Vacuum in a homogeneous gravitational field and excitation of a uniformly accelerated detector[1)]

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The theory of quantum effects in uniformly accelerated frames of reference and in a homogeneous gravitational field is discussed. Ways of describing processes of excitation of, and radiation from, a uniformly accelerated detector in a uniformly accelerated and in an inertial frame of reference are compared. Particular attention is devoted to a discussion of the equivalence principle for quantum phenomena in connection with the excitation of a uniformly accelerated detector and a detector at rest in a homogeneous gravitational field.

CONTENTS

1. INTRODUCTION

In classical physics, the concept of empty space was used; physically, this meant a certain spatial region devoid of particles and fields. Such an empty space can be regarded as a synonym for the vacuum of classical nonrelativistic physics. The "absolute space" introduced by Newton that "in its own nature, without relation to anything external, remains always similar and immovable," played the role of an ideal inertial frame of reference and simultaneously could be regarded as the realization of the classical vacuum. Using the words of Einstein, "the idea of independent existence of space and time could be expressed as follows: If matter were to disappear, there would remain only space and time (a kind of scene on which the physical phenomena unfold)".[1] However, a scene that is completely independent of matter—absolute space—is a metaphysical category, since it is not known how physical reality is to be associated with it. Therefore, in mechanics absolute space was actually replaced by practically realized inertial systems, in the first place, the astronomical frame of reference (the coordinate origin in this system is placed at the center of mass of the solar system, and the axes are directed to "fixed stars"; in this connection, see, for example, Ref. 2 and the literature quoted there). With the development of optics, electrodynamics, and the field concept the ideas of an ether, which had already been introduced earlier, came to the fore. The ether was assumed to be a particular substance that fills all space and through which the electromagnetic excitations propagate. Ultimately, Lorentz was forced to assume that the "parts of the ether are not displaced at all relative to each other. Thus, the ether appeared as the embodiment of a space absolutely at rest."[1)]

The fate of this immobile ether was, however, the same as that of Newton's absolute space; for the ether could not be given any physical content unless one considers the possibility of regarding the ether as an inertial frame of reference. In fact, the special theory of relativity legitimized this point of view—in the framework of special relativity one can assume that "the ether does not exist at all. Electromagnetic fields

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are not the states of a certain medium but independently existing realities that cannot be reduced to anything else and which, like the atoms of ponderable matter, are not associated with any carriers."[3] Therefore, it was usually accepted that special relativity had driven the ether out of physics, and the very word "ether" is not used in the modern literature. Against this one should not object, since the old concept of the ether was, one may say, discredited. However, the creation of special relativity by no means led to the disappearance of the notion of empty space or of inertial frames of reference different from noninertial systems. Einstein actually attempted to rehabilitate or, rather, reanimate the expression "ether," making it more precise. Thus, already in the speech we have quoted, "Äther und Relativitätstheorie," given in 1920, he said: "However, closer examination shows that the special theory of relativity does not require an unconditional rejection of the ether. One can accept the existence of an ether, but one must not worry about ascribing it a definite state of motion; in other words, abstracting, one must remove from it the last mechanical property that Lorentz left it." This idea is clarified in the same speech: "With regard to the mechanical nature of the Lorentz ether, one could say jokingly that Lorentz left it only one mechanical property—immobility. To this one may add that the only change introduced by the special theory of relativity in the concept of the ether was to deprive the ether of this, its last mechanical property."

Of course, the name is not the point. The expression "ether" was simply replaced by the term "vacuum" or "physical vacuum." As we have already emphasized, it is not possible to get by without this concept even in classical physics, especially when the part played by the gravitational field is taken into account. It is for this reason, as one can hardly doubt, that Einstein returned after the creation of the general theory of relativity to a discussion of the concept of the ether.[3] Instead of presenting the situation with our own words, we again return here to the same talk of Einstein, since what he said there is still perfectly accurate today; simultaneously, we achieve historical authenticity. Thus, according to general relativity, "the metrical properties of the space-time continuum in the neighborhood of individual space-time points are different and depend on the matter distribution outside the considered region. The idea of physically empty space is definitively eliminated by such space-time variability of scales and clocks. Accordingly, the recognition of the fact that "empty space" is not homogeneous and isotropic in the physical respect forces us to describe its state by means of ten functions—the gravitational potentials $g_{\mu\nu}$. But then the concept of the ether again acquires a certain content, which is quite different from the content of the ether concept in the mechanical theory of light. The ether of general relativity is a medium that by itself is devoid of *all* mechanical and kinematic properties but at the same time determines mechanical (and electromagnetic) processes." The general conclusion is as follows: "Summarizing, one can say that the general theory of relativity equips space with physical properties; thus, in this sense the ether exists. According to the general theory of relativity, space is unthinkable without the ether; indeed, in such a space not only would the propagation of light be impossible but there could not exist scales and clocks and there would not be any space-time distances in the physical sense of the word."[3]

Thus, although the expression "ether" was not retained, the concept of the physical vacuum that replaced it was already needed in prequantum physics. We believe it is appropriate to emphasize this circumstance not only in connection with the content of the present paper but also in view of the currently widespread identification of the physical vacuum with the physical vacuum of quantum field theory.[2)] However, the profound changes introduced into the concept of the physical vacuum by the creation of quantum field theory are in no doubt.[4] It is important that they already affect the physical vacuum in inertial frames of reference or, in other words, in Minkowski space-time, when $g_{00} = -1$, $g_{ij} = \delta_{ij}$ $(i,j = 1,2,3)$ and, in the widespread terminology, gravitational fields are absent (the meaning of this last assertion is well known—we are speaking of constancy of $g_{\mu\nu}$, i.e., homogeneity and isotropy of space-time). Already in Minkowski space-time the quantum vacuum is, in the graphic expression of I. Ya. Pomeranchuk, a "boiling operator liquid." Indeed, defining the vacuum as the lowest energy state in which all real particles (photons, electron-positron pairs e^-e^+, etc.) are absent, we know that in this state the fields are by no means zero but fluctuate around zero mean values; these fluctuations are the zero-point vibrations of the free fields. For interacting fields, one can speak of virtual photons, virtual e^-e^+ pairs, etc.

The idea of the zero-point vibrations of the quantum fields, which is already about 60 years old, is now well known. Nevertheless, bearing in mind the nature and aim of the present paper, which is basically intended for nonspecialists, it seems appropriate to recall some elementary facts and estimates. The energy of a zero-point vibration of the electromagnetic field corresponding to the "mode" (normal vibration) with frequency ω is[3)] $\hbar\omega/2$, and the spectrum of these vibrations in the interval $d\omega$ is proportional to $\hbar\omega^3 d\omega$. The transition to other inertial systems, i.e., a Lorentz transformation, does not change the spectrum of zero-point vibrations (see, for example, Ref. 6), as is necessary. The zero-point vibrations are perfectly real—they lead to the appearance of forces between bodies, to a change in the energy of their interaction. We are speaking here of van der Waals forces (see Ref. 7 and the literature given there), the best known being the attraction, already considered in 1948, between two perfectly conducting plates separated by an empty gap.[8] The existence of the zero-point vibrations is undoubtedly an exceptionally important and fundamental feature of quantum theory. It is true that this circumstance in conjunction with the fact that in the classical and quantum theories different approaches and methods are often used leads sometimes to confusion. For example, it is fairly widely believed that the spontaneous emission of light is due to the zero-point vibrations of the electromagnetic field, but this is incorrect.[9]

The concept and, one may say, content of the physical vacuum in quantum theory becomes even richer when one takes into account the interaction between fields, in particular, between the electromagnetic field and charged particles, i.e., in a different language, between the electromagnetic and electron-positron fields). Concretely, this interaction leads to the existence in the vacuum of virtual electron-positron (e^-e^+) pairs, which are continually appearing and being annihilated. The presence of these pairs immediately enables us to understand the existence of electric polarization of the vacuum and, generally, the influence of the vacuum of external electromagnetic fields. In such fields, the vacuum behaves like a nonlinear birefringent medium (the birefringence, i.e., the anisotropy of the optical properties, is obviously related to the existence of a distinguished direction—the direction of the external field). The heuristic richness of the notion of virtual electron-positron pairs can be demonstrated by an example. In a strong magnetic field the vacuum becomes, as we have said, birefringent, though there is no magnetic activity despite the fact that any medium in an external magnetic field in general becomes magnetoactive. The explanation is, obviously, that the vacuum contains the same number of virtual electrons (e^-) and positrons (e^+). These particles are rotated by the magnetic field in opposite directions, and the vacuum is similar to an exceptional medium like an e^-e^+ plasma (with equal concentrations of e^- and e^+), which is not magnetoactive but is birefringent.

An electromagnetic field, acting on the vacuum, may lead to the transformation of virtual e^-e^+ pairs into real pairs, i.e., to the creation of electrons and positrons. As examples of such processes we can take the production of e^-e^+ pairs when there is scattering of a photon (with frequency $\hbar\omega \geqslant 2mc^2$, where m is the mass of the electron and positron) by a Coulomb center or two photons collide. In the latter case, the reaction threshold is $\hbar\omega = mc^2$, where ω is the frequency of each of the photons in the center-of-mass system. Pairs can also be created in an external static electric field E. In the absence of an external field, the components e^- and e^+ of a virtual pair are usually at distances $l \lesssim l_c \equiv \hbar/mc = 3.86 \cdot 10^{-11}$ cm. The probability of finding them at a separation $l \gg l_c$ is suppressed by a factor $\exp(-l/l_c)$. The pair creation process takes place fairly strongly if $E \gtrsim E_0$, where $E_0 = mc^2/el_c$. The meaning of the expression for E_0 is obvious; for over the distance l_c the field E_0 does work mc^2 on an electron (or positron). In fields $E \ll E_0$, the probability of pair creation decreases in accordance with the law $\exp(-\pi E_0/E)$.

Before we turn to the direct subject of the present paper, we make some general remarks about quantum effects in the gravitational field. We are here speaking exclusively about a classical gravitational field, described moreover by the equations of general relativity (and not by some other theory of the gravitational field not yet excluded in principle). The neglect of quantum effects is admissible if the characteristic radius of curvature of space-time is much greater than the Planck length $l_g = (G\hbar/c^3)^{1/2} = 1.6 \cdot 10^{-33}$ cm, and therefore the effect of vacuum polarization by the gravitational field can be ignored. Also, we shall not consider space-time regions on the Planck scales $l \sim l_g$, $t \sim t_g = l_g/c$, in which the fluctuations of the gravitational field itself are large.[10–13,4)]

It is clear that a variable gravitational field can, generally speaking, create particle pairs from the vacuum. Assuming that the gravitational field is weak, we may make some remarks using a simple language. For example, the threshold of e^-e^+ pair creation for collision of two gravitons is $\hbar\omega = mc^2$, where ω is the graviton frequency. Pair creation by a variable gravitational field may play an important part in cosmology.[12,15]

We now consider the effect of a constant gravitational field on virtual e^-e^+ pairs. If the gravitational field is homogeneous, it cannot generate pairs, since the electron and positron will move in the same direction. Pairs can be created only by the tidal effect, i.e., in an inhomogeneous field. If g is the acceleration of free fall in a gravitational field, and L is the characteristic scale of its inhomogeneity, then the difference between the forces that act on e^- and e^+ is of order $f \sim mgl_c/L$, and the condition for creation, determined from the equation $fl_c \sim mc^2$, has the form $g/L \sim c^2/l_c^2$.

To be specific, we consider the creation of e^-e^+ pairs by a nonrotating (Schwarzschild) black hole with mass M. If a particle of mass m (in the given case, e^- or e^+) is to be able to escape from the neighborhood of a black hole "to infinity," an energy $\mathscr{E} \geqslant mc^2$ must obviously be expended. This energy can be obtained if one of the components of the separated virtual pair "is created" within the gravitational radius (at $r < r_g \equiv 2GM/c^2 = 1.48 \cdot 10^{-23}\,(M,\mathrm{g})$ cm $\approx 3M/M_\odot$ km or, as one usually says, below the event horizon). This particle is absorbed by the black hole and thus does work, while the other component of the pair, using this energy, escapes from the hole. For the region outside the black hole to which such escape of the particle is possible, the maximal value of the quantity g/L which occurs in the condition of creation is of order c^2/r_g^2. Therefore, one can expect intense production by a black hole of particles with a certain mass m only when $r_g \lesssim l_c = \hbar/mc$. For black holes with large mass, the creation probability will be exponentially suppressed. For massless quanta whose frequency at infinity is ω the condition of creation can be obtained if l_c is replaced by $\lambda/2\pi = c/\omega$. In other words, a black hole can create particles predominantly with energy $\mathscr{E} \sim \hbar c/r_g = \hbar c^3/2GM$. Such energy corresponds to a temperature

$$T_{\mathrm{b.h}} \sim \mathscr{E} k_B^{-1} \sim \hbar c^3 (2GMk_B)^{-1}, \tag{1.1}$$

where k_B is Boltzmann's constant.

These arguments and estimates appear to us as rather convincing evidence that when allowance is made for the quantization of the electromagnetic and other fields black holes must radiate photons and the "quanta" of these fields. Of course, this is a quantum effect, as is clear not only from the essence of the matter but also from the appearance of the quantum constant $\hbar$ in the expression (1.1) for $T_{\mathrm{b.h.}}$. There are other ways of justifying this conclusion; this question is examined more accurately and in more detail in, for example, Ref. 16. The conclusion of quantum "evaporation" of black holes was first drawn by Hawking in 1974,[17] and it was then unexpected. The calculations show that a Schwarzschild black hole with mass M radiates as a black body with temperature

$$T_{\mathrm{b.h}} = \frac{\hbar c^3}{8\pi k_B GM} \approx 10^{-7}\,\frac{M_\odot}{M}\ \mathrm{K}. \tag{1.2}$$

There is no doubt that this result of Hawking was a great achievement of theoretical physics.

The estimate (1.1) given above agrees with the exact expression for the temperature (1.2) of the Hawking radiation apart from a factor $1/4\pi$.

The field of a Schwarzschild black hole is empty and is a very important but special example of a gravitational field. It is clear that creation of various particles and, generally, quantum effects (in particular, vacuum polarization) will occur in general in all gravitational fields. Among such fields, a homogeneous gravitational field constant in time occupies a distinguished place. In such a field, all physical processes and phenomena occur in exactly the same way as in a uniformly accelerated frame of reference without any fields—this is the content of the equivalence principle, the basis of general relativity. How are quantum effects manifested in a homogeneous and constant gravitational field and how must we formulate the equivalence principle in the quantum domain? The fact that such a question is meaningful is already clear; for it is sufficient to point out that the Minkowski vacuum, which is invariant with respect to Lorentz transformations, as we have already said, is not invariant on the transition to an accelerated frame of reference. On the other hand, a homogeneous gravitational field is clearly distinguished, for in it there are no tidal forces and therefore e^-e^+ pairs will not be created.

A large number of studies, to which references will be given below in Sec. 2.3, has been devoted to the quantization of various fields in a uniformly accelerated coordinate system. We shall already mention here the result obtained by Unruh in 1976,[18] namely, it was found that a "detector" (atom, oscillator, etc.) at rest in a uniformly accelerated frame of reference will be excited in the same way as if it were in a thermal bath (or a field of thermal radiation) with temperature

$$T_a = \hbar a\,(2\pi k_B c)^{-1}, \tag{1.3}$$

where a is the constant acceleration of the system with respect to an inertial frame.

The expression (1.3) goes over into (1.2) if as acceleration we take the acceleration ("intensity" of the gravitational field) characteristic of a black hole—the so-called surface gravity of the black hole[5]:

$$\varkappa = \frac{GM}{r_g^2} = \frac{c^4}{4GM}\,. \tag{1.4}$$

The excitation of a "detector" at rest in a uniformly accelerated frame of reference is in some way related to the quantum radiation of black holes and has attracted much interest (see, for example, the book of Ref. 19, the reviews of Refs. 20–22, and the references given in them). The reason for the excitation of an accelerated "detector" and the nature of the change that this excitation induces in the state of the quantized field (massive or massless scalar field, electromagnetic field, etc.) that interacts with the detector are still being discussed and, it seems to us, the necessary clarity was not achieved until recently. Significant progress in understanding was reached in 1984 in Ref. 23, in which the excitation of a detector that is moving with uniform acceleration in Minkowski space was treated in an inertial frame of reference. It was found that the excitation of a "detector" that, say, initially is in its ground state in the Minkowski vacuum, i.e., in the absence of real quanta of the considered field, is accompanied by emission of a field quantum. Such a process—excitation with emission—is somewhat unusual and requires clarification. It is also necessary to explain the fact that the "detector" is excited with a thermal distribution with respect to its levels, the corresponding temperature being $T_a = \hbar a/2\pi k_B c$. The authors of the present paper pointed out that excitation with emission has long been known in the example of the so-called anomalous Doppler effect. In the case of an accelerated "detector," the situation is analogous and fully comprehensible. The reason for the excitation, characterized precisely by the temperature T_a, is also fairly clear—this result is due to fulfillment of the equivalence principle.

