Charmed and beautiful particles

M. A. Shifman

Institute of Theoretical and Experimental Physics, Moscow
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The present status of the theory of charmed and beautiful hadrons is briefly reviewed. The opportunities presented by these particles for the elucidation of fundamental questions, such as the number of generations of quarks and leptons and the mechanisms responsible for CP violation, are examined. Particular attention is devoted, within the framework of quantum chromodynamics, to the structure of hadrons with heavy quarks.

CONTENTS

1. INTRODUCTION

The leading edge of research in high-energy physics is advancing swiftly, leaving behind islands and even whole archipelagos of unsolved problems. Indeed, quantum chromodynamics itself is the clearest example of this. Only six or seven years ago, the theory of hadrons was regarded as problem number one, and some believed that its solution was imminent. However, time has passed, hopes were born and died, and the chreished goal—the creation of a complete theory of color trapping—was not reached. Other topics arose and developed in parallel, and the principal interests of theorists have gradually shifted to new promising fields such as supersymmetry, supergravity, and superstrings.

The result of all this has been that, having assumed their place in the extensive family of hadrons, charmed and beautiful particles have receded well into the theoretical rear.[1] The question therefore is whether further efforts toward understanding of their properties are now justified.

In my view, the answer is in the affirmative for three reasons. First, these particles are being investigated and will continue to be investigated experimentally for a long time. The potential importance of particular measurements can only be established theoretically. Second, some of the data—and I shall touch upon them later—can throw light on very fundamental questions lying outside the framework of the theory of hadrons, e.g., the number of generations of quarks and leptons. Finally, the third reason is more subjective. In the same way that nuclear physics became a separate discipline in the 1950s, quantum chromodynamics is becoming an independent subject, with its own specific methods, and continues to grow and develop. Since the confinement problem has not been solved, each new task in hadron physics is actually a kind of new adventure with an unknown outcome: will a reliable prediction emerge? Will it yield something new that will help us to understand the dynamics of large separations? And so on. Success is far from guaranteed, and it is precisely this uncertainty that, in the final analysis, makes this subject so interesting to theorists.

Charmed and beautiful particles occupy a special position in hadron physics because, on the one hand, the volume of available data on them is rapidly expanding and, on the other, we have an additional parameter, namely, the mass of the heavy quark m_Q:

$$m_{c,b,t} \gg \Lambda_{QCD}, \qquad (1)$$

where Λ_{QCD} is the parameter determining the behavior of the quark-gluon coupling constants.

In this review, we shall touch upon only a few of the questions that are frequently encountered in current literature. We shall confine our attention to hadrons containing only one c- or b-quark, leaving to one side the question of t-hadrons and hadrons containing two or more heavy quarks. If we use Q to denote a heavy quark (c, or b) and q to denote a light quark (u, d, s), the mesons and baryons that will be of interest to us will be of the form $Q\bar{q}$ and Qqq, respectively. In a channel with fixed quantum numbers, there are three charmed and three beautiful mesons[2]

$$D^0 = c\bar{u},\ D^+ = c\bar{d},\quad F = c\bar{s}, \qquad (2)$$

$$B_u = \bar{b}u,\ B_d = \bar{B}d,\ B_s = \bar{b}s; \qquad (3)$$

and we can construct the fifteen S-wave baryons whose quark composition and parameters are listed in Table I.

We open our review with a brief excursion into the processes and phenomena that could provide us with information about fundamental aspects of the theory. The extensive section headed "Charmed and beautiful hadrons in the chromodynamic kitchen" is devoted to the properties of mesons and baryons that are determined by the strong interaction. We shall summarize the achievements of the last two or three years under the following headings:

a) calculation of the mass spectrum and certain amplitudes from the QCD sum rules
b) inclusive nonleptonic widths (lifetimes)
c) meson annihilation constants f_D and f_B
d) exclusive hadronic decays of D- and F-mesons
e) strong and radiative decays of excited states.

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TABLE I. Baryons with one charmed quark.

Particle	Quark composition	Mass, MeV
Λ_c^+	$((ud)_{J=0}\, c)_{J=1/2}\, +$	2281
$\Sigma_c^0, \Sigma_c^+, \Sigma_c^{++}$	$((dd)_{J=1}\, c)_{1/2}\, +,\ ((ud)_{J=1}\, c)_{1/2}\, -$ $((uu)_{J=1}\, c)_{1/2}\, +$	2445 Preprint ITEP-51, 1986
$\Sigma_c^{*0}, \Sigma_c^{*+}, \Sigma_c^{*++}$	$((dd)_{J=1}\, c)_{3/2}\, +,\ ((ud)_{J=1}\, c)_{3/2}\, +,$ $((uu)_{J=1}\, c)_{3/2}\, +$	?
$\Xi_c^{0\,(A)},\ \Xi_c^{+\,(A)}$ *)	$((ds)_{J=0}\, c)_{1/2}\, +,\ ((us)_{J=0}\, c)_{1/2}\, +$	2460
$\Xi_c^{0\,(s)},\ \Xi_c^{+\,(s)}$ **)	$((ds)_{J=1}\, c)_{1/2}\, +,\ ((us)_{J=1}\, c)_{1/2}\, +$	?
$\Xi_c^{*0},\ \Xi_c^{*+}$	$((ds)_{J=1}\, c)_{3/2}\, +,\ ((us)_{J=1}\, c)_{3/2}\, +$	?
Ω_c^0 ***)	$((ss)_{J=1}\, c)_{1/2}\, +$	2740
Ω_c^{*0}	$((ss)_{J=1}\, c)_{3/2}\, +$	?

*)Old designation A^0, A^+.
**)Old designation S^0, S^+.
***)Old designation T.

2. FUNDAMENTAL ASPECTS

It has frequently been noted that the physics of c- and b-quarks is a potential source of indirect information about unknown interactions and particles, and about new properties of known objects. By studying decays, we can impose limits on:

—the existence of a fourth generation of quarks and leptons

—the existence of new particles, e.g., squarks, Higgs bosons, and so on

—the mass of the τ-neutrino

—the mechanism responsible for CP invariance violation.

Searches for the charged Higgs particle $H^\pm$ in the decay of the b-quark are probably the best known example of this. If this object exists with mass $M_H \leqslant m_b - m_c \sim 3$ GeV and coupling constant of the normal order of magnitude ($G_F^{1/2} m_b$), the b-quark will decay wholly via the semiweak process $b \to H^- c$ instead of the normal weak transition $b \to c\bar{u}d$ (see, for example, the review in Ref. 73):

$$\frac{\Gamma(b \to H^- c)}{\Gamma(b \to c\bar{u}d)} \sim \frac{6\pi^2}{G_F m_b^2} \sim 10^5. \tag{4}$$

Existing data on B-meson decays can, on the whole, be satisfactorily described in terms of the standard picture that excludes the existence of H^- with a coupling constant that is even lower (by an order of magnitude) than $G_F^{1/2} m_b$. We note in passing that, with a little bit of creative imagination, one can see in the above data a hint of the leakage of b-quarks into some unknown channels at the level of 20% of the total width (see Section 3.3).

Turning now to charmed hadrons, we must first recall the $D \to \tau\nu_\tau$ and $F \to \tau\nu_\tau$ decays, whose probabilities are sensitive to the mass of the ν_τ if this mass is of the order of 10 MeV (Refs. 3 and 4). The point is that the energy released in these decays is about 85 and 190 MeV, respectively, so that M_{ν_τ} of this magnitude would result in an appreciable reduction in the phase volume. The relative probabilities of these decays are:

$$\begin{aligned} \mathrm{Br}\,(D \to \tau\nu_\tau) &\approx 2.8\cdot 10^{-4} \left(\frac{f_D}{f_\pi}\right)^2, \\ \mathrm{Br}\,(F \to \tau\nu_\tau) &\approx 7.5\cdot 10^{-3} \left(\frac{f_F}{f_\pi}\right)^2, \end{aligned} \tag{5}$$

where f_D is the constant characterizing the annihilation of the D-meson by the corresponding axial current

$$\langle 0 \mid \bar{c}\gamma_\mu\gamma_5 d \mid D \rangle = i f_D p_\mu \tag{6}$$

(f_F is defined by analogy). The constant f_D is the analog of the well-known constant f_π ($f_\pi = 133$ MeV) that is frequently encountered in pion physics. Calculations of f_D will be discussed in Section 3.1. Here, we merely note that modern ideas suggest that[30,31]

$$f_D = 160 - 170 \text{ MeV}. \tag{7}$$

The mass of the τ-neutrino can be determined by measuring, say, the ratio[4] $\Gamma(D \to \tau\nu_\tau)/\Gamma(D \to \mu\nu_\mu)$. It is believed that this method produces the best upper limit, namely, $M_{\nu_\tau} \lesssim 10$ MeV, which is an improvement, by an order of magnitude, on the previous results.

Another important problem for researchers interested in D mesons is the direct measurement of the ratio $|V_{cd}/V_{cs}|$, where $V_{qq'}$ are the corresponding elements of the Kobayashi-Maskawa matrix. Within the framework of the present-day picture, i.e., three generations and very weak mixing of the third generation with the first and second,[3)] we may expect that

$$\left|\frac{V_{cd}}{V_{cs}}\right| = \left|\frac{V_{us}}{V_{ud}}\right|, \tag{8}$$

and this equation should be satisfied to within $O(10^{-3})$. The right-hand side of (8) is known with very good precision because the elements V_{ud} and V_{us} determine weak decays of the "old" hadrons, e.g., $\pi \to \mu\nu$ and $K \to \mu\nu$.

On the other hand, the ratio $|V_{cd}/V_{cs}|$ can be determined from the width ratio

$$\frac{\Gamma(D\to\rho e\nu)}{\Gamma(D\to K^* e\nu)} \quad \text{or} \quad \frac{\Gamma(D\to\pi e\nu)}{\Gamma(D\to K e\nu)}. \tag{9}$$

Accurate determination of $|V_{cd}/V_{cs}|$ from (9) would demand that, in both cases, theoretical analysis should take into account effects due to $SU(3)_{fl}$ violation, both in the phase volume (this is a trivial exercise) and in the matrix elements (a problem that is less trivial).

A deviation from (8) by more than 10^{-3}, if it is found, would be evidence for the existence of a fourth generation of quarks and leptons, exhibiting quite considerable mixing with the first three generations.[5]

I know of no other fundamental aspects that are accessible to investigation in charmed hadrons. In particular, CP-violation effects in the system $D^0 \leftrightarrow \bar{D}^0$ are expected to be exceedingly small[7,8] because $\Delta M_D/\Gamma \ll 1$ in the standard model. Neither the possible existence of new quark generations nor the existence of supersymmetric partners with known particles would affect this conclusion. The obvious reason is that the principal decay modes of the D^0-mesons are Cabibbo-allowed, whereas the $D^0 \leftrightarrow \bar{D}^0$ transitions are necessarily Cabibbo-forbidden.[7,8]

In this sense, the situation in the physics of b-hadrons would seem to be the most favorable. There are two neutral B-mesons, namely, B_d^0 and B_s^0, and both systems are natural probes for exploring the structure of the theory at distances less than or approximately equal to m_t^{-1}, where m_t is the mass of the (still undiscovered) t-quark.

Both the $B_d^0 \leftrightarrow \overline{B_d^0}$, $B_s^0 \leftrightarrow \overline{B_s^0}$ mixing and the CP-invariance violation in these systems have been frequently discussed in detail.[9-21] The key theoretical parameter determining the magnitude of all the effects is

$$x = \frac{\Delta M}{\Gamma}, \tag{10}$$

where ΔM is the off-diagonal element of the matrix and Γ is the total width of the meson. We note that, in contrast to the usual picture in the case of the K^0 meson, in which the K_{0L} and K_{0S} lifetimes are essentially different, the width difference can be neglected for the B-mesons. (We shall see in Section 3.3 that the difference between the lifetimes of all the b-hadrons is definitely not more than about 5%).

In the standard model, the main contribution to the $B^0 \leftrightarrow \overline{B^0}$ amplitude is provided by the square diagrams, such as those of Fig. 1a; they are, in fact, saturated by the contribution due to the t-quark. In the highest-order approximation in m_t^2, the graphs of Fig. 1a lead to the effective Lagrangian

$$\begin{gathered}\mathcal{L}_{eff} \sim G_F^2 m_t^2 (\bar{b}\Gamma_\mu q)(\bar{b}\Gamma_\mu q),\\ \Gamma_\mu = \gamma_\mu(1+\gamma_5), \quad q = d,\ s.\end{gathered} \tag{11}$$

We shall not pause to calculate the coefficient in (11) because this question has been examined in the literature (see, however, Section 3.4). This coefficient is determined by the elements V_{tb}, V_{ts}, V_{td} of the Kobayashi-Maskawa matrix and includes the so-called η-factors[4] that take into account renormalization due to hard gluons. If we use the factorization procedure for the matrix element over the states of the B^0, $\overline{B^0}$ (we shall return to this question in Section 3.4) then

$$x_{B_d} = \frac{\Delta M_B}{\Gamma} \approx 32\pi \frac{f_B^2 m_t^2}{m_b^4} \frac{|(V_{tb}^* V_{td})^2|}{|V_{cb}|^2} \frac{B_{sl}}{Z_c}, \tag{12}$$

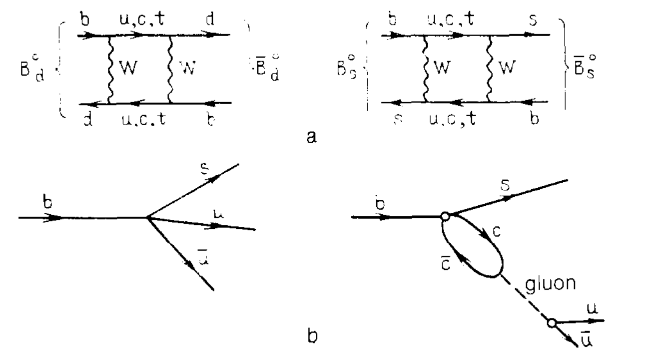

FIG. 1. a—Amplitudes of $B \leftrightarrow \bar{B}$ oscillations in the standard model. b—Interference between these two diagrams contains the CP-odd term which leads to a nonzero width difference $\Gamma(b\to u\bar{u}s) - \Gamma(\bar{b}\to u\bar{u}\bar{s})$.

where, to be specific, we give the result for B_d^0. In this expression, $B_{sl} \simeq 0.11$ is the relative probability of the semileptonic decay of the B-mesons and $Z_c = 0.3$–0.4 represents the suppression of semileptonic decays by the phase volume and gluonic corrections. It is also assumed that $m_t^2/M_W^2 \ll 1$ since, otherwise, we would have to take into account effects related to the W-boson propagators in Fig. 1a. The corresponding additional factor is equal to 3/4 for $m_t^2 = M_W^2$ and 1/4 for $m_t^2 \gg M_W^2$.

The numerical value of the parameter x_{B_d} is subject to considerable uncertainty, largely due to the uncertainty in V_{td}. If $m_t \sim 40$ GeV, it would appear that

$$x_{B_d} \sim 10^{-2}, \tag{13}$$

and the effect of mixing and oscillations due to mixing is very small in the standard model.

