Self-action of wave packets in a nonlinear medium and femtosecond laser pulse generation

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The review describes the present status of the physics of optical pulse self-action phenomenon. A brief analysis of linear propagation of ultrashort laser pulses in dispersive media and optical elements is given. Results on self-phase-modulation and on profile shock waves in media with various group velocity dispersion are presented. The problems of optical pulse compression and the possibilities to control their shape are discussed in detail. Particular attention is paid to the physical aspects of formation and interaction of optical solitons. Stochastic problems of temporal self-action are also discussed. In conclusion the promising directions in nonlinear optics of femtosecond laser pulses are discussed.

TABLE OF CONTENTS

INTRODUCTION

One of the most outstanding achievements of laser physics in recent years is, undoubtedly, the development of methods for generation and formation of optical pulses with pulse lengths $\sim 10^{-14}$ sec, i.e., pulses that contain only a few periods of optical oscillations under their envelopes. The consequences of the transition in optics to such femtosecond time scales are justly compared to the revolutionary changes in the spatial resolution of optical devices that followed the invention of the microscope.

Penetration into the area of the femtosecond time scale is the result of intense work of physicist and engineers that has been carried out during the last fifteen years. An impor-

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tant component of this progress has become the wide use of methods of nonlinear optics. The possibility of using fast nonresonance nonlinear response of condensed media for strong compression ("time focusing") of laser pulses was mentioned already in the papers published at the end of the 1960s. These ideas could be completely realized, however, only recently after the development of fiber-optic waveguides with very low losses.

Effective methods of optical pulse compression in fiber-optic waveguides are based on the *self-action effects of light* that are caused by dependence of the refractive coefficient of a medium on the intensity of a light wave. An obvious result of self-action of a laser pulse propagating in a nonlinear medium is the temporal *phase self modulation* (its velocity can be made very large). With the help of dispersive optical elements, the fast phase modulation can be transformed into amplitude modulation—this is the main idea of "time focusing," which is completely analogous in essence to the idea which is the basis of the usual spatial focusing of a light beam (a usual lens creates "fast" spatial phase modulation that changes into amplitude modulation during the propagation of a wave in free space). This compression technique is now successfully used in femtosecond pulse generators in the visible, infrared and near ultraviolet regions. A key aspect that, to a large extent, determines the effectiveness and reliability of such devices, is the stability of the transverse structure of the field in single-mode fiber optic waveguides up to intensities reaching 10^{10}–10^{11} W/cm^2. In other words, in fiber-optic waveguides the effects of spatial self-focusing and self-defocusing are practically absent even for high fields, and *temporal* self-action can be observed in pure form, without competition with other nonlinear effects. This fact gave new impetus to the theoretical and, particularly, to experimental investigations; last years have brought many exciting achievements in that direction. Undoubtedly, one of the most prominent places among them should be given to the work on the formation, propagation, interaction and amplification of optical solitons. Many new physical results were obtained in this area; at the same time there are more and more indications that optical solitons can be of direct interest for the methods of information processing and for optical communication systems.

The problems related to the propagation of high-power femtosecond pulses (with pulse lengths down to $\sim 10^{-14}$ sec, and intensities up to 10^{12}–10^{14} W/cm^2) have also led to new fundamental questions in the area of nonlinear optics; there are real situations when the already familiar quasioptical approximation is not valid any more, but the local nonlinear response of a medium can no longer be considered weak.

The goal of this review is to describe the present status of the physics of the self-action phenomena of optical pulses and to discuss new physical problems and possible future applications; in some sense this review can be considered as a continuation of the review published in Uspekhi in 1967.[1]

Section 1 "Linear Fourier-optics of wave packets" precedes the presentation of the main body of the material. It contains a summary of results on linear propagation of optical pulses in dispersive media and elements of optical systems. The analogy with Fourier-optics of wave beams has been found to be rather useful in linear Fourier-optics of wave packets; the spatial-temporal analogy has an obvious heuristic value also in nonlinear optics. How is it possible to get rid of the diffraction effects that are unavoidably present in optical experiments? The radical solution of the problem is provided by the use of weakly directing optical waveguides;[2] for this reason Sec. 1 is concluded with information on the dispersive properties of single-mode optical waveguides.

Section 2 gives a summary of theoretical and experimental results on the main effects of self-action for plane wave packets in media with a nonlinearity cubic in the field that leads to a broadening of the spectrum.

For strong fields it is possible to observe, together with the effect of self-phase-modulation (SPM) of a packet of constant shape, a dependence of group velocity on intensity. The broadening mechanism of a pulse spectrum in a medium with cubic nonlinearity is rather complicated, and only in the last few years it has become possible to separate the contribution of different effects.

Section 3 discusses mainly applied problems–the compression of ultrashort pulses and control over their shape.[3] At present many research groups perform experiments with fiber-optical methods of compression. For this reason we present the results of a detailed numerical experiment which allows one to optimize the compression, to determine the limits of its possibilities and the prospects for producing the shortest pulses possible. The same section also gives a rather detailed summary of experimental data.

The problems of physics and technology of optical solitons[4–5] are dealt with in Sec. 4; the emphasis is on problems directly related to experiments. The section discusses not only the formation of single solitons in a passive medium, but also the interaction of solitons, their amplification and soliton effects in generators (soliton lasers).

Finally, Sec. 5 puts an emphasis on new problems of the physics of temporal self-action phenomena. Special attention is paid to statistical problems in soliton theory, i.e., to problems directly related to the self-action of noise pulses.[6]

1. LINEAR FOURIER-OPTICS OF WAVE PACKETS

A group of problems related to the propagation of wave packets in linear dispersive media exists already in classical linear optics.[7–9] The greatest interest has been evoked by the dispersive spreading of a packet[10] and the formation of "precursors" during the interaction of a short pulse with a dispersive medium.[11] Until the late 1960s, however, experimental optics could not provide any contribution to the investigation of the aforementioned problems. Pulsed optical sources (including the first pulsed lasers) were in essence generators of relatively long bursts of optical noise. It is natural that under these conditions there could not be any possibility of studying the transformation of the envelope and phase during the propagation process and, even less, of any control over these parameters in time.

In contrast, the experimental technique for the formation and modification of *optical beams* has been progressing even before the development of lasers. The detailed development of Fourier-optics of wave beams became the theoretical basis of these methods.[12] Only in recent years have the techniques of optical pulse formation been augmented by the addition of effective methods of control over the envelope and phase of optical oscillations in the picosecond and femtosecond time scale. The principle of these methods is based on using a "non-resonant," nonlinear, "low-inertia" response of condensed media (see Refs. 13, 14). Progress was also made in the technique of measuring the profile and phase of short laser pulses.

Dispersive effects, similar to wave beam diffraction, can be used as a basis of various methods of compression and shape modification of such pulses. For this reason the Fourier-optics of wave packets propagating in a dispersive medium has recently undergone rapid development.[15,16] In this section we give a summary of results from the linear Fourier-optics of light pulses which are related primarily to the compression and formation of optical pulses of a specified shape.

1.1. Models of optical pulses

We write the electrical field intensity in the form

$$\mathbf{E}(t, z) = \mathbf{e}\frac{1}{2} A(t, z) \exp[i(\omega_0 t - k_0 z)] + \text{c.c.}, \tag{1.1}$$

where $\mathbf{e}$ is a polarization unit vector, the direction of the z-axis coincides with the direction of the wave vector, $k_0 = (\omega_0/c)n_0(\omega_0)$, ω_0 is the average spectral frequency of an initial wave, $n_0(\omega_0)$ is the linear refractive coefficient of a medium. The complex amplitude $A(t,z)$ is a slowly varying function of its arguments on a scale of the period of optical oscillations and the wavelength. The amplitude $A(t,z)$ is assumed to be given at the input $z = 0$ of a linear or nonlinear medium, i.e., $A(t,z=0) = A_0(t)$ is a known function.

We shall consider here the models of optical pulses that are most often used in laser physics.

1.1.1. Transform-limited pulses

A pulse the length τ_0 of which is completely determined by the inverse value of the width $\Delta\omega_0$ of its spectrum is usually called a *transform-limited* pulse. In this case there is no phase or frequency modulation of the pulse, i.e., the amplitude $A_0(t)$ can be assumed to be real; we denote it by $\rho_0(t)$. The phase is $\varphi_0(t) = \arg A_0(t) = 0$, $A_0(t) = \rho_0(t) \times \exp(i\varphi_0(t))$.

For *transform-limited* pulses the relationship

$$\tau_0 \Delta\omega_0 = K, \tag{1.2}$$

holds, where K is a constant ($K \approx 1$) that depends of the shape of the pulse envelope and the level at which τ_0 and $\Delta\omega_0$ are measured. Two types of pulses are considered most often:

$$\rho_0(t) = \rho_0 \operatorname{sech}\frac{t}{\tau_0}, \tag{1.3}$$

$$\rho_0(t) = \rho_0 \exp\left(-\frac{t^2}{2\tau_0^2}\right). \tag{1.4}$$

In the latter case the half-width of the pulse length τ_0 at the e^{-1} level from the maximum intensity is related to the pulse length at the half-width $\tau_{1/2}$ by the relationship $\tau_0 = (4 \ln 2)^{-1/2}\tau_{1/2}$. Having in mind this relation, we will in the future call the quantity τ_0 simply the pulse length.

For a Gaussian pulse (1.4)

$$\tau_0 \Delta\omega_0 = 2, \quad \tau_{1/2}\Delta\omega_{1/2} = 4 \ln 2, \tag{1.5}$$

where $\Delta\omega_0$ and $\Delta\omega_{1/2}$ are the spectral widths at the levels corresponding to the measurements of the pulse length.

1.1.2. Phase-modulated (PM) pulses

The phase $\varphi_0(t)$ of a pulse can be a complicated deterministic or a random function. In the following description a special role will be played by pulses for which $\varphi_0(t) = -(1/2)\alpha_0 t^2$, where α_0 is the frequency change rate. In this case, for a pulse with a Gaussian profile (1.4), the spectral width is

$$\begin{aligned} \Delta\omega_0^P &= \Delta\omega_0 [1 + (\alpha_0 \tau_0^2)^2]^{1/2}, \\ \tau_0 \Delta\omega_0^P &= 2[1 + (\alpha_0 \tau_0^2)^2]^{1/2}. \end{aligned} \tag{1.6}$$

Therefore, for PM pulses $\tau_0 \Delta\omega_0 \omega_0^P \gg 1$ when $\alpha_0 \tau_0^2 \gg 1$.

1.1.3. Pulses containing noise

Nonlaser sources of light and, in some cases, multimode lasers generate pulses with a complex amplitude of the form

$$A_0(t) = F(t)\, \xi(t), \tag{1.7}$$

where $F(t)$ is a regular function, and $\xi(t)$ is a random process; in general, $\xi(t)$ is a complex function. An example of the process (1.7) can be the "flashes" of optical noise.

In the following subsections of this section we discuss the propagation characteristics of optical pulses, described above, in linear dispersive media.

1.2. Propagation of short optical pulses in homogeneous dispersive media. Methods of description

The propagation of a plane wave packet in a linear isotropic dispersive medium is described by the wave equation

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} = 0, \tag{1.8a}$$

where

$$D(t, z) = \int_0^\infty \varepsilon_0(t')\, E(t - t', z)\, dt' \tag{1.8b}$$

is a linear electric displacement. The temporal dependence $\varepsilon_0(t) = n_0^2(t)$ is related to the frequency dependence of the linear dielectric permittivity.

When solving specific problems, it is common to use, instead of the system of integro-differential equations (1.8), the differential equations for complex amplitudes, using the slow change of the system during the period T of oscillations; practically this assumption is valid up to pulse lengths $\tau_0/T \approx 10$–100.

Substituting (1.1) in (1.8b) and taking into consideration the slow change of the complex amplitude $A(t - t', z)$

with time, we expand A in a series in t'. As a result we obtain

$$D(t, z)=\left[\varepsilon_0(\omega_0)A+\sum_{m=1}^{\infty}\frac{(-i)^m}{m!}\left(\frac{\partial^m\varepsilon_0(\omega)}{\partial\omega^m}\right)_{\omega_0}\frac{\partial^m A}{\partial t^m}\right] \times \exp[i(\omega_0 t-k_0 z)]+\text{c.c.} \quad (1.9)$$

Neglecting the derivatives $\partial^m\varepsilon_0(\omega)/\partial\omega^m$ in (1.9) is equivalent to ignoring the dispersion of a medium (zero-order approximation). In first-order approximation of dispersion theory we retain only the derivative $\partial\varepsilon_0(\omega)/\partial\omega$, neglecting the derivatives of higher orders. The derivatives $\partial^2\varepsilon_0(\omega)/\partial\omega^2$ are taken into account in the second-order approximation, etc. In other words, in this classification the order of derivatives of the dielectric permittivity $\varepsilon_0(\omega)$ determines the order of the approximation.

Substituting (1.1) and (1.9) into (1.8), we obtain the following equation for the amplitude $A(t,z)$:

$$\left[\frac{\partial}{\partial z}+\frac{1}{u}\frac{\partial}{\partial t}-i\cdot\frac{1}{2}k_2\frac{\partial^2}{\partial t^2}+i\frac{1}{2k_0}\left(\frac{\partial^2}{\partial z^2}-\frac{1}{u^2}\frac{\partial^2}{\partial t^2}\right)\right]A(t, z) -\frac{1}{2k_0}\sum_{m=3}^{\infty}\frac{(-i)^{m+1}}{m!}\left(\frac{\partial^m k^2}{\partial\omega^m}\right)_{\omega_0}\frac{\partial^m A}{\partial t^m}=0, \quad (1.10)$$

where u is the group velocity, and the parameter k_2 characterizes the dispersion of the group velocity

$$u=\left[\left(\frac{\partial k}{\partial\omega}\right)_{\omega_0}\right]^{-1}=c\left[n-\lambda_0\left(\frac{\partial n}{\partial\lambda}\right)_{\lambda_0}\right]^{-1},$$

$$k_2=\left(\frac{\partial^2 k}{\partial\omega^2}\right)_{\omega_0}=\frac{\lambda_0^3}{2\pi c^2}\left(\frac{\partial^2 n}{\partial\lambda^2}\right)_{\lambda_0}.$$

Equation (1.10) is exact in the sense that it takes into account the dispersive properties of a linear medium. At the same time, for many cases of propagation of ultrashort picosecond and femtosecond pulses the second-order approximation of dispersion theory is adequate. In that approximation the equation that is derived from (1.10) by neglecting the terms under the sum can be simplified further. By using a moving coordinate system ($z=z, \eta=t-z/u$), it is easy to show,[6,17,22] that the operator in parentheses gives values of a higher order of smallness than other derivatives.

As a result we obtain the equation

$$\frac{\partial A}{\partial z}-i\cdot\frac{1}{2}k_2\frac{\partial^2 A}{\partial\eta^2}=0, \quad (1.11)$$

that has the solution

$$A(\eta, z)=(i2\pi k_2 z)^{-1/2}\int_{-\infty}^{+\infty}A_0(t)\exp\left[\frac{i(\eta-t)^2}{2k_2 z}\right]dt. \quad (1.12)$$

Equation (1.11) is analogous to the parabolic equation used to describe the propagation of light beams in a quasioptical approximation. We note that for the first time the parabolic equation has been derived by M. A. Leontovich in 1944 during the investigation of propagation of radiowaves.

Within the framework of the described approximations, one can see a rather detailed spatial-temporal analogy between the behavior of wave packets and wave beams.[17,18,22] For this reason one often talks about the quasioptical approximation in the description of wave packets.

The value $k_2>0$ corresponds to the normal dispersion of group velocity, and $k_2<0$ corresponds to anomalous dispersion.

We emphasize once again that the parabolic equation (1.11) corresponds to the approximation of the dispersive properties of a medium by the following expression

$$k(\omega)=k(\omega_0)+\left(\frac{\partial k}{\partial\omega}\right)_{\omega_0}(\omega-\omega_0)+\frac{1}{2}\left(\frac{\partial^2 k}{\partial\omega^2}\right)_{\omega_0}(\omega-\omega_0)^2. \quad (1.13)$$

Dispersion of the group velocity ($k_2\neq 0$) leads to deformation of the wave packet.

1.2.1. Gaussian pulses

According to (1.12), for a *transform-limited* Gaussian pulse (1.4) we obtain

$$A(\eta, z)=V_0^{-1/2}(z)\rho_0\exp\left[-\frac{\eta^2}{2V_0^2(z)\tau_0^2}+i\varphi(\eta, z)\right], \quad (1.14)$$

where

$$V_0^2(z)=1+\left(\frac{z}{L_d}\right)^2, \quad l_d=\frac{\tau_0^2}{|k_2|},$$

$$\varphi(\eta,z)=\frac{(z/L_d)^2}{k_2 z V_0^2(z)}\eta^2-\frac{1}{2}\operatorname{arctg}\frac{k_2 z}{\tau_0^2}. \quad (1.15)$$

The length L_d is called the dispersive spreading length of a wave packet, it is completely analogous to the diffraction length $L_{dif}=(1/2)k_0 a_0^2$ of a wave beam (a_0 is the beam radius).

The length of a Gaussian pulse increases in a dispersive medium:

$$\tau_p(z)=V_0(z)\tau_0=\left[\tau_0^2+\left(\frac{k_2 z}{\tau_0}\right)^2\right]^{1/2}. \quad (1.16)$$

At the same time, the spectral width of a wave packet remains constant in a linear medium, and it is for this reason that a temporal phase (frequency) modulation arises, with the modulation frequency variation rate $\Omega=\partial\varphi/\partial\eta=\alpha(z)\eta$ equal, according to (1.15), to

$$\alpha(z)=\frac{2z}{k_2(z^2+L_d^2)}. \quad (1.17)$$

In a dispersive medium a transform-limited pulse, thus, transforms into a pulse with linear frequency modulation; the sign of the modulation is determined by the sign of k_2.

By analogy with diffraction of optical beams, it is common, in the treatment of dispersive spreading of an optical pulse, to introduce the near ($z\ll L_d$, $\tau_p\approx\tau_0$) and the far, or Fraunhofer, pulse zones ($z\gg L_d$, $\tau_p\approx(|k_2|/\tau_0)z$). For a Gaussian pulse the profile shape remains constant during pulse propagation. For a pulse of any other shape the profile shape will change during pulse propagation.

1.2.2. Phase-modulate pulse

We consider now the evolution of a PM pulse

$$A_0(t)=\rho_0\exp\left[-\frac{1}{2}(\tau_0^{-2}+i\alpha_0)t^2\right] \quad (1.18)$$

in a dispersive medium. In that case, in accordance with (1.12),

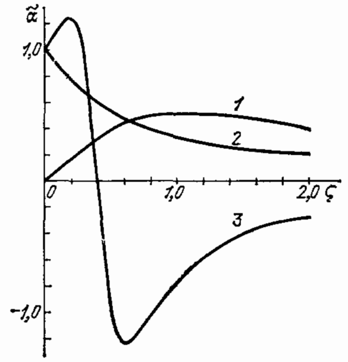

FIG. 1. Relative rate of variation of the frequency $\tilde{\alpha} = \alpha(z)/\alpha_0$ of an optical pulse as a function of the distance traversed $\zeta = z/L_d$ in a linear dispersive medium. 1—pulse without PM($\alpha_0 = 0$), here $\tilde{\alpha} = \alpha(z)\tau_0^2$; 2—PM pulse, $\alpha_0 k_2 < 0$, $\alpha_0\tau_0^2 = 2.0$; 3—PM pulse, $\alpha_0 k_2 > 0$, $\alpha_0\tau_0^2 = 2.0$.

$$A(\eta, z) = f^{-1/2}(z)\,\rho_0 \exp\left[-\frac{(\tau_0^{-2} + i\alpha_0)\,\eta^2}{2f(z)}\right], \tag{1.19}$$

where

$$f(z) = 1 - \alpha_0 k_2 z + i k_2 \tau_0^{-2} z.$$

According to (1.19), a pulse spreads in a medium if $\alpha_0 k_2 < 0$. In the case when $\alpha_0 k_2 > 0$, a PM pulse first gets compressed, and then broadens. The minimal duration of a pulse

$$\tau_{min} = [1 + (\alpha_0\tau_0^2)^2]^{-1/2}\,\tau_0 \tag{1.20}$$

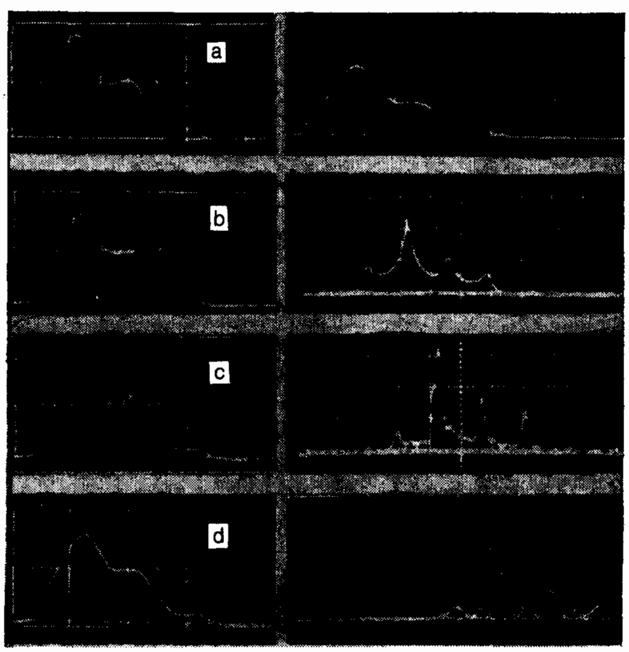

FIG. 2. Profile modification of short optical pulses propagating in a strongly dispersive medium.[19] The scale is 10 *ns* per division for input pulses, and 5 *ns* per division for output pulses. The pulses generated by a tunable laser had the mean frequency ω_0 close to the resonance frequency ω_P for the $^2P_{1/2}$ line in *Rb* vapor. The pulse phase was modulated in accordance with a harmonic law. Detuning from resonance is decreasing in going over from case *a* to case *d*; Fig. 2a—input pulse without PM.

is reached at the distance

$$L_K = \frac{(\alpha_0\tau_0^2)^2}{\alpha_0 k_2\,[1 + (\alpha_0\tau_0^2)^2]}. \tag{1.21}$$

At this distance the frequency modulation (FM) rate is $\alpha(L_k) = 0$, and the pulse length τ_{min} is determined by the total width of the spectrum (1.6). In other words, during optical compression a pulse becomes transform-limited. Pulse transition through the region of optimal compression is accompanied by a sign change of the FM rate (see Fig. 1, curve 3).

We note that experimental observation of the described effects became possible only after the development of tunable lasers able to generate well-formed short pulses. In Fig. 2 are shown oscillograms that illustrate profile deformation for short pulses propagating near narrow resonances in atomic vapors.[19] Here the dispersive effects (not only of the second order, but also of higher orders) are clearly noticeable already for pulse lengths of 10^{-8} sec. In Figs. 2b,c amplitudes of the largest peaks of emerging pulses are, respectively, 1.3 and 1.5 times larger than the amplitude of the input pulse.[19]

1.2.3. Propagation of a noise pulse. Temporal analogue of the Van-Zittert-Zernike theorem

In order to illustrate propagation characteristics of the noise pulses (1.7) in a dispersive medium, we assume that a pulse is described by a Gaussian regular function $F(t) = \rho_0 \exp[-t^2/2\tau_0^2]$. The correlation function $B_\xi(\theta)$ of a steady-state random process $\xi(t)$ is also assumed to have Gaussian form:

$$B_\xi(\theta) = \langle \xi(t)\,\xi^*(t+\theta)\rangle = \exp\left[-\left(\frac{\theta}{\tau_{K,0}}\right)^2\right], \tag{1.22}$$

where $\tau_{k,0}$ is the correlation time.

