Interactions of intense noise waves

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Experiments in statistical nonlinear acoustics are reviewed. Measurements of the average intensities of the harmonics of a narrow-band randomly modulated signal, the nonlinear transformation of broad aerodynamic–noise spectra, and different effects involving the interaction of regular and random waves (active suppression of noise by an intense signal, excess fading of a weak signal in noise fields, cascade-like broadening of spectra, formation of white noise, and other effects) are described. Theoretical explanations are given for the observed phenomena. An approximate method is developed for finding the stochastic solutions of equations of the Burgers and Khokhlov-Zabolotskaya type. Results on the diffraction of intense noise, taking into account the effects of the spatial and temporal statistics, the excitation of random waves by distributed sources, and the formation of steady-state spectra, are presented for the first time. The problems of the nonlinear transformation of the statistical characteristics of acoustic noise and other general questions are discussed.

CONTENTS

1. INTRODUCTION

The study of statistical phenomena accompanying the propagation of waves in nonlinear media is important for many areas of physics and technology. Amongst all the statistical problems in the physics of nonlinear waves, those concerning the interactions of intense noise disturbances in nondispersive media are of special interest. We shall refer to these problems as problems in statistical nonlinear acoustics, although many results obtained here are applicable to systems of a different physical nature—in the study of waves in plasmas, on the surface of a liquid, in particle streams, etc., in those situations when dispersion can be neglected or is very weak.[1,2]

From the theoretical standpoint, nondispersive nonlinear media are of interest because in such media all spectral components interact with comparable efficiency, since they are in resonance with one another. This leads to cascade-like multiplication of the spectral lines, nonlinear broadening of the spectra, and the appearance of a continuous component. In the time language, the distinguishing feature of the realization will be the formation in it of steep sections of the profile—discontinuities or shock waves with a front of finite width. After the appearance of the discontinuities the behavior of the disturbance is qualitatively different; nonlinear damping, which is independent of the dissipative properties of the medium become significant; saturation effects and a number of other effects appear. All these phenomena are specific precisely to nondispersive media.[3] Another characteristic feature of statistical nonlinear acoustics is the fact that it is necessary to study random isolated pulses with fluctuating parameters, and not only randomly modulated quasiperiodic signals as done for dispersive media.[4]

The behavior of strongly distorted profiles in acoustics is described adequately by simplified equations of the Burgers, simple-wave, and Khokhlov-Zabolotskaya type. These evolutionary equations are model equations for many physical systems. For this reason, in addition to the direct physical consequences of the theory it is also of interest to study the consequences of the formal mathematical analogy following from the analysis of the properties of the standard equations and their solutions.[5]

From the standpoint of applications these studies are important because of the fact that both in nature and technology there exist real sources which are essentially sources of noise waves. Detonation waves in the atmosphere and in the ocean,[6,7] acoustic shock pulses, noise from jet and other powerful engines,[8] and intense fluctuating sonar signals[9,10]

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are examples of low-frequency disturbances for which at definite distances nonlinear effects become significant. There also exist smaller sources whose emission spectrum lies in the ultrasonic band. These include, for example, thermooptical (laser)[11,12] and ordinary electromechanical transducers, whose field always contains fluctuations, and ensembles of microscopic radiators (cavitation noise,[13] acoustic emission,[14] etc.). A classical example of nonlinear statistical phenomena in acoustics are the processes involved in the establishment of thermodynamic equilibrium in solids, which occur as a result of the anharmonicity of the crystalline lattice[15-17]; the characteristic phonon frequencies here lie in the far hypersonic frequency band. Finally, there also exists intense noise of natural origin—thunder,[18] seismic waves,[19] and a number of other types of noise.[20]

We point out the obvious links between the statistical nonlinear acoustics and "nonwave" problems—models of turbulence,[21] problems of aeroacoustic interactions,[22] and stability in hydrodynamics.[23]

Thus the development of the physics of interactions of intense noise waves was stimulated by two factors: the diversity of real problems and the convenience of the apparatus of nonlinear evolutionary equations.[1,3,32] these equations were first obtained by R. V. Khokhlov[1] and his coworkers for the analysis of regular nonlinear problems. Weakly nonlinear waves, modulated in time and in space, are used in information transmission and processing, the diagnostics of the properties of a medium. The classification of sound-scattering objects, and the solution of other radiophysical problems in acoustics. The slow (on the wavelength scales) evolutionary character of the processes enables the use of approximate approaches, such as the slowly varying profile method, in order to simplify the problems.[3] These methods were developed and tested based on an idea proposed by R. V. Khokhlov.[1] The apparatus of simplifed equations is made even more necessary by the random character of the modulation of the waves. The complexity of the nonlinear statistical problems usually makes it impossible to solve them by other methods, and this path is the only possible one.

A number of publications on the theory of finite-amplitude random acoustic waves appeared at the end of the 1960s and beginning of the 1970s.[24-28] Systematic studies, however, were begun later.[29] The results of this period are reviewed in detail in Refs. 3–5. Extensive experimentation, largely stimulated by the development of the theory, began after 1975; a large number of studies was carried out in the USA, Japan, and a number of other countries.

Many researchers became interested in these problems as a result of a review presented by R. V. Khokhlov at the 5th Interational Symposium on Nonlinear Acoustics (Copenhagen, 1973). At the 6th Symposium, chaired by R. V. Khokhlov in 1975 in Moscow, Khokhlov presented a long and compete report; since then the physics of intense noise has occupied a central place in all-union and international conferences on nonlinear acoustics.

At the present time, statistical nonlinear acoustics exhibits all the features of a developed scientific field; 1) a specific theory and mathematical apparatus have been developed; 2) experimental studies are being vigorously pursued; 3) many scientific groups in different countries are studying the problem; 4) there is a range of applications; and, 5) links with other areas of physics and mechanics have appeared.

2. NONLINEAR DISTORTION OF THE SPECTRA OF ONE-DIMENSIONAL WAVES. BASIC THEORETICAL RESULTS

It is well known that for models of simple waves the measured statistical characteristics of nonlinear disturbances can be calculated exactly. The general expression for the correlation function of a stochastic process in an arbitrary section of a nonlinear medium in terms of the two-dimensional characteristic function of the input signal is presented in the reviews of Refs. 3 and 4. Formulas which relate the spectral density in the medium to the correlation function at the input were found for a normal (at the boundary of the nonlinear medium) process.

The approach to the analysis of the evolution of noise spectra developed in Refs. 30 and 31 can be generalized to more complicated models. We shall explain the idea of the method[32,33] for the important example of Burgers equation, which describes the propagation of intense plane acoustic disturbances in dissipative media:

$$\frac{\partial V}{\partial z}-V\frac{\partial V}{\partial \theta}=\Gamma\frac{\partial^2 V}{\partial \theta^2}. \qquad (1)$$

The following dimensionless variables are used in (1): $V=u/u_0$—the particles velocity, $z=x/x_{\rm disc}=\varepsilon c_0^{-2}\omega_0 u_0 x$—the distance traversed by the wave in units of the discontinuity distance x_d, and $\theta=\omega_0\tau=\omega_0(t-x/c_0)$. The ratio of the characteristic nonlinear and dissipative lengths $\Gamma=x_{\rm disc}/x_d=b\omega_0/2\varepsilon c_0\rho_0 u_0$ is the inverse acoustic Reynolds number. Here the constants u_0 and ω_0 are the characteristic values of the starting particle velocity and frequency of the signal; ρ_0,c_0 are the equilibrium density of the medium and the velocity of sound; and, ε and b are the nonlinear and dissipative parameters[3].

The solution of (1) with $\Gamma=0$ at distances up to the discontinuity distance has the form of a simple wave $V=f(T=\theta+zV)$. In order to generalize the methods of spectral analysis of stochastic simple waves we transform in (1) from z,θ to the new variables z,T:

$$\frac{\partial V}{\partial z}=\Gamma\frac{\partial^2 V}{\partial T^2}\left(1-z\frac{\partial V}{\partial T}\right)^{-2}. \qquad (2)$$

In the approximation $z|\partial V/\partial T|\ll 1$ Eq. (2) formally becomes the linear diffusion equation, which, however, includes nonlinear effects because of the implicit dependence $V[z,T(V,\ ...)]$. This approximation describes well the strongly distorted disturbance for $\Gamma\ll 1$ in the region up to the formation of discontinuities, as well as the opposite limiting case of weak nonlinearity $\Gamma>1$—in the entire region of propagation of the wave. For intermediate values $\Gamma\lesssim 1$ in the discontinuous region the processes are described only qualitatively, and even features which are associated with the fact that the structure of the shock fronts was not taken into account appear.

So when $z|\partial V/\partial T| \ll 1$ (the distances are short or the profile is very smooth), solving the linearized equation (2) we find the auxiliary function $V(z,T)$. The transition to the measured velocity $V(z,\theta)$ is made with the help of the nonlinear transformation[30,34]

$$V(z,\theta) = -\frac{i}{2\pi z}\int_{-\infty}^{\infty} e^{i\omega\theta}\frac{d\omega}{\omega}\int_{-\infty}^{\infty}(e^{i\omega z V(z,T)}-1)e^{-i\omega T}\,dT. \quad (3)$$

Since the behavior of the auxiliary function $V(z,T)$ is described by the linearized equation (2), the normal stationary stochastic process at the point of entry into $z = 0$ has the same properties for $z > 0$ also. This makes it possible to calculate the measured correlation function

$$R(z,\theta=\theta_1-\theta_2) = \langle V(z,\theta_1)V(z,\theta_2)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{(\omega z)^2}e^{i\omega\theta}e^{-(\omega z)^2\sigma^2(z)} \times \int_{-\infty}^{\infty}(e^{(\omega z)^2 R(z,T)}-1)e^{-i\omega T}\,dT. \quad (4)$$

Here $\sigma^2(z) = R(z,T=0)$, and $R(z,T=T_1-T_2) = \langle R(z,T_1)R(z,T_2)\rangle$ is an auxiliary correlation function, whose behavior in the model (2) is described by the equation

$$\frac{\partial R}{\partial z} = 2\Gamma\frac{\partial^2 R}{\partial T^2}. \quad (5)$$

Therefore, based on the given correlation function $R(0,\theta) = \langle f(\theta_1)f(\theta_2)\rangle$ of steady-state normal noise $f(\theta)$, the statistical characteristics of the wave can be calculated in an arbitrary section of the medium in two stages. It is first necessary to find the auxiliary function $R(z,T)$ as a solution of Eq. (5) with the boundary condition $R(0,T=\theta)$. Then it is necessary to make the nonlinear transformation (4) and determine the measured function $R(z,\theta)$ or the corresponding spectral density $S(z,\omega)$ (according to the Wiener-Khinchin theorem):

$$S(z,\omega) = \frac{1}{2\pi i\omega}e^{-(\omega z)^2\sigma^2(z)}\int_{-\infty}^{\infty}\frac{\partial R(z,T)}{\partial T}e^{(\omega z)^2 R}e^{-i\omega T}\,dT. \quad (6)$$

The approach[32,33] described above is applicable not only to the analysis of nonlinear random waves in a dissipative medium (1). The statistical characteristics can be calculated based on the formulas (4) and (6) for any disturbances of the type $V = V(z,T=\theta+zV,\mathbf{r})$; of course, it will be necessary to obtain the auxiliary function $R(z;\ T_1,T_2;\ \mathbf{r}_1,\mathbf{r}_2)$ from other linear equations, differing from (5). In particular, in Sec. 7 the method is used to analyze the evolution of the spectra of diffracted waves and of the spatial statistical characteristics.

We shall generalize the formulas (4)–(6) to the important problems of the propagation of spherically and cylindrically symmetric disturbances. For definiteness, we shall examine the case of converging waves with an initial radius of curvature of the wavefront r_0. The normalized distance z, traversed by the wave, in this case will vary from $z_0 = r_0/x_{\rm disc}$ to 0. Using the well-known[3] expressions for the reduced coordinates, we obtain a description of the energy spectrum (6) of spherical ($n = 2$) and cylindrical ($n = 1$) noise waves:

$$S(z,\omega) = -\frac{1}{\pi\omega}\int_0^{\infty}\frac{\partial R(z,T)}{\partial T} \times \exp\{-[\omega z\xi_n(z)]^2[\sigma^2(z)-R(z,T)]\}\sin\omega T\,dT; \quad (7)$$

here $\xi_1 = 2[1-(z/z_0)^{1/2}]$, $\xi_2 = \ln(z_0/z)$, $\sigma^2(z) = R(z,0)$; the auxiliary correlation function in (7) is calculated using the formula

$$R(z,T) = \frac{(z_0/z)^n}{[8\pi\Gamma(z_0-z)]^{1/2}} \times \int_{-\infty}^{\infty}R(z=z_0,T')\exp\left[-\frac{(T-T')^2}{8\Gamma(z_0-z)}\right]dT'. \quad (8)$$

The dispersion of the velocity of sound in acoustics is usually small and associated with some relaxation process with a characteristic time $t_{\rm rel}$. The propagation of intense plane waves in relaxing media is described by the equation[3]

$$\frac{\partial V}{\partial z} - V\frac{\partial V}{\partial\theta} = D\frac{\partial}{\partial\theta}\int_{-\infty}^{\theta}\frac{\partial V}{\partial\theta'}\exp\left(-\frac{\theta-\theta'}{\theta_{\rm rel}}\right)d\theta', \quad (9)$$

where $\theta_{\rm rel} = \omega_0 t_{\rm rel}$ and the number $D = (c_\infty^2 - c_0^2)/2\varepsilon c_0 u_0$ is the ratio of the discontinuity distance to the coherence length. The spectral energy density of the noise in such media can be calculated using the general formulas (4) and (6). the auxiliary correlation function $R(z,T)$, in this case, is determined from the linear equation

$$\frac{\partial}{\partial z}\left(\frac{\partial^2 R}{\partial T^2} - \frac{R}{\theta_{\rm rel}^2}\right) = -\frac{2D}{\theta_{\rm rel}}\frac{\partial^2 R}{\partial T^2}, \quad (10)$$

corresponding to the equation of nonlinear evolution (9). It is related to the starting noise spectrum $S(0,\omega)$ by the expression

$$R(z,T) = 2\int_0^{\infty}S(0,\omega)\exp\left(-2D\frac{\omega^2\theta_{\rm rel}z}{1+\omega_2\theta_{\rm rel}^2}\right)\cos\omega T\,d\omega. \quad (11)$$

In the limiting case of low-frequency spectra $\omega\theta_{\rm rel} \ll 1$, as is evident from the formulas (10) and (11), the function $R(z,T)$ will be determined from Eq. (5), in which the inverse Reynolds number Γ is replaced by the product $D\theta_{\rm rel}$.[32] In addition, the wave is described not by (9), but rather by the corresponding Burgers–Korteweig–de Vries equation.[3] In the other limit of high-frequency spectra we have the simple expression $R(z,T) = R(0,T)\exp(-2Dz/\theta_{\rm rel})$; it is also possible to obtain an exact solution, which can be obtained from (6) by making the substitution $(\omega z) \to (\omega\theta_{\rm rel}/D)[1-\exp(-Dz/\theta_{\rm rel})]$, in the case $\omega\theta_{\rm rel} \gg 1$.

