Phase phenomena in parametric amplifiers and generators of ultrashort light pulses[1)]

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This article reviews recent theoretical and experimental investigations in the development of femtosecond lasers based on optical parametric oscillators. Phenomena responsible for producing tunable high-power femtosecond pulses in media with quadratic susceptibility (parametric chirp, chirp reversal in real time, parametric self-compression of pulses with strong energy exchange) are discussed.

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1. INTRODUCTION

The development of sources of high-power tunable femtosecond light pulses is a problem of great importance in laser technology. Its solution opens up entirely new possibilities in the physics of fast processes. At the present time the techniques of generating extremely short light pulses (a few femtoseconds at the visible wavelengths) are based mostly on the formation of phase-modulated pulses in media with cubic nonlinearity, for instance in optical fibers,[1] and the subsequent compression of these pulses in devices with negative group velocity dispersion or by self-compression in an optical fiber.[2–4] Compressors based on optical fibers make it possible to compress pulses to 8 fs.[5] At the same time, mode-locked dye lasers have recently been developed that produce 30 fs pulses.[6] However, the energy in the pulses produced by these systems is usually small, not exceeding 10^{-8}–10^{-9} J, so that it is necessary to amplify these pulses, a task that is beset with great difficulties.[7]

We would prefer, in this review, to refer our readers to publications[1–10] devoted to the methods mentioned above, and concentrate on novel methods of controlling the phase of light waves, methods based on parametric interaction of the light waves. The most recent work has shown that these methods are very promising for the purpose of generating high-power femtosecond tunable pulses.

Substantial progress has been made over the past ten years in the development of picosecond and femtosecond radiation sources based on parametric amplification and generation of light waves in quadratically nonlinear media, the so-called optical parametric oscillators. Optical parametric oscillators combine a broad tuning range (from the ultraviolet to the infrared), a broad frequency band, and a high gain, with various possibilities of changing the amplitude and phase characteristics of the pulses.

The physical principles of optical parametric oscillators and possible schemes for tuning were outlined in 1962 by Akhmanov and Khokhlov[11] and also by Kroll[12] and Kingston.[13] Optical parametric oscillators in the nanosecond pulse-length range were developed for the first time in the work reported in Ref. 14. An important stage in the development of parametric generators of picosecond and femtosecond pulses was the appearance of theoretical investigations[15,16] which introduced the idea of parametric compression of light pulses under conditions of strong energy exchange and group velocity mismatch. Subsequently, parametric generation of light pulses in the picosecond[17,18] range, and somewhat later in the femtosecond[19,20] pulse-length range was attained. We stress that the pulse compression observed in these studies was the result of amplitude modulation and group velocity mismatch of the interacting light pulses. In accord with the theory, the maximum degree of compression did not exceed one order of magnitude, and the pulse lengths varied from 300 to 900 fs.[19,20]

In recent investigations[21–23] it was shown that substantial progress in the femtosecond pulse length range was possible by the use of three-wave parametric interactions of phase-modulated light pulses. The results of these investiga-

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tions and the ideas formulated in them make it possible to approach in a new way the problem of fast control of the phase of the light waves. To the present time studies have been made of the effect of phase modulation of the pumping light pulses on parametric amplification,[24,25] and processes such as conversion of quadratically phase modulated pumping (linear frequency variation—the frequency "chirp")[2)] into phase modulation of the generated pulses,[21,22] parametric generation of broad-band pulses (a continuum),[26–29] sign reversal of the frequency modulation, "chirp reversal" in parametric amplifiers,[23] parametric pulse compression under conditions of second order dispersion,[30–32] soliton formation,[31,33,34] and others.

The results that have been obtained leave no doubt as to the high degree of promise inherent in this direction of investigation. The balance of this review is devoted to a specific discussion of these results.

2. PARAMETRIC AMPLIFICATION IN A PHASE-MODULATED PUMPING FIELD

Let us examine how new features arise in parametric generation of light waves in a phase-modulated pumping field. The equations that describe the three-photon interaction of plane light waves in a nonlinear medium have the following form in the first approximation of dispersion theory[24]:

$$\frac{\partial A_1}{\partial z} = -i\sigma_1 A_3 (t - \nu_{31} z) A_2^*,$$
$$\frac{\partial A_2}{\partial z} + \nu_{21}\frac{\partial A_2}{\partial t} = -i\sigma_2 A_3 (t - \nu_{31} z) A_1^*, \qquad (1)$$

where A_j ($j = 1,2,3$) are the complex wave amplitudes,

$$\nu_{21} = \frac{1}{u_2} - \frac{1}{u_1}, \quad \nu_{31} = \frac{1}{u_3} - \frac{1}{u_1};$$

u_j are the group velocities of the waves, and σ_1 and σ_2 are the coupling constants. A coordinate system is chosen in which the first pulse is stationary. We note that Eqs. (1) are written in the approximation of a specified pumping field. The solution of (1) in general form involves the integrals of the Riemann function (see Ref. 25) and its analysis is difficult.

We shall analyze (1) for the case of Gaussian pump pulses with quadratic phase modulation $A_3 = a_{30} \exp[(-(t^2/\tau_3^2)(1 + i\gamma_3)]$. Let us introduce the dimensionless time $\eta = t/\tau_3$, the group interaction lengths $L_{21} = \tau_3/|\nu_{21}|$, and $L_{31} = \tau_3/|\nu_{31}|$, and change to the new functions

$$B_1 = A_1 \exp(-ip_2\gamma_3\eta^2), \quad B_2 = A_2 \exp\left[ip_2\gamma_3\left(\eta - \frac{zl}{L_{21}}\right)^2\right], \qquad (2)$$

where $p_2 = 1 - p_1$ and $p_1 = \nu_{32}/\nu_{12}$. We have as a result

$$\frac{\partial B_1}{\partial z} = -i\sigma_1 a_3 \left(\eta - \frac{z}{L_{31}}\right) \exp\left(-i\gamma_3\eta^2 - i\gamma_3 \frac{z^2}{L_{31}L_{32}}\right) B_2^*, \qquad (3)$$

$$\frac{\partial B_2}{\partial z} + \frac{1}{L_{21}} \frac{\partial B_2}{\partial \eta} = -i\sigma_2 a_3 \left(\eta - \frac{z}{L_{31}}\right) \times \exp\left(-i\gamma_3\eta^2 - i\gamma_3 \frac{z^2}{L_{31}L_{32}}\right) B_1^*.$$

We shall assume that $l \ll L_{31}$ and $\gamma_3 l^2/(L_{31}L_{32}) \ll 1$, where l is the length of the nonlinear medium. In this approximation the mismatches ν_{31} and ν_{32} are important only when the phase modulation of the pumping is taken into account. We thus obtain

$$\frac{\partial B_1}{\partial z} = -i\sigma_1 A_3(\eta) B_2^*, \quad \frac{\partial B_2}{\partial z} + \frac{1}{L_{21}} \frac{\partial B_2}{\partial \eta} = -i\sigma_2 A_3(\eta) B_1^*. \qquad (4)$$

The system of equations (4) are to be solved with the boundary conditions at $z = 0$: $B_{j0}(\eta) = A_{j0}(\eta) \exp(-ip_2\gamma_3\eta^2)$. The solution of Eqs. (4) is known (see Ref. 35). We shall assume that the amplification is large, $l \gg L_{nl}$ where $L_{nl} = 1/[(\sigma_1\sigma_2)^{1/2} a_{30}]$ and $L_{21} \gg l$. We note that the value of L_{21} is calculated for a pump pulse length τ_3. Recovering the functions A_1 and A_2, we have

$$A_1 = \frac{1}{2\sqrt{\pi}} \exp\left[\frac{l}{L_{nl}}(1 - \eta^2) - ip_1\gamma_3\eta^2\right] \times \int_{-\infty}^{\infty} \Big\{A_{10}(\eta - \alpha x) \exp[ip_1\gamma_3(\eta - \alpha x)^2]$$
$$+ \left(\frac{\sigma_1}{\sigma_2}\right)^{1/2} A_{20}^*(\eta - \alpha x) \exp[-ip_2\gamma_3(\eta - \alpha x)^2]\Big\} e^{-x^2}\, dx,$$
$$A_2 = \left(\frac{\sigma_2}{\sigma_1}\right)^{1/2} \exp(-i\gamma_3\eta^2) A_1^*; \qquad (5)$$

where $\alpha = \tau_c/\tau_3$, where $\tau_c = (lL_{nl})^{1/2}|\nu_{21}|\sqrt{2}$.