Using (1.12), is not difficult to calculate the noise pulse correlation function for $z > 0$. The pulse length and correlation time are equal, respectively, to[20,21]:

$$\begin{aligned} \tau_p(z) &= \left[1 + \left(\frac{z}{L_d'}\right)^2\right]^{1/2} \tau_0, \\ \tau_K(z) &= \left[1 + \left(\frac{z}{L_d'}\right)^2\right]^{1/2} \tau_{K0}, \end{aligned} \tag{1.23}$$

where L_d' is the dispersive spreading length of a noise pulse

$$L_d' = \left[1 + \left(\frac{\tau_0}{\tau_{K0}}\right)^2\right]^{-1/2} \left(\frac{\tau_0^2}{|k_2|}\right). \tag{1.24}$$

Expressions (1.23) completely coincide with the expressions for the beam radius $a(z)$ and the correlation radius $r_k(z)$ of random optical beams if τ_0, τ_{k0} and k_2 are substituted by a_0, Γ_{k0}, and k_0^{-1}, respectively.[6] In particular, a pulse with background noise spreads faster than a transform-limited pulse of the same length.

In the limiting case when $\tau_0 \gg \tau_{k0}$, the pulse correlation time in a medium is $\tau_k(z) \approx |k_2| z \tau_0^{-1}$. This result can be regarded as a consequence of the temporal analogue of the Van-Zittert-Zernike theorem.[6]

1.2.4. Effects of cubic dispersion

In cases when the parameter $k_2 = 0$ (see Sec. 1.6), it is necessary, in order to take into account the dispersive spreading of pulses, to start with higher-order approximations of the dispersion theory, i.e., to take into consideration the parameter (see (1.10))

$$k_3 = \frac{1}{2k_0}\left(\frac{\partial^3 k^2}{\partial\omega^3}\right)_{\omega_0} = \left(\frac{\partial^3 k}{\partial\omega^3}\right)_{\omega_0} + \frac{3k_2}{uk_0}. \quad (1.25)$$

In the third-order approximation of the dispersion theory pulse propagation is described in the general case by the equation (in a moving coordinate system):

$$\left(\frac{\partial}{\partial z} - i\frac{1}{2}k_2\frac{\partial^2}{\partial\eta^2} - \frac{1}{6}k_3\frac{\partial^3}{\partial\eta^3}\right)A(\eta, z) = 0. \quad (1.26)$$

Parameter k_3 can be related to a characteristic spreading length of a pulse

$$L_d^{(3)} = 2\tau_0^3 \mid k_3 \mid^{-1}. \quad (1.27)$$

The shape change of a regular pulse which follows from (1.26) for $k_2 = 0$ was analyzed in Ref. 16. It was shown that for the lengths $z \gg L_d^{(3)}$ the amplitude $A(\eta, z)$ of a pulse oscillates for $\eta < 0$, and decreases exponentially for $\eta > 0$.

Propagation of regular, phase-modulated and random pulses was studied within the framework of the third-order approximation in Refs. 23–25.

1.3. Deformation of optical pulses during reflection, the origins of phase modulation. Ultrashort optical pulses in a resonator

According to the results of Sec. 1.2, propagation of optical pulses in a homogeneous dispersive medium is described in the time-domain by (1.12), and in the spectral domain—by the expression

$$\tilde{A}(\Omega = \omega - \omega_0, z) = K(\Omega, z)\tilde{A}_0(\Omega), \quad (1.28)$$

where $\tilde{A}_0(\Omega)$ is the Fourier-spectrum of an initial pulse, $K(\Omega, z)$ is the frequency transfer-function, or simply the transmission coefficient. In the second approximation of dispersion theory we have

$$K(\Omega, z) = \exp\left(-i\cdot\frac{1}{2}k_2 z\Omega^2\right). \quad (1.29)$$

Consider the features of an ultrashort pulse reflection from an interferrometric mirror. A mirror reflection coefficient for the amplitude has the form:

$$K(\Omega = \omega - \omega_0) = r(\Omega) = \mid r(\Omega) \mid e^{i\varphi(\Omega)}. \quad (1.30)$$

Assuming that the phase $\varphi(\Omega)$ is a slowly varying function, we will describe the resonance properties of the mirror in the form (compare with (1.13))

$$\varphi(\Omega) = \varphi_0(\omega_0) + \varphi_0'\Omega + \frac{1}{2}\varphi_0''\Omega^2, \quad (1.31)$$

φ_0' and φ_0'' are, respectively, the first and second derivatives of the phase with respect to frequency at $\omega = \omega_0$.

Comparing (1.30) and (1.31) with the transmission coefficient of a dispersive medium (1.29) (see also (1.13)), it is easy to see that reflection of a short pulse from a mirror with a frequency-dependent, complex reflection coefficient leads to a phase shift, group delay and phase modulation of the pulse. For femtosecond pulses the onset of PM becomes quite significant.[26–29]

Reflection from a multilayer mirror is one of the examples of new non-steady-state linear problems,[181] that became important after the creation of femtosecond lasers. Ref. 30 mentions an interesting effect related to the total internal reflection of ultrashort pulses. Some features of the passage of ultrashort optical pulses through a Fabry–Perot interferometer are studied in Refs. 31–33.

1.4. Compression of phase-modulated optical pulses and focusing of optical beams

Compression of PM pulses briefly discussed earlier (see formulas (1.20) and (1.21)) is an important practical application of the deformation of a short optical pulse in a dispersive medium. The compression of PM pulses has become now, probably one of the most universal methods of obtaining femtosecond pulses in the visible, UV and IR regions. For this reason, we are going to discuss, without limiting the discussion to the solutions of the parabolic equation only, the physics of compression, paying special attention to the analogy and differences of this process from the spatial focusing of optical beams.

Basic ideas of the compression of optical pulses in optics were borrowed from the radar field.[34,35] In the first experiments on compression,[36] the relatively long He-Ne laser pulses ($\tau_0 \approx 0.5$ ns) were phase modulated with the help of an electrooptical modulator. The PM pulses were compressed by a dispersive device.[37]

We discuss now the major compression stages using both the time and the spectral terminology. Let a PM pulse have the form (1.18). The total phase of the pulse is $\Phi(t) = \omega_0 t - (1/2)\alpha_0 t^2$, and its instantaneous frequency

$$\omega(t) = \frac{d\Phi}{dt} = \omega_0 - \alpha_0 t \quad (1.32)$$

is a linear function of time.[1)]

The Fourier-transformation of the PM pulse has the form:

$$\tilde{A}_0(\Omega) = \left(\frac{\tau_0}{2\pi}\right)^{1/2}\rho_0 \exp\left\{-\frac{(\tau_0\Omega)^2}{2[1+(\alpha_0\tau_0^2)^2]} + i\varphi_0(\Omega)\right\}, \quad (1.33a)$$

where the phase is

$$\varphi_0(\Omega) = \frac{\alpha_0\tau_0^4\Omega^2}{2[1+(\alpha_0\tau_0^2)^2]}.$$

The Fourier-spectrum of a pulse on emerging from a dispersive medium is

$$\tilde{A}_K(\Omega = \omega - \omega_0) = \tilde{A}_0(\Omega)e^{i\varphi_K(\Omega)}, \quad \varphi_K(\Omega) = -\frac{1}{2}k_2 z\Omega^2. \quad (1.33b)$$

A simple qualitative picture of the compression physics of a FM *optical pulse* (1.18) can be given in the time domain.[38] The results required for this were obtained above (see expressions (1.12), (1.20), and (1.21)). In order to give their physical interpretation, a real FM pulse must be represented in the form of a train of pulses of constant frequency which monotonically varies from pulse to pulse (this is equivalent to a substitution of the linear dependence $\omega(t)$ (1.32) by a step-wise dependence). It is easy to see that if, for example, $\alpha_0 > 0$, then the condition of compression is $k_2 > 0$; in a medium with normal dispersion of group velocity the "higher" frequencies located on the falling edge of a pulse "catch up" with the low frequencies at the front edge of a pulse.

It is easy to find also the length of the "time focusing" (the compression length L_k). Over this distance the group delay time for highest and lowest pulse frequencies is $\Delta t_d \approx 2\tau_0$, i.e.,

$$2\tau_0 \approx \Delta t_d = \left(\frac{1}{u_1} - \frac{1}{u_2}\right) L_k$$

$$= \left[\left(\frac{\partial k}{\partial \omega}\right)_{\omega_1} - \left(\frac{\partial k}{\partial \omega}\right)_{\omega_2}\right] L_k = k_2 L_k \Delta\omega_0^p. \qquad (1.34)$$

Since in the considered case $\alpha_0\tau_0^2 \gg 1 \Delta\omega_0^p x 2\alpha_0\tau_0$ (see (1.6)), then $L_k = (\alpha_0 k_2)^{-1}$. The minimal duration of a compressed pulse is (see (1.20)) $\tau_{\min} = 1/\Delta\omega_0^p \approx (\alpha_0\tau_0)^{-1}$.

Let us formulate now the requirements analogous to expression (1.34) in the spectral domain. The answer is obvious for our model of a Gaussian PM pulse having the form (1.18). A PM pulse emerging from a dispersive medium will have the maximum amplitude and, therefore, the minimum duration under the condition that all its spectral components are precisely phase matched: $\varphi_0(\Omega) + \varphi_k(\Omega) = 0$ (see (1.33a), (1.33b)[2)]). It is easy to see that from this relationship follow expressions for L_k (1.21) and $\tau_{\min}$ (1.20). From the phasing condition for the pulse spectral components, using an approximation of geometrical optics ($\alpha_0\tau_0^2 \gg 1$), we obtain for L_k expression (1.34).

For optimum compression conditions, the pulse compression coefficient is

$$s = \frac{\tau_0}{\tau_{\min}} = \frac{\Delta\omega_0^p}{\Delta\omega_0} = [1 + (\alpha_0\tau_0^2)^2]^{1/2}, \qquad (1.35)$$

i.e., a pulse can be compressed by the same factor, by which its spectrum is broadened by frequency modulation.

Compression of FM pulses has a lot in common with the focusing of optical beams. Figure 3a shows the shapes of a beam and a wave front for typical cross sections of a medium. The profile shapes and results of FM for typical stages of pulse compression are shown in Fig. 3b. From comparison of both processes, it follows that one can talk about pulse compression as "time focusing," with the frequency modulator playing the role of a "time" lens. The region of optimum pulse compression is equivalent to the region of beam constriction. The focal distance of a "time" lens is the parameter

$$F = (\alpha_0 k_2)^{-1}. \qquad (1.36)$$

1.4.1. "Spectron"; pulse shape in the far-zone

Let us analyze the propagation of PM pulses in a dispersive medium for arbitrary initial shape of the profile $\rho_0(t)$.

At the output of a frequency-modulating device a pulse has the form

$$A_0(t) = \rho_0(t)\, e^{-i\alpha_0 t^2/2}. \qquad (1.37)$$

Evolution of this pulse in a dispersive medium in the second-order approximation of dispersion theory is described by the expression (1.12). In this case, at a distance $z = F = (\alpha_0 k_2)^{-1}$ we obtain

$$A(\eta, z) = (i \cdot 2\pi k_2 z)^{-1/2} \tilde{\rho}_0(\alpha_0\tau_0\eta)\, e^{i\alpha_0\eta^2/2}, \qquad (1.38)$$

$$\tilde{\rho}_0(\alpha_0\tau_0\eta) = \int_{-\infty}^{+\infty} \rho_0\left(\frac{t}{\tau_0}\right) e^{-i\alpha_0\eta t}\, dt. \qquad (1.39)$$

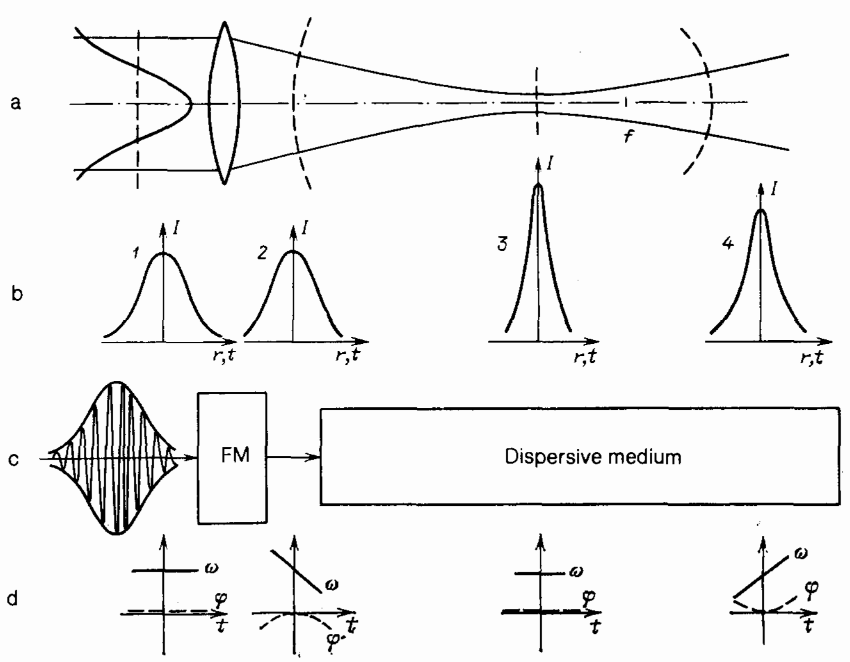

FIG. 3. Optical wave phase control in space (a–b) and in time (c–d). a—beam focusing by a lens; b—(for r): the solid lines — beam shape in front of the lens (1), immediately after the lens (2), in the area of beam constriction (3) and in the focal plane f of the lens (4); the dashed lines – wave front of a beam. c—Compression of FM pulse in a medium with normal dispersion. b—(for t): solid lines — pulse profile shape in front of the frequency modulator (1), at the compressor input (2), in the area of optimal compression (3) and at the "focal" plane (4). d—$\varphi(t)$ (dashed lines) and the frequency $\omega(t)$ (solid lines) in the same cross sections of the medium as in Fig. 3c.

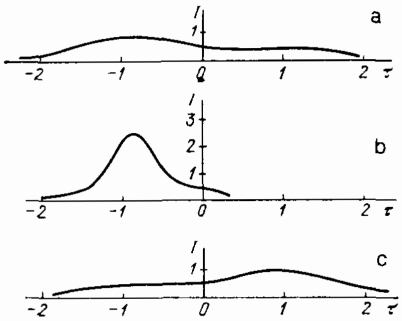

FIG. 4. Profile modification of an optical pulse in a dispersive linear medium. a—original pulse. b—"Spectron" (pulse at the focal plane of the "time" lens). c—Pulse reversed in time (pulse at the optically conjugated plane of the "time" lens).

From the obtained result it is possible to draw the following conclusions about the pulse in the "focal" plane of the "time" lens. The pulse shape is exactly the same as the Fourier-spectrum of the initial pulse.[34,39] Such pulses are called "spectrons."[20,40] The profile of pulses turns out to be symmetric independently of the shape of the actual initial profile $\rho_0(t)$ (see Fig. 4b). Also it turns out that a pulse has linear FM that has the same rate as the input FM, but with the opposite sign.[34,39] If $\alpha_0\tau_0^2 > 1$, the pulse is narrower than the initial pulse, and for $\alpha_0\tau_0^2 < 1$ it is wider. The described pulse properties are analogous to the properties of a light beam in the focal plane of a lens.[12]

In the absence of FM ($\alpha_0 = 0$), the pulses have the described properties in a dispersive medium in the far-field zone ($z \gg L_d$). In this case, it is possible to neglect the term $t^2/2k_2z$ in (1.38), and expression (1.38) takes the form similar to (1.39).

1.4.2. Modification of FM pulses of arbitrary shape; time-reversal of a pulse

The use of a frequency modulator makes it possible to build a number of interesting schemes for modification of optical pulses analogous in many respects to schemes for the formation of light beams and images.[12,41]

As an example we shall consider modification of an optical pulse by the system dispersive medium-frequency modulator-dispersive medium.[20] If the condition

$$\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{F}, \tag{1.40}$$

which is equivalent to the lens formula in the geometrical optics approximation, holds, then

$$A_0(\eta, z_2) = -i\frac{z_1}{z_2}\rho_0\left(-\frac{z_1}{z_2}\eta\right)\exp\left(i\alpha_0\frac{z_1}{z_2}\eta^2\right), \tag{1.41}$$

where z_1 and z_2 are the distances traversed by a pulse before and after the modulator.

We emphasize that the pulse shape does not change,[20] but the pulse is reversed in time relative to the initial pulse[42] (of the sign in the argument of ρ_0 and in Fig. 4c).

Transformation of optical pulses into pulses reversed in time provides the possibility to realize the convolution of optical pulses. A variation of convolution can be used, for example, for the reconstruction of a profile shape.[42]

1.5. Wave beams undergoing fast temporal modulation

In many practical situations the approximation of a plane wave packet cannot be used because of the mutual influence of spatial and temporal modulations. The high-frequency components of a pulse diffract slower than low-frequency components; for this reason even in a nondispersive medium at not too small values of the ratio $\Delta\omega_0/\omega_0$ one can expect, as was noted in Ref. 22, a deformation of an optical pulse.

The picture of mutual influence of spatial and temporal modulations is rather complicated in the general case. For this reason we consider only the particular case when $k_2 = 0$.[43–46] For a Gaussian beam (with a transverse intensity distribution $U^2(r) = \exp(-2r^2/a_0^2)$) and a Gaussian pulse (1.4) in the far-field zone of a beam $z \gg L_{\rm dif} = k_0a_0^2/2$ the pulse length varies according to the formula[45]:

$$\tau(z) = \left[1+\left(\frac{a_0 r}{v_0\tau_0 z}\right)^2\right]^{1/2}\tau_0, \tag{1.42}$$

where a_0 is the beam radius, and v_0 is the phase velocity.

According to (1.42), the pulse length increases towards the periphery of a beam, remaining constant along the axis. In Ref. 45, taking diffraction into account, it was discovered that the profile of an initially smooth pulse becomes modulated.

Changes in pulses can occur, certainly, also during the passage of light beams through different optical elements. In Ref. 46 it was found that in focusing of an ultrashort optical pulse by a zone plate its shape and length near the focal point are altered. In Ref. 47 in an approach, based on the spatial decomposition of a spectrum, possibilities are discussed of obtaining similar shortened or broadened pulses, and also pulses reversed in time.

1.6. Waves in single-mode fiber-optics waveguides. Realization of conditions for propagation of quasiplane wave packets

Conditions when the transverse structure of a wave packet remains practically unchanged at distances about 10^6 cm both in the linear and nonlinear propagation regimes are realized in single-mode optical waveguides. A typical fiber-optic waveguide is a cylinder made from fused quartz, approximately 100 μm in diameter, placed in a protecting cladding. In the area next to the cylinder axis there is an area, the core, where the refractive index is increased by addition of dopants. The typical diameter of the core of a single-mode waveguide is 5–10 microns. In the process of propagation of a wave beam along a waveguide, the core plays the role of a distributed lens that compensates the diffractive spreading of the beam.

The progress achieved in the manufacturing of fiber-optic waveguides, and their numerous applications are described in details in the review article of Ref. 2. For this reason we present here only a few results that are required for the following presentation.

Simultaneous effects of diffraction, linear refraction and dispersion are described by the equation

$$\left[\frac{\partial}{\partial z}+i\frac{1}{2k_{\rm cl}}\Delta_{\perp}-i\cdot\frac{1}{2}k_2\frac{\partial^2}{\partial\eta^2}+ik_{\rm cl}\frac{n_{\rm c}-n_{\rm cl}}{n_{\rm cl}}f(r)\right]A(\eta,r,z)=0, \qquad (1.43)$$

where the last term, responsible for refraction, includes the index of refraction of the cladding $n_{\rm cl}$, the maximum value of the index of refraction of the core $n_{\rm c}$, and the dimensionless function $f(r)$, describing the distribution of the index of refraction over the cross section of the waveguide; $k_{\rm cl}$ is the wave number in the cladding material. We note that Eq. (1.43) adequately describes the situation in the case of weakly directing optical waveguides[48] ($(n_{\rm c}-n_{\rm cl})\ll 1$) with smooth, on the scale of λ, variations of the index of refraction. A more general case is considered, for example, in Ref. 2. In practice the magnitude of $(n_{\rm c}-n_{\rm cl})$ is of the order of 10^{-2}–10^{-3}.

Diffractive spreading of a light beam is compensated by linear refraction on the longitudinal spatial scale of the order of the diffraction length $L_{\rm dif}=(1/2)k_0a_0^2\sim 10^{-1}$–$10^{-2}$ cm, while the dispersive phenomena in the case of picosecond pulses become apparent at distances $L_{\rm d}=\tau_0^2/|k_2|\sim 10^2$–$10^3$ m. This circumstance allows one to separate spatial and temporal effects and seek a solution of (1.43) in the form

$$A(\eta,r,z)=U(r)\psi(\eta,z)e^{-i\tilde{k}z}, \qquad (1.44)$$

where the function $U(r)$ describes the field distribution over the cross section of an optical waveguide, $\Psi(\eta,z)$ is a complex temporal amplitude, $\tilde{k}$ is the correction to the wave number $k_{\rm cl}$ $(0<\tilde{k}<(k_{\rm c}-k_{\rm cl}))$. Substitution of (1.44) into (1.43) leads to two independent equations

$$\tilde{k}U(r)=\frac{1}{2k_{\rm cl}}\Delta_{\perp}U(r)+k_{\rm cl}\frac{n_{\rm c}-n_{\rm cl}}{n_{\rm cl}}f(r)U(r),$$
$$i\frac{\partial\psi}{\partial z}=-\frac{1}{2}k_2\frac{\partial^2\psi}{\partial\eta^2} \qquad (1.45)$$

with the boundary conditions

$$\left.\begin{aligned}U(r)&\to 0\\ \psi(\eta,0)&=\psi_0(\eta),\end{aligned}\right.\quad \text{for } r\to\infty \qquad (1.46)$$

The first of the equations (1.45), together with the boundary conditions, leads to the problem of finding the eigenvalues $\tilde{k}_{j,m}$ and eigenfunctions $U_{j,m}(r)$, i.e., modes of an optical waveguide. The eigenfunctions of the weakly guiding optical waveguides, usually denoted in the literature[48] by LP_{jm}, constitute modes polarized in the direction perpendicular to the axis. For a number of practically important cases these distributions can be approximated by a Gaussian function.

Dispersive characteristics of fiber-optic waveguides are determined mainly by properties of the materials used (material dispersion). In experimental studies, as a rule, the dispersive parameter D, is used, defined by expression

$$D=\frac{1}{L}\frac{\partial t_d}{\partial\lambda}=-\frac{2\pi c}{\lambda^2}k_2, \qquad (1.47)$$

where t_d is the group delay at the distance L. The parameter

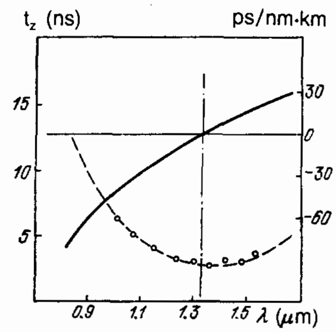

FIG. 5. Typical dependences of the temporal delay of an optical pulse (dashed line – experiment) and of a dispersion parameter (the solid line– calculations) in the vicinity of the wavelength corresponding to zero dispersion of the group velocity in a fiber-optics waveguide.[49]

D is usually expressed in ps/nm·km. Figure 5 shows the experimental dependence $t_{\rm d}(\lambda)$[49] and calculated values $D(\lambda)$. It can be seen that for optical waveguides for $\lambda_{\rm cr}\approx 1.3\,\mu$ the parameter D and, therefore, k_2 become zero. In the spectral region $\lambda>\lambda_{\rm cr}$ the anomalous group velocity dispersion $(k_2<0)$ is realized and for $\lambda<\lambda_{\rm cr}$ the normal dispersion. We note that in the vicinity of the point $\lambda_{\rm cr}$ the contribution of the waveguide dispersion can become significant.