The situation changes radically when we turn to B_s^0. The parameter x_{B_s} is then obtained from (12) by making the replacement $V_{td} \to V_{ts}$. We note that, for three generations, $|V_{tb}| \simeq 1$, $|V_{ts}/V_{cb}| \simeq 1$, and, consequently, the predictions for x_{B_s} are practically free of any dependence on the mixing angles. Numerically,

$$x_{B_s} \approx \left(\frac{f_{B_s}}{130\ \text{MeV}} \frac{m_t}{40\ \text{GeV}}\right)^2 \sim 1. \tag{14}$$

We must now say a few words about how the effects of $B^0 \leftrightarrow \overline{B^0}$ mixing could be observed experimentally. The case that is most frequently discussed in the literature is the following. Consider the creation of the $B^0\overline{B^0}$ pair in e^+e^--annihilation, followed by the semileptonic decay of each meson. We then have

$$b\bar{d} \to l^-\nu + \text{hadrons}, \quad \bar{b}d \to l^+\nu + \text{hadrons}. \tag{15}$$

If oscillations are turned off, then

$$e^+e^- \to B^0\bar{B}^0 \to l^+l^- + \ldots, \tag{16}$$

and we have the creation of a pair of leptons of different sign. When oscillations are turned on, the members of the lepton pairs have the same sign. This means that the yield of leptons of the same sign, n_{++} or n_{--}, is a measure of these oscillations. Quantitatively, in the case of e^+e^--annihilation,[23]

$$r_B = \frac{n_{++}+n_{--}}{n_{+-}} = \frac{x^2}{2+x^2}. \tag{17}$$

For small values of x, the quantity $r^B = O(x^2)$ is a measure of the fact that the created B^0 or $\overline{B^0}$ meson decays on a time scale that is shorter than the characteristic time of the oscillations. As $x \to \infty$,

$$r_B \to 1,$$

which is also very reasonable.

The CP-invariance violation leads to the fact that $A(B^0 \to \overline{B^0}) \neq A(\overline{B^0} \to B^0)$ and, consequently, $n_{++} \neq n_{--}$. However, it is well-known that, in contrast to oscillations, the CP-odd asymmetry

$$a = \frac{n_{++} - n_{--}}{n_{++} + n_{--}}$$

decreases with increasing m_t. Standard-model estimates show that

$$a \sim \begin{cases} 6\cdot 10^{-3} \left(\frac{40\ \mathrm{GeV}}{m_t}\right)^2 & \text{for } B_d^0, \\ 4\cdot 10^{-4} \left(\frac{40\ \mathrm{GeV}}{m_t}\right)^2 & \text{for } B_s^0. \end{cases} \tag{18}$$

One of the reasons for the suppression (18) is the kinematic factor $\sim m_c^2/m_t^2$, which appears unavoidably in the expression for a.

It is interesting that CP-violation leads to a difference between the inclusive widths of the b- and $\bar{b}$-quarks when we consider b and $\bar{b}$ decays into quarks with a given flavor, e.g.,

$$\Delta \equiv \Gamma(b \to u\bar{u}s) - \Gamma(\bar{b} \to \bar{u}u\bar{s}).$$

The fact that Δ is not zero is due to interference between the diagrams of Fig. 1b. The result for Δ was obtained in 1979 by Voloshin and Okun'. As proposed for the observable CP-odd effect, it is proportional to the parameter

$$\gamma = \cos\theta_1 \cos\theta_2 \cos\theta_3 \sin^2\theta_1 \sin\theta_2 \sin\theta_3 \sin\delta,$$

i.e., to the product of all three mixing angles in the Kobayashi-Maskawa matrix. (Voloshin and Okun' considered the standard model of CP-violation). Quantitatively, they show that

$$\Delta \sim \alpha_s(m_b) \frac{G_F^2 m_b^5}{192\pi^3} \gamma \sim 3\cdot 10^{5}\ \mathrm{s}^{-1}.$$

The relative correction Δ amounts to a few percent of the Cabibbo-forbidden modes of the form $b \to c\bar{c}d$.

The chances of finding CP-violating effects are probably improved if one studies exclusive decays of the B^0- and $\overline{B^0}$-mesons to some common final state f. Because of interference between the amplitudes

$$\begin{aligned} B^0 \to f:&\quad (B^0 \to f) + (B^0 \to \bar{B}^0 \to f), \\ \bar{B}^0 \to f:&\quad (\bar{B}^0 \to f) + (\bar{B}^0 \to B^0 \to f) \end{aligned} \tag{19}$$

there may be a difference between, say, the reaction cross sections

$$\begin{aligned} &\sigma(e^+e^- \xrightarrow{\Upsilon(4S)} B^0\bar{B}^0 \to f + (l^- + \ldots)), \\ &\sigma(e^+e^- \xrightarrow{\Upsilon(4S)} B^0\bar{B}^0 \to f + (l^+ + \ldots)). \end{aligned} \tag{20}$$

The most convenient final states in (19) and (20) that have been discussed in the literature are

$$f = J/\psi\, K_S^0,\ \rho K,\ f^+f^-,\ \text{etc.}$$

To stop the reader from gaining the erroneous conclusion that e^+e^--annihilation plays a unique part in searches for CP-violation, we note that alternative proposals have also been put forward. For example, Azimov *et al.*[20] have suggested the use of reactions in Σ^--beams. The point is that, in the incident-particle fragmentation region, the process Σ^- + nucleus → hadrons results mostly in the creation of s-quarks and, consequently, $s\bar{b}$-mesons but not $\bar{s}b$-mesons. This produces "labeled" B^0-mesons, and CP-violation manifests itself in oscillations in the yield of, say ρK as a function of the range of the B_s^0.

Although the relative probabilities of the above exclusive channels are small (the $B_s^0 \to \rho K$ decay is Cabibbo-forbidden), estimates of CP-violation effects range from 10 to about 100% (Refs. 16–20).

We note that the presence of a fourth generation may sharply alter the picture of CP-nonconservation in B_d^0 and, especially, in B_s^0-mesons.[24]

To conclude this section, I emphasize that I have presented a bird's-eye view of the subject, having deliberately omitted certain points that would have been useful in a review of $B\bar{B}$-oscillations or the problem of CP-violation. My aim here has been to illustrate the thesis formulated above about the importance of c- and b-hadrons as sources of indirect information about the overall structure of quark-lepton theory. A more technical discussion of this range of questions, and all the references to the very extensive original literature, can be found in the excellent specialist review by Azimov *et al.*[21]

3. CHARMED AND BEAUTIFUL HADRONS IN THE CHROMODYNAMIC "KITCHEN"

We now turn through 180° and consider hadrons containing c- and b-quarks, within the framework of the QCD. In other words, these hadrons are of interest to us not so much as instruments for studying other interactions, but in themselves: we shall discuss their structure, mass spectrum, decay properties, and so on. Nor is it the case that problems of this type are more "quiescent" and traditional in character. Many intersting results have been obtained in this area during the last year years, and I shall emphasize those advances that have not been adequately covered in other reviews. As usual my choice of the "basic" topics is subjective, and the order of the presenation relatively random. A further point of difference, as compared with the last Section, is that I shall examine in some detail certain relatively specialist questions, and will include in my discussion a number of technical points as well.

3.1. Annihilation constants f_D and f_B

The annihilation constants parametrize matrix elements of the form

$$\langle 0 \mid \bar{q}\Gamma Q \mid \text{meson}\rangle, \tag{21}$$

where Γ is a γ-matrix structure that secures the quantum numbers of the current $j_\Gamma = \bar{g}\Gamma Q$. From the point of view of applications, the most important is the coupling between the pseudoscalar mesons D, F, and B and the axial current[6)]

$$\langle 0 \mid \bar{q}\gamma_\alpha\gamma_5 Q \mid P\rangle = i f_P p_\alpha, \tag{22}$$

where P represents a pseudoscalar meson and p_α is its momentum. The current $\bar{q}\gamma_\alpha\gamma_5 Q$ is not conserved and, in contrast to the pion axial current, this nonconservation is large:

$\partial_\alpha j_{\alpha 5} = im_Q \bar{q}\gamma_5 Q$. Hence, when we determine the constant f_P, we must actually indicate the normalization point for the current $j_{\alpha 5}$ because the current operator evolves as the normalization point μ changes. If we use the analogy with the pion case, and introduce the constant f_P via the $P \to \mu\nu$ decay, it is not difficult to see that the amplitude for this decay literally includes the operator $j_{\alpha 5}$ normalized to a high point, $\mu \sim M_W$, so that the more precise form of (22) is

$$\langle 0 \mid [\bar{q}\gamma_\alpha\gamma_5 Q]_{\mu\sim M_W} \mid P\rangle = if_P p_\alpha.$$

When μ varies from M_W to m_Q, the operator $j_{\alpha 5}$ is not renormalized in the logarithmic approximation, but further reduction in μ introduces an additional logarithmic factor (see below).

The constant f_P is the analog of the celebrated pion constant f_π ($f_\pi = 133$ MeV) that determines the width $\Gamma(\pi \to \mu\nu)$ and appears in all relationships in low-energy pion physics. We note that f_P appears not only in widths of the form, $\Gamma(P \to \tau\nu)$, but also in the solution of many other problems, e.g. in the amplitude for $B^0 \leftrightarrow \overline{B^0}$ oscillations, which is proportional to f_B^2 [see (12)]. The pre-asymptotic corrections to the lifetime of mesons $Q\bar{q}$ and baryons Qqq contain $16\pi^2 f_P^2/m_Q^2$ (see Section 3.3).

The literature contains a whole spectrum of predictions for $f_{D,B}$, ranging from 100 to 500 MeV, and many authors who used this constant for purely utilitarian purposes, i.e., for substituting it into some particular formula, experience difficulties in choosing its numerical value (see, for example, the review by Thorndike[25]). It seems that the most reliable and accurate results for $f_{D,B,F}$ ensue from the QCD sum rules.[26] I shall not pause to consider early work[27–29] which made use of a number of approximations that tended to increase the theoretical uncertainties. The present situation is as follows. There are two independent publications,[30,31] in which a special analysis of f_D is carried out with allowance for the maximum number of terms in the sum rules and the minimum number of additional approximations. The predictions (160 and 170 MeV, respectively) are in excellent agreement with one another to within the theoretical uncertainty that amounts to 10–15 MeV. We shall therefore take as our starting point the value

$$f_D = 165 \pm 15 \text{ MeV}. \tag{23}$$

In the limit of $SU(3)_{fl}$ symmetry, the annihilation constants for the D- and F-mesons are identical, but comparison of f_π and f_K shows that the $SU(3)_{fl}$ violation may amount to about 20%. The F-mesons were studied in the context of the QCD sum rules in Ref. 32, where it was found that

$$f_F = 200 \pm 15 \text{ MeV}. \tag{24}$$

Although the question of the normalization point for the current $_{\alpha 5}$ was not explicitly discussed in Refs. 30–32, the method employed clearly shows that the results refer to $\mu \sim m_c$. The footnote to (22) indicates that these results are valid, at least in the leading logarithmic approximation (LLA), for $[j_{\alpha 5}]_{\mu\sim M_W}$. In other words, (23) and (24) are essentially estimates of physical constants for decays of the form $D \to \tau\nu$ and $F \to \tau\nu$. We note that calculations of $f_{D,F}$ based on nonrelativistic models provide the residues for the current normalized at the low point $\mu \sim R^{-1}$, where R is the confinement radius. Hence, to transform to the physical decay constants in this case, we must include the additional logarithmic factor representing the evolution of μ from R^{-1} to m_c (see below).

We now turn to the system of B-mesons. The limit $m_q = 0$, i.e., B_d- and B_u-mesons, has not been considered in the literature[28–31] devoted to the analysis of f_B. According to Ref. 31, $f_B \simeq 130$ MeV, whereas Zhitnitskiĭ *et al.*[30] find that $f_B \simeq 90$ MeV. However, terms $O(\alpha_s)$ are not considered in Ref. 30 in the theoretical part of the sum rules, and, according to Ref. 31, they increase the final result by 10%. There is therefore general agreement that

$$f_{B_{u,d}} = 115 \pm 15 \text{ MeV}. \tag{25}$$

When we pass from D-mesons to $B_{d,u}$-mesons, the annihilation constant is reduced by a factor of about 1.5. It is instructive to examine whether this behavior is consistent with theoretical expectations. We shall assume in our discussion that the masses m_c and m_b are large enough for the asymptotic dependnence on m_Q^{-1} to be valid.

In the limit as $m_Q \to \infty$, the quark Q becomes a fixed center of force with the light quark q "smeared out" around it. The size of the region in which this "smearing" takes place, is obviously independent of m_Q and is wholly determined by the confinement radius. If we use this picture, we can readily show that[33]

$$(f_P)_{\text{nonrel}} \sim m_Q^{-1/2}, \tag{26}$$

where the factor $m_Q^{-1/2}$ is essentially kinematic in character. Shuryak[29] has carried out a detailed analysis of sum rules in the $m_Q \to \infty$ limit. He found that

$$|\langle 0 \mid j_{\alpha 5} \mid P\rangle|^2 = 12n M_P, \quad n = |\psi(0)|^2, \tag{27}$$

and estimated the value n of the wavefuction at the origin which, as expected, was independent of m_Q in the limit as $m_Q \to \infty$.

In the nonrelativistic approach, f_P^2 is proportional to the quark annihilation probability with characteristic virtuality of the same order as for the mesons $Q\bar{q}$.[7] More formally, (27) must be rewritten in the form

$$(f_P^2)_{\text{nonrel}} M_P^2 = |\langle 0 \mid [j_{\alpha 5}]_{\mu\sim R^{-1}} \mid P\rangle|^2 = 12n M_P.$$

If the current that appears in the definition of the constants is normalized to a high point ($\mu \sim m_Q$), these (physical) annihilation constants contain the additional logarithmic dependence on m_Q

$$f_P \sim m_Q^{-1/2} (\alpha_s(m_Q))^{-2/b}, \tag{28}$$

where b is the first coefficinet in the Gell-Mann–Low function

$$b = \frac{11}{3} N_c - \frac{2}{3} N_f.$$

The existence of logarithmic corrections to (26) in the leading logarithmic approximation was recently discovered in Ref. 34. A series of the form $[\alpha_s \ln(m_Q^2 R^{-2})]^k$ appears due to the exchange of gluons with virtual momenta $R^{-2} \lesssim p^2 \lesssim m_Q^2$ (Fig. 2), where the logarithms have the relatively unusual "hybrid" structure and are both infrared and ultraviolet simultaneously. The momentum range $R^{-2} < p^2 < m_Q^2$ is ultraviolet for the light quark and infrared for the heavy quark. Direct evaluation of the graphs of Fig. 2

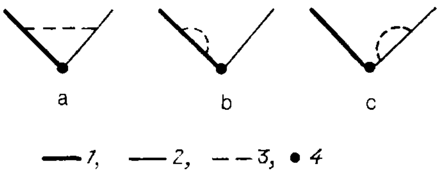

FIG. 2. Single-gluon correction to the current $\bar{Q}\Gamma q$ containing the hybrid logarithm $\ln(m_Q^2 R^{-2})$: 1—heavy quark; 2—light quark; 3—gluons; 4—Γ.

TABLE II. Masses of B-mesons (in GeV) determined[35] from the QCD sum rules.

J^P	$B_{u,d}$	B_s
0^-	5,31	5,42
1^-	5,38	5,46
0^+	6,13	6,29
1^+	6,17	6,34

leads[34] to $\gamma = 2$ for the anomalous dimension of any current of the form $\bar{q}\Gamma Q$ [see (28)].

The relation given by (28) is asymptotic. For $m_Q \to \infty$, the difference between the mass of the quark and the corresponding meson is insignificant. However, we would like to use it for real c- and b-quarks (D- and $B_{u,d}$-mesons), in which case it is numerically quite significant which mass appears in (28), i.e., the mass of the current quark, the block quark or the mass of the meson. Since, as already noted, $m_Q^{-1/2}$ is essentially a kinematic factor, the law given by (28) would seem to be more accurate if we rewrite it in the form

$$\frac{f_B}{f_D} = \left(\frac{M_D}{M_B}\right)^{1/2} \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{2/b}. \qquad (29)$$

When $\Lambda_{QCD} = 100$ MeV, we find that

$$\frac{f_B}{f_D} = 0.65, \qquad (30)$$

whereas (23) and (25) lead us to expect that $f_B/f_D = 0.7 \pm 0.1$. One should note, however, that the logarithmic correction in (29) is significant at the level of 10%, i.e., it lies within the limits of theoretical uncertainty.

To conclude this Section, we must warn the reader that, although the above estimates of $f_{D,B}$ seem reliable and self-consistent, the question is not finally settled. For example, quite recently, Reinders *et al.*[28] reanalyzed the sum rules for f_B (Ref. 35) and reduced the original result (276 MeV) to 190 ± 30 MeV. The discrepancy as compared with (25) persists even when the above uncertainties are taken into account; the reason for it has not been explained.

