}\left[\varepsilon^{-1}-\mathbf{v}^2+\frac{1}{2}\boldsymbol{\vartheta}^2(\tau)\right]\times\sin\left\{k\left[v\tau-\frac{1}{2}\int_0^{\tau}dt\boldsymbol{\vartheta}^2(t)+\frac{1}{2\tau}\left(\int_0^{\tau}dt\boldsymbol{\vartheta}(t)\right)^2\right]\right\}.\qquad(3.23)$$

For $\varepsilon=1$ this formula goes over into the corresponding result of Ref. 17, in which the influence of the polarization of the medium on the radiation was not taken into account.

The interest which equation (3.23) presents lies in its generality and in the possibility of discussing by means of this formula from a unified point of view radiation in various media and in external fields. In the latter case $\boldsymbol{\vartheta}(\tau)$ is a defined function of τ. If the radiation occurs in a medium, then Eq. (3.23) must be averaged over the random process which the scattering involves. This averaging must be carried out differently in amorphous and crystalline media.

4. THE LANDAU-POMERANCHUK EFFECT

4.1. The classical limit of the Bethe-Heitler formula

We shall first consider the case in which the motion occurs in an amorphous medium and the angle of scattering on the coherence length is rather small ($\omega l\vartheta_l^2\ll1$). In this case in Eq. (3.23) it is possible to expand the integrand in terms of the scattering angle. As a result we find[2)] that for $\varepsilon v^2<1$.

$$\frac{d\mathscr{E}}{d\omega}\approx\frac{e^2k}{2\pi}\int_{\delta}^{\infty}\frac{d\nu}{\nu^2}\left[1-2\frac{\delta}{\nu}\left(1-\frac{\delta}{\nu}\right)\right]|\mathbf{w}(\nu)|^2,\qquad(4.1)$$

where

$$\delta=\omega(1-v\varepsilon^{1/2}),\quad\mathbf{w}(\nu)=\int_{-\infty}^{\infty}dt\dot{\boldsymbol{\vartheta}}(t)\,e^{i\nu t}.$$

The variable of integration ν is related to the radiation angle θ in Eq. (3.5) as $\nu=\omega-\mathbf{kv}$.

Taking into account that at high energies the radiation develops in a length (along the particle momentum) which is considerably greater than the size of an atom, the quantity $|\mathbf{w}(\nu)|^2$ can be written in the form

$$|\mathbf{w}(\nu)|^2=\sum_{n,k}\boldsymbol{\vartheta}_n\boldsymbol{\vartheta}_k\exp[i\nu(t_n-t_k)],\qquad(4.2)$$

where ϑ_n is the scattering angle in the collision with the nth atom and t_n is the moment of time at which this collision occurred.

In an amorphous medium the collisions of a particle with various atoms are random, and therefore the quantity $|\mathbf{w}(\nu)|^2$ entering into (4.1) must be averaged over the random locations of the atoms in the medium. For a collision with an atom located at a point $\mathbf{r}_n=(\boldsymbol{\rho}_n,z_n)$ the scattering angle is

$$\boldsymbol{\vartheta}_n=-\frac{1}{E}\frac{\partial}{\partial\boldsymbol{\rho}}\int_{-\infty}^{\infty}dz\,u(\boldsymbol{\rho}-\boldsymbol{\rho}_n,\ z),$$

where $\boldsymbol{\rho}$ is the impact parameter and $u(r)$ is the potential energy of interaction of the particle with the atom. Substituting this relation into Eq. (4.1), we obtain after averaging over $\boldsymbol{\rho}_n$ and integration over ν the following expression for the spectral density of radiation:

$$\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle=\left(\frac{d\mathscr{E}}{d\omega}\right)_0\frac{1}{2\gamma^2(1-v\varepsilon^{1/2})}.\qquad(4.3)$$

Here the first factor is the usual expression for the radiation spectrum of a fast particle in an amorphous medium in the low-frequency region without inclusion of multiple scattering and polarization of the medium:

$$\left(\frac{d\mathscr{E}}{d\omega}\right)_0=\frac{2e^2nT}{3\pi}\gamma^2\int d^2\rho\boldsymbol{\vartheta}^2(\rho).\qquad(4.4)$$

In the case in which the potential of the atom is a screened Coulomb potential, the integral entering into (4.4) diverges in the region of small ρ. From the condition of applicability of Eq. (4.4) $\gamma\vartheta\lesssim1$ it follows that the integration over ρ in (4.4) should be limited by the value $\rho_{\min}\sim2Ze^2/m$, where $Z\,|e|$ is the charge of the nucleus of the atom. Quantum effects in the radiation appear, however, at $\rho_{\min}\sim1/m$, i.e., at distances larger than $2Ze^2/m$; therefore $\rho_{\min}$ must be set equal to $1/m$.[5] Here the quantity $(d\mathscr{E}/d\omega)_0$ coincides with logarithmic accuracy with the radiation spectrum given by the Bethe-Heitler formula[24,35]

$$\frac{d\mathscr{E}_{BH}}{d\omega}=\frac{16Z^2e^6nT}{3m^2}\left[\ln(183Z^{-1/3})+\frac{1}{12}\right],\quad\omega\ll E.\qquad(4.5)$$

The second factor in (4.3) describes the influence of the polarization of the medium on the radiation.[3)]

Equation (4.1) is valid if the condition $\omega l\vartheta_l^2\ll1$ is satisfied. For $\varepsilon=1$ in the case of an amorphous medium this inequality can be written in the form $\gamma^2ql_c\ll1$. The latter inequality is violated at sufficiently high energies of the particles.

4.2. The estimates of Landau and Pomeranchuk

If the condition $\gamma^2ql_c\ll1$ is not satisfied, then for determination of the radiation spectrum of the particle it is necessary to use the general formula (3.23), which must be averaged over the random values of the particle-scattering angle in the medium (this randomness is due to the random location of the atoms in an amorphous medium). The exact performance of the averaging is made difficult by the fact that the scattering angle enters into the spectral density of radiation in the argument of a sine function. Landau and Pomeranchuk, with the intention of obtaining estimates for $\gamma^2ql_c\gg1$, proposed replacing the average value of the sine by the sine of the average value of the quantity. Here the following formula is obtained for the averaged value of the spectral density of radiation[4)]:

$$\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle\approx\frac{e^2\omega T}{\pi\gamma^2}\int\limits_0^\infty\frac{d\tau}{\tau}\left(1+\frac{1}{2}\gamma^2q\tau\right)\times\sin\left[(1-v)\,\omega\tau+\frac{1}{12}\omega q\tau^2\right]. \quad (4.6)$$

The influence of the polarization of the medium on the radiation is not taken into account here, i.e., it is assumed that $\varepsilon=1$. In the case of interest to Landau and Pomeranchuk $\gamma^2ql_c\gg1$ one can neglect the term proportional to τ in the argument of the sine function. Here Eq. (4.6) takes the form

$$\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle=\frac{d\mathscr{E}_{LP}}{d\omega}=\frac{e^2}{2}\left(\frac{3\omega q}{2\pi}\right)^{1/2}T. \quad (4.7)$$

Comparison of this formula with the Bethe-Heitler result (4.5) shows that when the condition $\gamma^2ql_c\gg1$ is satisfied

$$\frac{d\mathscr{E}_{LP}}{d\omega}\ll\frac{d\mathscr{E}_{BH}}{d\omega}.$$

We see that the nature of the radiation of a fast particle in an amorphous medium changes substantially when $\gamma^2ql_c\sim1$, i.e., in the region of particle energies and frequencies of the radiated photons for which the mean square multiple scattering angle in a length l_c becomes comparable with the square of the characteristic angle of radiation of a relativistic particle $\theta^2\sim\gamma^{-2}$.

Equations (4.6) and (4.7) are suitable only for estimates. In spite of this, the conclusion presented reveals the essence of the effect—the effect is due to the curvature of the particle trajectory in the amorphous medium within the coherence length due to multiple scattering.

4.3. The Landau-Pomeranchuk effect and synchrotron radiation

An interesting correspondence exists between the Landau-Pomeranchuk effect and synchrotron radiation, i.e., the radiation of a particle moving along an arc in a uniform magnetic field. Indeed, as we have seen, the Landau-Pomeranchuk effect is due to the curvature of the particle trajectory due to multiple scattering. On the other hand, synchrotron radiation is due to the curvature of the particle trajectory due to the external magnetic field. In the latter case the acceleration of the particle is given by the well known formula

$$\dot{v}=\frac{eH}{E}$$

(H is the magnetic field strength). Here the angle of scattering in a time interval τ is

$$\vartheta(\tau)=\frac{eH\tau}{E}.$$

Substituting this expression into the general formula (3.23), we obtain the following expression for the radiation spectrum of a fast particle in a magnetic field:

$$\frac{d\mathscr{E}_H}{d\omega}=-\frac{2e^2\omega T}{\pi}\int\limits_0^\infty\frac{d\tau}{\tau}\left(1-v^2+\frac{1}{2}\frac{e^2H^2}{E^2}\tau\right)\times\cos\omega\tau\cdot\sin\omega\left(v\tau-\frac{1}{24}\frac{e^2H^2}{E^2}\tau^3\right). \quad (4.8)$$

This formula can be reduced by straightforward transformations to the well known formula for the synchrotron radiation spectrum,[37,38] which contains an Airy function. Equation (4.8), however, has the advantage that it permits one to see the relation between the Landau-Pomeranchuk effect and synchrotron radiation—in both cases the spectral density is represented in the form of an integral over the time τ and we can directly compare the formulas which determine the two effects.

In the Landau-Pomeranchuk effect in the argument of the sine function (4.6) there are terms proportional to τ and τ^2; on the other hand, in the case of synchrotron radiation there are terms proportional to τ and τ^3. In both cases at sufficiently low frequencies the terms linear in τ in the oscillating factors can be discarded. In this case in an amorphous medium the radiation spectrum is determined by Eq. (4.7), and in a magnetic field it is determined by (4.8),

$$\frac{d\mathscr{E}_H}{d\omega}\approx0.52e^2\left(\frac{eH}{E}\right)^{2/3}\omega^{1/3}T,\qquad\omega\to0. \quad (4.9)$$

Thus, in both cases the intensity of the radiation decreases with decrease of ω. The laws of this decrease, however, are different, since the argument of the sine in the synchrotron radiation formula involves a term with τ^3, while in the Landau-Pomeranchuk formula there is a term with τ^2. This difference in the powers of τ is due in turn to the difference in curvature of the trajectories due to multiple scattering and to a magnetic field.

4.4. Inclusion of the influence of multiple scattering on the radiation by the kinetic-equation method

In the previous sections it was shown that the radiation from a fast particle in an amorphous medium is weakened as a result of the influence of multiple scattering. We gave also estimates for the case in which the effect is significant. Now let us turn to the quantitative theory of the Landau-Pomeranchuk effect. There are two methods which permit development of a quantitative theory of this effect—the kinetic-equation method and the functional-integration method. We shall begin by describing the first method, which belongs to Migdal.[22]

We shall proceed from Eq. (3.5) for the spectral and angular density of the radiation and shall represent it in the form

$$\frac{d\mathscr{E}}{d\omega\,do}=\frac{e^2}{2\pi^2}\,\mathrm{Re}\int\limits_{-\infty}^{\infty}dt\int\limits_0^\infty d\tau\,[\mathbf{kv}]\,[\mathbf{kv}']\times\exp\left[-i\omega\tau+i\mathbf{k}\,(\mathbf{r}'-\mathbf{r})\right], \quad (4.10)$$

where $\mathbf{r}=\mathbf{r}(t)$, $\mathbf{v}=\mathbf{v}(t)$, $\mathbf{r}'=\mathbf{r}(t+\tau)$, $\mathbf{v}'=\mathbf{v}(t+\tau)$, and $\omega=|\mathbf{k}|$.