3. GENERATION OF THE HARMONICS OF A NARROW-BAND RANDOMLY MODULATED SIGNAL

We shall study at the point of entry into the nonlinear dissipative medium narrow-band normal noise with the correlation function $\sigma^2 R(x=0,t) = \sigma^2 b(t/t_c)\times\cos\omega_0 t$; here b is the slow ($\omega_0 t_c \gg 1$) envelope, and t_c is the correlation time. It is desirable to interpret the normalization constants ω_0, u_0, used in writing down Burgers equation (1) and the formulas (3)–(6), as the central frequency of the spectral line of the noise ω_0 and the variance σ.

The auxiliary function is determined from (5) and has the form

$$R(z, T) = b\left(\frac{T}{\omega_0 t_c}\right) \exp(-2\Gamma z) \cos T. \quad (12)$$

Substituting (12) into (4), it is possible to calculate the measured correlation function in a medium as a sum of the noise harmonic functions

$$R(z, \theta) = \sum_{n=1}^{\infty} \frac{2}{(nz)^2} \exp[-(nz)^2 e^{-2\Gamma z}] I_n \times \left[(nz)^2 e^{-2\Gamma z} b\left(\frac{\theta}{\omega_0 t_c}\right)\right] \cos n\theta. \quad (13)$$

It is interesting to follow the dynamics of the spectral lines. For example, for a Lorentz starting signal, setting $b = \exp(-|t|/t_c)$ and employing the asymptotic form of the modified Bessel function I_n, we find the widths of the spectra

$$\Delta\omega_n(z) \approx n\Delta\omega_1(z=0)\left[1 + \frac{n^4}{4(n+1)}(1 - e^{-2/n}) z^4 e^{-4\Gamma z}\right]. \quad (14)$$

Thus the characteristic width of the spectral line of the nth harmonic for small z is n times greater than that of the first harmonic. As the wave propagates, the widths of the spectral lines increase to some maximum values, achieved at the distance $z = 1/\Gamma$. Then the widths of the lines once again decrease. The broadening-narrowing process is observed in a nonlinear medium only when dissipative effects are taken into account (compare with Ref. 3); it is observed for small inverse Reynolds numbers Γ, and is all the stronger the higher the harmonic number n.

The result (13) permits establishing the difference in the rates of generation of the harmonics of monochromatic and randomly modulated waves. The comparison must be made assuming that the intensities are the same at the inlet (at $z = 0$); the center of the spectral line of the noise must coincide with the frequency ω_0 of the determinate wave. The average intensities of the second ($n = 2$) and third ($n = 3$) harmonics as function of the distance are shown in Fig. 1 by the solid ($I_n^{(N)}$, noise) and broken ($I_n^{(S)}$, regular signal) curves for inverse Reynolds numbers $\Gamma/\sqrt{2} = 0.01$ and 0.1. Analysis shows that depletion of the fundamental and growth of the higher-order harmonics occur at faster rates in the case of the noise. At short distances z, in the limit $\Gamma \to 0$, the ratio $I_n^{(N)}/I_n^{(S)} \approx n!$. The fact that in the noise field the nth harmonic is generated $n!$ times more efficiently was not observed in nonlinear optics[4,29]; this phenomenon is linked with the accentuation of large-amplitude surges in nonlinear transformations.

Linear damping, which increases with Γ, has a stronger effect on the harmonics of the determinate wave. It is evident from Fig. 1 that for $\Gamma/\sqrt{2} > 0.1$ the solid curves for the second and third harmonics of the noise in the entire domain of distances z pass above the corresponding broken curves. This is linked with the fact that for not very small $\Gamma \gtrsim 0.15$

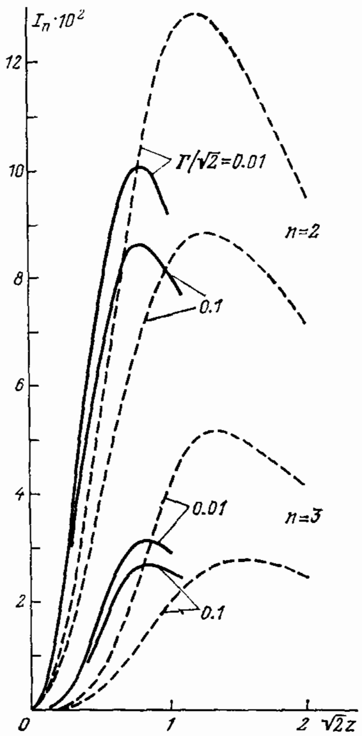

FIG. 1. Intensity of the second ($n = 2$) and third ($n = 3$) harmonics of narrow-band (solid curves) and of a harmonic signal with the same starting intensity (broken curves). An increase in the number Γ corresponds to intensification of linear dissipation.

the generation of the harmonics of the regular wave is appreciably suppressed by linear dissipation[32]; the noise, however, contains surges which are virtually unaffected by dissipation. Thus for moderate values of Γ the harmonics of the narrow-band noise are excited primarily owing to quasiperiodic oscillations with large amplitudes. The process is more efficient than in a regular wave with the same intensity.

These phenomena have been studied experimentally.[35] Noise with high intensity (up to 140 dB relative to $2 \cdot 10^{-5}$ N/m^2) propagated in a polyethylene tube 75 m long and 4.92 cm in diameter. The width of the spectral line constituted 6% of the central frequency 0.5, 1, 2, or 3.2 kHz. A tube whose wall was 0.56 cm thick was buried in sand, which suppressed the flexing modes. Thus conditions close to those of propagation of a plane wave in free space were obtained. In the tube, however, there was an additional mechanism of viscous-heat-conduction losses in the boundary layer at the walls.

The measurements of the acoustic pressure of harmonics with numbers $n = 2$–7 are shown in Fig. 2 for a monochromatic starting signal (solid curves) and narrow-band noise (circles). The ± 1 dB accuracy for $n > 3$ unavoidably decreased because of the overlapping of the spectral lines of the higher-order harmonics. The increase in the efficiency of nonlinear transformations in the field of the noise wave was clearly recorded in the experiment. Quantitative estimates of $I_n^{(N)}/I_n^{(S)}$ were obtained by extrapolating to small distances; the increase equals 2.2 ($n = 2$), 5.6 (3), 16 (4), 45 (5), 200 (6), and 500 ($n = 7$). For $n = 2$ and 3 good agreement is

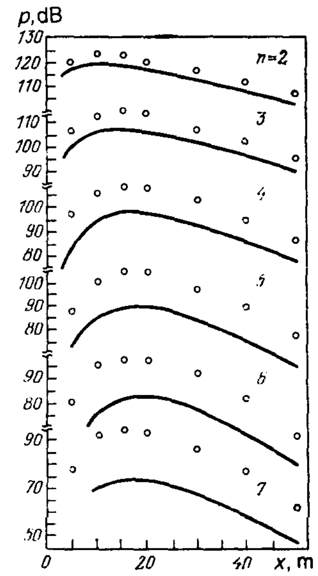

FIG. 2. Results of measurements of acoustic pressure at the frequencies of the harmonics of narrow-band noise.[35]

obtained with the theoretical value $n!$. the disagreement for $n \geqslant 4$, aside from the noted overlapping of the spectra, could be linked with the difficulties in carrying out the measurements in the near-field and with the effect of dissipation. We note that at the time the work was carried out[35] the theory of the phenomenon had not yet been developed. In later years, as far as we know, such experiments were not performed.

4. TRANSFORMATION OF BROAD SPECTRA—ACOUSTIC TURBULENCE

Interest in the characteristic features of the nonlinear evolution of broad acoustic spectra arose from the measurements of intense aviation noise. Anomalously low fading of the high-frequency wing of the spectrum is observed. In the band from 5 to 10 kHz the fading at 500 m is approximately 10 dB less than calculated.[36] Temperature nonuniformities of the atmosphere, humidity, and other properties along the path cannot explain the effect. It was therefore proposed that the anomalously high intensity of the high-frequency wing is caused by the pumping of energy from the intense low-frequency components of the noise spectrum of jet streams.[35]

The results of detailed measurmements of the parameters of stream noise under natural conditions are presented in Ref. 68. Figure 3a shows the behavior of the attenuation factor of noise waves excited by an aircraft with four jet engines. The receiver integrated over a 1/3-octave frequency bands. The distance varied from 262 to 345 m and from 345 to 501 m. It is obvious that the solid curves, constructed taking into account the nonlinear corrections, are in much better agreement with the experimental data (circles) than are the calculations based on the linear theory (broken curves). The linear theory, which does not take into account the pumping of energy upwards along the spectrum, overestimates the values of the attenuation factor.

The form of the spectra in octave frequency bands is

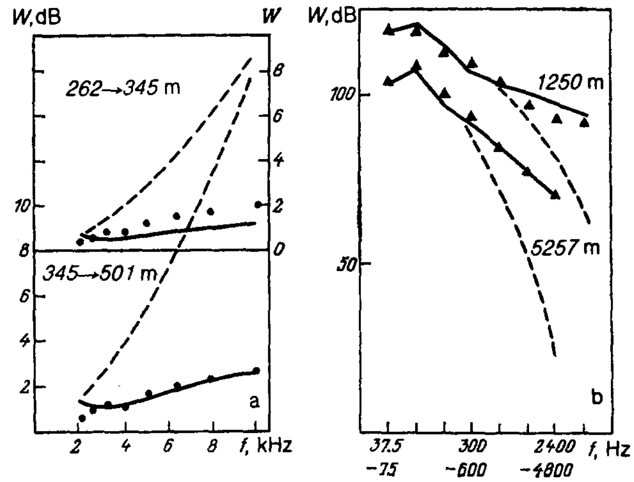

FIG. 3. Results of measurements of the attenuation factor (a) and spectrum (b) of intense stream noise under natural conditions.[68]

shown in Fig. 3b. Here the source of intense noise was the engine on the Atlas-D rocket. The points—data obtained from measurements of the sound intensity—in the frequency band > 1 kHz lie appreciably higher than the broken curves, representing the linear extrapolation of the noise spectrum measured near the source to large distances.

A series of special laboratory experiments was performed.[37,38] The measurements[37] were performed in a 29.3 m long air pipe with acoustic pressure levels up to 160 dB. Figure 4a shows the measured form of the disturbance near the source and far away from it (distances of 0.3 m and 25.9 m). Two processes are clearly manifested: increase in the slope of the leading fronts and the increase in the time scale of the oscillations. The first process gives rise to the formation of sharp jumps in the pressure—shock waves; in spectral language, this corresponds to the pumping of energy over to high frequencies, which was observed under natural conditions. The second process is associated with the motion of unsymmetric (relative to the zero level) shock fronts, their collisions, and coalescence[3]; it leads to pumping of energy from the center of the spectrum to the low-frequency band.

Thus the broad spectrum of an intense source is broadened by nonlinear interactions into both the high- and low-frequency regions. Figure 4b shows the relative acoustic noise intensities (in the 50-Hz band), measured at distances of 0.3 m, 15.0 m, and 22.3 m (curves 1–3).[37] Similar behavior of the spectrum was observed in Ref. 38, where, in addition, the statistic characteristics of the negative ("smoothed"; see Fig. 4a) slopes of the rectilinear sections of the wave profile were studied. Algorithms for the numerical simulation of the nonlinear evolution of noise disturbances were also proposed.[39]

We shall use the approach of Ref. 33 to describe the wide-band noise with the starting spectrum

$$S(z=0,\,\omega)=\frac{\omega^2}{4\pi^{1/2}}\exp\left(-\frac{\omega^2}{4}\right). \qquad (15)$$

The normalizing constants ω_0 and u_0, used in Burgers' equation (1) and the formulas (3)–(6), are interpreted here as

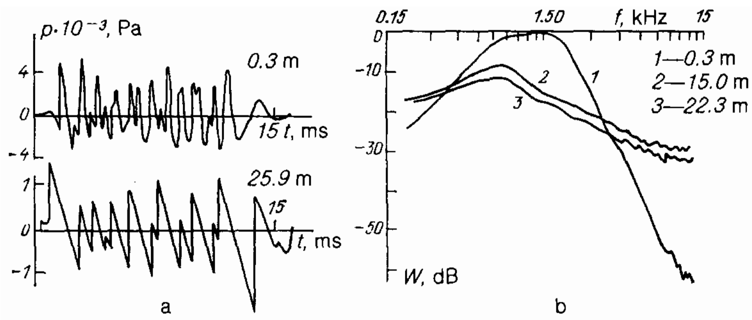

FIG. 4. Form of the disturbances (a) and power spectrum (b), measured in a laboratory experiment.[37]

the inverse correlation time t_c^{-1} and the variance σ. The auxiliary correlation function, according to (5), is given by

$$R(z,T)=(1+8\Gamma z)^{-3/2}\left(1-\frac{2T^2}{1+8\Gamma z}\right)\exp\left(-\frac{T^2}{1+8\Gamma z}\right),$$

$$\Gamma z=\frac{bx}{2c_0^3\rho_0 t_c^2}. \qquad (16)$$

Substituting (16) into (6) and calculating the integral approximately, we obtain

$$(1+8\Gamma z)S(z,\omega)=(1+3\beta)^{-7/2}\left[\frac{\omega_*^2}{4\pi^{1/2}}+\frac{9}{2\pi^{1/2}}\beta(1+3\beta)\right]$$
$$\times\exp\left[-\frac{\omega_*^2}{4(1+3\beta)}\right], \qquad (17)$$
$$\omega_*^2(z)=\omega^2(1+8\Gamma z),\quad \beta(z)=(\omega z)^2(1+8\Gamma z)^{-3/2}.$$

The dynamics of the transformation of the broad spectrum $S(z,\omega)$ (17) is shown in Fig. 5a for the inverse Reynolds numbers $\Gamma=0$ (solid curves) and $\Gamma=0.25$ (broken curves). As the distance z increases energy is observed to spread along the spectrum. At $\Gamma=0.25$, in addition, dissipation becomes significant. The low-frequency region is described by a quadratic law $S(z,\omega\to 0)\approx(\omega^2/4\pi^{1/2})\varphi(z)$. The slope $\varphi(z)$ of the parabola increases with z, reaches a maximum value $1+0.08\Gamma^{-2}$ at $z=1/2\Gamma$, and then once again decreases. We note that in a nondissipative medium the slope increases monotonically, while in a linear medium it remains constant.