3.2. Mass spectrum

The mass spectrum of c- and b-hadrons (both mesons and baryons) has been the subject of many different theoretical treatments. There have been many calculations based on potential models. Not being a specialist in this area, I refer the reader to published acounts[36–38] that summarize the results, present critical comparisons of different potential models, and refer the reader to earlier work.

Let us concentrate our attention on what is known about the masses, taking the QCD sum rules as the starting point. It seems that the first analysis was undertaken by Shuryak in Ref. 29, which we have already cited. In addition to qualitative results, numerical estimates were obtained in the asymptotic limit as $m_Q \to \infty$. The b-quark is most likely to lie inside this asymptotic region, but as far as charmed hadrons are concerned—and I wish to emphasize this particularly—the predictions reported in Ref. 29 are quantitatively invalid.

One of the most striking results presented in Ref. 29 is the large splitting between the S- and P-wave mesons (lowest states), which amounts to about 800 MeV. (In the limit as $m_Q \to \infty$, the spin interaction of the Q-quark is insignificant, so that we have degeneracy, say, between the 0^- and 1^- or between the 0^+ and 1^+ mesons.) A number close to about 800 MeV was obtained by Reinders *et al.*[35] for the S-P wave splitting in $b\bar{u}$ and $b\bar{d}$.

For the purposes of orientation, we reproduce a table, taken from Ref. 35, that summarizes the predictions for the B-meson masses that follow from the QCD sum rules (Table II). The precision is at a level of 50–100 MeV, so that the mass difference between the vector and pseudoscalar and between the axial and the scalar mesons is, of course, unreliable.

The analogous S-P wave splitting in D-mesons was determined by Blok and Eletskiĭ,[39] who considered the $J^P = 1^+$ axial meson. They found that its mass was

$$M(c\bar{u}, J^P = 1^+) = 2.55 \pm 0.1 \text{ GeV}, \qquad (31)$$

which agrees with the experimental result,[40] $M(D_1)_{J^P=1^+} = 2.42 \pm 0.006$ GeV.

Consequently, the mass difference beween the $J^P = 1^-$ and $J^P = 1^+$ states in the system $c\bar{u}$, $c\bar{d}$ is about 410 MeV. We note that the same result (~ 500 MeV) follows from the potential model[41] for systems with bare charm.

At this point, it will be convenient to pause and examine the mass difference between the S- and P-wave quark mesons in greater detail. We shall confine our attention to the lowest states in the channel with the given quantum numbers.

In the family of "old" hadrons, the SP-partners are, for example, the pairs (ρA_1), (ρf), ($\varphi f'$), etc. (Pions are excluded because of their special Goldstone nature.) In all cases, inspection of the PDG tables will readily reveal that the P-wave states are heavier than the S-wave states by about 500 MeV. The theoretical numbers for M_{A_1} and M_f, deduced from the QCD sum rules,[91] are in reasonable agreement with experiment.

The SP-splitting in charmonium is about 415 MeV:

$$M(c\bar{c}\,(^3P_1)) - M(c\bar{c}\,(^3S_1))$$

$$= M(\chi(3510)) - M(J/\psi) \approx 415 \text{ MeV},$$

which is also explained by the theory.[92] The analogous splitting in the $b\bar{b}$ family is approximately 435 MeV. Finally, $M_{D_1} - M_{D^*} \simeq 410$ MeV in D-mesons. The theoretical status of this result was discussed above.

Thus, the SP-wave splitting in all the known quark systems lies in the range 400–500 MeV. The prediction is[29,35] that the splitting will reach 800 MeV in the asymptotic state as $m_Q \to \infty$ and, if this is confirmed, our ideas based on the constituent quark model will be substantially reinforced.

Actually, these predictions[39,35] must be interpreted as follows. The interaction between the light valence quark and the quark condensate in vacuum depends significantly on the parameters of the second quark in the meson, $\bar{q}$, $\bar{c}$ or $\bar{b}$. [As we pass from $\bar{c}$ to $\bar{b}$, we must double (!) the SP-splitting. Undoubtedly, this proposition demands a degree of audacity.] A discussion of the problem in terms of the constituent quark model can be found in Ref. 93.

The replacement of the u- and d-quarks with the s-quark makes the meson heavier by 110–120 MeV. This fact was established in Ref. 32 for c-quarks and in Ref. 35 for b-quarks.

Of course, the mass differences $M_F - M_D$ and $M_{F^*} - M_{D^*}$ have long been determined experimentally and are in good agreement with our intuition, so that the main aim here is rather the reverse: to use these data to improve our information about the basic parameters of $SU(3)_{fl}$ violation, i.e., m_s and $f \equiv (\langle\bar{s}s\rangle/\langle\bar{u}u\rangle) - 1$. The quantity f is a measure of the $SU(3)_{fl}$ violation in vacuum condensates, and was first introduced and estimated by Ioffe.[42] Blok and Eletskiĭ[32] confirmed that $m_s = 150 \pm 30$ MeV and $f = -0.2 \pm 0.07$.

At the same time, the product $m_s\langle\bar{s}s\rangle$ can be found with much better precision:

$$m_s\langle\bar{s}s\rangle = -(210 \pm 5\ \mathrm{MeV})^4,$$

which was discovered earlier.[43]

Let us now consider the spectrum of charged baryons.[8)] We confine our attention to currents without derivatives, which create baryons of the form cqq (total of fifteen). In nonrelativistic quark theory, they can be interpreted as S-wave baryons. These fifteen baryons can be divided into a number of groups, according to their spin structure and the presence or absence of s-quarks.

We first consider baryons cqq, in which q = u or d, i.e., baryons without s-quarks. They can be constructed according to the following principle:

$$\left.\begin{aligned} \Lambda_c &= ((qq)_{J=0}\, c)_{J=1/2}, \\ \Sigma_c &= ((qq)_{J=1}\, c)_{J=1/2}, \\ \Sigma_c^* &= ((qq)_{J=1}\, c)_{J=3/2}. \end{aligned}\right\} \tag{32}$$

It is readily seen that Λ_c^+ is an isosinglet, and Σ_c and Σ_c^* in (32) are isotriplets, so that we have seven states.

The splitting between Σ_Q^* and Σ_Q is obviously related to the spin-spin interaction between the heavy quark and the light quark, and is therefore wiped out as m_Q increases. As far as the mass difference $M(\Sigma_Q) - M(\Lambda_Q)$ is concerned, this is largely determined by the interaction between the light quarks, so that, as $m_Q \to \infty$, the difference tends to a constant which, in QCD rules, is determined by vacuum condensates of the form

$$\langle \mathrm{vac} \mid \bar{q}q \mid \mathrm{vac}\rangle, \quad \langle \mathrm{vac} \mid \bar{q}\sigma_{\mu\nu}G^a_{\mu\nu}t^a q \mid \mathrm{vac}\rangle, \tag{33}$$

where $G^a_{\mu\nu}$ is the gluon field tensor.

The relative position of levels in the baryon families, and the general relationships formulated above, were predicted as far back as ten years ago in the model reported by De Rujula *et al.*[37] The corresponding qualitative picture is shown in Fig. 3.

The spectrum has been calculated[44,45] within the framework of the general method developed in Ref. 42. Of course,

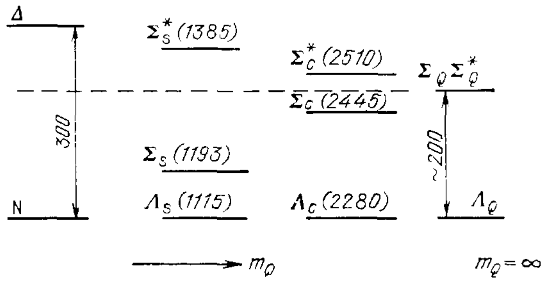

FIG. 3. Spin splitting in S-wave baryons Qqq, where q = u,d, as a function of m_Q (schematic).

the precision of the predicted masses in the QCD sum rules (~100 MeV in mass and ~25 MeV in the splitting) is much worse than in any of these potential models, but the results are derived from first principles of QCD and are reliable to within the indicated uncertainties. The basic aim of the analysis is to demonstrate the consistency between the QCD-vacuum ideas and the structure of the charmed baryon multiplets, and to relate the mass spread to fundamental parameters such as (33).

$SU(3)_{fl}$ violation effects, i.e., the inclusion of one or two strange quarks, were discussed in Ref. 45. In the case of one strange quark, we have three isodoublets: $\Xi_c^{(A)}$, $\Xi_c^{(S)}$, and Ξ_c^* (see Table I), i.e., a total of 6 states.

In the case of two strange quarks, we can construct two S-wave baryons

$$((ss)_{J=1}\, c)_{J=1/2} \text{ and } ((ss)_{J=1}\, c)_{J=3/2}. \tag{34}$$

There is little point in reproducing here in full all the results given in Refs. 44 and 45. They are in good agreement with existing data. To gain some idea about the precision of these results, we reproduce the mass splitting[45] between Λ_c^+ and the analogous baryon with one stange quark $\Xi_c^{+(A)}$:

$$M_{\Xi_c^{+(A)}} - M_{\Lambda_c^+} = 170 \pm 20\ \mathrm{MeV}. \tag{35}$$

This result must be compared with the experimental value of about 180 MeV.

It is interesting that the strange quark "weighs" about 170 MeV in charmed baryons, about 120 MeV in c-mesons, and about 100 MeV in b-mesons. The reduction in the "weight" of the s-quark that accompanies the change in its environment is a good illustration of the fact that the idea of the constitutent quark is only a rough approximation, and the composite quark model is unable to reproduce the finer details of hadron structure.

The overall mass splitting picture that has emerged from these calculations indicates that the four charmed baryons (Λ_c^+, $\Xi_c^{+(A)}$, $\Xi_c^{(0)A}$, and Ω_c^0) can decay only by virtue of the weak interaction, which is, of course, obvious. Less obvious is the fact that strong decays are apparently energetically inaccessible to the three baryons Ω_c^*, $\Xi_c^{+(S)}$, and $\Xi_c^{0(S)}$ and, for them, the main decay mode is the radiative transition

$$\Omega_c^* \to \Omega_c\gamma.$$

To conclude this Section, we return to mesons with bare charm and beauty. We draw attention to the fact that there is a little known mass calculation that was made several years

TABLE III. Masses of c- and b-mesons (in GeV) as reported in Refs. 46 and 47.

J^P	D	F	$B_{u,d}$	B_s
0^-	1,87	1,98	5,26	5,34
1^-	2,02	2,12	5,31	5,39
0^+	2,37	2,48	5,66	5,76
1^+	2,43	2,54	5,67	5,77
2^+	2,45	2,56	5,68	5,78

ago[46] and was based on a model combining Regge theory, topological expansion, and the idea of gluon "tubes" in the QCD vacuum.[47] The predicted spectrum obtained in this model is truly all-embracing; some of the predictions are listed in Table III. It seems to me that the most striking fact is that the masses of the axial mesons D_{1+} and F_{1+} in this table (which refers to 1982) agree to within 10 MeV with the respective experimental values, i.e., 2.42 ± 0.006 and 2.55 ± 0.06 GeV, obtained only this year![40,48]

A very significant result is the splitting between the S- and P-waves in b-mesons, e.g., $M(B_{J^P=0^+}) - M(B_{J^P=0^-})$. This will probably be the area of rivalry between the QCD sum rules and the approach developed in Refs. 46 and 47. Actually, the former method predicts[29,35] that the splitting between the S- and P-waves should increase as we pass from c-mesons (for which it is about 500 MeV) to the asymptotic region $m_Q \to \infty$, where it reaches ~ 800 MeV (as already noted, the b-quark lies in this asymptotic region). In the Kaĭdalov approach,[46,47] the reverse tendency obtains: the SP-splitting falls from about 500 MeV for c-mesons to about 400 MeV in systems including the b-quark.

3.3 Inclusive weak decays and lifetimes

This section could have been headed "History of superficial thinking by theoreticians and experimenters." Actually, the first theoretical estimates of the lifetimes of charmed particles were made before bare charm was discovered, and were based on the so-called spectator model, in which the presence of the light antiquark in the meson (diquark in the baryon) is relatively unimportant because it is merely a passive spectator, and the hadron lifetime is the same as the lifetime of the c-quark (Fig. 4). The natural expectation was that

$$\tau_{D^0} = \tau_{D^+} = \tau_F = \tau_{\Lambda_c} = \dots \quad (36)$$

The first experimental data indicated that $\tau(D^+)/\tau(D^0)$ was of the order of 5, or even 10 or more, and this was a real shock for theorists, none of whom foresaw this turn of events.

We must add that the ratio $[(\tau(D^+)/\tau(D^0))]_{exper}$ eventually evolved to a much more modest value and has now settled down at about 2–2.5.

Advances in the experimental situation have been parallelled by significant progress in the understanding of nonspectator effects in nonleptonic decays of c- and b-particles. We have in mind here mostly the power-type pre-asymptotic effects due to the interaction between the quark Q and the antiquark $\bar{q}$ (diquark qq), i.e., the spectator. They vanish as powers of m_Q^{-1} when $m_Q \to \infty$. Published analyses have been largely confined to the interference[49–54] and annihilation[55] pre-asymptotic mechanisms in inclusive decays. While early attempts to describe the observed spread in total width were mostly qualitative in character, we now have more quantitative approaches that have led to theoretically controlled estimates.

Basically, we now need answers to the following two questions:

(1) Are we in a position to explain the observed hierarchy of lifetimes of D^+, D^0, F, Λ_c^+, Ξ_c^+, Ω_c^0? What is the position of the as yet undiscovered Ξ_c^0 baryon in this hierarchy?

(2) What are the expectations with regard to $\Delta\Gamma_b$ and the spread in the widths of the b-hadrons?

We must admit straight away that a true theory, derivable from first principles of QCD, does not yet exist. In this sense the situation is fundamentally different from, say, the calculation of pre-asymptotic corrections to Euclidean correlation functions. In the decay of c- and b-particles, the kinematics is pseudo-Euclidean, and the particles succeed in reaching large distances. It follows that to formulate the procedure for expansion in powers of m_Q^{-1} we must introduce additional assumptions, the most important of which is quark-hadron duality at high energies. The use of the quark model for the evaluation of some of the amplitudes (see below) introduces a further element of uncertainty.

And yet, depsite all these reservations, the theory seems to be capable of answering both questions. A few explanations may not be out of place before we give these answers. The reader with a purely utilitarian purpose, who is disinterested in the intermediate steps, can find the answer straight away at the end of this Section.

A brief listing of the pre-asymptotic corrections looks as follows.

(a) Annihilation mechanism[55] (see, for example, Fig. 5). Modern estimates[56] shows that, for the decays of D^0- and F-mesons,

$$\frac{\Delta\Gamma_{ann}}{\Gamma_0} \approx 0.2, \quad (37)$$

where Γ_0 is the hadronic width in the parton spectator model, given by

$$\Gamma_0 = \frac{G_F^2 m_c^5}{64\pi^3}, \quad \left(\frac{N_c+2}{N_c}\Gamma_0\right)^{-1} \approx 7 \cdot 10^{-3}\,\mathrm{s}, \quad (38)$$

where $m_c = 1.55$ GeV, G_F is the Fermi constant, N_c is the

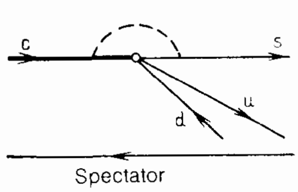

FIG. 4. Spectator mechanism in inclusive nonleptonic widths of charmed hadrons. The circle represents the Lagrangian $\mathscr{L}_W(\Delta C = 1)$.

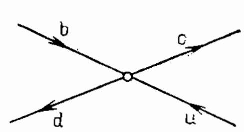

FIG. 5. Annihilation mechanism in the decay of B_d^0.

number of colors, equal to three in QCD, and the difference between $|V_{cs}|$ and unity is neglected. The annihilation contribution was discussed in Ref. 57 for B_d^0-mesons and, if $f_B \simeq 100$ MeV (see Section 3.1), then $\Delta\Gamma_{ann}/\Gamma_0 \lesssim 2\%$.