The problem consists of averaging this expression over all possible trajectories of the particle, i.e., determining the quantity

$$K=\langle\exp\left[i\mathbf{k}\,(\mathbf{r}'-\mathbf{r})\right]([\mathbf{kv}]\,[\mathbf{kv}'])\rangle,$$

where the angle brackets serve to indicate averaging.

We introduce into the discussion two probability functions: $W_1(\mathbf{r},\mathbf{v};t)$—the probability of values of the coordinates $\mathbf{r}$ and velocities $\mathbf{v}$ at a moment of time t and $W_2(\mathbf{r}',\mathbf{v}';\mathbf{r},\mathbf{v};\tau)$—the probability of values $\mathbf{r}'$ and $\mathbf{v}'$ at a later moment of time $t+\tau$ under the condition that these quantities at the moment of time t have values $\mathbf{r}$ and $\mathbf{v}$. Then the quantity K can be represented in the form

$$K=\int \mathrm{d}\mathbf{r}'\,\mathrm{d}\mathbf{v}'\,\mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{v}\,\exp[i\mathbf{k}(\mathbf{r}'-\mathbf{r})]\,([\mathbf{k}\mathbf{v}'][\mathbf{k}\mathbf{v}])\,W_1W_2. \tag{4.11}$$

The probability functions W_1 and W_2 satisfy the kinetic equation

$$\frac{\partial W}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}W =n\int \mathrm{d}\mathbf{v}'\sigma(\mathbf{v}'-\mathbf{v})\,[W(\mathbf{r},\,\mathbf{v}';\,t)-W(\mathbf{r},\,\mathbf{v};\,t)], \tag{4.12}$$

where $\sigma(\mathbf{v}'-\mathbf{v})$ is the cross section for elastic scattering of the particle by an individual atom in which its velocity changes from $\mathbf{v}$ to $\mathbf{v}'$ (here the scattering occurs by an angle $\vartheta=2\sin|\mathbf{v}'-\mathbf{v}|/2v$). In addition, the probability functions must satisfy the following initial conditions:

$$W_1(\mathbf{r},\,\mathbf{v};\,0)=\delta(\mathbf{r})\,\delta(\mathbf{v}-\mathbf{v}_0),$$
$$W_2(\mathbf{r}',\,\mathbf{v}';\,\mathbf{r},\,\mathbf{v};\,0)=\delta(\mathbf{r}'-\mathbf{r})\,\delta(\mathbf{v}'-\mathbf{v}),$$

where $\mathbf{v}_0$ is the initial velocity.

It follows from the kinetic equation and the initial conditions that W_2 depends only on the difference of the coordinates $\mathbf{r}'-\mathbf{r}$. In Eq. (4.11) there is the Fourier transform of W_2

$$W_{\mathbf{k}}(\mathbf{v}',\,\mathbf{v};\,\tau)=\int \mathrm{d}(\mathbf{r}'-\mathbf{r})\exp[i\mathbf{k}(\mathbf{r}'-\mathbf{r})]\times W_2(\mathbf{r}',\,\mathbf{v}';\,\mathbf{r},\,\mathbf{v};\,\tau),$$

and therefore Eq. (4.11) can be rewritten in the form

$$K=\int \mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{v}\,\mathrm{d}\mathbf{v}'\,[\mathbf{k}\mathbf{v}'][\mathbf{k}\mathbf{v}]\,W_1(\mathbf{r},\,\mathbf{v};\,t)\,W_{\mathbf{k}}(\mathbf{v}',\,\mathbf{v};\,\tau). \tag{4.13}$$

Using the Fourier transformation of the quantity W_2, it is obviously possible to represent the kinetic equation for W_2 in the form

$$\frac{\partial W_k}{\partial \tau}-i\mathbf{k}\mathbf{v}'W_k =n\int \mathrm{d}\mathbf{v}''\sigma(\mathbf{v}''-\mathbf{v}')\,[W_k(\mathbf{v}'',\,\mathbf{v};\,\tau)-W_k(\mathbf{v}',\,\mathbf{v};\,\tau)], \tag{4.14}$$

where $W_k(\mathbf{v}',\mathbf{v};0)=\delta(\mathbf{v}'-\mathbf{v})$.

The solution of this equation is now our main problem.

The characteristic angles of scattering and radiation of a relativistic particle in matter are small, and therefore in the last equation an expansion in these angles can be carried out. It is convenient to measure the angles from the direction of motion of the photon $\mathbf{k}=\mathbf{n}\omega$. In the approximation of small angles the following relations are valid:

$$\mathbf{v}\approx v\left(1-\frac{1}{2}\boldsymbol{\vartheta}^2\right)\mathbf{n}+\boldsymbol{\vartheta},\qquad \vartheta\ll 1,$$
$$\mathbf{v}'\approx v\left(1-\frac{1}{2}\boldsymbol{\vartheta}'^2\right)\mathbf{n}+\boldsymbol{\vartheta}',\quad \vartheta'\ll 1.$$

Substituting these relations into Eq. (4.14) and retaining the first two terms of the expansion in angle of the quantities entering into (4.14), we arrive at the Fokker-Planck equation

$$\frac{\partial}{\partial \tau}W_k-ikv\left(1-\frac{1}{2}\boldsymbol{\vartheta}'^2\right)W_k=\frac{1}{4}q\frac{\partial^2}{\partial \boldsymbol{\vartheta}'^2}W_k \tag{4.15}$$

with the initial condition $W_k(\boldsymbol{\vartheta}',\boldsymbol{\vartheta};0)=\delta(\boldsymbol{\vartheta}'-\boldsymbol{\vartheta})$, where q is given by Eq. (3.17).

The solution of Eq. (4.14) is to be sought in the form

$$W_k(\boldsymbol{\vartheta}',\,\boldsymbol{\vartheta};\,\tau)=\exp(\alpha\boldsymbol{\vartheta}'^2+\beta\boldsymbol{\vartheta}'\boldsymbol{\vartheta}+\gamma),$$

where α, β, and γ are certain functions which depend on τ and ϑ. We shall not give here the details of the solution of Eq. (4.15). This process has been described well in Refs. 5 and 16.

Obviously the solution must be substituted into Eq. (4.13). As a result, after integration over the angles of radiation, we obtain the Migdal formula[22] for the radiation spectrum of a fast particle in an amorphous medium

$$\left\langle\frac{\mathrm{d}\mathscr{E}}{\mathrm{d}\omega}\right\rangle=\left(\frac{\mathrm{d}\mathscr{E}}{\mathrm{d}\omega}\right)_0\Phi_{\mathrm{M}}(s), \tag{4.16}$$

where $(d\mathscr{E}/d\omega)_0=e^2qT/3\pi(1-v)$ is the spectrum of radiation without taking into account multiple scattering (see Eq. (4.4)), $s=\frac{1}{2}(1-v)(\omega/q)^{1/2}$, and $\Phi_M(s)$ is a function which takes into account the influence of multiple scattering on the radiation:

$$\Phi_{\mathrm{M}}(s)=24s^2\left(\int\limits_0^\infty \mathrm{d}x\,\mathrm{cth}\,x e^{-2sx}\sin 2sx-\frac{\pi}{4}\right). \tag{4.17}$$

We shall consider two limiting cases of Eq. (4.16). If $s\to\infty$, then $\Phi_M\to 1$. This case corresponds to low particle energies. Here multiple scattering does not influence the radiation.

For small values of s the function $\Phi_M\approx 6s$ and the expression for $\langle d\mathscr{E}/d\omega\rangle$ take the form

$$\left\langle\frac{\mathrm{d}\mathscr{E}}{\mathrm{d}\omega}\right\rangle=\frac{e^2}{\pi}(\omega q)^{1/2}\,T.$$

This formula differs only by a numerical factor from the estimated result of Landau and Pomeranchuk (4.7).

A plot of the function $\Phi_M(s)$ is shown in Fig. 3. We see that the kinetic-equation method which we have described permits not only a refinement of the coefficient in the Landau-Pomeranchuk formula, but also a description of the intermediate region of transition from the Bethe-Heitler result to the Landau-Pomeranchuk result.

Subsequently the kinetic-equation method has been used to study the effect of many other factors on the radiation, such as recoil on emitting radiation,[39,16] influence of the polarization of the medium,[5,36] effect of the target boundaries,[40] photon absorption, and others. The limits of applicability of the Fokker-Planck method have also been estimated for a given problem, and a procedure for increasing the accuracy of this method has been indicated.[41] The angular distribution of the radiation and its polarization

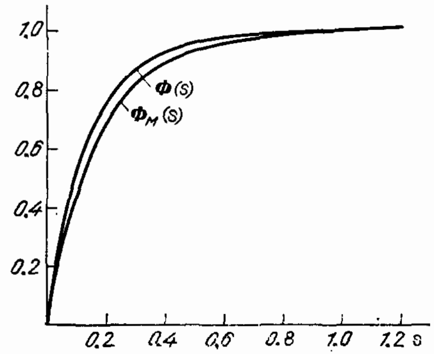

FIG. 3. Plots of the functions $\Phi_M(s)$ and $\Phi(s)$ which determine the influence of multiple scattering on radiation in an amorphous medium and in a crystal.

have also been investigated.[42] Here, however, the medium has always been assumed to be amorphous.

4.5. Averaging of the radiation spectrum over the trajectories by the functional-integration method

The general formula for the spectral density of radiation (3.23) is determined by the angles of scattering of the particle in matter. In view of the multiple scattering, these values are random, and therefore an average must be taken over them. In the Migdal method in order to carry out the averaging the kinetic equation for the particle distribution in coordinates and velocities was used.

The starting point in this problem, however, must be considered to be the concept of the particle trajectory, which is random. Therefore the averaging must, in essence, be carried out over random trajectories. This averaging can be carried out by means of the method of functional integration. At the present time this method has received extensive development in connection with various problems of field theory, not to mention the fact that by means of this method, as Feynman[43] has shown, it is possible to explain clearly the connection between quantum and classical mechanics. The possibility of use of the method of functional integration for averaging over random trajectories in problems involving radiation was established in Refs. 23 and 44. The importance of this approach lies in the fact that it permits one to take into account in a unified way the influence of random factors on the radiation in various problems, such as the influence of multiple scattering on radiation in amorphous media and in crystals and also radiation in external fields.

The procedure for functional integration can be carried out comparatively simply in the case in which the functional being averaged has a Gaussian form and the random process is Gaussian. This situation occurs in the problem considered here.