At high frequencies the behavior of the broad spectrum (17) is determined by the exponential factor. The exponential decreases by a factor of e at $\omega=\omega_b(z)$; the solid curves in Fig. 5b show the dependence of the "upper limit" of the spectrum on the distance. The broken curves illustrate the corresponding linear dependence. The limiting frequency $\omega_{\rm lim}(z)$ exhibits a complicated behavior: At first it decreases; then it increases, reaches a maximum, and once again begins to decrease. This behavior is due to the fact that the nonlinear or dissipative effects predominate at different stages of the propagation of the wave. For $\Gamma>0.29$ the width decreases monotonically—dissipation predominates for all z. At $\Gamma=0.25$ the nonlinearity is significant, whereas already at $\Gamma=0.5$ it is a small correction.[1)]

The problems of describing the transformation of broad spectra are usually classified with the problem of acoustic turbulence.[40,41] The most interesting problem is thought to be the problem of finding the equilibrium form of the spectral distribution or the universal law governing the decay of $S(z,\omega)$ in the limit $\omega\to\infty$. In the region of developed shock fronts the power spectrum decays as $\sim\omega^{-2}$, since the amplitudes of the harmonics forming the discontinuity decay as ω^{-1}. At higher frequencies linear dissipation is strongly manifested, and the spectrum decays according to the exponential law $S(z,\omega)=S(z=0,\omega)\exp(-2\Gamma\omega^2 z)$, following from Eq. (5). These questions are discussed in detail in the review of Ref. 5.

Among experiments concerning the power-law asymptotic behavior, we call attention to Refs. 84 and 86, as well as Ref. 87, where the ω^{-2} dependence is carefully checked.

5. INTERACTIONS OF A SIGNAL WITH NOISE. GENERATION OF A CONTINUOUS SPECTRUM. SUPPRESSION OF NOISE BY AN INTENSE SIGNAL

Among all the problems of statistical nonlinear acoustics, the problems of the interactions of regular and noise

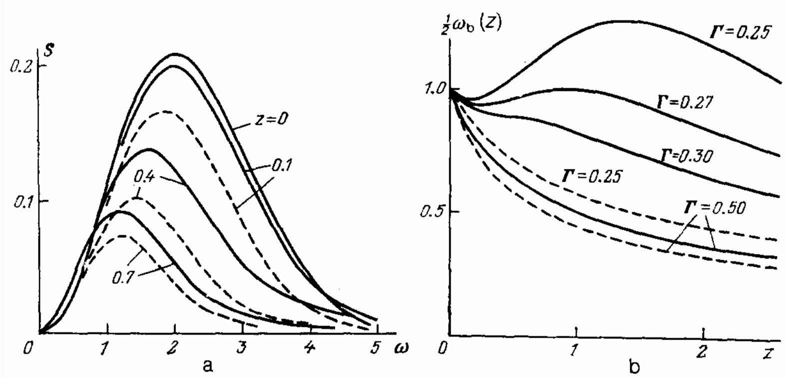

FIG. 5. Dynamics of the transformation of broad noise spectra for $\Gamma=0.25$ (broken curves) and $\Gamma=0$ (solid curves) (a) and of the "upper boundary" of the nonlinear spectrum (b).

waves are of greatest interest for applications. The directions of flow of acoustic energy in signal-noise interactions are determined by the relative intensities and the position of the spectra of the regular and random components of the disturbance. Some typical situations are discussed in Secs. 5 and 6.

The theory of the interaction of simple monochromatic and noise waves is developed in Ref. 31. We present an expression which generalizes the results of Refs. 3 and 31 to more complicated systems in the approximation of quasi-simple disturbances (see Sec. 1):

$$2\pi (\omega z)^2 S(z, \omega) = e^{-(\omega z)^2\sigma^2(z)} \int\limits_{-\infty}^{\infty} [e^{(\omega z)^2 R(z, T)} - 1]\, e^{-i\omega T}\, \overline{\exp\{i\omega z [s(z, T+T') - s(z, T')]\}}\, dT + \left| \int\limits_{-\infty}^{\infty} [e^{-(1/2)(\omega z)^2\sigma^2(z)} e^{i\omega z s(z, T)} - 1]\, e^{-i\omega T}\, dT \right|^2. \qquad (18)$$

Here the overbar denotes averaging with respect to the time T', which is necessitated by the presence of the regular signal $s(z,T)$.

We shall study, following Ref. 31, the simplest case of the interaction of plane waves in a nondissipative medium. If the noise at the inlet is mixed with a harmonic (at $x = 0$) disturbance $u = u_0 \sin \omega_0 t$, then in the solution (18) we must set $s = A \sin \Omega_0 T$ (here $s = u/\sigma$, $A = u_0/\sigma$, $\Omega_0 = \omega_0 t_c$). Then, from (18) we can obtain a formula which describes the distortion of the noise component:

$$S^{(N)}(\omega, z) = -\frac{1}{\pi} J_0^2 (A\omega z) \times \int\limits_0^{\infty} R'(T) \exp[(\omega z)^2 (R-1)] \frac{\sin \omega T}{\omega}\, dT. \qquad (19)$$

The result (19) differs from the corresponding expression (6) by the presence of the factor J_0^2, which accounts for the nonlinear losses in the noise spectrum owing to pumping of part of its energy upwards along the spectrum (due to the presence of the regular signal with $A \neq 0$).

The creation of new sections of the spectrum in the signal-noise interaction process is described by the expression

$$S^{(N,S)}(\omega, z) = -\frac{1}{\pi} J_1^2 (A\omega z) \times \int\limits_0^{\infty} R'(T) \exp[(\omega z)^2 (R-1)] \frac{\sin(\omega - \Omega_0) T}{\omega - \Omega_0}\, dT. \qquad (20)$$

Finally, the second term in the formula (18) contains information about the behavior of the harmonics of the regular disturbance. In particular, the amplitude of the wave at the fundamental frequency Ω_0 equals

$$S^{(S)}(\omega, z) = \frac{A^2}{2} e^{-(\omega z)^2} \left[\frac{2J_1(A\omega z)}{A\omega z}\right]^2 \delta(\omega - \Omega_0). \qquad (21)$$

It is evident that as a result of the nonlinear interaction with the noise component [this process is described by the exponential factor in (21)] and the generation of the characteristic harmonics (the factor in the brackets), the spectral density of the signal decreases, but the spectrum $S^{(S)}$ retains its delta-function form.

Let us examine an example illustrating the phenomena described. Let the noise correlation function at the inlet have the form $R(T) = (1 - 2T^2)\exp(-T^2)$, corresponding to the starting spectrum (15). Calculating the integrals (19) and (20) we arrive at the result shown in Fig. 6 for $\Omega_0 = 10$, $A = 0.5$. As z increases, the noise spectrum $S^{(N)}$ becomes distorted—energy flows from the center to the low and high frequency bands (see also Fig. 5).

It is of special interest to study the dynamics of the new region of the spectrum $S^{(N,S)}$ appearing in the vicinity of $\omega = \Omega_0$. The width of this "pedestal" at small distances equals approximately twice the width of the spectrum of the noise disturbance; each of the "wings" on both sides of Ω_0 has the same form as that of the starting noise spectrum. We call attention to the unsymmetric (relative to $\omega = \Omega_0$) form of the pedestal. Its high-frequency wing has a larger amplitude and grows more rapidly as a result of the nonlinear pumping of energy. The depression between the wings gradually vanishes (at $z \approx z_3$), and both wings of the pedestal coalesce into a single broad line, whose energy center is displaced into the frequency band $\omega > \Omega_0$. Later the amplitude of the line increases even more (up to $z = z_4$), and then the stage of spreading along the spectrum appears (see $z = z_5$); in addition, most of the energy is once again pumped into high frequencies.

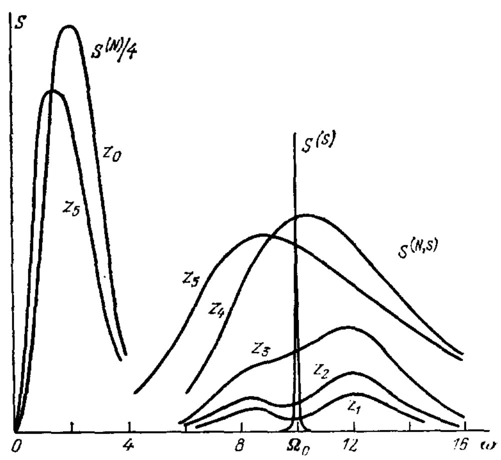

FIG. 6. Appearance of a section of the continuous spectrum near the first harmonic of the signal interacting with noise.

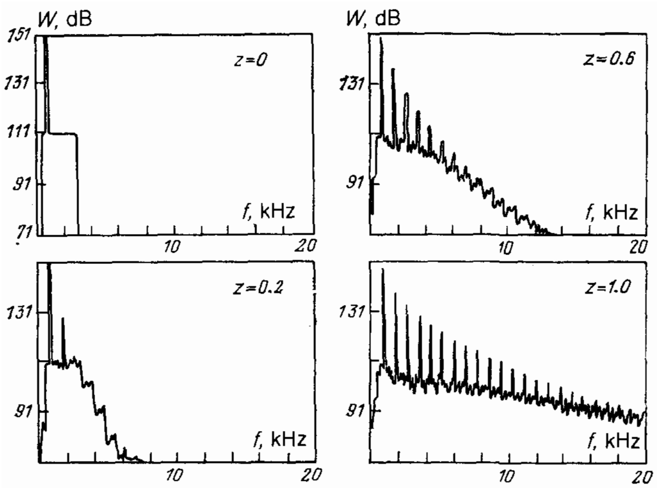

FIG. 7. Results of numerical modeling of the problem of signal–noise interaction.[42]

Figure 6 shows only part of the spectrum. The full picture is more complicated. Harmonics of the signal $n\Omega_0$ are created in the medium, and "pedestals" appear at the foot of each one.

For simplicity, we have talked about the position of the spectral line of the signal when there is no overlapping with the spectrum of the random disturbance. The solution (18) enables studying the general case also. If the signal and noise bands overlap, the dynamics become more complicated and the separation of the spectrum into the components $S^{(N)}$, $S^{(N,S)}$, $S^{(S)}$ for $z \neq 0$ becomes a formal procedure.

It is precisely the last case that is encountered in practice. Thus cavitation spectra,[13] noise from jet engines, and other sources of intense disturbances consist of discrete lines, against the background of wide-band noise. Since in accordance with Fig. 6 the broad spectrum is reproduced at the foot of each discrete component, it may be concluded that the continuous part of the spectrum grows rapidly as the sound propagates in the medium. This conclusion is confirmed by direct experiments,[42] which agree with the theory.[31] The intense formation of the continuous spectrum was also observed in Refs. 43 and 44; in particular, it was observed in Ref. 44 that the wide-band noise is intensified by 7 dB when a jet stream is irradiated with intense noise (130 dB).

The dynamics of this process is shown in Fig. 7. The spectra for different $z^{(s)}$ (the distance in units of the discontinuity lengths of the harmonic wave) were obtained in Ref. 42 in a numerical simulation of the physical experiment, which is described below. It is obvious that the propagation of the mixture of the discrete signal and the noise ($z^{(s)} = 0$) is accompanied by the appearance of harmonics of the signal (more than 30), which "stretch out" and intensify the continuous part of the spectrum. Thus at 20 kHz an intenstification by 25 dB was observed as the distance $z^{(s)}$ changed from 0.8 to 1.2.

The phenomena were studied experimentally in a tube filled with air. A noise wave was excited in the 1–3 octave band centered at the frequency 0.7–1.2 kHz. The average intensity of the harmonic signal reached 140–151 dB. Its frequency varied and could lie near the lower or upper boundary of the noise spectrum. Figure 8 shows the spectrograms of the noise with no signal (upper series) and in the presence of the full disturbance (bottom series); they were obtained at distances of $x = 0$ and $x = 7.38$ m with a receiver having an integration band of 100 Hz. It is obvious that when the signal is switched off the noise spectrum remains practically unchanged—its intensity is too low. In the presence of a 1.73 kHz signal, however, the broad spectrum becomes transformed: a continuous component extending to at least 50 kHz appears.

We note that the continuous spectrum does not always appear solely because of the wave nonlinearity. In specific systems other mechanisms can also operate. Thus, in the problem of acoustic cavitation nonperiodic pulsations under harmonic action were already observed for the simplest object—a single gas bubble. The nonlinear oscillator—the bubble—belongs to the class of Feigenbaum systems,[45] which are characterized by the presence of a stochastic attractor and by the fact that the motion becomes more complicated as

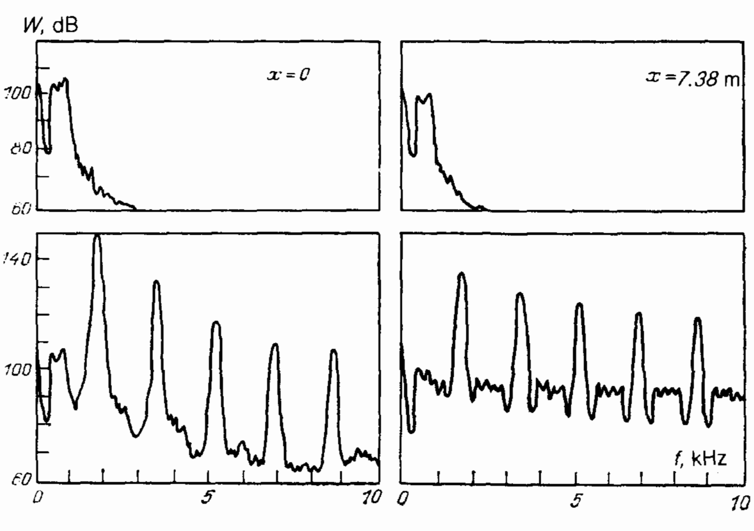

FIG. 8. Experimental spectrograms of noise in the absence of a signal (top series) and a mixture of an intense signal and a weak noise wave (bottom series).[42]

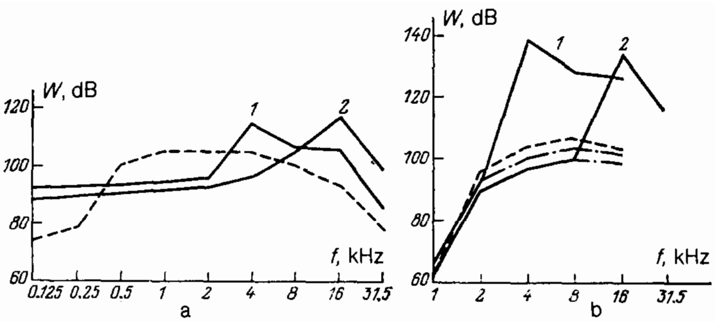

FIG. 9. Suppression of low-frequency noise by an intense high-frequency signal. a) Measured spectra; b) results of calculations in Ref. 48.

a result of period-doubling bifurcations.[46] The process of stochastization leads to the formation of a continuous spectrum of oscillations of a single bubble.[47] Of course, with the transition to a distributed system (cavitation region) a picture similar to that in Figs. 7 and 8 will be observed.