Let us note some regularities that can be inferred from (5). In the absence of phase modulation of the pumping, the waves at the output of the optical parametric oscillator are phase-conjugate. We shall discuss the fundamental importance of this fact later. Here we point out that the phase conjugation is maintained with accuracy to quantities of the order $\gamma_3\nu^2$ also for pumping with phase-modulated pulses. The quantity τ_c determines the parametric amplification bandwidth $\Delta\omega_a$ for monochromatic pumping. In particular, setting $\gamma_3 = 0$, $A_{10} \sim \delta(t)$, and $A_{20} = 0$, we have $A_1 \sim \exp(-t^2/\tau_c^2)$ and the amplification bandwidth, $\Delta\omega_a = 4\sqrt{\ln 2}/|\nu_{21}|\ 1/(lL_{nl})$, which is analogous to the results of Ref. 35. If the fact that the amplification bandwidth is bounded is not relevant ($\alpha \to 0$), then in the case $A_{20} = 0$ it follows from (5) that $A_1 = (A_{10}(\eta)/2) \exp[(l/L)_{nl}(1 - \eta^2)]$. Phase modulation of the pumping has no effect whatever on the signal (copropagating) wave. Moreover, it is noteworthy that phase modulation of the pumping is entirely superimposed on the idler (outgoing) wave, a result that agrees with the conclusions of Ref. 24. If it is of basic importance that the amplification band is bounded ($\alpha \neq 0$), then the effect of phase modulation of the pumping on wave generation in the optical parametric oscillator is more complicated. Using the Fourier transformation

$$A_{10}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{10}(\omega) \exp(i\omega t)\, d\omega,$$

we have

$$A_1(t) = \frac{1}{4\pi} \exp\left[\frac{l}{L_{nl}}\left(1 - \frac{t^2}{\tau_3^2}\right)\right] \times \int_{-\infty}^{\infty} S_{10}(\omega) \exp\left[i\omega t - \frac{\tau_K^2}{4}\left(\omega + \frac{2p_1\gamma_3 t}{\tau_3^2}\right)^2\right] d\omega. \qquad (6)$$

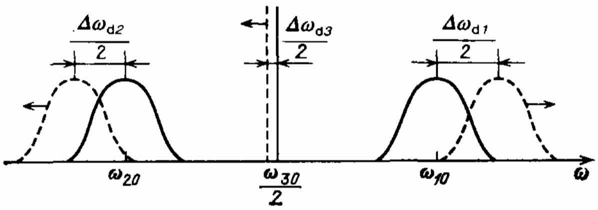

FIG. 1. Frequency shift of the amplification band of the signal wave ω_1 and idler wave ω_2 in an optical parametric oscillator pumped by a pulse with linear chirp.

It can be seen that amplification with pumping by phase-modulated pulses is equivalent to amplification with a time shift in the amplification frequency band (Fig. 1). We note that the physically similar effect of a signal of changing frequency on a resonant system has been studied previously (e.g., Ref. 36). The central frequency in the band contour of the signal wave amplification varies as $\omega_1 = \omega_{10} - 2p_1\gamma_3 t/\tau_3^2$. Taking into account that $\omega_3 = \omega_{30} - 2\gamma_3 t/\tau_3^2$, we obtain, for the frequency deviation $\Delta\omega_{d1} = p_1\Delta\omega_3$ and $\Delta\omega_{d2} = p_2\Delta\omega_3$. If $|p_1| \ll 1$ and $|p_2| \ll 1$, then the deviation of the central frequency of the amplification band will substantially exceed the frequency deviation of the pump pulse. The specific values of p_1 and p_2 are determined by the dispersion properties of the nonlinear medium. It is easy to estimate the value of p_1 for an interaction of the oo-e type near the degenerate mode. In this case, $p_1 \simeq F/(1-2\varkappa)$, where $F = \nu_{32}/\omega_3 k''$ and the degeneracy parameter is $\varkappa = \omega_2/\omega_3$. The values of F for a number of crystals and two pumping wavelengths are given in Table I. The nature of the effect of pumping on the amplification due to phase modulation is to a large extent governed by the ratio of the deviation $\Delta\omega_{d1}$ to the amplification band width $\Delta\omega_a$. Let us estimate this quantity, $\Delta\omega_{d1}/\Delta\omega_a$.

We have $\Delta\omega_{d1} \sim p_1\gamma_3/\tau_3$ and $\Delta\omega_a \sim \tau_c^{-1}$, from which follows $\Delta\omega_{d1}/\Delta\omega_a = \gamma_3(lL_{nl})^{1/2}/L_{32}$. Within the limits noted above, the shift of the frequency band is less than its width, $\Delta\omega_{d1} < \Delta\omega_a$, and so the phase modulation of the pumping will have little effect on the amplification of narrow-band signals, for which $\Delta\omega \ll \Delta\omega_a$. In the case of amplification of wide-band signals, $\Delta\omega \gg \Delta\omega_a$, the role of phase modulation of the pumping may prove to be significant. It would appear that the condition $\Delta\omega_{d1} \gg \Delta\omega_a$ can be realized if there are no limitations imposed by the conditions $l \gg L_{31}$ and $\gamma_3 l^2/L_{31}^2 \ll 1$.

Let us examine the characteristics of wave excitation in an optical parametric oscillator from the noise level for the case of pumping by pulses with a linear chirp (i.e., quadratic phase modulation). In this case the quantities A_{10} and A_{20} [expressions (5)] are the amplitudes of wide-band random processes ($\Delta\omega \gg \Delta\omega_a$) resulting from quantum noise in the parametric oscillator medium. Without loss of generality at high amplification $l \gg L_{nl}$, we may consider these random processes as being steady-state and Gaussian, with a correlation time $T_c \ll \tau_c$. It is of interest to calculate the correlation function $B_\Phi(\tau)$ of the quantity

$$\Phi(t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} A_{10}(t-\tau_c x) \exp\left[\frac{ip_1\gamma_3(t-\tau_c x)^2}{\tau_3^2} - x^2\right] dx.$$

The characteristic scale of the variation of A_{10} is of order T_c. For $t - \tau_c x \approx T_c$ we have $\exp[ip_1\gamma_3(t-\tau_c x)^2/\tau_3^2] \approx 1$ and

$$\Phi(t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} A_{10}(t-\tau_c x) e^{-x^2} dx.$$

Thus, the correlation function $B_\Phi(\tau) = \overline{\Phi(t)\Phi^*(t+\tau)}$ is given by the expression

$$B_\Phi(\tau) = \frac{1}{\pi} \iint_{-\infty}^{\infty} \exp(-x_1^2 - x_2^2) \times \overline{A_{10}(t-\tau_c x_1) A_{10}^*(t+\tau-\tau_c x_2)}\, dx_1\, dx_2. \qquad (7)$$

According to Ref. 35,

$$\overline{A_{10}(t-\tau_c x_1) A_{10}^*(t+\tau-\tau_c x_2)} = \frac{1}{2} \int_{-\infty}^{\infty} S_{10}^+(\omega_1+\Omega) \exp\{-i\Omega[\tau - \tau_c(x_2 - x_1)]\}\, d\Omega,$$

where S_{10}^+ is the noise intensity spectrum for positive frequencies. As a result we have

$$B_\Phi(\tau) = \frac{1}{2} \int_{-\omega_1}^{\infty} S_{10}^+(\omega_1+\Omega) \exp\left(-i\Omega\tau - \frac{1}{2}\Omega^2\tau_c^2\right) d\Omega. \qquad (8)$$