Indeed, according to Eq. (3.23) the spectral density of radiation is a functional $d\mathscr{E}[\boldsymbol{\vartheta}(\tau)]/d\omega$ of the random values of the scattering angle $\boldsymbol{\vartheta}(\tau)$ in which the angle $\boldsymbol{\vartheta}$ enters quadratically in the argument of a sine function, i.e., this functional has a Gaussian form. In regard to the random process related to multiple scattering, in the case of amorphous material this process is well known[5,34] to be Gaussian. This means that if at the initial moment of time the distribution of particles in angles is given by a delta function $P(\boldsymbol{\vartheta},0) = \delta(\boldsymbol{\vartheta})$, then at a time τ this distribution will have the form

$$P(\boldsymbol{\vartheta},\ \tau) = (\pi q\tau)^{-1}\exp\left(-\frac{\boldsymbol{\vartheta}^2}{q\tau}\right). \tag{4.18}$$

Using this formula, one can obtain the density of the probability that the scattering angles $\boldsymbol{\vartheta}_n = \boldsymbol{\vartheta}(n\Delta)$ at a time $t_n = n\Delta$ will lie in the intervals $(\boldsymbol{\vartheta}_n, \boldsymbol{\vartheta}_n + d\boldsymbol{\vartheta}_n)$ (Ref. 45)

$$d\mathscr{P}_N = \frac{d\boldsymbol{\vartheta}_1\ldots d\boldsymbol{\vartheta}_N}{(\pi q\Delta)^N}\exp\left[-\frac{\boldsymbol{\vartheta}_1^2}{q\Delta}-\frac{(\boldsymbol{\vartheta}_2-\boldsymbol{\vartheta}_1)^2}{q\Delta}-\ldots-\frac{(\boldsymbol{\vartheta}_N-\boldsymbol{\vartheta}_{N-1})^2}{q\Delta}\right], \tag{4.19}$$

where $\Delta = \tau/N$ and $n = 1,2,\ldots,N$.

With this probability one must also average the functional $d\mathscr{E}[\boldsymbol{\vartheta}(\tau)]/d\omega$, and here one must carry out the limiting transition to $N\to\infty$. Therefore the basic expression for the averaged radiation spectrum has the form

$$\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle = \lim_{N\to\infty}\int_{-\infty}^{\infty}\ldots\int d\mathscr{P}_N\frac{d\mathscr{E}[\boldsymbol{\vartheta}(\tau)]}{d\omega}. \tag{4.20}$$

Usually this expression is written in abbreviated form as a functional integral over the Wigner measure $d_W\boldsymbol{\vartheta}(\tau)$

$$\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle = \int d_W\boldsymbol{\vartheta}(\tau)\frac{d\mathscr{E}[\boldsymbol{\vartheta}(\tau)]}{d\omega}. \tag{4.21}$$

Noting that

$$\boldsymbol{\vartheta}^2(\tau) = \frac{\partial}{\partial\mu}\exp(\mu\boldsymbol{\vartheta}^2(\tau))|_{\mu\to 0},$$

we can easily see that all terms in the integrand of (3.23) which contain both components of the vector $\boldsymbol{\vartheta} = (\vartheta_x, \vartheta_y)$ are factorized, and therefore in calculation of $\langle d\mathscr{E}/d\omega\rangle$ it is sufficient to calculate the functional integral only over one of the components of the vector $\boldsymbol{\vartheta}$. Here the average value of the radiation spectrum can be represented in the form

$$\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle = -\frac{e^2k}{\pi}\,\mathrm{Im}\int_{-\infty}^{\infty}dT \times\int_0^{\infty}\frac{d\tau}{\tau}\left[e^{-i\delta\tau}\left(\frac{1}{\varepsilon}-v^2+\frac{1}{2}\frac{\partial}{\partial\mu}\right)Q_\omega^2 - e^{-2i\omega\tau}\left(\frac{1}{\varepsilon}-v^2+\frac{1}{2}\frac{\partial}{\partial\mu}\right)Q_{-\omega}^2\right]_{\mu\to 0}, \tag{4.22}$$

where

$$Q_\omega = \int d_W\vartheta(t)\exp\left[\mu\vartheta^2(\tau)-\frac{i\omega}{\tau}\int_0^{\tau}dt\,\vartheta^2(t)+\frac{i\omega}{2\tau}\left(\int_0^{\tau}dt\,\vartheta(t)\right)^2\right]. \tag{4.23}$$

The functional integral (4.23) has a Gaussian form, and therefore it can be calculated analytically by means of the well known procedure for calculation of such integrals.[45] We give here only the result of the calculation[23,44]

$$Q_\omega = \left\{D(0)\left[1-\frac{i\omega q}{2\tau}\int_0^{\tau}dt\,D^{-2}(t)\left(\int_t^{\tau}dt'\,D(t')\right)^2\right]\right\}^{-1/2}, \tag{4.24}$$

where $D(t) = \mathrm{ch}\,r(t-\tau) + (\mu q/r)\mathrm{sh}\,r(t-\tau)$, $r = (i\omega q/2)^{1/2}$.

Substituting (4.24) into (4.22), one can easily show that with the necessary accuracy (terms proportional to γ^{-2} and $(1-\varepsilon)$ are discarded),

$$\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle = \frac{2e^2\delta}{\pi}T\left[\mathrm{Im}\left(-r\int_0^{\infty}d\tau\,\mathrm{cth}\,r\tau\cdot e^{-i\delta\tau}\right)-\frac{\pi}{2}\right]. \tag{4.25}$$

Making here the substitution of variables $z = r\tau$ and going over from integration over the complex variable z to integration over the real variable $x = \mathrm{Re}\,z$ from 0 to ∞ (Fig. 4), we shall write the final expression for the radiation spectrum of a fast particle in an amorphous medium for $\varepsilon v^2 < 1$ in the form

$$\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle = \left(\frac{d\mathscr{E}}{d\omega}\right)_0\Phi_M(s_p)\frac{1}{2\gamma^2(1-v\varepsilon^{1/2})}, \tag{4.26}$$

where $\Phi_M(s_p)$ is the Migdal function introduced in the previous section, $s_p = \delta/2\sqrt{2}|r| = (1/4)\gamma^2(1-v\varepsilon^{1/2})(\omega/\omega_{LP})^{1/2}$, and $\omega_{LP} = q\gamma^4/4$.

Equation (4.26) takes into account, in addition to mul-

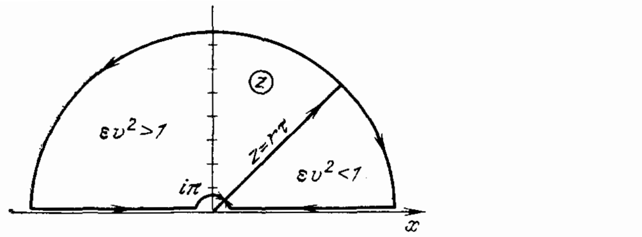

FIG. 4. Integration contours in Eq. (4.25) over the complex variable z for $\varepsilon v^2 < 1$ and $\varepsilon v^2 > 1$.

tiple scattering, the influence of the polarization of the medium on the radiation. For $\varepsilon = 1$ it goes over to Eq. (4.16). We therefore see that quantitative results concerning the influence of both multiple scattering and the polarization of the medium on the radiation can be obtained not only on the basis of the kinetic-equation method, but also on the basis of the functional-integration method.

4.6. Influence of multiple scattering on Cherenkov radiation

In derivation of Eq. (3.23) no specific dependence of the dielectric permittivity ε on the frequency ω was used, and therefore Eq. (3.23) can be used to study the radiation of fast particles in matter both for $\varepsilon v^2 < 1$ and in the case in which $\varepsilon v^2 > 1$. The first of these possibilities was discussed in the preceding sections.

Let us consider now certain features of the radiation of fast particles in an amorphous medium for $\varepsilon v^2 > 1$. In this case, as is well known,[46] Cherenkov radiation occurs. The length in which this radiation is formed in matter has macroscopic dimensions (for straight-line motion of a particle in the absence of absorption of the radiated waves this length actually is infinite; see Section 2.2), and therefore it is important to know how multiple scattering affects Cherenkov radiation.

In study of the influence of multiple scattering on the radiation of a fast particle in an amorphous medium for $\varepsilon v^2 > 1$ (and $\varepsilon \approx 1$) it is possible to use Eq. (4.25) which was obtained in the previous section. We shall immediately transform this equation to a form convenient for analysis.

For this purpose we shall carry out in (4.25) the replacement of the variable of integration τ by $z = r\tau$ and then go over from integration over the complex variable z to integration over the real variable $x = \operatorname{Re} z$ between the limits 0 and $-\infty$ (for $\varepsilon v^2 > 1$, in contrast to the case $\varepsilon v^2 < 1$, for convergence of the integral it is necessary to close the integration contour in the direction of the negative semiaxis $x = \operatorname{Re} z$; see Fig. 4). In this replacement of the integration contour it is necessary to take into account the residues at the singular points of the function $\operatorname{cth} z$. The resulting expression for the spectral density of radiation here can be written in the form[47,48]

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle = e^2 \omega v^2 \left(1 - \frac{1}{\varepsilon v^2}\right) + \frac{1}{\pi} e^2 (\omega q)^{1/2} \mathscr{F}(s_p), \qquad (4.27)$$

where

$$\mathscr{F}(s_p) = 4 s_p \left\{ \operatorname{Im}\left[\Psi\left(\frac{\beta}{2}\right) + \frac{1}{\beta} - \ln\frac{\beta}{2}\right] - 2\pi \operatorname{Re}\,(e^{-i\pi\beta} - 1)^{-1} \right\};$$

here $\beta = -2(1+i)\, s_p$ and $\Psi(x) = d \ln \Gamma(x)/dx$ is the logarithmic derivative of the Γ function.

The first term in (4.27) determines the Cherenkov radiation spectrum.[46] The second term describes the influence of multiple scattering on this radiation.

If $|s_p| \gtrsim 1$, then $\mathscr{F}(s_p) \approx -1/6|s_p|$. The correction to the spectral density of Cherenkov radiation due to multiple scattering is negative in this case.

For $|s_p| \to 0$ the function $\mathscr{F}(s_p) \to 1$ and consequently the radiation in this case turns out to be greater than Cherenkov radiation.

For arbitrary values of s_p a plot of the function $\mathscr{F}(s_p)$ is given in Fig. 5.

Therefore multiple scattering changes the spectrum of Cherenkov radiation, and there can be either a decrease or an increase of the radiation, depending on the value of the parameter s_p.

The formulas obtained are valid if the medium in which the radiation occurs is transparent, i.e., if the imaginary part of the dielectric permittivity is equal to zero. Effects associated with the absorption of the waves have been discussed in Ref. 48.

5. MULTIPLE SCATTERING OF FAST PARTICLES IN CRYSTALS

5.1. The possibility of stochastic motion of a charged particle in a crystal

We have considered the influence of multiple scattering on the radiation of a fast particle moving in an amorphous medium. Here it was shown that the effect depends substantially on the ratio between the mean square angle of multiple scattering of the particle on the coherence length and the square of the characteristic angle of its radiation, and increases with increase of the ratio of these quantities. Multiple scattering can influence the radiation of a charged particle moving not only in an amorphous medium, but also in a crystal. Here multiple scattering will occur not at individual atoms, but at groups of atoms, for example, at strings of atoms in the case of motion of a particle near a crystallographic axis.

Multiple scattering usually is associated with randomness and therefore at first glance it appears that it has no place in the motion of a particle in a crystal, which is a regular structure. In reality, however, randomness can arise even in motion in a crystal, but the reason for this will lie not in a disordered location of the atoms as occurs in an amorphous medium, but in the features of the very dynamics of the mo-

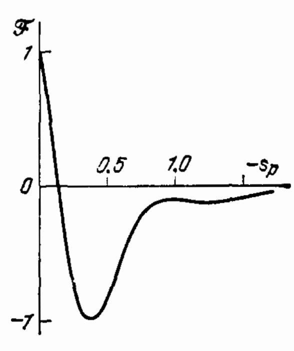

FIG. 5. Plot of the function $\mathscr{F}(s_p)$ which determines the influence of multiple scattering on Cherenkov radiation.

tion of the particle in the crystal. The point is that, as is well known,[49–51] even in particle motion in comparatively simple fields which depend on two coordinates, the motion may have a nature which is not necessarily regular, but stochastic. This question is determined entirely by the number of integrals of motion in the problem considered. For example, in the case of a system with two degrees of freedom, which has two integrals of motion, any finite motion will be regular and quasiperiodic. However, if there is only one integral, the motion will be stochastic.