In addition to being intensified, the noise spectrum, as follows from (19), can also be somewhat suppressed. To observe this, it is necessary to choose a sufficiently high signal frequency so that the noise spectrum and the "pedestal" for the first harmonic do not overlap. The supression of low-frequency noise by an intense signal is of great significance. Low-frequency noise has a harmful effect on structures and organisms; for this reason, safety requirements for operation of passenger aircraft are increased.[8] The unavoidable increase in the total sound intensity under the action of the intense signal does not present any danger, since it occurs in the high-frequency components which decay rapidly as they propagate.

This phenomenon was studied theoretically and experimentally in Ref. 48. The experimental arrangment consisted of a tube 11 cm in diameter, made up of five detachable sections each 1 m long. The gas–dynamic source of noise was placed on the axis of the tube; generators of the narrow-band signal were placed around it. The measurements were performed beyond the tube at a distance of 1 m from its output end at an angle of 45° to the axis. With the signal switched off the starting noise spectrum (broken curve in Fig. 9a) remained virtually unchanged at distances from 1 to 5 m. Because of the interaction with the signal there appeared a region in which the noise level decreased. Curve 1 corresponds to a signal frequency of 5.5 kHz; curve 2 corresponds to a signal frequency of 13.5 kHz. The tube was 5 m long. The tendency for the noise level to decrease increased with the intensity of the signal and the length of the interaction region.

Figure 9b shows the results of the calculations. The noise spectrum was taken in the form (15), and the inverse correlation time $t_c^{-1} = 8$ kHz. The ratio of the intensity of the noise component to that of the regular component equaled 0.16. The curves 1 and 2 were constructed for the distance $z = \varepsilon\sigma x/c_0^2 t_c = 0.8$. The dot-dashed curves show the spectrum of the suppressed noise component. Figures 9a and b agree only qualitatively. The form of the noise spectrum did not completely correspond to the observed form; diffraction of waves along the path from the output end of the tube to the sound receiver was not taken into account. In addition, the intense sound can change the structure of the turbulent flow and therefore that of the starting noise spectrum. For this reason, in such problems, aside from nonlinear interactions of sound with sound, it is also necessary to take into account acoustohydrodynamic phenomena of the sound-turbulence type.[22]

We call attention to the possibility of noise suppression as a result of a different process—the interaction of noise with a regular signal at a lower frequency. This possibility follows from a remarkable property of the solutions of Burgers' equation (1). Let $V^{(N)}(z,\theta)$ be an exact solution of (1). It is easy to verify that

$$V^{(S+N)} = \frac{\theta}{z_0[1-(z/z_0)]} + \frac{1}{1-(z/z_0)} V^{(N)}\left(\frac{z}{1-(z/z_0)}, \frac{\theta}{1-(z/z_0)}\right) \quad (22)$$

will also be an exact solution. In other words, from one solution it is possible to obtain another solution by superposing on it a linearly increasing or decreasing (depending on the sign of the constant z_0) function of time $\theta = \omega_0(t - x/c_0)$ in the comoving coordinate system. As follows from (22), for $z_0 > 0$ the wave $V^{(N)}$ is intensified on a rising slope and at distances $0 < z < z_0$ according to the law $1/[1-(z/z_0)]$, and its spectrum is shifted into the high-frequency region (the old variable θ transforms into $\theta/[1-(z/z_0)]$. Conversely, on a falling slope ($z_0 < 0$) the wave is suppressed $\sim 1/[1+(z/|z_0|)]$, and its spectrum is shifted into the low-frequency regon.[51,63]

The picture of this interaction is shown in Fig. 10. The starting (at $z = 0$) profile of the low-frequency signal $V^{(S)}$ has the form of an equilateral triangle; a noise disturbance $V^{(N)}$ is superposed on it (see Fig. 10a). The distorted profile of the resulting wave $V^{(S+N)}$ is shown on the left in Fig. 10b; it corresponds to the distance $z > 0$ [but $z < z_0$, since a shock front has not yet formed on the front side of $V^{(S)}$]. The noise component $V^{(N)}$ of the distorted wave is shown on the right in Fig. 10b. It is obvious that on the steep front slope of $V^{(S)}$ the noise is compressed and intensified, while on the gently sloping trailing side the noise is stretched out in time and suppressed.

The regular wave $V^{(S)}$ can contain only descending slopes. An example is the typical, for nonlinear acoustics, periodic saw-tooth wave, in which the linear sections of the profile are connected by shock fronts. It is obvious that such a "saw" will suppress high-frequency noise.

The question of the drop in the noise level accompany-

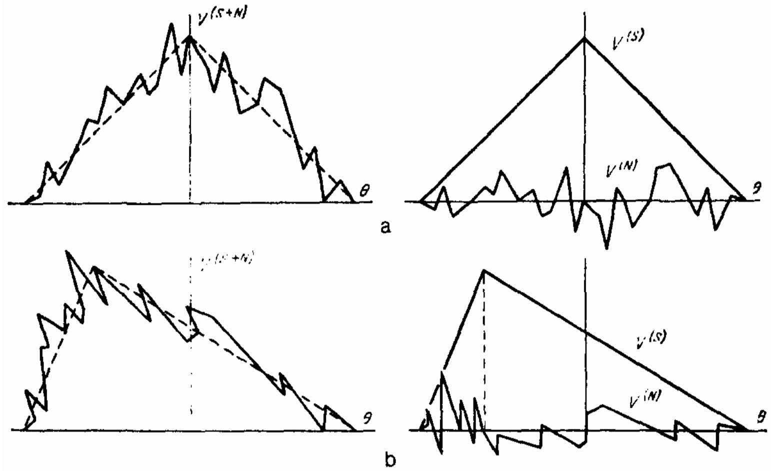

FIG. 10. Suppression of noise interacting with a low-frequency signal.

ing the interaction of the noise with an intense signal is also discussed in Ref. 49.

We emphasize that the possiblity of controlling the spectra, the shift in the frequency, and the amplification of weak signals in the field created by a powerful low-frequency pump, following from (22) and Fig. 10, should be of great importance not only for statistical problems, but also in different applications of nonlinear acoustics. This question is further discussed in Sec. 6 for the example of the problem of excess fading of a signal in a noise field.

6. FADING OF A SIGNAL INTERACTING WITH NOISE

The nonlinear interpretation of dissipative processes in the language of phonon–phonon interactions originated in the classical works of Landau, Rumer, and Akhiezer; it is mostly employed for calculating the sound attenuation factors in solids.[16] In many cases, however, this viewpoint is useful in the study of waves in liquids and gases. This is linked with the possibility of taking into account extraneous noise (whose presence can lead to anomalously high attenuation), the generation of harmonics, and other factors. Thus a hypothesis attributing the excess absorption of low-frequency sound in the ocean to the interaction with noise waves accumulating in the underwater sound channel was stated in Ref. 50. The results of Refs. 31 and 50 are linked with a number of experiments,[51–53] involving the comprehensive study of the nonlinear attenuation of a signal in a noise field. Theoretical studies in this direction have also been proposed.[54–57]

We shall first study the process of nonlinear attenuation of a weak signal with frequency ω_0 interacting with an intense low-frequency wave ω_L ($\omega_L \ll \omega_0, u_L \gg u_0$). Setting in the formula (3) $V=\exp(-\Gamma z)\sin T + (u_L/u_0)\sin(\omega_L T/\omega_0)$, we obtain for the particle velocity at the signal frequency ω_0

$$u = u_0 \exp\left(-\frac{b\omega_0^2}{2c_0^3\rho_0}x\right) J_0\left(\frac{\varepsilon}{c_0^2}\omega_0 u_L x\right)\sin\omega_0\tau. \qquad (23)$$

It is evident that the amplitude decreases for two reasons. The exponential in (23) describes the linear attenuation, and the Bessel function J_0 describes the attenuation associated with the presence of the low-frequency wave ($u_L \neq 0$). The argument of $J_0(y)$ can be written as $y=(x/x_{\text{disc},L})(\omega_0/\omega_L)$. Since the ratio of the frequencies is $\omega_0/\omega_L \gg 1$, the value of y (for distances $x \sim x_{\text{disc},L} = c_0^2/\varepsilon\omega_L u_L$—the discontinuity distance in a powerful low-frequency wave) can be large: $y \gg 1$. In addition, the amplitude of the signal, which decays as a function of the distance, will oscillate.

This behavior of the signal has been observed experimentally.[51,53] In Ref. 53 the measurements were performed

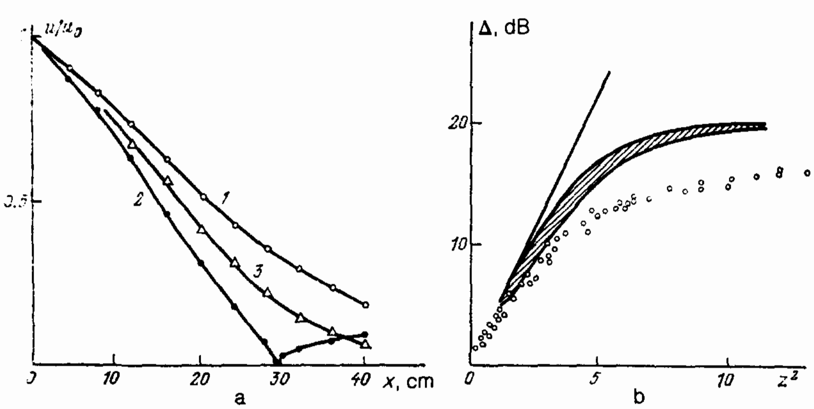

FIG. 11. Results of experiments on the nonlinear attenuation of an acoustic signal interacting with a noise wave in water.[53]

in a cell with water in a pulsed regime at megahertz frequencies. Curve 1 in Fig. 11a shows the experimental dependence of the amplitude of the signal, whose frequency equaled $\omega_0/2\pi = 11.5$ MHz, on the distance; the oscillatory velocity at the inlet was $u_0 \approx 8\cdot 10^{-2}$ cm/s. Here there are no nonlinear effects; damping occurs owing to viscosity and heat conduction. The experimental values of the amplitude of the signal interacting with the low-frequency wave $\omega_L/2\pi = 1.35$ MHz and $u_L \approx 6.5$ cm/sec are plotted on the curve 2. The attenuation increased appreciably, and the behavior of the curve qualitatively changed. Analogous oscillatory behavior of the amplitude was observed under natural conditions[51] at lower frequencies: $\omega_L/2\pi = 68$ kHz and $\omega_0/2\pi = 244$ kHz. We note that the curve 2 is described well by the formula (23). In particular, from the condition $y = 2.4$, corresponding to the first root of the Bessel fuanction $J_0(y)$, setting for water $\varepsilon = 4$, $c_0 = 1.5\cdot 10^5$ cm/s, we find the coordinate of the signal minimum ≈ 29 cm.

When the nonlinear attenuation of the signal occurs as a result of the interaction with noise, as follows from the formulas of Sec. 5 the amplitude decays exponentially

$$u = u_0 \exp\left[-\frac{b\omega_0^2}{2c_0^3\rho_0}x - \frac{1}{2}\left(\frac{\varepsilon}{c_0^2}\omega_0\sigma x\right)^2\right]\sin\omega_0\tau. \qquad (24)$$

In this case the additional damping is determined solely by the noise intensity σ^2 and is independent of the mutual position of the noise and signal spectra.

The results of experiments[53] on the attenuation of a signal in a noise field are shown in Fig. 11a (curve 3). The noise spectrum is concentrated in the band 0.6–1.9 MHz. The noise intensity 0.3 W/cm^2 ($\sigma = 4.6$ cm/s) was the same as that of the regular low-frequency wave when the points on curve 2 were measured. The behavior of curve 3 is described well by the formula (24).

Analogous measurements with high spectral resolution taking into account the finite width of the signal line were performed in Ref. 53. Under real conditions it often happens that the signal $S^{(S)}$ coalesces with the spectral complex $S^{(N,S)}$ (see Fig. 6), formed in the process of the interaction at the base of the line. Actually, a new and wider line, which cannot be separated into individual components, forms. Here it makes sense to follow only the peak value of the spectral density at the signal frequency.

The results of the measurements are shown in Fig. 11b. The straight line gives the excess fading $\Delta = 10 \lg e\cdot z^2 \approx 4.3(\varepsilon\omega_0\sigma x/c_0^2)^2$ of a delta-shaped line. The shading marks the region of theoretical values of Δ obtained taking into account the finiteness of the width of the signal spectrum and the error in the noise measurement. The disagreement with the experimental values (points) must apparently be attributed to the fact that diffraction phenomena in the interacting beams were ignored.

When the noise and signal waves propagate collinearly, the nonlinear attenuation is described by the formula (24). It is shown in Ref. 31 that when the weak noncollinear interactions are taken into account, the law $\exp(-\beta x^2)$ transforms into the standard dependence $\exp(-\alpha x)$. However, the model of an isotropic noise field better corresponds to real conditions. For this reason, a more accurate calculation must be performed not in the plane-wave approximation, but rather according to the theory of nonlinear acoustic beams.

We shall employ the model equation

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)u = -\frac{2\varepsilon}{c_0^3}\frac{\partial^2 u^2}{\partial t^2}, \qquad (25)$$

which is equivalent to the evolutionary Khokhlov-Zabolotskaya (KZ) equation

$$\left(\frac{\partial^2}{\partial x\,\partial\tau} - \frac{c_0}{2}\Delta_\perp\right)u = \frac{\varepsilon}{c_0^2}\frac{\partial^2 u^2}{\partial\tau^2}. \qquad (26)$$

In order to transform from (25) to (26) it is necessary to use the approximations of a slowly varying profile and quasi-optics,[58] i.e., in (25) we must set

$$u(x, \mathbf{r}_\perp, t) = u\left(\tau = t - \frac{x}{c_0},\ \mu x,\ \mu^{1/2}\mathbf{r}_\perp\right); \qquad (27)$$

here $\mathbf{r}_\perp$ is the coordinate in the plane perpendicular to the beam axis. $\Delta_\perp$ is the Laplacian with respect to the corresponding variables, x is the coordinate along the axis, and μ is a small parameter.