2. PHYSICS OF SELF-ACTION OF OPTICAL PULSES IN MEDIA WITH CUBIC NONLINEARITY. SELF-PHASE-MODULATION, PROFILE SHOCK WAVES (SELF-STEEPENING), DISPERSION EFFECTS

2.1. Light self-action in a medium with cubic nonlinearity

Physical causes giving rise to the dependence of the index of refraction on intensity can be quite diverse. This dependence can be due, for example, to the anharmonicity of electronic and vibrational responses, electrostriction, orientation of molecules in the field of light wave (Kerr effect), heating of the medium, etc. Phenomenologically, all these factors can be regarded as the manifestation of the nonlinear response of a medium, which includes only odd-power terms in the series expansion in terms of the electric field

$$\vec{\mathscr{P}}^{\rm nl}=\vec{\mathscr{P}}^{(3)}+\vec{\mathscr{P}}^{(5)}+\ldots=\hat{\chi}^{(3)}\mathbf{EEE}+\hat{\chi}^{(5)}\mathbf{EEEEE}+\ldots . \qquad (2.1)$$

The nonlinear corrections to the index of refraction n arise from the relationship $\mathbf{D}=\mathbf{E}+4\pi\vec{\mathscr{P}}$ in accordance with (2.1).

In the harmonic or quasiharmonic electric field (1.1), we have for the component describing the self-action caused by the cubic nonlinearity

$$\mathscr{P}_i^{(3)}(\omega)=\frac{1}{8}\chi_{ijkl}^{(3)}(\omega,\omega,\omega,-\omega)A_jA_kA_l^*. \qquad (2.2)$$

The fourth-rank tensor $\chi_{ijkl}^{(3)}$ has nonzero components not only in anisotropic, but also in isotropic media, and this explains the universality of self-action effects. The nonlinear corrections to the refractive index $\delta n=n_2|A|^2/2$ can be reliably registered by interferometric methods even in laser fields of moderate power (data on the values of n_2 for different materials can now be found in reference handbooks[50]).

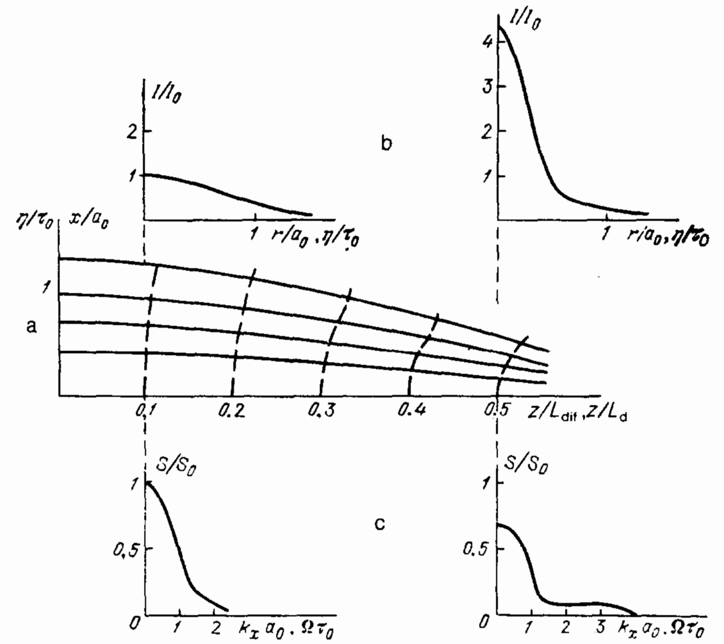

FIG. 6. Self-action of a collimated optical beam and a transform-limited wave packet in a medium with cubic nonlinearity ($n_2 > 0$). With the beam self-focusing: a—side view, beams (solid lines) and phase fronts for different z/L_{dif} (dashed lines); b—beam profile for different z/L_{dif}, angular beam spectrum. Same pictures are valid for the self-action of a wave packet ($k_2 < 0$): a—lines of equal intensity in the plane η, z (solid lines), "time" beams and self-modulation phase for different values of $\zeta = z/L_\alpha$ (dashed lines); b—pulse profile for different values of ζ; c—spectrum of a pulse after SPM.

One of the most remarkable demonstrations of light self-action phenomena is the spatial self-focusing of a light beam,[51] discovered experimentally for the first time in 1965.[52]

The physics of self-focusing of a beam of the form

$$\mathbf{E} = \mathbf{e}\,\frac{1}{2}\,A(r)\exp\left[i\,(\omega_0 t - k_0 z)\right] + \text{c.c.} \tag{2.3}$$

is illustrated in Fig. 6, which shows qualitatively the modifications of a wave front (spatial self-phase-modulation occurs), the cross-sectional intensity distribution, and the angular spectrum $S\ (k_x)$ of a beam as it propagates in a nonlinear medium with $n_2 > 0$ in the absence of nonlinear absorption.

Fig. 6 shows the situation, when nonlinear refraction can suppress the diffractive divergence of a beam. As is well known (see, for example, Ref. 1), this imposes limitation on the total beam power P, which must exceed the critical value

$$P_{\text{crit}} = \frac{c\lambda_0^2}{8\pi n_2} \tag{2.4}$$

for a Gaussian beam. When $P > P_{\text{crit}}$, the nonlinear refraction caused by self-phase-modulation in the first layers of a nonlinear medium, sharpens the transverse amplitude profile, and this, in turn, increases the steepness of the phase change, etc. As a result, a collapse of the beam takes place within a distance called the self-focusing length.[1]

A qualitative picture of the initial stages of self-focusing was traced out already in the first papers published in 1965–1966; a detailed discussion of the problem was given in the review of Ref. 1.

A similar graphic interpretation can be given to the main stages of self-action of a plane wave packet. Here there are many analogies with the picture of spatial self-focusing; there are, however, some significant differences. In a medium with the refractive index $n = n_0 + \tilde{n}_2 I$ the total phase of a wave is

$$kz = \frac{\omega_0}{c}\left[n_0 + \tilde{n}_2 I(t)\right] z = k_0 z + k_0 \tilde{n}_2 I(t)\, z,$$

i.e., there arises a nonlinear correction to the phase depending on time

$$\varphi(t,\ z) = -k_0 \tilde{n}_2 I(t)\, z. \tag{2.5}$$

The temporal self-phase-modulation leads, obviously, to broadening of the frequency spectrum (see Sec. 2.2). It is natural that, as is the case in spatial self-focusing, this must cause a change in the intensity profile.

The relevant simple considerations can be given by using results from Sec. 1.2. According to (2.5), the rate of frequency change, caused by self-action, is equal to

$$\alpha(t,\ z) = \frac{\partial^2 \varphi}{\partial t^2} = -k_0 \tilde{n}_2 \frac{\partial^2 I}{\partial t^2}\, z. \tag{2.6}$$

The behavior of a wave packet is determined, as is shown in Sec. 1.4, by the dispersion law of the medium. The case of $\alpha k_2 < 0$ is of special interest, since it demonstrates stages of optical pulse self-compression. SPM causes compression, and this, in turn, increases the tempo of self-modulation.

In the majority of experimental works carried out in the late 1960s-early 1970s with high-power pulsed lasers, the effects caused by spatial and temporal self-actions were closely inter-related, giving rise to their strong mutual influence. It is natural, that under these conditions the self-action process becomes significantly more complicated.[53–57,60]

In the mid-1970s, there appeared an opportunity to observe experimentally the spatial self-focusing and temporal self-actions separately from each other. Ref. 58 describes an elegant experiment on self-focusing of radiation from a tunable CW laser in atomic vapors. The opportunities to observe purely temporal self-actions became available in nonlinear optics only after creation of the single-mode optical-fiber waveguides with low losses.

2.2. Self-phase-modulation in a medium with "inertialess" nonlinearity

We shall start by considering the simplest problem of the propagation of a plane wave packet in a nonlinear isotropic medium with cubic nonlinearity

$$\frac{\partial^2 E}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 D}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial^2 \mathcal{P}^{(3)}}{\partial t^2}, \tag{2.7}$$

where the nonlinear polarization is given by the expression

$$\mathcal{P}^{(3)}(t,\ z)=\frac{3}{8}\chi^{(3)}|A|^2\exp[i(\omega_0 t-k_0 z)]. \tag{2.8}$$

As in the case of a linear medium, we seek a wave-packet solution of (2.7) in the form (1.1). Then the variation of the complex amplitude in a nonlinear medium is described in the second-order approximation of the dispersion theory by the equation (in the moving coordinates η, z; compare with (1.11))

$$\frac{\partial A}{\partial z}-i\cdot\frac{1}{2}k_2\frac{\partial^2 A}{\partial \eta^2}+i\beta_1|A|^2A=0, \tag{2.9}$$

where

$$\beta_1=\frac{3\pi\chi^{(3)}k_0}{2\varepsilon_0}=\frac{k_0 n_2}{2n_0}. \tag{2.10}$$

Equation (2.9) is approximate also from the point of view of taking into consideration the nonlinearity of the medium; we assume that the terms containing time derivatives of the slow part of the nonlinear polarization are of a higher order of smallness. We must emphasize, however, that below we shall consider two examples, when it is essential to take these terms into account. These terms are related to the appearance of profile shock waves (see Sec. 2.4), and the specifics of compression of high-power femtosecond optical pulses in optical-fiber waveguides (see Sec. 3.5).

In the first-order approximation of dispersion theory ($k_2=0$) the solution of Eq. (2.9) takes the form

$$A(\eta,\ z)=A_0(\eta)\,e^{-i\beta_1|A_0|^2 z}, \tag{2.11}$$

or, for the real amplitude $\rho(\eta,z)$ and phase $\varphi(\eta,z)$

$$\rho(\eta,\ z)=\rho_0(\eta),\quad \varphi(\eta,\ z)=-\tilde{\beta}_1 I_0(\eta)\,z, \tag{2.12}$$

where we assume that $\varphi_0(\eta)=0$, $\tilde{\beta}_1=k_0\tilde{n}_2/n_0$, $I_0=(cn_0/8\pi)|A_0|.^2$

From (2.12) it can be seen that the pulse profile propagates with the group velocity u ($\rho(\eta,z)=\rho_0(t-(z/u))$ and does not change. Conversely, the pulse phase varies in proportion to the distance traversed—self-phase-modulation occurs. The pulse frequency change due to self-action is

$$\delta\omega(t)=\frac{\partial\varphi}{\partial t}=-\tilde{\beta}_1\frac{\partial I_0(\eta)}{\partial\eta}z. \tag{2.13}$$

Let us consider the change in a Gaussian pulse caused by SPM. We introduce the maximum phase shift

$$\varphi_{\max}=\max_t|\varphi|=\tilde{\beta}_1 I_0(0)\,z \tag{2.14}$$

and the nonlinear SPM length—the distance at which $\varphi_{\max}=1$.

$$L_p=(\tilde{\beta}_1 I_0)^{-1}. \tag{2.15}$$

The range of the frequency change $\delta\omega(t)$ (2.13) increases with the increase of $\varphi_{\max}$.

Graphs in Fig. 7 show the temporal behavior of the frequency $\delta\omega(t)$ and the rate of its variation.

The maximum shift of the frequency $\delta\omega(t)$ for a Gaussian pulse is equal to

$$\delta\omega_{\max}=\left(\frac{2}{e}\right)^{1/2}\frac{\varphi_{\max}}{\tau_0}=0.43\,\varphi_{\max}\,\Delta\omega_0, \tag{2.16}$$

where $\Delta\omega_0$ is determined from (1.5).

Fig. 8a shows the shapes of the broadened spectrum of a Gaussian pulse for different values of the phase $\varphi_{\max}$. It can be seen that with an increase in the value of $\varphi_{\max}$ modulation appears in the pulse spectrum. Characteristics of nonlinear broadening of the spectrum for $\varphi_{\max}\gg 1$ were for the first time determined by Shimizu.[61] The main energy of the pulse is concentrated in the frequency band

$$\Delta\omega'=0.86\,\varphi_{\max}\Delta\omega_0. \tag{2.17}$$

A review of works performed at an early stage of research on the broadening of the spectrum in the case of self-action of optical picosecond pulses can be found in Ref. 55. We note that the correct interpretation of experimental data has always been complicated by competing nonlinear processes, first of all by self-focusing.

For the first time SPM of ultrashort pulses in the absence of self-focusing was realized by the authors of Ref. 62 in a capillary fiber-optic waveguide filled with CS_2. The "cleanest" experimental data on the pulse self-action from the point of view of comparison with the theory of SPM presented above were obtained in Ref. 59. The authors have

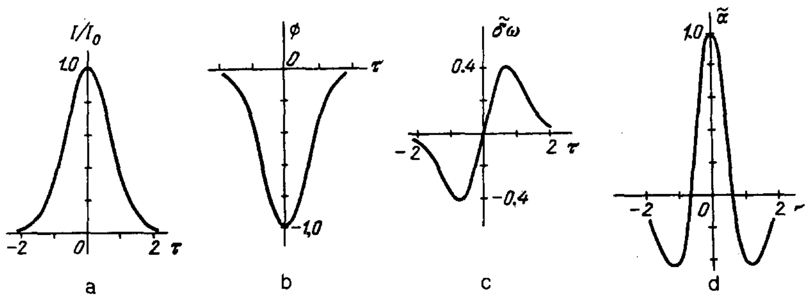

FIG. 7. Shape of a Gaussian pulse (a), the reduced phase $\varphi=\varphi/\varphi_{\max}$ (b), reduced frequency deviation $\delta\omega(t)=\delta\omega(t)/\delta\omega(0)$ (c) and reduced frequency change rate $\tilde{\alpha}(t,z)=\alpha(t,z)/\alpha(0,z)$ (d) as a function of time $\tau=\eta/\tau_0$. $\delta\omega(0)=2\varphi_{\max}/\tau_0$, $\alpha(0,z)=2\varphi_{\max}/\tau_0^2$; τ_0 is the pulse length.

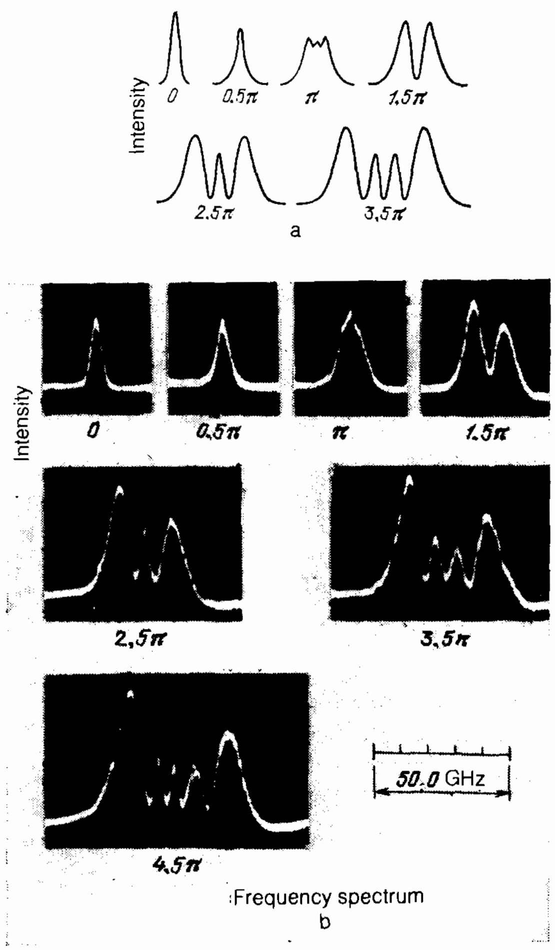

FIG. 8. Spectrum of a Gaussian pulse for different maximum values of φ_{max}.[59] a—theory, b—experiment.

studied the dependence of the spectrum shape at the output of a fiber-optical waveguide on the input pulse energy (Fig. 8b), and obtained good agreement with the theoretical expression (2.16).

If one neglects the fine structure of the broadened spectrum, then for its integral characteristic—a RMS width—we obtain the following expression[63] (compare with (2.17)).

$$\langle(\Delta\omega)^2\rangle = [1 + (0.88\varphi_{max})^2]^{1/2}\Delta\omega_0. \tag{2.18}$$

We note that the above results refer to SPM of symmetric pulses. It is easy to see that asymmetry of a pulse profile also leads to the asymmetry of the spectrum.

2.3. Self-phase-modulation in a medium with relaxation nonlinearity

The approximation of "inertialess" nonlinearity, on which the material of the previous section is based, is also valid, obviously, if the pulse length at the input to a nonlinear medium τ_0 is much larger than the relaxation time of the nonlinearity τ_r, i.e., $\varepsilon_0 \gg \tau_r$. Such a situation in fiber-optic waveguides holds up to $\tau_0 \approx 10^{-13}$ sec ($\tau_r < 10^{-14}$ sec). Conversely, if the high-frequency Kerr effect in liquids is used ($\tau_r \approx 10^{-12}$ sec), then it is essential to take into account

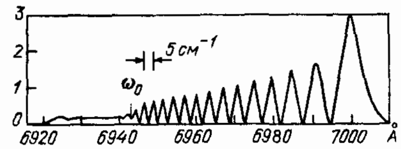

FIG. 9. Broadened spectrum of a Gaussian pulse with $\tau_0 = 2.7$ ps and the nonlinearity relaxation time 9 ps for $\varphi_{max} = 265$.[64]

the finite speed of the nonlinear response already in the picosecond region of pulse duration.

When a nonzero value of the relaxation time is taken into account, it is necessary to start from the non-steady-state equation for the nonlinear correction δn to the refractive index of the medium (see, for example, Ref. 1):

$$\tau_r \frac{\partial \delta n}{\partial t} + \delta n = \frac{1}{2} n_2 \rho^2(t, z). \tag{2.19}$$

In accordance with (2.19), the change in the frequency $\delta\omega = \partial\varphi/\partial\eta$, where $\varphi/\eta,z) = -k_0 z\delta n$, is determined by the expression ($\tau_0 \ll \tau_r$)

$$\delta\omega(\eta) = \frac{\partial\varphi}{\partial\eta} = -\frac{\omega_0 n_2 z}{2c\tau_p}\rho_0^2(\eta). \tag{2.20}$$

It can be seen that the frequency shift is negative over the entire pulse ($n_2 > 0$). And this means, that in the limiting case of "very slow nonlinearity" the pulse spectrum broadens towards the low-frequency Stokes region.

For a Gaussian pulse the maximum variation of the frequency is given by the relation

$$\delta\omega_{max} = -\frac{\omega_0 n_2 \rho_0^2 z}{2c\tau_p} \tag{2.21}$$

and it is inversely proportional to the relaxation time τ_r.

The broadened spectrum of a Gaussian pulse with a finite value of τ_r is shown in Fig. 9; as can be seen, the spectral distribution is essentially asymmetric relative the initial pulse frequency.

The temporal SPM picture of a constant shape pulse, on which the concepts presented above are based, corresponds to the real situation, naturally, only at the first stages of the self-action process. The shape of the pulse, undergoing SPM, was assumed to be unchanged.

What is the behavior of the broadened spectrum, profile, and phase in the case of the simultaneous presence of SPM and group-velocity dispersion? This problem is discussed in Sec. 2.5 and Secs. 3 and 4.

2.4. Profile shock waves (self-steepening)

In this section we consider the situation when significant nonlinear distortion of the profile is possible even in the absence of dispersion. We are dealing with the so-called profile shock waves, (self-steepening) arising during propagation of sufficiently powerful short pulses in a nonlinear medium.

For a theoretical description of the effect we must include higher-order approximations in the expression for a nonlinear source in the wave equation (2.7). Until now (see (2.9)) we were assuming

$$\ddot{\mathcal{P}}^{(3)} = \frac{\partial^2 \mathcal{P}^{(3)}}{\partial t^2} = -\omega_0^2 \mathcal{P}^{(3)}$$

$$= -\frac{3}{8}\,\omega_0^2 \chi^{(3)} |A|^2 A \exp[i(\omega_0 t - k_0 z)]. \qquad (2.22)$$

If the speed of a profile change and a nonlinearity are sufficiently large, then instead of (2.22) we must write the series

$$\ddot{\mathcal{P}}^{(3)} = -\frac{3}{8}\,\omega_0^2 \chi^{(3)} \left(|A|^2 A - i\,\frac{2}{\omega_0}\,\frac{\partial |A|^2 A}{\partial t} - \frac{1}{\omega_0^2}\,\frac{\partial^2}{\partial t^2}\,|A|^2 A \right) \times \exp[i(\omega_0 t - k_0 z)]. \qquad (2.23)$$

The smallness parameter in the expansion (2.23) is the ratio of the optical period to the pulse duration $\mu = T/\tau_0$. Then, in the first-order approximation in μ and the dispersion theory, instead of (2.9) we obtain

$$\frac{\partial A}{\partial z} + i\beta_1 |A|^2 A + \beta_2\,\frac{\partial |A|^2 A}{\partial \eta} = 0, \qquad (2.24)$$

where $\beta_2 = n_2/c$.

What is the consequence of the presence of an additional term in (2.24)? It leads, in particular, to the dependence of the group velocity on the intensity of the propagating pulse. This fact was noticed for the first time by Ostrovskiĭ.[65]

The nonlinear correction to the group velocity for a medium with $n_2 > 0$ ($n_2 < 0$) leads to the steepening of the trailing (forward) edge of a pulse in the course of its propagation. This phenomenon was studied in the absence of group-velocity dispersion in Refs. 66–68, and in the presence of group-velocity dispersion and nonlinearity relaxation in Refs. 69–71.

Let us turn to a specific analysis. By going over to the profile and phase, from (2.24) we obtain the system

$$\frac{\partial \rho}{\partial z} + 3\beta_2 \rho^2 \frac{\partial \rho}{\partial \eta} = 0, \qquad (2.25)$$

$$\frac{\partial \varphi}{\partial z} + \beta_2 \rho^2 \frac{\partial \varphi}{\partial \eta} = -\beta_1 \rho^2. \qquad (2.26)$$

Equation (2.25) is the equation for a simple wave (see, for example, Ref. 72).

In the theory of waves in weakly dispersive nonlinear media (nonlinear transmission lines, nonlinear acoustics), which is based on the method of a slowly varying profile, developed by R. V. Khokhlov (see Ref. 73), an equation of the type (2.25) is obtained for the field itself. This analogy allows one to transfer a number of results obtained for simple waves, for example, in the area of nonlinear acoustics, to simple profile waves.

2.4.1. Deformation of a profile

The solution of (2.25) has the implicit form

$$\rho(\eta, z) = \rho_0(\eta - 3\beta_2 z \rho^2(\eta, z)). \qquad (2.27)$$

Let us analyze (2.27) for a Gaussian pulse, for which

$$\rho^2(\eta, z) = \rho_0^2 \exp[-(\eta - 3\beta_2 z \rho^2)\,\tau_0^{-2}]. \qquad (2.28)$$

The pulse profile change in the course of propagation is illustrated by Fig. 10. It can be seen that the pulse deforms: the leading edge becomes flatter, and the trailing edge, on the contrary, steeper. "Self-steepening" of a front is taking place.