Just this situation exists in the motion of a fast charged particle in a crystal at a small angle ψ to one of the crystallographic axes (the z axis). In this case it is well known[52,53] that the variation of the impact parameter between successive collisions of a particle with the atoms of the lattice is small in comparison with the impact parameter. Under these conditions the motion of the particle in the crystal is determined mainly by the continuous potential of the strings of atoms located parallel to the z axis, i.e., by the lattice potential averaged over the coordinate z

$$U(\boldsymbol{\rho}) = \frac{1}{T} \int dz \sum_k u(\mathbf{r} - \mathbf{r}_k), \tag{5.1}$$

where $u(\mathbf{r} - \mathbf{r}_k)$ is the potential energy of interaction of the particle with a crystal atom located at the point $\mathbf{r}_k$ and $\boldsymbol{\rho} = (x, y)$ are the coordinates in the plane orthogonal to the z axis.

In this averaged field the particle momentum component p_z parallel to the z axis obviously will be conserved. However, in the plane perpendicular to the z axis the motion will be determined by the equation

$$\ddot{\boldsymbol{\rho}} = -\frac{1}{E_\parallel} \frac{\partial}{\partial \boldsymbol{\rho}} U(\boldsymbol{\rho}), \tag{5.2}$$

where $E_\parallel = (p_z^2 + m^2)^{1/2}$, $E_\parallel \approx E$.

We therefore arrive at the problem of two-dimensional motion of a particle in a field $U(\boldsymbol{\rho})$.

The potential $U(\boldsymbol{\rho})$ in which the particle motion in the crystal occurs is a periodic function of the coordinates x and y, and therefore the impression may be formed that the motion in this field can be only regular and quasiperiodic. However, this is not the case. The motion of the particle in the (x, y) plane can be either regular or random. This is determined entirely by the number of integrals of motion of Eq. (5.2).

One integral of motion of (5.2) is well known.[17,52,53] This is the integral of the energy of the transverse motion

$$E_\perp = \frac{1}{2} E\dot{\boldsymbol{\rho}}^2 - U(\boldsymbol{\rho}). \tag{5.3}$$

Depending on the value of $E_\perp$, the motion of the particle can be either finite (channeling) or infinite (superbarrier motion) in the (x, y) plane.

If in addition to $E_\perp$ there is a second integral of motion, then the variables in Eq. (5.2) separate and the motion of the particle in the field $U(\boldsymbol{\rho})$ will be regular. The existence of a second integral of motion in the problem considered, however, is not at all obligatory. On the contrary, as will be shown below, very frequently the second integral of motion will be absent. The motion of a particle in a crystal in this case will be random. Therefore even in a crystal irregular motion of a particle is possible, and this conclusion applies in equal degree to the motion of both channeled and superbarrier particles.

The discussion carried out above is classical. This discussion is valid if the effective constant of interaction of the particle with the atoms of the lattice within the coherence length is large in comparison with unity, and in addition, the number of quantum states which determine the motion of the particle in the crystal is large. These conditions are satisfied at sufficiently high energies E and small values of the angles of incidence of particles onto the crystal ψ.[10,18]

5.2. Regular and irregular motions of relativistic electrons in axial channeling

In this section we shall discuss the motion of a relativistic electron in a crystal under conditions of axial channeling and shall show that it can be either regular or irregular.

The potential energy $U(\boldsymbol{\rho})$ enters into the equation of motion (5.2) and we must have a specific expression for this function. As an illustration we shall consider the motion of a relativistic electron in a silicon crystal along the $\langle 110 \rangle$ axis. Equipotential surfaces of the continuous potential energy

$$U(\boldsymbol{\rho}) = \text{const} \tag{5.4}$$

in this case are shown in Fig. 6. The numbers by the lines correspond to values of $U(\boldsymbol{\rho})$ in electron volts. In the center of a cell this quantity can be taken equal to zero. The calculations were carried out with inclusion of thermal vibrations of the atoms in the lattice, corresponding to room temperature. As the potential of an individual atom of the lattice we used a Moliere potential.[53]

The function $U(\boldsymbol{\rho})$ has deep minima at the coordinate values determining the positions of the strings of atoms in the (x, y) plane, and also saddlepoints on the straight lines joining adjacent strings. Channeled electrons in this case, depending on the value of $E_\perp$, can move in the field of either one or two strings of atoms.

A change of the nature of the electron motion is naturally expected at those values of $E_\perp$ for which the electron has the possibility of entering into a region with negative curvature of the potential energy, where its motion is unstable.[54] The curvature is clearly negative in some vicinity of the

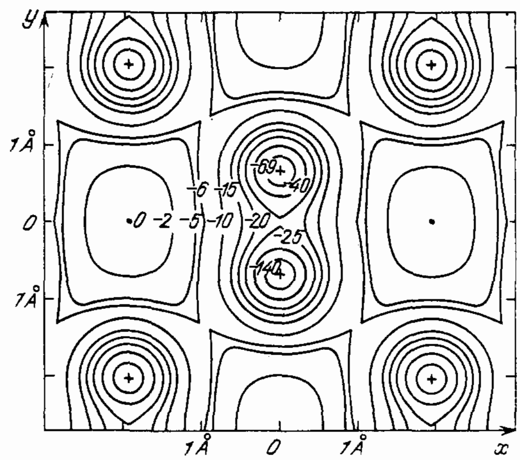

FIG. 6. Equipotential surfaces of the continuous potential energy $U(\rho)$ of interaction of an electron with a silicon crystal in motion of a particle along the $\langle 110 \rangle$ axis.

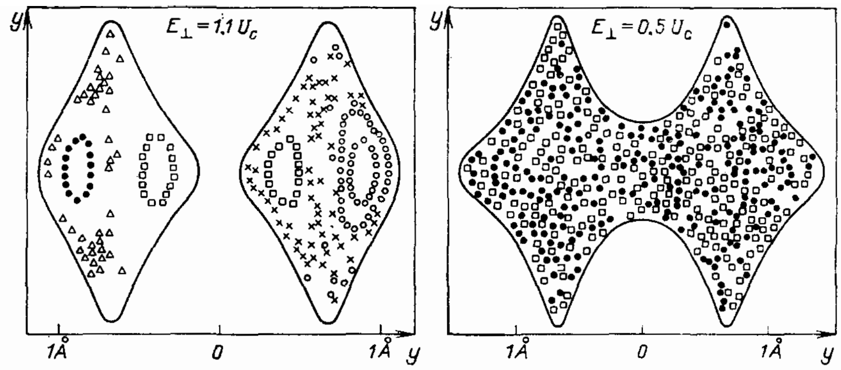

FIG. 7. Poincaré sections for axial channeling of electrons in a silicon crystal along the ⟨110⟩ axis.

saddlepoint, and therefore we shall consider the motion of an electron for transverse energies comparable with the value of the potential energy at the saddlepoint $U_c \approx -24$ eV.

In order to find out if there is a second integral of motion of Eq. (5.2) in the case considered $E_\perp \sim U_c < 0$, it is convenient to use the method of Poincaré sections.[50,51] This method is particularly effective for systems with two degrees of freedom, whose phase space is four-dimensional $(x, \dot{x}, y, \dot{y})$. In view of the conservation of the energy of the transverse motion, the phase trajectory of a particle will lie on a three-dimensional surface $E_\perp(x, \dot{x}, y, \dot{y}) = \text{const}$. Let us consider the points of intersection of the phase trajectory with an arbitrary plane, for example the plane $(y, \dot{y})$, i.e., we shall set $x = \text{const}$. Then in the case in which there is a second integral of motion J the set of consecutive intersections of the trajectory with a chosen plane will lie on some curve $y = y(\dot{y}, J)$ determined by this integral of motion. However, if there is no second integral, the points of intersection will be distributed randomly over a certain portion of this plane bounded by a separatrix. Poincaré sections can be constructed by numerical solution of the equation of motion. This problem was formulated and solved numerically in Ref. 55.

In Fig. 7 we have shown Poincaré sections $x = 0$ for $E_\perp = 1.1U_c$ and $E_\perp = 0.5U_c$. In these cases the motion of a particle will occur correspondingly in one and two potential wells. The various symbols correspond to various initial conditions, and the thin line shows the separatrix.

In Fig. 8 we have shown typical trajectories of a channeled electron in the (x, y) plane corresponding to various initial conditions with $E_\perp = 1.1U_c$ and $E_\perp = 0.5U_c$.

Our results show that at $E_\perp = 1.1U_c$, depending on the initial conditions, in addition to quasiperiodic motion there is random motion of the particle in a channel. With increase of $E_\perp$ the fraction of initial conditions for which the motion is random increases. For $E_\perp = 0.5U_c$ the phenomenon of dynamical chaos appears practically for all initial conditions. The randomization is due to instability of the trajectories of the channeled electron with respect to change of the initial conditions in the sense that a small change of the initial conditions leads to an exponential dispersal of initially close trajectories. At large times this instability leads to a motion which is perceived as random.

Let us note now some features of physical processes associated with the chaotic motion of an electron in a crystal.

First of all there is a change in the nature of the radiation under conditions of channeling. In quasiperiodic motion the electron radiation spectrum will contain sharp maxima at frequencies for which the coherence length is comparable with the length traveled by the electron during one oscillation (see for example Ref. 56). On the other hand, in chaotic motion of the electron in a channel there are no periodically repeated portions of the trajectory and consequently the radiation spectrum in this case will not contain sharp maxima.

Furthermore, in quasiperiodic motion along a trajectory of the rosette type the electron will not approach close to the nuclei of the lattice atoms. Therefore the yields of inelastic processes due to small impact parameters (nuclear reactions, scattering at large angles, and so forth) in such motion will be suppressed in comparison with the yields of the processes in random motion of the electron in a channel, when it can closely approach the nuclei. For the same reason allowance for the phenomenon of dynamical chaos should lead to a more rapid dechanneling of relativistic electrons in

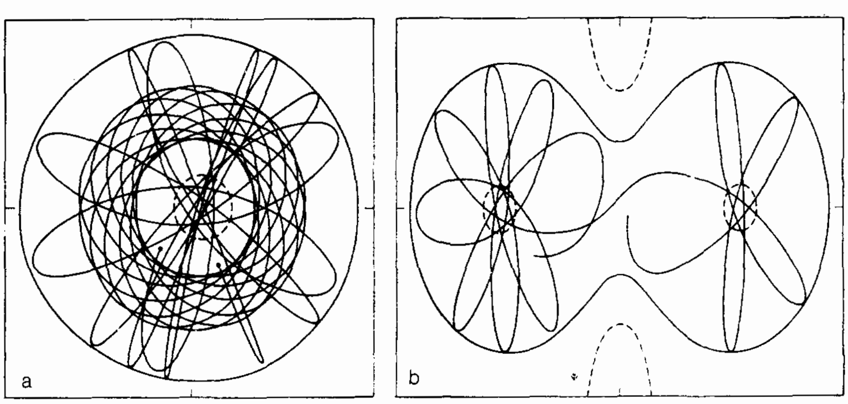

FIG. 8. Regular and chaotic trajectories of a channeled electron in the plane perpendicular to the channel axis for $E_\perp = 1.1U_c$ (a) and $E_\perp = 0.5U_c$ (b) (the dashed lines bound the regions with positive curvature of the potential energy).

comparison with the case[57] in which this phenomenon is not considered.