We shall represent the longitudinal component of the oscillatory velocity as

$$u = u^{(S)} + u^{(N)} + u^{(N,S)} \qquad \langle u^{(N)}\rangle = \langle u^{(N,S)}\rangle = 0; \qquad (28)$$

here $u^{(S)}$ is the regular signal (a harmonic plane wave), $u^{(N)}$ is the steady-state noise, and $u^{(N,S)}$ is a small fluctuation correction to the signal, appearing because of the interaction of the signal with the noise. We assume that the signal is weak and we neglect both the formation of its harmonics and the reaction on the noise field. Under these assumptions, from (25) we obtain the system of equations

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)u^{(N,S)} = -\frac{2\varepsilon}{c_0^3}\frac{\partial^2}{\partial t^2}(u^{(S)}u^{(N)}), \qquad (29)$$

$$\left(\frac{\partial}{\partial x} + \frac{1}{c_0}\frac{\partial}{\partial t}\right)u^{(S)} = \frac{\varepsilon}{c_0^2}\frac{\partial}{\partial t}\langle u^{(N,S)}u^{(N)}\rangle. \qquad (30)$$

An analogous system can be obtained from the KZ equation, but the subsequent calculations based on (29) and (30) are much simpler.

We shall write down the solution of (29) in terms of the retarded potential and set in the solution

$$u^{(S)} = \frac{1}{2}u_0\exp(i\omega_0 t - ik_0 x) + \text{c.c.},$$

$$u^{(N)}(t, \mathbf{r}) = \frac{1}{2}\int_{-\infty}^{\infty}\tilde{u}^{(N)}(\omega, \mathbf{r})\,e^{i\omega t}\,d\omega + \text{c.c.} \qquad (31)$$

This gives

$$u^{(N,S)} = -\frac{\varepsilon u_0}{4\pi c_0^3}\int_{-\infty}^{\infty}d\omega\int d\mathbf{r}'\,\frac{(\omega_0+\omega)^2}{|\mathbf{r}-\mathbf{r}'|} \times \exp\left[-ik_0x + i(\omega_0+\omega)\left(t - \frac{|\mathbf{r}-\mathbf{r}'|}{c_0}\right)\right]\tilde{u}^{(N)}(\omega, \mathbf{r}'). \qquad (32)$$

Substituting (31) and (32) into Eq. (30) and carrying out the averaging, we find the dispersion law

$$k_0 - \frac{\omega_0}{c_0} = \frac{\varepsilon^2\omega_0}{4\pi c_0^5}\int_{-\infty}^{\infty}(\omega_0-\omega)^2\,d\omega\int G(\omega, R) \times \exp\left[ik_0R\cos\theta - i(\omega_0-\omega)\frac{R}{c_0}\right]\frac{d\mathbf{R}}{R}, \qquad (33)$$

where $\mathbf{R}=\mathbf{r}-\mathbf{r}'$, $R\cos\theta=x-x'$, and $G(\omega, R) = \langle \tilde{u}^{(N)}\tilde{u}^{(N)*}\rangle = G(\omega)\times\sin(\omega R/c_0)/(\omega R/c_0)$ is the correlation function of the wave noise in a nondispersive medium. After integrating in (33) we obtain the attenuation factor

$$\alpha=\frac{\pi\varepsilon^2}{4c_0^3}\left[\int\limits_0^{\omega_0}\frac{\omega_0^2+\omega^2}{\omega}G(\omega)\,d\omega+2\omega_0\int\limits_{\omega_0}^{\infty}G(\omega)\,d\omega\right]. \quad (34)$$

The formula (34) was first derived by Westervelt.[56] Here, however, we give the computational scheme of Ref. 59, which is based on the simplified equations of nonlinear acoustics. This scheme is convenient for solving more complicated problems, in which it is necessary to take into account the dynamics of internal processes in the medium or its structural peculiarities, leading to the appearance of dispersion.

If the frequency of the signal is much higher than the characteristic frequency of the noise, the attenuation factor is proportional to the square of the frequency

$$\alpha\approx\frac{\pi\varepsilon^2}{4c_0^3}\omega_0^2\int\limits_0^{\infty}\frac{G(\omega)}{\omega}\,d\omega. \quad (35)$$

For the inverse ratio of the characteristic frequencies, α depends linearly on ω_0[3,31]:

$$\alpha\approx\frac{\pi\varepsilon^2}{2c_0^3}\omega_0\int\limits_0^{\infty}G(\omega)\,d\omega=\frac{\pi\varepsilon^2\mathscr{E}}{2c^3\rho_0}\omega_0; \quad (36)$$

here $\mathscr{E}=\rho_0\langle u^{(N)2}\rangle$ is the volume density of the energy in the noise waves. In Refs. 56 and 59 the formula (34) was used to describe the attenuation of first sound in superfluid helium at temperatures <0.6 K and the attenuation of low-frequency signals accompanying the interaction with dynamic noise in the ocean.

We shall indicate other schemes for calculating the dissipative coefficients in a noise field. In Ref. 60 a method based on the use of canonical variables for liquids or gases and the frequency distribution of the occupation numbers of the noise modes is described. Different modifications of models of weak acoustic turbulence and the random phase approximation are often used.[2,54,61] The fact that in acoustics only the quasicollinear waves interact efficiently is taken into account in these schemes already at the evaluation stage, and not in order to simplify the starting equations or calculations.

Returning to one-dimensional models, we note that the results of recent studies[62,63] of strong turbulence, described by Burgers' equation, are very useful for understanding the physics of noise–signal interactions.

7. DIFFRACTION OF INTENSE NOISE

The effect of the time-dependent and spatial statistics on the interaction of diffracting wave beams has been studied in detail in nonlinear optics.[4] The analogous phenomena in acoustics have not been studied. The complexity of nonlinear acoustic problems comes from the specific nature of these problems—the creation of many spectral components in a nondispersing medium. Under these conditions, one cannot regard waves as being quasiharmonic and one cannot transform from field equations to nonlinear parabolic equations for the amplitudes.

The basic equation of the nonlinear acoustics of finite beams is the KZ equation (26). We shall employ this equation to calculate the space-time-dependence correlation function

$$B(x,\tau=\tau_1-\tau_2,\mathbf{r}_1,\mathbf{r}_2)=\langle u(x,\tau_1,\mathbf{r}_1)\,u(x,\tau_2,\mathbf{r}_2)\rangle. \quad (37)$$

In accordance with the approximate method presented in Sec. 2, the solution of the nonlinear problem can be constructed in two stages. First, we find the auxiliary correlation function $B(x,\tau,\mathbf{r},\mathbf{R})$ from the linearized equation, corresponding to (26),

$$\frac{\partial^2 B}{\partial x\,\partial\tau}=\frac{c_0}{2}(\Delta_{\perp,1}-\Delta_{\perp,2})B=c_0\frac{\partial^2 B}{\partial\mathbf{r}\,\partial\mathbf{R}}, \quad (38)$$

where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$. Then, to obtain the physical result, the auxiliary function must be transformed by means of a nonlinear transformation of the type (6).

The solution (38) with the condition $B=B_0(\tau,\mathbf{r},\mathbf{R})$ at the boundary $x=0$ of the nonlinear medium has the form

$$B(x,\tau,\mathbf{r},\mathbf{R})=-\frac{1}{(2\pi)^2}\frac{1}{(c_0x)^2}\frac{\partial^2}{\partial\tau^2}\iint B_0\left(\tau-\frac{(\mathbf{r}-\mathbf{r}')(\mathbf{R}-\mathbf{R}')}{c_0x},\mathbf{r}',\mathbf{R}'\right)d\mathbf{r}'\,d\mathbf{R}'. \quad (39)$$

Let the starting ($x=0$) space–time-dependent correlation function be represented as a product of a time-dependent correlation function $B_0(t)$ and a space-dependent $B_\perp(\mathbf{r})$ correlation function

$$B_0(t,\mathbf{r},\mathbf{R})=B_0(t)\,B_\perp(\mathbf{r})\,I(\mathbf{R}); \quad (40)$$

Here $I(\mathbf{R})$ describes the distribution of the average intensity in the transverse cross section of the beam. Substituting (40) into (39) we see that for wide-band signals for $x>0$ $B(x,\tau,\mathbf{r},\mathbf{R})$ cannot be decomposed into the product of space- and time-dependent correlation functions, as was done for narrow-band randomly modulated waves.[4] Thus, in the process of propagation of acoustic noise the space- and time-dependent statistics are strongly coupled already in the linear approximation.

When the starting transverse correlation radius is less than other characteristics scales ($r_c\to 0$, the spatial correlation is approximated by a δ function) from (39) and (40) we obtain

$$B(x,\tau,\mathbf{r},\mathbf{R})=\frac{1}{\pi}\left(\frac{r_K}{2c_0x}\right)^2\times\int\limits_{-\infty}^{\infty}\omega^2 S_0(\omega)\,B_\perp(x,\omega,\mathbf{r})\exp\left[i\omega\left(t-\frac{\mathbf{r}\mathbf{R}}{c_0x}\right)\right]d\omega. \quad (41)$$

Here, in accordance with the Van-Zittert–Zernike theorems, the transverse correlation function is related to the initial intensity distribution by a spatial Fourier transform

$$B_{\perp}(x, \omega, \mathbf{r}) = \int I(\mathbf{R}') \exp\left(i \frac{\omega}{c_0 x} \mathbf{r}\mathbf{R}'\right) d\mathbf{R}'. \qquad (42)$$

It follows from the formula (41) that for wide-band signals a more general theorem holds: $B(x, \tau, \mathbf{r}, \mathbf{R})$ is determined by the frequency Fourier transform of the product $\omega^2 S_0 B_{\perp}$, where S_0 is the starting wave spectrum.

We shall examine the following example: at the inlet the beam I is Gaussian and the spectrum S_0 corresponds to a Gaussian correlation function $B_0(t) = \exp(-t^2/t_c^2)$.

$$I(\mathbf{R}) = u_0^2 \exp\left(-\frac{R^2}{R_0^2}\right), \quad S_0(\omega) = \frac{t_c}{2\sqrt{\pi}} \exp\left(-\frac{\omega^2 t_c^2}{4}\right). \qquad (43)$$

We shall calculate (41); concentrating on the correlation function near the axis of the beam, we set $\mathbf{r}_1 = 0$, $\mathbf{r}_2 = \mathbf{s}$. The linear space–time-dependent correlation function assumes the form

$$\tilde{B} = 2\left(\frac{c_0 x t_c}{r_c R_0 u_0}\right)^2 B(x, \tau, s)$$

$$= \left(1 + \frac{s^2}{r_{c1}^2}\right)^{-3/2} \left[1 - 2\frac{\left(\frac{\tau}{t_c} + \frac{s^2}{r_{c2}^2}\right)^2}{1 + \frac{s^2}{r_{c1}^2}}\right]$$

$$\times \exp\left[-\frac{\left(\frac{\tau}{t_c} + \frac{s^2}{r_{c2}^2}\right)^2}{1 + \frac{s^2}{r_{c1}^2}}\right]. \qquad (44)$$

There are two characteristic correlation radii:

$$r_{c1}(x) = \frac{c_0 t_c x}{R_0} \sim x, \quad r_{c2}(x) = (2c_0 t_c x)^{1/2} \sim x^{1/2}. \qquad (45)$$

In the limit $x \to 0$ the effective radius $r_c(x)$ increases linearly with the distance, after which a slower square-root growth appears.[64]

Figure 12 shows the normalized function $\tilde{B}(x, \tau = 0, s)$, constructed based on the formulas (44) for the values $\beta = (r_{c1}/r_{c2})^4 = 0.1; 0.2; 0.4; 0.7; 1.0$ (curves 1–5). Setting $\tau = 0$ we thereby assume that the function B is measured with two wide-band receivers (one of them lies on the axis of the beam, and the other is a distance s away from the axis), which record the total noise energy over the entire frequency spectrum. The change in the form of the curves with increasing β indicates the fact that aside from r_{c1}, the correlation radius r_{c2} also strongly affects the spatial correlation of the field. The broken curve 1′ in Fig. 12 shows the behavior of the effective correlation radius, determined at the 0.1 level [this is the dependence of $r_c/2R_0$ on the reduced distance $\sqrt{\beta} = (c_0 t_c/2R_0^2)x$. The curve 2′ describes the increase in the correlation radius $r_c(x)$, when the assumption that the starting field is δ-correlated breaks down and $r_c(0) = 0.2R_0$. Differences from the curve for $r_c(0) = 0$ are noticeable only at small distances.

The function $\tilde{B}$ (44) was calculated for wide-band noise, given at $x = 0$ in the form of a beam with a plane phase front. At short distances the increase in the correlation radius according to the law $r_{c1}(x)$ (45) is a result of diffraction. When the correlation radius begins to grow according to the law $r_{c2}(x)$, the diffraction process is mainly completed and the growth is determined by the spherical divergence of the components of the spectrum. It is precisely for this reason that the maximum correlation is observed [see (44)] for $\tau = -s^2/2c_0x$; the delay in the measurement of τ at a distance s from the axis must equal precisely the difference in the arrival times of the waves at the points located at distances x and $(x^2 + s^2)^{1/2} \approx x + (s^2/2x)$ from the sound source.

Figure 13 shows the "limits" of the correlation function (44) on the surface τ/t_c, s^2/r_{c1}^2 for different values of the parameter $\beta^{1/2} = (c_0 t_c/2R_0^2)x = 0; 0.1; 0.2$ (curves 1–3). these curves are formed by the intersection of the surface $\tilde{B}(\tau, s)$ with the surface at the level $\tilde{B} = 0.1$. It is evident that as x increases the region of the highest values of the function $\tilde{B}$ becomes localized along the curve $\tau = -s^2/2c_0x$.

Consider now the nonlinear problem. Neglecting for the time being the change in the spatial statistics ($s = 0$), we take into account its effect on the evolution of the spectrum of the intense diffracting noise. (The generation of noise acoustic harmonics in regular beams neglecting diffraction is studied in Ref. 65.) A new parameter appears here—the nonlinear length $x_{\text{disc}} = c_0^2 t_c/\varepsilon u_0$, so that it is convenient to transform to the variables used in (1) and (6): $z = x/x_{\text{disc}}$, $T = \tau/t_c$. We shall regard B and ω as dimensionless quanti-

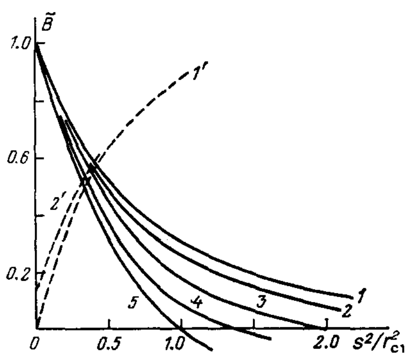

FIG. 12. Spatial correlation function of narrow-band acoustic noise and increase in the correlation radius.