Taking into account the condition $T_c \ll \tau_c$ we have

$$B_\Phi(\tau) = \left(\frac{\pi}{2}\right)^{1/2} \frac{1}{\tau_c} S_{10}^+(\omega_1) \exp\left(-\frac{\tau^2}{2\tau_c^2}\right). \qquad (9)$$

Therefore, the amplitudes of the generated waves at the output of the optical parametric oscillator can be written in the form

$$A_1 = \frac{1}{2} \exp\left[\frac{l}{L_{nl}}\left(1 - \frac{t^2}{\tau_3^2}\right) - \frac{ip_1\gamma_3 t^2}{\tau_3^2}\right] A_{1\xi} t, \qquad A_2 = \left(\frac{\sigma_2}{\sigma_1}\right)^{1/2} \exp\left(-\frac{i\gamma_3 t^2}{\tau_3^2}\right) A_{1\xi}^*(t), \qquad (10)$$

where $A_{1\xi}(t)$ is the stationary Gaussian noise, with the correlation function

$$B_\xi(\tau) = \left(\frac{\pi}{2}\right)^{1/2} \frac{1}{\tau_c} \left[S_{10}^+(\omega_1) + \frac{\sigma_1}{\sigma_2} S_{20}^+(\omega_2)\right] \exp\left(-\frac{\tau^2}{2\tau_c^2}\right).$$

It follows from (10) that the waves generated at the output of the optical parametric oscillator are noise pulses. Their amplitudes are equal to the product of two amplitudes: one, corresponding to the regular signal, which is determined by the envelope and the chirp of the pump pulse, and the other, corresponding to a random signal, the correlation time of

TABLE I.

Crystal	0.53 μm	1.06 μm
KDP	0.16	—
CDA	0.39	—
$LiNbO_3$	0.54	0.20
$LiIO_3$	0.56	−0.21

which is determined by the parametric amplification frequency band and does not depend on the temporal properties of the pumping noise. As can be seen, the quadratic phase modulation of the pulses that are produced can substantially exceed that of the pump pulses. In this connection the quantity p_1 (as well as p_2) can be called the quadratic phase modulation (linear chirp) gain. It is evident that in this case one can attain considerably more effective dispersive compression of the optical parametric oscillator pulses and in this way form pulses substantially shorter than the pump pulses.

3. PARAMETRIC AMPLIFICATION OF PHASE-MODULATED PULSES AND CHIRP REVERSAL IN REAL TIME

A unique feature of parametric light amplifiers is the possibility of effectively amplifying phase-modulated pulses. Let us specify the conditions for which compression of the amplified signal spectrum does not occur. This is important from the standpoint of forming femtosecond pulses. We shall assume that the pumping pulses are transform-limited. Then setting $\gamma_3 = 0$ and $A_{20} = 0$ in expressions (5), we obtain

$$A_1 = \frac{1}{2\sqrt{\pi}} \exp\left[\frac{l}{L_{\rm nl}}(1-\eta^2)\right] \int_{-\infty}^{\infty} A_{10}(\eta - \alpha x)\, e^{-x^2}\, dx,$$
$$A_2 = \left(\frac{\sigma_2}{\sigma_1}\right)^{1/2} A_1^{*}. \qquad (11)$$

It is seen, then, that two powerful phase-conjugate waves can be formed at the output of the parametric amplifier. In this approximation the mismatch of the group velocities of the waves has no effect whatever on the parametric gain. The mismatch ν_{21} determines the amplification frequency band. In particular, this approximation is valid near the degenerate mode (an interaction of the type e-oo). We note that if the mismatch ν_{31} is of fundamental importance, then some decrease or saturation of the gain will occur.

The fact that the phase modulation of the idler pulse is reversed in time relative to the phase modulation of the signal pulse should be considered to be the most important conclusion that follows from (11). Considering the high speed of response of second order electronic nonlinearities and the large spectral width of the parametric amplification band, which can be as high as several thousand cm^{-1} (Ref. 29), we can say that the parametric amplifier can produce phase conjugation of extremely short light pulses. We further assume that upon entering the medium the pulse amplitude has the form $A_{10} = a_{10}\exp[-(t^2/\tau_1^2)(1+i\gamma_1)]$. In this case $A_1(t)$ describes a phase-modulated Gaussian pulse:

$$A_1(t) = \frac{a_{10}}{2} \exp\left[\frac{l}{L_{\rm nl}} - \frac{t^2}{(\tau_1'')^2}(1+i\gamma_1'')\right] \times \left[1 + \left(\frac{\tau_c}{\tau_1}\right)^2 (1+i\gamma_1)\right]^{-1/2}, \qquad (12)$$

where

$$\tau_1'' = \frac{\tau_1'}{[1+(\tau_1'/\tau_0)^2]^{1/2}}, \quad \gamma_1'' = \frac{\gamma_1'}{1+(\tau_1'/\tau_0)^2},$$

$$\tau_1' = \tau_1\left\{\frac{[1+(\tau_c/\tau_1)^2]^2 + \gamma_1^2(\tau_c/\tau_1)^4}{1+(\tau_c/\tau_1)^2(1+\gamma_1^2)}\right\}^{1/2},$$

$$\gamma_1' = \frac{\gamma_1}{1+(\tau_c/\tau_1)^2(1+\gamma_1^2)}, \quad \tau_0 = \tau_3\left(\frac{L_{\rm nl}}{l}\right)^{1/2}.$$

If the pump pulse is sufficiently long (the approximation of monochromatic pumping, $\tau_0 \to 0$), then $\tau_1'' \approx \tau_1'$ and $\gamma_1'' \approx \gamma_1'$. Hence the nonmonochromaticity is unimportant if $(\tau_1'/\tau_0)^2 \ll 1$. For the conditions $(\tau_c/\tau_1)^2(1+\gamma_1^2) \ll 1$, $\tau_1' \approx \tau_1$, and $\gamma_1' \approx \gamma_1$, the pulse is amplified without amplitude or phase distortion. For $\gamma_1^2 \gg 1$ the condition obtained can be rewritten in the form

$$\tau_c \leqslant \frac{0.3\tau_1}{\gamma_1}. \qquad (13)$$

As a general statement the amplification takes place without distortion if the spectral width of the signal is less than the amplifier bandwidth.

If condition (13) is not satisfied, then as the quantity τ_1/γ_1 decreases, first there will be phase distortion and then amplitude distortion of the amplified pulse. It follows from (12) that in such a case the phase modulation of the pulse always decreases. It is easy to show that there is on the whole spectral filtering of the amplified pulse. In the limit $(\tau_c\gamma/\tau_1)^2 \gg 1$, with $\tau_1'' \approx \tau_c$, the duration of the pulse that is produced is determined entirely by the frequency band of the amplifier.

What sort of features are produced in the amplification of phase-modulated pulses in the field of the pump pulse? Let us assume that condition (13) is satisfied. In this case $\tau_1'' = \tau_1/[1+(\tau_1^2/\tau_0^2)]^{1/2}$ and $\gamma_1'' = \gamma_1/[1+(\tau_1^2/\tau_0^2)]$. As can be seen, in the amplification of the pulse, both its duration and chirp decrease. The spectrum of the pulse can either expand or contract. Compression does not occur if $\gamma_1 \leqslant \tau_1/\tau_0$. We note that in the amplification of a transform-limited pulse ($\gamma_1 = 0$) in the field of a pump pulse, the duration of the former pulse always decreases and its spectrum broadens. An experimental investigation of parametric amplification of phase-modulated pulses has been reported in Ref. 23.