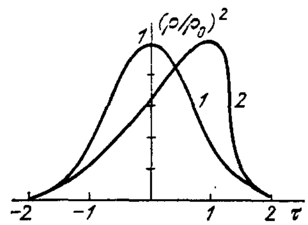

FIG. 10. Shape of a Gaussian pulse (curve 1) in a nonlinear medium (curve 2) for $3\beta_2 \rho_0^2 \tau_0^{-1} z = 1$.[71]

The steepening of the trailing edge of a pulse leads in the end to the formation of a discontinuity, for which $\partial\rho/\partial\eta = \infty$—a shock wave of the profile is formed. This occurs *at the distance*[71]

$$z = L_p = \left(\frac{e}{2}\right)^{1/2} \frac{\tau_0}{3\beta_2 \rho_0^2} \approx 7 v_0 \tau_0 \left(\frac{\delta n_{\max}}{n_0}\right)^{-1}, \qquad (2.29)$$

which is called the distance of the formation of discontinuity ($v_0 = c/n_0$, $\delta n_{\max} = (1/2) n_2 \rho_0^2$). We note that L_r approximately corresponds to the distance at which the pulse maximum is shifted by a distance equal to its half-width. The presence of attenuation δ_0 characterized by the length $L_\delta = (2\delta_0)^{-1}$ "delays" the formation of a shock wave; for $L_\delta < L_r$ a shock wave is not formed.

The first attempts to observe experimentally the profile shock waves in optics were made in the late 1960s (see Ref. 55); unfortunately, an unambiguous interpretation of the experimental results was complicated by a significant contribution of spatial self-focusing.

Grishkovskiĭ *et al.*[75] observed directly the shape distortion of a 10-ns dye laser pulse in Rb vapor, caused by the formation of a profile shock wave.

For picosecond and subpicosecond pulses, direct observations of the shape are not yet possible; information about self-action can be obtained from the form of the spectrum. It is easy to see that the form of the spectral broadening under the conditions of manifestation of the nonlinear correction to the group velocity described by Eqs. (2.25), (2.26), differs from the broadening under the conditions of "inertialess" SPM, corresponding to $\beta_2 = 0$. We shall illustrate this by the approximate calculations performed for a pulse of a constant shape ($z \ll L_r$, see also Ref. 76).

2.4.2. Spectrum broadening

We write a profile in the form $\rho^2(\eta, z) = \rho_0^2 \operatorname{sech}(\eta/\tau_0)$; then, according to (2.26), the relative change in frequency is[4]:

$$\frac{\delta\omega}{\omega_0} = \frac{1}{2}\{[1 + (Q^2 - 2Q \operatorname{sh}\tau)\operatorname{ch}^{-2}\tau]^{-1/2} - 1\}, \qquad (2.30)$$

where $Q = (n_2/c\tau_0)\rho_0^2 z$, $\tau = \eta/\tau_0$. The maximum shifts of frequency towards the Stokes $\delta\omega_{\max}^{s}$ and anti-Stokes $\delta\omega_{\max}^{a}$ regions are determined by relation ($Q > 0$)

$$\frac{\delta\omega_{\max}^{s,a}}{\omega_0} = \frac{1}{4}\left[(Q^2 + 4)^{1/2} \mp Q\right] - \frac{1}{2}. \qquad (2.31)$$

For $Q \ll 1$ from (2.31) follows the result $\delta\omega^{s,a}_{max} = \mp Q\omega_0/4$, coinciding with the result from the theory of self-phase-modulation described in the previous section, i.e., the broadening of the spectrum relative to the frequency ω_0 is symmetrical.

In the case when $Q \gg 1$, the maximum broadening towards the Stokes region is $\delta\omega^{s}_{max} \approx (Q^{-1} - 1)\omega_0/2$, and towards the anti-Stokes region–$\delta\omega^{a}_{max} \approx Q\frac{\omega_0}{2}$. Therefore, for $Q \gg 1$, the spectral pulse distribution becomes strongly asymmetrical; this asymmetry is connected with the presence of the term $\beta_2 \rho^2 \frac{\partial\varphi}{\partial\eta}$ in Eq. (2.26). The authors of Ref. 76, using the theory presented here, interpreted the results of experiments obtained by Fork *et al.*[77] Fork *et al.*[77], using 80-fsec pulses at a wavelength of 627 nm and with the intensity of $I_0 \approx 10^{14}$W/cm^2, observed the spectrum broadening towards the Stokes $\delta\omega^{s}_{max}/\omega_0 = -0.6$ and anti-Stokes $\frac{\delta\omega^{a}_{max}}{\omega_0} = 2.3$ regions; practically, a spectral continuum was generated. We remark, that the broadening of a spectrum analogous to the broadening considered here, was obtained in Ref. 79 on the basis of an analysis of Eq. (2.7). In Ref. 78, a picture of profile shock waves formation is used for an interpretation of broadening of the pulse spectrum in capillary fiber-optic waveguides.

In addition to the profile shock waves, the additional broadening of the spectrum of picosecond pulses (generation of light continuum[89,90,94]) can be caused by avalanche ionization of a medium,[91] four-photon parametric processes,[92,93] the motion of foci in the case of spatial self-focusing[54,55,57] etc.

2.5. Combined effect of nonlinearity and dispersion of a medium

Self-action of optical pulses in a nonlinear medium, accompanied by a broadening of the spectrum, can lead, as has been mentioned before, to the necessity to take into account the dispersion of the medium in the second- and higher-order approximations of dispersion theory. At the present time, this is the situation, which is most real and most often encountered in practice in connection with generation of optical pulses with pulse lengths $\sim 10^{-13}$–10^{-14} sec.

Let us consider self-action of a pulse in a dispersive medium, basing the analysis on Eq. (2.9).[5)] In a medium with $n_2 > 0 (\beta_1 > 0)$, the pulse frequency shift, determined by (2.13), varies, as shown in Fig. 7. On the other hand, a relative delay of various spectral components of a PM pulse is related to the dispersion parameter $k_2 \neq 0$. If $k_2 > 0$, the initial transform-limited pulse spreads faster than in a linear medium. If $k_2 < 0$, then self-compression of a pulse takes place[6)] (see Fig. 4b). These processes are analogous to self-defocusing ($k_2 > 0$) and self-focusing ($k_2 < 0$) of a two-dimensional beam.

The typical nonlinearity length

$$L_{nl} = \tau_0 (|k_2| k_0 \tilde{n}_2 I_0 n_0^{-1})^{-1/2} \tag{2.32}$$

is a temporal analogue of the beam self-defocusing or self-focusing lengths, respectively.

The regime of self-compression is of special interest. Under the condition $L_d = L_{nl}$, the dispersive spreading of a pulse is exactly compensated by compression. As a result, the pulse retains its shape—a soliton is formed.[83,84] The steady-state shape of a pulse can be found by setting A in (2.9) in the form $A = A_s(\eta)e^{i\Gamma z}$. Then for the amplitude $A_s(\eta)$ we obtain

$$\frac{1}{2} k_2 \ddot{A}_s + \Gamma A_s - \beta_1 A_s^3 = 0.$$

This equation is transformed to the form

$$\frac{1}{2} k_2 (\dot{A}_s)^2 + \Gamma A_s^2 - \frac{1}{2} \beta_1 A_s^4 = 0. \tag{2.33}$$

When $\beta_1 > 0$ and $k_2 < 0$ (2.33), we have the solution

$$A_s(\eta) = A_{s0} \operatorname{sech} \frac{\eta}{\tau_s}, \tag{2.34}$$

where the duration of the soliton τ_s and its amplitude A_{s0} satisfy the relation

$$2\Gamma = |k_2|/\tau_s^2 = \beta_1 A_{s0}^2. \tag{2.35}$$

The soliton energy density (J/cm^2)

$$W_{crit} = \frac{2|k_2|}{\tilde{\beta}_1 \tau_s} = \frac{2n_0|k_2|}{\tilde{n}_2 k_0 \tau_s} \tag{2.36}$$

is inversely proportional to its duration.

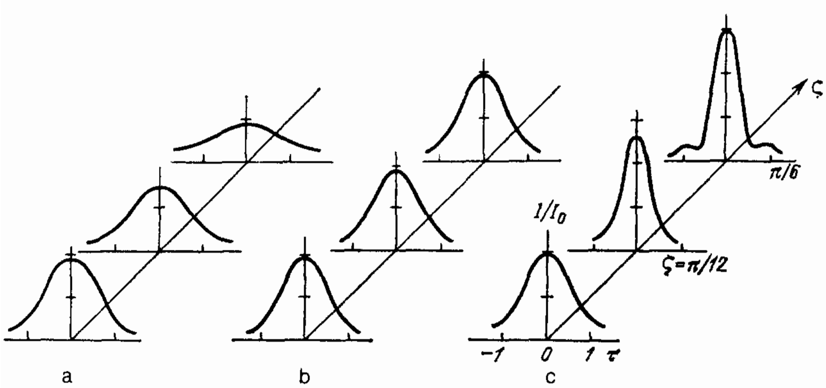

FIG. 11. Variation of pulse shape with distance for power densities $W < W_{crit}$ (a), $W = W_{crit}$ (b)—soliton regime and $W > W_{crit}$ (c).

At the first stage, the pulse with the energy density $W > W_{\text{crit}}$ compresses, and with $W < W_{\text{crit}}$ spreads (see Fig. 11). At the same time it is necessary to emphasize, that the soliton is a stable formation with respect to small perturbations (see Sec. 4). In this lies the radical difference between pulse self-focusing and self-focusing of three-dimensional beams, the instability of which was noticed for the first time by R. V. Khokhlov (see Ref. 85, p. 520). The stability problem of a plane wave in a medium with cubic nonlinearity was considered in detail in Ref. 86, and of a plane wave modulated in space and time—in Refs. 18, 87. The evolution of temporal perturbations of a plane wave was analyzed in Ref. 88.

The one-soliton regime of propagation of a pulse of the form (2.34), considered here, is a particular solution of Eq. (2.9). Other soliton regimes are discussed in Sec. 4.

3. FAST PHASE CONTROL, COMPRESSION AND FORMATION OF OPTICAL PULSES

In this section we shall concentrate on one of the most important applications of the temporal self-action effects, i.e., the physics and methods of generation of femtosecond pulses. As was shown in Sec. 1, for an efficient compression of a transform-limited pulse it is necessary to modulate its frequency according to a linear law. It is clear that if we talk about producing pulses with lengths comparable to a period of optical vibrations, the frequency scanning range must be comparable to the carrier frequency. The only practically available method of creating the required frequency modulation is, at the present time, self-phase-modulation in a medium with practically "inertialess" nonlinearity (electronic Kerr effect).

"Inertialessness" of a nonlinear response is related usually to the smallness of the nonlinear correction to the refractive index ($n_2 \approx 10^{-13}$ CGSE units); for this reason it is essential to have large interaction lengths. The optimum media for producing self-phase-modulation turned out to be optical-fiber waveguides (Fig. 12). This section starts with a brief discussion of their nonlinear characteristics. Further we will talk about the theory of optical compression, with special attention paid to the most urgent problems: an increase of energy efficiency, improvement of compressed pulse quality, generalization of optimum compression theory to randomly-modulated pulses, discussion of possibilities for adaptive control of the profile and spectrum of femtosecond pulses.

The section concludes with a review of experimental achievements on the compression of optical pulses and a discussion of prospects for generation of ultrashort pulses in the IR and UV regions.

3.1. Materials for nonlinear optical phase modulators. Nonlinear properties of fiber-optics waveguides

The idea of using optical nonlinearity for creation of a phase modulator, i.e., a "temporal lens", was expressed and realized at the end of the 1960s.[95] It is natural that as the nonlinear materials used at that time were liquids composed of molecules with anisotropic polarization, which have a relatively large nonlinearity of the index of refraction ($n_2 \approx 10^{-11}$) and a relaxation time of the order of several picoseconds.

Self-phase-modulation in liquids with $n_2 > 0$ gives rise to positive frequency sweeping of a pulse in those of its parts where the profile curvature is positive. In order to compress such pulses, it follows from Sec. 1.4 that one must have media with anomalous group-velocity dispersion. Cells with alkali metal vapors,[96] devices made of a pair of diffraction gratings[97] and some types of interferometers[36] were used as such media. Compression coefficients ~ 10 (from 20 ps to 2 ns[98] and from 100 ps to 7 ps[99]) were achieved in experiments. The drawbacks of compression schemes that use unbounded media, are related to the inhomogeneity of frequency sweeping in a transverse cross section of the beam and to the close relationship between spatial and temporal self-action effects, leading to instabilities of compressed pulse parameters.

A radical change in the situation has become possible due to the use of single-mode optical-fiber waveguides as nonlinear phase modulators. The magnitude of nonlinear correction to the index of refraction in quartz glasses is small ($n_2 \sim 10^{-13}$ CGSE units), however, the smallness of n_2 is more than compensated by the possibility of sustaining a stable transverse profile of the light beam (diameter 5–10 μ) over distances of the order of a characteristic absorption length δ_0^{-1} (in the optical range $\delta_0^{-1} \sim 10^4$–10^5 cm). In addition, one must emphasize the practically "inertialess" nature of the nonlinear response, the high beam "strength" and the stability of the geometry.

The magnitude of the nonlinear correction to the index of refraction $\delta n = \tilde{n}_2 I$, $\tilde{n}_2 3.2 \cdot 10^{-16}$ cm^2/W, becomes comparable in quartz fiber-optics waveguides with the difference of the refractive indices ($n_c - n_{cl}$) only for an intensity of $I \approx 10^{12}$ W/cm^2. If one works in an intensity interval 10^6–10^9 W/cm^2, the self-action model based on the assumption of constancy of the radiation mode structure in a waveguide is quite adequate[7]. From the conservation of the radiation

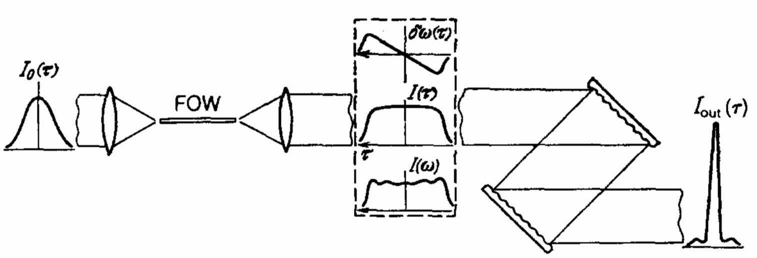

FIG. 12. Working schematic for compression of optical pulses using self-phase-modulation in a fiber-optics waveguide.

mode structure in the waveguide one more important advantage follows immediately—a high degree of homogeneity of frequency modulation in a transverse cross section of the beam. Real limitations on the frequency scanning range in a fiber-optics phase modulator are associated, above all, with competing nonlinear processes.[103,146]

3.2. Nondispersive and dispersive self-phase-modulation in fiber-optics waveguides

Self-phase-modulation of a real laser pulse leads to a complex phase time-variation law (see Fig. 7) even for a medium with inertialess nonlinearity (see Sec. 2.2). In other words, the "temporal lens", based on SPM, has, generally speaking, strong aberrations. It is easy to see, however, that the second-order dispersion can, to a large extent, correct the situation.[104] In this subsection we shall analyze the effects of dispersion on SPM, restricting the analysis to the area of normal dispersion ($\lambda < \lambda_{cr}, k_2 > 0$).

The second-order equation for the temporal complex amplitude (1.44) has, in accordance with (2.9), the form

$$i\frac{\partial\psi}{\partial\zeta} = -\operatorname{sign} k_2 \cdot \frac{1}{2}\frac{\partial^2\psi}{\partial\tau^2} + R|\psi|^2\psi - i\delta\psi; \qquad (3.1)$$

here $\tau = \eta/\tau_0$ is the normalized "running" time, the distance ζ is expressed in units of the dispersive length $\zeta = z/L_d$, the parameter $\delta = L_d\delta_0$ characterizes the absorption over a dispersion length.

The nonlinearity is characterized by the parameter $R = L_d/L_p$, where L_p is the length of self-phase-modulation $L_p = (\tilde{\beta}_1 I_{\rm eff})^{-1}$ (see (2.15)). Unlike the case of an unbounded medium, this length is determined by the effective value of radiation intensity in the waveguide

$$I_{\rm eff} = I_0\frac{\langle U^2\rangle}{\langle U^4\rangle}, \qquad (3.2)$$

where

$$\langle U^n\rangle = \int_0^{2\pi} d\varphi \int_0^{\infty} U^n(r)\, r\, dr,$$

and I_0 is the maximum value of the intensity. In practical calculations it is more convenient to use the expression

$$I_{\rm eff} = \frac{P_0}{S_{\rm eff}}. \qquad (3.3)$$

where P_0 is the peak value of pulse power, $S_{\rm eff} = \langle U^2\rangle^2/\langle U^4\rangle$ is the effective area of a mode, which differs only insignificantly from the area of the waveguide core[8)].

Typical modifications of a profile, spectrum and frequency modulation of a Gaussian pulse, obtained as a result of numerical solution of (3.1), are shown in Fig. 13.

The dynamics of the self-action process of a temporal profile is determined by relationships between characteristic lengths of self-phase-modulation L_p, the dispersion L_d and self-action L_{nl}. For experiments on the compression of transform-limited pulses with an initial length of several picoseconds[105] the situation is typical when the length L of a fiber-optics waveguide satisfies the inequality $L_p \ll L \sim L_{nl} < L_d$. In this case at the initial stage of pulse propagation the dominant process is the self-phase-modulation, leading to the broadening of the spectrum and the formation of linear frequency modulation within the pulse peak. Lowering of the frequency at the leading edge and its increase at the trailing edge under the conditions of normal group-velocity dispersion causes an additional, nonlinear spreading of the pulse and a flattening of its top. A result of the combined effect of dispersion and nonlinearity is the formation at the distance $z \sim 2L_{nl}$ of a practically rectangular pulse with linear frequency modulation.

Thus, the combined effect of nonlinearity and dispersion leads to a significant shape change of frequency modulation within a pulse: the region occupied by linear FM is significantly broadened (see Figs. 7 and 13). A decrease of the peak value of the intensity reduces the role of nonlinear effects, and the further evolution of the temporal profile is determined mainly by linear dispersive spreading.

3.3. Optical compressors

Since in fiber-optics waveguides $n_2 > 0$, an optical compressor must have anomalous dispersion[9)]. The simplest

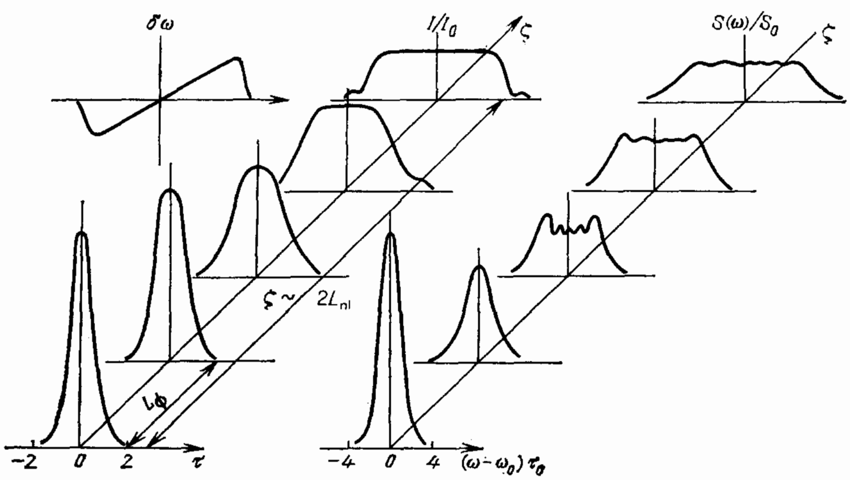

FIG. 13. Evolution of the profile and spectrum of an ultrashort pulse in the case of self-action in a medium with normal dispersion ($\tau = \eta/\tau_0$, $\zeta = z/L_\alpha$).

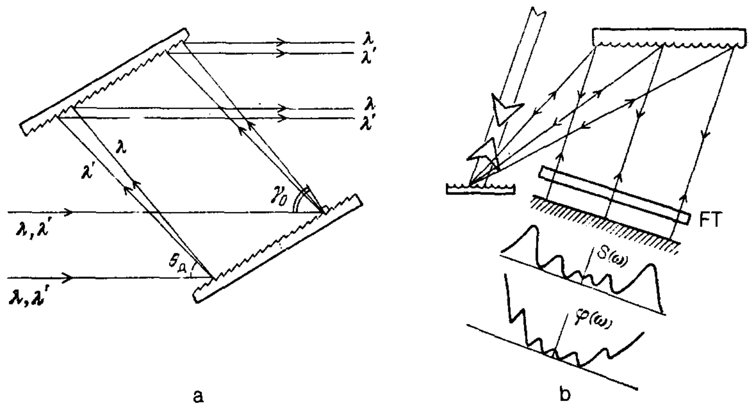

FIG. 14. Schematics of compressors with anomalous dispersion. a—grating type compressor, b—two-pass scheme (possibilities of controlling the amplitudes and phases of Fourier-components with the help of a phase transparency are shown).

optical compressor is shown in Fig. 14a. It consists of a pair of diffraction gratings placed parallel to each other. As can be seen from the diagram, the diffraction angle and, therefore, the length of the optical path of different spectral components of the pulse depends on the frequency. For $\Delta\omega/\omega_0 \ll 1$ the phase shift is

$$\varphi_K(\omega)=\varphi_0+\varphi_1(\omega-\omega_0)+\frac{1}{2}\varphi_2(\omega-\omega_0)^2+\dots, \qquad (3.4)$$

where the coefficients φ_1 and φ_2 depend on the parameters of a grating pair[97] in the following way:

$$\varphi_1=\frac{b(1+\cos\theta_D)}{c}, \quad \varphi_2=-\frac{b\lambda_0}{2\pi c^2}\,\frac{(\lambda_0/d)^2}{1-[(\lambda_0/d)-\sin\gamma_0)]^2}. \qquad (3.5)$$

In these expressions b is the distance between the centers of the gratings, γ_0 is the angle between the incident beam and the normal to the surface of the first grating, θ_d is the angle between the incident and diffracted beams.

Comparing expansions (3.4) with an expansion of the wave number k in series in powers of $(\omega-\omega_0)$, it is clear that a pair of gratings, placed at the distance b, is equivalent to an anomalously dispersive medium of the length b, with

$$k_2^{(K)}=\frac{\varphi_2}{b}. \qquad (3.6)$$

For typical values of the parameters $\gamma_0=60°$, $d=0.56\cdot 10^{-4}$ cm, $\lambda_0=0.5$ μm, the value of $k_2^{(K)}$ is of the order of 10^{-26} s^2 cm, and the effective dispersion length for $\tau_0\sim 1$ ps is $L_d=\tau_0^2/k_2^{(K)}\sim 1$ m.

Significantly larger values of dispersion can be achieved for glancing incidence of the light beam on the grating.[106] However, in that case the higher-order terms become important in the expansion, and the compressor ceases being a quadratic compressor.

We note that in the grating pair, shown in Fig. 14a, an undesirable effect develops—the spatial shift of high-frequency and low-frequency components. The indicated drawback can be removed by using a mirror that sends the radiation back to the grating pair. The spatial shift of the frequency components is compensated after a double passage of the beam[108] (see Fig. 14b).

For compression of frequency-modulated pulses with an initial pulse length in tens and hundreds of femtoseconds prism compressors were developed.[109] The advantages of prism compressors are related to low energy losses and the absence of a spatial frequency shift. The main applications area of prism compressors are intraresonator compression schemes.[110,111]

3.4. Modeling of compression systems—an optimum compressor

Practical realization of fiber-optics compression schemes requires the solution of a number of important questions, related to establishing optimum relations between the parameters of an initial pulse, optical waveguide and a compressor (see Refs. 112–114).