(b) Interference mechanism in the decay of the D^+-meson and certain baryons. The possibility of interference between the initial (spectator) and final quarks was noted in Ref. 49. This idea received its quantitative implementation in Refs. 51–54.

(c) cd→us- and bu→cd-scattering in baryon decays (see Fig. 5).

(d) Enhancement of nonleptonic Hamiltonian with $\Delta C = 1$ ($\Delta B = 1$). This effect is due to the dressing of the weak block with gluons. It was first analyzed in Ref. 58 in the leading logarithmic approximation. It is usually assumed (see, for example, classic textbooks[2,59]) that the effect can be reduced to the factors $C_\pm$ in the effective Lagrangian

$$\mathscr{L}_W = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left[\frac{C_+ + C_-}{2} (\bar{s}\Gamma_\mu c)(\bar{u}\Gamma_\mu d) + \frac{C_+ + C_-}{2} (\bar{s}_i \Gamma_\mu c^j \bar{u}_j \Gamma_\mu d^i) \right], \quad (39)$$

where, as an example, we have written out the Lagrangian with $\Delta C = 1$, $\Gamma_\mu = \gamma_\mu (1 + \gamma_5)$ and

$$C_- = \left(\frac{\alpha_s(m_c)}{\alpha_s(M_W)} \right)^{12/25}, \quad C_+ = \left(\frac{\alpha_s(m_c)}{\alpha_s(M_W)} \right)^{-6/25}. \quad (40)$$

When gluon exchanges are ignored, $C_+ = C_- = 1$ and only the first term survives in (39), in which the color of the c-quark is transmitted to the s-quark and the pair ($\bar{u}$d) is in the colorless state. We note that, because of the heavy gluons, $C_+ \neq C_- \neq 1$ and the second term appears in the Lagrangian with a different permutation of the color symbols. When weak decays are analyzed, it is occasionally useful to start, as a rough approximation, with quark diagrams that are meant to represent quarks and W-bosons; the corresponding "color-suppressing rules" are derived, and attempts are even made to verify them experimentally! These procedures completely ignore the fact that already hard gluons produce a redistribution of color, inducing the seond term in (39).

However, let us return to the question of enhancement. The total c-quark decay probability that follows from (39) is

$$\Gamma(c \to s u \bar{d}) = \frac{G_F^2 m_c^5}{192\pi^2} |V_{cs} V_{ud}^*|^2 N_c \left[\left(\frac{C_+ + C_-}{2} \right)^2 + \left(\frac{C_+ - C_-}{2} \right)^2 - \frac{1}{2N_c} (C_-^2 - C_+^2) \right]. \quad (41)$$

The negative third term in brackets represents interference between the two operators in the Lagrangian. This interference obviously vanishes in the limit as $N_c \to \infty$.

Using $\Lambda_{QCD} = 100$ MeV, we find the values of $C_\pm$, and are led to the conclusion that the standard enhancement factor in brackets in (41) is not very different from unity, i.e., $[\ldots] \simeq 1.1$ for the c-quark, and is even closer to unity for the b-quark.

For reasons that will be understood later, it is desirable for the nonleptonic decays of c- and b-quarks to be more enhanced. We therefore formulate the *nonstandard recipe*. We shall assume that N_c is large, and will discard terms proportional to N_c^{-1} in (41). We shall assume that the coefficients $C_\pm$ are fixed in this procedure.

Some justification of the recipe is given in Ref. 53. We shall not reproduce these arguments here, especially since they are not rigorous and the recipe itself is, to some extent, eclectic in character. Actually, if the argument were rigorous, then, for fixed m_Q and M_W and $N_c \to \infty$, we would have to put $C_\pm \to 1$ since, for example, $C_- = [\alpha_s(m_c)/\alpha_s(M_W)]^{4/b}$ and

$$b = \frac{11}{3} N_c - \frac{2}{3} N_f.$$

However, it is assumed that the factors N_c^{-1} in the coefficients $C_\pm$ are compensated by a large logarithm.

An analogous recipe for "discarding the $1/N_c$ contributions to the matrix elements" gradually emerged in Ref. 60–62, in which exclusive decays were considered on the basis of a phenomenological motivation. The most systematic implementation of the idea that the $1/N_c$ terms can be discarded can be found in Ref. 62, in which a wide range of exclusive decays was examined (we shall return to this point later).[9)] The QCD sum rules were subsequently used to verify[56] that the discarding of $1/N_c$ contributions was approximately valid, at least in D-meson decay (see Section 3.5).

If we adopt our prescription, the enhancement factor for the nonleptonic modes of c(b)-quark decay as compared with the parton result is

$$\left(1 + \frac{\alpha_s}{\pi}\right) \frac{C_+^2 + C_-^2}{2} = \begin{cases} 1.7 \text{ for the c-quark,} \\ 1.3 \text{ for the b-quark.} \end{cases} \quad (42)$$

We emphasize that, at this point, we are concerned with the overall enhancement of nonleptonic decays of c- and b-quarks, which appears even in the spectator model and does not produce a difference between the hadronic widths.

As far as the hierarchy of c- and b-hadron lifetimes is concerned, we recall effects (a)–(c), listed above. Effects (b) and (c) are numerically the most important, at least in systems containing c-quarks. We shall now show that the interference correction to $\Gamma(D^+)$ can be readily calculated.

The calculation of the inclusive nonleptonic width is conveniently divided into two stages. The first step is to find the operator that appears in the second order in $\mathscr{L}_W$. More precisely,

$$\mathscr{L}_{eff} = \mathrm{Im} \int d^4x\, iT(\mathscr{L}_W^+(x)\, \mathscr{L}_W(0)), \quad (43)$$

where the right-hand side is the imaginary part. The decay width of a hadron X obviously reduces to the matrix element

$$\Gamma_X = \frac{1}{M_X} \langle X | \mathscr{L}_{eff} | X \rangle \quad (44)$$

(Fig. 6). The second step is to calculate the hadronic matrix element. If we expand the correlator (43) in terms of local operators,[10)] we obtain an expansion for Γ_X in powers of m_Q^{-1}.

For example, Fig. 7a shows the usual spectator result

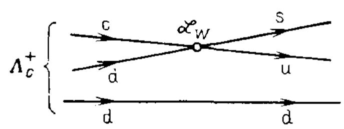

FIG. 6. Pre-asymptotic correction in the weak decay of Λ_c^+ due to cd→us scatterings.

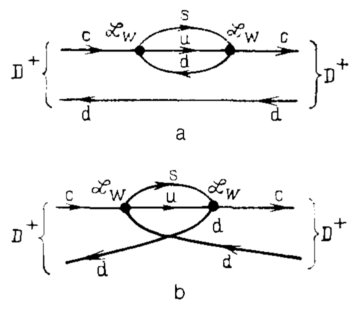

FIG. 7. The imaginary part of these graphs determines the nonleptonic width of the D^+ meson: a—spectator result; b—correction for interference between the initial and final $\bar{d}$.

for $\Gamma(D^+)$. To include interference, we use general rules to interchange lines corresponding to the initial and final $\bar{d}$-quarks. We thus arrive at Fig. 7b. From the computational point of view, this is the same as the diagram of Fig. 7a, except that the d-quark line is open.

The final result for the $\bar{d}$-interference correction is[52,53]

$$\Delta\Gamma_{\rm int}(D^+) = \frac{1}{m_D}\langle D^+ | \frac{G_F^2 m_c^2}{4\pi}\left[\frac{C_+^2+C_-^2}{2}(\bar{c}_i\Gamma_\mu d_j)(\bar{d}_j\Gamma_\mu c_i) + \frac{C_+^2-C_-^2}{2}(\bar{c}\Gamma_\mu d)(\bar{d}\Gamma_\mu c)\right] | D^+\rangle. \tag{45}$$

The next step is to evaluate the matrix element for the D^+-meson. In the absence of anything better, we use factorization, or vacuum "interlaying," the justification being that factorization becomes rigorous as $N_c \to \infty$. We then have

$$\frac{\Gamma_{\rm had}(D^+)}{\Gamma_0} \approx \left(1.7 - 16\pi^2 \frac{C_-^2 - 2C_+^2}{3}\frac{f_D^2}{m_c^2}\right). \tag{46}$$

It is now appropriate to make the following points. First, formulas (45) and (46) are not entirely accurate because they do not take into account the "hybrid" logarithmic renormalizations mentioned in Section 3.1. The normalization point μ of the operator in (45) is of the order of m_c. On the other hand, the characteristic virtualities of the quark in the D-meson are $\sim R^{-1}$, where R is the confinement radius. This means that, when we consider the matrix elements, we must take into account the evolution of the operators from $\mu \sim m_c$ to $\mu \sim R^{-1}$. This question will be discussed in greater detail in Section 3.4. Here, we merely note that the inclusion of the hybrid logarithm will numerically emphasize the factor $(C_-^2 - 2C_+^2/3$ in (46) and will remove the relatively strong compensation that occurs[54] in $C_-^2 - 2C_+^2$.

The $\bar{d}$-quark interference in the decay of the D^+ is destructive. It reduces the decay width as compared with the parton result Γ_0. At the same time, both the total lifetime $\tau(D^+)$ and the semileptonic relative probability $\mathrm{Br}(D^+ \to l\nu X)$ are in reasonable agreement with parton predictions. It is precisely for this reason that it is phenomenologically desirable to have an appreciable overall enhancement of nonleptonic decays of the c-quark [the term 1.7 in (46); in the parton approximation, the figure of 1.7 is replaced with unity]. The two effects in the D^+ decay (destructive interference and overall enhancement) are then found to cancel out almost completely, and we again have the parton picture. Numerically, as shown in Ref. 54,

$$\frac{\Delta\Gamma_{\rm int}(D^+)}{\Gamma_0} \approx -1. \tag{47}$$

Let us now also reproduce the two other terms in $\mathscr{L}_{\rm eff}$ that are responsible for interference and quark-quark scattering in charmed baryons:[54]

$$\begin{aligned}\mathscr{L}_{\rm eff} = -\frac{G_F^2 m_c^2}{8\pi}\Big[&\left(\frac{C_++C_-}{2}\right)^2\left(\bar{c}\Gamma_\mu c - \frac{2}{3}\bar{c}\gamma_\mu\gamma_5 c\right)\bar{u}\Gamma_\mu u\\ &+\frac{1}{4}(5C_+^2+C_-^2-6C_+C_-)\left(\bar{c}_i\Gamma_\mu c_j - \frac{2}{3}\bar{c}_i\gamma_\mu\gamma_5 c_j\right)\bar{u}_j\Gamma_\mu u_i\\ &+\left(\frac{C_+-C_-}{2}\right)^2\left(\bar{c}\Gamma_\mu c - \frac{2}{3}\bar{c}\gamma_\mu\gamma_5 c\right)\bar{s}\Gamma_\mu s\\ &+\frac{1}{4}(5C_+^2+C_-^2+6C_+C_-)\left(\bar{c}_i\Gamma_\mu c_j - \frac{2}{3}\bar{c}_i\gamma_\mu\gamma_5 c_j\right)\bar{s}_j\Gamma_\mu s_i\Big].\end{aligned} \tag{48}$$

These are obtained by cutting the u- and s-quark lines in Fig. 7a. The relatively unwieldly logarithmic factors are also omitted from this expression. For a complete answer, we refer the reader to Ref. 54, which gives an analysis of the pre-asymptotic corrections to the nonleptonic widths of c- and b-hadrons. In the first case, the corrections turn out to be very large. In fact, the expansion parameter

$$\xi_Q = 16\pi^2 \frac{f_Q^2}{m_Q^2} \tag{49}$$

is greater than unity for hadrons containing the c-quark. The origin of the factors f_Q^2/m_Q^2 and $4\pi^2$ is relatively clear. The former reflects the fact that all the non-spectator processes require the spatial coincidence of the Q-quark and the spectator ($\bar{q}$ in the meson and q in the baryon).

The factor $4\pi^2$ is due to the fact that the non-spectator contribution has one loop fewer than the spectator contribution (compare Figs. 7b and 7a). The further factor of four in (49) is due to the particular calculation.

If the parameter characterizing the pre-asymptotic effect is $\xi_c \gtrsim 1$, then, for charmed hadrons, we can at best expect a semiquantitative picture. The situation is complicated further by the fact that averaging of the four-fermion operators over the baryons Λ_c, Ξ_c, etc. introduces an additional uncertainty of the order of two. The factorization procedure cannot be used here, and different variants of the quark model yield numbers differing by factors of 2–2.5.[11]

And yet, with all the reservations imposed on the theoretical constuction described above, we can draw a particular conclusion. First, the difference between the nonleptonic widths of charmed hadrons by factors of 2–2.5 can be explained by the theory without too much effort. Next, the sign of the effect and, consequently, the hierarchy of lifetimes are apparently reliably established. According to Ref. 54,

$$\Gamma(\Omega_c^0) > \Gamma(\Xi_c^{0(A)}) > \Gamma(\Xi_c^{+(A)}) \approx \Gamma(\Lambda_c^+) > \Gamma(D^0) \approx \Gamma(F) > \Gamma(D^+), \tag{50}$$

where each inequality in the chain of nonleptonic widths signifies a jump by an amount of the order of $\Gamma_0 = G_F^2 m_c^5/64\pi^2$.

How does (50) look from the experimental point of view? The overall situation is now as follows. The lifetime data collected in Ref. 63 are reproduced in Fig. 8 here. As far as D, F, and Λ_c^+ are concerned, the hierarchy defined by (50) is valid. Next, we cannot exclude the fact that $\Gamma(\Xi_c^+) \simeq \Gamma(\Lambda_c^+)$. The greatest concern is caused by Ω_c^0. In

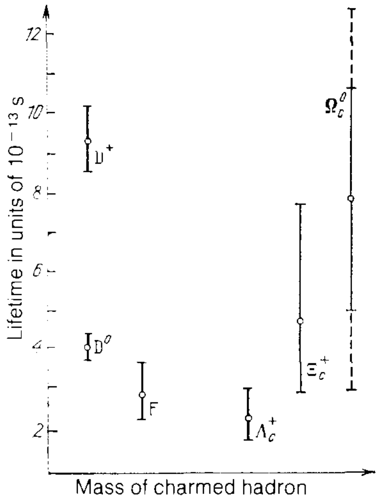

FIG. 8. Lifetimes of charmed particles.[61] The broken line shown for $\tau(\Omega_c^0)$ is a measure of the systematic uncertainty.

our picture, this baryon should be the shortest-lived, whereas the measured lifetime $\tau(\Omega_c^0)$, based on four events, turns out to be close to $\tau(D^+)$, i.e., the lifetime of the longest-lived charmed hadron. Moreover, because of the large statistical and systematic error, it is too early for any definite conclusions. The baryon Ξ_c^0 has not yet been discovered. It is predicted that its lifetime is shorter than $\tau(\Lambda_c^+)$, but greater than $\tau(\Omega_c^0)$.

The b-hadrons should have a much smaller lifetime spread. Actually, the parameter that governs $\Delta\Gamma$ is

$$\xi_b = \frac{16\pi^2 f_B^2}{m_b^2} \approx 0.08. \tag{51}$$

When we pass for ξ_c to ξ_b, we lose an order of magnitude in the mass ratio m_c^2/m_b^2, and a further factor of 1/2 is due to the ratio f_B^2/f_D^2. The natural estimate for $\Delta\Gamma$(b-hadrons) thus becomes $O(10\%)$. However, logarithmic renormalizations are constructed so that there is additional cancellation and suppression of pre-asymptotic corrections. If we take Ref. 54 literally, then $\Delta\Gamma(\text{b-hadrons})/\Gamma_0 \sim 1\%$. If we stretch all the quantities in the formulas in the upward direction, and introduce the additional "insurance factor" of 2, which represents the uncertainty in the matrix elements, we obtain the absolutely conservative estimate

$$\frac{\Delta\Gamma(\text{b-hadrons})}{\Gamma_0} \leqslant 0.05, \tag{52}$$

where Γ_0 is the mean width of the b-hadrons (see also Ref. 94).