We note in this connection that in a crystal randomness can result also from the interaction of a particle with irregularities of the crystal-lattice potential, and also with impurities. These factors also lead to destruction of the stability of motion of channeled electrons and, in particular, to acceleration of the dechanneling of particles.[58]

5.3. Scattering of a fast charged particle by strings of crystal atoms

We have considered motion of a fast electron under conditions of channeling and have shown that its motion in the channel can be either regular or chaotic. This conclusion, however, does not apply only to channeled particles. In fact, the motion of an electron in a channel becomes unstable if its trajectory passes through a region with negative curvature of the potential energy. For superbarrier particles this condition is satisfied, and therefore their motion in the field of the continuous potential of the atomic strings of a crystal also can be either regular or irregular. The nature of the motion of a superbarrier particle in a crystal in this case can be determined by means of the Poincaré-section method in the same way as was done in the preceding section.[55,59] The difference between the two types of motions in this case lies in the following.

A fast particle under conditions of superbarrier motion encounters atomic strings located parallel to the z axis along which the motion occurs. There may or may not be correlations between successive collisions. If there are correlations, the change of the impact parameter between successive collisions of a particle with atomic strings is small in comparison with the impact parameter, so that the trajectory will change smoothly with depth of penetration of the particle into a crystal. This case corresponds to regular motion.

Absence of correlations means that the change of the impact parameter is comparable with its value. This case corresponds to irregular, chaotic motion of the particle in the crystal. Here its collisions with different strings of atoms can be regarded as random.

A large spread of the impact parameters in successive collisions obviously corresponds to a pattern of random distribution of atomic strings—the strings remain parallel to each other and penetrate the entire crystal, but the distance between them and their relative location in the plane orthogonal to the z axis is in effect random. In other words, in this case one can proceed from a picture in which the collisions of the particle occur with irregularly located but nevertheless parallel strings. The motion of the particle in the field of an individual atomic string is determined by the continuous potential of the string. This potential in what follows we shall consider to be axially symmetric.

We shall now show that the relation between the impact parameter b and its variation Δb in successive collisions with atomic strings depends on the energy of the particle and on the orientation of the crystallographic axes relative to the particle momentum.

We note first that the approximation of continuous strings, when the latter appear as objects on which the scattering occurs, has meaning only in the case in which the angle ψ between the particle momentum and the string axis is sufficiently small, $\psi \ll R/d$.[17,52,53]

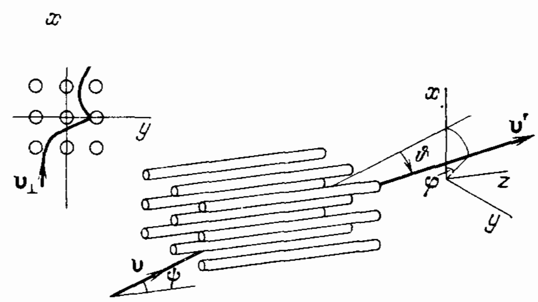

FIG. 9. Multiple scattering of a fast particle by atomic strings of a crystal.

In the field of the continuous potential of an individual atomic string $U_r(\rho)$ (potential in the form of a filament) the momentum component parallel to the string axis (the z axis) is conserved. Here scattering is possible only along the azimuthal angle φ in the plane orthogonal to the z axis. This angle is determined by the energy of transverse motion of the particle $E_\perp = E\psi^2/2$ and by the impact parameter of the string b (Refs. 17 and 60):

$$\varphi = \varphi(b) = \pi - 2b \int_{\rho_0}^{\infty} \frac{d\rho}{\rho^2} \left(1 - \frac{U_r(\rho)}{E_\perp} - \frac{b^2}{\rho^2}\right)^{-1/2}. \quad (5.5)$$

The angle of scattering of the particle by the string ϑ is related to the azimuthal angle φ by the equation

$$\vartheta = 2\psi \sin \frac{\varphi}{2}. \quad (5.6)$$

As a consequence of scattering by different strings of atoms there is a redistribution of the particles in the angles φ (Fig. 9). It is clear that correlations between the collisions of particles with strings of atoms can appear only in the case in which a particle is moving near one of the crystallographic planes, in which the strings are located periodically. We shall denote by α the angle between the particle momentum and this plane (Fig. 10). Then the change of the impact parameter will obviously be given by the relation

$$\Delta b \sim \frac{d_y}{\psi} \max(\alpha, \vartheta), \quad (5.7)$$

where d_y is the distance between strings in the (y,z) plane.

We are interested in the case in which $\Delta b \ll b$. This condition will occur only if $\alpha \ll (b/d_y)\psi$ and $|\varphi| \ll 1$. The latter inequality is valid for $E_\perp \gg |U_r|$. For such values of $E_\perp$, according to Eqs. (5.5) and (5.6),

$$\vartheta \approx \psi\varphi, \quad \varphi(b) = \frac{1}{2E_\perp} \frac{d}{db} \int_{-\infty}^{\infty} dy U_r((b^2 + y^2)^{1/2}). \quad (5.8)$$

Since scattering occurs mainly at $b \sim R$, the inequality $\Delta b \ll b$ will be valid under the conditions

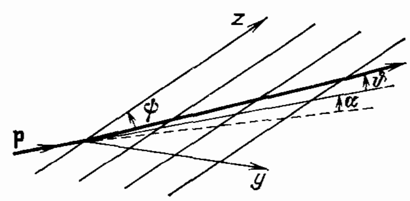

FIG. 10. The angles ψ and α which determine the orientation of crystallographic axes relative to the particle momentum.

$$\alpha \ll \frac{R\psi}{d_y}, \quad \psi_c\left(\frac{d}{R}\right)^{1/2} \ll \psi, \tag{5.9}$$

where $\psi_c = (2U_0/E)^{1/2}$ is the critical angle of axial channeling[52,53] and $U_0 = 2Ze^2/d$ (U_0 determines in order of magnitude the value of the potential energy $U_r(\rho)$ for $\rho \sim R$).

When these conditions are satisfied the particle trajectory will change smoothly in successive collisions with strings of atoms. The trajectory in this case will be determined by the continuous potential of the plane near which the motion occurs[52,53]:

$$\ddot{x} = -\frac{1}{E}\frac{\partial}{\partial x}U_p(x), \quad U_p(x) = \frac{1}{T_y}\int_{-\infty}^{\infty} dy U(\rho), \tag{5.10}$$

where x is the coordinate orthogonal to the (y,z) plane near which the motion occurs, T_y is the linear dimension of the crystal along the y axis, and $U(\rho)$ is the potential energy given by Eq. (5.1).

In this way we have arrived at the one-dimensional problem of motion of a particle in the field $U_p(x)$. It is clear that the motion in such a field will be regular.

If even one of the conditions (5.9) is violated, then in successive collisions of a particle with strings of atoms a substantial spread in the impact parameters will occur, $\Delta b \gtrsim b$. Here the collisions of a particle with different strings can be treated as a random process. We emphasize that we are everywhere here discussing the region of high energies of the incident particles. The motion of a particle with respect to atomic strings, as we see, can have either a regular or a random nature, and which of these possibilities occurs will depend on the angles of incidence of the particle with respect to crystallographic axes and planes and, generally speaking, on the initial conditions. On violation of the conditions (5.9) the interaction of a particle with strings can be considered as a random process and can be described by means of the kinetic equation.

We shall denote by $f(\varphi,z)$ the distribution function of particles in a crystal in the azimuth angle φ at depth z. This quantity changes as a consequence of scattering of particles by atomic strings in accordance with the following kinetic equation[61,62]:

$$\frac{\partial}{\partial z} f(\varphi, z) = n\, d\psi \int_{-\infty}^{\infty} db\,[f(\varphi + \varphi(b), z) - f(\varphi, z)]. \tag{5.11}$$

The solution of this equation with inclusion of the boundary condition $f(\varphi,0) = \delta(\varphi)$, where $\delta(\varphi)$ is a delta function, is a rather complicated function of the angle φ

$$f(\varphi, z) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \cos k\varphi \times \exp\left[-zn\, d\psi \int_{-\infty}^{\infty} db\,(1 - \cos\varphi(b))\right]. \tag{5.12}$$

Substantial simplifications appear at $\psi \gg \psi_c$ when the particle trajectory in the plane orthogonal to the z axis is close to a straight line. The distribution of the particles in angle in this case is Gaussian,[17,62]

$$f(\varphi, z) \approx (2\pi q_c z)^{-1/2} \exp\left(-\frac{\vartheta^2}{2q_c z}\right), \tag{5.13}$$

where $\vartheta \approx \psi \cdot \varphi$, $q_c = \Psi^2\bar{\varphi}^2$ is the mean square scattering angle of the particle in the crystal per unit length,

$$\overline{\varphi^2} = n\psi d \int_{-\infty}^{\infty} db\,\varphi^2(b), \tag{5.14}$$

and $\varphi(b)$ is given by Eq. (5.8).

If the potential of an individual atom of the crystal is taken as a screened Coulomb potential, then, as can easily be shown,

$$q_c = 4\pi^2 Z^2 e^4 n E^{-2}\frac{R}{\psi d}. \tag{5.15}$$

Comparing this quantity with the mean square multiple scattering angle q of the fast particle per unit length in an amorphous medium, we find that

$$\frac{q_c}{q} \approx \frac{\pi}{2\ln(183Z^{-1/3})}\frac{R}{\psi d}. \tag{5.16}$$

Therefore, over a wide range of the angles ψ ($\psi_c \ll \psi \ll R/d$—the condition of applicability of Eq. (5.16)) the mean square angle of multiple scattering of a fast particle in a crystal by atomic strings considerably exceeds the mean square multiple scattering angle in an amorphous medium. For this reason the influence of multiple scattering on the radiation in a crystal can be significantly greater than in an amorphous medium, and consequently the Landau-Pomeranchuk effect in the case of a crystal can appear much more intensely. This opens up new possibilities for investigation of the Landau-Pomeranchuk effect with the aid of contemporary accelerators.

6. INFLUENCE OF MULTIPLE SCATTERING ON THE RADIATION FROM ULTRARELATIVISTIC PARTICLES IN CRYSTALS

6.1. Influence of multiple scattering on coherent radiation at small azimuthal scattering angles

Multiple scattering of particles leads, as we have seen, to a suppression of bremsstrahlung at high energies in amorphous materials. It also leads to suppression of coherent radiation in crystals. In order to describe this effect in crystals it is necessary to average the general formula (3.23) for the spectral density of radiation over the scattering angles. In a crystal, in contrast to an amorphous medium, scattering occurs only in the azimuthal angle φ (see Fig. 9). If this angle is small ($\bar{\varphi}^2 l_c \ll 1$), then according to Eq. (5.13) the distribution of particles in angle is Gaussian. We shall begin the discussion with this case, which is the simplest.

