Gravitation, the general theory of relativity, and alternative theories

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The main stages in the construction of the theory of gravitation and prospects for its further development are discussed. The main attention is devoted to comparing the properties of the relativistic gravitational field and other physical fields. Two equivalent formulations of the general theory of relativity—the "geometrical" and the "field"— are considered in detail. It is explained why some of the field theories of gravitation developed in flat space-time are not different theories of the relativistic gravitational field but merely other formulations of general relativity.

Møller: ... Is this theory really Einstein's theory of gravitation in the sense that if you would have here many gravitons the equations would go over into the usual field equations of Einstein?
Feynman: Absolutely.
Møller: You are quite sure about it?
Feynman: ... There's no question that the thing is the Einsteinian theory. The classical limit of this theory ... is a nonlinear theory exactly the same as the Einsteinian equations .... It can't take care of the cosmological problem, in which you have matter out to infinity, or that the space is curved at infinity. It could be done I'm sure, but I haven't investigated it. I used as a background a flat one way out at infinity.
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The present note differs somewhat in its intention from the material usually included under the heading of Methodological Notes.

We wish to characterize general tendencies in the development of physical theory over several centuries and even make a small extrapolation into the future. At the center of our attention will be a comparison of the gravitational field with other physical fields and a clarification of the extent to which gravitation can be treated on an equal footing with other fields as a field defined in flat space-time. We shall see that the general theory of relativity admits a formulation as an exact and rigorous field theory on a flat-space background, and, moreover, as a theory possessing all the necessary attributes—action and equations of motion, energy-momentum tensor, and conservation laws, coordinate and gauge invariance. But we also analyze the question of measurements and observations in the presence of a gravitational field. This question is usually put on one side, but it is this question that forces us to the concept of a curved space-time. The universality of the gravitational interaction (which distinguishes it from other interactions) renders the flat space-time in the presence of a gravitational field unobservable, ghostlike, one can say, fictitious. One can compare it only with the grin that remains from the disappearing cat in the well-known story tale. We shall show that the attempt to interpret the metric relationships of the flat world as observables and concrete observational predictions based on this interpretation lead only to contradiction with experiment. We mention also possible ways of further development of gravitational theory, especially in connection with the construction of quantum gravity and its unification with other interactions. The present development of this science has stimulated a renewal of interest in alternative formulations of general relativity and has led to a much fuller understanding of their formal structure and connection with observations. We also distinguish alternative formulations and alternative theories. We show that some field theories of the gravitational field developed systematically in flat space-time may be, despite the wish of their authors, not alternative theories but, in essence, merely alternative formulations of general relativity, i.e., they lead to the same observational conclusions.

Thus, in the first stage Newton formulated clearly the law of mechanics, and the law of gravitation. The decrease of the force in inverse proportion to the square of the distance, i.e., to the surface area of a sphere, appeared very natural to his contemporaries. Newton's main achievement was the rigorous mathematical proof that for such a law the trajectories of the planets are closed curves—ellipses with the Sun at a focus.

The theory of gravitation was an example of a theory of *long-range interaction*. It seems to us that in Newton's lips the famous "Hypotheses non fingo"—"I frame no hypotheses" was tinged with regret rather than pride. We do not hear in these words pride over the fact that, having banished unreliable hypotheses, the author will construct his theory on a strong and eternal foundation. No, Newton uses the law of gravity and the theory of long-range interaction but at the same time recognizes that such a theory cannot be the last

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word. He recognizes that the long-range interaction itself requires a physical explanation but does not find (*cannot* find—this is not the same as *does not wish* to seek) the theory or at least a hypothesis that explains gravity.[1)]

There then followed the detailed development of analytical mechanics and, in particular, celestial mechanics, on the basis of Newton's laws. The work of Laplace and Poincaré is well known. This work, the concrete content of which changes, continues into out time—it is sufficient to mention the work of Kolmogorov, Arnol'd, and Moser. The conquest of space has also led to a further refinement of the observations. Apart from small corrections—to be discussed below—Newton's mechanics is confirmed magnificently by the observations.

One usually speaks of the scientific triumph of Leverrier and Adams, who predicted the existence of a previously unknown planet—Neptune. In reality, Leverrier predicted two planets—Neptune, beyond the orbit of Uranus, and Vulcan, situated between Mercury and the Sun.

We know today that the perturbations of the orbit of Mercury, which cannot be explained by Newtonian mechanics, are due to the effects of the general theory of relativity and not a hypothetical (and nonexistent) planet Vulcan.[1]

For a long time, celestial mechanics remained the paradigm of physical theory. The mathematical theory of potentials, the problems of the equilibrium of stars—at rest, rotating, binaries—these are the descendants of Newton's theory. In fact, the theoretical description of our contemporary "Friedmann," or "Hubble" expansion of the universe and the evolution of its structure could perfectly well have been obtained in the 18th or 19th century; there was only an absence of daring and of observational material—all the physical foundations were present. For more detail about this, see Ref. 2.

Electrostatics initially developed in the same direction. The universality of gravity and the differences of sign of the electrostatic interaction did not appear such serious details as to preclude similarity of these two theories.