It is of interest to evaluate the role of dispersion spreading during parametric amplification of broad-band light pulses. When a phase-modulated Gaussian pulse $A_1(t) = a_{10}\exp[-(t^2/\tau_1^2)(1+i\gamma_1)]$ propagates in a dispersive medium it spreads out. At a distance z its amplitude is given by the expression[37]

$$A_1(t, z) = \frac{a_{10}}{\psi^{1/2}} \exp\left\{-\frac{t^2}{\tau_1^2|\psi|^2} \times \left[1 + i\frac{z}{L_{\rm d1}}\left(1+\gamma_1^2+\gamma_1\frac{L_{\rm d1}}{z}\right)\right]\right\},$$

where $\psi = 1 - i(z/L_{\rm d1})(1+i\gamma_1)$. It can be seen that the pulse length and the phase modulation parameter are given by the expressions

$$\tau_1' = \tau_1\left[\left(1+\gamma_1\frac{z}{L_{\rm d1}}\right)^2 + \frac{z^2}{L_{\rm d1}^2}\right]^{1/2},$$

$$\gamma_1' = \frac{z}{L_{\rm d1}}\left(1+\gamma_1^2+\gamma_1\frac{L_{\rm d1}}{z}\right).$$

Let us consider the case $\gamma_1 \ll 1$. If $\gamma_1 z/L_{\rm d1} \ll 1$ we have $\gamma_1' \approx \gamma_1$ and $\tau_1' \approx \tau_1$, and the dispersion spreading of the pulse is unimportant. We note that for $\gamma_1 = 0$, this conditions takes the form $z \ll L_{\rm d1}$. It is easy to show that for a pulse with a given spectral width the dispersion spreading is less pronounced for phase-modulated pulses. If the length of a spectrally bounded pulse is τ_1 then its dispersion spreading is unimportant for $z \ll \tau_1^2/2|k''_\omega|$. In the case of a phase-modulated pulse of length τ_0 and a phase modulation parameter $\gamma_1 = \tau_0/\tau_1$, the spreading is negligible at distances $z \ll \tau_0^2/(2|k''_\omega|\gamma_1) = \tau_1^2\gamma_1/(2|k''_\omega|)$. It can be seen that when there is phase modulation, this distance is increased by a factor γ_1. Let us make an estimate. In a KDP crystal, for a pulse of length $\tau_1 = 10$ fs and $\lambda_1 = 1\ \mu$m, we find $L_{d1} \approx 0.4$ cm. If $\tau_0 = 1$ ps, then $\gamma_1 = 100$ and $L_{d1} \approx 40$ cm. As a general conclusion we can state that in amplifying femtosecond pulses with a given spectral width, amplification of phase-modulated pulses is preferable. In addition this tactic permits an increase in the amplitude saturation threshold.

4. PULSE SELF-COMPRESSION WITH STRONG ENERGY EXCHANGE

To a certain degree, pulse self-compression in quadratically nonlinear media is analogous to pulse self-compression that takes place in optical fibers and is a result of the simultaneous effect of cubic nonlinearity and of group velocity dispersion. The former effect may be understood in terms of nonlinear dispersion.[38] Under conditions of strong energy exchange between the waves there is substantial broadening of the spectrum of the interacting pulses, and their dispersion spreading is compensated by a compression resulting from the nonlinear dispersion. A series of papers have been devoted to these problems.[30–32] To achieve experimentally the compression of femtosecond pulses, optimization of this process with allowance for the actual parameters of the nonlinear medium is extremely important. It is therefore advisable to examine the results of an analysis of parametric compression under conditions of strong energy exchange and the dependence of the compression on the ratio of the dispersion spreading lengths of the interacting pulses. To this end we present the results of a numerical solution of a system of truncated equations which describe the three-wave interaction of light packets in nonlinear media, taking into account the dispersion spreading. We shall assume that the amplification takes place in the degenerate mode under conditions of group synchronism. The dispersion spreading is determined by the quantities $k''_{\omega j}$ ($j = 1,3$), which may be either positive or negative.[39] The number of parameters that enter into the problem can be reduced. In place of the dispersion spreading lengths L_{d1} and L_{d2} it is advantageous to introduce the quantities L_{d1}/L_{d3} and $L_{d1}L_{d3}$. In this case the regularities of the pulse compression are determined mainly by the ratio L_{d1}/L_{d3} and the sign of the product $k''_{\omega 1}k''_{\omega 3}$. A variation of $L_{d1}L_{d3}$ is equivalent to a variation in the coupling coefficient of the waves or in the initial pumping intensity. Such variations will mainly produce quantitative changes, but the overall behavior of the compression is unaltered. In order to observe compression the value of $L_{d1}L_{d3}$ must exceed a certain minimum which is determined by the conditions at the input to the nonlinear medium. Figure 2 shows the intensities at the peaks of the signal (curves 1) and pump (curves 2) pulses at the point of their maximum compression as a function of the ratio L_{d1}/L_{d3} for two cases: $k''_{\omega 1}k''_{\omega 3} > 0$ (the solid curves) and $k''_{\omega 1}k''_{\omega 3} < 0$ (the broken curves). The intensities and durations of the interacting pulses are normalized to the initial intensity and duration of the pump pulse. It can be seen that for $L_{d1} \ll L_{d3}$ compression of the pump pulse occurs to a greater degree than does that of the signal pulse. Profiles of the envelopes for $L_{d1}/L_{d3} = 0.1$ are shown in Fig. 3a. The envelopes of the initial pump pulses are shown by the dashed lines. In this case dispersion spreading of the pump pulse is insignificant and hence the picture of the mutual compression does not depend on the sign of $k''_{\omega 1}k''_{\omega 3}$. As the value of L_{d1}/L_{d3} increases the behavior of the mutual compression begins to depend strongly on the sign of $k''_{\omega 1}k''_{\omega 3}$.

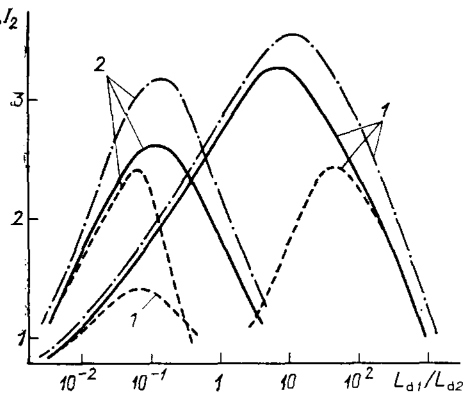

FIG. 2. Intensity at the peak of the signal (1) and pump (2) pulses at the point of maximum compression as a function of the ratio L_{d1}/L_{d2} for $k''_{\omega 1}k''_{\omega 2} > 0$ (solid curves, $L_{d1}L_{d3} = 10\,000$; dot-dash curves, $L_{d1}L_{d3} = 15\,000$) and $k''_{\omega 1}k''_{\omega 3} < 0$ (dashed curves, $L_{d1}L_{d3} = 10\,000$).

In the case $k''_{\omega 1}k''_{\omega 3} > 0$, for $L_{d1}/L_{d3} \gtrsim 1$, the intensity of the compressed signal pulse increases and that of the pump pulse falls off. As a result, the intensity of the signal pulse becomes larger than that of the pump pulse. If $L_{d1}/L_{d3} \gg 1$, then only the signal pulse is compressed. In this case the intensity of the pump pulse does not exceed the initial intensity, and the pulse itself splits up into several peaks whose widths may be substantially less than the initial width (Fig. 3b, $L_{d1}/L_{d3} = 10$). For the rest of this discussion we shall assume that pulse compression occurs if the pulse intensity exceeds the initial intensity of the pump pulse. In this connection, in Figs. 2 and 4 the pulse intensities and widths are shown only for the case of pulse compression.