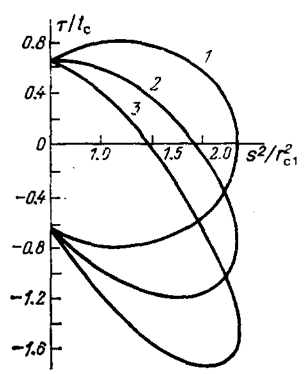

FIG. 13. Change in the form of the "boundary" (with respect to the level at 0.2 of the maxiumum) of the space-time correlation function of a broadband acoustic noise as a function of distance.

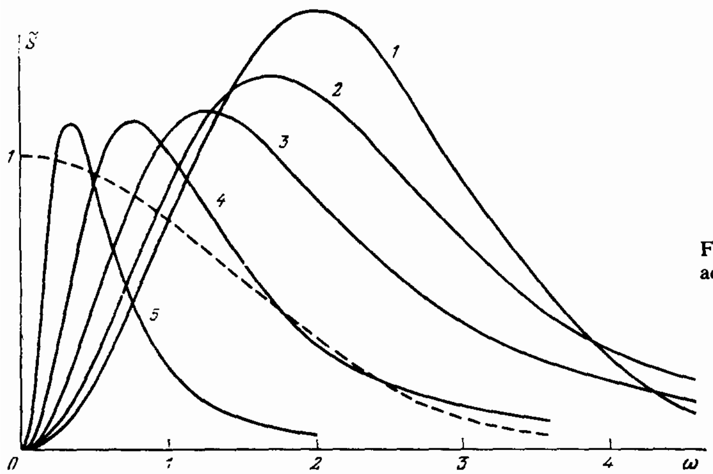

FIG. 14. Stationary spectra (in the far zone) of diffracting nonlinear acoustic noise.

ties; the normalizimg constants for them are u_0^2 and t_c^{-1}. The correlation function (44) assumes the form

$$B(z, T)=\frac{2}{(Nz)^2}(1-2T^2)\exp(-T^2). \quad (46)$$

The number $N = x_{disc}/x_{diff}$ is the ratio of the characteristic discontinuity distance to the characteristic diffraction distance $x_{diff} = r_c R_0/2c_0 t_c$. In the limit $N \to \infty$ the nonlinearity has virtually no effect, and for small N it affects mainly the evolution of the form and spectrum of the wave.[3]

Applying the nonlinear transformation (6) to (46), we find

$$\widetilde{S}=2\pi^{1/2}(Nz)^2 S(z, \omega)=\left[1+6\left(\frac{\omega}{N}\right)^2\right]^{-7/2} \times\left\{\omega^2+36\left(\frac{\omega}{N}\right)^2\left[1+6\left(\frac{\omega}{N}\right)^2\right]\right\} \times\exp\left\{-\frac{\omega^2}{4}\left[1+6\left(\frac{\omega}{N}\right)^2\right]^{-1}\right\}. \quad (47)$$

For weak ($N \gg 1$) and strong ($N \ll 1$) manifestations of nonlinearity we have

$$\widetilde{S}_{lin}=\omega^2\exp\left(-\frac{\omega^2}{4}\right), \quad \widetilde{S}_{nonlin}=\frac{36(\omega/N)^2}{[1+6(\omega/N)^2]^{5/2}}. \quad (48)$$

It is evident from the formulas (47) and (48) that in all cases in the far zone a stationary spectrum is formed—its form is independent of the distance z, and the values decrease as z^{-2}. The stationary form of the spectral distribution is shown in Fig. 14 for different degrees of manifestation of nonlinear effects. The nonlinearity intensifies as the increasing number of curves 1–5 increases ($N = \infty, 4\sqrt{6}, 2\sqrt{6}, \sqrt{6}, 1$), and in Fig. 14 the transition from $\widetilde{S}_{lin}$ to $\widetilde{S}_{nonlin}$ in (48) is observed. It is evident that nonlinear effects, together with diffraction, appreciably reduce the width and shift of the maximum of the spectral density into the low-frequency region. The broken curve corresponds to the starting spectrum (43) $\widetilde{S}(z=0, \omega) = \exp(-\omega^2/4)$; when the nonlinearity is not manifested, the spectrum in the far zone $\widetilde{S}_{lin}$ (48) (curve 1) equals the starting spectrum multiplied by ω^2 (the corrsponding correlation function equals the second derivative with respect to T of the starting function).

In order to follow the evolution of the form of the spectrum as a function of distance and the process of establishing the steady-state spectral distribution in the spherical divergence zone, it is necessary to drop the assumption that the correlation is of the δ-function form. In addition, the auxiliary function must be described by a more general expression than (46):

$$B=e^{-T^2}-\frac{\pi^{1/2}}{4}Nze^{(Nz)^2/4}\left\{e^{-NzT}\left[1-\Phi\left(\frac{Nz}{2}-T\right)\right]+e^{NzT}\left[1-\Phi\left(\frac{Nz}{2}+T\right)\right]\right\}; \quad (49)$$

where

$$\Phi(x)=\frac{2}{\pi^{1/2}}\int_0^x \exp(-\nu^2)\,d\nu$$

is the error integral. Calculations show that for a fixed diffraction length x_d the intensification of the nonlinearity (decrease in x_{disc}) causes an appreciable increase in the attenuation on the axis of the beam. This phenomenon has the following significance: the nonlinear interaction of the spectral components "feeds" the low-frequency region of the spectrum, from which the energy is extracted by diffraction.[2)]

We shall now consider how the nonlinearity affects the spatial statistics. To this end, we find the correlation function $B(z, \tau, s)$, applying the transformation associated with (6), to the auxiliary function (44):

$$B(z; \tau, s)=\frac{1}{(Nz)^2}\int_{-\infty}^{\infty}\frac{\partial\widetilde{B}}{\partial T}\Phi\left[\frac{(\tau/t_c-T)N}{2\sqrt{2}(1-\widetilde{B})^{1/2}}\right]dT. \quad (50)$$

Figure 15 shows the space–time-dependent correlation function at 0.1 of its maximum value.[3)] The calculation was performed for different manifestations of nonlinearity, the curves 1 correspond to the linear case ($N \to \infty$), the curves 2 correspond to $N = 2\sqrt{2}$, and the curves 3 correspond to $N = \sqrt{2}$. It is evident that as the nonlinearity increases, because of the mixing of the spectral components both the spatial and temporal correlations increase. The analogous effect of nonlinear smoothing of spatial surges has been well stud-

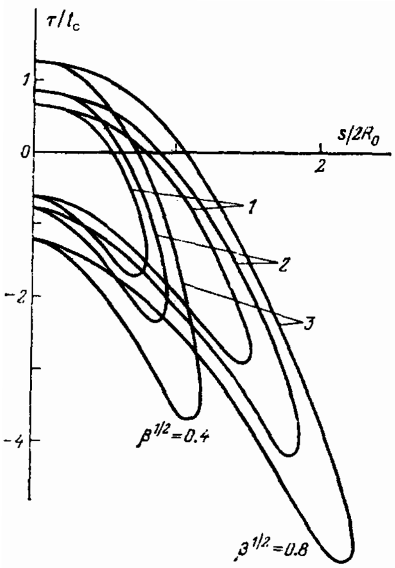

FIG. 15. Section of the space-time correlation function at different distances for different values of the diffraction-nonlinearity numbers.

ied for regular beams, described by the KZ equation.[3,10] The region of correlation expands also as a result of the accumulation of nonlinearity as the distance traversed by the wave increases. The two groups of curves in Fig. 14 correspond to a reduced distance $\sqrt{\beta}$ of 0.4 and 0.8.

The increase in the correlation radius, determined from the total energy (as in Fig. 12), is shown in Fig. 16. The curves 1–4 correspond to the values of N equal to $2\sqrt{2}$, $3\sqrt{2}/2$, $\sqrt{2}$, and $\sqrt{2}/2$. The broken curve corresponds to the linear case ($N\to\infty$, see curve 1′ in Fig. 12).

We note that this definition of the correlation radius is based on the idealized notion of point receivers, whose frequency characteristic is uniform over the entire frequency range. Of course, for wide-band noise this is not the only possible definition. If, for example, the receiver cuts out of

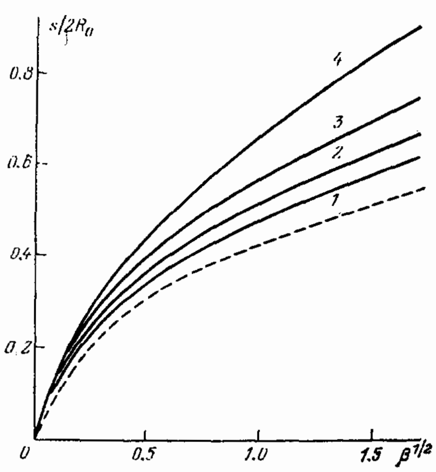

FIG. 16. Increase in the transverse-correlation radius during the propagation of a wave with a broad spectrum. The broken curve shows the linear dependence. The nonlinearity becomes stronger as the numbers on the curves increase.

the noise a narrow spectral band, then, in principle, the reverse effect is possible—accentuation of the surges and decrease in the correlation radius with increasing nonlinearity. This situation is encountered, for example, in nonlinear optics.[4] Moreover, real acoustic noise spectra are more complicated than the example studied here [the second formula in (43)] and can exhibit other, diverse properties.

In concluding this section, we point out that the problems of diffraction of intense noise, especially taking into account the spatial statistics, have not been previously studied. These problems are very complicated; they are difficult to solve even with the help of modern computers. In acoustics only the linear problems have been studied; an example is the analysis of the spatial correlation of noise created by the falling of raindrops on the surface of water.[66] At the same time, any real source of intense noise creates a spatially limited disturbance, and diffraction must in principle be taken into account in describing the propagation of this disturbance. The spatial statistics must affect the operation of powerful radiating systems with multicomponent mosaic antennas, the random transverse oscillations of the field here are linked with the spread in the parameters of separate elements. Another example is a cavitation layer with a random distribution of bubbles, emitting intense noise as they collapse. Many analogous sources exist in aeroacoustics and wave hydrophysics of the ocean,[67] where correlation measurements are primarily used.

Thus the study of nonlinear problems taking into account spatial statistics is only beginning; interesting results, both theoretical and experimental, can be expected here.

8. EXCITATION OF NONLINEAR RANDOM WAVES BY DISTRIBUTED SOURCES. STEADY-STATE SPECTRA

Thus far we have studied intense noise whose characteristics are fixed at the point of entry into the nonlinear medium. It happens, however, that often it is not the statistical properties of the starting noise field that are known, but rather those of spatially distributed sources exciting the wave. Such problems are described by inhomogeneous nonlinear equations, whose right side is a random function. The simplest such equations are the inhomogeneous Burgers equation and the equation describing simple waves.[69,70] This model is adequate for many situations realized in optoacoustics, mechanics, plasma physics, and the physics of electron beams (see Ref. 32). Some examples are the processes of excitation of nonlinear sound in a gas incident with nearly the velocity of sound on a laser beam or an uneven solid profile, the excitation of ocean waves by a traveling pressure wave, etc. In addition, inhomogeneous equations of the Burgers type are interesting as the simplest model of turbulence since, together with nonlinear mixing and dissipation, it includes a third fundamental factor—external energy sources.[63]

For definiteness, we shall talk about the excitation of sound by a moving field of random sources. Equations of the evolutionary type can be obtained if the sources move relative to the medium with the velocity c_1 close to the velocity c_0 of the characteristic wave: $|c_1 - c_0|/c_0 \ll 1$. In this case, two oppositely traveling waves efficiently excite only one wave, traveling synchronously with the sources. The equation for it[32] has the form

$$\frac{\partial V}{\partial z}-V\frac{\partial V}{\partial \theta}-\Gamma\frac{\partial^2 V}{\partial \theta^2}+\delta\frac{\partial V}{\partial \theta}=\frac{dF(\theta)}{d\theta}=f(\theta). \quad (51)$$

Here the variables $V=u/u_0$, $z=x/x_{\rm disc}$, $\theta=\omega_0 \times[\tau-x(c_1^{-1}-c_0^{-1})]$ are the same as those defined previously [see (1)], and $\delta=(c_1-c_0)/\varepsilon u_0$ is the dimensionless offset of the velocities. The sources are described by the function F with the normal distribution law and a fixed correlation function

$$\langle F(\theta_1)F(\theta_2)\rangle=R_F(\theta=\theta_1-\theta_2),\quad \langle F\rangle=0. \quad (52)$$

The normalizing "amplitude" u_0 depends on the intensity of the sources and is chosen so that $R_F(0)=1$.

The properties of the solutions of (51), corresponding to determinate physical formulations of the problems, are described in Ref. 32. The basic analytical results for the inhomogeneous Burgers equation (51) were obtained by linearizing it using a Hopf-Cole transformation. The behavior of the solutions of the inhomogeneous equation for simple waves [(51) with $\Gamma=0$] can be analyzed qualitatively in the phase plane. But these approaches[23] are inadequate for studying the excitation of intense noise waves.

An approximate (for large velocity offset $|\delta|\gg 1$) solution of the inhomogeneous equation for simple waves, corresponding to the absence of disturbances $V(z=0,\theta)=0$ at the boundary of the medium $z=0$, is proposed in Ref. 71:

$$V=\frac{1}{\delta}F(\theta)-\frac{1}{\delta}F\left(\theta-\delta z+zV-\frac{z}{\delta}F(\theta)\right)+O\left(\frac{1}{\delta^2}\right). \quad (53)$$

Employing (53), it is possible to carry out exactly the statistical averaging and calculate the correlation function $R_V(z,\theta=\theta_1-\theta_2)=\langle V(z,\theta_1)\times V(z,\theta_2)\rangle$ of the noise wave:

$$R_V=\frac{R_F(\theta)}{\delta^2}+\frac{1}{2\delta^2}\int_{-\infty}^{\infty}\frac{dR_F}{dT}\left[\Phi\left(\frac{T+\delta z-\theta}{\sqrt{2}\,z/\delta}\right)+\Phi\left(\frac{T+\delta z+\theta}{\sqrt{2}\,z/\delta}\right)-\Phi\left(\frac{T-\theta}{(2z/\delta)(1-R_F)^{1/2}}\right)\right]dT. \quad (54)$$

In the linear limit $(z/\delta\to 0)$ we have the expression $\delta^2 R_V\approx 2R_F(\theta)-R_F(\theta-\delta z)-R_F(\theta+\delta z)$. From here it is evident that the total intensity of the linear noise $R_V(z,0)=(2/\delta^2)[1-R_F(\delta z)]$ grows with increasing z and in the limit $\delta z\to\infty$ approaches the stationary value $2/\delta^2$, which decreases as the velocity offset $|\delta|$ increases.