One occasionally sees in the literature another point of view in relation to the $\tau(B^0)/\tau(B^+)$ problem (see, for example, Ref. 25). The logic of this is roughly as follows. In the spectator model, the relative probability of semileptonic decays $\mathrm{Br}(B \to l\nu X)$ is 15–16%. This result is actually obtained in the parton model for the b-quark decay $b \to l\nu c$ or, more precisely, the "standard" gluon enhancement factor $(C_-^2 + 2C_+^2)/3 \simeq 1.06$ is built-in for the b-quark. On the other hand, experiment shows that $\mathrm{Br}(B \to l\nu X)$ is appreciably less than 16%. Thus, according to the ARGUS group, $\mathrm{Br}(B \to l\nu X) = 11 \pm 1\%$, whereas the world average[25] is $11.7 \pm 0.6\%$. If the difference between 15–16 and 11% is ascribed to non-spectator effects (for example, annihilation corrections to B_d^0), the latter must be very significant, so that, finally, $\tau(B^+)/\tau(B^0) \sim 2$.

This line of argument is arbitrary in the light of the theory of pre-asymptotic power-type corrections, the outlines of which are now relatively clear. We must therefore insist on the estimate given by (52) and, taking the experimental data seriously, we must look for other ways of explaining the reduced value of $\mathrm{Br}(B \to l\nu X)$. One possibility is a nonstandard treatment of the problem of general enhancement of nonleptonic decays (see above), in which the standard factor $(C_-^2 + 2C_+^2)/3$ is replaced with (42). This is the so-called rejection rule for the $1/N_c$ contributions. The theoretical justification of this rule, is, of course, doubtful at the very least, but the rule is phenomenologically welcome. Actually, if we combine (42) with the proposition that the current quark masses ($m_b = 4.8$ GeV, $m_c = 1.35$ GeV, $m_s = 0.15$ GeV, $m_{u,d} = 0$) should be used in the spectator model[64] (see also Ref. 65), we can show that $\mathrm{Br}(B \to l\nu X)$ is about 12%, which, in principle, is not inconsistent with the data. A more fanciful hypothesis is that a few tens of percent of the decay width of the B mesons is due to some unusual unknown channels. (This hypothesis has been discussed by M. B. Voloshin, M. V. Danilov, and the present author.

3.4. Hybrid logarithms

Weak decays of heavy quarks have been under investigation by theorists for more than a decade. The effective Hamiltonian for nonleptonic processes is obtained by multiplying two currents together (see, for example, Fig. 9, which shows the amplitude for $c \to s u \bar{d}$). When the masses of the initial and final quarks are much smaller than the W-boson mass M_W—and we assume that this is the case—the W-boson propagator contracts to a point and the dependence on M_W reduces to $g^2/M_W^2 \sim G_F$ in the Born approximation, where g is the $SU(2)_{\text{weak}}$ gauge constant. It is noted in the pioneering Ref. 58 that the inclusion of gluon exchanges introduces an additional logarithmic dependence on M_W due to the anomalous dimension of the operator $(\bar{s}\Gamma_\mu c)(\bar{u}\Gamma_\mu d)$. In particular, the graph of Fig. 10 renormalizes the answer obtained in the Born approximation

$$\begin{aligned} &\frac{G_F}{\sqrt{2}} (\bar{s}\Gamma_\mu c)(\bar{u}\Gamma_\mu d) \\ &\to \frac{G_F}{\sqrt{2}} \Big[\Big(1 + \frac{\alpha_s}{4\pi}\Big) \ln \frac{m_W^2}{m_c^2} (\bar{s}\Gamma_\mu c)(\bar{u}\Gamma_\mu d) \\ &\qquad - \frac{3\alpha_s}{4\pi} \ln \frac{m_W^2}{m_c^2} (\bar{s}_i\Gamma_\mu c^j)(\bar{u}_j\Gamma_\mu d^i) \Big]. \end{aligned} \tag{53}$$

By summing the logarithms within the framework of the standard renormalization group, we obtain (39) and (40).

For us, however, another aspect is important at this point. The logarithmic factors in (53) are cut off at the lower

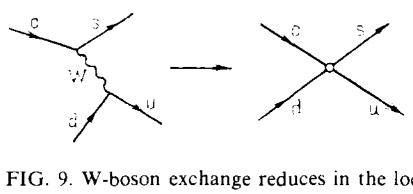

FIG. 9. W-boson exchange reduces in the local limit to the current × current operator.

FIG. 10. Gluon renormalization of the operator $(\bar{s}\Gamma_\mu c)(\bar{u}\Gamma_\mu d)$.

end by the mass m_c of the charmed quark. In other words, the evolution of the operator $(\bar{s}\Gamma_\mu c)(\bar{u}\Gamma_\mu d)$ ends, in the leading logarithmic approximation, with $\mu \sim m_c$, where μ is the normalization point. The reason for this is simple: the characteristic energy release by light quarks in the $c \to su\bar{d}$ transition is of the order of m_c. This absence of evolution as we reduce μ below m_Q in the operators containing the heavy quark is well known and common.

It is so common that, usually, the same formulas are written, unthinkingly, for the four-fermion operators with two heavy quarks. As an example, we mention the operator

$$(\bar{b}\Gamma_\mu d)\ (\bar{b}\Gamma_\mu d),$$

which is central to the analysis of the problem of $B^0\bar{B}^0$-oscillations. The so-called η-factors for this operator can be expressed[10,11] in terms of $\alpha_s(m_W)/\alpha_s(m_b)$. However, in this case, the statement that LLA involves only logarithms of the form $\ln(m_W^2/m_D^2)$, and we need take into account only the evolution from $\mu = m_W$ to $\mu = m_b$, is incorrect in principle. Careful inspection of the diagram in Fig. 11 will readily show that it includes the "hybrid logarithm" that accumulates from the region of virtual momenta defined by

$$R^{-2} \lesssim k^2 \lesssim m_Q^2.$$

This is an infrared region for the heavy quark Q and an ultraviolet region for the light quark q. In other words, the operator $(\bar{Q}\Gamma q)^2$ depends on the normalization point μ for $\mu < m_Q$ and, as usual, this reduces in LLA to factors of the form $[\alpha_s(m_Q)/\alpha_s(\mu)]^{\gamma/b}$. As far as I know, these factors were not previously discussed, since it was tacitly assumed that the evolution ended with $\mu = m_Q$.

The proposition that hybrid logarithms will appear can be readily verified directly. They are precisely the same as the well-known logarithm $\alpha \ln m_N/(m_n - m_p)$ that appears in the radiative correction to the β-decay of the neutron. In the latter case, we have to deal with the operator $\bar{p}\gamma_\alpha(1 + g_A\gamma_5)n\bar{e}\Gamma_\alpha\nu$, which is analogous to the four-quark operators.

Let us now consider a few typical examples that are frequently encountered in applications. We have already noted (Sec. 2) that, in the leading approximation, the $B^0 \leftrightarrow \bar{B}^0$ oscillations are determined by the matrix element of the operator $(\bar{b}\Gamma_\alpha q)^2$. Since the very earliest work,[66] the matrix element has been evaluated using factorization or vacuum "interlaying":

$$\langle B^0 \mid \bar{b}\Gamma_\alpha q\bar{b}\Gamma_\alpha q \mid \overline{B^0}\rangle = \frac{8}{3} \mid \langle B^0 \mid \bar{b}\gamma_\alpha\gamma_5 q \mid 0\rangle \mid^2. \qquad (54)$$

On the other hand, it is sometimes maintained that large deviations from factorization are possible (see, for example, the review in Ref. 67 and the references therein). Factorization becomes exact in the limit as $N_c \to \infty$ [in my view, this is a very strong argument in favor of (54)]. There are also indications[26] that, in the real world, the factorization procedure is valid for four-quark operators to within $O(10\%)$.

Logarithmic corrections are a potential source of breakdown of factorization. The reason is trivial: the factors $1/N_c$ for the nonfactorizable parts of the amplitudes may cancel with large logarithms of the form $\ln(M_W^2)/m_Q^2)$ or $\ln m_Q^2 R^{-2}$. Factorization will therefore be most accurate when, prior to factorization, all the logarithmic renormalizations are explicitly displayed and taken into account separately.

As far as the momentum range $m_b^2 \lesssim k^2 \lesssim m_W^2$ is concerned, the renormalization of the operator $(\bar{b}\Gamma_\alpha q)^2$ in this region was calculated a long time ago (these are the so-called η-factors; see, for example, Ref. 25). The operator $j_{\alpha 5}$ does not renormalize in the LLA in this region.

However, we already know from Section 3.1 that $j_{\alpha 5}$ is renormalized due to the hybrid logarithms below m_b. It follows that, before we write down (54), we must investigate the dependence of the operator $(\bar{b}\Gamma_\alpha q)^2$ on the normalization point μ for $\mu < m_b$, isolate the logarithmic renormalization, and only then use the factorization procedure for the operators at the point $\mu \sim R^{-1}$. Luckily, it was shown in Ref. 34 that the anomalous hybrid dimension of the operator $O = (\bar{b}\Gamma_\alpha q)^2$ is greater than the anomalous dimension of $j_{\alpha 5}$ by a factor of two

$$\gamma_O = 4, \quad \gamma_j = 2.$$

Hence, (54) is valid for any μ provided $\mu < m_b$.

The fact that $\gamma_O = 2\gamma_j$ is not trivial. Moreover, it depends on the specific structure of the operator. For example, for an operator of the form $(P - S) \times (P - S)$, which appears[10] in the amplitude for the $B^0\bar{B}^0$ oscillations with the factor $\sim m_b^2/m_t^2$, this equation is simply not valid and, if we were to use factorization in this case, the answer expressed in terms of the physical constant f_B would contain the additional renormalization $[\alpha_s(\mu \sim R^{-1})/\alpha_s(m_b)]^{(2\gamma_j - \gamma_O)/b}$.

This is precisely the situation in the case of operators that determine pre-asymptotic corrections to the widths of c- and b-hadrons (Section 3.3). A little thought will show that the situation is as follows.[54] All four-quark operators of the form $(\bar{Q}q)(\bar{q}Q)$ can be divided into two classes, depending on their color structure (the γ-matrix structure is unimportant). If the operator can be written so that the color does not change along the heavy quark line, for example,

$$(\bar{Q}\ \Gamma Q)\ \ (\bar{q}\Gamma q), \quad \Gamma \text{ — arbitrary},$$

then it is assigned to class one. Its anomalous hybrid dimension is zero. If the color is transferred from the heavy to the light quark,

$$(\bar{Q}\Gamma t^a Q)\ (\bar{q}\Gamma t^a q), \qquad (55)$$

FIG. 11. Gluon renormalization of an operator of the form $(\bar{b}\Gamma q)(\bar{b}\Gamma q)$, which leads to the logarithm $\ln(m_Q^2 R^{-2})$.

the operator belongs to class two and the anomalous hybrid dimension is 9/2:[12]

$$\langle (\bar{Q}\Gamma t^a Q)\,(\bar{q}\Gamma t^a q)_{\mu\sim m_Q}\rangle$$
$$= \langle (\bar{Q}\Gamma t^a Q)\,(\bar{q}\Gamma t^a q)_{\mu\sim R^{-1}}\rangle \left[\frac{\alpha_s(R^{-1})}{\alpha_s(m_Q)}\right]^{9/2b}, \tag{56}$$

where $\langle \ldots \rangle$ represents the matrix element evaluated over states with typical virtuality $\sim R^{-1}$.

Hybrid logarithmic corrections have an appreciable effect on numerical estimates of pre-asymptotic effects in nonleptonic widths of c- and b-hadrons.

3.5. Nonleptonic two-particle decays of the D- and F-mesons

Since the discovery of charmed particles[68] in 1976 and up until now, weak exclusive decays of charmed mesons (and, recently, also of the B-mesons) have been attracting the attention of theorists and experimenters. A very considerable volume of data has accumulated, covering literally tens of channels (see, for exmaple, the reivew in Ref. 69) and efforts in this direction are continuing. On the other hand, there is an equally abundant stream of theoretical papers on nonleptonic two-particle decays (summarized, to some extent, in Ref. 65). As always, this means that none of these papers provides a complete solution of the problem that actually (and not unexpectedly) turns out to be very complex.

Historically, the first model was the spectator model[70] (valence quark model), which is analogous to the model used in the analysis of nonleptonic decays of K-mesons and hyperons.[71] The main features of this model can be demonstrated by considering the example of the $D^+ \to \bar{K}^0\pi^+$ process. The corresponding amplitude $\langle D^+ | \mathscr{L}_W | \bar{K}^0\pi^+\rangle$ is illustrated somewhat symbolically in Fig. 12. The full point in this figure represents the Lagrangian (39) which includes heavy gluon exchange. Soft gluons that bind quarks and hadrons and, *inter alia*, are responsible for the interaction between π and K in the final state, are not shown explicitly.

If we "forget" the soft gluons for the moment (or, more precisely, assume that they are "concentrated" between pairs of lines indiated by the braces in Fig. 12), then

$$A\,(D^+ \to \bar{K}^0\pi^+)_{\text{fact}}$$
$$= \frac{1}{\sqrt{2}} G_F V_{cs} V_{ud}^* \,[C_1 \langle \bar{K}^0 \,|\, \bar{s}\Gamma_\mu c \,|\, D^+\rangle \langle \pi^+ \,|\, \bar{u}\Gamma_\mu d \,|\, 0\rangle$$
$$+ C_1 \langle \pi^+ \,|\, \bar{u}^j \Gamma_\mu c_i \,|\, D^+\rangle \langle \bar{K}^0 \,|\, \bar{s}^i \Gamma_\mu d_j \,|\, 0\rangle$$
$$+ C_2 \langle \bar{K}^0 \,|\, \bar{s}^i \Gamma_\mu c_j \,|\, D^+\rangle \langle \pi^+ \,|\, \bar{u}^j \Gamma_\mu d_i \,|\, 0\rangle$$
$$+ C_2 \langle \pi^+ \,|\, \bar{u}\Gamma_\mu c \,|\, D^+\rangle \langle \bar{K}^0 \,|\, \bar{s}\Gamma_\mu d \,|\, 0\rangle], \tag{57}$$

and if we use the factorization technique, we can reduce the matrix element of the four-fermion operator to the product of simpler matrix elements.

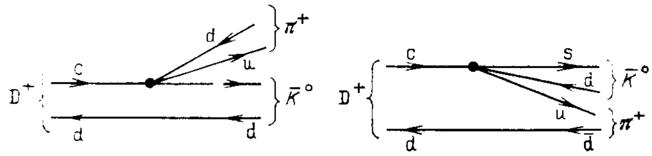

FIG. 12. $D^+ \to \bar{K}^0\pi$ decay in the spectator model.

Next, using the fact that

$$\langle \pi^+ \,|\, \bar{u}^i \gamma_\mu \gamma_5 d_j \,|\, 0\rangle = -\frac{i}{3} f_\pi \delta_j^i (p_\pi)_\mu,$$
$$\langle \bar{K}^0 \,|\, \bar{s}^i \gamma_\mu c_j \,|\, D^+\rangle = \frac{1}{3}\delta_j^i\,[f_+^K (p_D + p_K)_\mu + f_-^K (p_D - p_K)_\mu], \tag{58}$$

and substituting this in (57), we obtain

$$A\,(D^+ \to \bar{K}^0\pi^+) = \frac{1}{\sqrt{2}} G_F V_{cs} V_{ud}^* \,(-im_D^2)$$
$$\times\left[\left(C_1 + \frac{C_2}{3}\right) f_+^K f_\pi + \left(C_2 + \frac{C_1}{3}\right) f_+^\pi f_K\right], \tag{59}$$

where the constants f_+^K and f_π are defined in (58), and f_+^π and f_K are their analogs, obtained by the replacement $K \leftrightarrow \pi$. The figure of three in the denominator is actually equal to N_c.