In the present paper we shall consider both these aspects, and we shall make a number of calculations.[6)] This in fact is the aim of our paper. To prevent these calculations from obscuring the essence of the matter, we shall already note here the following. We shall consider a certain "detector"—an atom, oscillator, etc.—with discrete levels in three frames of reference: inertial, uniformly accelerated, and at rest in a homogeneous gravitational field. It is obvious that if the detector is in some level i, then the energy E_i of this level in the different frames of reference is different, but the actual fact of the detector's being in precisely level i does not depend on the frame of reference. The same applies to the distribution of an ensemble of identical detectors over their levels. Further, suppose that the considered detector in an inertial system has a certain constant acceleration $\mathbf{a}$, and that the velocity of the detector is $\mathbf{v} = 0$ at the time $t = 0$. Then in a uniformly accelerated frame of reference, with acceleration $\mathbf{a}$ with respect to the inertial frame, the acceleration of the detector will always be zero, and its velocity can also be taken to be zero. For the distribution of the detector over the levels, it is clear from what we have said that the calculations "from the point of view" of the inertial or uniformly accelerated frames of reference must lead to the same result. However, in the given case it was found that in a number of cases the use of the accelerated frame of reference leads more readily to the goal; in such a system one can more readily, and, moreover, in more general form, reach the conclusion that the distribution function of the detector is thermal (Boltzmann), and with the temperature (1.3). Similarly, to describe the behavior of the detector at rest in a homogeneous gravitational field it is convenient to use a frame of reference at rest in this field. To show the validity of this, in Sec. 2 we shall quantize a field in flat space-time (in Minkowski space) in the inertial and uniformly accelerated frames of reference, and also in a homogeneous gravitational field. Then, in Sec. 3, we consider the behavior of a uniformly accelerated detector and a detector at rest in a homogeneous gravitational field in frames of reference with respect to which these detectors are at rest. In Sec. 4, the processes of excitation and emission of a uniformly accelerated detector are considered in an inertial frame of reference. In Sec. 5, we consider the excitation of a detector that is moving in a medium with constant but superluminal velocity, when the Doppler effect is both normal and anomalous. Finally, in Sec. 6 we shall discuss the equivalence principle when allowance is made for quantum phenomena.

2. THE VACUUM IN MINKOWSKI SPACE-TIME TREATED IN INERTIAL AND UNIFORMLY ACCELERATED FRAMES OF REFERENCE. THE VACUUM IN A STATIC HOMOGENEOUS GRAVITATIONAL FIELD

2.1. Quantum field theory in an inertial frame of reference

The theory of free (i.e., noninteracting) quantum fields in Minkowski space has been treated in many textbooks (see, for example, Refs. 26 and 27). We here recall briefly only some of the main points of this theory in the simplest example of a scalar massless field.

We shall assume that we know a certain inertial frame of reference, and, abstracting somewhat, we can speak of a realization of Minkowski space, in which we shall work (it would be inappropriate to discuss this question at the classical level in more detail; for the literature, see, for example, Ref. 2). The position of any particle with respect to this inertial frame of reference can be characterized by the Cartesian coordinates $X^\mu = (cT, X, Y, Z)$. In these coordinates, the element of length ds^2 between two events X^μ and $X^\mu + dX^\mu$ can be expressed in the form (we use notation in which the Greek indices take the values 0, 1, 2, 3, the Latin indices 1, 2, 3)

$$ds^2 = \eta_{\mu\nu}\, dX^\mu dX^\nu = -c^2\, dT^2 + dX^2 + dY^2 + dZ^2. \quad (2.1)$$

For brevity, we shall sometimes call such an inertial system an I system, and the Cartesian coordinates X^μ the coordinates associated with the I system.

In the Minkowski space, the scalar massless field φ is described by the equation

$$\Box\varphi \equiv \eta^{\mu\nu}\frac{\partial}{\partial X^\mu}\frac{\partial}{\partial X^\nu}\varphi = 0. \quad (2.2)$$

The general solution of Eq. (2.2) can be expressed in the form

$$\varphi(X) = \int d^3k\, [\Phi_k(X)\, a_k + \bar{\Phi}_k(X)\, a_k^*], \quad (2.3)$$

where $\bar{\Phi}_k(X) = \overline{\Phi_k(X)}$, $k^\mu = (\omega_k/c, \mathbf{k}), \omega_k = c|\mathbf{k}|$, and

$$\Phi_k(X) = e^{-i\omega T}\tilde{\Phi}_k(X) = \hbar^{1/2}[(2\pi)^3\cdot 2\omega_k]^{-1/2}\, e^{ik_\mu X^\mu} \quad (2.4)$$

are the positive-frequency solutions of Eq. (2.2), and the bar denotes complex conjugation. One can say that the functions $\bar{\Phi}_k(X)$ correspond to the negative frequencies $-\omega_k$. In the classical theory, a_k and $a_k^* = \overline{a_k}$ are complex conjugate functions; in the quantum theory $\hat{a}_k$ and $\hat{a}_k^* = (\hat{a}_k)^*$ are Hermitian-conjugate operators satisfying the commutation relations

$$[\hat{a}_k, \hat{a}_{k'}^*] \equiv \hat{a}_k\hat{a}_{k'}^* - \hat{a}_{k'}^*\hat{a}_k = \delta(\mathbf{k}-\mathbf{k}'),$$
$$[\hat{a}_k, \hat{a}_{k'}] = [\hat{a}_k^*, \hat{a}_{k'}^*] = 0. \quad (2.5)$$

The state of a field described by the vector $|0; M\rangle$ in Hilbert space determined by the relations

$$\hat{a}_k|0; M\rangle = 0, \quad (2.6)$$

is the state with the lowest energy. All the remaining states, which arise from the action of the operators $\hat{a}_k^*$ on $|0; M\rangle$, have a higher energy and describe "excitations" of the system. The state $|0; M\rangle$, in which there are no excitations, is called the vacuum. The states $|1_k; M\rangle \equiv \hat{a}_k^*|0; M\rangle$ correspond to single-particle states, for which there is in the Minkowski space one quantum of the field φ possessing energy $\hbar\omega_k$ and momentum $\hbar\mathbf{k}$ and described by the wave function $\Phi_k(X) = \langle 0; M|\hat{\varphi}(X)|1_k; M\rangle$. The many-particle states are interpreted similarly. The operators $\hat{a}_k^*$ and $\hat{a}_k$ are called the operators of creation and annihilation of particles in the states $\Phi_k(X)$.

One can show that the scheme of quantization we have described and, in particular, the choice of the vacuum are Poincaré invariant, i.e., do not depend on the particular choice of the original inertial frame of reference. The vacuum $|0; M\rangle$ is not changed if one uses instead of the plane-wave expansion (2.4) an expansion with respect to any other complete system of solutions of Eq. (2.2) provided that these solutions are, as before, taken to be positive-frequency solutions with respect to the time T in the inertial frame of reference. Of course, the solutions themselves can be expressed in any, not necessarily Cartesian coordinates. The use of basis solutions that have positive frequency with respect to the time T guarantees that the corresponding vacuum is the state of the system with lowest energy and, therefore, is identical to the Minkowski vacuum $|0; M\rangle$.[7)]

It is quite a different matter if, using a curvilinear four-dimensional system of coordinates x^μ in which x^0 depends nonlinearly on $X^0 = cT$, we make an expansion with respect to positive- and negative-frequency solutions with respect to x^0. For such quantization, which corresponds under certain conditions to quantization in noninertial frames of reference, the results (in particular, the choice of the vacuum) are different from those described above corresponding to quantization in inertial frames of reference. We shall discuss this in more detail in Sec. 2.3.

2.2. Uniformly accelerated frame of reference and frame of reference at rest in a homogeneous gravitational field

We now describe the properties of the simplest noninertial frame of reference, namely, a frame that is uniformly accelerated with respect to an inertial frame. To this end, we introduce in Minkowski space new coordinates $x^\mu = (\eta, \rho, y, z)$ related to the Cartesian coordinates $X = (cT, X, Y, Z)$ by[8)]

$$cT = \rho\,\mathrm{sh}\,\eta, \quad X = \rho\,\mathrm{ch}\,\eta, \quad Y = y, \quad Z = z. \quad (2.7)$$

These new coordinates cover the part R_+ of Minkowski space in which $X > c|T|$. The two null planes $\mathscr{H}^+ (X = cT)$ and $\mathscr{H}^- (X = -cT)$ form the boundary of R_+. The coordinate lines $\eta(\rho, y, z = \text{const})$ are hyperbolas, the asymptotes to which are null lines on $\mathscr{H}^+$ and $\mathscr{H}^-$ (Fig. 1). Suppose that along such a coordinate line η there moves a certain particle. For such a particle, the parameter η is related to its proper time τ by [see (2.9) below] $\tau = \rho\eta c^{-1}$, and its 4-velocity u^μ and 4-acceleration a^μ in the coordinates X^μ are

$$u^\mu = (c\,\mathrm{ch}\,\eta,\ c\,\mathrm{sh}\,\eta,\ 0,\ 0), \quad (2.8)$$
$$a^\mu = (c^2\rho^{-1}\,\mathrm{sh}\,\eta,\ c^2\rho^{-1}\,\mathrm{ch}\,\eta,\ 0, 0).$$

In other words, in the inertial frame of reference such a particle moves with an acceleration directed along the X axis, the

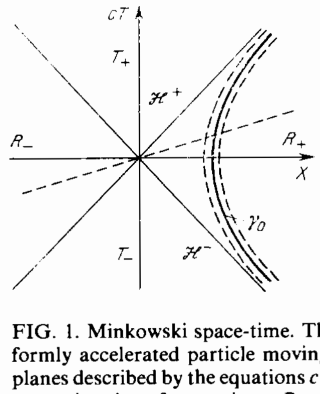

FIG. 1. Minkowski space-time. The curve γ_0 is the world line of a uniformly accelerated particle moving along the X axis; $\mathscr{H}^\pm$ are the null planes described by the equations $cT \mp X = 0$, which divide the complete space-time into four regions: R_+, R_-, T_+, and T_-, in each of which corresponding Rindler coordinates can be introduced. The uniformly accelerated (A) coordinate system associated with the γ_0 particle covers the region R_+.

magnitude of the acceleration, $a \equiv |a_\mu a_\mu|^{1/2} = c^2\rho^{-1}$, being constant (uniformly accelerated motion). Charged particles move, for example, along such trajectories in a homogeneous constant electric field if their velocity is directed along the field.[9)]

With such a particle, one can associate a rigid uniformly accelerated frame of reference. Namely, suppose that the particle "carries" a standard clock and that to it are attached three rigid rulers directed along the axes X, Y, and Z. The rigidity of the rulers means that their length measured in the frame of reference of the uniformly accelerated particle does not change with the time. In the considered situation, it is not difficult to formulate necessary and sufficient conditions for rigidity of bodies. Indeed, we fix the position X_0^μ of the particle at the time η_0. Then it is easy to show that the locus of the points (events) X^μ simultaneous with X_0^μ from the point of view of the accelerated observer coincides with the plane that passes through X_0^μ and through the "line" $\rho = 0$; this plane is described by the equation $\eta = \eta_0$ (see Fig. 1). If $x^\mu(\eta)$ are the coordinates of the end of a ruler whose zero point is fixed at the particle $(x_0^\mu(\eta))$, then its length l is

$$l = [(\rho - \rho_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2}.$$

Therefore, if the world lines that describe the beginning and end of the ruler coincide with the coordinate lines of the time η, then the length of such a ruler is constant and it is rigid. In other words, physical rigidity in the considered case is equivalent to "coordinate" rigidity, i.e., constancy of the coordinates ρ, y, z of the beginning and end of the ruler; the frame of reference we have introduced is nonrotating.

One can say that the system of coordinates x^μ is a canonical realization of a uniformly accelerated frame of reference in the sense in which the Cartesian system of coordinates X^μ is associated with the inertial frame of reference. For brevity, we shall refer to this uniformly accelerated coordinate system as the A system.

We also draw attention here to a fairly important property of rigid accelerated moving bodies (rulers, etc.)—different points of them move with different accelerations. To see this, it is sufficient to consider two points of the body possessing different coordinates ρ. The points with larger ρ move with smaller acceleration. This leads, in particular, to the consequence that the maximal length of a rigid accelerated body in the direction opposite to the acceleration, measured from a chosen point moving with acceleration a, cannot exceed c^2a^{-1} (c^2a^{-1} is the distance in the system A from the hyperbola to the coordinate origin; see Fig. 1). To avoid confusion, we emphasize that by the acceleration of some uniformly accelerated frame of reference we always mean the acceleration a of the particle with which this system is associated; when considering accelerated bodies (atoms, etc.), we shall assume that their lengths in the direction determined by the acceleration vectors a are much less than c^2a^{-1}.

Having in mind the discussion below (in Sec. 6) of the equivalence principle in connection with quantum effects in accelerated frames of reference, we also describe here the frame of reference at rest in a homogeneous static gravitational field (for a more detailed discussion of this question, see, for example, Refs. 28–31). We note first of all that the line element (2.1) takes in the coordinates x^μ the form

$$dS^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = -\rho^2\, d\eta^2 + d\rho^2 + dx^2 + dy^2. \quad (2.9)$$

It can be assumed that this metric $g_{\mu\nu}$ describes a static homogeneous gravitational field. Instead of ρ and η one frequently uses the associated coordinates $x = \rho - c^2a^{-1}$ and $\tau = c\eta a^{-1}$, in terms of which the metric (2.9) becomes

$$ds^2 = -(1 + c^{-2}ax)^2\, c^2\, d\tau^2 + dx^2 + dy^2 + dz^2. \quad (2.10)$$

A body at rest under the influence of certain "external forces" at the point $x = 0$ ($\rho = c^2a^{-1}$) has in this static gravitational field a 4-acceleration equal to a. Therefore, $\mathbf{g} = -\mathbf{a}$ is the acceleration of free fall at this point with respect to the chosen body at rest. In the Newtonian approximation, when $|\varphi|/c^2 \ll 1$,

$$-g_{00} = 1 + 2c^{-2}\varphi, \quad (2.11)$$

where φ is the Newtonian potential. Comparing (2.10) with (2.11), we see that the metric (2.10) corresponds to the case of a homogeneous gravitational field in the Newtonian approximation with potential

$$\varphi = -\mathbf{gr} = |g|\, x, \quad \mathbf{r} = (x, y, z). \quad (2.12)$$

With a body at rest at the point (ρ, y, z) we can associate a rigid frame of reference in exactly the same way that we did for the uniformly accelerated particle. We shall call this frame of reference the frame of reference at rest in the homogeneous static gravitational field or, abbreviated, G system.

The significance of the G system is particularly clear in the case when the static gravitational field is created by a massive gravitating body. For example, we can consider the gravitational field near the surface of the Earth, Sun, a neutron star, etc. In a region with dimensions $l \ll L$, where L is the characteristic radius of curvature of the space-time, we can use the metric (2.10) to describe the gravitational field of a massive source of this kind, and the G system is distinguished by the property that it is at rest relative to the surface of the massive body (for example, a neutron star).

It is not difficult to introduce coordinates analogous to (2.7) in the three remaining quadrants of M: R_-, T_+, and T_- (see Fig. 1). The connection between these coordinates (which we shall also denote by η, ρ, y, z) and the Cartesian coordinates in all four regions can be written in the form

$$\left.\begin{array}{l} cT = \varepsilon\rho\, \mathrm{sh}\, \eta, \quad X = \varepsilon\rho\, \mathrm{ch}\, \eta, \\ \quad Y = y, \quad Z = z \end{array}\right\} \quad \text{in } R_\varepsilon\text{: } \varepsilon X > c\,|T|,$$

$$\left.\begin{array}{l} cT = \varepsilon\rho\, \mathrm{ch}\, \eta, \quad X = \varepsilon\rho\, \mathrm{sh}\, \eta, \\ \quad Y = y, \quad Z = z \end{array}\right\} \quad \text{in } T_\varepsilon\text{: } \varepsilon cT > |X|, \quad (2.13)$$

where $\varepsilon = \pm 1$. The coordinates (η, ρ, y, z) were used by Rindler,[30] and they are usually called Rindler coordinates. To shifts $\eta \to \eta + \psi$ in these coordinates there correspond Lorentz transformations in the Minkowski space of the form

$$cT' = cT\, \mathrm{ch}\, \psi + X\, \mathrm{sh}\, \psi, \quad X' = cT\, \mathrm{sh}\, \psi + X\, \mathrm{ch}\, \psi. \quad (2.14)$$

These transformations are symmetry transformations, and according to Noether's theorem, a definite conservation law corresponds to them. In the considered case, the corresponding conserved quantity, which is often called the Rindler energy (we denote it by K), can be written in the form

$$K = K^{(+)} + K^{(-)}, \quad K^{(\varepsilon)} = \varepsilon \int\limits_{\substack{\eta=\mathrm{const} \\ R_\varepsilon}} T^{\eta}_{\eta}\rho \, d^3 x; \qquad (2.15)$$

where T^{ν}_{μ} is the energy-momentum tensor of the field.

2.3. Quantum field theory in a uniformly accelerated frame of reference

A large number of studies has been devoted to the construction of the quantum theory of physical fields in a uniformly accelerated frame of reference; among these, we mention Refs. 18–23 and Refs. 32–45 (some further references will be given below).

Following the plan of our exposition, we now consider the main points of quantum field theory in a uniformly accelerated frame of reference, taking the example of the scalar massless field φ. Let $x^\mu = (\eta, \rho, y, z)$ be Rindler coordinates covering the region R_ε ($\varepsilon = \pm$), their connection with the Cartesian coordinates being given by (2.13). It is easy to show that the functions

$$\begin{aligned} u^{(\varepsilon)}_{\nu\mathbf{q}}(x) &= e^{-i\varepsilon\nu\eta} U_{\nu\mathbf{q}}(x), \quad \nu \geqslant 0, \\ U_{\nu\mathbf{q}}(x) &= \left(\frac{\hbar}{c}\right)^{1/2} \frac{\mathrm{sh}^{1/2}(\pi\nu)}{2\pi^2} e^{i\varepsilon\mathbf{q}\mathbf{z}} K_{i\nu}(q\rho) \end{aligned} \qquad (2.16)$$

are solutions of the equation $\Box\varphi = 0$ in the region R_ε. [For convenience here and below we use the following notation: $\mathbf{z} = (y, z)$ is a two-dimensional vector in the y, z plane, $\mathbf{q} = (q_y, q_z)$, $\mathbf{qz} = q_y y + q_z z$, $q = (q_y^2 + q_z^2)^{1/2}$, $K_a(\xi)$ is a Macdonald function.] For real values of ν and ξ the function $K_{i\nu}(\xi)$ is real and satisfies the condition $K_{-i\nu}(\xi) = K_{i\nu}(\xi)$. Note that in R_ε the Rindler frequency ν is related to the normal frequency ω in the uniformly accelerated (with acceleration a) frame of reference (i.e., to the frequency ω relative to the standard clocks in this system) by $\omega = \varepsilon a \nu c^{-1}$. It is obvious that the solutions which we have introduced have positive frequency with respect to the proper time $\tau = \varepsilon c \eta a^{-1}$ in this frame of reference.