The conditions for pulse compression for the case $k''_{\omega 1}k''_{\omega 3} < 0$ are more stringent. When $L_{d1}/L_{d3} \geqslant 0.04$ the compressed pulses decrease in intensity. If L_{d1} and L_{d3} are comparable, the phenomenon of parametric decompression of the interacting pulses occurs.[31] This condition also produces a broad dip in the curve describing the dependence of the intensity of the compressed signal pulse on L_{d1}/L_{d3} (see Fig. 2). Note that in order to observe decompression of the interacting pulses it is still not sufficient that the condition $k''_{\omega 1}k''_{\omega 3} < 0$ hold, as was suggested in Ref. 31. The profiles of the pulse envelopes in the case of decompression are shown in Fig. 3c ($L_{d1}/L_{d3} = 2.5$). The widths of the envelopes of the interacting pulses are increased and their intensities are substantially reduced. The pulses are split into separate

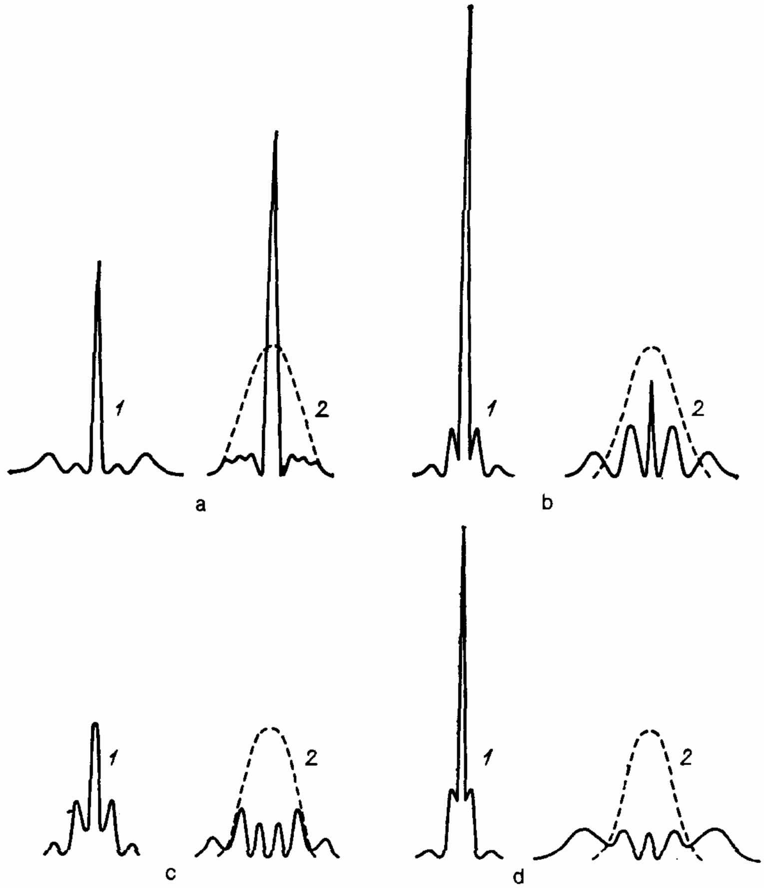

FIG. 3. Envelope profiles for signal (1) and pump (2) pulses for $k''_{\omega1}k''_{\omega3} > 0$ for $L_{d1}/L_{d3} = 0.1$ (a), $L_{d1}/L_{d3} = 10$ (b), and $k''_{\omega1}k''_{\omega3} < 0$ for $L_{d1}/L_{d3} = 2.5$ (c) and $L_{d1}/L_{d3} = 40$ (d).

peaks. As L_{d1}/L_{d3} is further increased to values > 2, compression is again possible, but only for the signal pulse. Typical pulse profiles are shown in Fig. 2d. If $L_{d1}/L_{d3} \gg 1$, then the spreading of the signal pulse becomes unimportant and the compression of the pulse does not depend on the sign of $k''_{\omega1}k''_{\omega3}$.

Figure 4 shows the length of the signal pulse (curves 1) and the pumping pulse (curves 2) at the point of maximum compression as a function of the ratio L_{d1}/L_{d3} for $k''_{\omega1}k''_{\omega3} > 0$ (the solid curves) and $k''_{\omega1}k''_{\omega3} < 0$ (the dashed curves). On the whole, the behavior of the curves in Fig. 4 is closely related to that of the curves in Fig. 2. The minimum lengths of the compressed pulses are obtained in the region of maximum intensity. It can be seen that the lengths of the interacting pulses as a result of compression can be an order of magnitude smaller than the length of the initial pump pulse.

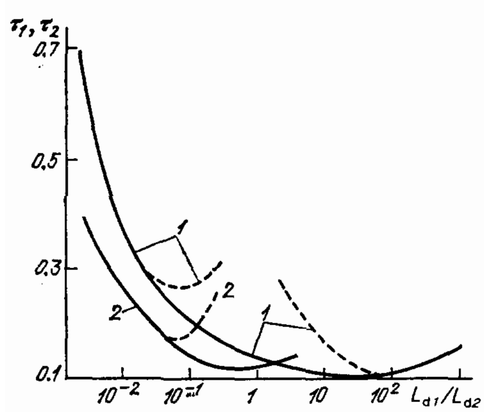

FIG. 4. Length of the signal (1) and pump (2) pulses at the point of maximum compression as a function of the ratio L_{d1}/L_{d3} for $k''_{\omega1}k''_{\omega3} > 0$ (solid curves) and $k''_{\omega1}k''_{\omega3} < 0$ (dashed curves).

In conclusion we note that conditions for pulse compression via their three-wave interaction can be obtained for a wide range of dispersion spreading lengths and can be exploited for substantial decrease in the lengths and increase in the intensities of the pulses.

5. EXPERIMENTAL RESULTS

Experiments on parametric conversion of phase-modulated pumping, parametric amplification of phase-modula-

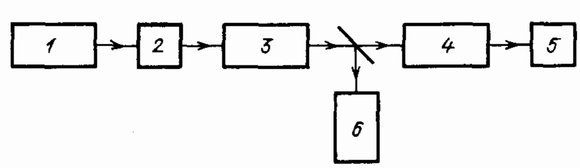

FIG. 5. Block diagram of the apparatus for studying parametric chirp and compression of optical parametric oscillator pulses. 1) laser, 2) second harmonic generator, 3) optical parametric oscillator 4) compressor, 5) pulse length measuring device, 6) dynamic interferometer.

TABLE II.

a) Laser

λ, μm	E_p, J	E_{out}, mJ	τ, ps
1.069 0.534	50	3 1	12 10

b) Optical parametric oscillator

E_{th}, mJ	E_{out}, mJ	τ, ps
0.3	0.05	3.5

ted pulses, chirp reversal in real time, and dispersive pulse compression have been carried out with various kinds of laser pumping, various schemes for parametric light generation, and various kinds of nonlinear crystals.[21,22,23,40] These investigations had as their purpose the determination of the principles involved in the parametric interaction of phase-modulated pulses and the determination of the parametric amplification and light generation schemes best suited for the formation of subpicosecond light pulses.

5.1 Experiments in parametric amplification with phase-modulating pumping

A generalized block diagram of the apparatus is shown in Fig. 5. The optical parametric oscillator (3) is pumped by the radiation of the picosecond laser (1). The radiation from the optical parametric oscillator is directed into a pulse compressor (4) and then into a pulse-length measuring device (5). To study the phase characteristics, part of the radiation from the optical parametric oscillator is diverted into a dynamic interferometer (6) which includes a Michelson interferometer and an "Agat SF-3" streak camera.

Parametric amplification of pump chirp and compression of pulses from a superradiant optical parametric oscillator based on a $LiNbO_3$ crystal pumped by the second harmonic of a passive-mode-locked YAG:Nd^{3+} laser was first investigated in Ref. 21. In an extension of this work, experiments were carried out with a resonator-type optical parametric oscillator employing a $LiNbO_3$ crystal pumped with the second harmonic of a quasi-cw YAG:Nd^{3+} laser. The comparatively long pulse length ($\sim$60 ps) and the very extended pump train ($\sim$30 pulses) made it possible to trace graphically the dynamics of the conversion of the phase-modulated pumping into phase modulated waves generated by the optical parametric oscillator.

Let us examine in more detail the results of the latter experiments on the effect of chirp amplification in an optical parametric oscillator with synchronous pumping. The resonator-based optical parametric oscillator uses a $LiNbO_3$ crystal, which is pumped with a passive-mode-locked $La_2Be_2O_5$:Nd^{3+} laser. The main energy and time parameters of the laser radiation, of the second harmonic, and of the optical parametric oscillator are presented in Table II.