If the azimuthal scattering angle φ is small, then the particle scattering angle ϑ which enters into Eq. (3.23) (see Fig. 9) will be given by $\vartheta \approx \psi\varphi$. The scattering in the angles ϑ therefore will also be Gaussian. Here the probability density that the scattering angles in the crystal $\vartheta_n = \psi\varphi(n\Delta)$ at a moment of time $\tau_n = n\Delta$, where $n = 1,2,\ldots,N$, $\Delta = \tau/N$, and $N \gg 1$, will lie in the intervals $(\vartheta_n, \vartheta_n + d\vartheta_n)$ is given by the relation[45]

$$d\mathscr{P}_N = \frac{d\vartheta_1 \ldots d\vartheta_N}{(2\pi q_c \Delta)^{N/2}} \exp\left[-\frac{\vartheta_1^2}{2q_c\Delta} - \ldots - \frac{(\vartheta_N - \vartheta_{N-1})^2}{2q_c\Delta}\right]. \tag{6.1}$$

Using this expression, we can represent the average value of the radiation spectrum of a fast particle in a crystal in the form of a functional integral[23,44]

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle_c = \lim_{N\to\infty} \int_{-\infty}^{\infty} \ldots \int d\mathscr{P}_N \frac{d\mathscr{E}\,[\vartheta(\tau)]}{d\omega} = \int d_W \vartheta(\tau)\, \frac{d\mathscr{E}\,[\vartheta(\tau)]}{d\omega}. \qquad (6.2)$$

Equation (6.2) differs from the corresponding formula for an amorphous medium (4.21) by the fact that in a crystal the scattering angle $\vartheta(\tau)$ has only one component, whereas in an amorphous medium $\boldsymbol{\vartheta}(\tau)$ contains two components. The average scattering angles in a crystal and an amorphous medium, as is shown by Eq. (5.6), can differ considerably. It can be concluded from this that the formula for the average value of the radiation spectrum in a crystal will differ from the formula for the average value of the radiation spectrum in an amorphous medium in that in Eq. (4.22) it is necessary to make the replacements $Q^2_{\pm\omega} \to Q_{\pm\omega}$ and $q \to 2q_c$. Therefore we arrive at the following expression for $\langle d\mathscr{E}/d\omega \rangle_c$ (Refs. 23 and 44):

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle_c = \frac{d\mathscr{E}_{\mathrm{coh}}}{d\omega}\, \Phi(s_c)\, \frac{1}{2\gamma^2(1-v\varepsilon^{1/2})}, \qquad (6.3)$$

where

$$\frac{d\mathscr{E}_{\mathrm{coh}}}{d\omega} = \frac{2e^2}{3\pi}\, \gamma^2 q_c T$$

is the corresponding result of the theory of coherent radiation of a fast charged particle in a crystal,[5,17]

$$\Phi(s_c) = 48 s_c^2 \left\{ -\frac{\pi}{4} + \int_0^{\infty} dx\, \frac{\exp(-2s_c x)}{(x\, \mathrm{sh}\, x)^{1/2}} \times \left[\sin 2s_c x + \frac{1}{2}\, \frac{\mathrm{ch}\, x - 1}{\mathrm{sh}\, x} (\sin 2s_c x + \cos 2s_c x) \right] \right\} \qquad (6.4)$$

and $s_c = (1/4)\gamma^2(1 - v\varepsilon^{1/2})(\omega/\omega_c)^{1/2}$; $\omega_c = q_c\gamma^4/2$.

Equation (6.3) determines the effect both of multiple scattering and of the polarization of the medium on the intensity of the coherent radiation of a fast particle in a crystal for $\varepsilon v^2 < 1$.[5)]

At small and large values of s_c the function $\Phi(s_c)$ has the following asymptotic forms:

$$\Phi(s_c) \approx \begin{cases} 1, & s_c \gg 1, \qquad (6.5a) \\ 6\eta s_c, & s_c \ll 1, \qquad (6.5b) \end{cases}$$

where

$$\eta = \int_0^{\infty} dx\, x^{-1/2} (\mathrm{ch}\, x - 1)(\mathrm{sh}\, x)^{-3/2} \approx 1.33.$$

Therefore for $s_c \gg 1$ Eq. (6.3) goes over into the corresponding result of the theory of coherent radiation of relativistic particles at atomic strings of the crystal with inclusion of the effect of polarization of the medium on the radiation

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle_c = \frac{d\mathscr{E}_{\mathrm{coh}}}{d\omega}\, \frac{1}{2\gamma^2(1-v\varepsilon^{1/2})}. \qquad (6.6)$$

For $s_c \to 0$ Eq. (6.3) gives a more accurate value of the coefficient in the corresponding result of Ref. 12, which was found on the basis of qualitative estimates. In this limiting case, according to Eq. (6.5b),

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle_c \approx \frac{d\mathscr{E}_{\mathrm{coh}}}{d\omega} \left(\frac{\omega}{\omega_c} \right)^{1/2}. \qquad (6.7)$$

We see that for $s_c \ll 1$ ($\omega \ll \omega_c$) there is a significant suppression of the coherent radiation due to multiple scattering.

For arbitrary values of s_c the function $\Phi(s_c)$ is given in Fig. 3. The curves in this figure show that the functions $\Phi(s_c)$ and $\Phi_M(s)$ are very close to each other. On the other hand, the variables s_c and s for given E and ω can differ considerably, and therefore the conditions at which a change in the nature of the radiation occurs are different in a crystal and in an amorphous medium.

Let us compare now the main characteristics of the radiation of fast particles in a crystal and in an amorphous medium in the low-frequency region.

We note first of all that the quantity $d\mathscr{E}_{\mathrm{coh}}/d\omega$ is related to the spectral density of radiation of a particle in an amorphous medium (4.5) by the equation

$$\frac{d\mathscr{E}_{\mathrm{coh}}}{d\omega} = \frac{q_c}{q}\, \frac{d\mathscr{E}_{\mathrm{BH}}}{d\omega}. \qquad (6.8)$$

In the range of angles ψ of interest to us ($\psi_c \ll \psi \ll R/d$), according to Eq. (5.16), we have $q_c \gg q$, and therefore $\mathscr{E}'_{\mathrm{coh}} \gg \mathscr{E}'_{\mathrm{BH}}$.

In the frequency region $\omega \gg \omega_p$, as is well known,[3] the dielectric permittivity is given by the relation $\varepsilon = 1 - (\omega_p^2/\omega^2)$, where $\omega_p = [4\pi n Z e^2/m)^{1/2}$ is the plasma frequency. The quantity $2\gamma^2(1 - v\varepsilon^{1/2})$ which enters into Eqs. (4.26) and (6.3) in this region of frequencies can be written in the form

$$2\gamma^2(1 - v\varepsilon^{1/2}) \approx 1 + \frac{\gamma^2\omega_p^2}{\omega^2}.$$

Equations (4.26) and (6.3) show that both in an amorphous medium and in a crystal the polarization of the medium affects the radiation in the frequency region $\omega \lesssim \gamma\omega_p$. In addition, multiple scattering in an amorphous medium and in a crystal affects the radiation respectively at $\omega \lesssim \omega_{LP}$ and $\omega \lesssim (2q_c/q)\omega_{LP}$. For $\psi_c \ll \psi \ll R/d$, according to Eq. (5.16), we have $q_c \gg q$, and therefore the change in the character of the radiation in a crystal occurs at lower energies and in the region of higher frequencies than in an amorphous medium.

In Fig. 11 we have shown the results of a calculation of the radiation spectrum of electrons with $E = 1$ and 10 GeV in an amorphous medium (dashed curves) and in a crystal (solid curves) in the case in which the beam enters a tungsten crystal at an angle $\psi = 2$ mrad to the $\langle 100 \rangle$ crystallographic axis. As the potential of an individual atom of the medium in the calculations we used a screened Coulomb potential.

Our results show that in emission of radiation in a crystal the frequency region in which multiple scattering has an

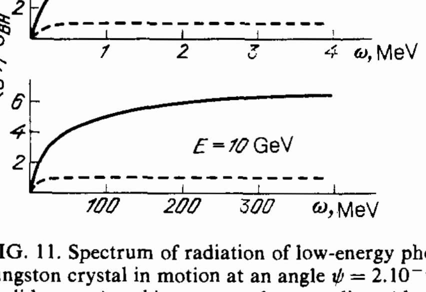

FIG. 11. Spectrum of radiation of low-energy photons by electrons in a tungsten crystal in motion at an angle $\psi = 2.10^{-3}$ rad to the $\langle 100 \rangle$ axis (solid curves) and in an amorphous medium (dashed curves).

important effect on the radiation is significantly larger than the corresponding frequency region for an amorphous medium. It is also important that at not very high particle energies in an amorphous medium the effect of multiple scattering on the radiation cannot be discussed independently of the influence of the polarization of the medium, whereas in a crystal such a treatment is possible. For this purpose it is necessary to satisfy the conditions

$$\omega_{\rm LP} \ll \gamma\omega_{\rm p} \ll \frac{2q_{\rm c}}{q}\,\omega_{\rm LP}.$$

These inequalities, in particular, are satisfied in the case in which an electron with $E = 1$ GeV is moving in a tungsten crystal at an angle $\psi = 2$ mrad to the $\langle 100\rangle$ axis. For $E = 10$ GeV, as can easily be verified, the conditions

$$\omega_{\rm LP} < \omega_{\rm LP} \ll \frac{2q_{\rm c}}{q}\,\omega_{\rm LP}$$

are satisfied and consequently for this energy there is a frequency region in which, both in a crystal and in an amorphous medium, the influence of multiple scattering on the radiation can be discussed independently of the influence of the polarization of the medium.

Therefore in the motion of ultrarelativistic particles in a crystal the suppression of the coherent radiation due to multiple scattering (the analog of the Landau-Pomeranchuk effect of suppression of bremsstrahlung in an amorphous medium) can appear at significantly lower particle energies than in an amorphous medium.

Up to this point we have been discussing dynamic stochasticity which arises in motion of a particle in a given potential. However, in a crystal there is also another kind of stochasticity due to thermal vibrations of the atoms in the lattice. It also leads to a change of the radiation spectrum—to a small decrease (by 10–25%) of the incoherent part of the radiation cross section in comparison with the Bethe-Heitler result. In what follows we shall not discuss this question. It was first investigated by Ter-Mikaelyan[1] and has been treated in detail in his book.[5] The influence of channeling and superbarrier motion of fast particles in a crystal on the incoherent part of the radiation cross section has been investigated in Refs. 64 and 65.

6.2. Influence of multiple scattering on coherent radiation at large azimuthal scattering angles

In the preceding section we have considered the influence of multiple scattering on the coherent radiation of fast particles in a crystal at small values of the azimuthal scattering angle. In passage of a particle through a crystal the conditions can also be such that the characteristic values of the azimuthal angles of its scattering by atomic strings will not be small in comparison with unity. In this case the scattering will not be a Gaussian process, and therefore a special study is required. Such an investigation can be carried out in the general case if the radiation has a dipole nature, i.e., if the condition $\gamma^2\overline{\vartheta}_l^2 \ll 1$ is satisfied.[6)]

The spectral density of the radiation from a relativistic particle in the dipole approximation is given by Eq. (4.1). We shall be interested in what follows in the radiation in the frequency region for which the coherence length $l_c = 2\gamma^2/\omega$ is large in comparison with the length $2R/\psi$ in which the acceleration of a particle in collision with each atomic string is nonzero. In this frequency region the quantity $|\mathbf{w}(\nu)|^2$ which enters into Eq. (4.1) is determined by the relation (4.2), in which we understand the angle $\boldsymbol{\vartheta}_k$ to be the scattering angle in the collision with the kth string and t_k is the moment of the collision.