The functions $u^{(+)}_{\nu\mathbf{q}}$ defined by (2.16) in the region R_+ can be extended to the complete Minkowski space M as solutions of the equation $\Box\varphi = 0$ by requiring that in R_- these solutions vanish. Similarly, assuming that $u^{(-)}_{\nu\mathbf{q}} = 0$ in R_+, we can define in M the solutions $u^{(-)}_{\nu\mathbf{q}}$.

Using the basis functions that we have introduced—they form a complete system[10]—we can write down an expansion of the operator $\hat{\varphi}$ with respect to them in the form

$$\hat{\varphi}(x) = \sum_{\varepsilon = \pm} \int\limits_0^\infty d\nu \int d\mathbf{q} \, [u^{(\varepsilon)}_{\nu\mathbf{q}}(x) \hat{b}^{(\varepsilon)}_{\nu\mathbf{q}} + \overline{u}^{(\varepsilon)}_{\nu\mathbf{q}}(x) \hat{b}^{(\varepsilon)*}_{\nu\mathbf{q}}], \quad (2.17)$$

where $\overline{u}^{(\varepsilon)}_{\nu\mathbf{q}} \equiv \overline{u^{(\varepsilon)}_{\nu\mathbf{q}}}$. It is sometimes convenient (and we shall use this) to denote the set of indices $\nu\mathbf{q}$ by the collective index J and use the notation

$$\sum_J \equiv \int\limits_0^\infty d\nu \int d\mathbf{q}, \quad \delta_{JJ'} \equiv \delta(\nu - \nu')\,\delta(\mathbf{q} - \mathbf{q}'). \qquad (2.18)$$

In this notation, (2.17) takes the form

$$\hat{\varphi}(x) = \sum_\varepsilon \sum_J (u^{(\varepsilon)}_J(x) \hat{b}^{(\varepsilon)}_J + \overline{u}^{(\varepsilon)}_J(x) \hat{b}^{(\varepsilon)*}_J). \qquad (2.19)$$

It is easy to see that the operators $\hat{b}^{(\varepsilon)*}_J$ and $\hat{b}^{(\varepsilon)}_J$ satisfy the standard commutation relations for creation and annihilation operators:

$$\begin{aligned} &[\hat{b}^{(\varepsilon)}_J, \hat{b}^{(\varepsilon')*}_{J'}] = \delta_{\varepsilon\varepsilon'}\delta_{JJ'}, \\ &[\hat{b}^{(\varepsilon)}_J, \hat{b}^{(\varepsilon')}_{J'}] = [\hat{b}^{(\varepsilon)*}_J, \hat{b}^{(\varepsilon)*}_{J'}] = 0. \end{aligned} \qquad (2.20)$$

Note that although we have particularized to the scalar massless field, relations of the form (2.19)–(2.20) also hold for other boson fields except that φ, $u^{(\varepsilon)}_J$, and $\overline{u}^{(\varepsilon)}_J$ are not scalar but vector or tensor functions satisfying appropriate field equations, and the index J contains not only ν and $\mathbf{q}$ but also additional quantum numbers (for example, for massless fields with nonzero spin there is the helicity).

By means of the expansion (2.19) we can show that the operator $\hat{K}$ corresponding to the Rindler energy K [see (2.15)] can be written in the form

$$\hat{K} = \hat{K}^{(+)} + \hat{K}^{(-)}, \quad \hat{K}^{(\varepsilon)} = \sum_J \varepsilon\nu \hat{b}^{(\varepsilon)*}_J \hat{b}^{(\varepsilon)}_J. \qquad (2.21)$$

Finally, we define the Rindler vacuum $|0; R\rangle$ as the state in which there are no Rindler quanta:

$$\hat{b}^{(\varepsilon)}_J |0; R\rangle = 0. \qquad (2.22)$$

The complete Fock space H of the Rindler states arises by applying the creation operators $\hat{b}^{(\varepsilon)*}_J$ of the Rindler particles to $|0; R\rangle$.

For points x lying in one of the regions R_ε (to be specific, we shall assume, for example, that this is the region R_+), it is sufficient in the expansion of the operator $\hat{\varphi}(x)$ to make a restriction to the terms for which ε has the corresponding sign (+). Accordingly, we can define the state $|0; R_+\rangle$ by the relations $\hat{b}^{(+)}_J |0; R_+\rangle = 0$ and construct the Fock space $H^{(+)}$ by applying the operators $\hat{b}^{(+)*}_J$ to $|0; R_+\rangle$. The operator $\hat{K}^{(+)}$ can be regarded as the Hamiltonian in this state space. The space $H^{(-)}$ and Hamiltonian $\hat{K}^{(-)}$ are constructed similarly. The complete Fock state space H is, as the mathematicians say, the tensor product of the spaces $H^{(+)}$ and $H^{(-)}$. Accordingly, we shall sometimes write the Rindler vacuum $|0; R\rangle$ in the form $|0; R\rangle = |0; R_+\rangle |0; R_-\rangle$.

Our systems of functions $u^{(\varepsilon)}_J$, $\overline{u}^{(\varepsilon)}_J$ can be expanded with respect to the plane waves (2.4). The converse is also true—the plane waves (2.4) can be expressed in the form of expansions with respect to the Rindler modes. Using these expansions and the representations (2.13) and (2.17) for the operator $\hat{\varphi}$, we can establish a connection between the operators $\hat{a}_\mathbf{k}$ and $\hat{a}^*_\mathbf{k}$, on the one hand, and the operators $\hat{b}^{(\varepsilon)}_J$ and $\hat{b}^{(\varepsilon)*}_J$, on the other. This connection has a relatively complicated form, and we shall not give the explicit expressions here (they can be found, for example, in Ref. 21). We merely note here that the vacuum $|0; M\rangle$, which is determined in the inertial frame of reference by Eqs. (2.6), satisfies the equations

$$(e^{\pi\nu/2} \hat{b}^{(\varepsilon)}_{\nu\mathbf{q}} - e^{-\pi\nu/2} \hat{b}^{(-\varepsilon)*}_{\nu\mathbf{q}}) |0; M\rangle = 0, \qquad (2.23)$$

these holding both for $\varepsilon = +$ as well as for $\varepsilon = -$. To prove this, it is sufficient to show that the functions

$$e^{\pi\nu/2} u^{(\varepsilon)}_{\nu\mathbf{q}} + e^{-\pi\nu/2} \overline{u}^{(-\varepsilon)}_{\nu\mathbf{q}}$$

when expanded with respect to the plane waves (2.4) contain only positive frequencies with respect to the time T in the inertial frame of reference (for more details about this, see Ref. 21).

The relation (2.23) shows that the ordinary vacuum $|0; M\rangle$ in Minkowski space [see (2.6)] is not identical to the Rindler vacuum $|0; R\rangle$ [see (2.22)]. Moreover, (2.23) makes it possible to find an expansion of $|0; M\rangle$ with respect to a basis of many-particle Rindler states. Namely, one can show that[21]

$$|0;M\rangle = \prod_J \sum_{n_J=0}^{\infty} c_J e^{-\pi n_J \nu} |n_J; R_+\rangle |n_J; R_-\rangle,$$

$$c_J = (1 - e^{-2\pi\nu})^{-1/2}. \qquad (2.24)$$

To describe physically measurable quantities, one considers, in particular, the expectation values of operators that describe observables in the chosen quantum state. Let $\hat{Q}$ be such an operator and consider $\langle \hat{Q} \rangle_M \equiv \langle M; 0|\hat{Q}|0; M\rangle$, i.e., the expectation value of $\hat{Q}$ in the Minkowski vacuum. The usual way of calculating $\langle \hat{Q} \rangle_M$ consists of replacing the operator $\hat{\varphi}$, on which $\hat{Q}$ depends (we assume that $\hat{Q}$ is a function of $\hat{\varphi}$), by its expansion (2.3) with subsequent use of the commutation relations (2.5). However, one can also proceed differently, using a different basis—the Rindler basis; for this, it is necessary to use the expansion (2.19), the commutation relations (2.20), and the expression (2.24) for $|0; M\rangle$. Of course, the results of these two calculations of $\langle \hat{Q} \rangle_M$ are identical. However, let us consider in somewhat more detail the second method of calculation in the special case when the operator $\hat{Q} = \hat{Q}^{(+)}$ of the observable does not depend on the values of the field $\hat{\varphi}(x)$ outside the region R_+. It is obvious that then the observable $\hat{Q}^{(+)}$ depends only on $\hat{b}_J^{(+)}$ and $\hat{b}_J^{(+)*}$ and does not contain a dependence on $\hat{b}_J^{(-)}$ and $\hat{b}_J^{(-)*}$. This enables us, after replacement of $|0; M\rangle$ by the expression (2.24), to carry out a partial averaging over the $|n_J; R_-\rangle$ states. As a result of this, the expression for $\langle \hat{Q}^{(+)} \rangle_M$ is reduced to the form[11]

$$\langle \hat{Q}^{(+)} \rangle_M = \mathrm{Sp}_{(+)} (\hat{\rho}^{(+)} \hat{Q}^{(+)}), \qquad (2.25)$$

where

$$\hat{\rho}^{(+)} = \rho_0 e^{-2\pi \hat{K}^{(+)}} \equiv \rho_0 e^{-\hat{K}_a^{(+)}/k_B T_a}; \qquad (2.26)$$

where $\hat{K}_a^{(+)} = \hbar a c^{-1} \hat{K}^{(+)}$, $\hat{K}^{(+)}$ is the operator of the Rindler energy defined by (2.21), $\rho_0 = \prod_J [1 - \exp \times (-2\pi\nu_J)]$ is a normalization constant, and T_a is given by

$$T_a = \frac{\hbar a}{2\pi k_B c}. \qquad (2.27)$$

The operation of taking the trace in (2.25) is done in the space $H^{(+)}$.

We emphasize that the appearance of the density matrix $\hat{\rho}^{(+)}$ in the problem of calculating the expectation values of the operator $\hat{Q}^{(+)}$ in the pure state $|0; M\rangle$ is due to the fact that the chosen observable $\hat{Q}^{(+)}$ actually depends only on the states of a distinguished subsystem of the considered complete system. In accordance with the rules of quantum mechanics, the state of such a subsystem is described by a density matrix. Moreover, it follows from the relations (2.26) and (2.27) that the density matrix $\hat{\rho}^{(+)}$ itself corresponds to black-body radiation with temperature T_a.

Thus, as long as we consider the calculation of the values of local observables that depend only on the field $\hat{\varphi}$ in the region R_+ the result of calculating the vacuum expectation value for this observable in the Minkowski vacuum agrees with the result of calculating it in the thermal gas of Rindler particles (or, it would be better to say, quasiparticles) with temperature (2.27). If it is borne in mind that the trajectories of the "detectors" at rest in the uniformly accelerated frame of reference remain at all times in the region R_+, we can expect that the results of measurements made on the field $\hat{\varphi}$ in the uniformly accelerated frame of reference will lead to the same results as measurements in a corresponding thermal bath with temperature T_a at rest in this uniformly accelerated frame of reference. In other words, the Minkowski vacuum in the accelerated frame of reference appears as a gas of Rindler quanta of the corresponding field heated to the temperature T_a. We emphasize that this conclusion, which is based ultimately on the relation (2.23), is not related to the specific form of the considered field and is of a general nature.

It must be emphasized at once that although the distribution with respect to the "energy" $\hbar\omega = \hbar\nu a c^{-1}$ of the Rindler quanta has a thermal nature in the considered state the wave functions $u_{\nu\mathbf{q}}^{(+)}$ differ significantly from plane waves. This difference is due to the fact that free quanta in the Minkowski space move relative to the accelerated frame of reference with an acceleration, i.e., as if they were acted on by a force (an inertia force). This has the consequence that comparison of the thermal gas of the Rindler particles with a state in a thermal bath is valid only under the additional stipulation that this thermal bath and its contents are in a field of external forces ("inertia" forces) or, as will be shown in the following section 2.4, in a homogeneous static gravitational field.

2.4. Vacuum in a static homogeneous gravitational field

The quantization scheme developed in the foregoing subsection for a uniformly accelerated frame of reference can be to a large extent transferred formally to the case of quantum field theory in a static homogeneous gravitational field. We now briefly describe this theory, leaving aside for the moment the discussion of more subtle questions of the formulation of the equivalence principle for quantum phenomena. We shall return to this question in Sec. 6.

We recall that the metric of a homogeneous gravitational field in the coordinates $x = (\eta, \rho, y, z)$ associated with the G system has the form (2.9). To construct the quantum theory of the scalar field in this system we use an expansion of the field operator $\hat{\varphi}(x)$ of the form

$$\hat{\varphi}(x) = \int_0^{\infty} d\nu \int d\mathbf{q}\, (u_{\nu\mathbf{q}}(x)\, \hat{c}_{\nu\mathbf{q}} + \bar{u}_{\nu\mathbf{q}}(x)\, \hat{c}^*_{\nu\mathbf{q}}), \qquad (2.28)$$

where

$$u_{\nu\mathbf{q}}(x) = e^{-i\nu\eta} U_{\nu\mathbf{q}}(x) = \left(\frac{\hbar}{c}\right)^{1/2} \frac{\mathrm{sh}^{1/2}(\pi\nu)}{2\pi^2} \exp(-i\nu\eta + i\mathbf{q}\mathbf{z}) \times K_{i\nu}(q\rho), \quad \nu > 0. \qquad (2.29)$$

and the operators of creation $\hat{c}^*_{\nu\mathbf{q}}$ and annihilation $\hat{c}_{\nu\mathbf{q}}$ satisfy the standard commutation relations. We define the vacuum $|0; B\rangle$ in the homogeneous gravitational field by the conditions

$$\hat{c}_{\nu\mathbf{q}} |0; B\rangle = 0. \qquad (2.30)$$

Formally, such a scheme of quantization does not differ in any way, except for the obvious change of notation $\hat{b}_{\nu q}^{(+)} \to \hat{c}_{\nu q}^{(-)}$, $\hat{b}_{\nu q}^{(+)*} \to \hat{c}_{\nu q}^{*}$, $|0; R_+\rangle \to |0; B\rangle$, from the scheme of quantization in the uniformly accelerated frame of reference. We note however that the problem of quantization in a homogeneous gravitational field, like that of the realization of a G system, acquires a real physical significance in the case where there is a massive body generating such a field. Then the metric (2.9) describes the gravitational field outside this body (for $\rho > \rho_0$; if $\rho = \rho_0$ is the equation of the boundary of the body). Of course, in a real situation in which the body has a large but finite size the metric (2.9) is approximate, valid in a region with scale $l \ll L$, where L is the characteristic radius of curvature of the space-time. Within the body, the metric will already have a different form. Because of the change of the metric and the boundary conditions, the wave functions $u_{\nu q}(x)$ used to expand the operator $\hat{\varphi}(x)$ are different and, in general, not identical to (2.29). It is important, however, that if the field is static it is still meaningful to make an expansion of the form (2.28) using positive- and negative-frequency functions with respect to the "time" η. The state $|0; B\rangle$ defined in a static gravitational field of general form by the relations (2.23) has become known as the Boulware vacuum. For this state, the energy of the quantized field calculated with allowance for the work of the gravitational field is minimal. (Boulware himself considered such vacuum states in the description of quantum effects in the gravitational field of static black holes.[46-48]). For more details of the definition and properties of the vacuum in a static gravitational field, see Refs. 15 and 16 and the literature cited there; see also Ref. 49.

We emphasize especially that in the case when the static gravitational field is generated by a static massive body space-time regions different from R_+ are absent (Fig. 2). Therefore, there is no need to introduce additional solutions [like $u_{\nu q}^{(-)}$]. Such (additional) regions in space-time arise when the massive body generating the gravitational field is unstable and its contraction (collapse) results in the formation of a black hole. The question of the definition of the vacuum in the gravitational field of a black hole has been investigated frequently and in detail in the literature (a detailed exposition of this question and references to the corresponding studies can be found, for example, in Refs. 16, 21, and 50).

To conclude this section, we note that the complete

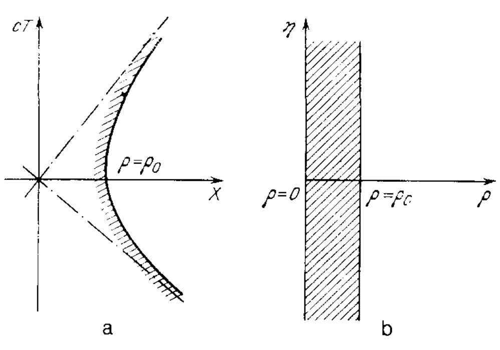

FIG. 2. The space-time in a frame of reference at rest in a homogeneous static gravitational field (G system). The region occupied by the matter that creates the gravitational field ($0 \leqslant \rho \leqslant \rho_0$) is hatched.

state space in a static homogeneous gravitational field is generated, as usual, by applying the creation operators $\hat{c}_{\nu q}^{*}$ to the vacuum $|0; B\rangle$. One can also consider different mixed states. An example of this kind is the state described by the density matrix

$$\hat{\rho}_\theta = \rho_\theta^{(0)} \exp\Big[-\sum_J \hbar\omega_J \hat{c}_J^* \hat{c}_J (k_B\theta)^{-1} \Big], \qquad (2.31)$$

where $\omega_J = |\mathbf{g}|\nu_J c^{-1}$ is the frequency of mode J measured in a G system in which the acceleration of free fall is $\mathbf{g}$. This distribution describes an equilibrium gas of black-body radiation from scalar massless particles, and the arbitrary parameter θ is the temperature of this gas measured in the G system. We recall that the local temperature θ_{loc} of an equilibrium radiation gas in a static gravitational field depends on the position in such a way that $\theta_{loc}(\mathbf{x})\, |g_{00}(\mathbf{x})|^{1/2}$ is constant. Therefore, to fix the value of this temperature we must specify the point at which it is considered. In our case, this is the coordinate origin $x = \rho - c^2|\mathbf{g}|^{-1} = 0$ of the system, i.e., θ is the temperature at this point. We note that in the special case when $\theta = T_a$ the density matrix (2.31) is equal to (2.26) if we set $\mathbf{g} = -\mathbf{a}$, $\hat{c}_J = \hat{b}_J^{(+)}$, $\hat{c}_J^* = \hat{b}_J^{(+)*}$. The state in the homogeneous static gravitational field described by such a density matrix is called the Hartle-Hawking "vacuum" and denoted by $|0; H\rangle$.[12] In the case of a homogeneous field $a = c^2\rho^{-1}$, and therefore in such a state $|0; H\rangle$ the equality $\theta_{loc}(\mathbf{x}) = T_{a(\mathbf{x})}$ holds not only at the origin of the G system but also in the entire space.