An essentially new stage in physics began when the experiments of Faraday and the theory of Maxwell combined electricity and magnetism into the unified theory of the electromagnetic field. This theory also included free (far from the sources) electromagnetic waves.[2)] The electromagnetic theory of light was created. Long electromagnetic waves were deliberately generated and used for communication by Hertz, Popov, and Marconi. There soon developed the well-known contradiction between the Galileo transformation to a moving coordinate system (in Newtonian mechanics) on the one hand and the group properties of Maxwell's equations and Michelson's experiments on the other.

The special theory of relativity was created! It then became obvious that the theory of gravitation must also be relativistic.

Here, like Sheherazade in the Arabian Nights, we interrupt the story of the development of physics and consider how the related science of mathematics, or, more precisely, geometry, developed.

Lobachevskiĭ, Bolyai, and Gauss showed that a nontrivial non-Euclidean geometry is in principle possible, i.e., is not self-contradictory. The idea of the possibility of existence (and rotation and displacement) of an infinitely rigid body made these men limit themselves to spaces with homogeneous and isotropic curvature. The next step was taken by Riemann, who considered spaces of any number of dimensions with curvature that depends at a given point on the orientation of the considered infinitesimal surface and varies from point to point. He introduced the concept of the metric tensor (with two indices, $g_{\mu\nu}$) and the curvature tensor (with four indices, $R_{\alpha\beta\mu\nu}$).

Entirely naturally, there soon appeared general suggestions to the effect that the geometry of space must depend on the physical properties of the bodies (or fields) that fill a given space. The special theory of relativity developed at the beginning of the century required the unification of three-dimensional space with time into a single complex. In the simplest case, Minkowski's concept of a pseudo-Euclidean flat world was developed.

The culmination of this development was the creation by Einstein of the general theory of relativity. The idea of an influence of particles and fields on the curvature of space-time and also of the motion of particles and fields in this curved space-time proved to describe all properties of gravitation magnificently. From the historical point of view, a remark of Einstein's is very interesting. He pointed out that the greater part of his work (on the theory of Brownian motion and photons and the special theory of relativity) flowed in the stream of topical problems of his time. Within two or three years, this work would have been done by others if he has not done it himself. However, for the general theory of relativity he made an exception. In Einstein's opinion, general relativity is to such a degree nontrivial that its creation could have been delayed for another 50 years. It is noteworthy that it was precisely in the sixties that there appeared studies that led to general relativity in a regular manner without the illuminating and genial idea of the connection between the geometry of space-time, gravitation, and the equivalence principle that Einstein advanced at the beginning of the century. Thus, the estimate of 50 years was confirmed with satisfying accuracy.

The use of the adjective "magnificent" for the description of gravitation by means of general relativity can be expanded upon in various ways:

1. General relativity predicts astronomical effects such as the corrections to the trajectories of the planets, the change in the frequency of light, the bending of light rays, and the time delay in the propagation of radio signals. Direct observations confirm these predictions with ever increasing accuracy.

2. General relativity explains the most general properties of the universe as a whole; on this topic, see any modern review of cosmology. Black holes were predicted and are used today to explain phenomena in x-ray binary systems and in the nuclei of galaxies and quasars.

3. Gravitational waves were predicted and their emission is revealed by the motion of binary stars, including the binary pulsar.

4. The geometrical formulation of the theory of gravitation automatically includes the possibility of introducing locally inertial coordinates at any point of the space-time manifold and along the world line of any freely moving observer. In such a coordinate system, there is weightlessness, and the nonvanishing gravitational influence of the surrounding medium has the nature of a tidal deformation. In the theory, there is a local principle of equivalence between the gravitational field and accelerated motion of the coordinate system. Experiment confirms the equivalence principle.

5. The equations of gravitation impose certain restrictions on the motion of matter and the propagation of fields that fill space. In particular, for a point particle the equations of motion are themselves a consequence of the space-time geometry. In the general case, the constraints take the form of balance equations for the energy, momentum, and angular momentum with allowance for the effect of the gravitational forces.

Each of these results is an ornament of general relativity. Thus, general relativity is an entirely satisfactory theory of gravitation. In reality, there are no internal reasons and no discrepancies with experiment and observations that require changes in the theory.

Why then is the question of alternative theories of gravitation posed?! One may here distingush two tendencies, two formulations of the question.

The first tendency declares that general relativity is incorrect and unsatisfactory in the very field of classical (nonquantum) gravitation for which it was created. Within this direction, there are nuances. It has been claimed that there are definite numerical discrepancies between some particular observed quantity calculated by means of general relativity and experimental data. Such suggestions have usually been very short lived.