Figure 6 shows the results of a dynamic interferometric study of pump chirp and parametric light generation at various places in the pulse train, It can be seen that for $\varkappa = 0.467$ there is a tenfold amplification of the pump chirp, producing at the termination of the pulse train a chirp of up to 8 cm^{-1}/ps. This is in good agreement with the theoretical calculations.[21]

Since the idler pulses of the optical parametric oscillator emission have negative chirp, the compression of these

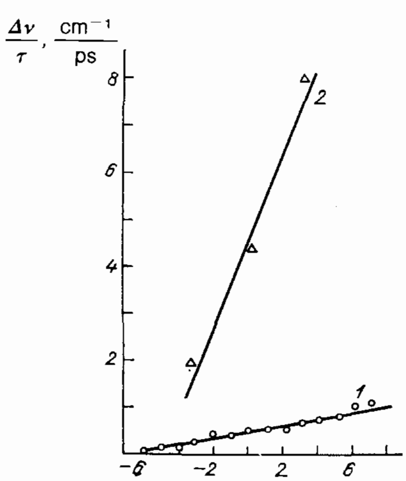

FIG. 6. Pump chirp (1) and chirp of $LiNbO_3$ optical parametric oscillator emission (2), $\lambda = 1.02\ \mu$m, as a function of the pulse number in the train.

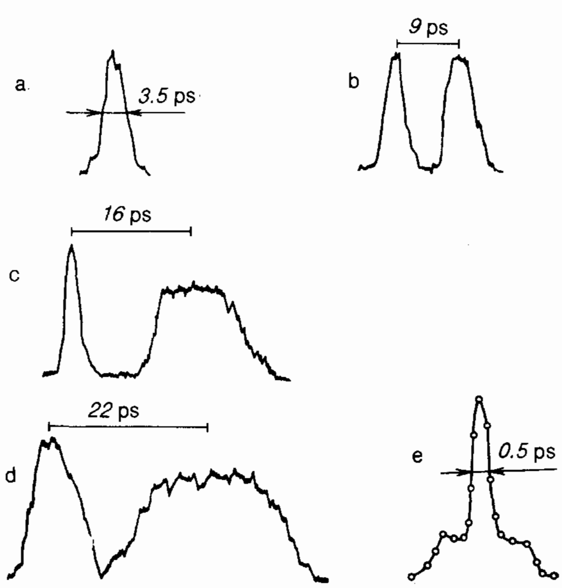

FIG. 7. a) Typical streak camera pictures of the idler and signal pulses at the input to the compressor; b) at the output of the compressor for a less-than-optimal compressor length; c) for optimal compressor length; and d) greater than optimal length. e) correlation diagram of compressed pulse for optimal compressor length.

pulses was studied in a medium with positive group velocity dispersion. The use of such media helps to increase the throughput of the compressors and makes it possible to eliminate the need for expensive diffraction gratings, which are used for compression of pulses with positive chirp. For this medium we chose a KRS-6 crystal, which has a rather high dispersion. The crystal length necessary for optimal pulse compression was chosen experimentally. The results of the experiment are shown in Fig. 7. Signal pulses ($\lambda_1 = 1.02$ μm) and idler pulses ($\lambda_2 = 1.12$ μm), having pulse lengths ~ 3.5 ps (Fig. 7a) and chirp of opposite signs were fed simultaneously into the compressor. The pulse lengths were measured with an "Agat SF-3" streak camera. We studied the pulses from the end of the train, which have the largest chirp (8 cm^{-1}/ps). As the signal and idler pulses pass through the compressor, they become separated in time and change their lengths in the KRS-6 crystal as a result of the group velocity dispersion: the signal pulse broadens and the idler pulse narrows (Fig. 7b). This change in the lengths of the signal and idler pulses occurs over approximately a path length in the crystal ~ 35 cm. Figure 7c shows the situation after the pulses have traversed a path of 35 cm in the KRS-6 crystal. The pulses have separated by 16 ps and the signal pulse length is 13 ps and the idler pulse length is < 2 ps. Figure 7e shows, averaged over the pulse train, a correlation diagram of the compressed idler pulse, taken for the optimum compressor length and measured by the method of noncollinear second harmonic generation. If the length of the compressor is further increased the idler pulse also begins to broaden (Fig. 7d).

We have also studied chirp amplification and pulse compression in an optical parametric oscillator based on an $LiNbO_3$ crystal synchronously pumped with the second harmonic of an active-mode-locked Q-switched YAG:Nd^{3+} laser operated at a pulse repetition rate up to 10 kHz.[22] The principal energy and time parameters of the laser radiation, of the second harmonic, and of the optical parametric oscillator are given in Table III.

Figures 8a–8c show dynamic interferograms of the pump pulses at the beginning (a), middle (b), and end (c) of the train, measured with an interferometer with a free dispersion range of 3.5 cm^{-1}. As can be seen, the pump chirp has been increased and linearized. Figures 8d–8f show the dynamic interferograms of the corresponding signal pulses of the optical parametric oscillator (the free dispersion range of the interferometer is 50 cm^{-1}). At the beginning of the train pulses are formed with a wide spectrum determined by the random phase modulation (Fig. 8d). The increase of the effective interaction length and of the pump chirp in the middle of the train leads to a narrowing of the spectrum and formation of regular phase modulation (Fig. 8e). At the end of the train the spectrum is dominated by the regular component, which attains a width ~ 26 cm^{-1} determined by the pump chirp amplification coefficient $p_1 = 7$ (Fig. 8f). In the compression of pulses from the end of the pulse train of the optical parametric oscillator, a 20-fold compression of the signal pulses to 2.5 ps was attained [this is close to the resolution limit of the image converter camera used (Fig. 9)].

TABLE III.

Radiation source	τ, ps	η, %	P, kW	Number of pulses in train
Laser	70	—	450	40
Second harmonic	60	40	190	30
Optical parametric oscillator	50	17	30	20
Optical parametric oscillator after the compressor	2.5	5	20	20

5.2 Experimental studies on parametric amplification of phase-modulated pulses and chirp reversal in real time

The experiments were carried out by the method of almost-collinear parametric amplification (Fig. 10).[23] A single 5-ps pump pulse, of energy ~ 3 mJ, was formed in a passive-mode-locked phosphate glass laser. A signal with a

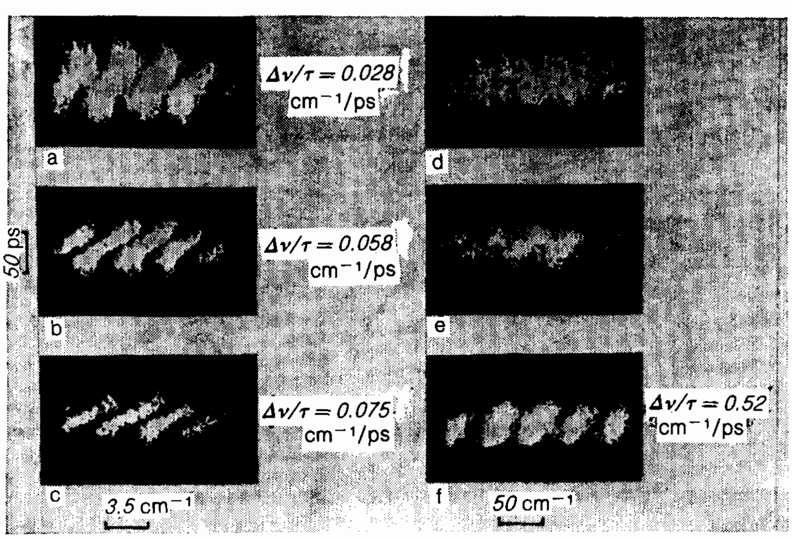

FIG. 8. Dynamic interferograms of the pump pulses (a,b,c) and the optical parametric oscillator emission ($\lambda = 0.99$ μm;d,e,f) at the beginning (a,d), middle (b,e) and end (c,f) of the pulse train.