3. DETECTORS AT REST IN A UNIFORMLY ACCELERATED FRAME OF REFERENCE AND IN A STATIC HOMOGENEOUS GRAVITATIONAL FIELD

3.1. Model description of a detector

Hitherto, our attention has been concentrated on the description of the state of the quantized field. The different descriptions of the same physical state (in particular, the Minkowski space vacuum $|0; M\rangle$) in different frames of reference warrant study in their own right, but they acquire real physical interest in connection with the problem of the interaction of accelerated bodies ("detectors") with the corresponding quantized field. A specific feature of such problems (in what follows, we shall have in mind, unless stated otherwise, only uniformly accelerated detectors) is that to describe the state of the bodies it is convenient to use a uniformly accelerated frame of reference, whereas to describe the vacuum of the quantized field with which the body interacts it is more convenient to use an inertial frame of reference. Of course, if we are calculating any invariant quantities the result will not depend on the choice of the particular frame of reference. Ultimately, the choice of such a system is a question of convenience. Specifically, if we are interested in the way in which the state of a body changes as a result of interaction with the field it is convenient to consider the process in the uniformly accelerated frame of reference. The change in the state of the radiation field is more readily described in the inertial frame of reference.

In the present section, we consider the behavior of a uniformly accelerated "detector" and the behavior of a "detector" at rest in a homogeneous static gravitational field. By "detector" we shall (as is usual) understand a body ("system") with internal degrees of freedom such that the interaction with the quantized field changes the state of these internal degrees of freedom. In principle, the role of the detector could be played by molecules, atoms, etc. Initially, however, we shall not specify any particular realization of the detectors but will describe certain general features of their behavior, deferring to the end of the section the consideration of the possibility of using as such detectors real physical objects (ions, elementary particles, etc.). Since the question of the behavior of a uniformly accelerated detector has often (beginning with Unruh's paper[18]) and in detail been considered in the literature (see, for example, Refs. 19–22 and the references given there), we shall give here the necessary results only briefly.

Suppose the body that we regard as a detector is initially at rest in an inertial frame of reference, has a set of discrete energy levels, and is in some stationary state $|i\rangle$. Of course, if this is not the ground state and the nonmoving detector interacts with the quantized field, then transitions are possible and a strictly stationary state is impossible. For the moment, we shall not consider these processes, making the assumption that the interaction of the detector with the field is weak, and formally we set the coupling constant characterizing this interaction equal to zero. In other words, we describe initially the state of the detector, and we then take into account its interaction with the field as a perturbation. We now put the detector into a state of uniformly accelerated (with acceleration a) motion. If the acceleration takes place adiabatically, levels different from i are not excited, and the system will still remain in the state $|i\rangle$. Since the detector moves with an acceleration, an external force must obviously act on it and deform it to a certain degree. In general, the wave functions that describe the stationary state will also be deformed, and the energy levels of the detector will themselves change. However, we shall assume that the levels remain discrete—in the uniformly accelerated frame of reference that we consider below and in which the detector is at rest, they correspond to the Rindler energies E_i, and the wave functions $|i\rangle$ in this state are characterized by the time dependence $\exp(-iE_i\tau/\hbar) = \exp(-i\varepsilon_i\eta)$, where $\varepsilon_i = cE_i/\hbar a$, τ is the proper time in the uniformly accelerated frame, and $\eta = a\tau/c$ is the Rindler time. With regard to the spatial behavior of the detector wave functions, we shall merely make the assumption that the dimensions of the detector are much less than c^2a^{-1} and, accordingly, the wave functions are significantly nonzero only in a certain neighborhood of the world line $\rho = \rho_0 = c^2a^{-1}$, whose scale l (in the self-frame) satisfies the condition $l \ll c^2a^{-1}$.

Let $\hat{K}_D^{(+)}$ be the Hamiltonian that describes the evolution of the noninteracting detector with variation of the Rindler time η; then

$$\hat{K}_D^{(+)}|i\rangle = \varepsilon_i|i\rangle. \tag{3.1}$$

Under the condition that the states of the detector are orthogonal and form a complete system, i.e.,

$$\langle i|j\rangle = \delta_{ij}, \quad \sum_i |i\rangle\langle i| = I, \tag{3.2}$$

this Hamiltonian can be described in the form

$$\hat{K}_D^{(+)} = \sum_i \varepsilon_i|i\rangle\langle i|. \tag{3.3}$$

We define $\hat{d}_i^*$ and $\hat{d}_i$ as operators of creation and annihilation of the detector state $|i\rangle$, setting

$$\hat{d}_i^*\hat{d}_j \equiv |i\rangle\langle j|, \tag{3.4}$$

and then the detector Hamiltonian (3.3) in the employed second-quantization representation can be expressed in the form (see, for example, Ref. 53)

$$\hat{K}_D^{(+)} = \sum_i \varepsilon_i \hat{d}_i^{(*)}\hat{d}_i. \tag{3.5}$$

In the absence of interaction of the uniformly accelerated detector with the quantized field, its behavior is trivial: If initially it was in the state $|i\rangle$, then it always remains in this state. The same is true of the stationary (including the vacuum) states of the quantized field. Inclusion of an interaction between the detector and the field changes the situation qualitatively; for the state $|i\rangle|0;M\rangle$ of the complete system (detector plus radiation field) will not in general be an eigenstate for the complete (with allowance for the interaction) Hamiltonian. In the detector transitions take place from level to level (the detector "detects" field quanta), and the state of the radiation field changes.

3.2. Interaction of an accelerated detector with the vacuum. Description in a uniformly accelerated frame of reference

We consider the simplest case when the interaction of the quantized scalar massless field $\hat{\varphi}$ with the detector is described by the following addition to the action:

$$S_I = \int (-g(x))^{1/2}\lambda\hat{\varphi}(x)\hat{\Psi}^*(x)\hat{\Psi}(x)\,d^4x, \tag{3.6}$$

where

$$\hat{\Psi}(x) = \sum_i \hat{d}_i\Psi_i(\mathbf{x})\,e^{-i\varepsilon_i\eta},$$
$$\hat{\Psi}^*(x) = \sum_i \hat{d}_i^*\overline{\Psi_i(\mathbf{x})}\,e^{i\varepsilon_i\eta}, \tag{3.7}$$

$\mathbf{x} = (\rho,y,z)$, λ is the coupling constant, and $\Psi_i(\mathbf{x}) = \langle\mathbf{x}|i\rangle$ is the wave function of the detector in the state $|i\rangle$.

Note that the wave functions $\Psi_i(\mathbf{x})$ are significantly nonzero only in a small neighborhood of the world line $\mathbf{x} = \mathbf{x}_0 = (c^2a^{-1}, 0, 0)$, and therefore the integration in (3.6) is effectively over this region. It is convenient to expand $\hat{\varphi}(\eta,\mathbf{x})$ in a Taylor series of the form

$$\hat{\varphi}(\eta, \mathbf{x}) = \sum_{l=0}^{\infty} \frac{1}{l!}\Delta\mathbf{x}^{a_1}\dots\Delta\mathbf{x}^{a_l}\hat{\varphi}_{a_1\dots a_l}(\eta), \tag{3.8}$$

where $a_k = 1, 2, 3$,

$$\hat{\varphi}_{a_1\dots a_l}(\eta) = \left(\frac{\partial^l}{\partial x^{a_1}\dots\partial x^{a_l}}\hat{\varphi}(\eta, \mathbf{x})\right)_{\mathbf{x}=\mathbf{x}_0}, \tag{3.9}$$

$$\Delta\mathbf{x}^a = \mathbf{x}^a - \mathbf{x}_0^a. \tag{3.10}$$

If we substitute the expansion (3.8) in (3.6), we obtain

$$S_I = -\int \hat{K}_I\,d\eta,$$
$$\hat{K}_I = -\lambda\sum_{l=0}^{\infty}\sum_{i,j}\frac{1}{l!}\sum_{a_1\dots a_l}\exp[-i(\varepsilon_j-\varepsilon_i)\eta] \times D_{ij}^{a_1\dots a_l}\hat{\varphi}_{a_1\dots a_l}(\eta)\,\hat{d}_i^*\hat{d}_j, \tag{3.11}$$

where

$$D_{ij}^{a_1\cdots a_l}=\int_0^\infty d\rho\int d^2z\,\rho\Delta x^{a_1}\ \ldots\ \Delta x^{a_l}\overline{\Psi}_i(\mathbf{x})\Psi_j(\mathbf{x}). \quad (3.12)$$

The operator $\hat{K}_I$ is the interaction Hamiltonian in the uniformly accelerated frame. The terms of the sum with $l=0,1,\ldots$ correspond to the contribution of the monopole, dipole, and higher terms of the multipole expansion of the interaction Hamiltonian. One can show that in the case when the inequalities $|\varepsilon_i-\varepsilon_j|\ll 1$ are satisfied the main contribution to the detector transition amplitude is, in general, made by the monopole term,

$$\hat{K}_{I,0}=-\lambda\sum_{i,j}\exp[-i(\varepsilon_j-\varepsilon_i)\eta]D_{ij}\hat{\varphi}(\eta)\hat{d}_i^*\hat{d}_j. \quad (3.13)$$

Of course, in the case of interaction with the transverse magnetic field the main term will usually be the dipole term in the expansion of the interaction Hamiltonian (in this case, the monopole term vanishes).

Using the standard perturbation theory in the interaction representation, we can readily show that in the framework of the monopole approximation (3.13) the amplitude $A_{iI\to fF}$ for transition of the system from the state $|i\rangle|I\rangle$ ($|i\rangle$ and $|I\rangle$ are the initial states of the detector and the field) to the state $\langle f|\langle F|$ ($\langle f|$ and $\langle F|$ are final states of the detector and the field) during the complete time of "operation" of the detector has the form

$$A_{iI\to fF}=-i\hbar^{-1}\int\lambda(\eta)e^{-i(\varepsilon_i-\varepsilon_f)\eta}D_{fi}\langle F|\hat{\varphi}(\eta)|I\rangle d\eta. \quad (3.14)$$

Averaging $|A_{iI\to fF}|^2$ over all the final states of the radiation field and taking into account the condition of completeness of these states,

$$\sum_F|F\rangle\langle F|=I, \quad (3.15)$$

we obtain the following expression for the probability $W_{i\to f}^I$ of transition of the detector from the initial state i to the final state f:

$$W_{i\to f}^I=\hbar^{-2}\int d\eta\int d\eta'\lambda(\eta)\lambda(\eta')\exp[-i(\varepsilon_i-\varepsilon_j)(\eta-\eta')]\times G_I(\eta',\eta)|D_{fi}|^2, \quad (3.16)$$

where $G_I(\eta,\eta')=\langle I|\hat{\varphi}(\eta)\hat{\varphi}(\eta')|I\rangle$. In the more general case in which the initial state is described by some density matrix $\hat{\rho}_I$, the expression (3.16) for the transition probabilities is unchanged except for the difference that

$$G_I(\eta,\eta')=\mathrm{Sp}(\hat{\rho}_I\hat{\varphi}(\eta)\hat{\varphi}(\eta')). \quad (3.17)$$

In the special case with which we must deal in what follows $G_I(\eta,\eta')=g_I(\eta-\eta')$, i.e., the function G_I depends only on the difference $\eta-\eta'$, and in place of the total probability $W_{i\to f}^I$ we introduce the probability $W_{i\to f}^I$ of transition of the detector in unit proper time $\tau=c\eta a^{-1}$. One can show (see, for example, Ref. 19) that if $\lambda=\lambda_0=$ const then

$$w_{i\to f}^I\equiv\frac{dW_{i\to f}^I}{d\tau}=\lambda_0^2|D_{fi}|^2F^I(\varepsilon_f-\varepsilon_i), \quad (3.18)$$

where

$$F^I(\varepsilon)=\frac{a}{c}\hbar^{-2}\int_{-\infty}^{\infty}e^{-i\varepsilon\eta}g_I(\eta)\,d\eta. \quad (3.19)$$

The spectral distribution function $F^I(\varepsilon)$ is completely determined by the state of the radiation field, whereas the matrix elements $|D_{fi}|^2$ depend only on the structure and properties of the detector. We can similarly obtain an expression for the probability of detector transitions in unit proper time analogous to (3.18) in the case when the main contribution is made by the terms of the multipole expansion (3.11) of order l and the transitions associated with the lower multipoles are suppressed.

The spectral function $F^M(\varepsilon)$ corresponding to uniformly accelerated motion of the detector in the vacuum $|0;M\rangle$ can be obtained using either the picture of Rindler quanta, i.e., by making the calculations in the A system) or by the more usual method in which the calculations are made in an inertial system (I system). Let us compare these methods.

Since the world line of the detector motion, described by the equation $\mathbf{x}=\mathbf{x}_0=$ const, lies entirely in R_+, to calculate $G_M(\eta,\eta')=\langle 0;M|\hat{\varphi}(\eta)\varphi(\eta')|0;M\rangle$ we can use the relation (2.25). Substituting the expansion (2.17) for $\hat{\varphi}$, we find

$$G_M(\eta,\eta')=\sum_J[\{\exp[i\nu(\eta-\eta')]\bar{n}_J+\exp[-i\nu(\eta-\eta')]\times(\bar{n}_J+1)\}|U_J(\mathbf{x}_0)|^2], \quad (3.20)$$

where

$$\bar{n}_J=\mathrm{Sp}_{(+)}(\hat{\rho}^{(+)}\hat{b}_J^{(+)*}\hat{b}_J^{(+)})=\bar{n}_\nu=[\exp(2\pi\nu)-1]^{-1}=\left(\exp\frac{\hbar\omega}{k_BT_a}-1\right)^{-1} \quad (3.21)$$

are the mean population numbers of the Rindler particles in mode J. The relation (3.20), shows, in particular, that $G_M(\eta,\eta')$ actually depends only on the difference $\eta-\eta'$. Substituting (3.20) in (3.19) and making some simple manipulations, we find

$$F^M(\varepsilon)=\Gamma_a^M(|\varepsilon|)[\theta(\varepsilon)\bar{n}_\varepsilon+\theta(-\varepsilon)(\bar{n}_{|\varepsilon|}+1)], \quad (3.22)$$

where

$$\Gamma_a^M(|\varepsilon|)=\frac{2\pi}{\hbar^2}\sum_J\delta(\nu-|\varepsilon|)|U_J(\mathbf{x}_0)|^2=\frac{2\pi}{\hbar^2}\frac{a}{c}\int|U_{|\varepsilon|q}(\mathbf{x}_0)|^2d^2q=\frac{a^3}{2\pi c^6\hbar}|\varepsilon|. \quad (3.23)$$

The last equation is obtained by direct calculation of the integral for the wave functions (2.16). It is interesting to note that for a scalar massless field Γ_a^M (the phase space in the frame of reference with acceleration **a**) is equal to Γ_θ, the phase space in a thermal bath with temperature $\theta=T_a$ at rest in the inertial frame. This result is not universal; it does not hold for a scalar massless field in a space-time different from 2 and 4 dimensions, and also for other fields even in 4-dimensional space.[42,43]

In other words, the probability of transitions of the uniformly accelerated detector from level i to level f in unit proper time τ is proportional to the number density $\bar{n}_{\Delta\omega_{if}}$ of the Rindler quanta with energy $\hbar\Delta\omega_{if}=\hbar(\omega_f-\omega_i)$ and to the "phase space" Γ_a^M of the Rindler particles with this energy at the point $\mathbf{x}_0$ on the detector world line (we recall that in the accelerated frame the detector is at rest at the point $\mathbf{x}=\mathbf{x}_0$).

The spectral function $F^M(\varepsilon)$ for other boson fields has a form analogous to (3.22), while for Fermi fields $\bar{n}_\varepsilon$ is replaced by $\bar{n}_\varepsilon^F=[\exp(2\pi\varepsilon)+1]^{-1}=[\exp(\hbar\omega/$

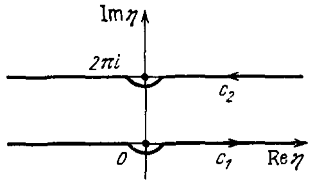

FIG. 3. Contours of integration in the plane of the complex variable η.

$k_B T_a) + 1]^{-1}$ (see, for example, Ref. 22). The explicit expression for the phase space Γ_a^M depends, of course, on the type of field.