The expression for the spectrum of the intensity of the wave corresponding to (54) has the form

$$S_V(z,\omega)=\frac{S_F(\omega)}{\delta^2}\left\{1-2\exp\left[-\left(\frac{\omega z}{\delta\sqrt{2}}\right)^2\right]\cos\delta\omega z\right\}+\frac{1}{\delta^2}\frac{1}{2\pi i\omega}\int_{-\infty}^{\infty}\frac{dR_F}{dT}\exp\left[-\left(\frac{\omega z}{\delta}\right)^2(1-R_F)\right]\times\exp(-i\omega T)\,dT; \quad (55)$$

here

$$S_F=\frac{1}{2\pi}\int_{-\infty}^{\infty}R_F(\theta)e^{-i\omega\theta}\,d\theta$$

is the spectrum of the intensity of the distributed sources. Comparing the formulas (55) and (6) we note that the second term in (55) is the spectral density of a freely propagating simple wave, whose starting spectrum (at the boundary $z=0$) is $S_F(\omega)/\delta^2$. The high-frequency asymptotic behavior determined by this term in the nonlinear medium ω^{-3}, as is well known,[3] is associated with the inaccurate description of discontinuities by solutions of the type (53). At large distances the spectrum of the disturbance formed is proportional to the spectral density of the sources: $S_V(z\to\infty,\omega)=S_F(\omega)/\delta^2$.

The dynamics of the formation of the intensity spectrum of the wave is illustrated in Fig. 17 for $S_F=(1/2\pi^{1/2})\exp(-\omega^2/4)$. Figure 17a shows the spectrum $\tilde{S}_V=2\pi^{1/2}\delta^2 S_V$ for values of the velocity offset $\delta^2=4$ at distances $z/\delta=0.15$, 0.3 (2), and 0.5 (3) (broken curves). The corresponding solid curves 1–3 were constructed using the linear theory (the curves 1 are practically identical). It is obvious that in the process of excitation of the wave the nonlinearity increases the redistribution of energy over the spectrum. In Fig. 17b analogous curves are given for $\delta^2=10$

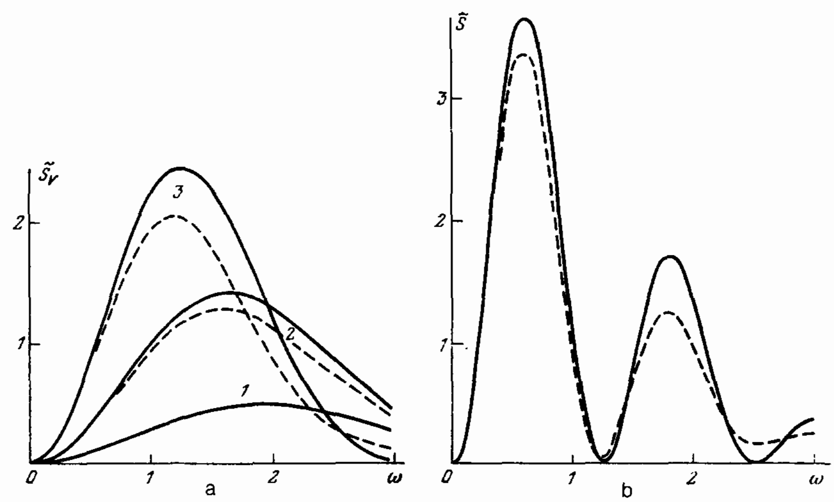

FIG. 17. The spectrum of an intense wave, excited by a traveling field of distributed sources, as a function of distance.

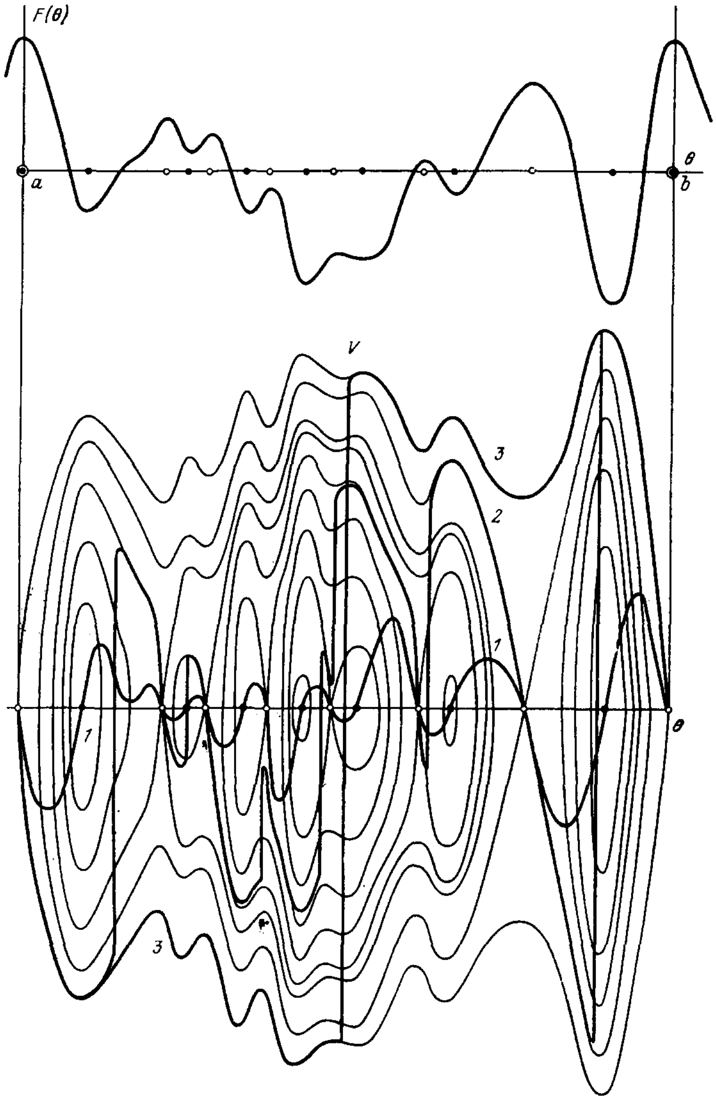

FIG. 18. Segment of the realization of the original function of random sources, the phase-plane diagram, and the profiles of the excited wave with motion and merging of discontinuites.

and $z/\delta = 0.5$. Here, oscillations owing to the larger frequency offset δ are clearly seen; the nonlinearity partially "smooths" them.

Interesting phenomena in intense noise fields, excited by random sources, occur after the formation in the realization of sections with a steep profile—sequences of shock waves.

To study these phenomena we shall employ the method of qualitative analysis of the solutions of Eq. (51) (for $\Gamma \ll 1$) in the phase plane.[32] We shall study a segment of the random function $F(\theta)$, describing the right side of (51), which lies between the two maximum surges $F(\theta = a)$ and $F(\theta = b)$ Fig. 18. To simplify the discussion, we shall set the velocity offset $\delta = 0$ and we shall assume that the surges are quite large and almost identical in magnitude ($F(a) \approx F(b) \gg \langle F^2 \rangle^{1/2} = 1$). In addition, the neighboring sections of the realization ($\theta < a$, $\theta > b$), as will be explained below, will not affect for a long time the nonlinear processes within the segment under study $a \leqslant \theta \leqslant b$. Here a quasistationary field, which does not vary as z increases until a moving discontinuity appears from the outside in the segment $[a,b]$—from the vicinity $\theta = c$ of the larger maximum: $F(c) > F(a)$. However, for $F(a) \gg \langle F^2 \rangle^{1/2}$ this event is unlikely.

Figure 18 shows a segment of the realization of $F(\theta)$ between two small peaks [for simplicity $F(a) = F(b)$]; the phase-plane diagram (fine lines) and the waveform (curves 1–3) for three values $z_1 < z_2 < z_3$ are constructed beneath them. Curve 3, corresponding to the traversed distance z_3, shows the quasistationary profile.

The phase-plane method is based on the properties of the characteristics of Eq. (51)

$$\frac{V^2}{2} + F(\theta) = C, \quad \frac{d\theta}{dz} = -V. \tag{56}$$

The first of the formulas (56) describes the family of phase trajectories corresponding to different values of the constant C. The second formula in (56) is the differential law of motion of each image point along its trajectory. Knowing the coordinates $\theta_n, V_n(\theta_n)$ of this point at the distance

$z_n = n\Delta z$, it is possible to find its position $\theta_{n+1} = \theta_n - V_n \Delta z$ at the distance z_{n+1}. The aggregate of image points for a given z_n forms a curve—the wave profile. The nonuniqueness of the profile indicates the appearance of a discontinuity, whose position is determined in the same manner as for freely propagating disturbances—from the rule of "equal intercepted areas." [3]

At short distances the wave is weak, and grows according to a linear law $V \approx z f(\theta)$: its profile reproduces the derivative $\mathrm{d}F/\mathrm{d}\theta = f$ (curve 1). It is obvious that the statistical characteristics of the disturbance are described by the normal distribution, and the intensity spectrum is given by

$$S_V(z, \omega) \approx z^2 S_f(\omega) = z^2\omega^2 S_F(\omega). \qquad (57)$$

During the growth process the wave form becomes distorted because of the increasing effect of the nonlinearity. One discontinuity forms between each pair of neighboring saddle points (they are marked by circles in the phase-plane diagram in Fig. 18). These discontinuities are nonsymmetric relative to the zero level $V = 0$ (curve 2); for this reason, they move and collide with one another like absolutely inelastic particles. As a result of the set of collisions on the segment one "large particle" forms—a discontinuity (curve 3), corresponding to the absolute maximum of the random function $F(\theta)$.

It is not difficult to see that the quasistationary profile consists of the segments of two separatrices $V = \pm [2(C_{\max} - F(\theta))]^{1/2}$, passing from the saddle point $\theta = a$ to the saddle point $\theta = b$, and a discontinuity whose position is determined by the integral equation (51) ($\delta = 0$, $\Gamma \to 0$):

$$\int_a^b V(z, \theta)\, \mathrm{d}\theta = z[F(b) - F(a)]. \qquad (58)$$

In constructing the phase-plane diagram we set $F(a) = F(b) = C_{\max}$, so that in Fig. 18 the discontinuity is drawn so that the area under the curve 3 exactly equals zero.

If the magnitudes of the peaks are close, but not identical, for example, $F(b) > F(a)$, the area (58) under the curve $V(z,\theta)$ will grow linearly with increasing z owing to the displacement of the discontinuity to the left, when the wave traverses a distance

$$z_{st} = \int_a^b \{2[F(b) - F(\theta)]\}^{1/2}\, \mathrm{d}\theta \cdot [F(b) - F(a)]^{-1},$$

the discontiniuty will reach the left boundary $\theta = a$ of the segment under study and will leave the segment. A stationary wave with no discontinuities and whose form coincides with the top separatrix $V = \{2[F(b) - F(\theta)]\}^{1/2}$ will be established in the segment $[a,b]$. In the other case $F(b) < F(a)$, the discontinuity will pass beyond the boundary at the right $\theta = b$, and the stationary wave in the segment $[a,b]$ will assume the form of the bottom separatrix: $V = -\{2[F(a) - F(\theta)]\}^{1/2}$.

In all cases the quasistationary wave is described by the formula

$$V \approx \pm [(2C_{\max})^{1/2} - (2C_{\max})^{1/2} F(\theta)]^{1/2}, \qquad (59)$$

where $C_{\max}$ can be a slowly increasing function of the distance z. It is now understandable that at large z the statistics of the noise field once again becomes Gaussian with the mean $\pm (2C_{\max})^{1/2}$ and variance $(2C_{\max})^{-1}$. Thus the values of V are localized in a narrow region around the center, determined by the magnitude of the largest maximum $C_{\max}$.

The intensity spectrum acquires the form

$$S_V(z \gg 1, \omega) \approx \frac{S_F(\omega)}{2C_{\max}}. \qquad (60)$$

Comparing the formulas (57) and (60) we can see that in the quasistationary spectrum the low-frequency components are accentuated. This is linked with the enlargements of the scales of the field owing to the coalescence of moving discontinuities. In addition, the nonlinearity determines the high-frequency asymptotic behavior, associated with the structure of the front of "long-lived" shock waves. This asymptotic behavior has the form ω^{-2} in the limit $\Gamma \to 0$ of vanishingly small viscosity (see the end of Sec. 4). When the finiteness of the width of the shock-wavefront, described by the expression $V = (2C)^{1/2}\, \mathrm{th}[\theta(2C)^{1/2}/2\Gamma]$, at high frequencies is taken into account we have

$$S_V(z, \omega) \sim \Gamma^2\, \mathrm{sh}^{-2} \frac{\omega\pi\Gamma}{(2C)^{1/2}}. \qquad (61)$$

For small but finite Γ the power-law dependence ω^{-2} in the limit $\omega \to \infty$ transforms into the exponential dependence $\sim \exp(-\beta\omega)$.

The qualitative picture of the formation of quasistationary noise, described with the help of Fig. 18, permits stating plausible hypotheses that the statistical characteristics of the process are close to Gaussian, and that the correlation of the field $V(z,\theta)$ and the right side $f(\theta)$ of Eq. (51) (for large z) is weak. These hypotheses are sufficient for uncoupling the correlations and obtaining a closed stochastic equation.

$$\frac{\partial^2 R_V}{\partial z^2} - (2\Gamma)^2 \frac{\partial^4 R_V}{\partial\theta^4} + \frac{\partial^2}{\partial\theta^2}(R_V - 1)^2 = 2R_f. \qquad (62)$$

It is not difficult to find the stationary solution of (62) for $\Gamma = 0$. Let, for example, the correlation function $R_f = \langle f(\theta_1) f(\theta_2)\rangle$ and the corresponding spectral density S_f equal

$$R_f = 2e^{\ 2|\theta|} - e^{-|\theta|}, \quad \pi S_f = \frac{1}{1 + (\omega^2/4)} - \frac{1}{1 + \omega^2}. \qquad (63)$$

The sources (63) excite a wave with the statistical characteristics

$$R_V = e^{-|\theta|}, \quad \pi S_V = \frac{1}{1 + \omega^2}. \qquad (64)$$

The physical constants were chosen so that R_V and R_f would equal unity at $\theta = 0$.