There are other objections, these relating essentially, not to the content, but to the form of general relativity. In ordinary field theories developed in flat space-time, for example, in electrodynamics, the energy-momentum tensor of the field is a locally well-defined quantity with appropriate transformation and conservation laws. In the standard "geometrical" formulation of general relativity, the localization of gravitational energy (like the other components of the energy-momentum pseudotensor) remains indefinite. The prefix *pseudo* means that the corresponding quantity does not behave like a tensor under arbitrary coordinate transformations. This unusual circumstance is sometimes regarded as an original sin of general relativity. However, as is shown, for example, in Ref. 4 and, in more detail, in Ref. 5, the properties of the pseudotensor do not prevent determination of the total energy and other quantities, albeit with certain reasonable restrictions. The appearance of the pseudotensor in the formalism of the theory cannot be taken as a reason for demanding the replacement of general relativity by a different theory. All the observational conclusions can, in principle, be obtained directly from the field equations without recourse to the pseudotensor.

Another answer to these objections (possibly, more convincing) is the fact that general relativity admits a completely equivalent "field" formulation in which there is a tensor (and not pseudotensor!) $t_{\mu\nu}$ for the energy and momentum of the gravitational field, this tensor satisfying ordinary conservation laws. Such a theory, an alternative to general relativity in form but not in content, can be formulated on the "background" of a flat Minkowski space with all rigor and the necessary attributes (for more details about the properties of the theory and the tensor $t_{\mu\nu}$, see Ref. 6 and later in this paper, particularly in Appendix 1).

There is however a quite different source that feeds the attempts to find alternative formulations and alternative theories; this is a tendency of a different kind. As already noted, general relativity is a nonquantum theory of gravitation. It is obvious that at the microscopic level it is necessary to construct a quantum theory of gravitation.

The quantum theory of weak gravitational fields can be constructed trivially with the small perturbations of the space-time metric being regarded as field variables on the background of the unperturbed Minkowski metric. Fundamental problems do not arise here, at least not in the linear approximation. The first calculations were already made by the Leningrad physicist M. P. Bronshteĭn as early as 1936. It is well known that gravitons are massless particles. The projection of the spin of the particle onto the direction of motion is ± 2, in contrast to the quanta of the electromagnetic waves, photons, with spin ± 1.

Difficulties arise on the transition to microscopic scales of the order of the Planck scales (10^{-33} cm, 10^{-43} sec) and energies of order 10^{19} GeV. At this level, the ordinary quantization scheme breaks down, since the fluctuations of the space-time metric are too large.

One possible approach to quantum gravity is to use the Feynman method of quantization. One considers all possible scenarios of four-dimensional geometry.[3)] For each scenario ("path") the action integral S is calculated, and then, finally, to determine the probability of transition from one state to another one adds the amplitudes, which are calculated as the exponentials $\exp(iS)$. This is the path, logically irreproachable but technically rather difficult, followed by Wheeler, DeWitt, Regge, and others. It is being applied most directly to cosmology by Hawking (see, for example, Ref. 7).

There is however a different approach to the construction of a quantum theory of gravitation. In this, one first introduces a fictitious[4)] Minkowski space-time (which we shall abbreviate to FM). In this FM, we construct equations for the field variables that characterize the gravitational field. Besides the free gravitational field, we consider other fields (electromagnetic, fermionic $e^{\pm}$, $\mu^{\pm}$, for example, etc.) and their interaction with the gravitational field.[5)]

Without any calculations, one can establish two extremely important properties of such a gravitational field:

1) it is a tensor field of second rank corresponding to the fact that it effectively describes a change of the metric, i.e., $g_{\alpha\beta}(x)$, the second-rank tensor in the expression $ds^2 = g_{\alpha\beta}\,dx^\alpha\,dx^\beta$ of the "old," or "geometrical," general relativity;

2) the field equations are nonlinear; the gravitational

field interacts with the energy and momentum of the other fields; hence, it must also interact with its own energy and momentum.

The task is to construct in the FM a theory that exactly, identically, gives all the observational conclusions given by the standard general relativity. The importance of this seemingly formal problem has increased sharply in the last decade or two in connection with the problem of supersymmetry and the construction of unified field theories.

Theories that unite bosonic and fermionic fields, fields with integer and half-integer spin, have been discovered. In particular, a field with spin 2 (tensor field) is connected by a definite supertransformation to a field with spin 3/2. Thus, the graviton field, whose existence one cannot doubt, is associated with a field of particles with spin 3/2—the so-called (hypothetical) gravitinos.

Thus, for the real existence of such a connection it is very desirable that gravitation (spin 2) be described in field form and not by the geometrical general relativity. The connection between the field form of the theory of gravitation and supersymmetry becomes even more intimate in the concrete study of the quantum theory. The point is that in the "pure" quantum theory of gravitation the nature of the field and its interaction with the energy-momentum density lead to the appearance of uneliminable infinities when the quantum corrections are calculated. It has been found that the introduction of the supersymmetric partner, the gravitino, eases the infinity problem.

One can say briefly that the field formulation of the theory of gravitation makes supersymmetry *possible* but at the same time supersymmetry is in a certain sense *necessary* for the quantum theory of gravitation.