The expression (3.22) for the spectral distribution can be obtained by making the calculations in the I frame. It is sufficient to note that $G_M(\eta, \eta')$ is equal to the value of the positive-frequency function $G_M(x,x') = \langle 0; M|\hat{\varphi}(x)\hat{\varphi}(x')|0; M\rangle$ calculated for points x and x' lying on the detector world line. Using the explicit expression for $G_M(X, X')$ in Cartesian coordinates,

$$G_M(X,X') = \{4\pi^2 \hbar c^{-1}[\eta^{\mu\nu}(X_\mu - X'_\mu)(X_\nu - X'_\nu) + i\zeta(T - T')]\}^{-1}, \qquad (3.24)$$

where $\zeta > 0$ is an infinitesimally small correction, going over to the Rindler coordinates, and substituting the detector equation of motion $\rho = \rho_0 = c^2 a^{-1}$, $y = z = 0$, we obtain

$$G_M(\eta, \eta') = g_M(\eta - \eta') = -\frac{a^2\hbar}{16\pi^2 c^5}\,\mathrm{sh}^{-2}\left[\frac{1}{2}(\eta - \eta' - i\zeta)\right]. \qquad (3.25)$$

The integral in (3.19) can be readily calculated. The function $g_M(\eta)$ is periodic (with period $2\pi i$) in the complex η plane. Integrating the function $\exp(-i\varepsilon\eta)g_M(\eta)$ around a closed contour containing the straight lines C_1 and C_2 (Fig. 3), and calculating the integral by means of the residues, we obtain an expression for $F^M(\varepsilon)$ that is identical to (3.22).

It is an interesting fact that in a space-time with an odd number of dimensions $d + 1$ (d is the number of spatial dimensions) the element of phase space Γ_a^M for bosons contains the factor $[\exp(2\pi\varepsilon) - 1][\exp(2\pi\varepsilon) + 1]^{-1}$ and $F^M(\varepsilon)$ takes a form similar to the spectral function for fermions.[43] Similarly, in the case of fermions in these spaces an additional factor $[\exp(2\pi\varepsilon) + 1][\exp(2\pi\varepsilon) - 1]^{-1}$ arises in Γ_a^M (Ref. 22). Of course, no real change of the statistics occurs in these cases.[54] (On the connection of this effect with the absence of Huygens's principle in spaces with an odd number of dimensions, see Ref. 55.)

We point out a further feature of the "thermal gas" of Rindler quanta corresponding to the vacuum $|0; M\rangle$. It is that in the system there is a distinguished direction (the direction of the acceleration vector $\mathbf{a}$). This has the consequence that the distribution of the Rindler quanta is anisotropic. This anisotropy may be manifested, in particular, in the fact that the probabilities of detector transitions of dipole type will, in general, depend on the orientation of the detector relative to the acceleration vector.[36,37,41] In the discussed monopole case, the anistropy will not, of course, be manifested.

We now note the following general property of the transition probabilities $w_{i\to f}^M$ that follows from the representation (3.22) for the spectral function, namely, the transition probability $w_{i\to f}^M$ and the probability of the reverse transition, $w_{f\to i}^M$, are connected by

$$\frac{w_{i\to f}^M}{w_{f\to i}^M} = \frac{F^M(\varepsilon_f - \varepsilon_i)}{F^M(\varepsilon_i - \varepsilon_f)} = \exp[2\pi(\varepsilon_i - \varepsilon_f)] = \exp\frac{\hbar(\omega_i - \omega_f)}{k_B T_a}. \qquad (3.26)$$

This relation is known as the principle of detailed balance. It means, in particular, that if the distribution of the uniformly accelerated detector with respect to the levels is described by an equilibrium thermal density matrix with temperature T_a ("internal degrees of freedom of the detector heated to temperature T_a"), then such a state of a detector when coupled to a quantized field in the state $|0; M\rangle$ will not be changed. It is assumed here, of course, that we can ignore the back reaction on the detector of the changes that it makes in the state of the field. In the case we consider, such an assumption is sensible and can be justified, since the field quanta created when the detector transitions occur propagate freely and escape to infinity (this process is described in more detail in Sec. 4).

The relation (3.26) also means that a thermal density matrix with temperature T_a describes the state of the detector after the establishment of the equilibrium which arises as a result of prolonged motion with constant acceleration during which the detector interacts with the vacuum $|0; M\rangle$. This final equilibrium state does not depend on the chosen initial state. The characteristic time $\Delta\tau$ of establishment in the A system of a thermal distribution of the detector with respect to the levels is of order

$$\Delta\tau \sim \hbar\frac{(c^2/a)^3}{\lambda_0^2|D|^2}, \qquad (3.27)$$

where $\lambda_0^2|D|^2$ is the characteristic value of the transition matrix elements $\lambda_0^2|D_{fi}|^2$ (the "cross section" of the detector).

We emphasize that the principle of detailed balance (3.26) has a universal nature. It is satisfied not only for fields of different types but is also independent of the multipolarity of the interaction Hamiltonian $\hat{K}_I$ [see (3.11)]. Ultimately, this relation is a consequence of the thermal nature of the density matrix $\hat{\rho}^{(+)}$ (2.26) describing the distribution of the Rindler quanta in the R_+ region which corresponds to the vacuum $|0; M\rangle$. It is universal to the extent that the conclusion of a thermal nature of the density matrix $\hat{\rho}^{(+)}$ is universal. This universality, like the fulfillment in the general case of the principle of detailed balance in the case of uniformly accelerated motion, can be established particularly easily if the treatment is made in the uniformly accelerated frame of reference. It is here that the advantages mentioned above of such a description for these problems are, in particular, manifested.

We emphasize once more that, of course, when we calculate quantities that are invariant (that do not depend on the frame of reference) we can use all frames of reference. In particular, we could obtain the relation (3.26) by making all the calculations in the inertial frame of reference. However, the occurrence each time of relations of the type (3.26) in the different variants of the theory would appear as rather remarkably fortuitous. But when the treatment is made in the uniformly accelerated frame the appearance of such relations is, as was emphasized above, rather natural.

3.3. Behavior of a detector at rest in a static homogeneous gravitational field

To describe the interaction of a detector at rest in a static homogeneous gravitational field with the quantized field $\hat{\varphi}$ we can use methods that to a large degree are analogous to the ones employed to describe a uniformly accelerated detector. Therefore, omitting the details, we shall dwell only on some basic points.

We shall consider a detector of the same form as in Sec. 3.1, assuming this time that it is adiabatically introduced into a static gravitational field and placed at the point $\mathbf{x} = \mathbf{x}_0$, at which the acceleration of free fall is $\mathbf{g}$. We shall assume that the levels of this detector are discrete; they correspond to the energy (calculated with allowance for the work of the gravitational forces) E_i, so that the wave functions $|i\rangle$ in this relation are characterized by a time dependence $\exp(-iE_i\tau/\hbar) = \exp(-i\varepsilon_i\eta)$, where τ is the proper time in the G system in which the detector is at rest, $\varepsilon_i = cE_i/g\hbar$ and $\eta = g\tau c^{-1}$. In this case, the detector Hamiltonian $\hat{K}_D^{(+)}$ in the second-quantization representation has the previous form (3.5).

Suppose the interaction of the detector with the quantized field $\hat{\varphi}$ is described by the expression (3.6). Then in the monopole approximation the probability of transition of the detector in unit proper time τ from level i to level f is given by the expressions (3.18)–(3.19), where

$$g(\eta) = \langle \hat{\varphi}(\eta, \mathbf{x}_0)\, \hat{\varphi}(0, \mathbf{x}_0)\rangle. \tag{3.28}$$

The expectation value in (3.28) is calculated with respect to the state of the quantized field. We denote by $\bar{n}_J = \langle \hat{c}_J^{+}\hat{c}_J\rangle$ the mean number of "Boulware" quanta in the mode u_J for this state. If $\bar{n}_J$ depends only on the energy ν of these quanta ($\bar{n}_J = \bar{n}_\nu$), then for the spectral function $F(\varepsilon)$, which occurs in the expression (3.18) for the transition probability $w_{i\to f}$, we can obtain a representation analogous to (3.22)–(3.23):

$$F(\varepsilon) = \Gamma_g(|\varepsilon|)\, [\theta(\varepsilon)\bar{n}_\varepsilon + \theta(-\varepsilon)(\bar{n}_{|\varepsilon|} + 1)], \tag{3.29}$$

where

$$\Gamma_g(|\varepsilon|) = c\hbar^2 \int |U_{|\varepsilon|\mathbf{q}}(\mathbf{x}_0)|^2\, d\mathbf{g} \tag{3.30}$$

and $e^{-i\nu\eta}U_{\nu\mathbf{q}}(\mathbf{x})$ are the wave functions of the mode with quantum numbers $\nu\mathbf{q}$ of the scalar field in the given external static gravitational field.

We apply this expression to the case when the detector is at rest in the static homogeneous gravitational field and the quantized field is in its ground state described by the Boulware vacuum $|0; B\rangle$. For this state, we obviously have $\bar{n}_\varepsilon^B = 0$ and

$$F^B(\varepsilon) = \theta(-\varepsilon)\, \Gamma_g(|\varepsilon|). \tag{3.31}$$

If the detector was in the ground state, it will not be excited. This becomes obvious if one notes that for ground state i the energy differences $\varepsilon_f - \varepsilon_i$ are always positive (if $i \neq f$). This result is valid for any static (not necessarily homogeneous) gravitational field provided one chooses the vacuum $|0; B\rangle$.

As a different example we consider the behavior of the detector in the static gravitational field in the case when there is an equilibrium gas of black-body radiation with temperature θ (measured in the G system). This state is described by the density matrix (2.31), and for it

$$\bar{n}_\varepsilon^\theta = [\exp(2\pi\beta\varepsilon) - 1]^{-1} = \{\exp[\hbar\omega(k_B\theta)^{-1}] - 1\}^{-1}, \tag{3.32}$$

where $\omega = c^{-1}|g|\varepsilon$, $\beta = \hbar|g|(2\pi k_B c\theta)^{-1}$.

For the transition probabilities of the detector interacting with this thermal gas the principle of detailed balance holds,

$$\frac{w_{i\to f}^\theta}{w_{f\to i}^\theta} = \exp[\hbar(\omega_i - \omega_f)(k_B\theta)^{-1}], \tag{3.33}$$

and the distribution with respect to the levels of such a detector will after a time become the Boltzmann distribution with the same temperature θ as the local temperature of the ambient gas at the point at which the detector is at rest. Therefore, such a detector in an arbitrary static gravitational field can, in particular, play the role of a thermometer measuring the local temperature $\theta_{loc}(\mathbf{x})$ of the gas, this temperature being described in the equilibrium state by the relation $\theta_{loc}(\mathbf{x}) = \theta_0|g_{\tau\tau}(\mathbf{x})|^{-1/2}$, where θ_0 is a constant.[56]

In the case we consider of a homogeneous gravitational field, the wave functions $u_{\nu\mathbf{q}}$ of the scalar field are described by the expression (2.29), and for the spectral function (3.29) we have

$$F^\theta(\varepsilon) = \frac{|g|^3}{2\pi c^6\hbar}\, \varepsilon\, (\exp(2\pi\beta\varepsilon) - 1]^{-1}, \tag{3.34}$$

where $\beta = \hbar|g|/2\pi k_B c\theta = T_g/\theta$. Obviously, for the state $|0; H\rangle$ (the Hartle-Hawking vacuum), for which $\beta = 1$, this spectral function is identical to the spectral function $F^M(\varepsilon)$ described by the relations (3.22)–(3.23). In other words, a detector at rest in a static homogeneous gravitational field at a point at which the acceleration of free fall is $\mathbf{g}$ behaves in the case of interaction with the quantized field in the state $|0; H\rangle$ in exactly the same way as an analogous detector moving with acceleration $\mathbf{a} = -\mathbf{g}$ in ordinary Minkowski space provided that the quantized field in this space is in the vacuum state $|0; M\rangle$.

For the convenience of the readers, the different vacuum states used in the present paper are compared in Table I. Summarizing the arguments given above, we can say that the detectors at rest in the I, A, and G systems detect the M, R, and G quanta, respectively.

3.4. Elementary particles in the role of detectors

Naturally, the question arises of whether we can under real conditions observe excitation of an accelerated "detector" and also whether this problem is of interest in any physical investigations. We note first of all that it follows from Eq. (1.3) for temperature T_a that a temperature of 1°K corresponds to the huge acceleration $a_{1°K} = 2.4\cdot 10^{22}$ cm/sec^2. It is easy to show that in the case of motion of a macroscopic body with such acceleration the work done by the accelerating force on the constituent particles of the body over the characteristic distance of the order of the distance between the particles greatly exceeds the characteristic interaction energy of the particles.[13)] Therefore, it is evidently impossible to impart to a macroscopic body such an acceleration without destroying it. The possibility of observing the effects in which we are interested is just as problematic for accelerated ions. It appears that the most realistic objects for use as accelerated detectors are elementary particles.[33] First of all,

TABLE I.

Vacuum	Notation	Region in which vacuum is defined	Definition of positive-frequency solutions or connection with other states	Conditions under which the detector does not detect "quanta" in the chosen vacuum
Minkowski	$\|0; M\rangle$	Minkowski space	$\exp(-i\omega T)$, T is the time in the I system	Detector at rest in the I system (I is an inertial frame of reference)
Rindler	$\|0; R\rangle$	R_+ region of Minkowski space (see Fig. 1)	$\exp(-i\nu\eta)$, $\tau = \eta c/a$ is the time in the A system, a is the acceleration of the A system	Detector at rest in the A system (A is a uniformly accelerated frame of reference)
Boulware	$\|0; B\rangle$	Static homogeneous gravitational field	$\exp(-i\nu\eta)$, $\tau = \eta c/g$ is the time in the G system, g is the acceleration of free fall in the G system	Detector at rest in the G system (G is the frame of reference at rest in a static gravitational field)
Hartle-Hawking	$\|0; H\rangle$	Static homogeneous gravitational field	The state corresponds to equilibrium thermal (with temperature $T_g = \hbar g/2\pi k_B c$) distribution of the B quanta in the G system (g is the acceleration tion of free fall in the G system)	Detector at rest in a freely falling frame of reference

Note. In a homogeneous gravitational field the behavior of detectors in the vacuum state $|0; B\rangle$ in the G system is identical to the behavior of the same detectors in the vacuum state $|0; R\rangle$ in the A system. Detectors that are identical as regards their structure and state of motion behave in exactly the same way in the vacuum state $|0; M\rangle$ in the A system in Minkowski space and in the vacuum state $|0; H\rangle$ in the G system in the homogeneous gravitational field.

we must explain how elementary particles, which in a certain sense are structureless, can play the part of detectors, i.e., bodies for which one characteristically requires the presence of different internal states. The point is that the motion of such a particle is characterized not only by momentum or acceleration but also by a complete set of quantum numbers, for example, the quantum number s_z that determines the projection of the spin of the particle onto a certain axis z. If such a particle moves under the influence of an external force with a constant fixed acceleration, then it can additionally execute a uniform motion with transverse momentum $p_\perp$ in the direction perpendicular to the acceleration. At the same time, the particle may have different spin orientations, etc. These additional degrees of freedom, characterized by the values of s_z and $p_\perp$, can in a certain sense be regarded as "internal," and one can study the distribution of the particle with respect to the energy levels corresponding to these degrees of freedom. It is in this sense that elementary particles can be used as "detectors."

As simplest cases we note that, as the calculations of Ref. 57 show, the motion of an electron in the vacuum $|0; M\rangle$ under the influence of a constant homogeneous electric field of intensity **E** is accompanied by "thermal" (with temperature $T_{a(E)} = \hbar a(E)/2\pi k_B c$, $a(E) = eE/m_e$) excitation of the "energy levels" corresponding to its motion in the direction at right angles to the field. If along the electric field **E** there is also a magnetic field **H**, so that the energy levels corresponding to the transverse motion of the electrons are quantized, then accelerated motion of the electron will be accompanied by thermal [with temperature $T_{a(E)}$] excitation of the corresponding Landau levels. [14)] A thermal distribution over the levels also arises for the energy levels corresponding to interaction of the spin of an accelerated electron with a magnetic field.[33]

The estimates show[33,60] that the "thermal" corrections which arise from uniformly accelerated motion of electrons at the energies of the existing accelerators or ones under construction could in principle lead to observable effects. The main obstacle is the length of the time during which the "thermal equilibrium" is established. We note that whereas the characteristic time τ_{eq} of establishment of thermal equilibrium over the levels of the accelerated detector measured in the uniformly accelerated frame in which it is at rest appreciably exceeds c/a (a is the acceleration), the time measured in the laboratory (inertial) frame is of order $\Delta T_{eq} \sim ca^{-1}\exp(\tau_{eq}a/c)$; see (2.35). The exponential nature of this dependence has the consequence that with increasing τ_{eq} the value of ΔT_{eq} becomes so great that observation in the inertial frame of reference of the process of establishment of equilibrium in the case of electrons accelerated in linear accelerators becomes effectively impossible.

Estimates show[33,60] that the situation is much more favorable in the case of ring accelerators. We note that for motion of an electron in such an accelerator the motion is not uniformly accelerated, since the acceleration of constant magnitude constantly changes its direction. There have been studies (among which we mention those of Refs. 33 and 60–64) in which the interaction of a detector moving uniformly in a circle with a quantized field has been analyzed. It was found that although the probabilities of transitions of such a detector no longer satisfy the principle of detailed balance, so that the equilibrium distribution of the detector over the levels will not be thermal, many of the features of uniformly accelerated motion remain. Estimates show[33,60] that, for example, for the ring accelerator SPEAR at Stanford for electrons with energy ~ 4 GeV the time of establishment of equilibrium in the case of interaction of the electron spin with the quantized field is of order 10 min and that these effects are in principle measurable. These effects of interaction of an electron with a vacuum could explain the observed depolarization of electron beams (for more detail about this, see Refs. 33 and 60).

It goes without saying that we do not at all regard it as necessary to test specially the results presented above relating to excitation of a uniformly accelerated detector. From the experimental point of view, only the opposite formulation of the problem is of interest—it is possible that in certain cases the formula obtained for T_a and other expressions may be suitable for the analysis and interpretation of experiments already made and future experiments.