In the limit of $SU(3)_{fl}$ symmetry, it is clear that (59) reduces to

$$A\,(D^+ \to \bar{K}^0\pi^+) = -\frac{im_D^2}{\sqrt{2}} G_F V_{cs} V_{ud}^* f_+^K f_\pi \frac{4}{3}(C_1 + C_2), \tag{60}$$

where the numerical value of f_+^K was determined earlier in the QCD sum rules:[72]

$$f_+^K = 0.5 \pm 0.1. \tag{61}$$

It is readily verified that the spectator model then gives $\mathrm{Br}(D^+ \to \bar{K}^0\pi^+) \sim 6\%$, which is greater than the experimental value by a factor of 2.5. A still more dramatic discrepancy between the spectator model and experiment (by two or three orders of magnitude!) occurs for the ratio $\Gamma(D^0 \to \bar{K}^0\pi^0) \times \Gamma^{-1}(D^0 \to K^-\pi^+)$. This well known circumstance is discussed, for example, in the review given in Ref. 73 (see Section 3.5).

Many authors have tried to improve the theoretical description by improving the spectator model and introducing into it additional elements, including the so-called non-spectator contributions.[13] A critical analysis of all these papers is hardly possible here, and I shall examine only some of them, with apologies to authors whose work has not been included for one reason or another.

Because partial widths are relatively very sensitive to the magnitude of the coefficients C_+, it has been suggested that, in reality, these coefficients are very different from the values given by (40) (Ref. 74) and, in particular, we have the so-called sextet enhancement,[50,75] in which $C_-/C_+ \gg 1$. The difference as compared with (40) must be due to nonperturbative gluons because the QCD corrections are small in the next-order approximation after LLA.[76]

Several workers have considered charmed mesons on the basis of phenomenological Lagrangians constructed from charmed and light meson fields.[77] The interaction constants were determined from experiment.

A significant step forward was the inclusion of the interaction between the particles in the final state.[61,78,79]

The so-called annihilation mechanism[55,79,80] (Fig. 13), which was ignored in early work because the corresponding diagrams are compressed as compared with the graphs of Fig. 9 by the factor $\sim 4\pi^2 d_D^2 m_s^2/m_c^4 \sim 10^{-2}$, have attracted considerable attention. The emission of a gluon by the initial quark removes the chiral suppression m_s^2/m_c^2 because the quark-antiquark system can be in a spin 1 state after the

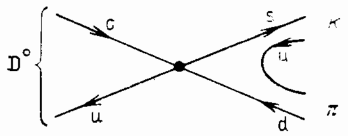

FIG. 13. Annihilation contribution to the $D^0 \to K\pi$ amplitude.

emission of the gluon. However, it may be shown that the relative contribution of the annihilation mechanism does not exceed a few percent (for present-day values of the parameters) in the case of the hard (perturbative) gluon.

It has since become clear that the annihilation mechanism plays a considerably greater part when soft gluons are considered.[79,80] The analysis of the entire set of two-particle decays, given in Ref. 79, is very instructive from this point of view and demonstrates the enhancement of the annihilation diagrams, such as Fig. 13, with the emission of a gluon with $k^2 \sim (300\text{–}500\ \text{MeV})^2$.

Finally, we note Ref. 62, which we have already cited and in which it was suggested that terms suppressed as $1/N_c$ could be discarded in the evaluation of the partial widths in the valence quark model. It was noted that agreement with experiment was very much better for most of the modes. (The dynamic reason for this is discussed below).

Despite the advances that have been made, at least at the qualitative level, there is now even greater necessity for a systematic approach. A finite challenge to this theory has been presented by the new data, e.g., the $D^0 \to \bar{K}^0\varphi$ decay, which is forbidden in the spectator model. It turns out[81] that its relative probability is not at all small and amounts to about 1%. Quantum chromodynamics must supply a computational scheme based on first principles, and with a minimum number of additional assumptions, that will provide a unified way of treating all the two-particle decays, yielding predictions whose precision could be specified in advance.

An approach based on the QCD sum rules[26] was developed in Ref. 56[14)] From the fundamental point of view, the problem is not very different from problems that have already been solved by the method of three point sum rules (see, for example, Ref. 83), but the technical difficulties are much greater. We shall discuss below the basic theoretical results, but let us first note that partial widths have been found for about 50 channels of the form

$$D \to PP,\ D \to PV,\ F \to PP,\ F \to PV,$$

where P is a pseudoscalar and V a vector meson consisting of u-, d-, and s-quarks:

$$P = \pi,\ K,\ \eta,\ \eta',\quad V = \rho,\ \varphi,\ \omega,\ K^*.$$

As usual, each particle in the QCD sum rules is replaced with the corresponding current, e.g., $j^{(D)} = i\bar{c}\gamma_5 d$ for the D^0 meson, $j^{(\pi)}_{\mu 5} = \bar{d}\gamma_\mu\gamma_5 u$ for the π^+-meson, and $j^{(P)}_\mu = \bar{u}\gamma_\mu d$ for the ρ^+-meson.

The central object is the current correlator in the Euclidean region or, more precisely, the *four-point* Green's function of the form

$$\Pi_{\mu\nu} = \int d^4x d^4y d^4z \exp(iQ_2x + iqy) \times \langle T\{j^{(A)}_\mu(y)\, j^{(B)}_\nu(x)\, j^{(D)}(0)\, \mathscr{L}_W(z)\}\rangle, \quad (62)$$

where $\mathscr{L}_W$ is the effective Lagrangian with $\Delta C = 1$, given in (39), A is the pseudoscalar meson (momentum q), B is the pseudoscalar or vector meson (momentum Q_2), and the momentum entering the current $j^{(D)}$ is denoted by Q_1. The first complication is thus the appearance of the Lagrangian $\mathscr{L}_W(z)$ in (62).

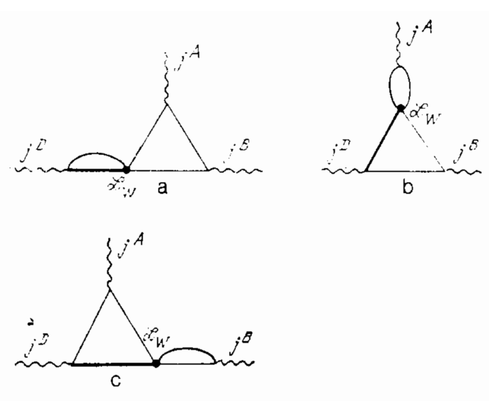

FIG. 14. Skeleton diagrams in the sum rules for weak nonleptonic decays $D, F \to PP, PV$.

The correlation function $\Gamma_{\mu\nu}$ is calculated for

$$Q_1^2,\ Q_2^2,\ q^2 \sim -(1.5\ \text{GeV})^2 \quad (63)$$

using the Wilson operator expansion, in which operators up to the sixth dimension are taken into account. There are three skeleton diagrams defining $\Gamma_{\mu\nu}$ (Fig. 14). The word, "skeleton," means that one or more of the quark lines can be cut and directed into the vacuum and, moreover, gluon lines forming the gluon condensate $\langle 0|\alpha_s G^a_{\mu\nu} G^a_{\mu\nu}|0\rangle$ can be added. Figure 15 shows an example of a "dressed" diagram for $D^0 \to \bar{K}^0\varphi$.

When the gluon lines lie wholly inside the two- or three-cornered block of Fig. 14, it is relatively clear that the sum rules give the same answer as the spectator model (factorization). This contribution must, of course, be taken into account in the amplitude, but deviations from the spectator model are more interesting. For example, we must concentrate our attention on diagams in which the gluon line joins the two- and three-angle blocks (see Fig. 15). If we mentally draw such a gluon line in the graphs of Fig. 14, Fig. 14a will represent the annihilation mechanism and Figs. 14b and c the unfactorized part of the spectator mechanism.

The correlator (62) can be evaluated reliable in the Euclidean region (63) in terms of the quark, gluon, and mixed condensates for each of these three types of graph. Although this evaluation is relatively laborious, it presents no fundamental difficulty. However, we are interested in amplitudes in the physical region. As is usual in sum rules, the extrapolation is performed using dispersion relations, saturated by

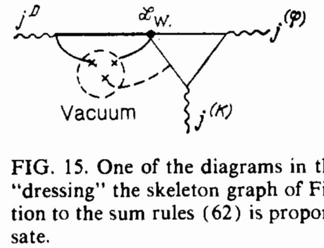

FIG. 15. One of the diagrams in the $D^0 \to \bar{K}^0\varphi$ amplitude, obtained by "dressing" the skeleton graph of Fig. 14a. The corresponding contribution to the sum rules (62) is proportional to the mixed vacuum condensate.

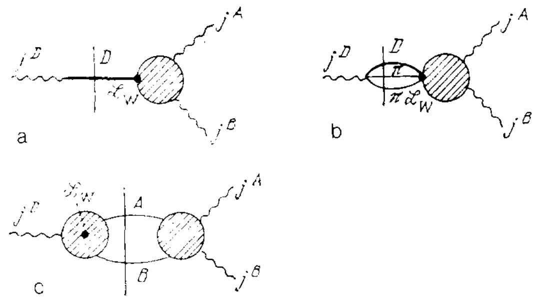

FIG. 16. Imaginary parts of different origin in the correlator (62) when the jump in Q_1^2 is considered.

the resonance contribution in which we are interested, plus the background due to higher states, which is partly minimized by borelization. Unfortunately, this saturation and the treatment of sum rules, present us with a special problem that is much more complicated than traditional problems.[26,83] It is here that we encounter the beginnings of the main source of theoretical uncertainty.

Let us examine this in more detail. Consider the jump in the variable Q_1^2, which, the reader will recall, corresponds to the current $j^{(D)}$. This jump is shown symbolically in Fig. 16.

Figure 16a represents the imaginary part corresponding to the D-meson. It is precisely this part that is the required useful quantity in the phenomenological part of the sum rules. Figures 16b and c show the "contamination."

Figure 16b represents the contribution of excited D-states and of the charmed continuum. We are used to this kind of background[26,83] and we can control it effectively within the framework of the borelization procedure. The exponentially suppressed effect can be taken into account as a small correction to the contribution of Fig. 16a, using the usual parton model of the continuum.

Figure 16c shows the "irregular" or "parasitic" imaginary part, in which there are no particles containing a heavy quark. Actually, it is due to the interaction between the light mesons A and B in the final state. The corresponding contribution to the phenomenological part of the sum rules is not suppressed by the Borel exponential, and can hardly be estimated accurately (at least, as far as I know). This is the most disagreeable point which, in principle, places a question mark over the possibility of theoretical control.

The "parasitic contamination" aparently reduces the precision of the calculation, but this precision is still acceptable (it is estimated in Ref. 56 that the uncertainty definitely does not exceed a factor of 1.3–1.5 in the amplitude). The physical argument is as follows.

The amplitude (62) as a function of Q_1^2 contains two components. One is the slow component that varies smoothly with Q_1^2 from m_D^2 to the Euclidean value $\sim -m_c^2$. (The corresponding imaginary part of the dispersion relation is shown in Figs. 16a and b.) This slow component has superimposed upon it a modulation due to interaction in the final state. The characteristic scale of this modulation in Q_1^2 is of the order of $m_c R^{-1}$. As Q_1^2 varies from m_D^2 to $\sim -m_c^2$, unitary effects due to the rescattering of "light" particles are found to vanish.

By evaluating the amplitudes in the Euclidean region, we "feel" only the slow component. Extrapolation to the physical region within the framework of the usual procedures (dispersion relation + borelization + saturation of the imaginary part by the graphs of Figs. 16a and b) yields not the true value of the amplitude at $Q_1^2 = m_D^2$, but the amplitude averaged over some near region, so that the scattering phases are averaged in the final state.

To summarize, if the "parasitic" imaginary part is discarded in the analysis, as was done in Ref. 56, the sum rules produce "semifinished" amplitudes which, in principle, must be corrected for interaction in the final state if this interaction is significant. This last step was not discussed in Ref. 56.

The result of the analysis given in Ref. 56 can be formulated in a few words, even though the original papers are quite voluminous. Three universal numbers are extracted from the QCD sum rules (more precisely, three for the D, F→PP decays and three others for the D, F→PV decays). These numbers parametrize annihilation effects and the nonfactorizable part of the spectator diagrams in the amplitudes for the two-particle decays. Suppose that, for the D→PP decays, the amplitude M_1 corresponds to the skeleton diagram of Fig. 14a (annihilation mechanism), whereas M_2 and M_3 correspond to the skeleton diagrams of Figs. 14b and c (unfactorizable part in the spectator mechanism):

$$M_1 = -0.09 \pm 0.02\ \text{GeV}^3, \qquad M_2 \approx M_3 \approx 0.11 \pm 0.03\ \text{GeV}^3. \tag{64}$$

The significant point is that the amplitude for any process D, F→PP can be written in the form

$$A\,(\mathrm{D} \to \mathrm{A} + \mathrm{B}) = A_{\text{fact}} + \sum_{i=1,2,3} \alpha_i^{\mathrm{AB}} M_i, \tag{65}$$

where the entire information about the quark composition of the final mesons is concentrated in the numerical coefficients α_i^{AB}. It is a trivial matter to establish these coefficients for each specific channel. Nontrivial dynamic information is encoded in the amplitudes M_i.

In particular, instead of (50), we now have

$$A\,(\mathrm{D}^+ \to \overline{\mathrm{K}^0}\pi^+) = -\frac{iG_F}{\sqrt{2}} V_{cs} V_{ud}^* \left[\frac{4}{3} m_D^2 f_+^K f_\pi (C_1 + C_2) - (C_1 M_3 + C_2 M_2)\right]. \tag{66}$$

The relative decay probability is then about 2.5%, which must be compared with the experimental result $2.9 \pm 0.4 \pm 0.6\%$. Predictions are obtained for the absolute values of the partial widths, and the following lifetimes can be used to convert these into relative probabilities (some of the results are collected in Table IV):

$$\tau\,(\mathrm{D}^+) = 9\cdot 10^{-13}\,\text{s}, \quad \tau\,(\mathrm{D}^0) = 4.5\cdot 10^{-13}\,\text{c}, \quad \tau\,(\mathrm{F}) = 3\cdot 10^{-13}\,\text{s}. \tag{67}$$

We note particularly that the predicted value for the celebrated ratio $\Gamma(\mathrm{D}^0 \to \overline{\mathrm{K}}^0\pi^0)/\Gamma(\mathrm{D}^0 \to \mathrm{K}^-\pi^+)$ is 0.25 ($\sim 10^{-2}$–10^{-3} in the naive spectator model!), whereas experiment yields 0.35 ± 0.1 (Ref. 84). There is also excellent agreement in the case of the $\mathrm{D}^0 \to \overline{\mathrm{K}^0}\varphi$ decay, which is a measure of the departure from the spectator model.

As expected,[53] nearly half (by probability of the two-particle meson decays contain ρ^+ and K^{*0} in the final state.

If we sum over all these channels (plus the semileptonic

TABLE IV. Relative probabilities of two-particle decays of D- and F-mesons.

Decay	Relative probability,[56] %	Spectator model, %	Experiment, %
$D^+ \to K^0\rho$	14	9	14,1±4,1±2,7
$D^0 \to K^-\pi^+$	6,4	2,6	5,4±0,4
$D^0 \to \overline{K^0}\pi^0$	1,5	0	2,2±0,6±0,4
$D^0 \to K^-\rho^+$	15	4,5	15,1±3,2±3
$D^0 \to \overline{K^0}\varphi$	1,3	0	1,5±0,4
$F^+ \to \overline{K}^{*0}K^+$	1,5	0	
$F^+ \to \eta\rho$	4,5	2,3	
$F^+ \to \pi^+\eta$	2,5	1,2	

modes), we find that we have to assign about 60% of the total width to D^0 and D^+. The remaining 40% appear to "leak" into the $D \to VV$, $D \to SP$ modes (where S is the scalar meson) and the three-particle modes.