In a dispersion-free regime of SPM the calculation of compression can be carried out easily if one assumes certain values of τ_0 and $I_{\rm eff}$ and the waveguide length L for the parameters of an initial pulse. Indeed, using formula (2.18) for the nonlinear broadening of a Gaussian pulse, we obtain for $\varphi_{\max}\gg 1$

$$\Delta\omega\approx 0.88\,\varphi_{\max}\Delta\omega_0. \qquad (3.7)$$

An approximate expression for the compression degree s has the form (see (1.35))

$$s\approx 0.88\tilde{\beta}_1 I_{\rm eff}L. \qquad (3.8)$$

The optimum distance between the gratings (see (1.21)) is expressed in the following way:

$$b\approx 1.14\tau_0^2(k_2^{(K)}\tilde{\beta}_1 I_{\rm eff}L)^{-1}. \qquad (3.9)$$

The given expressions illustrate the physical principles of compression, but do not allow calculations of such important characteristics as the compressed pulse shape, its maximal power, etc. Real quantitative laws of the dispersive regime of compression were established in the papers of Refs. 112, 113 by methods of mathematical modeling. The models were based on Equation (3.1).

The results of numerical studies of compression for the system optical waveguide-quadratic compressor are presented in Fig. 15c in the form of the dependence of compression on the length of the fiber-optics waveguide for different values of the nonlinearity parameter R (see Equation (3.1)). It can be seen that for each value of the nonlinearity

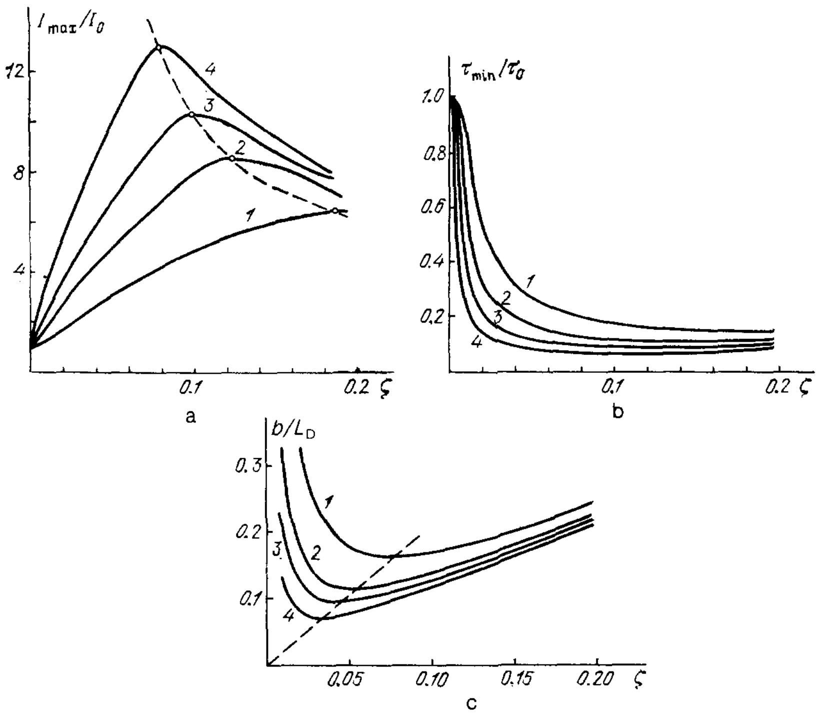

FIG. 15. Optimal conditions of compression. Calculated results: a—relative intensity of a compressed pulse as a function of $\zeta = z/L_\alpha$; b—length of a compressed pulse as a function of the reduced waveguide length ζ; c—optimal distance between the compressor gratings b/L_α as a function of ζ. Parameter of the curves is the ratio P/P_{crit}: 100 (1), 200 (2), 300 (3) and 500 (4).[114]

parameter there is an optimal length L_{opt} of a waveguide, at which the maximum compression degree s_{max} is reached. The existence of an optimum is due to the fact that for $z < L_{opt}$ the process of nonlinear broadening of the spectrum is not yet completed. At distances $z > L_{opt}$ the peak intensity of a pulse is significantly reduced, and this leads to the suppression of self-phase-modulation, while the continuing dispersive spreading of the pulse fronts reduces the possibility of its compression.

The analysis of the dependence of s on ζ and R given in Ref. 112 has made it possible to establish simple empirical formulas for the optimum length of a waveguide and the intensity gain

$$L_{opt} = CL_{nl} \quad \frac{I_{max}}{I_0} \approx C^{-1}R^{1/2}. \tag{3.10}$$

The constant C entering these expressions varies insignificantly with variation of the profile. For Gaussian pulses $\psi_0 = \exp(-\tau^2/2)$ the value of C is about 1.79; for pulses with a profile in the form of a hyperbolic secant $\psi_0 = \mathrm{sech}\,\tau$, $C \approx 1.84$. Figure 15c illustrates the dependence of the optimal distance between the gratings, expressed in units of L_d, on the waveguide length L and the nonlinearity parameter R. The case of larger nonlinearities is discussed in Ref. 128.

Expression (3.9) for the degree of compression has been obtained without taking into consideration competing nonlinear processes. In real experimental situations the maximum degree of compression is limited, as a rule, by the process of stimulated Raman scattering.

The process of stimulated Raman scattering, developing from the level of random noise, leads to a significant degradation of pumping under the condition[146] that

$$g_{c\ \mathrm{eff}} z \approx 16, \tag{3.11}$$

where g_c is the signal gain coefficient at the Stokes frequency, having in the optical frequency region the order $g_c \sim 2\cdot 10^{-11}$ cm/W. For $z < L_{nl}$ formula (3.8) leads to the limitation $s \leqslant 14k_0\tilde{n}_2/g_c$. This limitation has a fundamental character, since the quantity s is determined, finally, by the ratio of the real and imaginary parts of the cubic susceptibility.

Of course, the real picture of the Raman frequency shift is much more complicated, since the pulses at the main and Stokes frequencies "run apart" because of the differences in group velocities.

3.5. Special features of self-action of high-power femtosecond pulses

With a decrease of the length of initial pulses and an increase of input intensity to the level 10^{12} W/cm^2,[115,116] the character of nonlinear processes becomes significantly more complicated. The transition to new temporal scales leads to the necessity to reconsider initial assumptions, which were absolutely natural in the picosecond range of pulse lengths. Among these are the assumptions about slow variation of the complex amplitude, quasi-steady-state nature of the nonlinear response, negect of higher-order dispersion (1.10), etc. Recently, all these questions began attracting the attention of researchers.[71,117,118] However, the analysis, carried out in relation to the problems of fiber-optics compression for typical values of the parameters $\tau_0 \approx 50$–100 fs, $I_{eff} \sim 10^{10}$–10^{12} W/cm^2, and distances 1–15 cm, has shown that the main role here is played by the effects related to the rate of change of the nonlinear polarization (see Sec. 2.4 and Eq. (2.24)). The initial equation, written in dimensionless units, has the form

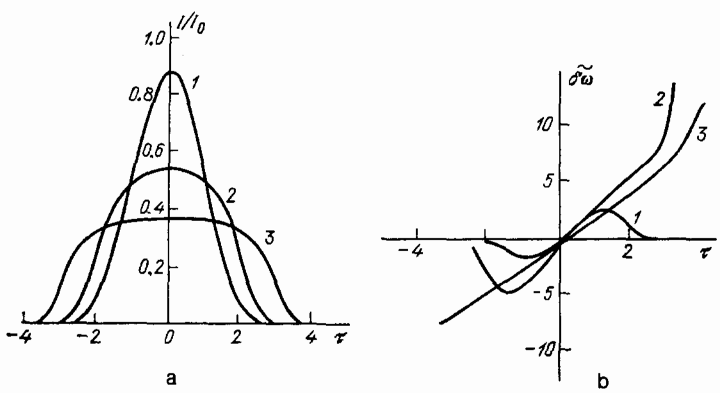

FIG. 16. Pulse shape (a) and the dependence of the instantaneous frequency $\tilde{\delta\omega} = \delta\omega\tau_0$ on the time (b) for different distances from the input to the waveguide $\tilde{z} = z/L_{np}$: 0.4 (1), 1.2 (2) and 2 (3).[118]

$$i\frac{\partial\psi}{\partial\zeta} = -\frac{1}{2}\frac{\partial^2\psi}{\partial\tau^2} + R|\psi|^2\psi - i\mu R\frac{\partial}{\partial\tau}(|\psi|^2\psi), \qquad (3.12)$$

where the parameter of "non-stationarity" $\mu = T_0/\pi\tau_0$ is proportional to the ratio of the optical period of oscillations $T_0 \sim 2$ fs to the initial length of the pulse and is of the order of 10^{-2}. The necessity of taking into account the last term in Eq. (3.12) is due to the fact that $\mu R \sim 1$.

Special physical characteristics of self-action under these conditions were determined with the help of computer simulation.[118] At distances $z < L_{nl}$ (Fig. 16) an increase of the group delay for the pulse peak leads to the steepening of the pulse trailing edge.

Further propagation of the pulse is accompanied by a flattening of its top and an increase of the rate of frequency modulation. At this stage, the frequency sweeping rate is decreasing at the leading edge of the pulse and is increasing at the trailing edge. The influence of this process on the pulse spectrum is shown in Fig. 17a. The spectrum becomes asymmetric, and develops a "wing" in the region of high frequencies, and the intensities of the spectral components in the Stokes region are slightly increasing.

The influence of all these processes on the compression degree that can be achieved is shown in Fig. 17b, which shows the dependence of s on the waveguide length $\tilde{z} = z/L_{nl}$ for a fixed value of the nonlinearity parameter and various values of the nonstationarity parameter μ. It can be seen that with an increase of μ a decrease of compression degree and a shift of the optimum compression point towards the area of larger waveguide lengths is taking place.

Thus, taking into account the rate of change of the nonlinear polarization makes it possible to interpret a number of special features of the compression of femtosecond pulses mentioned in the experimental studies (Ref. 115, 116).

3.6. Compression of random pulses

Typically, real systems have amplitude and phase fluctuations that influence the self-action of pulses and their compression. Some principles regulating these processes have been analyzed in Ref. 119 by methods of computer simulation. The Monte-Carlo method was taken as the basis of the numerical investigation.[120]

We shall now discuss some results of computer modeling of pulses with random phase-amplitude modulation of the form

$$\tilde{\psi}(\tau, 0) = \psi_0(\tau)\,[1 + \sigma\xi(\tau)], \qquad (3.13)$$

where $\Psi_0(\tau) = \exp(-\tau^2/2)$ is the deterministic profile, and $\xi(\tau) = \xi_R + i\xi_I$ is a random process, the quadrature components of which are distributed according to the normal law with a zero average value and variance equal to 1; the parameter σ characterizes the noise level. The autocorrelation function was assumed to be Gaussian (1.22). Figure 18 shows the transformation of temporal intensity distribu-

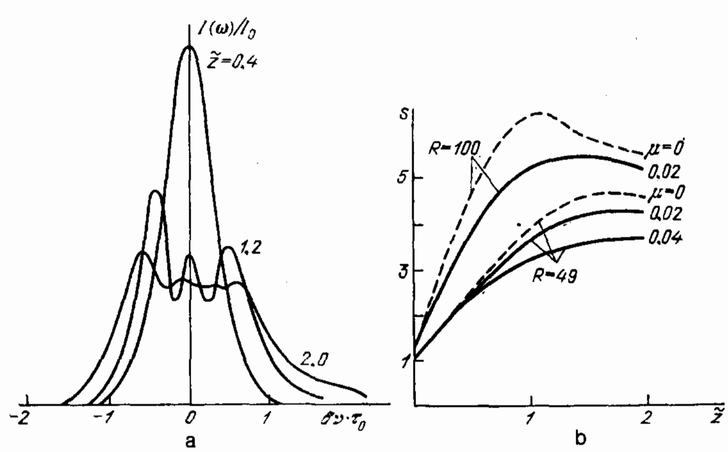

FIG. 17. Spectrum modification of a femtosecond pulse with distance (a) and the dependence of the degree of compression s on the correct length $\tilde{z} = z/l_{np}$ of the waveguide (b) for different values of parameter μ.[118]

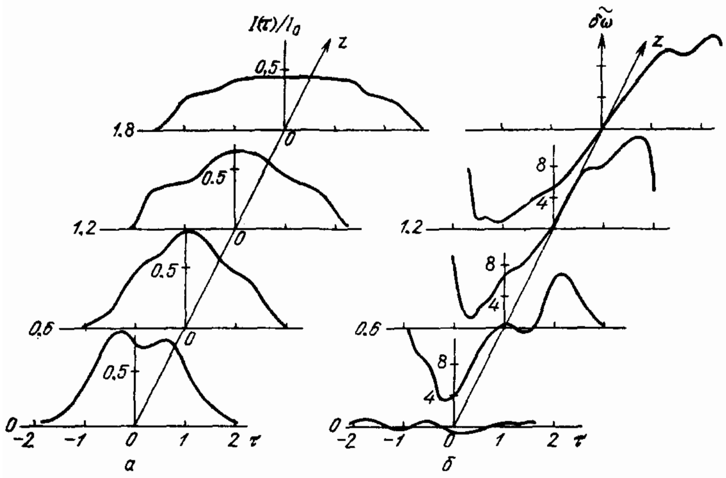

FIG. 18. Modification of the shape of a realization of a random pulse (a) and of the distribution of the instantaneous frequency $\delta\tilde{\omega} = \delta\omega\tau_0$ (b) with the distance $\tilde{z} = z/L_{np}$.

tions and the instant value of the frequency with an increase in the distance traversed by one of the random pulses along fiber-optics waveguide. It can be seen how the smoothing of intensity fluctuations and linearization of frequency modulation in the central part of the pulse take place during the self-action process. Fluctuations of intensity and frequency modulation are localized mainly at the leading and trailing pulse edges that correspond to low- and high-frequency components of the spectrum.

Figure 19 shows the dependence of the compression degree $\bar{s}$ averaged over the ensemble of realizations on the length of the fiber-optics waveguide, expressed in units of L_{nl}. For comparison, the corresponding dependence calculated for a transform-limited pulse ($\sigma = 0$) is shown by a dashed line. An increase of the noise fluctuation variance σ^2 leads to a decrease of the average value of the maximum degree of compression, which can be observed, as before, at $z \sim 2L_{nl}$. A decrease of the correlation time $\tau_{K,0}$ also leads to a decrease of the value of s.

The special features of self-action of partially coherent pulses noted in numerical experiments—the "pushing out" of fluctuations toward high- and low-frequency wings of the spectrum—permits the stabilization of parameters of compressed pulses by spatial filtration of their spectral components in a grating compressor. The simplest spectral filtration is achieved by placing a diaphragm in the beam at the plane of beam-returning mirror (see Fig. 14b).

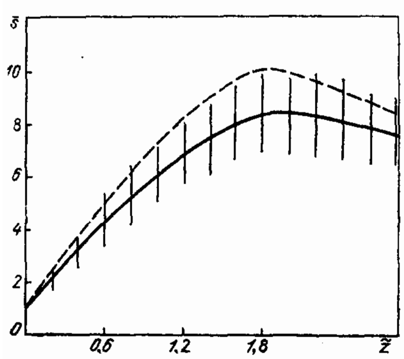

FIG. 19. Dependence of the mean compression of random pulses on the length $\tilde{z} = z/L_{np}$ of the waveguide.[119] The dashed line corresponds to a transform-limited pulse ($R = 300$, $\sigma = 0.14$, $\tau_c = 0.64\,\tau_0$). The standard deviation is indicated.

Computer modeling shows that the addition of a frequency filter with a rectangular transmission function $K(\omega) = 1$ for $\omega_1 < \omega < \omega_2$, where the transmission band $\Delta\omega_{tr}$ was chosen to be equal to the value of the spectral broadening of a deterministic pulse, lowers the fluctuation level of the degree of compression approximately by a factor of two. The established regularities have been confirmed by experimental results.[119,121]

3.7. Adaptive optics of ultrashort pulse generators

Spatial separation of the pulse spectrum (Fig. 14b) in a compressor allows constructing schemes for profile control on a femtosecond time scale.[119,121] The greatest opportunities here are opened up by the action on the phase of the pulse Fourier-components.[122] Let us briefly discuss the problems of spectrum control as applied to the problem of an "ideal" compressor. Essentially, we are discussing a device performing complete phase matching of all the spectral components of the pulse and, therefore, forming a pulse of shortest possible length. The expression for transfer coefficient has the form:

$$K(\omega) = |K(\omega)| e^{i\varphi_K(\omega)}, \tag{3.14}$$

where, for a passive linear compressor, $|K(\omega)| \leqslant 1$.

The spectrum of a pulse which has undergone self-phase-modulation, can be written in the following form

$$\tilde{A}(\omega) = |\tilde{A}(\omega)| e^{i\varphi(\omega)}.$$

At the output of a compressor we have

$$\tilde{A}_K(\omega) = |K(\omega)|\,|\tilde{A}(\omega)| e^{i(\varphi+\varphi_K)}.$$

For an ideal compressor (see also Sec. 1.4) we have

$$\varphi_K(\omega) = -\varphi(\omega), \quad |K(\omega)| = 1.$$

As was shown in Sec. 3.3, real grating and prism compressors perform phase matching of spectral harmonics in a parabolic approximation. The dependences $\varphi(\omega)$, arising in

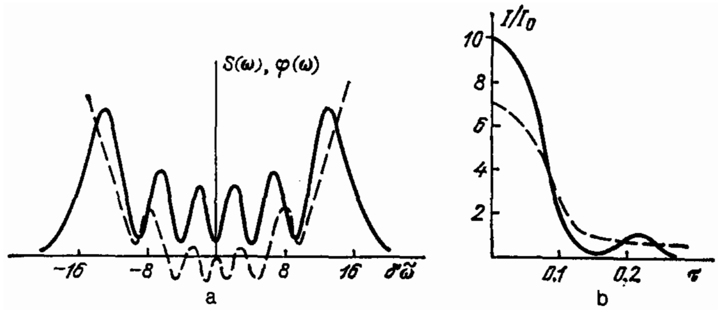

FIG. 20. a—Spectral power density and phases of Fourier components (dashed line) of a pulse after non-dispersive SPM (a) for $R\zeta = 18$. b—Shape of the compressed pulse after an "ideal" (continuous curve) and a quadratic compressor (dashed line).

the process of self-phase-modulation, are more complicated. As an illustration, Fig. 20a shows the dependences of $|\tilde{A}(\omega)|^2$ and $\varphi(\omega)$ for the case of a pulse which had experienced dispersion-free self-phase-modulation.

Figure 20b shows the shape of a compressed pulse for the case of ideal and quadratic compressions. It can be seen that the use of an ideal compressor gives significant advantage in the degree of compression and guarantees higher radiation contrast.

In practice, an ideal compressor can be realized with the aid of a usual grating pair and a phase transparency, placed in the plane of the "returning" mirror (see Fig. 14b). The function of the transparency is to remove phase aberrations—deviations of the real dependence $\varphi(\omega)$ from parabolic dependence. The controlled phase transparencies based on liquid crystals are successfully used at the present time for schemes for phase correction of coherent optical beams.

Combination of amplitude and phase methods of control of a spectrum broadened by SPM, allows one not only to achieve "ideal" compression, but also to solve a number of other profile control problems, in particular, to form from one pulse of a signal generator a train of several pulses.

In conclusion we indicate that the use in optical compression schemes of fast controllable elements makes it possible to develop adaptive systems that control spectral and temporal characteristics of ultrashort optical pulses, and systems for controlling "light by light".

3.8. Experimental results. Generation of pulses with pulse length down to 10^{-14} sec

Among the experimental investigations of recent years, there are three characteristic directions oriented towards applications in the area of fast-process spectroscopy and in optical information systems.

The first of these directions is the high-efficiency compression of quasicontinuous pulses from solid-state lasers with active mode synchronization with an initial pulse length in tens of picoseconds to the subpicosecond region.[108,123,124]

The second direction is the compression of frequency-tunable pulses from dye lasers with synchronous pumping, with initial pulse lengths of several picoseconds to tens of femtoseconds.[105,125]

The third direction is generation of the shortest possible pulses (8–30 fs) by compression of ring dye laser pulses with an initial pulse length of 40–100 fs.[115,116,126,127]

We shall now proceed to analyze the results obtained in these three directions. The authors of Ref. 108 have achieved an 80-fold compression of pulses of the second harmonic of quasicontinuous generation (repetition rate 100 MHz) of a YAG:Nd^{3+} laser with active synchronization of modes. The initial pulse length is ~ 33 ps, the peak power is ~ 240 W. The unusual scheme of a grating compressor used in the experiment has made it possible to avoid a diffractive shift of the beams and to obtain at the exit of the system pulses with a length of 410 fs and a peak power exceeding the input power by more than a factor of five.

Obvious advantages are opened up by the use of a signal generator built on the basis of a YAG:Nd^{3+} laser, operating in the regime of double modulation—active mode synchronization and Q-switching.[124] Frequency doubling was produced in a KDP crystal with an efficiency of 50%, and this made it possible to produce transform-limited pulses with a pulse length of 33 ps and a peak power of 130 kW. Efficient compression of such pulses down to 1 ps has been produced by using a short section of a single-mode optical waveguide (with a length of 10 m) and a grating compressor. The maximum degree of compression in these experiments was limited by the process of stimulated Raman scattering.

An alternative approach is the compression of YAG:Nd^{3+} laser radiation on the main frequency. Using this approach, the authors of Ref. 123 produced 45-fold pulse compression of an 80 ps initial pulse down to 1.8 ps. After frequency doubling in the KDP crystal, an average power of 500 mW was obtained at a repetition rate of 82 MHz.

An efficient 15-fold compression of radiation from a YAG:Nd^{3+} laser with mode synchronization and Q-switching was achieved in Ref. 129 with the help of a short section of an optical waveguide of 10 m length. Hybrid schemes are promising, in which solid-state laser pulses, compressed with the help of fiber-optics waveguides, are used for synchronous pumping of dye lasers.[130]

The second of the mentioned directions—compression of picosecond pulses from dye lasers with synchronous pumping served as a basis for creation of spectra-analytical systems with femtosecond temporal resolution.

Typical orders of magnitude for quantities involved in experiments are as follows. Synchronously pumped dye lasers generate transform-limited pulses with pulse lengths of

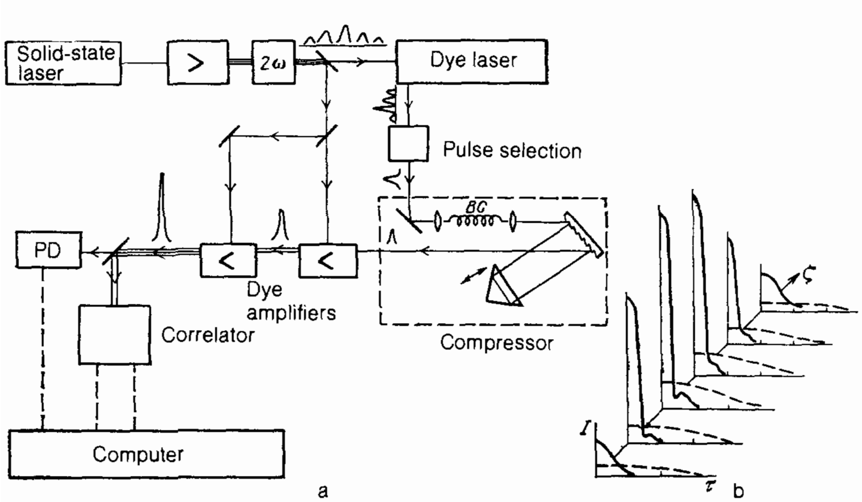

FIG. 21. a—Schematic diagram of generation of frequency-tunable transform-limited subpicosecond pulses using a fiber-optics compressor. b—Compression results for a train of pulses: a pulse at the optical waveguide exit (dashed lines), and after compressor (solid lines).[112]

5—6 ps, peak power ~ 1 kW and repetition rate ~ 4 MHz. After traveling a distance of 30 m along a fiber-optics waveguide, the profile acquires a typical rectangular shape (see Fig. 13), and the length is increased by approximately a factor of five. At the output of a grating compressor one obtains pulses with $\tau_{min} \sim 450$ fs and peak power ~ 3 kW.