Overall, the problem of quantum theory and phenomena at the scale of the Planck parameters is at the present time still far from a final and experimentally confirmed solution. There also exists a further direction of search in which one considers spaces with a larger number of dimensions ($D = 10$, 11, or 26). It is assumed that the "redundant" $D - 4$ dimensions are in some manner compactified. A long thin tube gives a good picture of this. The coordinate along the tube axis is a "real" coordinate. The tube radius (which depends on the coordinate) plays the part of a field variable. Thus, a multidimensional space makes it possible to describe several fields in a space of a smaller number of dimensions. The complete theory of "our," four-dimensional, space is the low-energy limit of a theory with a larger ($D > 4$) number of dimensions.

In this low-energy region a diversity of colors and flavors (not only the colors of the quarks!) does indeed reign. This picture cannot be reduced to the geometry of a four-dimensional continuum. In the first half of the century, attempts to construct a unified field theory did not succeed. But the theoreticians may now be on the correct path.

Currently of greatest promise is the theory of superstrings, which aims at a unified and complete description of all interactions. It begins roughly with the following propositions: "In a flat 10-dimensional space-time, with one time and nine spatial dimensions, there propagates a one-dimensional object, a string ...," etc. One may suppose that in this theory the classical gravitational field is "made up" of elementary excitations of a set of superstrings and is then combined with the flat metric of the "background" manifold into the metric of the curved observed world.

But let us return to a field theory of gravitation in a four-dimensional fictitious Minkowski space in the classical region, at large wavelengths, without quantum effects. What properties does such a theory possess if it is exactly equivalent to geometrical general relativity?

The most important point is that now the influence of gravitation on particles and on the electromagnetic field is described by interaction with the tensor gravitational field that we introduce. Thus, Einstein's equivalence principle is now not so obvious—it must still be discovered.

The frequencies of the oscillations of an oscillator or the frequency of light emitted by a hydrogen atom depend on the strength of the gravitational fields. The local velocity of light also depends on the strengths of the gravitational fields, which, in addition, bend the ray. The usual process of measuring the time between two events consists of measuring the number of repeated periodic events (swings of a pendulum, oscillations of an oscillator). In the field theory, such a method gives a value that depends not only on the elapsed time as measured in the FM but also on the gravitational field. The same applies to the measurement of distance between particles.

All material fields and particles interact with the tensor gravitational field in the same universal manner. There are no particles neutral with respect to the gravitational field whose world lines could "trace" for us the geometry of the Minkowski world. The object of pride—flat space-time—turns out to be unobservable. It is for this reason that we say that the flat Minkowski world on whose background the gravitational theory is developed is unphysical and fictitious.

In the presence of a gravitational field, the attempt to give the coordinate intervals Δx and Δt in the Minkowski world a direct observational meaning leads to a contradiction with experiment. (For more details about measurements in the field theory, see Appendix 2.)

Thus, analysis of the process of measurement in the field theory of gravitation necessarily leads to the concepts of "true," or "physical," durations and lengths, in contrast to the time and coordinates of the FM. The theory is geometrized.

In general relativity, the question of the topology of the manifold arises naturally together with the question of the Riemannian curvature. Usually, one constructs solutions with a single valued (and without identifications) time but with nontrivial topology of the spatial sections. An example is the closed Friedmann universe, which has a finite volume. One can say[12] that the field theory on the background of the FM gives a solution of this kind too. The Minkowski world obviously has an infinite volume in the section $t = \text{const}$, since each coordinate, for example, x, varies in the range $-\infty \leqslant x \leqslant +\infty$. However, the coordinates in the FM by themselves do not yet have any significance—they are lifeless. The transition to the physical volume is such that the physical observed volume of the universe is in the given case finite.

Let us summarize. We do not depart from the firm conviction that as a theory of the classical gravitational field general relativity is correct. At the least, there are neither theoretical nor experimental reasons for doubting this. However, this conviction does not rule out the possibility of constructing a completely adequate field theory of gravitation on the background of a fictitious Minkowski metric. Such a theory exists, is helpful in the classical region, and may, especially, be suitable for considering quantum processes and supersymmetry.[6)]

The lucidity of the equivalence principle is lost, but the possibility of describing a closed world and black holes remains.

Only time—the coming years or decades—will show whether all fundamental physics is to be transformed into geometry. Then, naturally, the theory of gravitation will remain geometrical. But even then the field approach will keep its value as a low-energy limit of the fundamental theory, and the field theory of gravitation with the concepts of the Newtonian potential, gravimagnetic field, etc., will be helpful for astronomy and other applications.

In working on this paper, we had in mind the shining example of E. M. Lifshitz, who was totally dedicated to science. To his memory we dedicate this paper.

APPENDIX 1.

Field formulation of general relativity and the energy-momentum tensor of the gravitational field

In the geometrical formulation of general relativity, the energy characteristic of the gravitational field is the energy-momentum pseudotensor.