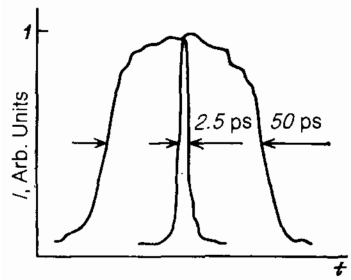

FIG. 9. Streak camera picture of pulse before and after compression.

linear positive chirp was prepared in a single-mode optical fiber 1.3 m long. To form this pulse, a pulse of energy $\sim 1\,\mu J$ ($\lambda = 1.054\,\mu m$, insertion efficiency $\sim 40\%$) was fed into the optical fiber. As a result of phase self-modulation the spectrum of the pulse broadened on the average to 400 cm^{-1} and the pulse length increased to 10 ps. Since the phase-modulated pulse has a broadened spectrum ($\Delta\nu\tau \gg 1$), to obtain parametric amplification we used, for the quadratically nonlinear medium, a CDA crystal ($l = 4$ cm, e-oo interaction) having 90-degree synchronism and an exceptionally broad-band amplification (to 2000 cm^{-1}, Ref. 23). Estimates show that the group velocity mismatch and the dispersion spreading of the pulses in the crystal were insignificant in the experiment. The signal and pump pulses were fed through matching delay lines (not shown in Fig. 10) into the crystal, in which an energy gain of $\sim 10^4$ was attained. Since the pump pulse "cut out" a corresponding spectral band from the wider signal pulse, after the amplifier the frequency deviation between the signal and the idler pulses was reduced on the average to 200 cm^{-1}. The time behavior of the phase was studied by the dynamic interferometric method. Figure 11 shows dynamic interferograms of the pulse at the output of the single-mode optical fiber (a) as well as of the signal (b) and idler (c) pulses at the output of the parametric amplifier, these interferograms being taken with an interferometer with a free spectral range of 555 cm^{-1}. The direction of inclination of the bands depends on the sign of the chirp. As can be seen from the interferograms shown in Figs. 11b and 11c, the bands slope in different directions. This indicates that the phase characteristics of the signal and the idler pulses are conjugate. In the compression of parametrically amplified phase-modulated pulses a compression to 280 ps was obtained (Fig. 11d). The power of the compressed pulses reached 1 Gw.

6. CONCLUSIONS

In summing up we may state that three-wave parametric interaction opens up new possibilities for the formation and conversion of femtosecond pulses. The most important advantage is the possibility of obtaining high-energy light pulses at the output of parametric amplifiers and light generators. Quite recently this potential was demonstrated in the picosecond range[41] (the energy of the pulses of the parametric oscillator, pumped by a wide-aperture beam, was 2.3 J). The wide tuning range of the optical parametric oscillators and the possibility of operation in the infrared are extremely important. The latter of these capabilities permits the effective use of optical parametric oscillators in studying nonlinear optical processes in optical fibers.[3,42] Of particular interest are methods of forming ultrashort light pulses in quadratically nonlinear media and controlling their phase characteristics.

In the first place, the extremely wide parametric amplification bands that can be realized in a number of crystals (KDP, $LiIO_3$, $LiNbO_3$, CDA, etc.) solve the problem of am-

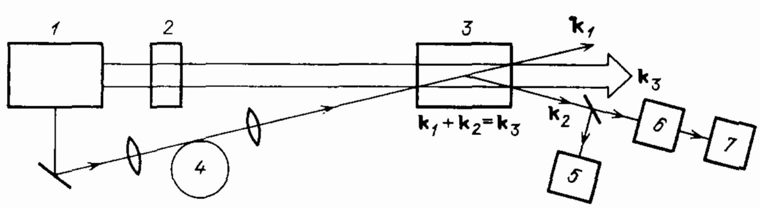

FIG. 10. Diagram of experiment. 1) laser; 2) second harmonic generator; 3) parametric amplifier; 4) single-mode optical fiber, 5) dynamic interferometer; 6) compressor; 7) pulse length measuring device.

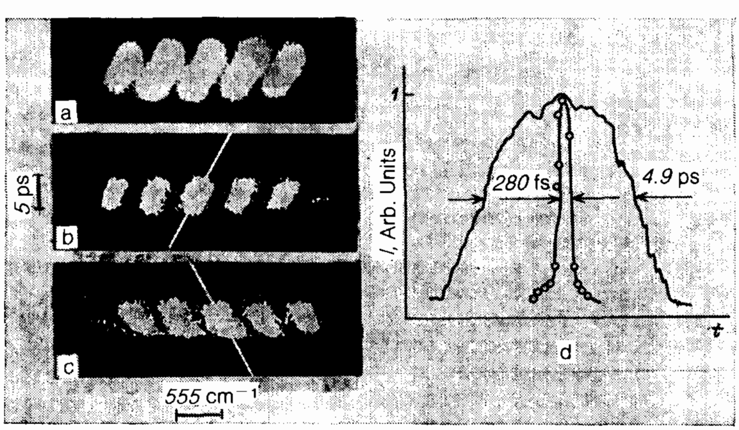

FIG. 11. Dynamic interferogram of the pulse at the output of the single-mode optical fiber a); of the signal pulse b) and idler pulse c) at the output of the parametric amplifier. d) streak camera picture of the pulse at the input to the compressor, and correlation diagram of pulse at the output of the compressor.

plifying weak phase-modulated signals (like those that enter from an optical fiber) by five to six orders of magnitude without distorting their phase characteristics. This capability makes it possible in principle to form, at the output of non-aperture-limited, dispersion-type compressors, femtosecond pulses of power exceeding tens of gigawatts. We note that in contrast to unique femtosecond high-power dye laser systems,[43] parametric amplification and generation of light makes it possible to construct unified systems based on solid state active and nonlinear elements.

In the second place, pulse phase conjugation, which accompanies parametric amplification, makes it possible to achieve broad-band pulse chirp reversal in real time. In principle, chirp reversal is the time analogue of phase conjugation. The problem of chirp reversal of light signals in the process of four-wave parametric interaction has been discussed theoretically in Refs. 44 and 45 where the suggestion was made to use *spectral component conjugation to compensate for phase distortion* introduced by the group velocity dispersion. The principal requirement for obtaining reversal over the entire frequency band $\Delta\nu$ of the phase-modulated pulse is formulated as $\Delta\nu \ll 1/\tau_R$, where τ_R is the characteristic response time of the nonlinear medium. Three-wave parametric processes operate on electronic nonlinearity, the response time of which is $\tau_R < 10^{-14}$ s, so that phase conjugation is possible for extremely broad spectral bands. The method of control by means of the sign of the chirp plays a decisive role in the optimization of femtosecond pulse compression, and it allows the use of media with either negative or positive group velocity dispersion in compressors. Moreover, phase-conjugate pulses with linear chirp opens up new possibilities in four-photon phase spectroscopy, dynamic holography of space-time events, as well as in information processing systems.

In the third place, study of the conversion of the phase characteristics of the pumping during parametric light generation had led to a new method of controlling the steepness of the chirp of ultrashort light pulses. It is noteworthy that the steepness of the chirp of pulses from an optical parametric oscillator can exceed by several others of magnitude the steepness of the pump chirp. Futhermore, the coefficient of chirp conversion has no relation whatever to the pumping intensity and is determined only by the dispersion characteristics of the crystal. Apparently this phenomenon will find application in the subsequent development of lasers in the femtosecond range.

Regarding the significance of numerical experiments for studying parametric self-compression of light pulses in a quadratically nonlinear medium, we shall note that they undoubtedly stimulate further interest in setting up real experiments. At the same time, it is evident that the expected effects will appear most clearly only in the fields of pump pulses whose duration is some hundreds of femtoseconds. Only then does the group velocity dispersion in nonlinear crystals begin to play an important role.

In final summary, it should be stated that the creation of parametric femtosecond lasers is at present in a state of intense development and investigation. In the near future we should expect new and fundamental progress in this direction.

[1)] This article is published in connection with the 25th anniversary of the invention of the laser (1960), and it completes the first publication of the series of articles comprising the entire January 1986 issue of Usp. Fiz. Nauk (Vol. **148**, No. 1) [Sov. Phys. Usp. **29**, No. 1] and the articles by E. M. Dianov and A. M. Prokhorov, and N. G. Basov in the February issue (Vol. **148**, No. 2) [Sov. Phys. Usp. **29**, No. 2] (*Edit. note*).

[2)] This English term, frequency chirp, has become generally accepted in recent years.