The relation (4.2) must be averaged over the scattering angles $\boldsymbol{\vartheta}$. As before, we shall be interested in the radiation in the case in which the collisions of the particle with various strings are random. Taking into account that the scattering in the collision with each string involves a change of the azimuthal angle φ (see Eq. (5.5)), we obtain after averaging over φ (Refs. 44 and 66)

$$\langle|\mathbf{w}|^2\rangle = 2\psi^2 \sum_{m,k} \cos\nu(t_m - t_k)\left(\langle\cos\varphi\rangle^{|m-k|} - \langle\cos\varphi\rangle^{|m-k+1|}\right), \tag{6.9}$$

where

$$\langle\cos\varphi\rangle = (\bar{a})^{-1}\int db\,\cos\varphi\,(b)$$

and $\bar{a}$ is the average distance between strings, $\bar{a} = (nd)^{-1/2}$.

Note that in an amorphous medium after averaging $|\mathbf{w}|^2$ over angles, the double sum over the scattering centers (4.2) has gone over to a single sum. As a result of this, the dependence of $\langle|\mathbf{w}|^2\rangle$ on t_k has dropped out. However, in the case of a crystal the double sum after averaging over angles is still present and consequently the dependence of $\langle|\mathbf{w}|^2\rangle$ on t_k is retained. Therefore Eq. (6.9) must still be averaged over the random moments of time of the particle collision with atomic strings t_k.

The procedure for averaging Eq. (6.9) over t_k has been described in detail in Ref. 44 and we shall not dwell on this question here, but shall only give the resulting expression for the average value $|\mathbf{w}|^2$ on the assumption that the target thickness T is significantly greater than the coherence length l:

$$\langle|\mathbf{w}|^2\rangle = 4T\psi^2\frac{\langle\sin^2(\varphi/2)\rangle}{\bar{\tau}}\,\frac{\nu^2}{\nu^2 + 4(\langle\sin^2(\varphi/2)\rangle/\bar{\tau})^2}, \tag{6.10}$$

where $\overline{\tau} = \bar{a}/\psi$ is the average time of free travel of the particle between successive collisions with atomic strings. Here the average value of the spectral density of radiation takes the following form:

$$\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle_{\rm c} = T\,\frac{e^2\psi\omega_{\rm d}}{3\pi\gamma^2(1-v\varepsilon^{1/2})}\,F(x), \tag{6.11}$$

where $\omega_d = 4\gamma^2\langle\sin^2(\varphi/2)\rangle/\overline{\tau}$, $x = (2\omega/\omega_d)(1 - v\varepsilon^{1/2})$, and

$$F(x) = x\left(\frac{3}{2} - x^2\right)\operatorname{arcctg} x + 3x^2\left[1 - \frac{1}{2}\ln(1 + x^{-2})\right]. \tag{6.12}$$

This formula is valid for arbitrary values of the azimuthal scattering angle. It is required only that the conditions $l \gg 2R/\psi$ and $\gamma^2\,\overline{\vartheta_l^2} \ll 1$ be satisfied.

Let us consider some limiting cases of Eq. (6.11) for the case $\omega \gtrsim \gamma\omega_p$.

The function $F(x)$ at small and large values of x has the following asymptotic behavior:

$$F(x) \approx \begin{cases} 1 - \dfrac{7}{20x^2} + \ldots, & x \gg 1, \quad (6.13\text{a}) \\ \dfrac{3}{4}x(\pi + 2x\ln(ex^2) + \ldots), & x \ll 1. \quad (6.13\text{b}) \end{cases}$$

The argument of this function in the frequency region of interest can be written in the form $x = (\omega/\omega_d)[1 + (\gamma^2\omega_p^2/$

$\omega^2)$]. From the latter relation and also from the asymptotic behavior (6.13) it follows that the spectral density of radiation from a fast particle in a crystal will depend substantially on the relation between the frequencies ω, $\gamma\omega_p$, and ω_d.

If the condition $\omega_d \ll \gamma\omega_p$ is satisfied, then according to (6.11) and (6.13a) we have

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle_c = T \frac{2e^2\psi^2\omega_d}{3\pi\,[1+(\gamma^2\omega_p^2/\omega^2)]}. \tag{6.14}$$

If in addition to the condition $w_d \ll \gamma\omega_p$ the inequality $\psi \gg \psi_c$ is satisfied, then this formula goes over into the corresponding result of the theory of coherent radiation (6.6) with inclusion of the influence of the polarization of the medium on the radiation.

The frequency ω_d depends on the ratio between ψ and ψ_c. The maximum of ω_d is achieved at $\psi \lesssim \psi_c$, i.e., when the characteristic values of the azimuthal scattering angles are comparable with unity. In this region of the angle ψ we have in order of magnitude $\omega_d \sim 4\gamma^2 ndR\psi_c$. Comparing this value of ω_d with $\gamma\omega_p$, we find that

$$\left(\frac{\omega_d}{\gamma\omega_p}\right)^2 \sim 4\gamma ndR^2.$$

This relation shows that for $\psi \lesssim \psi_c$ in the region of sufficiently large γ the inequality $\omega_d \gg \gamma\omega_p$ can always be satisfied. In this case there are three regions of the frequency ω in which the radiation differs substantially: $\omega > \omega_d$, $\omega_d > \omega > \gamma\omega_p$, and $\gamma\omega_p > \omega$.

In the frequency region $\omega > \omega_d$ (as before we assume satisfaction of the condition $l \gg 2R/\psi$) Eq. (6.11) goes over into Eq. (6.14). In this frequency region multiple scattering and polarization of the medium do not have an effect on the radiation. The radiation here is determined only by the features of the particle interaction with the field of an individual atomic string.

For $\omega_d > \omega > \gamma\omega_p$, according to Eqs. (6.11) and (6.13b),

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle_c = \frac{1}{2} T e^2 \omega \psi^2. \tag{6.15}$$

In this frequency region multiple scattering of a particle by atomic strings leads to a rapid decrease of the spectral density of the radiation with decrease of the frequency of the radiated photon.

We note that Eq. (6.15) can be obtained easily from the approximate formula (3.16). In fact, for $\psi \lesssim \psi_c$, according to Eqs. (6.6) and (5.12), we have $\overline{\vartheta^2} \approx 2\psi^2$.[17] Substituting this relation into Eq. (3.16), we arrive at the dipole approximation with accuracy up to the numerical coefficient in Eq. (6.15).

Comparing this result with the corresponding result for an amorphous medium (3.18), we see that in motion in a crystal multiple scattering of a particle by atomic strings has a significant influence on the radiation not only for $\gamma^2\overline{\vartheta^2} \gg 1$, as was the case in an amorphous medium, but also in the case in which $\gamma^2\overline{\vartheta_l^2} \ll 1$. This is due to the violation of the condition of a Gaussian distribution of the particles in the crystal in angle for $\psi \lesssim \psi_c$.

For $\omega < \gamma\omega_p$ Eq. (6.11) goes over to Eq. (6.14). In this frequency region the polarization of the medium has a significant effect on the radiation of a particle in a crystal.

The formulas obtained in this section are valid if the condition of dipole radiation of the particle in the crystal is satisfied, $\gamma^2\overline{\vartheta_l^2} \ll 1$. For $\psi \sim \psi_c$ we have in order of magnitude $\overline{\vartheta_l^2} \sim \psi_c^2$ and the inequality $\gamma^2\psi_c^2 \ll 1$ leads to a restriction on the particle energy E. Note in this connection that there is an interval of energies E in which the conditions $\gamma\psi_c \ll 1$ and $\gamma\omega_p \ll \omega_d$ are satisfied simultaneously. Specifically, these inequalities are satisfied if $(4ndR^2)^{-1} \ll \gamma \ll md/4Ze^2$. Therefore there is an interval of the energies E in which multiple scattering of a fast particle in a crystal considerably affects its radiation if the condition of dipole radiation is satisfied.

6.3. Influence of multiple scattering on radiation in thin layers of matter

The results obtained up to this time have concerned the case in which the target thickness is large in comparison with the coherence length: $T \gg l$. However, the length in which the radiation of a fast particle in matter is formed increases rapidly with increase of the particle energy and with decrease of the frequency of the radiated photon. Therefore at sufficiently high energies E and small ω and T one can have the condition $T \ll l$, in which the radiation is formed in a region larger than the target thickness T. We shall show that multiple scattering in this case, like the case $T \gg l$, can have a substantial influence on the radiation.[11,30,40]

The spectral density of radiation for $T \ll l$, according to Eq. (3.12), is determined only by the angle of scattering of the particle by the target ϑ. The angles of scattering of different particles are different, and therefore Eq. (3.12) must be averaged over the distribution $f(\vartheta)$ of particles emerging from the target in angle,

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle = \int d\vartheta\, f(\vartheta)\, \frac{d\mathscr{E}(\vartheta)}{d\omega}. \tag{6.16}$$

Note that for large target thicknesses $T \gg l$ we carried out an averaging over the particle scattering angles inside the target. On the other hand, in the case being considered the averaging is carried out over the scattering angles at the exit from the target.

For values of the mean square scattering angle of the particle $\overline{\vartheta^2} = \int d\boldsymbol{\vartheta} f(\vartheta)\boldsymbol{\vartheta}^2$ large and small in comparison with γ^{-2}, Eq. (6.16) has the following asymptotic behavior:

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle = \frac{2e^2}{3\pi} \begin{cases} \gamma^2\overline{\vartheta^2}, & \gamma^2\overline{\vartheta^2} \ll 1, & (6.17\text{a}) \\ 3\ln\gamma^2\overline{\vartheta^2}, & \gamma^2\overline{\vartheta^2} \gg 1. & (6.17\text{b}) \end{cases}$$

The equations (6.17) show that for small and large values of the parameter $\gamma^2\overline{\vartheta^2}$ the radiation differs substantially. This is due to the fact that the phases of the waves radiated by an electron in directions close to the momenta of the incident and scattered particles are opposite, and therefore, depending on how these waves interfere with each other, the radiation will also be different. The interference of these waves is determined by the parameter $\gamma\vartheta$ (see Section 3.2). Equation (6.16) is valid both for radiation in an amorphous medium and in a crystalline medium. The difference between the radiation spectra in these cases will be due only to the specific form of the distribution function $f(\vartheta)$ over which the averaging is carried out.

In an amorphous medium the distribution of the particles in angle is Gaussian with a mean square multiple scattering angle (3.17) proportional to the target thickness T. Equation (6.17a) in this case gives with logarithmic accuracy the Bethe-Heitler result (4.5).

In motion in a crystal at a small angle ψ to one of the crystallographic axes, as was shown in Section 5.3, over a wide range of the angles ψ ($\psi_c \ll \psi \ll R/d$) the average values of the scattering angles are substantially greater than in an amorphous medium. Equation (6.17a) in this case gives a result which coincides with the corresponding result of the theory of coherent radiation (6.8), according to which in the low-frequency region the intensity of the radiation from electrons in a crystal $\mathscr{E}'_{\rm coh}$ significantly exceeds (in order of magnitude by $R/4\psi d$ times) the intensity of radiation $\mathscr{E}_{\rm BH}$ in an amorphous medium. Therefore the enhancement of the radiation from an electron in a crystal in comparison with that in an amorphous medium is due to the increase in the average scattering angles of the particle in a crystal in comparison with the average scattering angles in an amorphous medium.