4. EXCITATION OF AND RADIATION FROM A UNIFORMLY ACCELERATED DETECTOR. DESCRIPTION IN AN INERTIAL FRAME OF REFERENCE

Thus, a uniformly accelerated detector interacting with the quantized field $\hat{\varphi}$ in the state $|0; M\rangle$ and described in the uniformly accelerated frame in which it is at rest behaves in the same way as if it were surrounded by an equilibrium thermal gas of Rindler quanta with temperature T_a. Moreover, the excitation of the detector is accompanied by the absorption of a Rindler quantum from the thermal "bath," and the transition to a lower energy level is accompanied by emission of a Rindler quantum. What will then be "seen" by an observer at rest in the inertial frame? An answer to this question was given in Ref. 23 and is as follows. In the absence of real quanta (the state $|0; M\rangle$) and for the description in the inertial system transition of the detector to either a higher or a lower level will be accompanied by emission of a quantum of the field $\hat{\varphi}$, i.e., in the electromagnetic case by emission of a real photon.

To see this, we consider the simplest variant of a monopole detector possessing just two levels: an "upper" one (state $|t\rangle$) and a "lower" one (state $|l\rangle$), and we assume that the difference of the Rindler energies of these levels is $\Delta\varepsilon = \varepsilon_t - \varepsilon_l$. We shall also assume that $D_{tt} = D_{ll} = 0$ [see (3.13)]; then, denoting $D = D_{tl} = \overline{D_{lt}}$, we write the Hamiltonian (3.13) of the interaction of such a detector with the quantized field in the form

$$\hat{K}_{I,0} = -\lambda\,(D e^{i\Delta\varepsilon\eta}\hat{d}_t^*\hat{d}_l + \overline{D} e^{-i\Delta\varepsilon\eta}\hat{d}_l^*\hat{d}_t)\,\hat{\varphi}(\eta). \quad (4.1)$$

Let $|\Psi\rangle_{\text{in}}$ be the initial (before the detector is switched on) state of the total system, i.e., of the detector and the radiation field. Then the final state $|\Psi\rangle_{\text{out}}$ of this system that arises from the chosen initial state as a result of interaction of the detector with the radiation field can be expressed in the interaction representation that we employ in the form

$$|\Psi\rangle_{\text{out}} = |\Psi\rangle_{\text{in}} - i\hbar^{-1}\int \hat{K}_{I,0}\,d\eta\,|\Psi\rangle_{\text{in}}. \quad (4.2)$$

Here as earlier in Sec. 3.2, we ignore the contributions of higher order in λ. (The conditions under which this is possible were discussed in Sec. 3.2). We apply the relation (4.2) for cases when the detector before the switching on of the interaction was in one of the states (l or t), and the state of the field was the vacuum state $|0; M\rangle$. We denote these states by

$$|\Psi_{l,0}\rangle_{\text{in}} = |0; M\rangle\,|l\rangle,\quad |\Psi_{t,0}\rangle_{\text{in}} = |0; M\rangle\,|t\rangle.$$

Then by means of the relations (4.1) and (4.2) we obtain

$$|\Psi_{s,0}\rangle_{\text{out}} = |\Psi_{s,0}\rangle_{\text{in}} + iD_s\int d^3k f_s(k)\,|1_{\mathbf{k}}; M\rangle\,|\bar{s}\rangle, \quad (4.3)$$

where $s = l$ or t and $\bar{s} = t$ or l, respectively; here, $D_l = D$, $D_t = \overline{D}$,

$$f_s(\mathbf{k}) = \hbar^{-1}\int d\eta\,\lambda(\eta)\,e^{i\Delta\varepsilon_s\eta}\overline{\Phi}_{\mathbf{k}}(X(\eta)), \quad (4.4)$$

$\Delta\varepsilon_l = \Delta\varepsilon$, $\Delta\varepsilon_t = -\Delta\varepsilon$, and $X^\mu = X^\mu(\eta)$ is the equation of the detector's world line. The relations (4.3) confirm what we said above. The elementary process associated with transition of the detector from the state t to the state l, like the reverse process ($l \to t$), is accompanied in the inertial frame by emission of field quanta. Of course, it cannot be otherwise, since in the state $|0; M\rangle$ there are no free quanta and they can only be emitted by the detector.

The mean number density of the quanta of the field $\hat{\varphi}$ with momentum $\mathbf{k}$ emitted as a result of the process is

$$\bar{n}_s(\mathbf{k}) = {}_{\text{out}}\langle\Psi_{s,0}|\hat{a}_{\mathbf{k}}^*\hat{a}_{\mathbf{k}}|\Psi_{s,0}\rangle_{\text{out}} = |D|^2|f_s(\mathbf{k})|^2. \quad (4.5)$$

In the case $\lambda = \text{const}$, the functions $f_s(\mathbf{k})$ can be calculated explicitly. If we denote

$$F(\mathbf{k};\ \varepsilon) = \int_{-\infty}^{\infty} \exp(-i\varepsilon\eta + i|\mathbf{k}|\,cT(\eta) - i\mathbf{k}\mathbf{X}(\eta))\,d\eta, \quad (4.6)$$

then

$$f_s(\mathbf{k}) = \frac{\lambda}{[(2\pi)^3 2|\mathbf{k}|\hbar c]^{1/2}}F(\mathbf{k};\ -\Delta\varepsilon_s). \quad (4.7)$$

To calculate the integral (4.6), it is convenient to introduce the notation

$$k_\perp = (k_y^2 + k_z^2)^{1/2},\quad \text{th}\,\eta_0 = k_x|\mathbf{k}|^{-1},\quad \varkappa_\perp = k_\perp c a^{-1}, \quad (4.8)$$

and we then have

$$F(\mathbf{k};\ \varepsilon) = \exp(-i\varepsilon\eta_0)\int_{-\infty}^{\infty}\exp[-i\varepsilon(\eta-\eta_0) + i\varkappa_\perp\,\text{sh}(\eta-\eta_0)]\,d\eta$$
$$= 2\exp(-i\varepsilon\eta_0)\exp\left(\frac{\pi\varepsilon}{2}\right)K_{i\varepsilon}(\varkappa_\perp). \quad (4.9)$$

The obtained expression enables us, in particular, to show that

$$\frac{\bar{n}_l(\mathbf{k})}{\bar{n}_t(\mathbf{k})} = e^{-2\pi\Delta\varepsilon}. \quad (4.10)$$

We note that the function $F(\mathbf{k};\varepsilon)$, and therefore $\bar{n}_s(\mathbf{k})$, does not in fact depend on the momentum vector $\mathbf{k}$ of the quantum but only on $k_\perp$. The situation here is completely analogous to that which obtains in the case of radiation of a uniformly accelerated electric charge, for which the spectral energy density of the radiation depends only on $k_\perp$ (for more detail about this, see, for example, Ref. 59). This feature is due to the invariance of the problem under shifts with respect to the Rindler time η. As in the case of the radiation of a uniformly accelerated charge, we can conclude on the basis of (4.9) that the radiation with a given value of the wave number is formed in a definite section of the detector's trajectory; this is called the "formation zone" or the "coherence interval," and its position (the parameter η_0) is determined by the ratio $k_x/k_\perp$.

As was already noted in Sec. 3.2, in the case of prolonged motion of the detector reverse transitions begin to play a part, and the conditions of applicability of the considered approximation (the first order in λ) are violated. In the simplest case of the two-level detector, these processes admit a fairly complete description. However, we shall not dwell on this in more detail and merely restrict ourselves to considering the limiting case when as a result of interaction of the detector with the quantized field the detector attains a state of equilibrium (which, as we have shown, is a thermal one for a uniformly accelerated detector) with distribution over the levels l and t characterized by the probabilities (populations)

$$w_l = [1+\exp(-2\pi\Delta\varepsilon)]^{-1},\quad w_t = \exp(-2\pi\Delta\varepsilon)\, w_l. \tag{4.11}$$

The mean number density of quanta with momentum $\mathbf{k}$ radiated by such a detector is

$$\bar{n}_a(\mathbf{k}) = \bar{n}_l(\mathbf{k})\, w_l + \bar{n}_t(\mathbf{k})\, w_t = \frac{\lambda^2|D|^2}{4\pi^3\hbar c|\mathbf{k}|}[\mathrm{ch}(\pi\Delta\varepsilon)]^{-1}|K_{i\Delta\varepsilon}(\varkappa_\perp)|^2. \tag{4.12}$$

We recall once more (see Sec. 3.2) that, by assumption, the radiation escapes freely "to infinity" and does not have a back reaction on the detector.

5. EXCITATION OF AND RADIATION FROM AN ACCELERATED DETECTOR AND THE ANOMALOUS DOPPLER EFFECT. SOME ADDITIONAL COMMENTS

The circumstance that an accelerated detector in an inertial frame of reference radiates a field quantum not only in the case of transitions from upper to lower levels but also when the reverse transitions associated with its excitation occur seems somewhat unusual and warrants a more detailed discussion. This applies particularly to the case when the detector is initially in its lowest energy level. In this connection, the present authors pointed out in Ref. 24 that actually analogous processes of radiation by a detector when it is excited have long been known and discussed, in particular in the study of the anomalous Doppler effect.[65]

Let us briefly recall the essence of this effect. (A more detailed discussion of it and corresponding references to the literature can be found in the book of Ref. 5, Chaps. 6 and 7.) Suppose a two-level detector (lower level l and upper level t) moving with constant velocity $\mathbf{v}$ in a medium with refractive index $n(\omega)$ $(vn > c)$ radiates a photon with momentum $\hbar k^\mu = (\hbar\omega/c, \hbar\mathbf{k})$, $\hbar k = \hbar\omega n/c$. To derive the conditions under which such radiation is possible, it is convenient to use the energy-momentum conservation law, writing it in the form

$$p_1^\mu - \hbar k^\mu = p_2^\mu, \tag{5.1}$$

$$p_i^\mu = (E_i c^{-1} \equiv [(m_0+m_i)^2c^2+\mathbf{p}_i^2]^{1/2},\ \mathbf{p}_i), \tag{5.2}$$

where p_i^μ is the 4-momentum of the detector before $(i=1)$ and after $(i=2)$ the radiation. Squaring Eq. (5.1), we can obtain

$$-\Delta\varepsilon\,(2m_0+m_1+m_2) = 2\frac{E_1}{c^2}\hbar\omega\left(1-\frac{vn}{c}\cos\vartheta\right) + \frac{\hbar^2\omega^2}{c^2}(n^2-1), \tag{5.3}$$

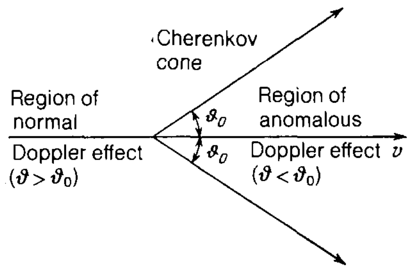

FIG. 4. Normal and anomalous Doppler effect. The figure shows the regions of the normal and anomalous Doppler effect in the case of motion of a particle with velocity $\mathbf{v}$ in a medium with refractive index $(vn/c > 1$, $\cos\vartheta_0 = c/vn$, ϑ_0 is the opening angle of the Cherenkov cone).

where $\mathbf{v} = \mathbf{p}_1 c/E_1$, $\mathbf{k}\cdot\mathbf{v} = kv\cos\vartheta$, and $\Delta\varepsilon = (m_2 - m_1)c^2$ is the change in the energy of the detector measured in its frame of reference. In the approximation in which $m_{1,2} \ll m_0$ and the recoil is negligibly small $(\hbar\omega/m_0c^2 \ll 1)$, we obtain from (5.3)

$$\hbar\omega = -\Delta\varepsilon\left(1-\frac{v^2}{c^2}\right)^{1/2}\left(1-\frac{vn}{c}\cos\vartheta\right)^{-1}. \tag{5.4}$$

We denote by ϑ_0 the Cherenkov angle determined by the condition $\cos\vartheta_0 = c/vn$. It follows from the relation (5.4) that in the region of the normal Doppler effect we have $\Delta\varepsilon < 0$ for $\vartheta > \vartheta_0$ and, therefore, $\varepsilon_1 = \varepsilon_t$, $\varepsilon_2 = \varepsilon_l < \varepsilon_t$, i.e., the emission of photons propagating outside the Cherenkov cone is associated with transition of the detector from the level t to level l. For photons emitted by the detector within the Cherenkov cone $(\vartheta < \vartheta_0)$, the situation is different, $\Delta\varepsilon > 0$, $\varepsilon_1 = \varepsilon_l$, $\varepsilon_2 = \varepsilon_t$, i.e., such emission is accompanied by excitation of the detector. The regions of the normal and anomalous Doppler effect are shown (for a nondispersive medium) in Fig. 4. Of course, the energy conservation law is not violated here, since for motion with constant velocity under these conditions a force must act on the body, and its work covers the necessary energy loss.

Thus, for motion with constant velocity $v > c/n$ in a medium a detector that was initially in the ground state $(|l\rangle)$ begins to be excited due to its interaction with the quantized electromagnetic field, and at the same time it radiates real photons, i.e., to a certain degree it behaves like an accelerated detector. We emphasize that in both cases in which we are considering excitation from the ground level the effect has a purely quantum nature. A classical oscillator or other classical system under analogous conditions, i.e., in the ground state for motion with constant velocity $v > c/n$ in a medium or with constant acceleration in a vacuum, is not excited. For further details of the interaction of a uniformly accelerated oscillator with the electromagnetic field, see Refs. 38 and 40.

In the same way as was done for the anomalous Doppler effect, we can, using the energy-momentum conservation laws, obtain conditions for radiation from a uniformly accelerated detector in vacuum. To make the treatment more specific, we assume that the detector as a whole possesses an electric charge Q and that its acceleration is due to a constant homogeneous electric field $\mathbf{E}$ directed along the X axis acting on this charge. Of course, under these conditions the detector is a source of electromagnetic radiation associated with the accelerated motion of the charge. We shall not here consider this radiation but concentrate our attention on the radiation associated with the transitions of the detector from state 1 to state 2. We can even assume that the detector serves for detection of only a scalar massless field and, therefore, in the absence of transitions scalar radiation does not arise.

In the considered case, the energy-momentum conservation law can be written in the form

$$P_1^\mu = P_2^\mu + \hbar k^\mu, \tag{5.5}$$

where

$$P_i^\mu = p_i^\mu + q_i^\mu,$$
$$q_i^\mu = QF^\mu{}_\nu X^\nu, \quad F_{\mu\nu} = E\,(\delta_\mu^0\delta_\nu^1 - \delta_\mu^1\delta_\nu^0); \tag{5.6}$$

where δ_μ^ν is the ordinary Kronecker delta, $\hbar k^\mu = (\hbar\omega \equiv \hbar|\mathbf{k}|, \hbar\mathbf{k})$ is the 4-momentum of the radiated quantum, and p_i^μ and X^μ are the 4-momentum and coordinate (in the inertial system) of the detector before $(i = 1)$ and after $(i = 2)$ the emission. [The expression for p_i^μ is given in (5.2).] It follows from (5.5) that

$$\hbar\omega = \frac{-\Delta\varepsilon\,(2m_0 + m_1 + m_2)\,c^2 + 2p_1^\mu\Delta q_\mu - \Delta q_\mu\Delta q^\mu}{2E_2\,(1 - v_2\cos\vartheta\cdot c^{-1})}, \tag{5.7}$$

where $\Delta q^\mu = q_2^\mu - q_1^\mu$, $\mathbf{v}_2 = \mathbf{p}_2 c/E_2$, $\mathbf{v}_2\mathbf{k} = v_2(\omega/c)\cos\vartheta$, $\Delta\varepsilon = (m_2 - m_1)c^2$. If the emission is to be accompanied by excitation of the detector $(\Delta\varepsilon > 0)$, the numerator on the right-hand side of (5.7) must be positive. In the approximation $m_{1,2} \ll m_0$, $\hbar\omega/m_0c^2 \ll 1$ this condition has the form

$$\Delta\varepsilon < p_1^\mu\Delta q_\mu\,(m_0 + m_1)^{-1} = QE\Delta X, \tag{5.8}$$

where ΔX is the distance between the positions of the detector before and after the time of emission, measured in the inertial frame in which the detector is at rest before emission of the quantum. Of course, in this frame the relation (5.8) can be obtained directly from such considerations. In the initial state (before the emission) the detector energy is $(m_0 + m_1)c^2$, while in the final state it is $(m_0 + m_2)[c^2 + (v^2/2)]$ and the potential energy of the detector in the field has changed by $-QE\Delta X$ and a quantum with energy $\hbar\omega$ has appeared. The momentum of the detector was initially zero; at the end it is $(m_0 + m_2)\mathbf{v}$, and the momentum of the quantum is $\hbar\mathbf{k} = \hbar c^{-1}\omega\mathbf{k}k^{-1}$. The energy of the quantum is $\hbar\omega = 0$ for $\mathbf{v} = 0$, and $(m_2 - m_1)c^2 \equiv \Delta\varepsilon = QE\Delta X$. This in fact is the threshold for radiation, since $\hbar\omega > 0$ when $\Delta\varepsilon < QE\Delta X$.

The condition (5.8) means that the energy needed for emission of a quantum and excitation of the detector is taken from the electric field that accelerates the detector.

The cases considered above of uniform motion in a medium with velocity $v > c/n$ and uniformly accelerated motion in a vacuum are, of course, only special cases of motion of the detector in which it can be excited from the ground state with emission of a quantum. Such an effect also occurs in the case of motion with arbitrary (variable in direction or magnitude) acceleration. As an interesting special example, we mention the already quoted case of motion of a detector with constant speed in a circle, which was considered in Refs. 61 and 62.