This means that in a given process, i.e., in an unambiguously specified 4-geometry determined by the metric tensor $g_{\alpha\beta}(x)$, it is not possible to define in a sensible manner an energy-momentum tensor of this geometry. The quantity that arises naturally is not a tensor but a pseudotensor. It is changed if we describe a given process, i.e., a given 4-geometry, by means of a different set of coordinates. The most popular pseudotensor—the Landau-Lifshitz pseudotensor[4]—contains the metric $g_{\alpha\beta}$ and its first derivatives. All components of the pseudotensor vanish in a locally inertial coordinate system, along the world line of a freely falling observer. As Landau and Lifshitz themselves note,[4] even in flat space-time it is possible to obtain nonvanishing components of the pseudotensor by appropriate choice of curvilinear coordinates. In other words, the components of the pseudotensor do not possess a tensor transformation law with respect to arbitrary coordinate transformations, but do behave as the components of a tensor with respect to a smaller class of transformations, to which the Lorentz transformations belong.

This property is a fundamental difference of the gravitational field, identified with the space-time metric $g_{\alpha\beta}$, from, for example, the electromagnetic field. In the latter case, the density of electromagnetic energy is the component of a tensor. Under Lorentz transformations, and indeed under arbitrary coordinate transformations, it transforms in accordance with a linear law, being mixed with the other components of the energy-momentum tensor—the Poynting vector and the tension tensor. But no coordinate transformation can make the entire electromagnetic energy-momentum tensor vanish, in contrast to the gravitational pseudotensor. The true uneliminable tensor in general relativity is the curvature $R_{\alpha\beta\mu\nu}$. This corresponds to the fact that even in the state of weightlessness tidal forces cannot be eliminated.

As already noted above, in the main text of the paper, there are physicists who regard this situation as unsatisfactory. They include the authors of the studies of Ref. 15. In the opinion of the authors of the present paper, the appearance of the pseudotensor in general relativity does not by itself spoil the theory. Indeed, this is not the first case of the use in a theory of quantities that cannot be defined without arbitrariness. In the Newtonian theory of gravitation, we use a potential, to which a constant or a function of the time can be added. In the theory of electromagnetism, we use not only the measured fields **E** and **H** but also the vector potential A_α, although we know that we may add to A_α the gradient $\nabla_\alpha \varphi$ of an arbitrary scalar function $\varphi(x,y,z,t)$. The appearance of the pseudotensor does not give rise to ambiguity in the observational predictions of general relativity, and therefore we can readily accept it.

Nevertheless, as a psychological phenomenon, a negative attitude to the pseudotensor exists and is one of the reasons for the search for an alternative field theory of gravitation in flat space-time. And, indeed, in such a theory the energy-momentum tensor of the tensor gravitational field, calculated in accordance with the field formulas, is a true tensor $t_{\mu\nu}$; the prefix *pseudo* is deleted. It would seem that the aim has been achieved and that this by itself justifies the field theory of gravitation. Moreover, one can also get the impression of a fundamental departure from general relativity, leading to new observational consequences. However, "...chase nature out through the door and it will go in through the window... ." The tensor $t_{\mu\nu}$ is noninvariant with respect to gauge transformations, and in this sense the numerical value of $t_{\mu\nu}$ remains nonunique. However, none of the experimentally verified conclusions depend on this, and they are the same as in standard general relativity. To analyze the situation further, we require some details (for still further details, we refer the reader to Ref. 6).

The point of departure of the field theory of gravitation is that in the flat world there is a tensor gravitational field $h^{\mu\nu}$ and other (nongravitational) fields. The coordinate system in the flat space can be taken to be Lorentzian, and then the line element takes the form

$$d\sigma^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu = c^2\, dt^2 - dx^2 - dy^2 - dz^2. \quad (1)$$

In arbitrary curvilinear coordinates, the components of the metric are not so simple as in the expression (1) (in the general case, we shall write $\gamma_{\mu\nu}$ instead of $\eta_{\mu\nu}$), but the curvature tensor constructed from $\gamma_{\mu\nu}$ is, of course, identically equal to zero. The Lagrangian of the theory consists of a gravitational part L^g and a material part L^m. The universality of the coupling of the material fields to gravity appears in the fact that the field $h^{\mu\nu}$ occurs in L^m only in the form of the sum

$$(-\gamma)^{1/2}(h^{\mu\nu}+\gamma^{\mu\nu})\equiv(-g)^{1/2}g^{\mu\nu}. \tag{2}$$

Even without penetrating into the details of the varied formulations of the equivalence principle, one can guess that it is contained in the theory here (cf. Ref. 16).

The variational principle, applied to $L^g + L^m$, leads to the gravitational field equations

$$\frac{1}{2}(h_{\mu\nu;\alpha}{}^{;\alpha}+\gamma_{\mu\nu}h^{\alpha\beta}{}_{;\alpha;\beta}-h^{\alpha}_{\nu;\mu;\alpha}-h^{\alpha}_{\mu;\nu;\alpha})$$
$$=\frac{8\pi G}{c^4}(t_{\mu\nu}+\tau_{\mu\nu}), \tag{3}$$

where the covariant differentiation and raising and lowering of the indices are done by means of $\gamma_{\mu\nu}$. On the right-hand side of Eqs. (3) we have $t_{\mu\nu}$, the energy-momentum tensor of the gravitational field calculated by varying L^g with respect to $\gamma_{\mu\nu}$. We can calculate $\tau_{\mu\nu}$ from L^m similarly. The field equations (3) contain the differential conservation laws

$$(t_{\mu\nu}+\tau_{\mu\nu})^{;\nu}=0$$

with the integral conservation laws to which they lead; these reflect the fact that the flat world admits a 10-parameter group of motions (the Poincaré group).