With increase of the target thickness, the condition $\gamma^2\overline{\vartheta^2} \ll 1$ is violated, and in a crystal this condition is violated much more rapidly than in an amorphous medium. For $\gamma^2\overline{\vartheta^2} \gg 1$, according to Eq. (6.17b), the intensity of radiation is practically independent of T. This means that for $\gamma^2\overline{\vartheta^2} \gg 1$ the spectral density of the radiation essentially does not depend on the number of collisions of the particle with atoms of the medium, i.e., in this case there is a suppression of the radiation (bremsstrahlung in an amorphous medium, and coherent radiation in a crystal) from fast particles in a thin layer of matter. Note that Eq. (6.17b) differs from the corresponding formulas (4.7) and (6.7) which describe the suppression of radiation in a thick layer of amorphous or crystalline material. Equations (4.7) and (6.7) depend substantially on T, E, and ω ($\mathscr{E}'_{\rm LP} \sim \langle\mathscr{E}\rangle'_c \sim T\omega^{1/2}/E$), whereas in (6.17b) these dependences are essentially absent.

In Fig. 12 we have given the dependence on the target thickness T of the radiation spectrum from ultrarelativistic electrons in a thin layer ($T \ll l$) of amorphous and crystalline material. The calculations were carried out by means of Eq. (6.16) on the assumption that the beam enters a silicon crystal at an angle $\psi = 0.5$ mrad to the $\langle 111\rangle$ axis (curve 1) and a disoriented crystal (amorphous medium, curve 2).

These results show that in interaction of particles with a crystal more favorable conditions can be created for study of the suppression of radiation from relativistic-particles in a thin layer of matter than in the interaction with an amorphous target. Specifically, the suppression of coherent radiation appears at smaller values of T and E and in a larger range of frequencies ω than does the suppression of bremsstrahlung.

We have discussed up to now the case in which the angles of incidence of particles onto the crystal are large in comparison with the critical channeling angle. However, the equations (6.17) can be used also for $\psi \lesssim \psi_c$. It is required only that the condition $T \ll l$ be satisfied. The average values of the square of the angle of scattering of the particle by the crystal in this case are equal in order of magnitude to ψ_c^2.[52,53] Here the formulas (6.17), which are valid for $T \ll l$, differ substantially from the formulas for the radiation spectrum of channeled particles in the case $T \gg l$. In the latter case the radiation spectrum will depend on T and ω ($\mathscr{E}'_{\rm ch} \sim T\omega$; see Refs. 17 and 67), whereas for $T \ll l$ these dependences do not occur.

We mention in conclusion that the condition $T \ll l$ can be satisfied in a number of experiments similar to those carried out in recent years in study of the radiation of channeled particles of ultrahigh energies in thin crystals.[68–70] For this it is required only that one consider radiation in crystals thinner than the crystals used in the experiments of Refs. 68–70 and that one investigate the radiation at lower frequencies than the characteristic frequencies of channeled-particle radiation; in the experiments of Refs. 68–70 the principal attention was devoted to study of the radiation at these characteristic frequencies. In particular, in channeling of positrons with $E = 20$ GeV through a silicon crystal of thickness $T = 100\,\mu$m (the conditions of the experiments of Refs. 68 and 70) the requirement $T \ll l$ is satisfied in the frequency region $\omega \lesssim 5$ MeV. However, with use of thinner crystals the frequency region in which the condition $T \ll l$ is satisfied is extended.

We mention in conclusion that at the present time extensive studies are being carried out on the interaction of relativistic particles with crystals also from the experimental point of view. The data obtained are in qualitative agreement with the results set forth in the present review. We note some of these results.

The measured orientation dependences of the angular distributions of particles scattered by a crystal indicate that over a wide range of angles of incidence of particles on a crystal relative to crystallographic axes which are closely packed with atoms, scattering occurs mainly along the azimuthal angle (Fig. 13), and that the average values of the

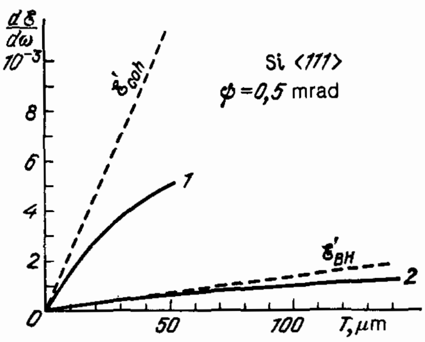

FIG. 12. Spectral density of radiation of high-energy electrons $\langle d\varepsilon/d\omega\rangle$ in a thin layer of crystalline (1) and amorphous (2) material as functions of the target thickness.

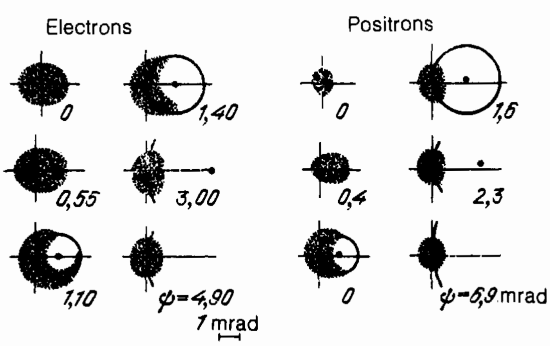

FIG. 13. Angular distributions of electrons and positrons with energy 1 GeV scattered in a silicon crystal of thickness 185 μm for various values of the angles of incidence ψ of the particles onto the crystal with respect to the $\langle 110\rangle$ axis.[71] The points are the direction to the axis.

particle-scattering angles in the crystal are substantially greater than the average scattering angles in an amorphous medium.[71,72]

The orientation dependences of the spectral and polarization characteristics of the radiation indicate that in passage of ultrarelativistic electrons and positrons through crystals, conditions can be created in which correlations appear between successive collisions of a particle with atomic strings, and also conditions in which there are no correlations.[5,68-70,73-75] The former of these possibilities is realized in passage of particles through a crystal along closely packed atomic strings of crystallographic planes and shows up in the existence of sharp peaks in the spectral and angular distributions of the radiation. A typical example of spectral distributions of radiation in this case is shown in Fig. 14, parts *E* and *F*.

The second possibility is realized in motion at a small angle to one of the crystallographic axes but far from crystallographic planes which are closely packed with atoms (see Fig. 14, Part B). In this case in the low-frequency region the radiation substantially exceeds the radiation of a particle in an amorphous medium, but there are no sharp peaks in the radiation spectrum. There are also experimental indications of the existence of the suppression of coherent radiation of fast particles at atomic strings of a crystal in the low-frequency region (Fig. 14B and Fig. 15).

It must be mentioned, however, that we are speaking only of qualitative agreement of the results mentioned above, and also of many other experiments, with the results of the theory which has been set forth. This is due to the fact that the experiments, as a rule, have been carried out with certain additional restrictions which are complicated to take

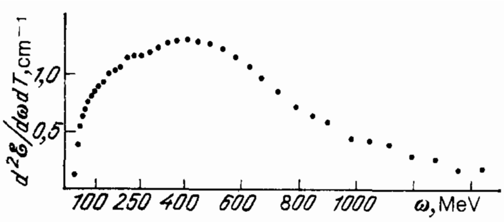

FIG. 15. Radiation spectrum of 4.5-GeV electrons entering a diamond crystal of thickness 1.7 mm at an angle $\psi = 0.52$ mrad to the $\langle 100\rangle$ axis.[76]

into account in a quantitative theoretical analysis. For example, the orientation dependence of the average scattering angles of electrons in a crystal was measured[72] under the condition that the particle radiate a photon in passing through the crystal, whereas in passing through a thin crystal most of the particles of the incident beam pass through the target without radiation. In the experiments of Refs. 71, 72, 74, and 76 the disorientation of the crystallographic axes with respect to the incident beam was accomplished within one of the crystallographic planes, so that in disorientation by relatively large angles (ψ of the order of several critical channeling angles ψ_c) the particles can execute both random and regular motion relative to atomic strings.

This arrangement of the experiments is due to the fact that the principal attention up to the present time has been devoted to study of the interaction of particles with crystals under conditions of channeling (both axial and planar channeling) and no special experiments have been set up to study the interaction of superbarrier particles with crystals. The fact is, as we have shown above, that in the interaction of superbarrier particles with atomic strings many important and interesting effects should appear at ultrahigh energies. In addition, in recent years experimental data have appeared which indicate that even in passage of ultrahigh-energy electrons through crystals along crystallographic axes there is very rapid dechanneling of particles, i.e., the transition of the particles from subbarrier to superbarrier states (for electrons with 1 GeV traversing a silicon crystal the dechanneling length is several tens of microns[77,78]), and therefore in passage of electrons through crystals whose thickness exceeds the dechanneling length the contribution of superbarrier particles to physical processes can be not only significant, but also dominant. For these reasons it is desirable to carry out goal-oriented experiments on study of the features of the interaction of superbarrier particles with atomic strings of a crystal. Such experiments are required both for a quantitative check of the theory and for discovery of the conditions under which superbarrier particles will play a dominant role in interactions.

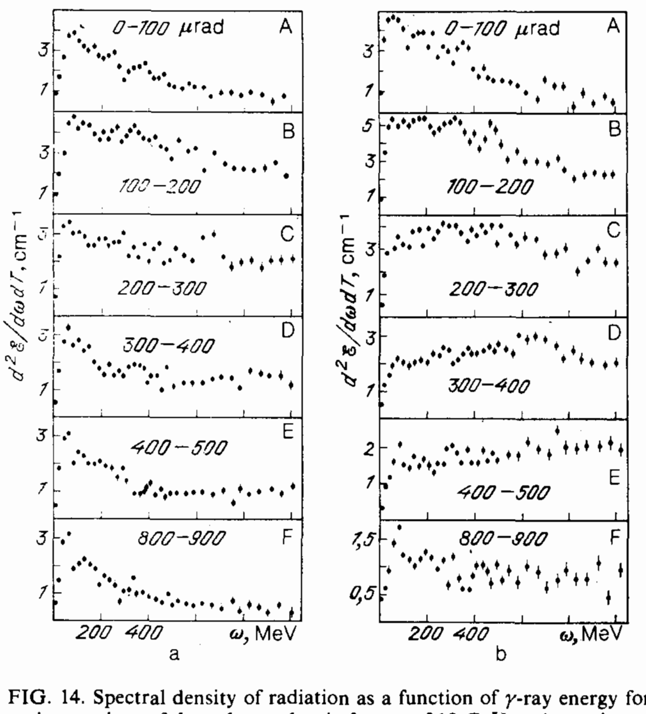

FIG. 14. Spectral density of radiation as a function of γ-ray energy for various regions of the polar angles ψ of entry of 10-GeV positrons into a silicon crystal relative to its $\langle 111\rangle$ axis.[74] The crystal thickness is 113 μm. The limits of the angular regions in μrad are shown in the plots. We have selected positrons entering the crystal near the (110) plane (a) and near the (112) plane (b).

1)The idea of a semibare electron was introduced by E. L. Feinberg[33] in studying the evolution in time of the state vector of the (electron + photon) system after scattering of an electron by an atom at a large angle.

2)Here and in what follows it is assumed for simplicity that ε is close to unity.

3)The influence of the polarization of the medium on the radiation of a fast particle in an amorphous medium was first established by Ter-Mikaelyan.[36]

4)This formula differs by the factor in front of the sine function from the corresponding formula of Ref. 2 (see the review in Ref. 17).

[5)]For $\varepsilon = 1$ a similar result has been obtained recently[63] by means of the kinetic-equation method.

[6)]This condition corresponds to smallness of the transverse displacement of the radiating particle in the coherence length $l\vartheta_l$ in comparison with the radiated wave length λ.[35]

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