There is a further class of phenomena, namely, radiation from moving macroscopic polarized bodies, related to the radiation from a moving detector. From the microscopic point of view, any body is a system with many levels. Polarizability of a body means that under the influence of an external electric field, say, the body acquires a certain dipole moment. In the general case, we are considering the interaction of a body with a field, in particular, a zero field. Therefore, if in the initial state the body is in the lowest level (for example, at temperature $T = 0$) and there is no radiation, then in the case of superluminal motion in a medium or acceleration in a vacuum excitations will arise in the body and the corresponding radiation will also appear. In other words, a macroscopic body can be regarded as a detector, with all the consequences that flow from this. However, in this case fixing the state of the body (the detector) is difficult and in general one will only be able to observe the radiation corresponding to it. A well-known example of a phenomenon of this kind is the quantum radiation from accelerated "mirrors" (i.e., bodies with infinitely large polarizability[19]; for the radiation from accelerated bodies with finite polarizability, see Ref. 66). The quantum radiation from a neutral polarizable particle moving uniformly with velocity $v > c/n$ in a medium, considered in Ref. 67, is also analogous to the anomalous Doppler effect. The quantum radiation from an absorbing cylindrical body rotating around its symmetry axis, described in Refs. 68 and 69, is similar to the excitation and emission from a detector in circular motion.[61,62] We note that interest may also attach to the class of problems with motion of a detector or a polarizable body with acceleration not only in vacuum but also in a medium under conditions when the velocity of the detector exceeds the velocity of light in this medium, $v > c/n$. Combination of the anomalous Doppler effect and acceleration may change the equilibrium distribution over the levels and lead, in particular, to the appearance of population inversion. It may also be important that the features of motion in a medium are also manifested in the case when a source (detector) moves in a vacuum gap or channel in a medium or near a medium (for more details, see Ref. 5). Such a situation is particularly important under conditions when a detector moving in a condensed medium may break up or be strongly decelerated (obviously, both these effects are absent in the case of motion in an empty gap or channel or in vacuum near a medium). The excitation of a detector with emission of a "photon" and the quantum radiation from a polarizable body are also possible in the case of motion with constant subluminal velocity $v < c/n$ in a medium but under the condition that the refractive index of the medium changes along the direction of the motion due to a change of n in space and (or) time.[70] This quantum effect is a particular analog of the well-known effect of transition radiation.[5,71]

6. THE EQUIVALENCE PRINCIPLE IN THE QUANTUM DOMAIN AND THE "THERMAL NATURE" OF THE VACUUM IN A UNIFORMLY ACCELERATED FRAME OF REFERENCE

In the discussion and comparison of the behavior of detectors at rest in a uniformly accelerated frame of reference and in a static homogeneous gravitational field (Sec. 3), we came right up against the question of the applicability of the equivalence principle to phenomena for which their quantum nature is important. In this section, we shall discuss in detail this important question.

The equivalence principle, the basis of the general theory of relativity, was first formulated by Einstein in Ref. 72 published in 1907, and it was the first step in the creation of general relativity. In it, in particular, he wrote: "Let us consider two frames of reference Σ_1 and Σ_2. Suppose Σ_1 moves with acceleration in the direction of its X axis and suppose its acceleration (constant in time) is equal to γ. Suppose that Σ_2 is at rest but is in a homogeneous gravitational field that imparts to all bodies the acceleration $-\gamma$ along the X axis.

It is well known that with respect to Σ_1 the physical laws do not differ from the laws referred to Σ_2; this is due to the fact that in a gravitational field all bodies are accelerated in the same manner. Therefore, at the present state of our knowledge there are no grounds for believing that the frames of reference Σ_1 and Σ_2 differ in any respect from one another, and in what follows we shall assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the frame of reference.

This assumption extends the relativity principle to the case of uniformly accelerated rectilinear motion of the frame of reference. The heuristic value of this assumption is that it enables us to replace a homogeneous gravitational field by a uniformly accelerated frame of reference, which to a certain degree is amenable to theoretical treatment."

There is reflected here the transition from the previously known and experimentally directly verified assertion (often called the "weak equivalence principle") of the equivalence of the laws of mechanics in a uniformly accelerated frame of reference and in a frame of reference at rest in a homogeneous static gravitational field to the general requirement of the equivalence of all physical phenomena in these systems (which makes up the content of the so-called Einstein equivalence principle). This transition was the necessary link that enabled Einstein to construct general relativity, in which gravity is described by the metric tensor $g_{\mu\nu}$ alone. Einstein frequently emphasized the heuristic value of the equivalence principle in his subsequent work too. For example, in 1911 he wrote[73]: "As long as we restrict ourselves to purely mechanical phenòmena for which Newton's mechanics is valid, we are assured of the equivalence of the systems K and K'.[15)] However, our idea will be sufficiently deep only if the systems K and K' are equivalent with respect to all physical phenomena, i.e., if the laws of nature in the system K are completely identical to the laws of nature in the system K'. Assuming this, we obtain a principle that has great heuristic value if it is indeed correct." To avoid the confusion that is encountered often, we also emphasize that one cannot in any way reduce gravitation to kinematics, i.e., to the choice of some accelerated frame of reference. Einstein emphasized this already before the completion of general relativity in 1915. For example, in the already quoted paper of 1911 he wrote[73] "Of course, it is not possible to replace an arbitrary gravitational field by a state of motion of the system without a gravitational field any more than one can transform all points of an arbitrarily moving medium to rest by means of a relativistic transformation."

Of course, like any other physical principle, the equivalence principle requires proper experimental verification. In fact, the testing of the weak equivalence principle with the greatest possible accuracy for bodies of different chemical composition makes it possible to draw certain conclusions about the accuracy with which the Einstein equivalence principle is satisfied (for more details about this, see Ref. 74 and the literature given in it).

It is obvious that at the time when Einstein first formulated the equivalence principle he was mainly concerned with the laws of classical physics, although his formulation admits obvious extension to quantum physical laws. The question of whether or not the equivalence principle holds for the description of phenomena for which their quantum nature is important is by no means trivial. This can be seen by comparing the behavior of a detector that is uniformly accelerated in an inertial frame of reference (and, therefore, is at rest in the uniformly accelerated frame) with the behavior of a detector at rest in a homogeneous gravitational field, say, near the surface of a cold ($T = 0$) neutron star (Fig. 5). The behavior of such detectors is different; for in the first case the detector is excited from the ground state, while in the second it remains in the ground state (see Table I in Sec. 3.3). This difference is also obviously preserved in the case when the acceleration of free fall **g** in the gravitational field and the acceleration of the motion of the detector with respect to the I system in Minkowski space, **a**, are related by $\mathbf{g} = -\mathbf{a}$. But it is precisely the identity of all the physical laws in the A system (uniformly accelerated frame with acceleration **a**) and in the G system (frame of reference at rest in a homogeneous gravitational field $\mathbf{g} = -\mathbf{a}$) that is the content of the equivalence principle.

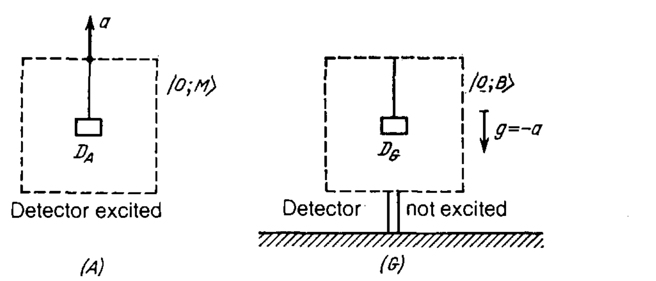

FIG. 5. A detector D_A attached to a transparent box ("Einstein lift") and moving with it with uniform acceleration **a** in the vacuum $|0; M\rangle$ in Minkowski space-time is excited (Fig. 5, A); an analogous detector D_G at rest near a cold ($T = 0$) "neutron star" (state of the field $|0; B\rangle$) is not excited (Fig. 5, G).

How can we reconcile the different behaviors of the detectors at rest in the A and G systems with the equivalence principle and do we not have here a violation of the principle for quantum phenomena? To avoid confusion, we answer immediately that the equivalence principle is satisfied for the considered phenomena. With regard to the particular question of the behavior of the detectors at rest in the A and G systems, the apparent contradiction is due, not to violation of the equivalence principle, but to an insufficiently accurate use of it. It is another matter that the equivalence principle in the quantum domain needs to be verified, generally speaking, to the same extent as in the classical domain. Ultimately, here too experiment will play the decisive role. We merely note that at the present time there are no indications of invalidity of the equivalence principle.

The identity of the laws of physics in the frames of reference A and G that we discussed in the formulation given above of the Einstein relativity principle means in particular that the time evolution of the physical objects in these frames of reference A and G will be the same, but provided that the initial and boundary conditions in them are specified in the same way. (In this respect, the equivalence principle is analogous, for example, to the relativity principle, which guarantees the same evolution in time of the physical objects in different inertial frames of reference only when the initial and boundary conditions in these systems are chosen appropriately.[16)]) In this connection, see, for example, the discussion of the question of the radiation from a moving electron in an accelerated frame of reference and in a homogeneous static gravitational field.[75] Neglect of this circumstance could lead, and in a number of cases has actually led, to a seeming contradiction with the equivalence principle.

In the case in which we are interested, the physical system (object) consists of a detector and a quantized field, with which it interacts. Therefore, the comparison of the behavior of the detectors in the A and G systems must be done in a way that ensures identical initial and boundary conditions for not only the detectors but also the quantized field. It was this last condition that was violated when, choosing the vacuum state $|0; M\rangle$ as the initial state of the quantized field in the A system, we chose, without particular grounds (although at first glance it did seem natural) the initial state $|0; B\rangle$ for the field in the G system (see Table I).

To illustrate the validity of the equivalence principle for these quantum phenomena, we consider the following thought experiment. Suppose that in an I system there are at rest two isolated "boxes" with specular walls, i.e., ones that completely reflect the field quanta, in each of which there is a detector that is fixed ("suspended") relative to them. In order to ensure the same choice of the initial states, we assume that in both "boxes" the state of the quantized field is the vacuum state, i.e., there are no real quanta ("photons") with which the detectors may interact, and that the detectors themselves are in the ground state.

It is obvious that as long as the "boxes" are at rest the detectors in them are not excited. We now set one of the "boxes" into a state of uniformly accelerated (with acceleration $\mathbf{a}$) motion and place the other in a homogeneous gravitational field at a point at which the acceleration of free fall is $\mathbf{g} = -\mathbf{a}$ (Fig. 6). Of course, in both cases the transition of the system to the new state is associated with application to it of forces that depend on the time. Moreover, in the case of motion with variable acceleration of the specular walls of the "box" quantum radiation can occur (see, for example, Ref. 19). However, if the process occurs sufficiently slowly ("adiabatically"), so that for its description we can use the adiabatic approximation, then it can be shown that the detectors will still be in the ground state (in the one case, this is the state with the lowest Rindler energy; in the other, it is the state with the lowest energy when allowance is made for the work of the gravitational forces). With regard to the states of the quantized field, the state within the "box" at rest in the A system is identical to the Rindler vacuum $|0; R\rangle$, while the state within the "box" at rest in the G system is identical to the Boulware vacuum $|0; B\rangle$.[17)] As a result of the interaction with the quantized field the detectors will behave in the same way in the two cases—they will not be excited (see Table I in Sec. 3.3), in agreement with the equivalence principle. At the same time, as we see, the state of the field within the accelerated "box" is not at all identical to the vacuum state $|0; M\rangle$ in the surrounding Minkowski space. Therefore, if the insulation of the "box" is destroyed, for example, by opening a "door" in its wall, the state of the field within the box, i.e., the state $|0; R\rangle$, will begin to be readjusted, and ultimately it will be changed in such a way as to come into equilibrium with the surrounding state $|0; M\rangle$. Of course, in this process real field quanta may be radiated, but we shall assume as before that they escape to infinity and do not change the state of the field within the "box." As a result of the change of the state in the accelerated "box," the detector in the A system begins to be excited, whereas in the G system the detector still remains unexcited even after the opening of a "door" in the "box" (the "box" is placed near a cold, $T = 0$, neutron star, so that the state of the quantized field outside is $|0; B\rangle$, i.e., the same as inside; see above).

An interesting question is the following: How can we explain in the inertial frame of reference the difference of the final state within the accelerated "box" (before the "door" in it is opened) from the initial state $|0; M\rangle$? The answer to this question reduces to the following. In the process of acceleration, the specular walls of the "box" move with variable acceleration with respect to the I system and, therefore, are a source of quantum radiation (see, for example, Ref. 19). If the accelerations of the front and rear walls were equal, the radiation fluxes from them within the box would exactly cancel each other. However, since the box is rigid, the front (in the direction of acceleration) wall of it always moves with smaller acceleration then the back wall, and therefore such compensation of their radiation does not occur. As a result, the state of the quantized field within the box will be different from $|0; M\rangle$. This process in relation to the analogous problem of the behavior of detectors in the gravitational field of a black hole is considered in detail in Ref. 76.

As another example illustrating the fulfillment of the

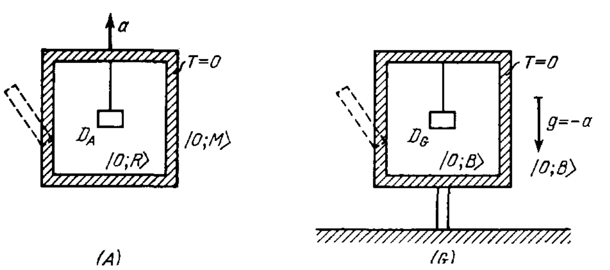

FIG. 6. Before the opening of the "doors" the identical detectors D_A and D_G at rest in insulated boxes, one of which moves with uniform acceleration $\mathbf{a}$ (Fig. 6, A) while the other is at rest in a homogeneous gravitational field (at a point at which the acceleration of free fall is $\mathbf{g} = -\mathbf{a}$, as in Fig. 6, G), behave in the same way and are not excited.

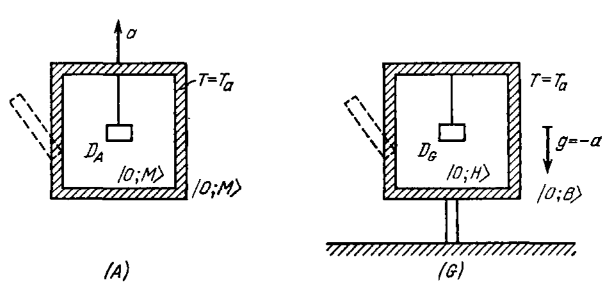

FIG. 7. The detector D_A (at rest in the A system) and the detector D_G (at rest in the G system) also behave in the same way (are excited) if the walls of the insulated "boxes" in which they are placed are heated to the same temperature T_a.

equivalence principle, we consider the following modification of the thought experiment described above. Namely, we shall assume that as **a** and **g** = − **a** increase so does the temperature of the walls of both "boxes," this occurring, moreover, in such a way that the equation $T = T_a \equiv \hbar a / 2\pi k_B c$ holds (Fig. 7). For the systems within these "boxes," the "boxes" are essentially thermal baths, and accordingly the detectors in both "boxes" (in the final state, i.e., uniformly accelerated and at rest in the gravitational field) will be excited and measure the temperature T_a. The equivalence principle is again satisfied. If the "doors" of the "boxes" are opened, then, in contrast to the case considered above, the state of the field within the uniformly accelerated box (it is described by the thermal density matrix for the Rindler quanta with temperature T_a) will be in equilibrium with the ambient quantized field in the state $|0; M\rangle$. At the same time, for the state of the field within the "box" at rest near the cold ($T = 0$) neutron star the equilibrium will be disturbed after the opening of the "door." Equilibrium near the surface of the star is possible if outside the "box" there is an equilibrium thermal gas of "photons" provided the temperature of this gas at the point at which the "box" is placed is equal to the temperature T_a of the "box" walls. In other words, for equilibrium to hold the state of the quantized field outside the box must be identical to the Hartle-Hawking vacuum $|0; H\rangle$.

In the cases considered above, it was assumed that the walls of the "box" ensure complete insulation from the surrounding world.[18] These boundary conditions make it possible to guarantee identity of the states of the quantized field within the "boxes." Of course, nothing is changed if the walls of the "boxes" are made completely transparent for the quantized field provided appropriate states for these fields outside the "boxes" are chosen. In particular, in this sense the Minkowski vacuum $|0; M\rangle$ in the A system corresponds to the Hartle-Hawking vacuum $|0; H\rangle$ in the G system. Similarly, the Rindler vacuum $|0; R\rangle$ in the A system corresponds to the Boulware vacuum $|0; B\rangle$ in the G system (see Table I). For such a choice of the states, the behavior of detectors at rest or having the same motion with respect to the two frames of reference will be the same, as required by the equivalence principle.[19]

We shall not consider in detail all possible forms of motion of the detectors but limit ourselves to one example. Suppose a detector in a box with transparent walls moves inertially, i.e., uniformly and rectilinearly in an I system in the vacuum $|0; M\rangle$, while another detector in the vacuum $|0; H\rangle$ falls freely in a homogeneous gravitational field (Fig. 8a). Then in neither the first nor the second case will the detector be excited from the ground state. Some other variants of behavior of the detector for different choices of the state of its motion and (or) for other vacuum states are shown in Figs. 8b–8d. In each of these cases, the detectors in the A and G systems have the same behavior.