The functions (63) and (64), illustrated in Fig. 19, exhibit the behavior discussed in connection with the formulas (57) and (60). If the spectral density of the sources S_f in the limit $\omega \to 0$ behaves like $\sim \omega^2$, then the low-frequency components in the spectrum of the wave S_V will be accentuated: $S_V(\omega \to 0) \to$ const. This phenomenon is not limited to the example studied (63) and is of a general character. When

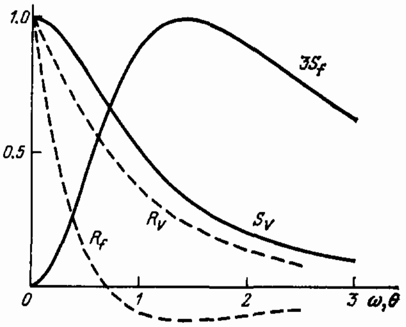

FIG. 19. Spectra (solid curves) and correlation functions (broken curves) of distributed sources and the nonlinear random wave excited by them.

the spectrum of the sources $S_f(\omega\to 0)\sim\omega^n$, where $n<2$, a singularity appears in the noise wave ($S_V(\omega\to 0)\to\infty$: the quasistationary spectrum is not formed). Physically, this is linked with the fact that the high-frequency dissipation cannot prevent the accumulation of energy at low frequencies; the diffraction of nonlinear noise could be a real competing effect here (see Sec. 7).

The steady-state solutions of (62) with $\Gamma=0$ also exhibit ω^{-2} high-frequency asymptotic behavior. This is linked with the presence of a break (at the point $\theta=0$) in the noise correlation function, exhibiting the universal behavior $R_V(\theta\to 0)\approx 1-|\theta|+\ldots$, while $R_f(\theta\to 0)\approx 1-a\theta^2+\ldots$ Of course, taking viscosity into account ($\Gamma\neq 0$) smooths out this break; the solution of the singularly perturbed problem [Eq. (62) with the small parameter $(2\Gamma)^2$ in front of the highest, fourth order derivative] must describe the transition of the power-law asymptotic behavior to the exponential type behavior (61).

In concluding this section, we point out that the statistical theory of problems of intense noise emission, based on the nonlinear equations of the type (51), is actually only beginning to be constructed. In the near future it will apparently be possible to obtain here more detailed quantitative results. To this end, the theory of surges of stochastic processes and especially the computer modeling of nonlinear discontinuous waves are promising.[32]

9. NONLINEAR TRANSFORMATION OF THE STATISTICAL CHARACTERISTICS OF ACOUSTIC NOISE

A method for calculating distribution functions for nonlinear noise waves, based on the property of ergodic steady-state processes, is proposed in Ref. 72. This property is usually used in the experimental determination of statistical characteristics, and consists of the fact that the relative time that the process spends in the interval $(u, u+\Delta u)$ converges to a one-dimensional distribution function.

Consider a segment of the narrow-band random process u (Fig. 20). We take on the axis of the oscillatory velocity u the point u_i and a small neighborhood Δu_i around it; this neighborhood corresponds to the set $\Delta\theta_k^{(\nu)}(A_k,\varphi_k)$ on the dimensionless-time axis $\theta=\omega_0(t-x/c_0)$. The index k refers to the number of the period; $\nu=1$ or 2 depending on whether or not u_i is on the front or back slope of the wave;

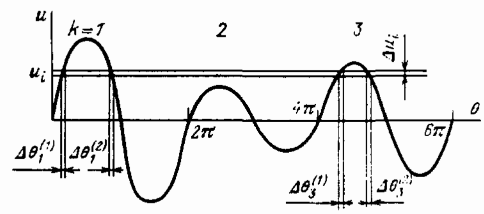

FIG. 20. Segment of the stochastic process; the quantities used in deriving the nonlinear functions of the distribution based on ergodicity.

A_k,φ_k are the random values of the amplitude and phase. Then the distribution function is given by

$$W(u_i)=\lim_{\substack{\Delta u_i\to 0\\ N\to\infty}}(2\pi N\Delta u_i)^{-1}\sum_{\nu;\,k=1}^{N}|\Delta\theta_k^{(\nu)}(A_k,\varphi_k;x)|. \quad (65)$$

The notation in (65) is convenient because the distortions of acoustic waves are most simply described in terms of the change in the increments $\Delta\theta_k^{(\nu)}(A_k,\varphi_k;x=0)$ as the distance x increases.

Thus if the plane wave does not contain discontinuities and its propagation in the low-viscosity medium ($\Gamma\ll 1$) is described by Burgers equation (1), then

$$\theta_k^{(1)}=\arcsin\frac{u_i}{A_k}-\frac{\varepsilon}{c_0^2}\omega_0 u_i x,\quad \theta_k^{(2)}=\pi-\arcsin\frac{u_i}{A_k}-\frac{\varepsilon}{c_0^2}\omega_0 u_i x. \quad (66)$$

It is evident that in the region prior to the formation of discontinuities the sum $|\Delta\theta_k^{(1)}|+|\Delta\theta_k^{(2)}|=2(A_k^2-u_i^2)^{-1/2}\Delta u_i$ does not change as x increases. For this reason, the form of the one-dimensional distribution function $W(u)$ (65) remains the same, though the form of the signal changes substantially.[72] This fact is discussed for starting signals of arbitrary form in Ref. 73.

After discontinuities form in the wave, the nonlinear distortions cause the smooth sections of the profile to creep onto the shock front. At first the point u_0, lying on the front slope, appears on the shock front; $\Delta\theta_k^{(1)}$ immediately vanishes, while the value of $\Delta\theta_k^{(2)}$, corresponding to the back slope, will increase as before. Then the point u_i on the descending section of the profile will collide with the shock front. The nonlinear distortions of the one-dimensional distribution function are linked precisely with the vanishing of $\Delta\theta_k^{(\nu)}(u_i)$ when u_i reaches the shock wavefront.

For initially Gaussian noise, after averaging the sum of the increments $\Delta\theta_k^{(\nu)}(A_k\varphi_k;x)$ (with a Rayleigh distribution law for the amplitude A_k and a uniform distribution of the phase φ_k) we obtain the function

$$W(V)=\frac{e^{-V^2}}{2\sqrt{\pi}}\left[1+\Phi(V\operatorname{ctg}zV)+\frac{z}{\pi^{1/2}}e^{-V^2\operatorname{ctg}^2 zV}\right],$$
$$|V|<\frac{\pi}{z}, \quad (67)$$

and $W(V)=0$ for $|V|>\pi/z$; here $V=u/\sigma\sqrt{2}$, $z=\varepsilon\sigma\omega_0 x/\sqrt{2}c_0^2$. The nonlinear transformation of the starting normal distribution is shown by the solid curves 1–3 ($z=0,\pi/2,\pi$) in Fig. 21. The form of $W(V)$ (67) becomes

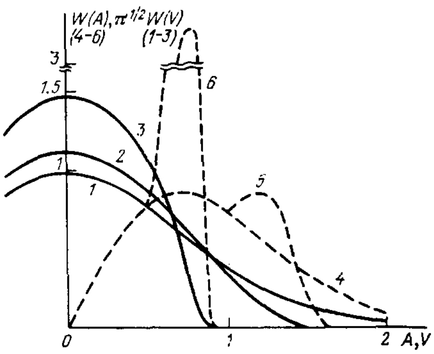

FIG. 21. Nonlinear transformation of the distributions of the signal $W(A)$ (curves 1–3) and its maxima $W(V)$ (broken curves 4–6).

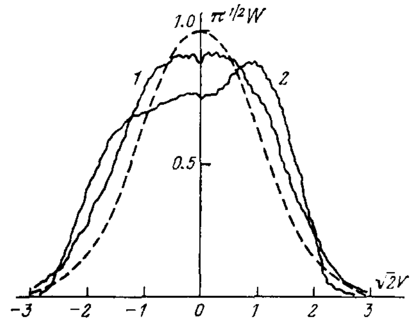

FIG. 22. Experimental observation of the tendency of a normal distribution to transform into a uniform distribution at the stage of formation of shock fronts.[77]

strongly deformed. Because of the nonlinear damping the probability of large values of $|V|$ decreases (the function W vanishes completely at finite values $|V| = \pi/z$; as a result of this the probability for observing small $|V|$ increases).

The corresponding changes in the Rayleigh distribution of the peaks are shown in Fig. 21 by the broken curves 4–6 ($z = 0, \pi/2, \pi$).

Thus because of the formation of discontinuities in the nonlinear medium and the subsequent damping the probability of small values of the signal V and its envelope A increases owing to the decrease in the probability of large surges. For arbitrarily small z the distributions $W(V)$ and $W(A)$ are bounded for large values of the arguments and the intensity of the signal $\langle V^2 \rangle$ is less than at the point of entry into the medium. The behavior of the average intensity and variance of the process in a nonlinear medium is analyzed in Refs. 65 and 74 with the help of the formula (67).

Later, the method of Ref. 72 was successfully used in the solution of other statistical problems.[73–76]

The characteristics described in Refs. 76 and 77 were checked experimentally. In Ref. 77 noise with an intensity of 152 dB was excited in a muffled aluminum tube 20 m long and 5 cm in diameter. The average frequency of the spectrum with a 1/3-octave band equaled 1 kHz. The results of the analysis of the measurements are presented in Fig. 22. The distribution function of the noise at the input was nearly Gaussian (broken curve). As the noise propagated ($x = 7.5$ and 17.5 m—curves 1 and 2) the distribution approached the uniform distribution. It was found that this tendency is linked with the formation of discontinuities. The asymmetry of the curve can be explained by the weak dispersion of the wave in the tube; as is well known, it leads to asymmetric nonlinear distortion of the compression and rarefaction phases.[3]

The distribution function of narrow-band noise was measured in Ref. 76 on an electrical model of a nonlinear weakly-dispersive medium. The model consisted of an artificial transmission line of the low-frequency filter type with varicaps serving as the nonlinear capacitors. The theory of Ref. 72 was refined in Ref. 76 so as to incorporate the conditions of the experiment. The function $W(V)$ was calculated taking into account two additional factors: dissipation and weak cubic nonlinearity. It turned out that the transformation of the normal distribution $W(V)$ into a uniform distribution owing to losses is somewhat smoothed. The cubic nonlinearity, as well as the dispersion, leads to asymmetry of $W(V)$ relative to positive and negative surges. The measurements are in good agreement with the more accurate theory.

We note that the hypothesis of ergodicity and stationarity of the stochastic process in a nonlinear medium[72] was later provided with a rigorous mathematical substantiation. It is shown in Refs. 32 and 78 for an initially normal stationary process $V(z=0,\theta)$, whose evolution is described by Burgers equation (1), that the solution $V(z,\theta)$ will be ergodic for any z. This permits calculating the statistical characteristics on a computer by averaging one realization of the solution.

In Refs. 79 and 80 a numerical method is presented for calculating the characteristics of nonlinear noise waves, and the uncertainties of the approximations obtained are estimated. In order to use this method it is necessary to model numerically the realization of the starting normal process with a given correlation function, calculate the realization of the solution in the required z section with the help of a difference scheme, and average the realization with respect to θ, under the assumption of ergodicity, in order to obtain the statistical characteristics of the solution.

In particular, the one-dimensional probability density of an initially normal process with the correlation function $R = \exp(-\theta^2/4)\times\cos 10\,\theta(\Gamma = 0.01)$ was calculated in Ref. 80. For small z the distribution remains Gaussian; then it acquires the form of a plateau-shaped function, close to a uniform function; finally, at $z \sim 10$ the distribution is smoothed out and once again becomes similar to a Gaussian distribution. The stage of formation of discontinuities is clearly reflected in the realizations. However, the fronts are slightly asymmetric, are shifted relative to one another only slightly, and are washed out owing to the increasing effect of dissipation. It is precisely the high-frequency absorption that led here to smoothing and, possibly, to normalization of the distribution function.

There should be no difficulties in obtaining the analogous results (numerical and analytical) for intense narrow-band noise, described by different evolutionary equations—taking into account relaxation processes (9), diffraction (26), and other factors. It is also possible to take into account the displacement of the shock fronts,[75] associated with their weak asymmetry.

However, when the starting noise is confined to a narrow band and the realization consists of many strongly differing triangular pulses with moving and repeatedly colliding, sharply asymmetric discontinuities,[81] the analytic methods of the type used in Ref. 72 are ineffective. Here it was possible to employ the approach used in Refs. 63, 82, and 83, which is based on the asymptotic analysis of the general solution of Eq. (1). It was shown in Refs. 82 and 83 that at large distances x, where the merging of shock fronts is significant, the statistical characteristics become self-similar. They depend solely on one parameter $\tau_{\rm out}(x)$—the outer scale of acoustic turbulence, characteristic for the time interval between neighboring discontinuities. The self-similar state forms for $\tau_{\rm out}(x) \gg t_c$ (t_c is the correlation time at $x = 0$), i.e., as a result of the merging of a large number (~ 10–10^2) of discontinuities. Because of the motion of the fronts the field at a fixed time τ is determined by the values of the starting field from a region much larger than t_c. This "nonlinear mixing" results[81–83] in the normalization of the one-dimensional distribution. The two-dimensional distribution $W(V_1, V_2)$ in this case, however, differs substantially from the Gaussian distribution.

The initial stage in the establishment of the self-similar state was observed in an experiment[84] performed in a tube 12 m long at an average sound frequency of 1.5 kHz at a 150 dB. Full self-similarity, however, did not occur, since the condition for repeated merging of discontinuities was not satisfied because of the small size of the medium.

It should be noted that the merging of $\gtrsim 10$ discontinuities has apparently not yet been observed. It could possibly be observed in specially formulated numerical experiments or in experiments with very long gas-filled tubes, where there is no diffraction and special measures are taken to reduce dissipative losses (in the gas volume and in the layer near the wall). Nevertheless, the normalizing of the distribution under conditions of nonlinearity is an important asymptotic result. We assume that under real conditions the absorption of the wave, diffraction, dispersion, and other processes capable of mixing weakly correlated sections of the field will intensify the normalizing process. Moreover, the nonlinearity can only lead to broadening of the spectra and thereby create conditions for strong mixing owing to the indicated linear mechanisms. In all cases, it is necessary to estimate the average intensities, the characteristic distances, the frequencies and other parameters in order to determine the realizability of these states in experiments.

10. CONCLUSIONS

In this review we attempted to indicate the place of statistical nonlinear acoustics among adjoining areas of research, to present the basic ideas, and to indicate the special peculiarities of the problems arising in this field. The review was based on the results of recent experimental work and the corresponding theoretical explanations. Some well-known and some special results were omitted. In particular, the theory of parametric radiators of acoustic noise[88–90] was not discussed, and the methodological questions of nonlinear statistics and other problems were discussed only briefly. Many of these problems are discussed in the review of Refs. 3–5, 63, and 85, whose bibliographies substatially supplement the list of references presented here.

I am grateful to S. A. Akhmanov for the idea of writing this review and for his support of research in the physics of nonlinear random waves, carried out over many years in the physics department at Moscow State University.

[1)]The transformation of the spectrum (15) was recently studied by the method of numerical modeling.[80] The results of the calculation for $\Gamma > 0.1$ differ from those obtained analytically[33] by not more than 10% for any z.

[2)]The dynamics of the spectra was studied in detail by B. S. Azimov.

[3)]The results in Figs. 15 and 16 were obtained by V. A. Khokhlova.

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