We point out immediately that this theory is completely equivalent to the ordinary "geometrical" general relativity. Using the connection (2) and regarding $g_{\mu\nu}(x)$ as the components of the metric of a curved space-time, we return to the geometrical formulation of general relativity, i.e., to the Hilbert action and Einstein's equations. Equations (3) are transformed exactly into the Einstein equations

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{8\pi G}{c^4}T_{\mu\nu}.$$

The tensor $t_{\mu\nu}$ itself contains $\gamma_{\mu\nu}$ in an essential manner and does not reduce to a function of only the $g_{\mu\nu}$.

The field formulation is given in covariant form; it admits arbitrary coordinate transformations. But there is one further symmetry, which may be called a gauge, or internal, symmetry. In the same coordinate system, one can change $h^{\mu\nu}$ in accordance with a definite law,

$$h^{\mu\nu}\to\bar{h}^{\mu\nu}=h^{\mu\nu}+\Delta(h^{\mu\nu},\ \gamma^{\mu\nu},\ \xi^{\alpha}), \tag{4}$$

where the additional terms Δ (not necessarily infinitesimally small—in the general case they are finite) depend on the arbitrary functions ξ^α and their derivatives. The nongravitational dynamical fields change in accordance with an analogous law. The equations of the dynamical fields are invariant with respect to such a substitution, i.e., if $h^{\mu\nu}$ is a solution, then so is $\bar{h}^{\mu\nu}$. The transformation (4) recalls the gauge transformation in electrodynamics. Combining in accordance with the rule (2) the same $\gamma^{\mu\nu}$ with $h^{\mu\nu}$ or $\bar{h}^{\mu\nu}$, we obtain "different" $g^{\mu\nu}$, which, however, can be exactly transformed into each other by a coordinate transformation and, therefore, describe the same 4-geometry. The actual form of this coordinate transformation is determined by the functions $\xi^\alpha(x)$ and all their derivatives, but we shall not use it here. Thus, the gauge symmetry of the field theory is intimately related to the coordinate symmetry of the geometrical theory.

The tensor $t_{\mu\nu}$ is not gauge invariant. The field theory in the FM saves the covariance of $t_{\mu\nu}$ with respect to coordinate transformations but comes up against noninvariance with respect to the gauge transformations. The construction of an absolutely irreproachable energy-momentum tensor of the gravitational field is illusory.

One could advance the idea of making $t_{\mu\nu}$ unambiguous by "fixing" the gauge. One of the convenient choices of $\xi^\alpha(x)$ is the one that achieves the conditions

$$h^{\mu\nu}{}_{;\nu}=0. \tag{5}$$

For the functions $g^{\mu\nu}(x)$, these conditions take by virtue of (2) the form[7]

$$[(-g)^{1/2}\ g^{\mu\nu}]_{;\nu}=0, \tag{6}$$

and if the functions $g^{\mu\nu}(x)$ are considered against the background of the flat world in the Lorentz coordinates (1), then Eqs. (6) reduce to the harmonic conditions

$$[(-g)^{1/2}\ g^{\mu\nu}]_{,\nu}=0, \tag{6'}$$

which were so fruitfully used by Fock.[17]

It should be emphasized that Eqs. (5) still leave an arbitrariness in $t_{\mu\nu}$. There are transformations (4) that do not violate (5) but nevertheless change $t_{\mu\nu}$. In the same way, the harmonic coordinates (6′) and transitions between them do not yet transform the gravitational pseudotensor into a tensor. But this is not really the point.

It is clear from what was said above that the prescription for fixing the gauge in the description of $h^{\mu\nu}(x)$ has the same value as the prescription for using certain coordinates rather than others for the description of $g^{\mu\nu}(x)$. The addition of the conditions (5) (or other such conditions) to Eqs. (3) does not change the physical content of the theory, does not preclude any solutions of Einstein's equations for $g^{\mu\nu}(x)$, and does not make any solutions preferable to any others. *Any* solution of Einstein's equations (not necessarily expressed in harmonic coordinates) satisfies the conditions (6) if the functions $\gamma_{\mu\nu}(x)$ are written down appropriately, namely, we must have

$$\gamma_{\mu\nu}=\eta_{\alpha\beta}f^{\alpha}{}_{,\mu}f^{\beta}{}_{,\nu},$$

where $f^\alpha(x)$ are found as solutions of the equations

$$[f^{\alpha}{}_{,\mu}(-g)^{1/2}g^{\mu\nu}]_{,\nu}=0.$$

Finally, the presence or absence of the conditions (5) or (6) does not change any of the experimentally verified conclusions. The question of the gauge transformations and the observable quantities arises and can be solved already in the weak-field approximation. In Appendix 2, we consider the question of measurements in the field theory in more detail.