Thus, in the simple thought experiments considered above the equivalence principle is satisfied. Of course, we have described only schematically how this occurs. The complete mathematically rigorous proof of our arguments is by no means simple and would require lengthy calculations. Such an analysis, made moreover in different frames of reference and permitting clarification of the "mechanism" of fulfillment of the equivalence principle as applied in different

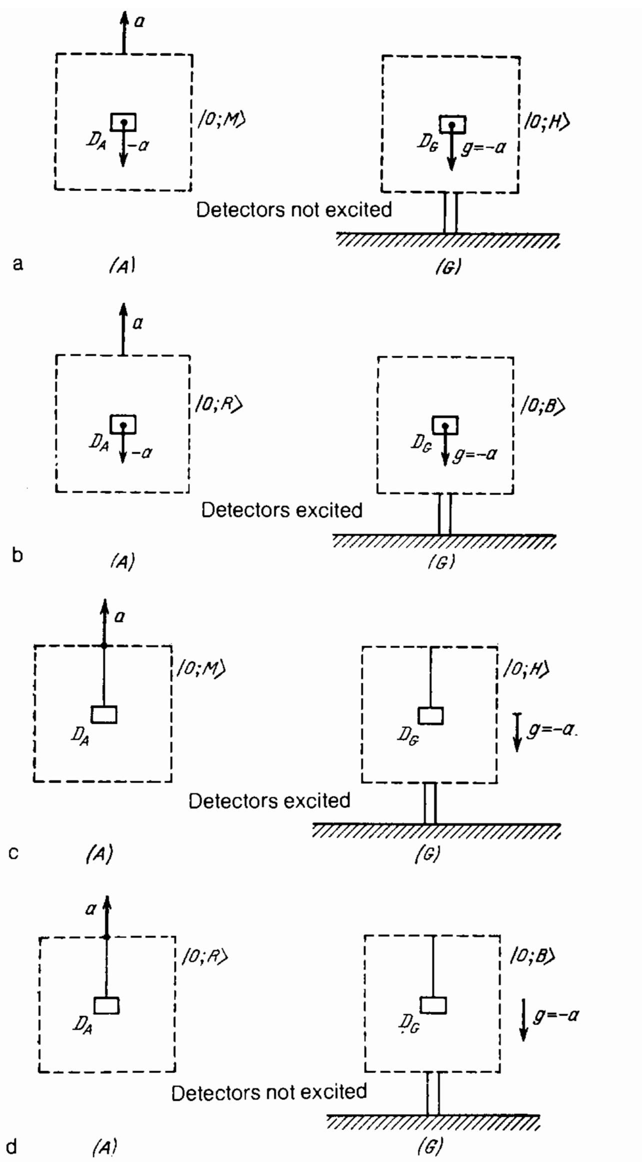

FIG. 8. Figures 8a, 8b, 8c, and 8d show different types of motion of the detectors D_A and D_G with respect to the corresponding (A and G) frames of reference for different but corresponding ("matched") choices of the vacuum states; the behaviors of such detectors are then identical. Detectors D_A and D_G are not excited in the situations shown in Fig. 8a (free fall of a detector, the states $|0; M\rangle$ and $|0; H\rangle$) and in Fig. 8d (detectors at rest in the A and G systems, states $|0; R\rangle$ and $|0; B\rangle$). The detectors D_A and D_G are excited in the situations shown in Fig. 8b (detectors moving freely, states $|0; R\rangle$ and $|0; B\rangle$) and in Fig. 8c (detectors at rest in the A and G systems, states $|0; M\rangle$ and $|0; H\rangle$). Of course, the situation shown in Fig. 8c (A) is identical to the one shown in Fig. 5 (A). Similarly, the situations shown in Fig. 8d (G) and Fig. 5 (G) are identical.

physical situations, is undoubtedly important and helpful. There is, however, another side to the question, namely, the great heuristic value of the equivalence principle, which Einstein so strongly emphasized, in particular, in the quotations given earlier. If one adopts this principle, regarding it as valid, then it becomes possible to connect phenomena that occur in a uniformly accelerated frame of reference with phenomena in a homogeneous static gravitational field.[20] Below, as an illustration, we show how the equivalence principle can lead to the conclusion that the vacuum $|0; M\rangle$ considered in a uniformly accelerated frame of reference A has a thermal nature (see also Refs. 21 and 78).

To this end, we consider a gas of "photons" (or any other quanta, which, for simplicity, we shall also call "photons") in a homogeneous static gravitational field. It is well known that if there is even a very small interaction between the "photons" equilibrium will be established in such a system after a certain time has elapsed; moreover, the equilibrium distribution will be described by the thermal density matrix $\hat{\rho}_\theta$ [see (2.31)][21)] (see, for example, Ref. 56; a more rigorous proof is given in Ref. 79). The following question arises: What additional properties of the considered state distinguish the parameter value $\beta = \hbar a/2\pi k_B c\theta$ ($\theta = T_a$, $\beta = 1$), for which the considered state is identical to the Hartle-Hawking vacuum in a static homogeneous field?

We consider in more detail the properties of the equilibrium gas. To this end, we introduce the function

$$G^\beta(x,\ x') = \mathrm{Sp}\,[\hat{\rho}_\theta \hat{\varphi}(x)\,\hat{\varphi}(x')]$$
$$= \sum_J [u_J(x)\,\bar{u}_J(x')\,\bar{n}_J^\theta + \bar{u}_J(x)\,u_J(x')\,(\bar{n}_J^\theta + 1)], \tag{6.1}$$

the analog in the considered case of the positive-frequency function (3.26). The quantities $\bar{n}_J^\theta$ are determined by the relation (3.32). This function $G^\beta(x,x')$ can be calculated explicitly, and it has the form[80]

$$G^\beta(x,\ x') = \frac{1}{8\pi^2\rho\rho'}\,\frac{1}{\beta\,\mathrm{sh}\,\lambda}\,\frac{\mathrm{sh}\,(\lambda/\beta)}{\mathrm{ch}\,(\lambda/\beta) - \mathrm{ch}\,[(\eta - \eta' - i\zeta)/\beta]}, \tag{6.2}$$

where

$$x = (\eta,\ \rho,\ y,\ z),\ x' = (\eta',\ \rho',\ y',\ z'),$$
$$\mathrm{ch}\,\lambda = [\rho^2 + \rho'^2 + (y - y')^2 + (z - z')^2]\,(2\rho\rho')^{-1}. \tag{6.3}$$

By means of this function, we can calculate the energy-momentum tensor for the considered gas of scalar massless quanta. We shall not go into the details of the calculation; the final result is

$$\langle T_\mu^\nu\rangle^{\mathrm{ren}} = \frac{\hbar c}{\rho^4}\,\frac{\beta^{-4} - 1}{1440\,\pi^2}\,(\delta_\mu^\nu - 4\delta_0^\nu\delta_\mu^0). \tag{6.4}$$

We now note that for all $\beta \neq 1$ this tensor depends essentially on ρ, and the energy-momentum distribution described by it is manifestly inhomogeneous. This is not remarkable, since, as we have already noted, the local temperature of this thermal gas depends on the position. What may surprise is a different fact: For $\beta = 1$, the tensor (6.4) vanishes, and the energy-momentum distribution described by it becomes completely homogeneous. The expressions for the energy-momentum tensors of other quantized fields possess a similar property.

How can one explain this, at first glance, strange behavior of $\langle T_\mu^\nu\rangle^{\mathrm{ren}}$? The equilibrium thermal gas at rest in the G frame of reference is in an external static gravitational field. The effect of this external field leads to a change in the wave functions that describe the motion of the particles of the gas, and this, in its turn, has the consequence that the values of the local observables (for example, $\langle T_\mu^\nu\rangle^{\mathrm{ren}}$) depend on the characteristics of the field. This effect is well known as vacuum polarization by an external field. Because of the symmetry of the problem in the considered case the contribution to the energy-momentum tensor describing the vacuum polarization,

$$\langle T_\mu^\nu\rangle^{\mathrm{pol}} \equiv \langle T_\mu^\nu\rangle^{\mathrm{ren}}|_{\theta=0} = -\frac{\hbar c}{1440\,\pi^2\rho^4}\,(\delta_\mu^\nu - 4\delta_0^\nu\delta_\mu^0), \tag{6.5}$$

has the same tensor structure as the contribution determined by the temperature,

$$\langle T_\mu^\nu\rangle^\theta = \frac{\hbar c}{\rho^4}\,\frac{\beta^{-4}}{1440\pi^2}\,(\delta_\mu^\nu - 4\delta_0^\nu\delta_\mu^0), \tag{6.6}$$

but opposite sign. This makes possible the complete mutual canceling of these contributions for a certain value of the parameter β. This property distinguishes the parameter value $\beta = 1$, i.e., the symmetry of the state described by the density matrix $\hat{\rho}_\theta$ is higher for the value $\theta = T_a$ ($\beta = 1$) than for all other values of this parameter. In particular, going over, for example, from Rindler to Cartesian coordinates, one can show that the function $G^1(X,X')$ is identical to (3.24), and thus show that for $\beta = 1$ this function, which describes the physical properties of the system, is invariant with respect to Poincaré transformations. The state uniquely distinguished by this property is the Hartle-Hawking vacuum $|0;H\rangle$. If in accordance with the equivalence principle we now require the analogous property of Poincaré invariance to hold for the state corresponding to $|0;H\rangle$ in the A system we find that the vacuum $|0;M\rangle$ in Minkowski space is singled out by this requirement.

According to the equivalence principle, all physical observables in the A system for this chosen initial state $|0;M\rangle$ of the quantized field must be identical to the analogous physical observables in the G system for the state $|0;H\rangle$. If we bear in mind that this last describes an equilibrium thermal gas with temperature T_g, we may conclude that in a uniformly accelerated frame of reference the Minkowski vacuum $|0;M\rangle$ behaves like a thermal equilibrium gas with temperature $T_{a=g}$.

7. CONCLUDING REMARKS

We make some further remarks of a general nature concerning the overall physical significance of the questions considered in the present paper.

After the creation of the general theory of relativity (1915) and quantum mechanics (1925–1927), the problem naturally arose of constructing a theory that unifies them and permits the description of both quantum and classical phenomena with allowance for the gravitational interaction. This problem is still not completely solved at the present time; it is indeed, the most important problem in theoretical physics.

Work on this problem began with discussion of general questions of the quantization of the gravitational field as well as of the simplest quantum-gravitational effects. As an example of the latter we can take graviton-graviton scattering. The quantum effects in gravity were shown to be small for scales appreciably exceeding the Planck scales (in the case of length, we obviously have here the Planck length $l_g \sim 10^{-33}$ cm).

At the end of the sixties and beginning of the seventies, one can say that a new stage of development commenced in the quantum theory of gravity. On the one hand, it was recognized that the theory of gravity is a gauge field theory with all the consequences that flow from this. In particular, one can use the quantization formalism developed for such theories. In the framework of gauge theories one can also attempt to unify gravity with other fields with a view to creating variants of a unified field theory (the currently most popular candidate is supergravity, and more recently the theory of strings and superstrings has become popular). On the other hand, interest arose in quantum effects in strong classical gravitational fields in connection with the discussion of the part played by quantum effects in the early stages in the evolution of the universe and in the gravitational field of black holes. In 1974, Hawking showed that when quantization of the fields is taken into account black holes radiate quanta of these fields with a thermal spectrum. This established a remarkable connection between the theory of gravitation, quantum theory, and thermodynamics already for the example of a static gravitational field. As was shown by Unruh (1976), this connection is also in essence preserved in the limit of a homogeneous gravitational field, and also in a uniformly accelerated frame of reference. This last result is rather natural in view of the equivalence principle.

These questions of quantum effects in uniformly accelerated frames of reference and in a homogeneous gravitational field could, evidently, have been clarified long ago—after the creation of the theory of gravitation and quantum field theory. But as has already happened more than once in physics, this occurred only after a different and more complicated problem had been solved—that of quantum effects in the gravitational field of a black hole. It is possible that the problem of the connection between gravity, quantization, and thermodynamics has a fundamental significance that goes beyond the framework of just the quantum theory of gravity and may be important for the further development of physics.

By virtue of what we have said, it is clear that the problem of quantization and the "choice" of the vacuum in a homogeneous gravitational field and in an accelerated frame of reference has great importance, at least methodologically. We therefore hope that there is a justification for the publication of the present paper devoted to detailed consideration of quantum theory in uniformly accelerated frames of reference and in a homogeneous gravitational field, as well as analysis of the equivalence principle in the quantum domain.

We would like to thank A. I. Nikishov, I. D. Novikov, and V. I. Ritus for comments they made on reading the draft.

[1)]Extended exposition of paper read May 29, 1986 at the Joint Scientific Session of the Section of General Physics and Astronomy and the Nuclear Physics Section of the USSR Academy of Sciences.

[2)]For example, only "vacuum, physical" in quantum field theory occurs in the Fizicheskiĭ Éntsiklopedicheskiĭ Slovar' [Physics Enclyclopedia, published by Sovetskaya Éntsiklopediya, Moscow (1983)].

[3)]Quantization in the M space can also be done in such a way that the energy of the zero-point vibrations is zero (see, for example, Ref. 5). However, the physics results are not changed by such a choice of the energy scale.

[4)]We must, it is true, bear in mind that modern high-energy physics has "probed" space experimentally only to scales $l \sim 10^{-16}$ cm. The assumption that "everything is in order" with space-time down to the Planck scales is thus a huge extrapolation. Nevertheless, it is usually made and, generally speaking, this is natural as long as there are no indications of the existence of any new fundamental length $l_f > l_g$ (if such a length l_f exists, general relativity and the entire physical theory known to us could be incorrect). One should not forget that in principle a length $l_f > l_g$ could exist,[13,14] but here this possibility will be ignored. We could in fact assume that we are interested in only the domain $l \gg l_f, t \gg t_f = l_f/c$.

[5)]A particle at rest near a nonrotating Schwarzschild black hole at the point r is not moving geodesically and, therefore, has an acceleration. The magnitude of this acceleration is $a(r) \equiv |a_\mu a^\mu|^{1/2} = GMr^{-2}\alpha(r)^{-1}$, where $\alpha(r) \equiv |g_{00}(r)|^{1/2} = [1-(2GM/c^2r)]^{1/2}$ is the red shift factor (radiation that at distance r has frequency ω_1 will have after escaping "to infinity" the frequency $\omega_2 = \alpha(r)\omega_1$). The surface gravity $\varkappa = GM/r_g^2$ can be defined as the value on the surface of the black hole of the quantity $\alpha(r)a(r)$. If near the black hole there is equilibrium (thermal) radiation with local temperature $T(r) \approx \hbar a(r)/2\pi k_B c$, then "at infinity" (with allowance for the red shift), this radiation will correspond to the temperature $T_{b.h.} = \hbar\varkappa/2\pi k_B c$, a value identical to (1.3).

[6)]In the present variant of the paper these calculations are kept to a minimum because of lack of space. The complete text of the paper is published elsewhere.[25]

[7)]For a more detailed discussion of these questions, see, for example, Ref. 25. Individual problems connected with quantization are also considered in the book of Ref. 5.

[8)]*Translator's Note.* The Russian notation for the trigonometric, inverse trigonometric, hyperbolic functions, etc., is retained here and throughout the article in the displayed equations.

[9)]If the motion does not occur along one direction (in the given case, the X axis) but the acceleration is constant, a = const, then we are dealing with a more general type of accelerated motion (see, for example, Ref. 5). Uniformly accelerated motion is also sometimes called hyperbolic motion.

[10)]With regard to the completeness of the system of functions $u_{\nu q}^{(\varepsilon)}$, $\bar{u}_{\nu q}^{(\varepsilon)}$ see, for example, Ref. 45. As basis functions it is often convenient to use, not the solutions $u_{\nu q}^{(\varepsilon)}$, $\bar{u}_{\nu q}^{(\varepsilon)}$, but a complete system of solutions constructed from them and having the nature of wave packets, possessing, in particular, regular behavior in the neighborhood of the horizons $H^{\pm}$ (see, for example, Ref. 22).

[11)]For a detailed derivation of this relation, see, for example, Ref. 25.

[12)]In Ref. 51, Hartle and Hawking considered the state of a quantized field in equilibrium with a black hole. If the black hole does not rotate, then in such a state there is a black-body radiation with temperature $T_{b.h.}$ far from the black hole [see (1.2)]. The corresponding state became known as the Hartle-Hawking vacuum. In the limit of a homogeneous field, this state is identical to the state $|0; H\rangle$ that we have introduced. For more detail about this, see Ref. 52 (and also Refs. 16 and 25).

[13)]The work Mad for acceleration $a \sim 10^{22}$ cm/sec^2 for $M \sim m_p \sim 10^{-24}$ g and $d \sim 10^{-8}$ cm is of order 100 eV.

[14)]This can be shown by using, for example, the relations given in Ref. 58 (see also Ref. 59).

[15)]The system K is at rest in a homogeneous gravitational field, while K' is uniformly accelerated. In the present paper, we have used G and A, respectively to denote them. (Note of the authors.)

[16)]Considering the simplest example, we recall that the trajectory and velocity of a material point in two inertial frames of reference I and I′ are the same only if the initial position and velocity of the point with respect to the systems I and I′ were taken to be the same.

[17)]We emphasize that the wave functions $u_{\nu q}$ of the quantized field within "specular boxes" differ from (2.16) and (2.29), since they satisfy different boundary conditions. However, it is important that the corresponding stationary states have the previous form of the time dependence $\exp(-i\nu\eta)$, where $\tau = c\eta/|a|$ is the proper time in the frame of reference in which the "box" is at rest. For such a choice of the positive-frequency solutions, we use for the corresponding vacuum states the same notation as in the absence of boundaries (see Table I). This same stipulation concerning the change of the wave functions within the box due to the boundary conditions also holds for the other states of the quantized field in the "boxes" considered below. We emphasize that although the dimension of the detector, like the dimension of the box, in the direction of the acceleration vector is bounded by $l = c^2/a$, the dimensions in the transverse direction can be arbitrarily large. In the case of the G system, these dimensions are limited only by the size of the region in which the gravitational field can be assumed to be homogeneous.

[18)]Of course, such an idealized formulation of the problem cannot be realized in any experiments. In fact, it is obviously sufficient if the walls of the box are opaque merely for the frequencies that determine the transitions of the detector.

[19)]In the situation considered by Boyer in Ref. 77 the condition of the choice of the corresponding states in the uniformly accelerated frame A and in the frame G at rest near a cold neutron star is not satisfied. Therefore, Boyer's conclusion of thermal excitation of a detector at rest (fixed) in the G frame is incorrect. For a discussion of the question of the equivalence principle for quantum phenomena, see also Refs. 49, 52, and 78.

[20)]In Ref. 52, the equivalence principle was, for example, used to calculate the energy-momentum fluxes of a quantized field into a black hole.

[21)]The inclusion of a weak interaction is important only to establish equilibrium. After this equilibrium has been established, one can assume that the particles of the thermal gas are practically free, since the corrections associated with their interaction are small, and the gas can be described by means of the density matrix $\hat{\rho}_\theta$.

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