There are practically no reliable data on the F-mesons, so that all the results are essentially pure predictions. $F^+ \to \varphi\pi^+$, $\eta\rho^+$, $\eta\pi^+$ have the largest weights among the two-particle channels.

The situation as far as the F-meson is concerned is somewhat puzzling. According to Ref. 56, the two-particle modes PP and PV account for only about 12% of the total width, which is much less than the result for D^0- and D^+-mesons. The question is: why are these D and F decays so dissimilar? Is there some unusual aspect of the theory that has escaped our attention so far, but lies behind this fact?

On the other hand, the problem would have been partially solved (or, at least, made easier) if the F^+ lifetime were to be greater than that given by (67). As noted in Section 3.3, it is more likely that $\tau(F^+) \simeq \tau(D^0)$. The relative probabilities for the F-mesons in Table IV must then be multipled by 1.5.

In conclusion, let us consider the rule for discarding the $1/N_c$ contributions,[53,60–62] which has alrady been mentioned above. Dynamic analysis[56] actually provides a theoretical basis for this rule. Thus, it is readily verified that the $1/N_c$ terms in the factorizable part of the spectator diagrams are almost completely canceled numerically by the unfactorizable amplitudes M_2 and M_3 in almost all cases ($M_{2,3}$ are, of course, parametrically $\sim 1/N_c$). It is predicted in Ref. 56 that the cancellation does not occur in a few rare exceptions ($D^0 \to K^*\pi^+, \overline{K^0}\omega$). As the experimental precision of $\mathrm{Br}(D^0 \to K^{*-}\pi^+ \lesssim \overline{K^0}\omega)$ improves, it will in principle become clearer which of these two approaches is the more successful, namely, the phenomenological prescription[62] or the sum rule predictions.[56]

The annihilation mechanism remains uncompensated and must be taken into account separately. Analysis[56] shows that it should increase the hadron width D^0 by about 20%.

3.6. Strong and radiative decays

It goes without saying that most c- and b-hadrons are excited states that decay via the strong interaction. Some best known processes of this kind are

$$D^* \to D\pi \quad \text{or} \quad \Sigma_c \to \Lambda_c\pi.$$

It is clear that experimental data will become much more numerous in the next few years, and will provide an excellent testing ground for different theoretical methods and approaches.

Unfortunately, the subject is not well advanced from the theoretical point of view. The $D^* \to D\pi$ decays were considered in Ref. 85 within the framework of the QCD sum rules. It was found that

$$\Gamma(D^{*+} \to D^0\pi^+) = 8.5 \pm 2.5\ \text{keV},$$
$$\Gamma(D^{*+} \to D^+\pi^0) = 3.6 \pm 1.5\ \text{keV},$$
$$\Gamma(D^{*0} \to D^0\pi^0) = 5.7 \pm 2\ \text{keV}. \qquad (68)$$

It is important to note that Ref. 85 was confined to D^* decays, although the techniques used in that paper are very effective for any other two-particle decays involving the emission of a pion and very little energy release. For those who are familiar with sum rules, we add that the authors of Ref. 85 analyze the correlation function

$$\Pi_\mu^A(p, q) = i \int d^4x e^{-ipx} \langle T\{\bar{c}\gamma_\mu u(x), \bar{u}\gamma_5 c(0)\}\rangle_A \qquad (69)$$

in the external field $A_\mu^a(x)$, where a is the isotopic symbol and the QCD Lagrangian includes the additional term

$$\Delta\mathcal{L}_A = A_\mu^a e^{iqx}\bar{q}\tau^a\gamma_\mu\gamma_5 q, \qquad (70)$$

where τ^a are the Pauli matrices operating in the isotopic space, and $q = u,d$. A similar technique is employed in Refs. 86–88 and reduces substantially the volume of the necessary computations. It is open to further applications without further additional modification. One would hope that this method will soon be used to solve other problems, e.g., baryon decays of the form $\Sigma_c \to \Lambda_c\pi$.

Since this has not yet been done, a simplified estimate of the $\Sigma_c \to \Lambda_c\pi$ decay probability has been proposed by Voloshin. His line of argument is as follows. To be specific, let us concentrate our attention on the transition $\Sigma_c^{++} \to \Lambda_c^+ + \pi^+$ and consider the matrix element

$$\langle\Lambda_c^+ | \bar{d}\gamma_\mu\gamma_5 u | \Sigma_c^{++}\rangle \approx g_A \bar{\psi}_f\gamma_\nu\gamma_5\psi_i\left(g_{\mu\nu} - \frac{g_\mu q_\nu}{q^2}\right),$$

where $q = p_{\Sigma_c} - p_{\Lambda_c}$ and, for small q^2, the parameter g_A on the right-hand side becomes a constant. ψ_i and ψ_f are the wave functions of the initial and final baryons, respectively. The trasversality of the right-hand side follows from the conservation of the axial current $\bar{d}\gamma_\alpha\gamma_5 u$ in the chiral limit.

If we parametrize the $\Sigma_c \to \Lambda_c\pi$ transition in the form

$$A(\Sigma_c^{++} \to \Lambda_c^+ + \pi^+) = g_\pi\bar{\psi}_f\gamma_5\psi_i\varphi_\pi,$$

we find that g_π will obviously satisfy the Goldberger-Treiman type condition

$$(M_{\Sigma_c} + M_{\Lambda_c})g_A = g_\pi f_\pi.$$

Next,

$$\Gamma(\Sigma_c^{++} \to \Lambda_c^+\pi^+) = g_\pi^2 \frac{|\mathbf{q}|}{8\pi} \frac{(M_{\Sigma_c} - M_{\Lambda_c})^2 - m_\pi^2}{M_{\Sigma_c}^2}.$$

Substituting $g_\pi = (M_{\Sigma_c} + M_{\Lambda_c})g_A f_\pi^{-1}$, we find that

$$\Gamma(\Sigma_c^{++} \to \Lambda_c^+\pi^+) \approx 6.5 g_A^2\ \text{MeV}.$$

An estimate for g_A in this expression can be obtained by comparing the two matrix elements $\langle\Lambda_c^+|\bar{d}\gamma_\mu\gamma_5 u|\Sigma_c^{++}\rangle$ and $\langle\Lambda_s^0|\bar{d}\gamma_\mu\gamma_5 u|\Sigma_s^+\rangle$. In the naive quark model it is natual to expect that the constants g_A in these matrix elements are the same; in the latter case, g_A ($\Sigma_s \to \Lambda_s$) is known from experiment and is approximately equal to 0.65. If this is so, then

$$\Gamma(\Sigma_c^{++} \to \Lambda_c^+ + \pi^+) \approx 2.7\ \text{MeV}.$$

A future and more accurate analysis must, in my view, confirm the validity of this simple estimate to within at least a factor of two.

A totally different approach to the problem was developed in Refs. 89 and 90, where the three-meson constants are extracted from the Regge pole theory (in combination with duality ideas and the picture of gluon tubes[46,47]).

Since I am not a specialist, I cannot estimate the precision of this method, and simply reproduce the results without comment:[15]

$$\Gamma(D^{*+} \to D^0\pi + D^+\pi^0) = \begin{cases} 20\ \text{keV}^{89}, \\ 35\ \text{keV}^{90}. \end{cases} \qquad (71)$$

Since the experimental absolute widths are unknown, (71) can only be compared with the prediction obtained from the sum rules, i.e., about 12 keV [see (68)]. The discrepancy is by up to a factor of 2–3, and the reason for this must be explained.

Pion decays of b-mesons have also been discussed in Ref. 90. Because the spin effects are small, the $B^* - B$ mass splitting (see Section 3.2) and pion decays of the B^*-mesons are forbidden by energy considerations, and only electromagnetic processes of the form $B^* \to B\gamma$ are allowed. Definite predictions were obtained in Ref. 90 for the width $\Gamma(B^{**} \to B\pi)$ of the $J^P = 2+$ mesons. We note, however, that the estimated value of $M_{B^{**}}$ obtained in Ref. 90 is even lower than that in Refs. 46 and 47 (see Table III). This last result is, in turn, lower by 400 MeV than that prediced by the sum rules.

I must now draw attention to one problem that is important and interesting in practice although it does not appear to have been examined in the literature. The discovery of the (cs)-meson with probable quantum numbers $J^P = 1^+$ in a neutrino reaction was reported in Ref. 48. This axial meson was found in the two-photon cascade.

$$F_1 \to F^* + \gamma,$$
$$\hookrightarrow F + \gamma,$$

and it is claimed that $M_{F_1} = 2.55 \pm 0.060$ GeV, while the photon mode of the decay $F_1 \to F^*\gamma$ contributes an appreciable fraction ($\sim 100\%$) to the total width. On the other hand, when $M_{F_1} > 2.51$ GeV, only strong decays are possible:

$$F_1 \to F\pi\pi, \quad F_1 \to D^*K.$$

If the first of these is highly suppressed by the Zweig rule (plus additional suppression due to the emission of soft pions), then, in the process $F_1 \to D^*K$, there is no apparent reason for special dynamic suppression.[16] If $M_{F_1} = 2.55$ GeV, which is theoretically the most likely result, provided[40] $M_{D_1} = 2.420 \pm 0.006$ GeV, the phase volume for $F_1 \to D^*K$ is lower by a factor of only 2.5 than for $F_1 \to F^*\gamma$. We then ask: how can we reconcile this situation with the suggested[48] dominance of the radiative mode? It is clear that a more accurate theoretical calculation of $\Gamma(F_1 \to F^*\gamma)$ and $\Gamma(F_1 \to D^*K)$ must be carried out, and unexpected features may then emerge.

The excited states of c- and b-hadrons differ from the "old" hadrons by the fact that radiative decays play a more important role in these systems. The B^* mesons have just been mentioned in this connection. An analogous situation is the energy-forbidden pion decay and the existence of a single open channel, i.e., the radiative channel, which may also occur for the baryons Ω_c^*, $\Xi_c^{0(S)}$, and $\Xi_c^{+(S)}$ (see Section 3.2). As far as the D^*-meson is concerned, it may undergo strong decay to $D\pi$, but the $D\gamma$ channel successfully competes with this because it involves little energy release.

The theoretical advances made in the description of radiative decays are very modest. In principle, the traditional nonrelativistic potential model could be used provided, of course, it is properly evaluated and its adjustable parameters are determined from other characteristics that are well established eperimentally. In particular, in the language of the nonrelativistic quark model, the $D^* \to D\gamma$ decays constitute an M_1-transition, for which the probability calculation is a trivial matter and the probability itself does not even depend on the quark wave functions because the overlap integral is equal to unity in the nonrelativistic limit. Specifically,

$$\Gamma(D^* \to D\gamma)_{\text{nonrel}} = \frac{\alpha}{3} k^3 \left(\frac{Q_c}{M_c} - \frac{Q_{\bar{q}}}{M_q}\right)^2, \qquad (72)$$

where **k** is the photon momentum, Q_c and Q_q are the charges of the quarks c and $\bar{q}$, respectively (for example, $Q_c = 2/3$), and M_c and M_q are the masses of the "block" quarks. If we assume that $M_c = 1.55$ GeV and $M_u = M_d = 380$ MeV, then $\Gamma(D^{*0} \to D^0\gamma) \simeq 29$ keV and $\Gamma(D^{*+} \to D^+\gamma) \simeq 1$ keV. For comparison, here are the results obtained from the QCD sum rules:[83]

$$\Gamma(D^{*0} \to D^0\gamma) = 14 \pm 7\ \text{keV},$$
$$\Gamma(D^{*+} \to D^+\gamma) = 0.67 \pm 0.2\ \text{keV}. \qquad (73)$$

Despite the relatively large uncertainty, there is an indication of a discrepancy between the nonrelativistic estimates and (73).

We note in conclusion of this Section that an independent analysis of some of the pion and photon decays was made in Ref. 93 using the constituent quark model.

4. CONCLUSION

Several of the problems that I have not touched upon in this review seem important from the theoretical point of view, but have not, so far, found satisfactory (final) solution. Following established tradition, let us enumerate some of them.

Exclusive weak decays of b-hadrons constitute a problem that, to all intents and purposes, has not even begun to be investigated. It involves features and difficulties that have not yet been overcome. It is interesting that the B-meson can decay into a baryon-antibaryon pair—a situation that we encounter for the first time. As far as I know, there are no reliable theoretical predictions. It has become clear that we understand relatively little, even in the case of inclusive decays. Indeed, the M_X spectra in the $B \to J/\psi + X$, $B \to l\nu + X$ decays, where X is a hadronic system, are apparently significantly different from the parton spectra, despite the fact that the mass of the b-quark (~ 4.8 GeV) is large enough and existing ideas suggest that it lies in the asymptotic region in which the quark-hadron duality is already valid. This impedes the solution of a practically important problem, namely, the determination of the poorly known elements of the Kobayashi-Maskawa matrix from V_{bu} data.[17)]

Moreover, the theoretical picture of strong and radiative decays in excited states can hardly be regarded as complete.

This review has been an attempt to demonstrate that the physics of charmed and beautiful hadrons is an interesting and lively subject that will continue to develop for some time. Theory and experiment are advancing side by side, assisting one another. New problems that are difficult to foresee will undoubtedly arise in the course of research. It is precisely this aspect, i.e., the hope that something unusual and unexpected will turn up, is, of course, the strongest stimulus for both experimenters and theorists.

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1) We assume that the reader is familiar with basic ideas on c- and b-hadrons, which can be found in standard textbooks, for example, in Refs. 1 and 2.

2) According to the new nomenclature, proposed by the Particle Data Group (PDG), the F-mesons are to be denoted by the symbol D_s. In this review, we shall use the earlier designation for the F-mesons.

3) A discussion of the Kobayashi-Maskawa matrix is outside the scope of my present task; see the recent review by Buras.[6] I merely note that $|V_{bu}|$ and $|V_{td}| < 0.02$ and $|V_{bc}|$ and $|V_{ts}| < 0.07$.

4) It is possibly worthwhile to recall that analogous η-factors were first calculated in Refs. 22 for the transition with $\Delta S = 2$.

5) By factorization, we understand the vacuum saturation of all possible channels in the matrix element $\langle \bar{B}^0|(\bar{b}\Gamma_\mu q)(\bar{b}\Gamma_\mu q)|B^0\rangle \to (8/3)|\langle 0|(\bar{b}\Gamma_\mu q)|B^0\rangle|^2$. In the literature, the fraction 8/3 on the right-hand side is sometimes replaced by $8B/3$, where the coefficient B must parametrize deviations from factorization. It is possible to produce arguments (sufficiently convincingly, in my view) showing that $B \simeq 1$; see, in particular, Section 3.4. We assume that $B = 1$ in (12).

6) In the language of the nonrelativistic quark model, the constant f_P is proportional to the wave function at the origin; see below.

7) In other words, the momenta of the external quark lines are $p_q^2 \sim R^{-2}$ and $p_Q^2 - m_Q^2 \sim R^{-1} m_Q$.

8) As far as we know, the QCD sum rules have not been used to perform such calculations for beautiful baryons. Shuryak[29] has reported asymptotic results ($m_Q \to \infty$) for the mass splitting between baryons Λ and Σ, but this is subject to considerable uncertainty: $M(\Sigma_Q) - M(\Lambda_Q) = 400 \pm 250$ MeV. This mass difference should not depend on m_Q as $m_Q \to \infty$, and other estimates show that it is ~ 200 MeV.

9) The question of inclusive decays was touched upon in Ref. 62 in the context of the $1/N_c$ approach. As in Ref. 53, it is noted in Ref. 62 that this recipe leads to a considerable overall enhancement of nonleptonic decays of D-mesons [see (42)] as compared with the standard treatment. This is a desirable feature from the phenomenological point of view.

10) As already mentioned, because of the pseudo-Euclidean kinematics this is not a true Wilson operator expansion. The quark-hadron duality must be introduced at this point.

11) For example, the estimates of $\langle \Lambda_c^+ | \mathscr{L}_{\rm eff} | \Lambda_c^+ \rangle$, given in Refs. 53 nd 54, differs from the estimate based on the nonrelativistic potential model for Λ_c^+ by a factor of about 2.3. A.M. Badalyan *et al.* have obtained results for the quantities in which we are interested, using the potential model.