APPENDIX 2.

Measurements in the field theory

As we have already said, in the field theory all material fields interact with gravity in a universal manner. The equations of motion can be interpreted either as equations in a curved space-time with metric $g^{\mu\nu}$ (geometrical formulation) or as equations in a flat world in the presence of a gravitational field $h^{\mu\nu}$ (field formulation). We introduce Lorentzian coordinates x, y, z, t in the FM and consider, for example, the equations of the electromagnetic field in the approximation of geometrical optics. Gravitational forces constructed from $h^{\mu\nu}(x)$ influence the motion of an idealized photon. Such is the field interpretation of the equations of a null geodesic in the metric $g^{\mu\nu}(x)$. The velocity of light in the coordinates of the FM, defined as $\Delta x/\Delta t$, will depend on the gravitational potentials and vary from point to point. But how does this velocity change at a given place and how can one show that it really is as the theory predicts? For it is necessary to bear in mind that every oscillator, as a model of a clock, and every rod, as a model of a linear scale, are also subject to the action of the gravitational field. (As will be explained below, the behavior of scales is determined by the solution of quantum problems, and it is necessary to take into account $\hbar$.) Thus, suppose that in a weak gravitational field with Newtonian potential φ the velocity of light $\Delta x/\Delta t$ is

$$\frac{\Delta x}{\Delta t}=c\left(1+\frac{2\varphi}{c^2}\right).$$

But any clock at this position measures $\Delta\tau = \Delta t[1 + (\varphi/c^2)]$, and any rod will have length $\Delta l = \Delta x[1 - (\varphi/c^2)]$. We thus find that the velocity of light determined by real standards, i.e., $\Delta l/\Delta\tau$, and not by means of the unobservable coordinates x, y, z, t, is always equal to c (for more details, see, for example, Ref. 19). All this confirms the assertion made above concerning the unobservability of the FM.

Even the self-propagating gravitational field, i.e., gravitational waves, cannot help us to make the geometry of the flat world observable. Let us consider Eqs. (3). The main element of the left-hand side of these equations is the ordinary d'Alembertian applied to $h_{\mu\nu}$, i.e., a combination of the form

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}\right)h_{\mu\nu}.$$

The characteristics of the d'Alembertian are isotropic (null) geodesics of the Minkowski metric. In the linear approximation, the rays of gravitational waves are thus images of straight lines of the flat world. But the right-hand side of Eqs. (3) (the tensor $t_{\mu\nu}$) also contains second derivatives in the form of terms of the type $h^{\alpha\beta}h_{\mu\nu;\alpha;\beta}$. Therefore, in a nonlinear situation, for example, in the gravitational field of a body, the gravitational waves, like the electromagnetic waves and, in general, material fields described by L^{m}, are also deflected and undergo a change of velocity relative to the FM.

It should be pointed out that the velocity of light occurs already in the classical, i.e., nonquantum theory. The situation with regard to the units of length and duration separately is more complicated. In order to determine these units, it is necessary to solve quantum problems in the presence of the tensor gravitational field; it is necessary to know how the masses of the elementary particles (electrons and protons) used in laboratory standards of frequency and length[19] are transformed in this case. This can be seen formally from the fact that a quantity with the dimensions of length or time can be constructed only by using $\hbar$ and m. This difficult task is usually avoided by showing the equivalence of the field theory and the geometrical theory.

If one gives the coordinates of the FM the significance of directly observable quantities, this leads to imaginary differences in the predictions of the geometrical and field theories. We consider the Schwarzschild solution in ordinary and harmonic coordinates. The line element in the first case is

$$ds^2=\left(1-\frac{2Gm}{r}\right)dt^2-\left(1-\frac{2Gm}{r}\right)^{-1}dr^2-r^2(d\theta^2+\sin^2\theta\, d\varphi^2) \qquad (7)$$

and in the second

$$ds^2=\left(\frac{r-Gm}{r+Gm}\right)dt^2-\left(\frac{r+Gm}{r-Gm}\right)dr^2-(r+Gm)^2(d\theta^2+\sin^2\theta\, d\varphi^2). \qquad (8)$$

In the field approach, the solutions (7) and (8) give different $h^{\mu\nu}$ connected by a gauge transformation (4), the solution (8) satisfying the subsidiary conditions (6).[8)] (When (8) is substituted in (6′), it is necessary to take into account the connection between r, θ, φ and x, y, z.)