[1]D. Grischkowsky and A. C. Balant, Appl. Phys. Lett **41**, 1 (1982).
[2]E. B. Treacy, J. Quantum Electron. **QE-5**, 454 (1969).
[3]E. M. Dianov, A. Ya. Karasik, P. V. Mamyshev, G. I. Onishchukov, A. M. Prokhorov, M. V. Stel'makh, and A. A. Fomichev, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 148 (1984) [JETP Lett. **40**, 903 (1984)].
[4]L. F. Mollenauer and R. H. Stolen, Opt. Lett. **9**, 13 (1984).
[5]J. A. Valkmanis, R. L. Fork, and J. P. Gordon, Opt. Lett. **10**, 131 (1985).
[6]W. H. Knox, R. L. Fork, M. C. Downer, R. J. Stolen, and C. V. Shank, Appl. Phys. Lett. **46**, 1120 (1985).
[7]R. L. Fork, C. V. Shank, R. Yen, and C. A. Hirlimann, IEEE J. Quantum Electron. **QE-19**, 500 (1983).
[8]C. V. Shank, R. L. Fork, R. Yen, and R. H. Stolen, Appl. Phys. Lett. **40**, 761 (1982).
[9]J. G. Fujimoto and A. M. Weiner, and E. P. Ippen, Appl. Phys. Lett. **44**, 832 (1984).
[10]J. M. Halbout and D. Grishkowsky, Appl. Phys. Lett. **45**, 1281 (1985).
[11]S. A. Akhmanov and R. V. Khokhlov, Zh. Eksp. Teor. Fiz. **43**, 351 (1962). [Sov. Phys. JETP **16**, 252 (1962)].
[12]N. M. Kroll, Phys. Rev. **127**, 1207 (1962).
[13]R. H. Kingston, Proc. IRE **50**, 472 (1962).
[14]J. A. Giordmaine and R. C. Miller, Phys. Rev. Lett. **14**, 973 (1965).
[15]S. A. Akhmanov, A. I. Kovrigin, A. P. Sukhorukov, R. V. Khokhlov, and A. S. Chirkin, Pis'ma Zh. Eksp. Teor. Fiz. **7**, 237 (1968) [JETP Lett. **7**, 182 (1968)].
[16]S. A. Akhmanov, A. S. Chirkin, K. N. Drabovich, A. I. Kovrigin, R. V. Khokhlov, and A. P. Sukhorukov, IEEE J. Quantum Electron. **QE-4**, 598 (1968).
[17]A. G. Akhmanov, S. A. Akhmanov, A. S. Chifkin, K. N. Drabovich, A. I. Kovrigin, A. S. Piskarskas, and A. P. Sukhorukov, IEEE J. Quantum Electron. **QE-4**, 598 (1968).
[18]K. P. Burneĭko, M. V. Ignatavichyus, V. I. Kabelka, A. S. Piskarskas, and A. Yu. Stabinis, Pis'ma. Zh. Eksp. Teor. Fiz. **16**, 365 (1972) [JETP Lett. **16**, 257 (1972)].
[19]A. Fendt, W. Kranitzky, A. Laubereau, and W. Kaiser, Opt. Commun. **28**, 142 (1979).
[20]B. Bareĭka, G. Dikchyus, A. Piskarskas, V. Sirutkaĭtis, and Ya. Yasevichyute, Tezisy dokladov III Vsesoyuznoĭ konferentsii "Optika lazerov" [Abstracts of papers to the 3rd All-Union Conference "Laser Optics], S. I. Vavilov State Optical Institute, Leningrad, (1981) p. 197.
[21]A. Piskarskas, A. Stabinis, and A. Yankauskas, Kvantovaya Elektron. **12**, 1781 (1985) [Sov. J. Quantum Electron. **15**, 1179 (1985)].
[22]A. Piskarskas, A. Stabinis, A. Umbrasas, and A. Yankauskas, Kvantovaya Elektron. **12**, 2335 (1985) [Sov. J. Quantum Electron. **15**, 1539 (1985)].
[23]R. Danelyus, A. Piskarskas, V. Sirutkaĭtis, A. Stabinis, and A. Yankauskas, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 101 (1985) [JETP Lett. **42**, 122 (1985)].
[24]A. P. Sukhorukov, and A. K. Shchednova, Zh. Eksp. Teor. Fiz. **60**, 1251 (1971) [Sov. Phys. JETP **33**, 677 (1971)].
[25]A. P. Sukhorukov, Avtoreferat Dokt. Dissertatsii [Author's abstract of doctoral dissertation] Moscow State University, Moscow (1972).
[26]R. Danelyus, V. Kabelka, A. Piskarskas, and V. Smil'gyavichyus, Kvantovaya Elektron. **5**, 679 (1978) [Sov. J. Quantum Electron. **8**, 398 (1978)].
[27]A. J. Campillo, R. C. Hyer, and S. L. Shapiro, Opt. Lett. **4**, 357 (1979).
[28]V. N. Krylov and C. B. Papernyi, Zh. Tekh. Fiz. **50**, 442 (1980) [Sov. Phys. Tech. Phys. **25**, 268 (1980)].
[29]B. Bareĭka, A. Birmontas, G. Dikchyus, A. Piskarskas, V. Sirutkaĭtis, and A. Stabinis, Kvantovaya Elektron. **9**, 2534 (1982) [Sov. J. Quantum Electron. **12**, 1654 (1982)].
[30]Yu. N. Karamzin and A. P. Sukhorokov, Zh. Eksp. Teor. Fiz. **68**, 834 (1975) [Sov. Phys. JETP **41**, 414 (1975)].
[31]T. S. Filipchuk, Avtoreferat kand. dissertatsii [Author's abstract of candidate's dissertation] Moscow State University (1980).
[32]V. Vasilyauskas and A. Stabinis, Kvantovaya Elektron. **12**, 1881 (1985) [Sov. J. Quantum Electron. **15**, 1243 (1985)].
[33]Yu. Karamzin, A. P. Sukhorukov, and T. S. Filipchuk, Izv. vuzov Ser.

"Radiofizika" 21, 456 (1978).
[34]Yu. N. Karamzin, A. P. Sukhorukov, and T. S. Filipchuk, Vestn. Mosk. Univ. Fiz. Astron. 19, 91 (1978).
[35]S. A. Akhmanov, Yu. E. D'yakov, and A. S. Chifkin, Vvedenie v statisticheskuyu radiofiziku i optiku [Introduction to Statistical Radiophysics and Optics] Nauka, M., (1981).
[36]A. A. Kharkevich, Spektry i analiz [Spectra and Analysis], Fizmatqiz M., 1962.
[37]M. B. Vinogradova, O V. Rudenko, and A. P. Sukhorukov, Teoriya voln [Wave Theory] Nauka, Moscow (1979).
[38]A. P. Sukhorukov and A. K. Sukhorukova, Pis'ma Zh. Eksp. Teor. Fiz. 34, 200 (1981) [JETP Lett. 34, 191 (1981)].
[39]A. Virmontas, V. Vasilyauskas, A. Piskarskas, and A. Stabinis, Kvantovaya Elektron. 12, 1191 (1985) [Sov. J. Quantum Electron. 15, 787 (1985)].
[40]V. Vasilyauskas, A. Piskarskas, V. Sirutkaĭtis, A. Stabinis, and A. Yankauskas, Izv. Akad. Nauk. SSSR Ser. Fiz. 49, 493 (1986) [Bull. Acad. Sci. USSR Phys. Ser. 49, No. 3, 73 (1986)].
[41]S. A. Akhmanov, I. A. Begishev, A. A. Gulamov, E. A. Erofeev, B. V. Zhdanov, V. I. Kuznetsov, L. N. Rashkovich, and T. B. Usmanov, Kvantovaya Elektron. 11, 1701 (1984) [Sov. J. Quantum Electron. 14, 1145 (1984)].
[42]K. Mochizuki, Y. Namihira, and H. Wakabayashi, IEEE J. Quantum Electron. QE-18, 278 (1982).
[43]R. L. Fork, C V. Shank, and R. T. Yen, Appl. Phys. Lett. 41, 223 (1982).
[44]J. H. Marburger, Appl. Phys. Lett. 32, 372 (1978).
[45]A. Yariv, P. Fekete, and D. Pepper, Opt. Lett. 4, 52 (1979).

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