12) This statement is valid to within the "penguin" graphs, which are numerically suppressed (see Ref. 54 for further details).

13) Factorization seems to work with reasonable precision for K-mesons and hyperons.[7] Here, this hypothesis at least reproduces all the consequences of PCAC, which is a theoretical justification. Similar considerations, relying on the low-energy release and PCAC in K-meson and hyperon decays, show that interaction in the final state can be neglected. In two-particle decays of charmed mesons, the energy released is large, and these ideas do not work.

14) A conceptually similar method was used previously in Ref. 82 to analyze hyperon decays.

15) Some theoretical details are treated in Refs. 89 and 90 differently.

16) The trivial hypothesis $M_{F_1} < 2.51$ GeV is not inconsistent with Ref. 48, but is hardly acceptable from the theoretical point of view. Actually, in this case, the mass difference $M_{F_1} - M_{D_1}$ would be less than 90 MeV, which is absolutely unthinkable (we recall that $m_s \approx 150$ MeV).

17) It is interesting that the authors of Ref. 5 foresaw the serious difficulty involved in extracting V_{bu} from inclusive leptonic spectra in the energetically forbidden region for the $b \to c$ transition. They proposed to determine V_{bu} from $\Gamma(B^+ \to \nu_\tau \tau^+)$. The corresponding relative probability is expected to be 10^{-4}. Only experimenters can decide whether this project is too fanciful.

[1] F. E. Close, *An Introduction to Quarks and Partons*, Academic Press, London, 1979.
[2] L. B. Okun', Leptony i kvarki, Nauka, M., 1981 [English transl., Leptons and Quarks, North-Holland, Amsterdam, 1982].
[3] Ya. I. Azimov and V. A. Khoze, Usp. Fiz. Nauk **132**, 379 (1980) [Sov. Phys. Usp. **23**, 699 (1980)].
[4] E. D. Zhizhin *et al.*, Yad. Fiz. **36**, 930 (1982) [Sov. J. Nucl. Phys. **36**, 545 (1982)].
[5] N. G. Ural'tsev and V. A. Khoze, Usp. Fiz. Nauk **146**, 507 (1985) [Sov. Phys. Usp. **28**, 617 (1985)].
[6] A. Buras, Talk at the EPS Conf., Bari, 1985; Preprint MPI-PAE/Pth64-85, 1985.
[7] M. K. Gaillard, B. W. Lee, and J. Rosner, Rev. Mod. Phys. **47**, 277 (1975); A. L. Kingsley, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D **11**, 1919 (1975).
[8] Ya. Azimov and A. Ioganssen, Yad. Fiz. **33**, 388 (1981) [Sov. J. Nucl. Phys. **33**, 205 (1981)].
[9] J. Ellis *et al.*, Nucl. Phys. B **131**, 285 (1977).
[10] J. S. Hagelin, *ibid.* **193**, 123 (1981).
[11] F. J. Gilman and J. S. Hagelin, Phys. Lett. B **133**, 123 (1981).
[12] A. A. Anselm and Y. I. Azimov, *ibid.* **84**, 72 (1979).
[13] C. Bernabeau and C. Jarskog, Z. Phys. G **8**, 233 (1981).
[14] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980).
[15] A. B. Carter and A. I. Sanda, Phys. Rev. D **23**, 1567 (1981).
[16] I. I. Bigi and A. I. Sanda, Nucl. Phys. B **193**, 85 (1981).
[17] I. I. Bigi and A. I. Sanda, Phys. Rev. D **29**, 1393 (1984).
[18] L. L. Chau and H.-Y. Cheng, Phys. Rev. Lett. **53**, 1037 (1984).
[19] L. L. Chau, Phys. Lett. B **165**, 429 (1985).
[20] Ya. I. Azimov, N. G. Ural'tsev, and V. A. Khoze, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 317 (1986) [JETP Lett. **43**, 409 (1986)].
[21] Y. I. Azimov, N. G. Uraltsev, and V. A. Khoze, Preprint LINP-600, Leningrad, 1986.
[22] A. I. Vaĭnshteĭn *et al.*, Yad. Fiz. **23**, 1024 (1976) [Sov. J. Nucl. Phys. **23**, 540 (1976)]. V. A. Novikov *et al.*, Phys. Rev. D **16**, 223 (1977); Yu. P. Malakyan, Yad. Fiz. **25**, 441 (1977) [Sov. J. Nucl. Phys. **25**, 238 (1977)].
[23] L. B. Okun, B. M. Pontekorvo, and V. I. Zakharov, Lett. Nuovo Cimento **13**, 218 (1975).
[24] A. Anselm *et al.*, Phys. Lett. B **156**, 102 (1985); U. Türke *et al.*, Nucl. Phys. B **285**, 313 (1985); X.-G. He and S. Pakvasa, Phys. Lett. B **156**, 236 (1985); I. I. Bigi, Z. Phys. C **27**, 303 (1985).
[25] E. H. Thorndike, Ann. Rev. Nucl. Part. Sci. **35**, 195 (1985).
[26] M. Shifman, A. Vaĭnshteĭn, and V. Zakharov, Nucl. Phys. B **147**, 395 (1979).
[27] V. Novikov *et al.*, Proc. Intern. Conf. Neutrinos-78, ed. by E. Fowler, Purdue Univ., 1978, p. 278.
[28] L. Reinders, S. Yazaki, and H. Rubinstein, Phys. Lett. B **104**, 305 (1981).
[29] E. Shuryak, Nucl. Phys. B **198**, 83 (1982).
[30] A. R. Zhitnitskiĭ, I. R. Zhitnitskiĭ, and V. L. Chernyak, Yad. Fiz. **38**, 1277 (1983) [Sov. J. Nucl. Phys. **38**, 775 (1983)].
[31] T. M. Aliev and V. L. Eletskiĭ, Yad. Fiz. **38**, 1537 (1983) [Sov. J. Nucl. Phys. **38**, 936 (1983)].
[32] B. Yu. Blok and V. L. Eletskiĭ, Yad. Fiz. **42**, 1246 (1985) [Sov. J. Nucl. Phys. **42**, 787 (1985)].
[33] Ya. I. Azimov, L. L. Frankfurt, and V. A. Khoze, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 373 (1985) [JETP Lett. **24**, 338 (1985)].
[34] M. B. Voloshin and M. A. Shifman, Yad. Fiz. **45**, No. 2 (1987) [Sov. J. Nucl. Phys. **45**, No. 2 (1987)].
[35] L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
[36] P. Taxil, Proc. Twentieth Conf. Moriond., January 1985, La Plague, France, 1985; A. Martin, Preprint CERN-TH-4382, Geneva, 1986.
[37] A. De Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. D **12**, 147 (1975).

[38] P. Hazenfratz *et al.*, Phys. Lett. B **94**, 401 (1980); J. M. Richard, *ibid.* **100**, 515 (1981); **139**, 408 (1984); A. Martin and J. M. Richard, *ibid.* **115**, 323 (1982); J. M. Richard and P. Taxil, *ibid.* **128**, 453 (1983); J. L. Basdevant and S. Bourkas, Z. Phys. G **30**, 103 (1986).
[39] E. Yu. Blok and V. L. Eletskii, Yad. Fiz. **44**, 774 (1986) [Sov. J. Nucl. Phys. **44**, 500 (1986)].
[40] H. Albrecht *et al.*, ARGUS Collab.; Phys. Rev. Lett. **56**, 549 (1986).
[41] E. Eichten *et al.*, Phys. Rev. D **21**, 203 (1980).
[42] B. L. Ioffe, Nucl. Phys. B **188**, 317 (1981); **191**, 591.
[43] L. J. Reinders and H. R. Rubinstein, Phys. Lett. B **145**, 108 (1984).
[44] V. M. Belyaev and Yu. B. Blok, Z. Phys. C **30**, 151 (1986).
[45] B. Yu. Blok and V. L. Eletsky, *ibid.* **30**, 229 (1986).
[46] A. V. Kaidalov, *ibid.* **12**, 63 (1982).
[47] A. B. Kaidalov, Proc. ITEP Winter School of Physics, 1986, Vol. 1, p. 51.
[48] A. Asratyan *et al.*, Proc. Intern. Conf. on Neutrinos-86, 1986.
[49] R. D. Peccei and R. Rück, Proc. Ahrenshoop Symposium on Special Topics in Gauge Field Theories, Zeuten, DDR, 1981, p. 8; B. Guberina *et al.*, Phys. Lett. B **89**, 111 (1979).
[50] T. Kobayashi and N. Yamazaki, Prog. Theor. Phys. **65**, 775 (1981); Y. Koide, Phys. Rev. D **20**, 1739 (1979); K. Jagannathan and V. S. Mathur, *ibid.* **21**, 3165 (1980).
[51] G. Altarelli and L. Maiani, Phys. Lett. B **118**, 414 (1982); H. Sawayanagi *et al.*, Phys. Rev. D **27**, 2107 (1983).
[52] N. Bilić, B. Guberina, and J. Trampetić, Nucl. Phys. B **248**, 261 (1984).
[53] M. B. Voloshin and M. A. Shifman, Yad. Fiz. **41**, 187 (1985) [Sov. J. Nucl. Phys. **41**, 120 (1985)].
[54] M. B. Voloshin and M. A. Shifman, Zh. Eksp. Teor. Fiz. **91**, 1180 (1986) [Sov. Phys. JETP **64**, 698 (1986)].
[55] W. Bernreuter, O. Nachtmann, and B. Stech, Z. Phys. c **4**, 257 (1980); M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **44**, 7, 962 (1980); H. Fritzsch and P. Minkowski, Phys. Lett. B **909**, 455 (1980).
[56] B. Yu. Blok and M. A. Shifman, Yad. Fiz. **45**, 1-3 (1987).
[57] J. Leveille, Proc. CLEO Collaboration Workshop on B-meson, CLEO Preprint 81/05, Univ. of Rochester, 1981, p. 3.
[58] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33**, 608 (1974); G. Altarelli and L. Maiani, Phys. Lett. B **52**, 351 (1974).
[59] M. B. Voloshin and K. A. Termartirosyan, Theory of Gauge Interactions Between Elementary Particles (in Russian), Energoatomizdat, M., 1984.
[60] N. Deshpande, M. Gronau, and D. Sutherland, Phys. Lett. B **90**, 431 (1980); D. Tadic and J. Trampetić, *ibid.* **114**, 179 (1982).
[61] M. Bauer and B. Stech, *ibid.* **152**, 380 (1985); B. Stech, Proc. Moriond Workshop of Flavour Mixing and CP-Violation, ed. by J. Tran Tranh. Van, La Plagne, 1985, p. 151.
[62] A. J. Buras, J. M. Gerard, and R. Buckl, Nucl. Phys. B **268**, 16 (1986).
[63] C. Casco and M. C. Touboul, Preprint CERN EP/85-176; Rev. Mod. Phys. (in press).
[64] C. Davies and S. H. Tye, Cornell Preprint CLNS-85/635, 1985.
[65] R. Rückl, Proc. Twenty-Second Intern. Conf. on High Energy Physics, ed. by A. Meyer, Leipzig, 1984, p. 135.
[66] A. I. Vaĭnshteĭn and I. B. Khriplovich, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 141 (1973) [JETP Lett. **18**, 83 (1973)]. M. K. Gaillard and B. W. Lee, Phys. Rev. D **10**, 897 (1974).
[67] R. D. Peccei, Preprint DESY 85/121, 1985.
[68] G. Goldhaber *et al.*, Phys. Lett. **37**, 255 (1976).
[69] D. Hitlin, Intern. Symposium on Production and Decays of Heavy Flavours, ed. by K. Schubert and K. Waldi, Heidelberg, 1986, p. 99.
[70] M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. **47**, 277 (1975); J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B **100**, 313 (1975); N. Cabibbo and L. Maiani, Phys. Lett. B **73**, 418 (1978); D. Fakirov and B. Stech, Nucl. Phys. B **133**, 315 (1978).
[71] A. I. Vaĭnshteĭn, V. I. Zakharov, and M. A. Shifman, Zh. Eksp. Teor. Fiz. **72**, 1275 (1977) [Sov. Phys. JETP **45**, 670 (1977)].
[72] T. M. Aliev, V. L. Eletskiĭ, and Ya. I. Kogan, Yad. Fiz. **40**, 823 (1984) [Sov. J. Nucl. Phys. **40**, 527 (1984)].
[73] V. A. Khoze and M. A. Shifman, Usp. Fiz. Nauk **140**, 3 (1983) [Sov. Phys. Usp. **26**, 387 (1983)].
[74] L. S. Dulyan and A. Knodjamirian, Preprint EFT 410(17)-80, Erevan, 1980; M. Deshpande, M. Gronau, and D. Sutherland, Phys. Lett. B **90**, 431 (1980); Y. Igarashi, M. Kuroda, and S. Kitakado, *ibid.* **93**, 125; M. Gronau and D. Sutherland, Nucl. Phys. B **183**, 367 (1981).
[75] S. P. Rosen, Phys. Lett. **44**, 4 (1980); I. I. Bigi, Z. Phys. **5**, 313 (1980); Phys. Lett. B **90**, 177 (1980); G. Eilam and M. Gronau, *ibid.* **96**, 391.
[76] G. Altarelli *et al.*, Phys. Lett. B **99**, 141 (1981).
[77] R. G. Miller and B. McKellar, Phys. Rev. D **27**, 1142 (1983); A. Kamal, *ibid.* **31**, 1055 (1985).
[78] H. J. Lipkin, Phys Rev. Let. **44**, 710 (1980); A. N. Kamal and E. P. Cooper, Z. Phys. C **8**, 67 (1981); A. N. Kamal, Preprint SLAC-PUB-3443, 1984.
[79] V. L. Chernyak and A. R. Zhitnisky, Nucl. Phys. B **201**, 492 (1982).
[80] B. Guberina, D. Tadic, and J. Trampetic, *ibid.* **B202**, 317.
[81] H. Albrecht *et al.*, ARGUS Collab. Phys. Lett. B **158**, 525 (1985); C. Bedek *et al.*, CLEO Collab. Preprint CLNS 86/715 CLEO-86-2, 1986.
[82] V. Khatsimovskiĭ, Yad. Fiz. **42**, 1236 (1985) [Sov. J. Nucl. Phys. **42**, 781 (1985)]. Nucl. Phys. B **277**, 298 (1986).
[83] B. L. Ioffe and A. V. Smilga, Phys. Lett. B **114**, 353 (1982); V. A. Nesterenko and A. V. Radyushkin, *ibid.* B **115**, 410.
[84] J. Hauser, Preprint CALT-68-1275, 1985.
[85] V. L. Eletsky and Y. I. Kogan, Z. Phys. C **28**, 155 (1985).
[86] B. L. Ioffe and A. V. Smilga, Nucl. Phys. B **232**, 109 (1984).
[87] U. Balitsky and A. Yung, Phys. Lett. B **129**, 328 (1983).
[88] V. M. Belyaev and Ya. I. Kogan, Yad. Fiz. **40**, 1035 (1984) [Sov. J. Nucl. Phys. **40**, 659 (1984)]. Phys. Lett. B **135**, 273 (1984).
[89] P. E. Volkovitskiĭ and A. B. Kaĭdalov, Yad. Fiz. **35**, 1556 (1982) [Sov. J. Nucl. Phys. **35**, 909 (1982)].
[90] K. Hikasa and K. Igi, Phys. Rev. D **23**, 2027 (1981).
[91] L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Nucl. Phys. B **196**, 125 (1982); M. Shifman and T. Aliev, Phys. Lett. B **112**, 401 (1982).
[92] L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Nucl. Phys. B 186, 109 (1981).
[93] J. Rosner, Preprint EFI 85/91, 1985; Comm. Nucl. and Part. Phys. (in press).
[94] M. B. Voloshin, N. G. Ural'tsev, V. A. Khoze, and M. A. Shifman, Yad. Fiz. **45** (1987).

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