Writing down the equations for propagation of light, one can find the time t for propagation of a signal from one point to another and back again. This problem gives a model description of radar location of Mercury from the Earth in the gravitational field of the Sun. Putting the Earth at $r = r_e$ and Mercury at $r = r_p$ and using (7) or (8), we can obtain two formulas for the delay time of a radio signal, these differing both analytically and numerically. It would seem that the predictions of general relativity are ambiguous, while the field theory based on (3) and (5) gives a unique prediction confirmed by experiment (see Ref. 15). In fact, of course, there is no ambiguity in the predictions of general relativity and no contradictions with it. The experimentally verifiable predictions that follow from (7) and (8) are identical.[20] The difference in the formulas is entirely explained by the fact that identical numerical values of r in (7) or (8) correspond to somewhat different physical differences. Specifically, the connection between the coordinates r in (7) and (8) is such that

$$r_7 = r_8 + Gm,$$

where r_7 corresponds to (7) and r_8 to (8). In other words, the same numerical value of r in the two cases corresponds to different circular orbits. Planets in these orbits have different periods of revolution around the Sun, and these periods can be directly observed from the Earth and are unambiguously determined. Of course, the transition in (7) and (8) to identical, operationally defined quantities gives identical observational predictions for the delay time, and these have been confirmed experimentally.[21, 9)]

The incorrectness of giving the meaning of observable quantities to the coordinate intervals Δx and Δt in FM can be seen particularly clearly in the case of the effect of the change of the frequency of light in a gravitational field. Let us take the solution (8). We regard the coordinates t, r, θ, φ as inertial coordinates in the Minkowski world. At fixed r, θ, φ we place a source of radiation (for example, an atom) and at other r, θ, φ a detector. Suppose the atom emits a wave train containing N oscillations and lasting for a time Δt. The detector also observes N oscillations. The gravitational field (8) being static, the theory asserts that at the point of reception the oscillations occupy exactly the same time interval Δt. If we were to give an observational significance to the intervals Δt, we should arrive at the conclusion that the frequencies $\nu = N/\Delta t$ of the signal at the point of emission and the point of reception are the same. As is well known, this contradicts the experiments in which we observe a change in the frequency of a signal in a gravitational field. We do not know of any fields or particles that do not interact with the gravitational field and we do not assume their existence. In the field theory the experimental fact of the frequency shift is explained by the fact that any oscillator that measures the time is subject to the action of the gravitational field and oscillates differently at the point of emission and the point of reception. In other words, the explanation of the effect of the change in the frequency of the signal in the field theory amounts to the recognition that Δt is not observable.

Summarizing, we assert once more that the variants of the field theory of gravitation on the background of a flat world considered here are in fact a field formulation of general relativity. The attempt to interpret the metrical relationships of the flat world as observables leads to a contradiction with experiment.

[1)]We give another example of ambiguous interpretations of sayings of the classics. Newton's words "If I have seen further it is by standing on ye sholders of giants" (Newton's letter to Hooke, 15th Febr., 1676) are given as an example of modesty and respect for predecessors. In reality, this phrase was said in the polemic with Hooke and should be seen rather as a gibe—Newton was tall and large while Hooke was almost a dwarf. The desire to ascribe all the high moral principles without exception to the great people of the past is also present in us of this age as the desire to have a well-defined unambiguous energy density in a theory; about this, see below.

[2)]In the article of Ref. 3, a remarkable fact is recorded: In 1832, Faraday deposited a letter with the inscription "Open after 100 years." In it, he advanced the hypothesis of an electromagnetic nature of light. It became part of science 40 years after the letter was written.

[3)]The 4-geometry contains identically the law of variation of the 3-geometry with time; it is also necessary to take into account the nonuniqueness in the method of identifying "time" in a given 4-geometry.

[4)]The reason why we call it fictitious will be given a little later.

[5)]The idea of constructing a relativistic theory of gravitation on the basis of a flat world has a rich history. A list of some early studies in this direction is given in Ref. 8. We mention in particular Ref. 9, which discusses the formal structure of the theory, and Ref. 10, in which the connection with physical measurements is established. An analysis of this whole direction, corresponding to the state at the time of writing, is contained in the book of Ref. 11. The same idea is used in the studies of Ref. 15. The value of the field approach for quantum gravity is also correctly noted there. However, we can in no way agree with some of the assertions of Ref. 15, especially the criticism of general relativity, about which we shall have more to say below.

[6)]Although the field formulation is not mandatory, it is entirely meaningful and assists in the analysis in concrete investigations. As an example, we may mention the problem of radiative deceleration of gravitating bodies and the derivation of the formula for the loss of energy of a radiating system[13] and also the calculation of the quantum conformal anomaly for gravitons.[14]

[7)]The relativistic theory of gravitation developed in Ref. 15 contains exactly the same Lagrangian L^g and L^m as are discussed here and in Ref. 6 and lead to the gravitational field equations (3). However, in Ref. 15 one necessarily requires not only Eqs. (3) but also fulfillment of the conditions (5) or the equivalent conditions (6), which are raised to the status of universal gravitational field equations.

[8)]The functions $g^{\mu\nu}$ in (7) also satisfy the generally covariant equations (6) if one makes a "rearithmetization" of the spatial coordinates in the flat background world and writes its metric in the form $d\sigma^2 = c^2dt^2 - dr^2 - (r - Gm)^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$.

[9)]There are also no ambiguities obtained in observable quantities in the more general case when the coefficient of the angular part of ds^2 is written in the form $[r + (\lambda + 1)Gm]^2$, where λ is an arbitrary parameter.[15] The cases considered above correspond to $\lambda = -1$ and $\lambda = 0$.

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