Creation of a source of high-power frequency tunable pulses is reported in Ref. 112; this source uses a pulsed solid-state laser with mode synchronization as a signal generator. The special feature of this scheme is that the compressed pulse is amplified by a two-cascade amplifier, and the same signal generator serves as a pumping source for the synchronously excited laser and for the dye amplifiers (see Fig. 21). Such an approach allows one to reduce the noise level and to avoid a number of technical difficulties, related to synchronization of the amplifying cascades. The energy of the compressed pulse after two cascades of amplification was 4.2 μJ, which corresponds to a peak power of 7 MW.

Significantly higher degress of compression can be obtained with the help of cascade schemes.[125] The main idea of cascade compression is shown in Fig. 22. A further improvement of this scheme by the introduction of an intermediate dye amplifier pumped by a YAG:Nd^{3+} laser has made it possible to achieve the shortest possible output pulse lengths of 16 fs with a peak power of 88 kW and a repetition rate of 1 kHz. The total compression coefficient of the two cascades was $s \simeq 340$.[131]

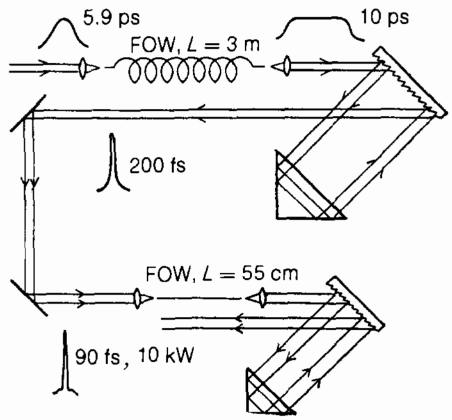

FIG. 22. Schematic diagram of two-cascade compression.[125]

Another approach to the formation of maximally shortest pulses is based on the use of fiber-optics compressors for compression of pulses from dye ring-lasers. The first experiments in that direction were carried out in 1982. The authors of Ref. 126 have obtained 30 fs pulses by a three-fold compression of radiation from a dye laser. The dispersive self-phase-modulation has been carried out in a single-mode optical waveguide with a length of 15 cm. The results of later experiments on compression of femtosecond pulses are given in Table I. They clearly illustrate the progress in methods of generation, amplification, and compression of femtosecond pulses. We note that the 8 fs pulses obtained recently (Fig. 23) correspond to approximately four periods of optical oscillations; therefore, in the spectral region covered by dye lasers experimental results have approached very closely to the theoretical limit.

3.9. Generation of femtosecond pulses in the IR and UV regions

The problem of generation of high-power pulses in the IR and UV regions is interesting from the point of view of fundamental applications in laser photochemistry, plasma physics, semiconductors, etc. Application of traditional fiber-optics compression methods common in the visible and near IR regions is complicated by technological difficulties of making single-mode optical waveguides with small losses. In the infrared region ($\lambda = 2$–11 μm) significant progress was achieved over the last several years.[132] In the ultraviolet region ($\lambda \sim 0.25$–0.35μ) optical losses, caused by Rayleigh scattering ($\sim \lambda^{-4}$) and by absorption, reach hundreds of decibels per kilometer.

Still, the idea itself of using SPM and dispersive compression turned out to be quite productive also in the IR region. The starting point here were experiments of Corkum.[133] In these experiments pulses from a CO_2-laser with an initial length $\tau_0 \approx 2$ ps entered a high-pressure ($p = 10$

TABLE I. Compression of femtosecond pulses.

Initial pulse length, fs	Input power kw	Waveguide length cm	Output pulse length length, fs	Compression coefficient	Repetition rate Hz	Gear references
90	6,7	15	30	3	10	1982 [126]
65	55	0,8	16	4	10	1984 [127]
110	260	1,5	12	8	800	1984 [115]
40	250	0,7	8	5	5000	1985 [116]

atm) regenerative amplifier. In the process of formation of a train of output pulses it was possible to observe their shortening from 2 ps to 600 fs at the characteristic values of peak intensity $\sim 10^{12}$ W/cm^2. The author of Ref. 133 made the assumption that the observed shortening of pulses is related to the formation of an electron density wave. An increase in the concentration of free charge carriers induced by radiation causes a change in the real and imaginary parts of the index of refraction and, therefore, leads to frequency modulation. During propagation of a frequency-modulated pulse in a medium with anomalous dispersion (in the experiment under discussion these were laser windows made of NaCl crystal) the pulse compresses.

In subsequent theoretical papers,[134,135] an analysis of mutual influence of high-power pulsed radiation from a CO_2 laser and the electron density waves induced by it has been carried out. It was shown that if the intensity of the laser pulse is large enough to cause a variation of free electron concentration N_e, the resulting change in the index of refraction causes an increase of the instantaneous value of the frequency with time (positive "chirp") at the leading edge of the pulse and an increase in absorption at the trailing edge.

Various physical mechanisms, leading to an increase of N_e-"heating" of electrons by radiation with subsequent ionization by electron collisions and photoionization of electronically-excited atoms in the field of intense radiation are analyzed in Ref. 135. We note that questions of avalanche ionization during propagation of high-power pulses have been discussed by Yablonovich and Bloembergen (see, for example, the review of Ref. 55).

Significant SPM arises during propagation of infrared picosecond pulses in semiconductors. This was convincingly demonstrated in recent experiments.[136] During the passage of a pulse with an energy of $W_0 \sim 1 \mu$J ($\tau_0 \sim 8$ ps, λ_0 in the range 4–9 μm) through an InSb sample of a thickness of 300 μm, a decrease in the index of refraction of $\Delta n = 4 \cdot 10^{-2}$ was observed during the pulse period. Theoretical analysis showed that the principal mechanism leading to the increase of transparency and the change in Δn is the dynamic shift of the edge of the absorption band.

In conclusion we shall discuss briefly the possibilities of generating high-power picosecond pulses in the UV region. The most obvious prospects here are related to frequency-doubling of pulses from synchronously-pumped dye lasers and their subsequent amplification in excimer amplifiers. At the present time, pulses with pulse lengths of 2 ps and 5 ps[137,138] and energy ~ 10 mJ have already been produced in such systems.

Further advances into the range of femtosecond pulses is associated with a decrease in the length of initial dye laser pulses. Figure 24 shows the modified scheme of the experimental arrangement developed by the authors.[138] As new elements, it includes a fiber-optics compressor analogous to the one shown in Fig. 21, and a two-cascade dye amplifier, pumped by an Xe-Cl laser. We note that the amplifier serves also as a selector of a single pulse from the train of dye laser pulses. Therefore, a transform-limited dye laser pulse ($\tau_0 \sim 8$ ps) was amplified, after 20-fold compression, to an energy of 50 μJ. After frequency-doubling in a thin KDP crystal, the resulting pulse ($\lambda = 0.308\ \mu$m) entered an excimer amplifier. The wide XeCl amplification band ($\Delta\nu \sim 160$ cm^{-1}) allowed one to obtain high-power UV pulses with lengths of ~ 350 fs.

Thus, during the last several years there was laid the foundation for realization of efficient generators of femto-

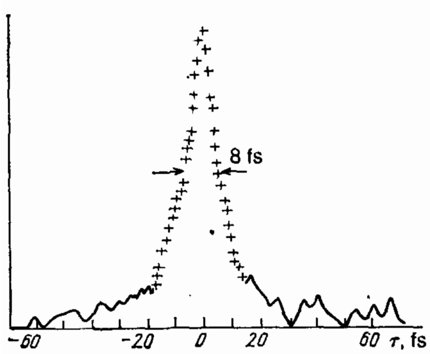

FIG. 23. Experimental autocorrelation function for intensity of a femtosecond pulse.[116]

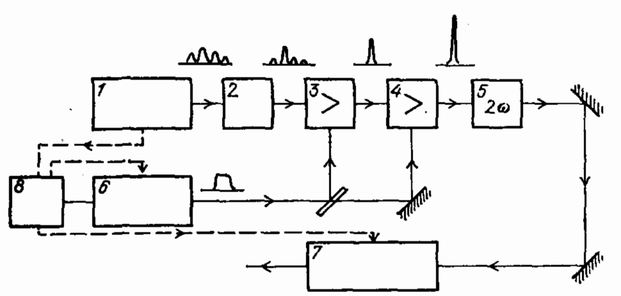

FIG. 24. Schematic block—diagram of a subpicosecond pulse generator for the UV region. 1—dye laser; 2—compressor; 3, 4—dye amplifiers; 5—frequency doubler; 6—excimer pump laser; 7—excimer amplifier; 8—synchronization scheme.

second pulses over the wide spectral range from 0.3 to 10 μm.

4. OPTICAL SOLITONS

The possibility in principle to realize experimentally[9] soliton propagation regime for picosecond pulses in single-mode fiber-optics waveguides was indicated by the authors of Ref. 139. The first correct experiments, in which optical solitons were observed, were performed in 1980.[4] Subsequent theoretical and experimental investigations brought out not only a number of subtle peculiarities in the formation, propagation, and interaction of solitons under real conditions, i.e., in the presence of perturbing factors, but also indicated the prospects of numerous technical applications (see the review articles of Refs. 2, 4, 140, 141). In this sense, optical experiments play a leading role in the physics of solitons.

This section contains a summary of the principal results observed in the course of investigating Schrödinger-type solitons. The main attention is paid to the physical problems directly related to the applications of soliton effects to information transmission in fiber-optics waveguides and for producing ultrashort pulses. However, for the sake of completeness of presentation, we start with an analysis of some properties of one-soliton and multi-soliton solutions of the Schrödinger equation.

4.1. One-soliton and multi-soliton solutions of the nonlinear Schrödinger equation

We shall neglect optical losses in Eq. (3.1), and by renormalizing $q(\tau,\zeta) = R^{1/2}\psi(\tau,\zeta)$, we write (3.1) (for $k_2 < 0$) in the form convenient for mathematical analysis

$$i\frac{\partial q}{\partial \zeta} = \frac{1}{2}\frac{\partial^2 q}{\partial \tau^2} + |q|^2 q. \qquad (4.1)$$

One of the soliton solutions $q = \operatorname{sech}\tau$ was obtained in Sec. 2.5. It is easy to see that the solution can be generalized in the following form:

$$q(\tau, \zeta) = \varkappa \operatorname{sech}(\varkappa\tau)\, e^{-i\varkappa^2\zeta/2}, \qquad (4.2)$$

where $\varkappa$ is the form-factor, determining the pulse amplitude and its length $\tau_0/\varkappa$. The condition of pulse stationarity $R = 1$ leads to the expression for the critical power (see Sec. 2.5)

$$P_{\text{crit}} = \frac{k_2}{\tau_0^2}\,\frac{S_{\text{eff}}n_0}{k_0\tilde{n}_2}. \qquad (4.3)$$

Significant stability of soliton pulses towards regular and noise perturbations is supported by results of theoretical and numerical analysis.[142,143] In particular, if the initial conditions have the form:

$$q(\tau, 0) = q_0 \operatorname{sech}\tau, \qquad (4.4)$$

where $q_0 = 1 + \xi$, $-0.5 < \xi < 0.5$ is the perturbation of the amplitude, then for $\zeta \to \infty$ the solution (4.1) has the form (4.2) with the form-factor $\varkappa = 1 + 2\xi$.[143]

The possibility of obtaining solitons from pulses of different shape and their stability towards perturbations are of fundamental significance for their use as carriers of information.

Another quite important class of solutions of Eq. (4.1) are the bound states of solitons corresponding to the initial conditions (4.4) for $q_0 = N$, where $N \geqslant 2$ is an integer. The principal properties of such pulses are analyzed in Refs. 142, 143. By using the inverse scattering problem method, it was shown that the pulses with a profile in the form $N \operatorname{sech}\tau$ constitute a nonlinear superposition of solitons with form-factors $\varkappa_n = (2n-1)$, where $n = 1, 2, \ldots, N$. For $q_0 = N$ the shape of the profile q_N can be found for arbitrary τ, ζ from the solution of a linear system of N equations. In addition, it was shown that the solution q_N is stable with respect to small perturbations of the amplitude q_0 of the type $q_0 = N + \xi$.

An important characteristic of N-soliton pulses is their

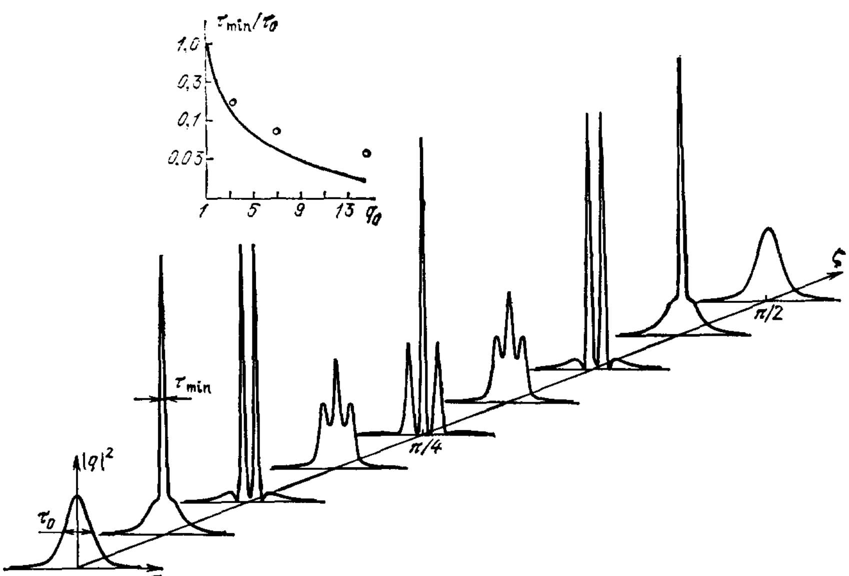

FIG. 25. Self-compression of an N-soliton pulse for $N = 4^5$ and (on the insert) the dependence of the minimal pulse length on N.[155]

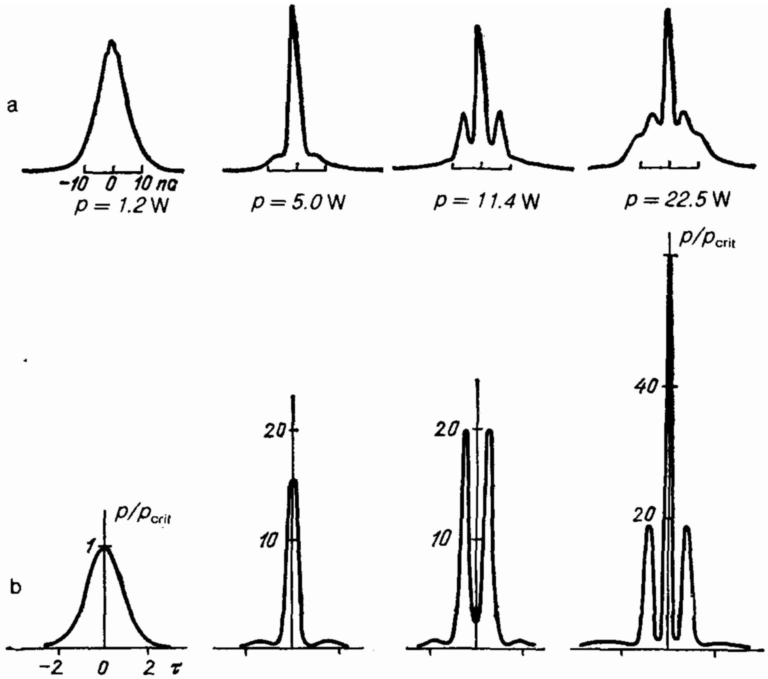

FIG. 26. Experimental profiles of autocorrelation functions for intensity[4] (a) and calculated intensity profiles for $N = 1$, 2, 3 and 4 (b) (from left to right).

periodicity in ζ, with the period, when expressed in dispersion lengths, equal to $\pi/2$. It is essential that for an arbitrary $q_0 > 1$, the initial propagation stage of a transform-limited pulse corresponds to self-compression, which indicates a possibility of creating fiber-optics compressors for transform-limited pulses in the near IR region. Profile modification in the process of propagation is illustrated in Fig. 25 for $N = 4$. All the described characteristics of soliton pulse self-action were confirmed experimentally.[4] Figure 26a shows the experimentally obtained temporal profiles of the autocorrelation function for the intensity at the waveguide output and the calculated intensity profiles for $N = 1, 2, 3$ and 4, and $\zeta = \pi/4$ corresponding to them.

Successful experiments with optical solitons, the results of which agree with the theory not only qualitatively, but also quantitatively, have stimulated the emergence of three important directions of research: 1) solitons in real communication lines, 2) control over the profile and spectrum of picosecond pulses, including their compression down to the range of femtosecond lengths, and 3) creation of soliton lasers.

We shall now discuss the main results obtained in these three directions.

4.2. Propagation of solitons over super-large distances—the problems and prospects

In order to find the limiting capabilities of optical information systems, it is necessary to take into account a number of such factors of perturbation as optical losses, interaction of solitons in the train of pulses, higher-order dispersion, competing nonlinear processes, etc.

The effects of low optical losses on the dynamics of soliton pulses can be estimated by the perturbation method developed in Ref. 140. In the presence of losses, the energy of a pulse

$$W(\zeta) = \int_{-\infty}^{\infty} |q|^2 \, d\tau$$

decreases with distance according to the exponential law:

$$W(\zeta) = W_0 e^{-2\delta\zeta}. \tag{4.5}$$

If optical losses over a distance of the order of the dispersion length are small, then the form-factor of a soliton will decrease, and its length will increase:

$$\begin{aligned} \varkappa(\zeta) &= \varkappa_0 e^{-2\delta\zeta}, \\ \tau(\zeta) &= \tau_0 e^{2\delta\zeta}. \end{aligned} \tag{4.6}$$

For example, for losses ~ 0.2 db/km the pulse length will increase approximately by a factor of 2.7 over a distance of about 20 km for an initial pulse length $\tau_0 \sim 10$ ps.

More complicated is the question concerning the influence of large optical losses ($\delta\zeta > 1$) on the dynamics of one-soliton and N-soliton pulses.[144,145] In this case the modification of the profile is investigated by the methods of computer simulation. Figure 27 shows the ζ-dependences obtained by the authors of Ref. 145 of the mean square of the length of an N-soliton pulse ($N = 2$) for different levels of losses. The dashed lines correspond to the case $N = 1$. With an increase of the parameter δ one can observe the increase of the spatial period of pulsations $\langle \tau^2(\zeta) \rangle$ and the increase of modulation depth. In numerical experiments[145] a decay of the bound state of solitons into two pulses moving in opposite directions was discovered also.

The broadening of pulses caused by optical losses can be reduced to a minimum and even completely removed by using the process of stimulated Raman scattering.[147] In the approximation of a given pumping field the power of a

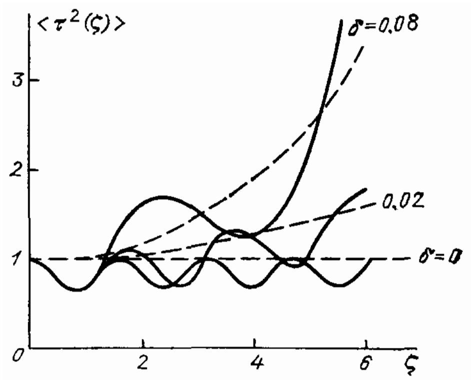

FIG. 27. Solitons in an absorbing medium. Dependence of the RMS pulse length on the distance $\zeta = z/L_\alpha$ for different levels of losses $\delta = L_\alpha \delta_0$[145]; solid lines—$N = 2$, dashed lines—$N = 1$.

Stokes wave increases exponentially with distance[146]:

$$P_s(z) = P_s(0)e^{g_s P_p z/S_{\rm eff}}, \tag{4.7}$$

where the amplification coefficient g_s is of the order of 10^{-11} cm/W for a frequency shift $\eta_p - \eta_s \sim 440$ cm^{-1}. The effective area $S_{\rm eff}$ in formula (4.7) is determined by the overlap integral $\langle v_p^2\rangle\langle v_s^2\rangle/\langle v_p^2 v_s^2\rangle$ and in a single-mode optical waveguide differs only slightly from $S_g = \pi a_0^2$.

The possibility of compensating for optical losses by Raman amplification has been demonstrated convincingly in recent experiments.[147] Transform-limited pulses from a color-center laser ($\lambda_s = 1.56\,\mu$m, $\tau_{1/2} = 10$ ps) were introduced into a single-mode fiber-optics waveguide with a length $L = 10$ km. Continuous pumping radiation ($\lambda_p = 1.46\,\mu$m, $P = 125$ mW) was introduced from the output end of the waveguide. In the absence of pumping, the length of an output pulse increased by approximately a factor of 1.5 (Fig. 28), however, the use of Raman amplification has made it possible to compensate for the broadening of the pulse completely (dashed line in Fig. 28).

In Ref. 148 the transmission of soliton trains over super-large distances was studied numerically in the presence of periodically located amplifying areas of a fiber-optics waveguide. It is shown that with the optimal choice of system parameters it is possible to reach the information transmission rate ~ 10 Gbits/sec for a distance of $L \sim 10^3$ km.

Questions of nonlinear stabilization of pulse lengths over relatively small distances $\zeta < 1$ are discussed in Ref. 149.

We shall now discuss the influence of higher-order dispersion on the dynamics of a temporal profile. By including cubic terms in the expansion (3.2) we obtain the following equation for the complex amplitude of a temporal profile (see (1.26)):

$$i\frac{\partial\psi}{\partial\zeta} = -\frac{1}{2}\frac{\partial^2\psi}{\partial\tau^2} + \frac{1}{6}i\mu_1\frac{\partial^3\psi}{\partial\tau^3} + R|\psi|^2\psi, \tag{4.8}$$

where the parameter $\mu_1 = 2L_d/L_d^{(3)}$ characterizes the relative contribution of the third-order dispersion. In the area of maximum transparency of quartz glass $\lambda \sim 1.5\,\mu$m this parameter is small ($\mu_1 \sim 10^{-2}$ for $\tau_0 \sim 1$ ps), and the third-order dispersive effects are estimated with the help of perturbation theory. The authors of Ref. 140 have shown that in this case there appear insignificant distortions of the profile and a correction to the group velocity of the order of μ_1. Qualitative results of perturbation analysis are confirmed by results of numerical experiments,[150] even for $\mu_1 \sim 1$. Typical intensity profiles are given in Fig. 29a for different values of ζ ($R = 1$, $\psi_0 = \operatorname{sech}\tau$, ζ is expressed in terms of the dispersion length).

However, as the radiation length λ approaches the value where the group velocity has zero dispersion ($\lambda_{\rm crit} = 1.3\,\mu$m; see Fig. 5) and the nonlinearity parameter R increases, the physical picture of self-action changes significantly. An irreversible decay of the initial pulse into fragments takes place, and its total width and additional group delay increases significantly. Characteristic intensity profiles are shown in Fig. 29b. The orders of magnitude of the quantities involved here are the following: the length of a picosecond pulse is doubled due to self-action at a distance of ~ 6 km for the peak power $P_0 \sim 1$ W.

For high-power subpicosecond pulses a significant perturbing factor is the nonlinear correction to the group velocity, which is responsible for the formation of a profile shock wave (self-steepening) (see Secs. 2.4 and 3.5). The presence of a second-order anomalous dispersion, however, leads to the stabilization of the steepness of the trailing edge. In Ref. 71 a soliton-type solution of Eq. (3.12) was obtained for $k_2 < 0$, a special characteristic of which is the presence of nonlinear frequency modulation. The authors of Ref. 151 in numerical experiments have studied in detail the influence of the rate of change of the nonlinear polarization amplitude on

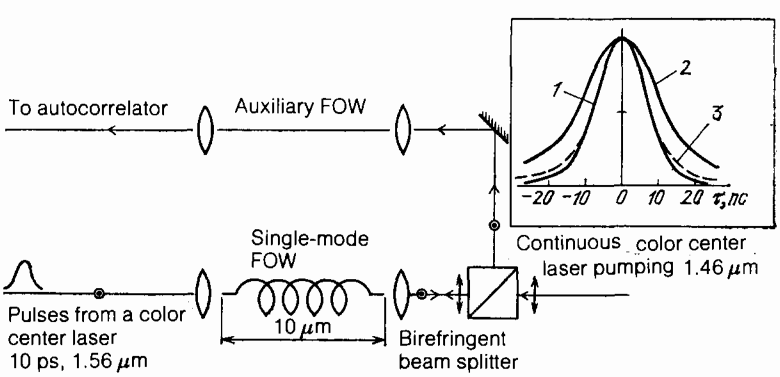

FIG. 28. Schematic arrangement for the compensation of losses during soliton propagation in long optical waveguides using stimulated Raman scattering.[147] On the insert are shown profiles of autocorrelation intensity functions: 1—input pulse, 2—output pulse in a waveguide with losses, 3—after compensation of losses.

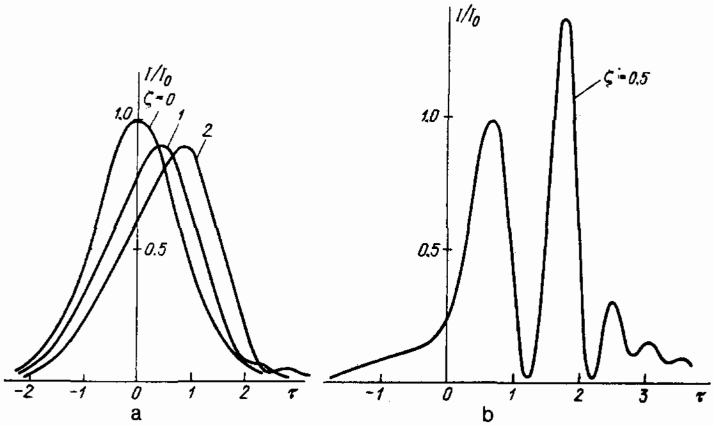

FIG. 29. Nonlinear propagation of ultrashort pulses for $\lambda \sim \lambda_{crit}$ ($k_2 \approx 0$).[150] a—Modification of pulse profile with distance for $P = P_{crit}$. b—Same for $P = 10\,P_{crit}$.

the propagation of N-soliton pulses and have shown the possibility of their decay.

Transmission of information along fiber-optics communication lines is planned to be accomplished by a sequence of solitons, and therefore the problems of soliton interaction are quite important. The physical picture of a Schrödinger-type soliton interaction is discussed in Ref. 152 by perturbation theory methods, and in Ref. 153 by the inverse problem method of scattering theory. An important result is that the propagation dynamics of a soliton pair of the form

$$q(\tau, 0) = \text{sech}(\tau + T_s) + \text{sech}(\tau - T_s)\, e^{i\Delta\varphi} \quad (4.9)$$

with an initial pulse sequence period $2T_s$ can be described in terms of quasiparticles, between which acts an exponentially decreasing "force". The magnitude and sign of this "force" strongly depend on the phase difference $\Delta\varphi$. When $\Delta\varphi$ changes from 0 to π "attraction" changes to "repulsion". Estimates for the merging point L_s of two "attracting" solitons were obtained; in particular, for $2T_s \sim 80$ ps and $\tau_0 \sim 4$ ps, L_s is ~ 100 km.

An effective approach that allows one to select a soliton component for an arbitrary sequence of N pulses and to study its evolution with distance is developed in Ref. 154. As an illustration, Fig. 30 shows the motion trajectories for the maxima of the function $|q|^2$ for the case $\Delta\varphi = 0$, $N = 2$ for different T_s [initial conditions (4.9)]. The authors of Ref.

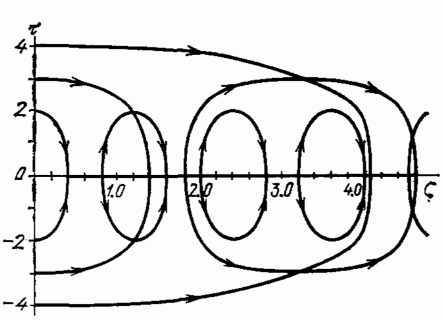

FIG. 30. Interaction of two solitons.[154] Trajectories of motion of intensity maxima for the initial conditions (4.11) are shown.

145 have analyzed the influence of optical losses on the character of soliton interaction.

Thus, the detailed analysis of the physical picture of soliton propagation along real optical waveguides carried out over the last several years has confirmed the usefulness of utilizing solitons in optical information systems.

4.3. Self-compression of high-power picosecond pulses in fiber-optics waveguides

In the previous section we considered the question of how, using methods of nonlinear optics, one can minimize the change in width and energy of an optical pulse at greatest possible distances. Now we consider the problems of using nonlinear effects for producing pulses of the shortest possible duration. As has already been mentioned earlier, the self-action of a transform-limited N-soliton pulse always starts with its compression. However, the experimental realization of fiber-optics compressors requires the solution of the practically important problems of the optimal length of a fiber-optics waveguide and the greatest achievable degree of compression.

Simple estimates of these important quantities can be obtained from considerations similar to those given in Sec. 3. Starting from the magnitude of the nonlinear spectral broadening $\Delta\omega \sim \tau_0^{-1} z/L_p$ and the condition $\Delta t_3 \sim z k_2 \Delta\omega = 2\tau_0$, it is easy to show that the trailing edge of a pulse that has undergone SPM, "catches up" with the leading edge at the distance $z \sim L_{nl}$ [see (2.32)]. Thus, the length of an optical waveguide should be of the order of a nonlinear length.

To estimate the degree of compression s we use the fact that $s \sim \dfrac{\Delta\omega}{\Delta\omega_0}$. If a pulse at the input to a medium is transform-limited, the estimate for the degree of compression has the form

$$s \sim \frac{L_{nl}}{L_p} = \tau_0 \left(\frac{k_0 \tilde{n}_2 I_{eff}}{|k_2|} \right)^{1/2} \quad (4.10)$$

The complete information on the dependence of s and L_{opt} on the radiation characteristics and waveguide parameters can be obtained only by numerical experiments. The authors of Ref. 155 studied the dependence of the optimal waveguide length, the degree of compression and the amount of energy contained in the central peak (see Fig. 25) on the pulse amplitude q_0 for the case $q(\tau,0) = q_0 \operatorname{sech} \tau$.

The experimental results on self-compression are given in Ref. 155. A transform-limited pulse from a color-center laser ($\lambda \approx 1.5\,\mu$m, total pulse length at half-height is ~ 7 ps) with synchronous pumping was introduced into a fiber-optics waveguide of length $L \approx 320$ m. At a peak power of radiation of $P_0 \approx 200$ W, which corresponds to $q_0 \approx 13$, the pulse length at the output of the waveguide decreased to 310 fs. In a shorter waveguide with $L = 100$ m, and at higher powers it was possible to realize a degree of compression $s = 27$, and to obtain pulses with a length ~ 260 fs.

Recently, among the methods for the formation of ultrashort pulses in the near IR range there appeared another promising approach—compression using Raman frequency shift. The approach is based on the transformation of an N-soliton pumping pulse into a high-power one-soliton pulse at the Stokes frequency. In the theoretical papers of Refs. 156, 157 the physics of nonlinear transformation of solitons in a fiber-optics waveguide with a small number of modes was examined under the condition of complete or partial compensation of the detuning of group velocities at the frequencies ω_p and ω_s due to the waveguide dispersion.

The result of the nonlinear transformation is a soliton at the Stokes frequency with an energy practically equal to the energy of the multi-soliton pumping pulse. Thus, for example, the pumping pulse $q_p = N \operatorname{sech} \tau$ for $N = 3$ is transformed into a Stokes soliton $q_s = \varkappa \operatorname{sech}(\varkappa\tau)$ with the form-factor $\varkappa = 7$.

The authors of Ref. 158 have realized experimentally compression of pulses in a single-mode waveguide using the Raman frequency shift. As a source they used a parametric light generator with synchronous pumping ($\tau_0 \sim 30$ ps, $\lambda \sim 1.5$–$1.65\,\mu$m). For an input pulse power $P_0 \sim 900$ W, and a waveguide length of 250 m, it was possible to observe at the waveguide output 200 fs pulses at the Stokes frequency with a power of 55 kW (the magnitude of the Stokes shift was ~ 55 cm^{-1}). The physical mechanism of compressed pulse formation based on Raman amplification of a "priming" pulse at the frequency arising during the SPM at the leading edge of the pulse, was also considered.

The possibility to achieve, in principle, a high degree of compression in the system fiber-optics waveguide-amplifier is shown in Ref. 159. It is essential that self-compression, in combination with amplification, allows one to form high-power single pulses without a background signal.

4.4. Soliton laser

This is how the authors of Ref. 160 called the source of stable, tunable frequency and length, pulses with a profile in the form $q = \varkappa \operatorname{sech}(\varkappa\tau)$. A diagram showing the principle of operation of a soliton laser is given in Fig. 31. The laser consists of two coupled resonators—the main one (the mirrors M_1, M_2, M_0) and the auxiliary one (the mirrors M_0, M_3). The main resonator constitutes a color-center laser with synchronous pumping, generating in an autonomous regime transform-limited 8 ps pulses with a mean power of ~ 1 W and a repetition frequency of ~ 100 MHz. In the soliton laser regime a part of the radiation is introduced through the semitransparent mirror M_0 (transmission coefficient $\sim 30\%$) and a beam-splitter into the second resonator, containing a single-mode fiber-optics waveguide, which does not change polarization. The power level P_2 in the second resonator and the fiber-optics waveguide length are chosen in such a way that the self-compressed pulse is injected into the main resonator synchronously with pumping, and this leads to generation of a shorter pulse etc., until the steady-state regime is reached. In the steady-state regime a pulse was reproduced after it passed twice through the auxiliary resonator and represented a bound state of two solitons. Its parameters could be found from the condition

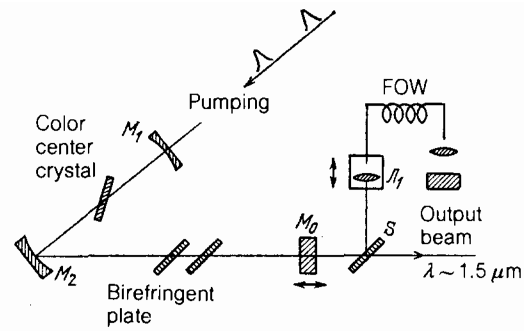

FIG. 31. Block–diagram of a soliton laser.[160] A color center laser with a resonator formed by the mirrors $M_1 - M_0$, and an auxiliary resonator $(M_0 - M_3)$ containing a fiber optics waveguide are shown.

$$2L = \frac{\pi}{2} L_d, \quad P = 4P_{\text{crit}}, \tag{4.11}$$

where L is the waveguide length, and P_{crit} is the critical power, determined by the relation (4.3). Thus, in addition to the condition of balance between amplification and losses usual for a steady-state regime of generation, one adds a condition of balance between the dispersive spreading and nonlinear self-compression, which determines the duration of the generated pulse.

Typical orders of magnitude are the following: for the optical waveguide length $L \sim 30$ m a soliton laser generated pulses of length ~ 3 ps and a peak power of ~ 30 W. When a short optical waveguide with $L \sim 60$ cm was used, pulses with a length of 210 fs and a peak power of 2 kW were generated.

The authors of Ref. 160 considered that an entirely realistic goal can be generation of 100 fs length pulses. Soliton lasers are quite promising sources for scientific and technical applications. A detailed presentation of the theory of soliton lasers is given in Ref. 161.

5. STATISTICAL PROBLEMS IN OPTICAL SOLITON THEORY

Advances of experimental techniques and important applications of self-action effects posed new questions before the theory of self-action. In Sec. 3 we have already discussed briefly a modification of truncated equations that describe the self-action of high-power femtosecond pulses. Next we shall discuss the development of the theory of temporal self-action taking into account higher-order nonlinearities (see the conclusions).

Generalization of the results, obtained for the deterministic case, to the case of randomly-modulated fields[10], is of considerable interest both from the point of view of the development of the theory and for the applications. We had an example of such a situation in Sec. 3 which discussed compression of randomly-modulated pulses.

A natural statistical generalization of self-action effects of regular pulses discussed in Sec. 2 is the problem of self-action of noise waves[172–174] (see also Ref. 6).

The statistical problems of optical soliton theory are of a more fundamental character. One of the main questions here is—under what conditions is it possible to form a soliton from optical noise? In Secs. 5.1–5.3 the process of such formation is discussed step by step; Sec. 5.1 deals with a calculation of the energy threshold for the formation of solitons from flashes of optical noise. Section 5.2 is devoted to the influence of random modulation of an optical pulse on propagation in the near-field. Finally, evolution of solitons in the far-field is discussed in Sec. 5.3; analysis in it differs from analysis of previous sections in that it is based on the inverse problem method of scattering theory.

5.1. Threshold conditions for the formation of optical solitons from pulses of noise

In this subsection we concentrate our attention on the estimate of the threshold energy flux density at which the formation of an optical soliton from a pulse having both regular and noise components becomes possible.[165,166] We also shall study the influence of dispersion and the correlation time of the noise component of the input signal on the critical value of the nonlinearity parameter (see Sec. 4.1). The analysis is based on the method of moments.[167]

We carry out the discussion using as an example noise pulses of the form (3.14) with the correlation function of the random component $\xi(\tau)$ in the form (1.22). We introduce the moments of the intensity distribution $|\psi(\tau)|^2$

$$\Theta_m = \langle \tau^m \rangle = \int\limits_{-\infty}^{+\infty} \psi^*(\tau)\, \tau^m \psi(\tau)\, d\tau. \tag{5.1}$$

Averaging over the variable τ will be denoted in the future by angular brackets $\langle ... \rangle$. According to Eq. (3.1), the change of the moment θ_2 (mean square of the pulse length) is described by the system of equations

$$\frac{d\Theta_2}{d\zeta} = \frac{i}{2} \langle \dot{\psi}\tau\psi^* \rangle + \text{c.c.}, \tag{5.2}$$

$$\frac{d^2\Theta_2}{d\zeta^2} = 2H_0 + R\,\langle (\psi\psi^*)^2 \rangle, \tag{5.3}$$

where $\dot{\psi} = \dfrac{\partial\psi}{\partial\tau}$, $H_0 = \langle \dot{\psi}\dot{\psi}^* \rangle - R\,\langle (\psi\psi^*)^2 \rangle$ is an integral of Eq. (3.1), having the meaning of a Hamiltonian.

Substituting the boundary condition (3.13) into (5.3), we obtain the approximate polynomial representation

$$\Theta_2(\zeta) = C_0 + C_1\zeta + C_2\zeta^2 + \dots, \tag{5.4}$$

where

$$C_0 = \Theta_2(0), \quad C_1 = \frac{i}{2} \langle \dot{\psi}_0 \tau \psi_0^* \rangle + \text{c.c.},$$

$$C_2 = H_0 + \frac{1}{2} R\,\langle (\psi_0\psi_0^*)^2 \rangle. \tag{5.5}$$

The applicability region of the representation (5.4) is determined by the inequality $R\zeta^2 < 1$, which is equivalent to the condition $z < L_{nl}$.

Substituting the specific expression (3.13) into (5.4) and performing statistical averaging (it is denoted by a bar), we obtain

$$\overline{\Theta}_2 = \overline{C}_0 + \overline{C}_1\zeta + \overline{C}_2\zeta^2, \tag{5.6}$$

where $\overline{C}_0 = \overline{\Theta}_2(0)$, $\overline{C}_1 = 0$, and the coefficient

$$\overline{C}_2 = \overline{H}_0 + \frac{1}{2} R\,\overline{\langle (\psi_0\psi_0^*)^2 \rangle}. \tag{5.7}$$

The condition of constancy of the duration $\overline{\Theta}_2$ for the statistically averaged pulse takes the form: $\overline{C}_2 = 0$. From here for a pulse with the profile $\rho(\tau) = \operatorname{sech}\tau$ we obtain the relationship

$$R_{\text{crit}} = -\frac{2\overline{H}_0}{\langle |\psi_0|^4 \rangle} \approx 1 + 3\sigma^2 \left[2\left(\frac{\tau_0}{\tau_{\text{K},0}}\right)^2 - 1 \right], \tag{5.8}$$

which is valid in the approximation of weak fluctuations, $\sigma^2 \ll 1$. An analysis of this expression shows that in the case of "fast" fluctuations $\tau_{K0} < \sqrt{2}\tau_0$ the critical energy flux density grows with an increase of the fluctuation variance σ^2.

For slow fluctuations $\tau_{K0} > \sqrt{2}\tau_0$ expression (5.8) predicts a decrease of critical power. This, at first glance, paradoxical result is associated with the specifics of the pulse (3.13) itself: with an increase of the fluctuation variance σ^2 the average energy of the input pulse $\overline{W} = W_0(1+\sigma^2)$ also increases. Pulses with $R > R_{\text{crit}}$ will on the average be compressed, and pulses with $R < R_{\text{crit}}$ will spread.

5.2. Self-action of randomly-modulated pulses in the near zone

How do the statistical characteristics of pulses with random modulation vary in a nonlinear medium? This section is devoted to an answer to that question. We shall analyze the evolution of the pulse length and correlation time of random pulses in the case of self-action in the area of anomalous dispersion of the group velocity. A rigorous solution of the aforementioned statistical problems is not possible. For this reason, the analytical results obtained here[168–170] are restricted to the initial stage of nonlinear propagation ($z < L_{nl}$); calculations are carried out in the so-called approximation of the given channel with the use of Feynmann path integrals.

5.2.1. Path integration; approximation of the given channel

We shall use the nonlinear Eq. (3.1). Considering $|\psi(\tau,\zeta)|^2$ as a potential, in which a pulse propagates the solution (3.1) can be written in the form[168] (see also Ref. 171)

$$\psi(\tau, \zeta) = \int\limits_{-\infty}^{+\infty} \psi_0(\theta)\, G(\theta, \tau; \zeta)\, d\theta, \tag{5.9}$$

where $\psi_0(\theta)$ is given at the input of a nonlinear medium.

The function $G(\theta,\tau)$ is determined by the path integral

$$G(\theta,\ \tau;\ \zeta)=\int \exp\left[-\int_0^{\zeta} \mathscr{L}(\tau(x),\ \dot{\tau}(x))\,dx\right] D\tau(x),$$

$$\mathscr{L}(\tau(x),\ \dot{\tau}(x))=\frac{1}{2}\dot{\tau}^2(x)+R|\psi(\tau(x),\ x)|^2, \tag{5.10}$$

where $\dot{\tau}(x = d\tau(x)/dx$, integration is done along an infinite number of trajectories, connecting the points with the coordinates θ, 0 and τ, ζ, where $\theta = \tau(0)$ and $\tau = \tau(\zeta)$.

Thus, from Eq. (3.1) we go over to the path-integral Eq. (5.9), (5.10). These expressions turn out to be convenient for an approximate calculation of the statistical characteristics of a randomly modulated pulse in a nonlinear medium. An obvious method for solving (5.9), is to use the iteration method.

We shall demonstrate a method of solving (5.9) and (5.10) using as an example a regular PM pulse. The maximum contribution to (5.10) is given by the trajectories satisfying the Euler equation

$$\frac{\partial}{\partial x}\frac{\partial \mathscr{L}}{\partial \dot{\tau}}-\frac{\partial \mathscr{L}}{\partial \tau}=0, \tag{5.11}$$

or, in our case, the equation

$$\ddot{\tau}-R\frac{\partial}{\partial \tau}|\psi(\tau(x),\ x)|^2=0, \tag{5.12}$$

which can be solved only for a specific form of $\psi(\tau,x)$.

As a zero-order approximation of the iteration method we can take the solution $\psi_{D0}(\tau,x)$ corresponding to $R = 0$ in (3.1) or $\psi_{nl}(\tau,x)$ which corresponds to neglecting the dispersion term. Having in mind the analysis of nonlinear propagation of high-power optical pulses, we shall use the second approach; in that case

$$\psi^0(\tau,\ x)=\psi_{nl}(\tau,\ x)=\psi_0(\tau)\exp\left[-iR|\psi_0(\tau)|^2 x\right],$$

$$|\psi^0(\tau,\ x)|^2=|\psi_0(\tau)|^2. \tag{5.13}$$

The use of the substitution (5.13) in (5.10) means that at the following stage of solution of (5.9) we consider the propagation of the pulse in a non-steady-state medium, the parameters of which are determined by the initial pulse. Such an approximation is usually called the "given channel approximation" (GCA); it can be used at distances $z < L_{nl}$.

In the case of a Gaussian pulse $\psi_0^2(\tau) = \exp(-\tau^2)$ Eq. (5.12) for optimal trajectories can be solved in the paraxial approximation ($\tau < 1$). In that case (5.10) takes the form

$$G(\theta,\ \tau;\ \zeta)=\left(-i\cdot 2\pi\frac{\sin h\zeta}{h}\right)^{-1/2},$$

$$\exp\left\{-i\frac{h}{2\sin h\zeta}\left[(\theta^2+\tau^2)\cos h\zeta-2\tau\theta\right]-iR\zeta\right\}, \tag{5.14}$$

where $h = (2R)^{1/2}$. Expression (5.14) is, in fact, the "Green's function" of (5.9).

Assume that the initial pulse has the quadratic PM (1.18). It is easy to see that the phase modulation of the pulse does not influence parameters of the given channel since the function (5.10) is determined by the expression (5.14).

For the normalized intensity of the pulse we obtain the expression

$$|\psi(\tau,\ \zeta)|^2=V^{-1}(\zeta;\ \alpha_0)\,e^{-(\tau/V(\zeta;\ \alpha_0))^2}, \tag{5.15}$$

$$V^2(\zeta;\ \alpha_0)=1+\left[(h^{-2}-1)(1+\gamma^2)\sin^2(h\zeta)+\gamma h^{-1}\sin(2h\zeta)\right] \tag{5.16}$$

where $\gamma = \alpha_0\tau_0^2$.

In the absence of phase modulation ($\alpha_0 = 0$, $\gamma = 0$) steady-state propagation of the pulse corresponds to the value $R = 1/2$ ($h = 1$).

According to (5.15), when $\gamma = 0$, but $h \neq 1$, the maximum intensity $I_{max} = V^{-1}(\zeta;0)I_0$ and the pulse length $\tau_\mu/\zeta) = V(\zeta;0)\tau_0$ oscillate with the distance, and this agrees qualitatively with the results of Sec. 4.1.

For a PM pulse with $h = 1$ the function $V^2(\zeta;\alpha_0) = 1 + \gamma\sin 2\zeta$ and the pulse length $\tau_\mu/\zeta) = V(\zeta;\alpha_0)\tau_0$ varies periodically, and in the initial stage of propagation either increases ($\alpha_0 > 0$), or decreases ($\alpha_0 < 0$). The contribution of phase modulation to (5.16) is rather complicated and at the initial stage cannot be compensated by SPM. Calculations show that there exists a critical value of the initial frequency modulation rate α_{crit}; for $\alpha_0 > \alpha_{crit}$ and $\zeta > L_d$, a PM pulse spreads, and for $\alpha_0 < \alpha_{crit}$ a soliton with the form-factor $\varkappa < 1$ is formed.

5.2.2. Pulse with random phase modulation

Let us consider self-action of a pulse with a random phase

$$\varphi_0(\tau)=\exp\left[-\frac{\tau^2}{2}+i\varphi_0(\tau)\right], \tag{5.17}$$

where $\varphi_0(\tau)$ is a steady-state Gaussian process with the correlation function in the form (1.22), dispersion σ_p^2 and correlation time τ_{cp}.

The pulse correlation function in a nonlinear medium is determined, according to (5.9), (5.14), and (5.17), by the expression

$$B(\tau_2,\ \tau_1;\ \zeta)$$

$$=V_2^{-1}(\zeta)\exp\left\{-\frac{1}{2V_2^2(\zeta)}\left[\tau_1^2+\tau_2^2-2\left(\frac{\sigma_P}{\tau_P}\right)^2(\tau_2-\tau_1)^2\right]\right\}, \tag{5.18}$$

where $\tau_p = \dfrac{\tau_{cp}}{\tau_0}$ and

$$V_2^2(\zeta)=1+\left\{\left[1+4\left(\frac{\sigma_P}{\tau_P}\right)^2\right]h^{-2}-1\right\}\sin^2(h\zeta). \tag{5.19}$$

According to (5.18), the pulse length averaged over the ensemble and the correlation time vary according to the same law: $\tau_\mu(\zeta) = V_2(\zeta)\tau_0$, $\tau_c = V_2(\zeta)\tau_{cp}/\sigma_p$. These quantities do not vary when

$$h_{crit}=\left[1+4\left(\frac{\sigma_P\tau_0}{\tau_{cp}}\right)^2\right]^{1/2}. \tag{5.20}$$

In this case the statistically average pulse propagates in a steady-state manner. This result differs from the expression (5.15) for a pulse with regular PM. The distinction is due to the fact that in individual pulses (realizations) the existence of PM leads to additional broadening or narrowing of a pulse in such a way, that under a certain condition the average pulse length and correlation time can remain constant.

Random PM leads to an increase of the threshold h_{crit}

(5.20) for steady-state propagation and, therefore, to an increase in the threshold energy density. In Ref. 169 calculations are presented of the intensity correlation function of a pulse with random PM in a nonlinear medium.

5.2.3. Self-action of a noise pulse

In the given channel approximation the problem of finding the correlation function for a noise pulse in a nonlinear medium

$$B(\tau_1, \tau_2; z) = \int\!\!\int_{-\infty}^{+\infty} d\theta_1 d\theta_2 \int\!\!\int K(\theta_1, \theta_2; \tau_1, \tau_2)$$

$$\times \exp\left[-i\frac{1}{2}\int_0^{\zeta}(\dot{\tau}_1^2(x)-\dot{\tau}_2^2(x))\,dx\right] D\tau_1(x)\, D\tau_2(x) \quad (5.21)$$

is reduced to calculating the correlator

$$K(\theta_1, \theta_2; \tau_1, \tau_2)$$

$$= \Big\langle \psi_0(\theta_1)\psi_0^*(\theta_2)$$

$$\times \exp\left\{-iR\int_0^{\zeta}[|\psi_0(\tau_1(x))|^2 - |\psi_0(\tau_2(x))|^2]\,dx\right\}\Big\rangle. \quad (5.22)$$

We consider self-action of a noise-pulse in the form $\psi_0(\tau) = \xi(\tau)F(\tau)$ with a regular profile $F(\tau) = \exp(-\tau^2/2)$ and a random Gaussian process $\xi(\tau)$, with the correlation function (1.22).

It is convenient to carry out the averaging (5.22) separately for two propagation regimes: 1) coherent regime $(z < L_{\rm coh} = \tau^2_{\rm co}/|k^2|)$ and 2) incoherent regime $(z > L_{\rm coh})$. We consider here the coherent propagation regime; in this case it is possible to perform in (5.22) the substitution

$$\int_0^{\zeta} |\psi_0(\tau)(x))|^2\,dx \approx |\psi_0(\tau(0))|^2\zeta = |\psi_0(\theta)|^2\zeta. \quad (5.23)$$

The correlation function (5.21) in paraxial approximation is calculated in Ref. 170. According to Ref. 170, the pulse length and correlation time of a noise pulse in a nonlinear medium are equal to

$$\tau_P(\zeta) = V_3(\zeta)\tau_0, \quad \tau_c(\zeta) = [1 + (2R\zeta)^2]^{-1/2} V_3(\zeta)\,\tau_0. \quad (5.24)$$

The magnitudes of $\tau_p(\zeta)$ and $\tau_c(\zeta)$ can both decrease or increase with an increase of ζ, and there can be a situation, when the correlation time is decreasing, while the average pulse length is increasing.

Analysis shows[170] that in the incoherent regime of self-action $(L_{\rm coh} < z < L_{\rm nl})$ of a noise pulse its average length and correlation time are always increasing with an increase of ζ. Intensity statistics of noise pulses has been studied in Ref. 175–176. The influence of noise on the nonlinear propagation regime of a regular pulse was considered in Ref. 168. It was shown that the coherent properties of a pulse deteriorate and the steady-state regime of pulse propagation is absent at the initial stage.

5.3. Soliton statistics. The inverse problem method

The approximate methods described above make it possible to follow the evolution of the profile and of the statistical characteristics at the early stages of propagation $(z < L_{\rm nl})$. The same is true for methods using numerical solutions of the Cauchy problem for the nonlinear Schrödinger equation.[177] With the shift to the far-zone $(z > L_{\rm nl})$, the drawbacks of the direct numerical methods associated with grid dispersion or an artificial periodization of the solution in the spectral approach become apparent. At the same time, such practically important problems as an estimate of the influence of initial pulse fluctuations on the pass band of soliton communication systems require the development of adequate methods of analysis of the wave fields in the far zone. In fact, at the initial pulse length $\tau_0 \sim 6$ ps and $\lambda \approx 1.5$ μm, the far zone corresponds to a waveguide length of $L \sim 1$ km, and for $\tau_0 \sim 1$ ps $L \sim 30$ m. At the present time transmission of solitons has been achieved over a distance of 50 km.[147]

The analysis of fields in the far zone is based on the method of the inverse problem in scattering theory.[142] We shall give here some results of this method required for further presentation. Let us consider the Cauchy problem for a nonlinear Schrödinger equation written in the traditional mathematical form (4.1) with the initial condition $q(\tau,0) = q_0(\tau)$. It is required to find $q(\tau,\zeta)$ from the known q_0 for q from a class of functions decreasing in the absolute value when $\tau \to \pm\infty$ faster than any power function.

According to the inverse problem method, instead of the nonlinear evolution of the complex amplitude $q(\tau,\zeta)$, one considers an auxiliary linear scattering problem, in which the required solution enters in the form of a potential

$$\frac{\partial}{\partial\zeta}\Phi^{\pm} = i\Lambda\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\Phi^{\pm} + i\begin{pmatrix}0 & q_0\\ q_0^* & 0\end{pmatrix}\Phi^{\pm}, \quad (5.25)$$

where $\Phi^{\pm}$ is a matrix-function of τ and of the spectral parameter $\Lambda \in R$, q^* is the complex conjugate of q. The functions $\Phi^{\pm}$ satisfy the asymptotic relations

$$\Phi^{\pm}(\tau) \to \exp\left[i\Lambda\tau\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right] \quad (5.26)$$

when $\tau \to \pm\infty$, respectively, since the potential $q_0 > 0$ when $\tau \to \pm\infty$. The relationship between the functions Φ^- and Φ^+ is established with the help of the scattering matrix $\hat{S}$

$$\Phi^- = \Phi^+\hat{S}, \quad \hat{S} = \begin{pmatrix}a^* & b\\ -b^* & a\end{pmatrix}. \quad (5.27)$$

The coefficient $a = a(\Lambda)$ depends only on the spectral parameter Λ, and the coefficient B can be presented in the form $b = B^0(\Lambda)\exp(i2\Lambda^2\zeta)$. The coefficients are related by the normalization condition $aa^* + bb^* = 1$. Additionally, $a(\Lambda) \to 1$ when $\Lambda \to \pm\infty$, $\Lambda \in R$.

As is shown in Ref. 142, the coefficient $a(\Lambda)$ can be analytically continued into the upper half-plane $\operatorname{Im}\Lambda > 0$, and in order to find the soliton component of q_0 it is necessary to find the zeros $\Lambda_m = \operatorname{Re}\Lambda_m + i\operatorname{Im}\Lambda_m$ of the coefficient $a(\Lambda)$ for $\operatorname{Im}\Lambda > 0$. The soliton solution corresponding to Λ_m, has the form:

$$q_m(\tau, \zeta) = \varkappa_m \operatorname{sech}[\varkappa_m(\tau - \tau_{0m} - v_m\zeta)] \times \exp\left[-iv_m\tau + i\frac{1}{2}(v_m^2 - \varkappa_m^2)\zeta + \varphi_{0m}\right], \quad (5.28)$$

where $\varkappa_m = 2 \operatorname{Im} \Lambda_m$ is the soliton amplitude, $v_m = 2 \operatorname{Re} \Lambda_m$ is its velocity; τ_{0m} and φ_{0m} are the initial coordinates of the maximum and the phase.

If the velocities of all the solitons included in the solution (4.1) are different, then at $\zeta \to \infty$ an N-soliton solution is determined by the trivial linear superposition

$$q_s = \sum_{m=1}^{N} q_m,$$

where q_m is expressed by formula (5.28).

At finite distances or in the presence of coinciding velocities (see, for example, Fig. 25), the superposition of solitons is essentially nonlinear. Its reconstruction can be reduced to the solution of a system of N linear equations.[142] It was found that a nonsoliton part of the solution is essential only in the initial stages of propagation, since for $\zeta \to \infty$ it decreases as $\zeta^{-1/2}$. Using the graphic analogy with the problem of diffraction of a slit-shaped beam in a nonlinear medium, solitons in the far zone correspond to bright stripes with an intensity that does not depend on the distance, and the non-soliton part corresponds to a linearly diffracting diffuse background.

An energy relationship between the soliton and non-soliton parts of the solution can be found using a nonlinear generalization of the Parseval theorem,[179] according to which

$$\int_{-\infty}^{+\infty} |q|^2 d\tau = 2 \sum_{m=1}^{N} \varkappa_m + \frac{1}{\pi} \int_{-\infty}^{+\infty} \ln\left[1 + \frac{bb^*}{aa^*}(\xi)\right] d\xi. \quad (5.29)$$

The first term on the right side of this equation corresponds to the energy of the soliton part of the solution (a discrete spectrum), and the second term—to the non-soliton part (continuous spectrum). This theorem allows one to establish an analogy between a Fourier-spectrum and its nonlinear analogue, determined by the coefficients a and b.

In a few particular cases it is possible to calculate the soliton spectrum analytically.[143] A pulse with the profile $q_0(\tau) = N \operatorname{sech} \tau$ discussed in Sec. 4 can serve as an example. At the present time effective numerical methods have been developed[178] that allow one not only to find the "soliton spectrum" of an arbitrary pulse $q_0(\tau)$, but also to calculate the soliton component of the solution $q_s(\tau,\zeta)$ for arbitrary ζ.

A number of useful results can be obtained with the help of perturbation theory. The authors of Ref. 143 have found that the small real variations $|\xi| < \frac{1}{2}$ of the amplitude of the one-soliton pulse $q_1 = (1 + \xi)\operatorname{sech} \tau$ lead to a linear change of the form-factor of the soliton $\varkappa_1 = 1 + 2\xi$ and do not cause corrections to its group velocity. From here, in particular, it follows that if the amplitude of the initial pulse $q_0 = 1 + \xi$ is a random function ($\bar{\xi} = 0, \overline{\xi^2} = \sigma^2$), then during the transition to the far zone ($z > L_d$) the distribution law and the mean value of $\varkappa_1$ remain constant, but the standard deviation of the fluctuations will be doubled.

In Ref. 179 it was shown that purely phase perturbations of initial conditions (for the model of the diffusive drift of the phase) leads in the first order of perturbation theory to "stochastization" of soliton velocity. A random correction to the group velocity has a normal distribution with the variance $\sim\sigma^2$. The second-order correction to the eigenvalue Λ_m is purely imaginary—it corresponds to random variations of the soliton amplitude.

The formation of optical solitons from pulses of partially-coherent radiation was studied in Ref. 180 by the method of statistical testing based on the approach described in Ref. 178. An illustration showing one of the realizations of the initial conditions is given in Fig. 32. A "noise flash" in the form

$$q_0(\tau) = \sigma\xi(\tau)\, e^{-\tau^2/2}, \quad (5.30)$$

where $\xi(\tau)$ is the complex Gaussian noise ($\bar{\xi} = 0$, $\overline{\xi(\tau)\xi^*(\tau+\theta)} = \exp(-\theta^2/\tilde{\tau}_{c,0}^2)$, $\sigma = 4$, $\tilde{\tau}_{c,0} = 0.4$) is shown in Fig. 32a. Separation of the soliton component has shown that this flash gives rise to two solitons with the parameters $\varkappa_1 = 2.4$, $v_1 = -0.71$ and $\varkappa_2 = 0.61$, $v_2 = 2.8$, where the velocities are expressed in units of L_d/τ_0.

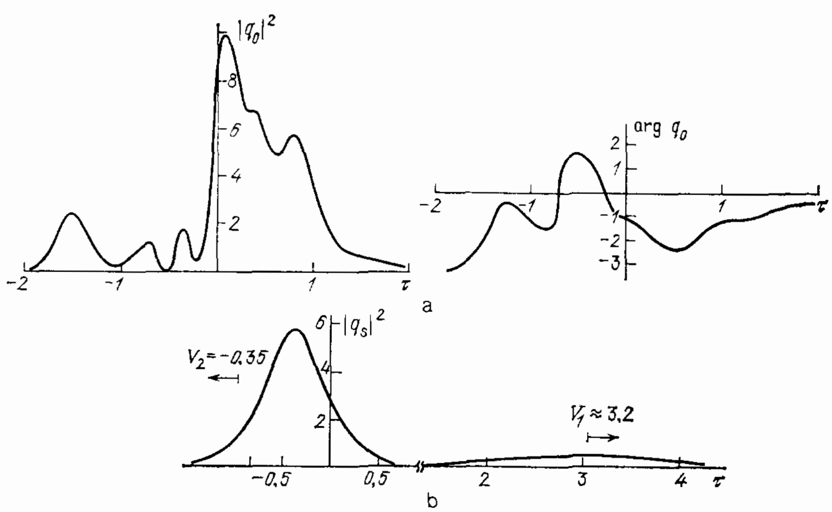

FIG. 32. Noise pulse (a) and its soliton component obtained by the inverse problem method at the distance $\zeta = 1$ (b).

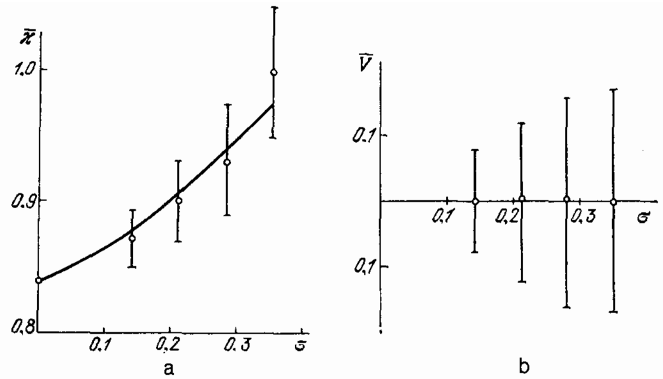

FIG. 33. Formation of a soliton in the presence of noise.[180] a—Dependence of the mean value of the form-factor on the noise amplitude σ. b—Dependence of the mean correction to the group velocity on σ (standard deviations are shown in the picture).

The soliton part of the solution $|q_s|^2$ is shown in Fig. 32b for $z=L_d$.

Fig. 33 shows the influence of additive Gaussian noise on the parameters of a soliton, formed for $\zeta\to\infty$ from the pulse with the profile

$$q_0(\tau)=e^{-\tau^2/2}+\sigma\xi(\tau). \tag{5.31}$$

The dependence of the mean value of the amplitude $\overline{\varkappa_1}$ and the velocity $\overline{v_1}$ on the parameter $\sigma(\bar{\xi}=0,\ \bar{\tau}_{c0}=0.4)$ obtained by averaging over realizations, is shown. It is possible to see that the increase of noise variance leads to some increase of the average value of the amplitude $\bar{\varkappa}$. The presence of complex Gaussian noise leads to fluctuations of soliton velocity (see Fig. 33); naturally the magnitude of these fluctuations increases with an increase of σ. Note, that fluctuations of the group velocity lead to the necessity of increasing the interval between pulses in order to use solitons in communication lines.

Therefore, at the present time there exist a number of effective mathematical methods allowing one to study in detail the stochastic self-action of pulses both in the near and far zones.

CONCLUSIONS

1. The development of efficient generators of picosecond and femtosecond optical pulses and the use of new nonlinear media have greatly increased the interest in the physics of self-action of wave packets and nonlinear optical methods for control of shape and length of ultrashort laser pulses. Successes of nonlinear-optics compression methods have made it possible to develop unified principles for construction of high-power femtosecond laser systems in the visible, IR and UV regions of the optical spectrum.

A typical laser system, using nonlinear optical compression, consists of a signal generator of frequency tunable, high-quality transform-limited picosecond pulses, a fiber-optics compressor, a nonlinear optical frequency converter, and a power amplifier. The use of such systems opens up unique opportunities for studying nonequilibrium states in atoms, molecules, and condensed media.[190] Experiments of this kind put very stringent requirements on the stability of formed pulses. The solution of the problem is connected, in our opinion, with the efficient use of methods of adaptive optics, discussed in Sec. 3.7. Practically, the situation here is similar in many respects to the situation, arising during focusing of high-power light beams in a nonlinear, statistically inhomogeneous medium. The results that can be achieved here by methods of adaptive optics are well known.[183,184] The methods described in Ref. 184, can be effectively used for adaptive control over shape and length of laser pulses in a grating compressor.

Undoubtedly, the problem of direct registration of temporal behavior of the profile and, especially, of the phase of optical packets, is becoming more urgent. Over the last few years some interesting work has been done in that area; we note, in particular, Ref. 182 in which registration of the temporal behavior of the phase was accomplished using the response of an ensemble of two-level atoms.

Speaking of the use of femtosecond pulses in high-speed systems for processing and transmission of information, we emphasize once again the fundamental necessity of using optical nonlinearity for stabilization of such pulses. Experiments on the formation, interaction, and amplification of optical solitons carried out recently are quite encouraging in this sense.

2. Practically "inertialess" (relaxation time $\tau_r\sim10^{-14}$ sec) cubic nonlinearity of optical fibers is quite small $\chi^{(3)}\sim10^{-14}$ CGSE units. On the other hand, in semiconductors, especially under resonant conditions, it is possible to obtain $\chi^{(3)}\approx10^{-2}$–10^{-3} CGSE units. Such "giant" cubic nonlinearities of semiconductors are widely used lately for wave front reversal and creation of optically bistable devices.[185,186] However, as a rule, the price for the large value of nonlinearity is a sharp increase of the inertia of nonlinear response. At the same time we indicate that for some semiconductors it is possible to realize a relatively large cubic nonlinearity with $\tau_r<10^{-12}$ sec (see Ref. 136); the compression with their help of ultrashort pulses in the infrared range is, undoubtedly, of great interest.

3. In this review we could not discuss the characteristic effects of self-action caused by quadratic nonlinearity $\hat{\chi}^{(2)}$.[187] The simplest nonlinear effect in a medium with $P^{(2)}\sim\hat{\chi}^{(2)}\mathbf{E}^{(2)}$ is generation of the second optical harmonic. When the intensity of the second harmonic becomes comparable with the intensity of exciting radiation, there arises a reverse reaction of the harmonic on the main wave. In this situation, the first manifestation is the reaction of the harmonic on the phase of the main wave. This latter reaction is equivalent to the appearance of a dependence of its phase velocity on intensity, i.e., a characteristic kind of self-action. Optical rectification in electro-optical crystals also leads to a similar effect.[188]

Reverse reaction on the phase in parametric processes[189], taking place because of quadratic nonlinearity, underlies many interesting wave effects. Numerical experiments, performed by Sukhorukov and his co-authors[191–193] show the possibility of existence of stable multi-frequency formations—the so-called parametric solitons, effects of parametric compression, etc. Now the time has come for experimental verification of these effects.

Of course, parametric processes open up also many other possibilities for the formation and compression of optical pulses. Attention to prospects of fast control and compression of pulses in media with quadratic nonlinearity was drawn for the first time already in Ref. 13; for the present status of this technique see Refs. 194–195. The technique of parametric compression (four-frequency interactions using cubic nonlinearity[196] can also be used here) apparently can be especially useful for the formation of the shortest possible pulses (see Refs. 13, 196).

4. The problem of wave propagation in media with a strong local nonlinear response has become a fundamental problem of nonlinear optics which has been attracting increasing attention in recent years (see the review of earlier work in Ref. 197). There are increasing indications of the fact that such a situation can be realized near exciton resonances in semiconductors and in artificial, inhomogeneous nonlinear media.[198] Non-resonance optical bistability and multistability, and stochastic self-modulation of propagating optical waves can become manifestations of a strong local nonlinearity[11)].[199–201]

[1)]Linear frequency modulation is often called in English "chirp," and the Russian direct transliteration of this word is increasingly often used in the Russian literature.

[2)]In the general case the phase matching condition $\varphi_0(\Omega) + \varphi_k = 0$ guarantees only that the resulting amplitude will be maximum, since the concept of a pulse length is ambiguous for pulses of complex shape.

[3)]We note that in a two-level medium the steepness increase of the pulse leading edge is possible due to its preferential amplification.[80–81]

[4)]Expression (2.30) differs from the corresponding formulas of Ref. 76 by a factor of 2. This difference, in our opinion, is due to the fact that Ref. 76 takes into account only the dependence of group velocity on intensity.

[5)]Equation (2.9) is often called the nonlinear Schrödinger equation.

[6)]Note that in Ref. 82 the possibility was shown of self-compression of short optical pulses in metal halide vapors.

[7)]The problems of simultaneous analysis of spatial-temporal effects are discussed in Refs. 100–102.

[8)]Note that the efficiency of self-action depends also on the polarization of the radiation.[59]

[9)]There are discussions on the possibility of making light-induced phase gratings in fiber-optics waveguides by using two auxiliary waves propagating in the opposite directions.[107] The possibility in principle of realization of anomalous group velocity dispersion in the visible region of wavelengths has been shown for these systems.

[10)]A closely related group of problems arises in the investigation of self-action in a medium with fluctuations of dielectric permittivity.[162,163] In particular, in fiber-optics waveguides the fluctuation of the nonlinear parameter $R = R(\zeta)$ can be caused by fluctuations of fiber diameter.[164]

[11)]In optical resonators these phenomena are well studied; we emphasize that in the presence of feedback controlled by a resonator they can be observed even in the case of a weak local nonlinearity.

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